Consistent parametrizations of dark matter self-interactions

Pollica physics centre

Self-Interacting Dark Matter: Models, Simulations and Signals

June 20, 2023

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Outline

- Motivation: dark matter scattering versus nucleon scattering
- Towards the effective range theory
- Parametrizing SIDM
- Conclusions

Self-interacting dark matter hypothesis

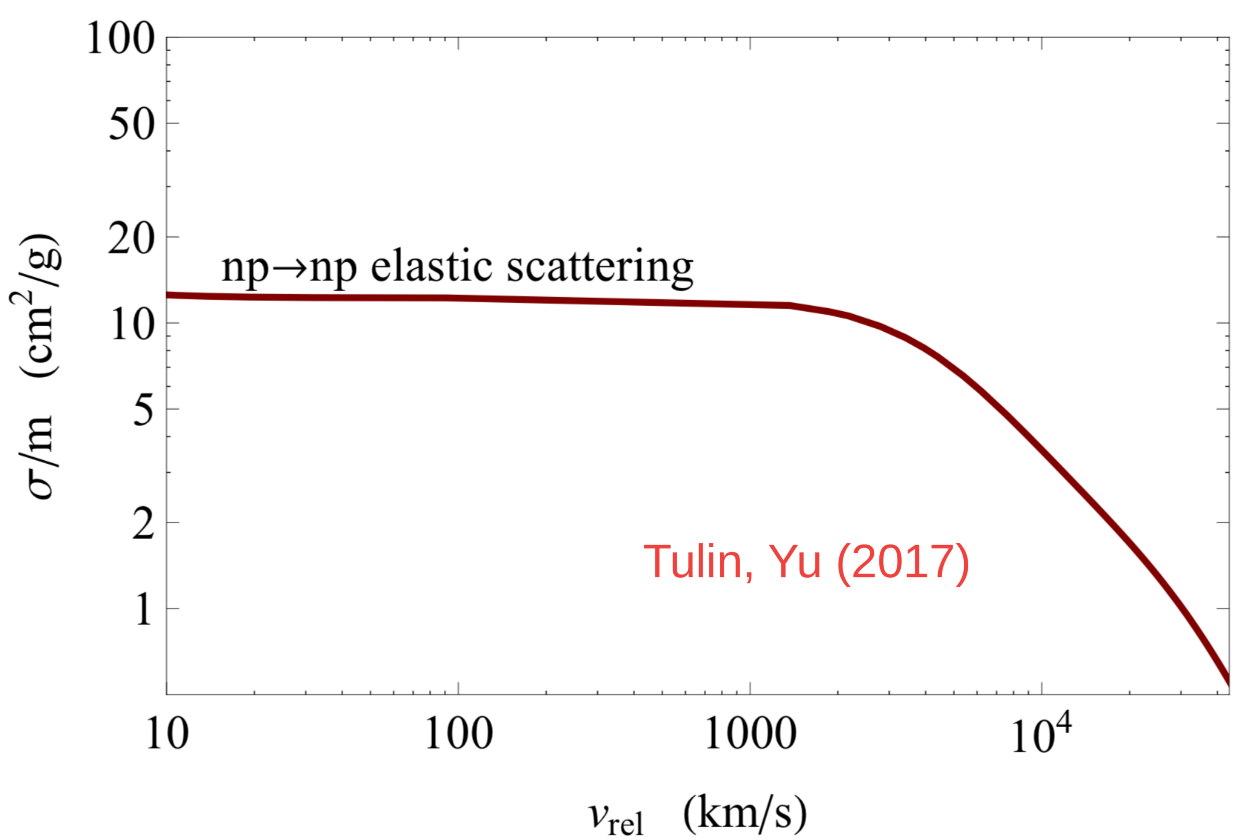
To be more specific, we suggest that the dark matter particles should have a mean free path between ~1 kpc to 1 Mpc at the solar radius in a typical galaxy (mean density 0.4 GeV/cm^3), for reasons to be explained below. For a particle of mass m_x, this implies an elastic scattering cross section of

$$\sigma/m \sim 1\ \mathrm{cm}^2/\mathrm{g} \tag{1}$$

intriguingly similar to that of an ordinary hadron.

Observational Evidence for Self-Interacting Cold Dark Matter

David N. Spergel and Paul J. Steinhardt
Phys. Rev. Lett. **84**, 3760 – Published 24 April 2000



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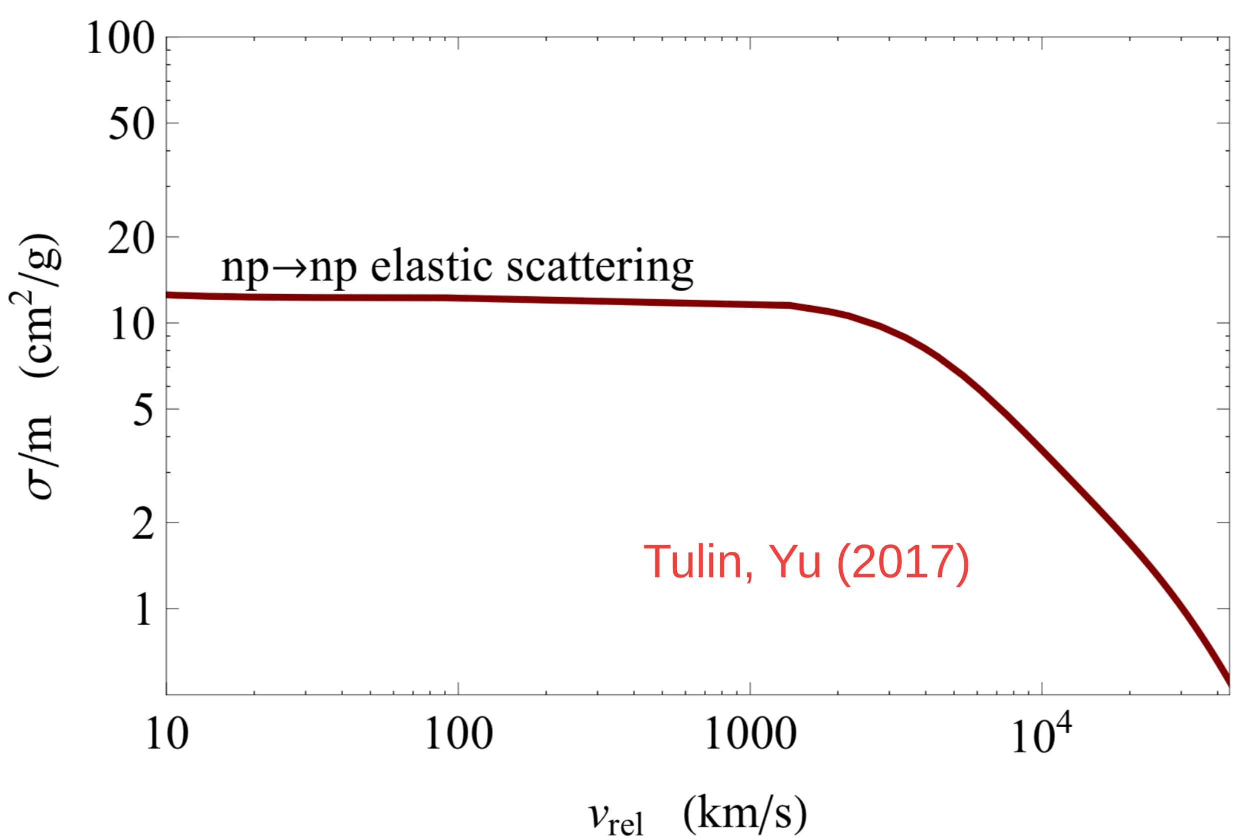

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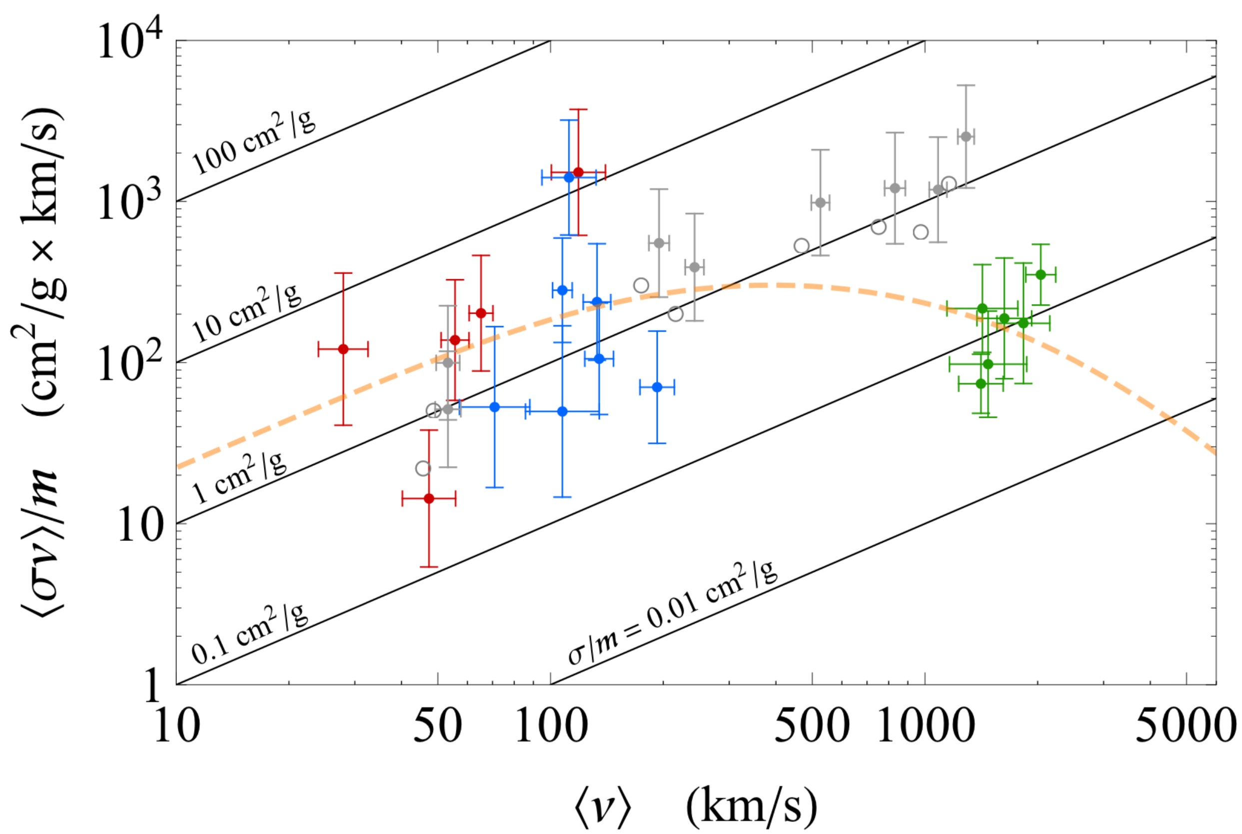
Dark Matter Halos as Particle Colliders: Unified Solution to Small-Scale Structure Puzzles from Dwarfs to Clusters

Manoj Kaplinghat, Sean Tulin, and Hai-Bo Yu
Phys. Rev. Lett. **116**, 041302 – Published 28 January 2016



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Light mediator

On the Interaction of Elementary Particles. I.

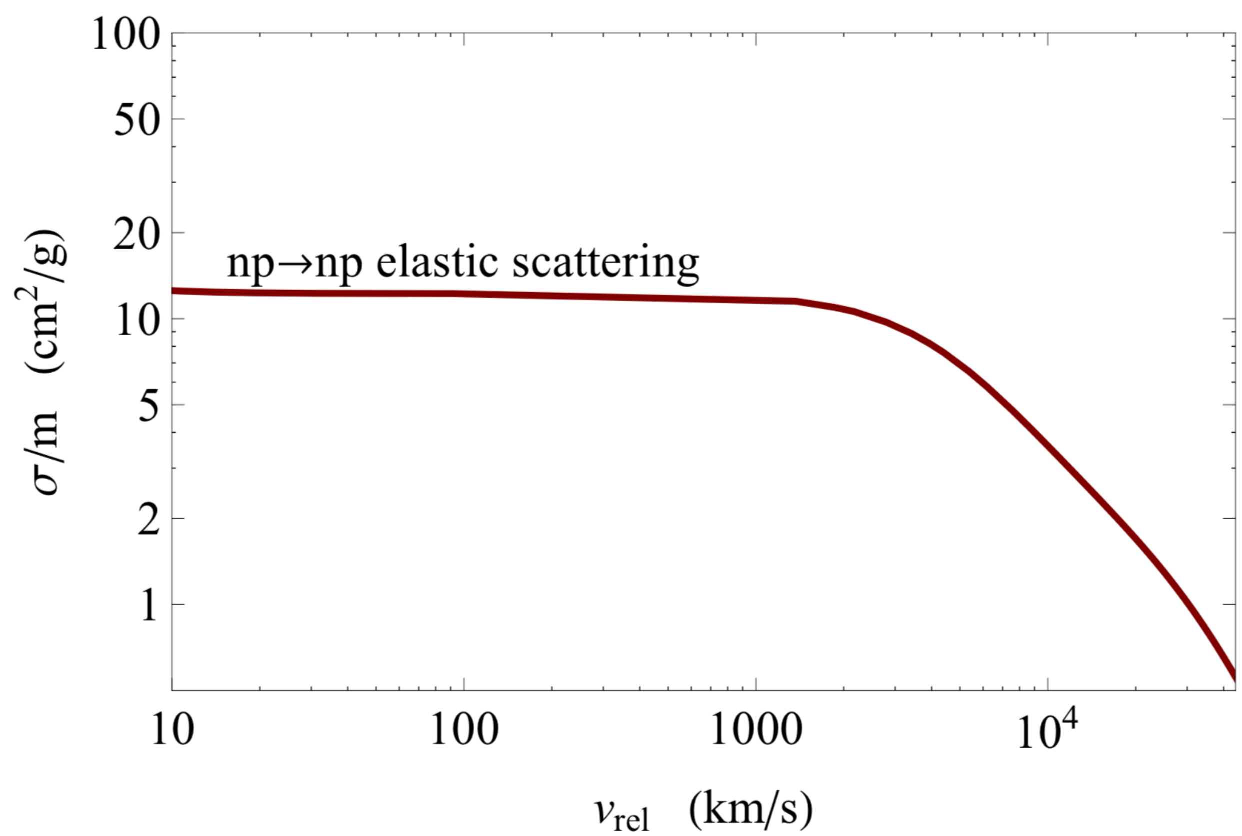
By Hideki YUKAWA.

(Read Nov. 17, 1934)

The potential of force between the neutron and the proton should, however, not be of Coulomb type, but decrease more rapidly with distance. It can be expressed, for example, by

$$+ \text{ or } -g^2\frac{e^{-\lambda r}}{r}, \tag{2}$$

so that the quantum accompanying the field has the proper mass $m_U = \frac{\lambda h}{c}$.



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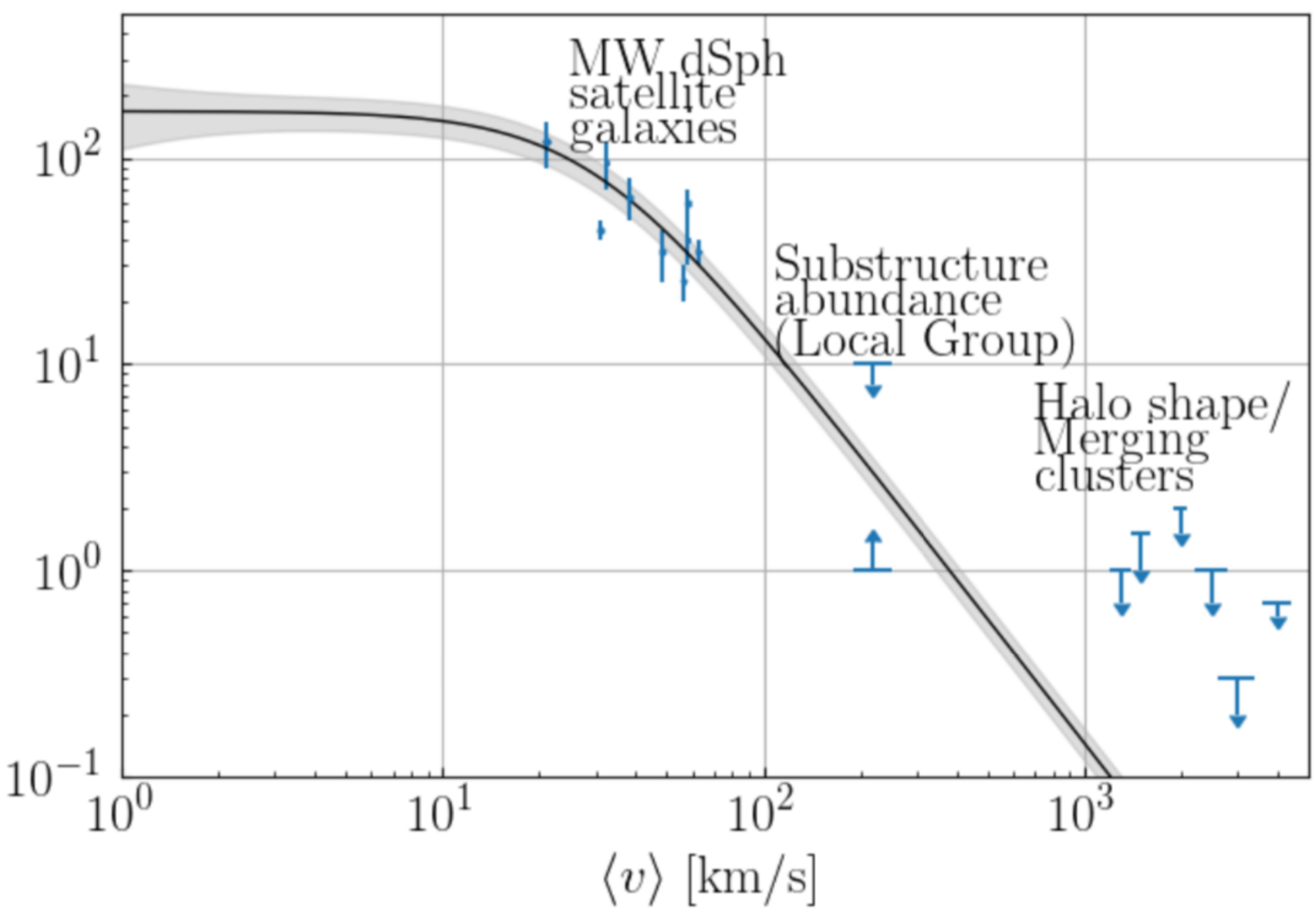
Light mediator

Constraining Velocity-dependent Self-Interacting Dark Matter with the Milky Way's dwarf spheroidal galaxies

Camila A. Correa[1]*



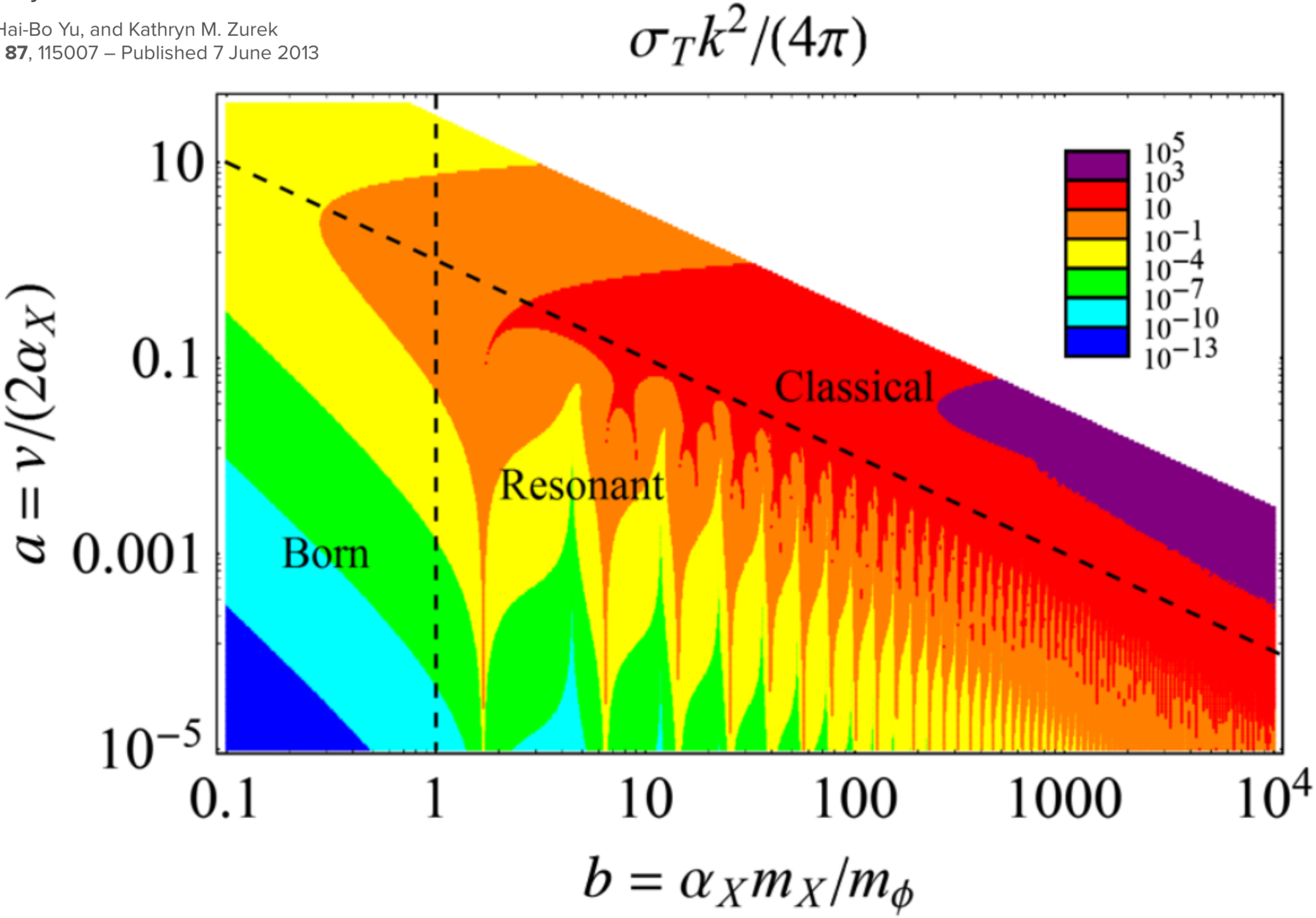
Born approximation of the Yukawa potential

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_0}{4\pi}\left[1 + \frac{v^2}{w^2}\sin^2\left(\frac{\theta}{2}\right)\right]^{-2},$$

Light mediator

Beyond collisionless dark matter: Particle physics dynamics for dark matter halo structure

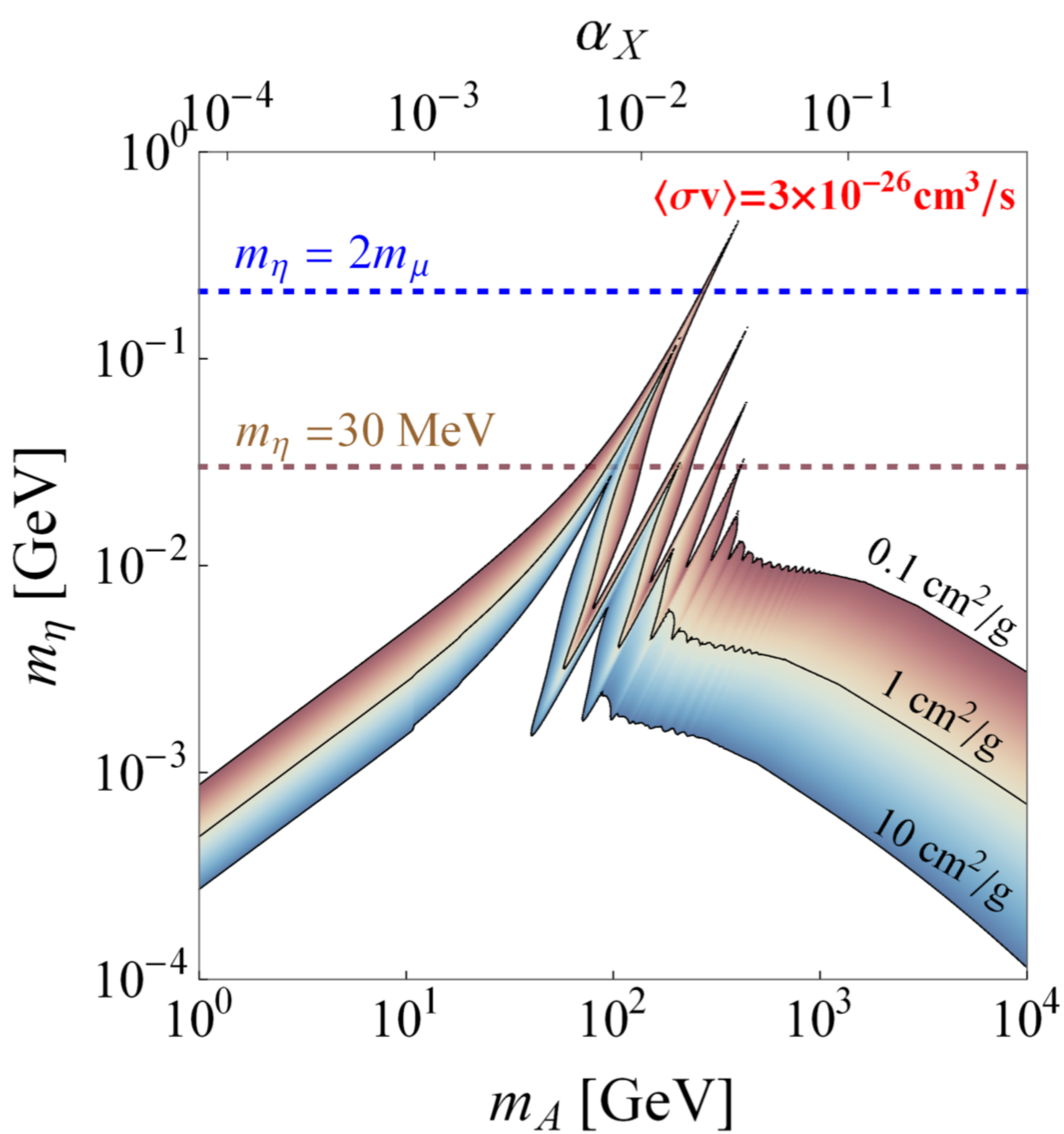
Sean Tulin, Hai-Bo Yu, and Kathryn M. Zurek
Phys. Rev. D **87**, 115007 – Published 7 June 2013



Light mediator

Production regimes for Self-Interacting Dark Matter

omas Hambye[3] and Bryan Zaldivar[3,4]



Accounting for all cosmological and astrophysical constraints is not trivial (for freeze out)

talk by Thomas Hambye

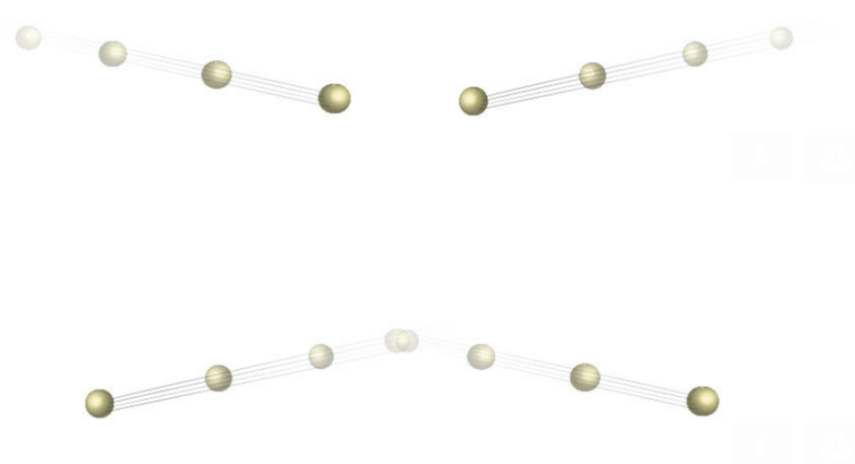
Resonant self-interacting dark matter

Velocity Dependence from Resonant Self-Interacting Dark Matter

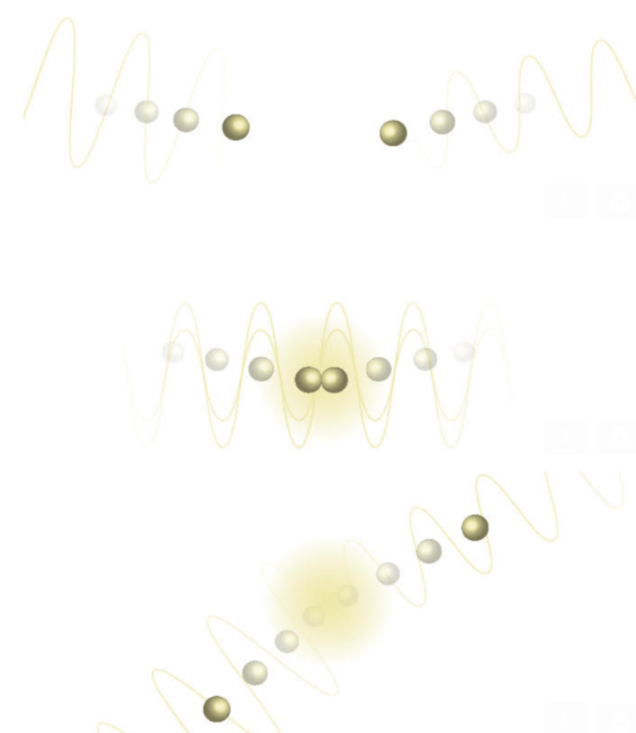
Xiaoyong Chu, Camilo Garcia-Cely, and Hitoshi Murayama
Phys. Rev. Lett. **122**, 071103 – Published 22 February 2019

$$\sigma \;=\; \sigma_0 + \frac{4\pi\, S}{mE(v)} \cdot \frac{\Gamma(v)^2/4}{\left(E(v) - E(v_R)\right)^2 + \Gamma(v)^2/4}\,, \quad \Gamma(v) = m_R \gamma v^{2L+1}$$

At clusters of galaxies



In galaxies



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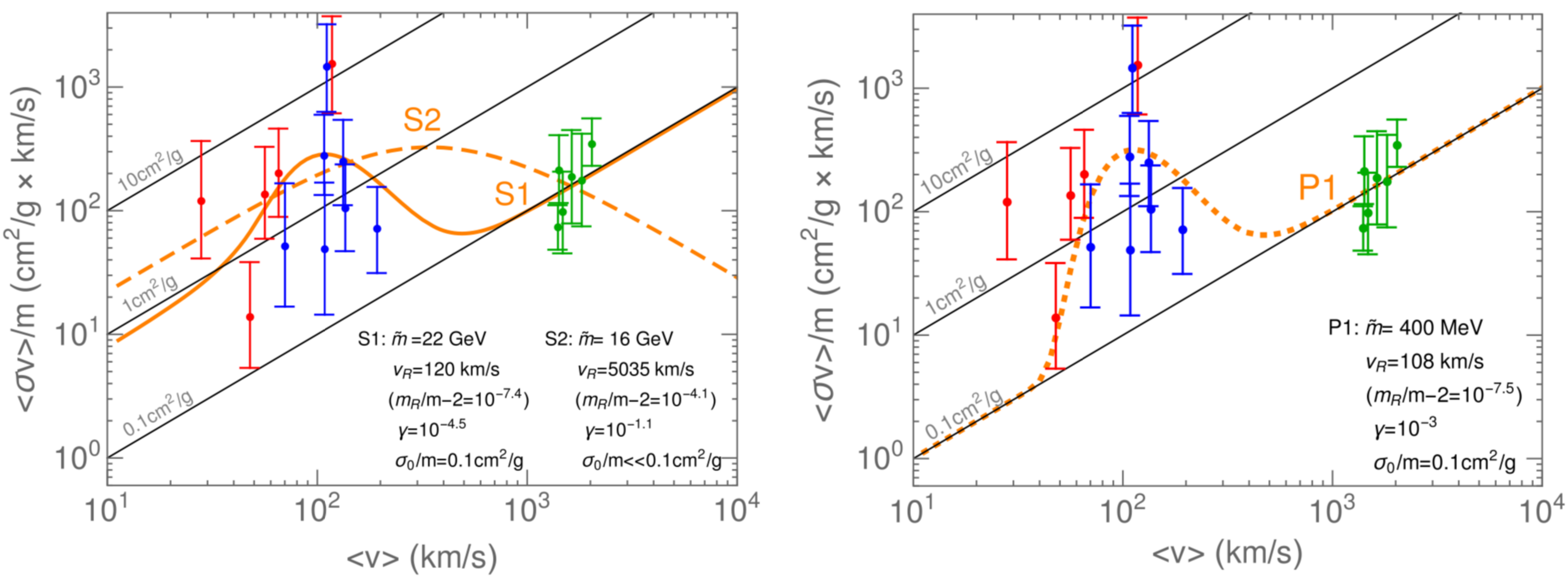

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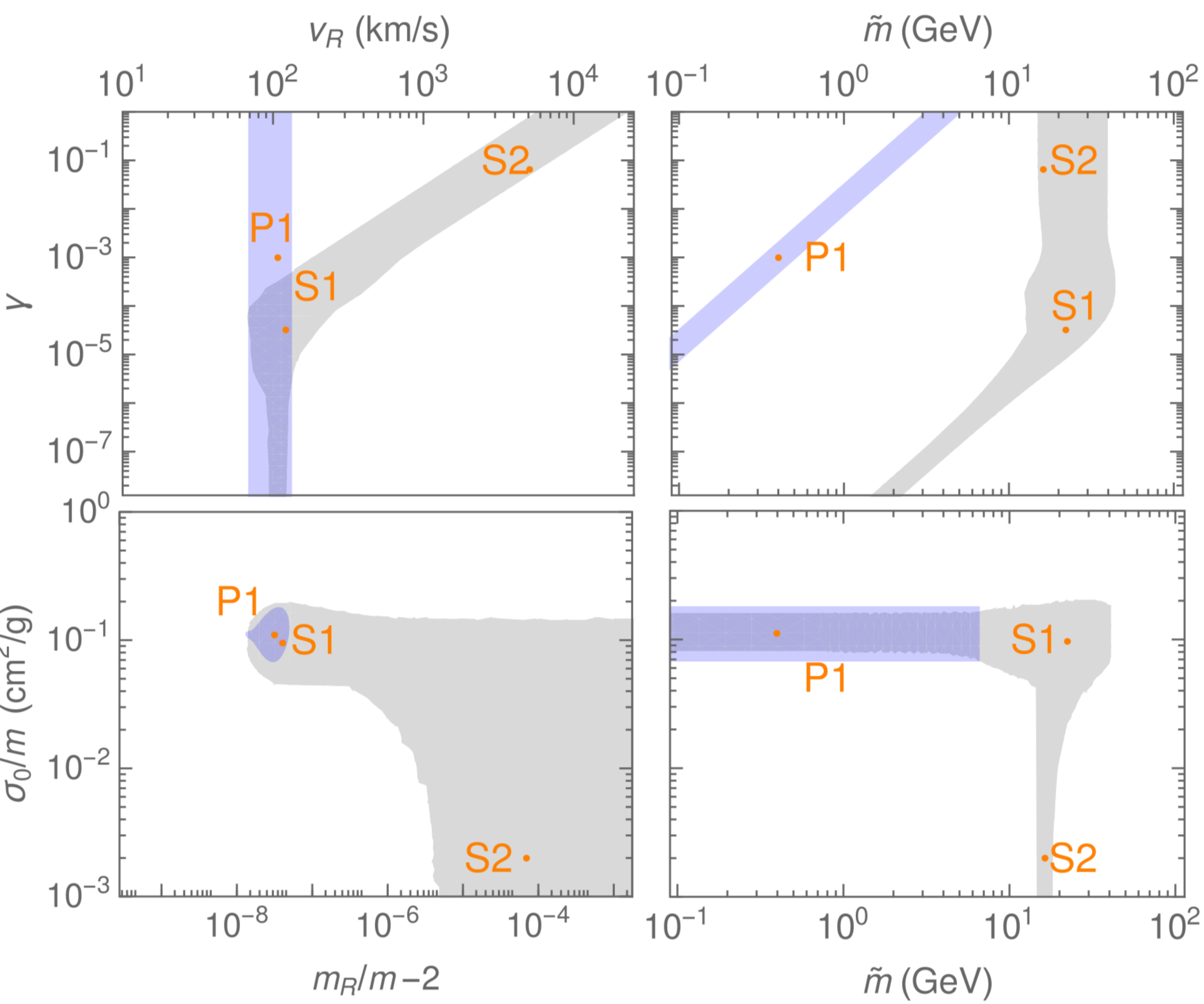


Scenario	Interaction Lagrangian	L	$J_{\rm DM}$	J_R^P	S	γ
I	$gR\overline{\rm DM}\gamma^5{\rm DM}$	0	$\frac{1}{2}$	0^-	$\frac{1}{4}$	$g^2/32\pi$
IIa	$gR{\rm DM}^i{\rm DM}^i$	0	0	0^+	$\frac{1}{3}$	$g^2/16\pi m_R^2$
IIb	$g\epsilon_{ijk}R^i_\mu{\rm DM}^j\partial^\mu{\rm DM}^k$	1	0	1^-	1	$g^2/384\pi$
III	$(1/\Lambda)R_{\mu\nu}\mathcal{T}^{\mu\nu}_{\rm DM}$	2	0	2^+	5	$m_R^2/30720\pi\Lambda^2$

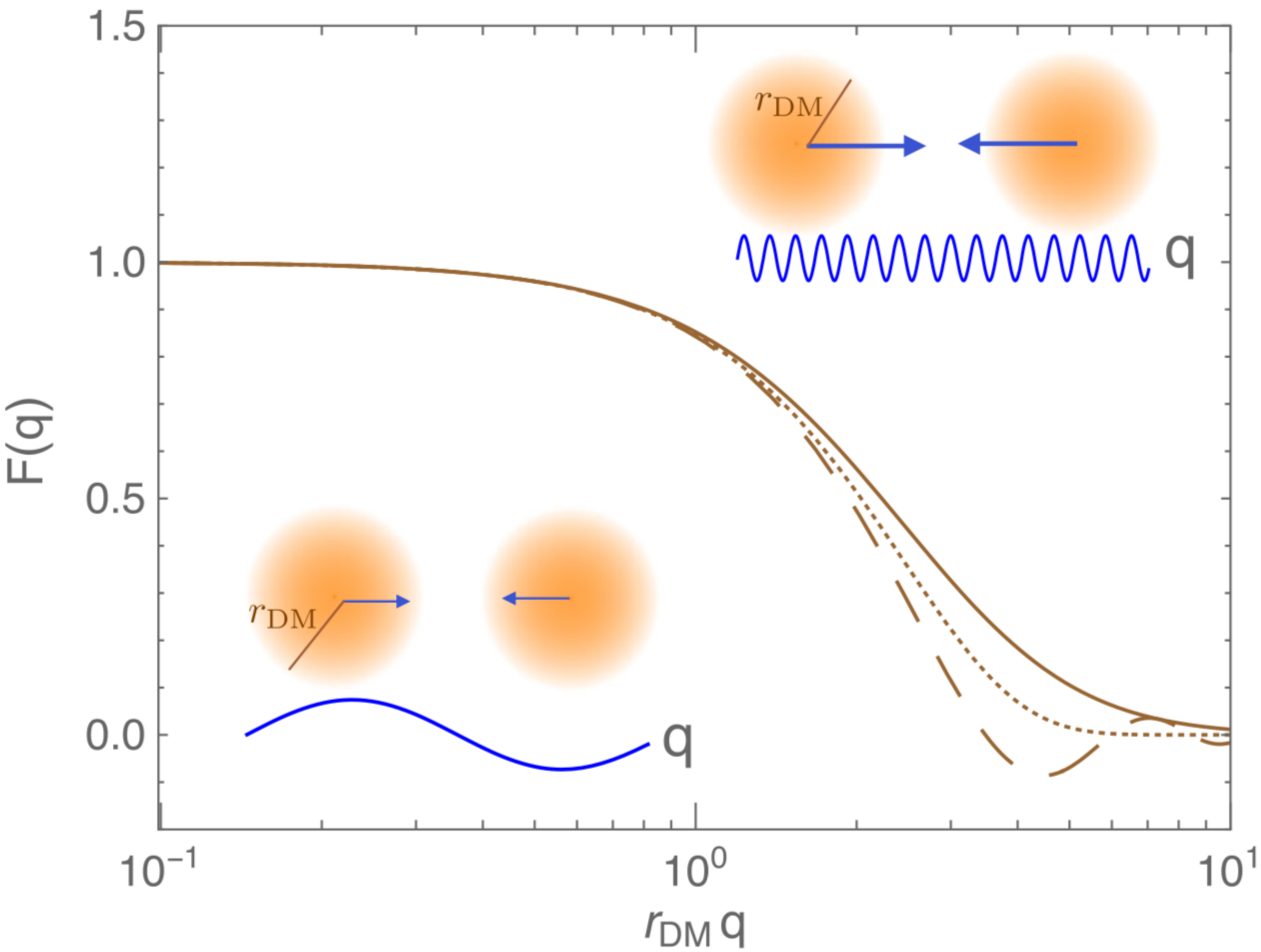
The way the non-relativistic cross section varies with the velocity is largely independent of the internal structure

Simple parametrization

Finite-size dark matter

Finite-Size Dark Matter and its Effect on Small-Scale Structure

Xiaoyong Chu, Camilo Garcia-Cely, and Hitoshi Murayama
Phys. Rev. Lett. **124**, 041101 – Published 28 January 2020



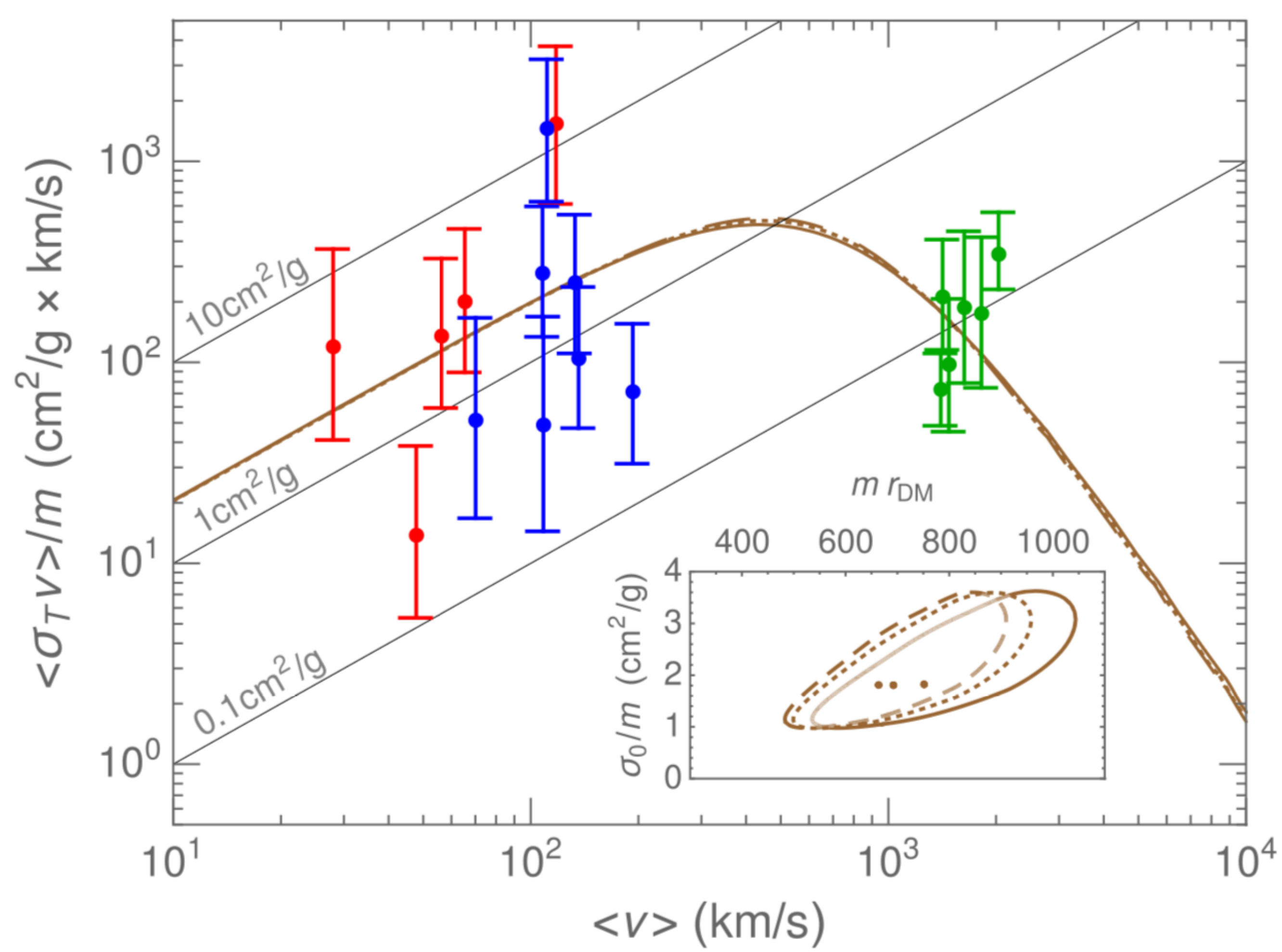


TABLE I. Form factors for different density distributions.

Shape	$\rho(r)$	$r_{\rm DM}$	$F(q)$
Top hat	$(3/4\pi r_0^3)\theta(r_0 - r)$	$2\sqrt{3}r_0$	$\{(3[\sin(r_0 q) - r_0 q\cos(r_0 q)])/(r_0^3 q^3)\}$
Dipole	$[(e^{-r/r_0})/8\pi r_0^3]$	$\sqrt{3/5}r_0$	$\{1/[(1 + r_0^2 q^2)^2]\}$
Gaussian	$[1/(8 r_0^3 \pi^{3/2})]e^{-r^2/(4r_0^2)}$	$\sqrt{6}r_0$	$e^{-r_0^2 q^2}$

The way the non-relativistic cross section varies with the velocity is largely independent of the internal structure

Simple parametrization

Theory of the Effective Range in Nuclear Scattering

H. A. BETHE
*Physics Department, Cornell University, Ithaca, New York**
(Received February 28, 1949)

The scattering of neutrons up to about 10 or 20 Mev by protons can be described by two parameters, the scattering length at zero energy, a, and the effective range, r_0. A formula (16), expressing the phase shift in terms of a and r_0 is derived; it is identical with one previously derived by Schwinger but the derivation is very much simpler. Reasons are given why the deviations from the simple formula are very small, as shown by the explicit calculations by Blatt and Jackson.

The theory is then applied to proton-proton scattering, with a similarly simple result. Moreover, a method is developed to compare proton-proton and proton-neutron scattering without explicit calculation of a nuclear potential.

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Effective range theory

Light mediators

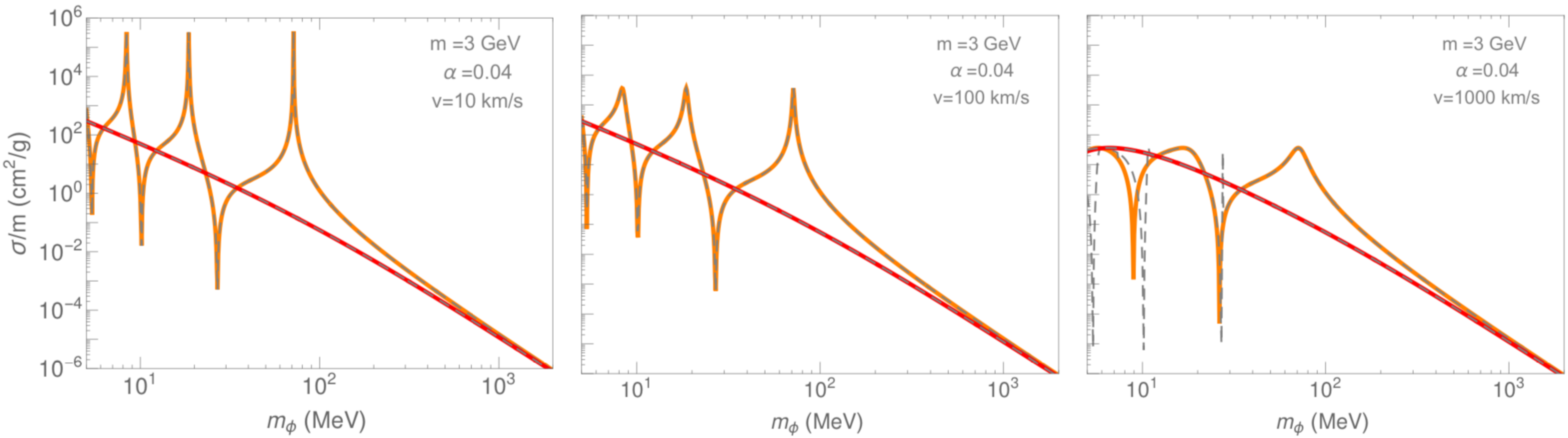
For short-range interactions, regardless of the potential, the non-relativistic **s-wave** scattering cross section can be approximated by means of

$$\sigma(v) = 4\pi a^2 \left(\left(1 - \frac{1}{8}\frac{r_e}{a}(mav)^2\right)^2 + \frac{1}{4}(mav)^2 \right)^{-1}$$

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

A practical and consistent parametrization of dark matter self-interactions

Xiaoyong Chu,[a] Camilo Garcia-Cely[b] and Hitoshi Murayama[c,d,e,b,1]



Effective range theory

Breit-Wigner resonances, bound-states, etc.

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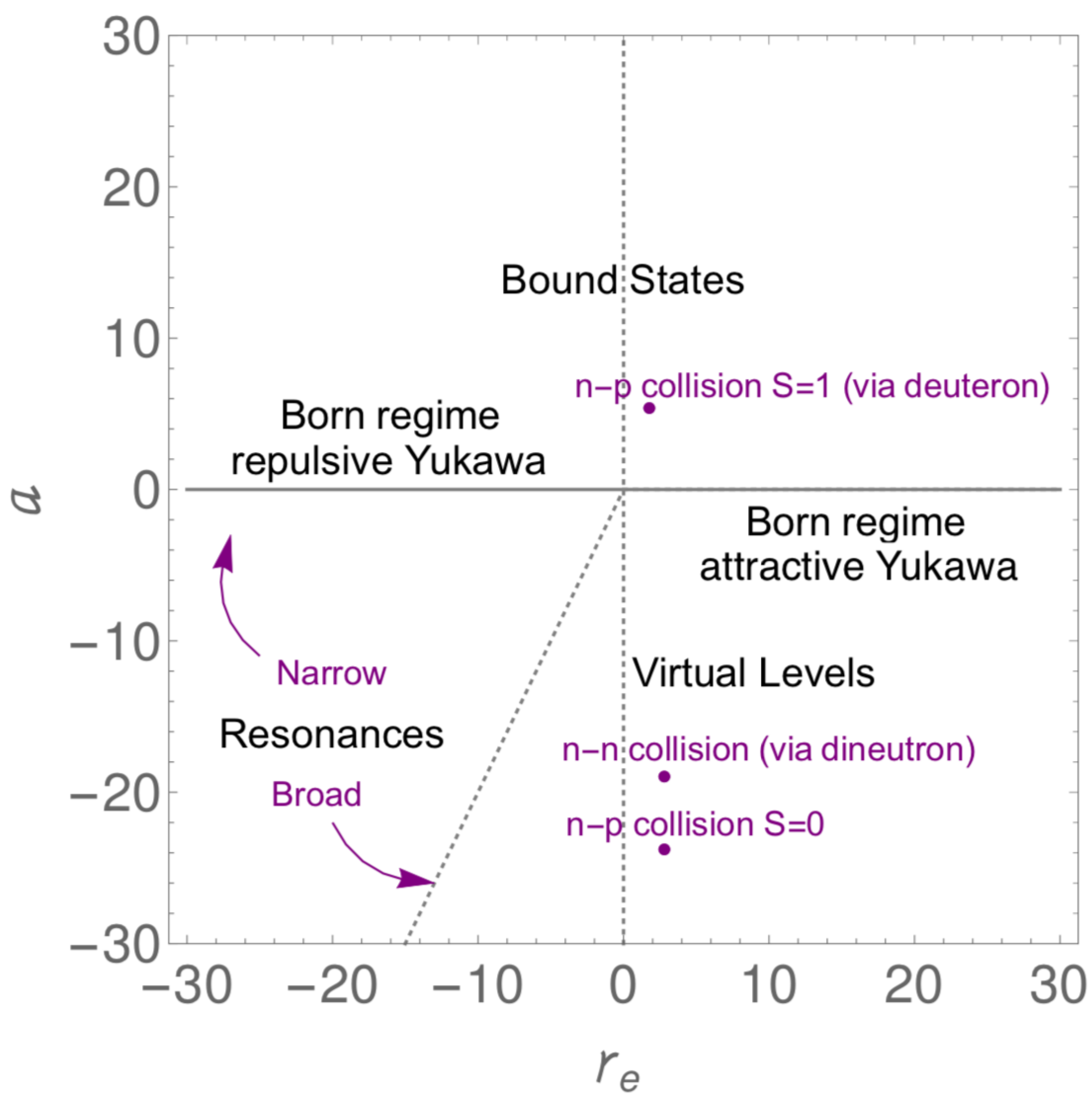

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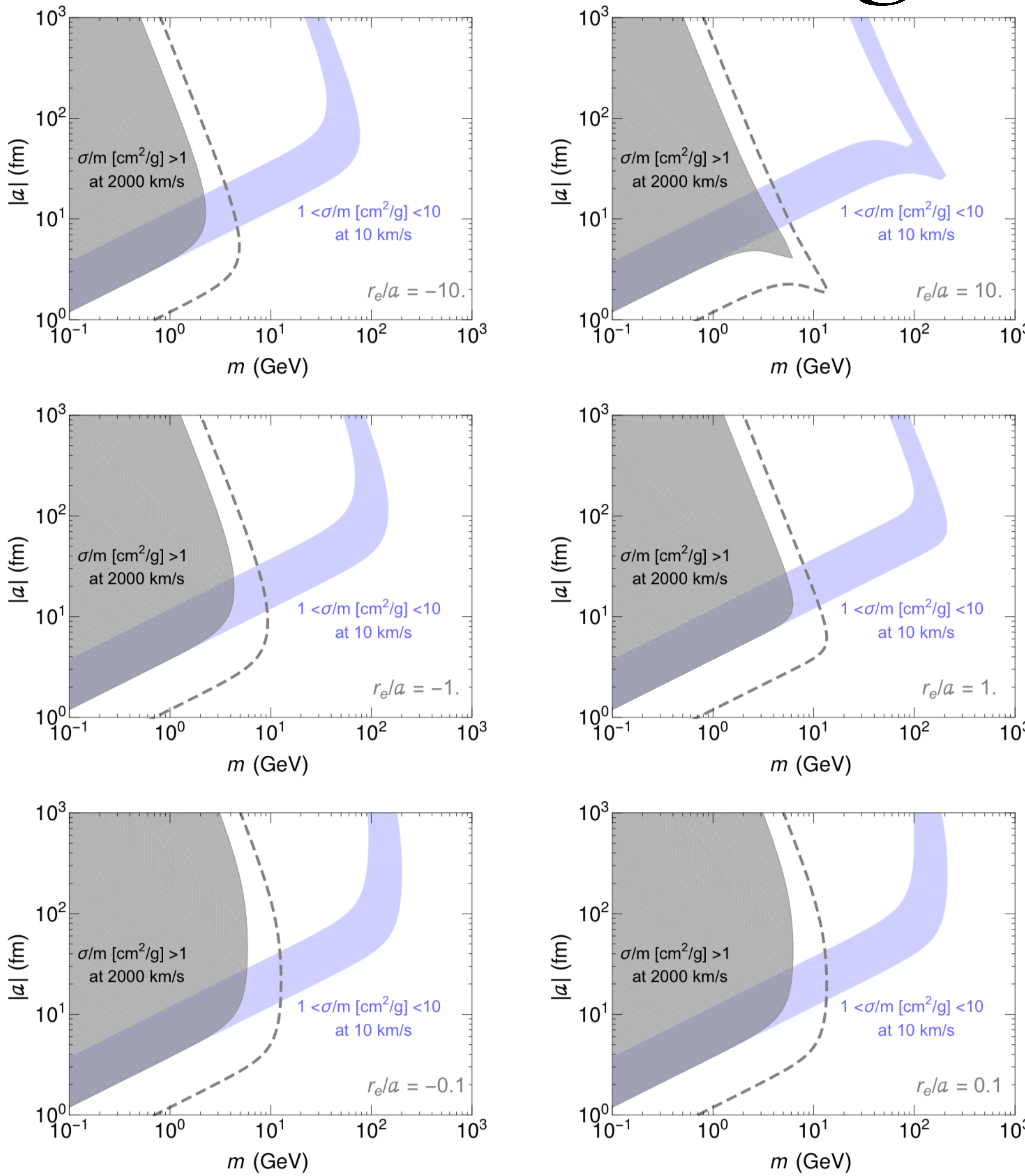

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Lessons

- The sign of either parameter cannot be fixed by the velocity dependence of the cross section.

- From this point of view, models based on light mediators are as well motivated as those arising from strongly coupled sectors.

- Studies based of light mediators can be adapted to other models . For example

$$\sigma = \frac{\sigma_0}{1 + \frac{v^2}{\omega^2}} \quad \longrightarrow \quad r_e = 0 \quad \text{and} \quad \sigma_0 = 4\pi a^2$$

When does it fail?

- Anti resonances
- Resonances with a potential barrier
- Semi-classical regime

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Parametrization can be modified easily

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Parametrization can be modified easily

- Many partial waves and therefore challenging
- Angular dependence becomes important

See e.g. Colquhoun, Heeba, Kahlhoefer, Sagunski and Tulin, 2020

Conclusions

- Resonant SIDM is a viable model giving velocity-dependent scattering cross sections. Scenarios in which DM has a finite size are another alternative.
- The velocity dependence of the scattering cross section is given by the effective range theory.
- This leads to a model-independent parametrization of the velocity dependence of dark matter scattering in astrophysical halos.
- This parametrization is able to simultaneously describe resonances, light mediators and strongly coupled systems.

Effective range theory

Light mediators