An exact AdS/CFT duality

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Pollica summer workshop
New connections between Physics
and Number Theory
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Based mainly on work with Rajesh Gopakumar

AdS / CFT duality

Much recent progress in string theory has been related to AdS/CFT duality

[Maldacena '97, ...]

superstrings on
$\mathrm{AdS}_5 \times S^5$

=

SU(N) super Yang-Mills theory in 4 dimensions

4d non-abelian gauge theory similar to that appearing in the standard model of particle physics.

AdS/CFT correspondence

The relation between the parameters of string theory on AdS and the dual CFT is schematically

$$g_s \sim g_{\mathrm{YM}}^2 = \frac{\lambda}{N}$$

string coupling constant

$$\frac{R}{l_s} \sim g_{\mathrm{YM}}^2 N = \lambda$$

AdS radius in string units

‘t Hooft parameter

AdS/CFT correspondence

For example, in the large N limit of gauge theory at large ‘t Hooft coupling

$$\underset{\substack{\big\uparrow \\ \text{small}}}{g_s} \sim g_{\rm YM}^2 = \frac{\lambda}{N} \qquad\qquad \frac{R}{\underset{\substack{\big\uparrow \\ \text{large}}}{l_s}} \sim g_{\rm YM}^2 N = \lambda$$

supergravity (point particle) approximation is good for AdS description. This is interesting since it gives insights into strongly coupled gauge theories using supergravity methods.

Strategy for a proof

There is very good evidence that at least the original duality is correct. However, we have not yet succeeded in proving it (and we therefore also do not know to which extent it generalises).

In order to make progress in this direction, our strategy will be as follows:

- **Consider tensionless regime**
- **Lower dimensional version (AdS3)**
- **Generalise AdS3 to AdS5**

Weakly coupled gauge theory

The **tensionless regime** arises in another corner of parameter space where the gauge theory is weakly coupled

$$\underset{\substack{\uparrow \\ \text{small}}}{g_s} \sim g_{\mathrm{YM}}^2 = \frac{\lambda}{N} \qquad \qquad \frac{R}{\underset{\substack{\uparrow \\ \text{small}}}{l_s}} \sim g_{\mathrm{YM}}^2 N = \lambda$$

$l_{\mathrm{s}} \to \infty$ `tensionless strings' [Sundborg '01] [Witten '01] [Sezgin,Sundell '01]

Tensionless limit

This is the regime where AdS/CFT becomes perturbative:

tensionless strings on AdS ↔ weakly coupled/free SYM theory

- very stringy (far from sugra)
- higher spin symmetry
- maximally symmetric phase of string theory

Tensionless limit

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- maximally symmetric phase of string theory

Could it have a free worldsheet description?

Lower dimensions

The **other key ingredient** is to consider the lower-dimensional case, in particular, string theory on AdS3.

The advantage of going to the 3d case is that

- Solvable world-sheet theory for strings on AdS3 exists [sl(2,R) WZW model]

[Maldacena, (Son), Ooguri ’00 - ’01]
[Berkovits, Vafa, Witten ’99]

- Much better control over 2d CFTs

AdS3

It has long been suspected that the CFT dual of string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

is on the same moduli space of CFTs that also contains the symmetric orbifold theory

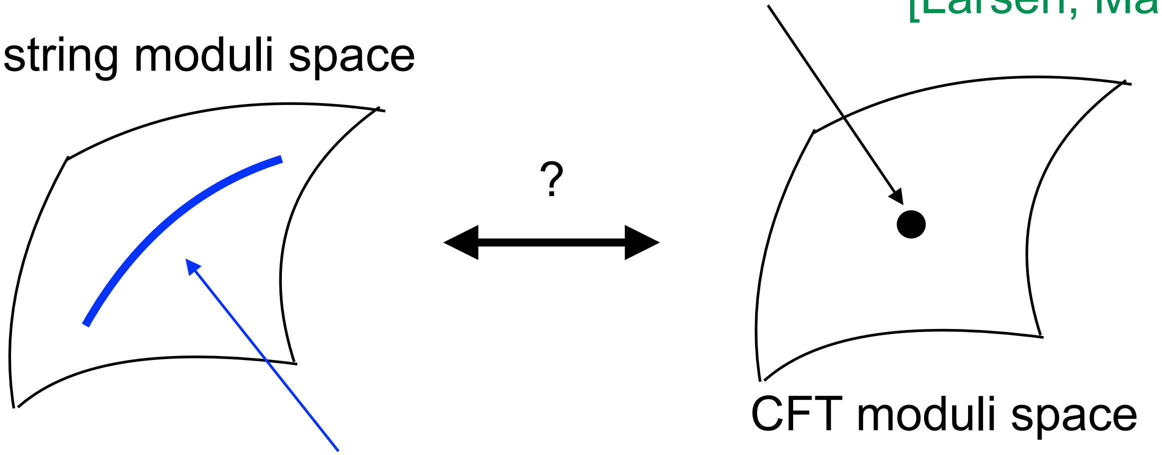
$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^N / S_N$$

[Maldacena ’97], …. see e.g. [David et.al. ’02]

AdS3

However, it was not known what precise string background is being described by the symmetric orbifold theory itself.

[Larsen, Martinec ’99]



string moduli space

?

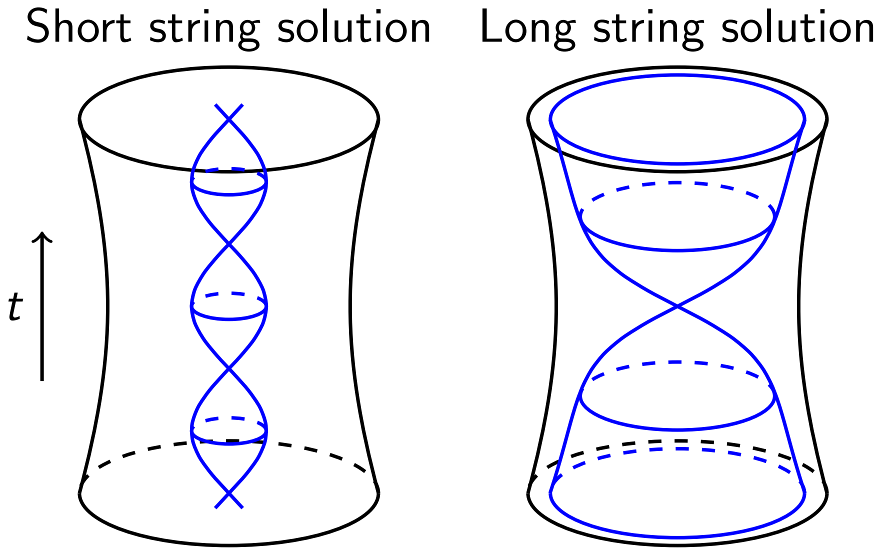
CFT moduli space

Conversely, it was not known what the precise CFT dual of the explicitly solvable worldsheet theory for strings in terms of an sl(2,R) WZW model is.

[Maldacena, (Son), Ooguri ’00 & ‘01]

AdS3

In fact, the only consensus was that the actual symmetric orbifold theory **cannot** be dual to the WZW model…



The basic reason for this is that the WZW model describes the background with pure NS-NS flux, which is known to have long string solutions.

[Seiberg, Witten '99], [Maldacena, Ooguri '00]

AdS3

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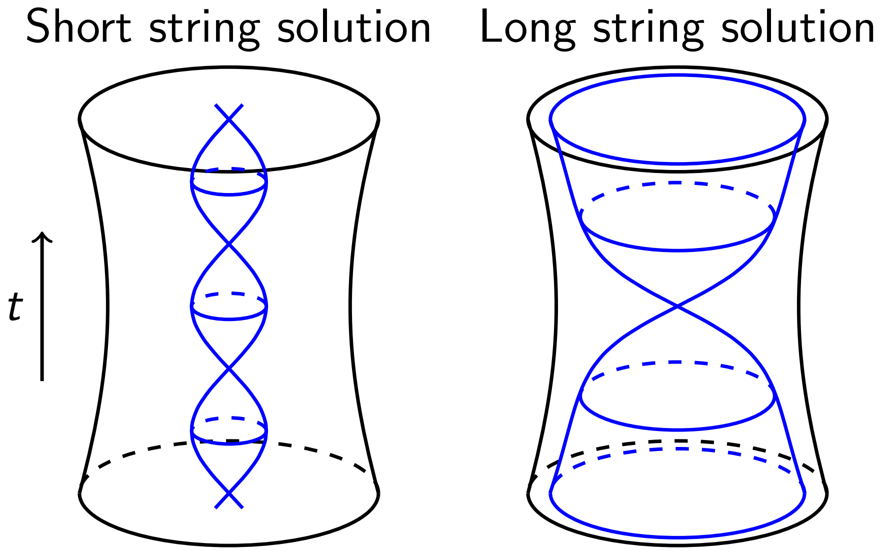


These long strings give rise to a continuum of excitations that are not present in the actual symmetric orbifold theory.

[Seiberg, Witten ’99], [Maldacena, Ooguri ’00]

A concrete proposal

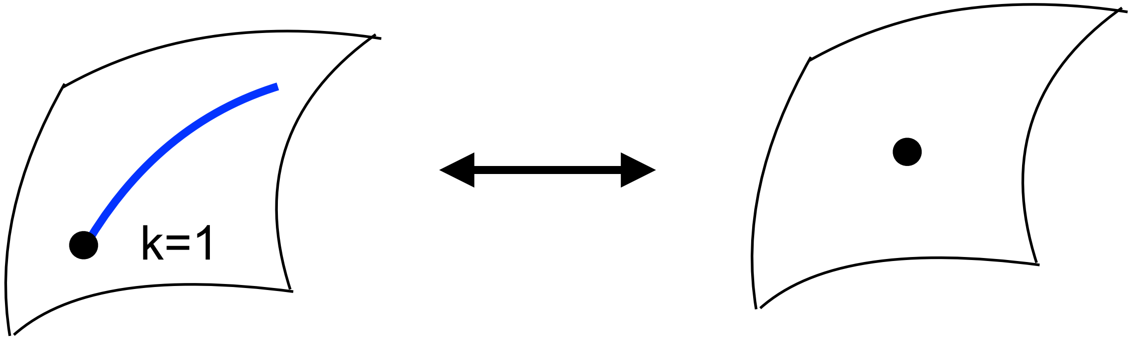
Since the symmetric orbifold theory has a higher spin symmetry algebra, it should be dual to string theory at the tensionless point. [MRG, Gopakumar ’14]

The tensionless limit arises when the spacetime geometry is of string size, i.e. in the **deeply stringy regime**.

In the context of the WZW description, this should be the situation where the level of the sl(2,R) affine theory takes the smallest possible value, i.e. k=1.

WZW model

This led us to study the spacetime spectrum of the k=1 sl(2,R) WZW model systematically.



We have shown that this worldsheet description is indeed exactly dual to the symmetric orbifold.

An exact AdS/CFT duality

[MRG, Gopakumar ’18]
[Eberhardt, MRG, Gopakumar ’18]
[Eberhardt, MRG, Gopakumar ’19]
[Dei, MRG, Gopakumar, Knighton ’20]

Plan of talk

1. **Introduction and Motivation**
2. Matching the spectrum in AdS3/CFT2
3. Matching correlators in AdS3/CFT2
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Hybrid formalism

[Berkovits, Vafa, Witten '99]

World-sheet theory is best described in terms of the hybrid formalism: for pure NS-NS flux WZW model based on

$$\mathfrak{psu}(1,1|2)_k$$

together with the (topologically twisted) sigma model for T4. For generic k, this description agrees with the NS-R description a la Maldacena-Ooguri.

[Troost '11], [MRG, Gerigk '11]
[Gerigk '12]

Hybrid formalism

For the following it will be important to understand the representation theory of

$$\mathfrak{psu}(1,1|2)_1$$

The bosonic subalgebra of this superaffine algebra is

$$\mathfrak{sl}(2)_1 \oplus \mathfrak{su}(2)_1$$

Thus only **n**=1 and **n**=2 are allowed for the highest weight states.

Short representations

A generic representation of the zero mode algebra $\mathfrak{psu}(1,1|2)$ has the form

rep of sl(2,R)

$$j = \tfrac{1}{2} + is \quad (s \leftrightarrow \text{cont.})$$

$$(C^{j}_{\alpha}, \mathbf{n})$$

$$(C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \qquad (C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1}) \qquad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \qquad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1})$$

$$(C^{j+1}_{\alpha}, \mathbf{n}) \qquad (C^{j}_{\alpha}, \mathbf{n+2}) \qquad 2 \cdot (C^{j}_{\alpha}, \mathbf{n}) \qquad (C^{j}_{\alpha}, \mathbf{n-2}) \qquad (C^{j+1}_{\alpha}, \mathbf{n})$$

$$(C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \qquad (C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1}) \qquad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \qquad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1})$$

$$(C^{j}_{\alpha}, \mathbf{n})$$

rep of su(2) of dim = n+1.

Short representations

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rep of sl(2,R) $\quad j = \frac{1}{2} + is \quad (s \leftrightarrow \text{cont.})$

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$$(C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \quad (C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1}) \quad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \quad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1})$$

$$(C^{j+1}_\alpha, \mathbf{n}) \quad (C^j_\alpha, \mathbf{n+2}) \quad 2\cdot(C^j_\alpha, \mathbf{n}) \quad (C^j_\alpha, \mathbf{n-2}) \quad (C^{j+1}_\alpha, \mathbf{n})$$

$$(C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \quad (C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1}) \quad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n+1}) \quad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{n-1})$$

$$(C^j_\alpha, \mathbf{n})$$

Thus for k=1 need
a short rep!

Short representations

In fact, the only representations that are allowed are

$$(C^{j}_{\alpha}, \mathbf{2})$$

$$(C^{j+\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{1}) \qquad (C^{j-\frac{1}{2}}_{\alpha+\frac{1}{2}}, \mathbf{1})$$

and the shortening condition actually implies that this is only possible provided that

$$j = \frac{1}{2} \quad \longrightarrow$$ **NO CONTINUUM!**

[Eberhardt, MRG, Gopakumar '18]

Free field realisation

The level k=1 theory has a free field realisation

$$\mathfrak{u}(1,1|2)_1 \;\cong\; \left\{ \begin{array}{l} 4 \textbf{ symplectic bosons } \xi^{\pm}\,,\;\eta^{\pm} \\ 4 \textbf{ real fermions } \psi^{\pm}\,,\;\chi^{\pm} \end{array} \right.$$

with

$$\boxed{\{\psi^{\alpha}_{r}\,,\chi^{\beta}_{s}\} = \epsilon^{\alpha\beta}\,\delta_{r,-s}} \qquad \boxed{[\xi^{\alpha}_{r}\,,\eta^{\beta}_{s}] = \epsilon^{\alpha\beta}\,\delta_{r,-s}}$$

Generators of $\mathfrak{u}(1,1|2)_1$ are bilinears in these free fields.

In order to reduce this to $\mathfrak{psu}(1,1|2)_1$ one has to gauge by the `diagonal' u(1) field

$$Z = \tfrac{1}{2}\left(\eta^{-}\xi^{+} - \eta^{+}\xi^{-} + \chi^{-}\psi^{+} - \chi^{+}\psi^{-}\right)\,.$$

R sector representation

The ground states of the above short representation arise from the `R-sector' where the zero modes act as

$$\begin{aligned} \xi_0^+ |m_1, m_2\rangle &= |m_1, m_2 + \tfrac{1}{2}\rangle \ , & \eta_0^+ |m_1, m_2\rangle &= 2\, m_1\, |m_1 + \tfrac{1}{2}, m_2\rangle \ , \\ \xi_0^- |m_1, m_2\rangle &= -\, |m_1 - \tfrac{1}{2}, m_2\rangle \ , & \eta_0^- |m_1, m_2\rangle &= -2\, m_2\, |m_1, m_2 - \tfrac{1}{2}\rangle \ , \\ \chi_0^+ |m_1, m_2\rangle &= 0 \ , & \psi_0^+ |m_1, m_2\rangle &= 0 \ . \end{aligned}$$

The relevant charges are then $(Z_0 = U_0 + V_0 \cong 0)$

$$\boxed{\begin{array}{ll} J_0^3\, |m_1, m_2\rangle = (m_1 + m_2)\, |m_1, m_2\rangle & \\ K_0^3\, |m_1, m_2\rangle = \frac{1}{2}\, |m_1, m_2\rangle & L_0\, |m_1, m_2\rangle = 0 \\ U_0\, |m_1, m_2\rangle = (m_1 - m_2 - \frac{1}{2})\, |m_1, m_2\rangle & V_0\, |m_1, m_2\rangle = 0 \end{array}}$$

Spectral flow

The full worldsheet spectrum consists of this R-sector representation, together with its spectrally flowed images. Here spectral flow comes from

[Henningson et.al. ’91]
[Maldacena, Ooguri ’00]

$$\tilde{\xi}^{\pm}_{n} = \xi^{\pm}_{n\pm\frac{w}{2}} \ , \qquad \tilde{\eta}^{\pm}_{n} = \eta^{\pm}_{n\pm\frac{w}{2}} \ , \qquad \tilde{\psi}^{\pm}_{n} = \psi^{\pm}_{n\mp\frac{w}{2}} \ , \qquad \tilde{\chi}^{\pm}_{n} = \chi^{\pm}_{n\mp\frac{w}{2}} \ ,$$

consider above
R sector rep. for
tilde modes

interpret in terms
of `untilde’
modes

For w>1: not highest weight representation any longer.

Spectral flow

The full worldsheet spectrum consists of this R-sector representation, together with its spectrally flowed images. Here spectral flow comes from

[Henningson et.al. '91]
[Maldacena, Ooguri '00]

In particular, the various Cartan generators transform as

$$J^3_m = \tilde{J}^3_m + \frac{kw}{2}\delta_{m,0} \ ,$$

$$K^3_m = \tilde{K}^3_m + \frac{kw}{2}\delta_{m,0} \ ,$$

$$L_m = \tilde{L}_m + w(\tilde{K}^3_m - \tilde{J}^3_m) \ .$$

Physical states (bosons)

Bosonic oscillators:

$$\begin{aligned} \mathbb{T}^4 : &\quad \text{4 bosons} \\ \mathfrak{su}(1,1|2)_1 : &\quad \text{4 symplectic bosons} \end{aligned}$$

Physical state conditions:

$$\begin{aligned} Z_n = 0 \;\rightarrow &\quad \text{removes 2 bosons} \\ L_n = 0 \;\rightarrow &\quad \text{removes 2 bosons} \end{aligned}$$

Thus only the 4 torus bosons, say, survive.

Physical states (bosons)

Consider thus the state $\alpha^{i_1}_{-n_1} \cdots \alpha^{i_l}_{-n_l} |m_1, m_2\rangle$

Zero mode conditions:

$$(N = \sum_{i=1}^{l} n_l)$$

$$Z_0 = 0 \;\; \rightarrow \quad m_1 - m_2 = \tfrac{1}{2}$$

$$L_0 = 0 \;\; \rightarrow \quad N + w(\tilde{K}^3_0 - \tilde{J}^3_0) = N + w(\tfrac{1}{2} - \tilde{J}^3_0) = 0$$

fixes $m_1 + m_2$

Thus spacetime conformal dimension is

$$\boxed{J^3_0 = \tilde{J}^3_0 + \frac{w}{2} = \frac{N}{w} + \frac{w+1}{2}}$$

Physical states (bosons)

$$\boxed{J_0^3 = \tilde{J}_0^3 + \frac{w}{2} = \frac{N}{w} + \frac{w+1}{2}}$$

Thus we have the correspondence

$$\alpha^{i_1}_{-n_1} \cdots \alpha^{i_l}_{-n_l} \, |m_1, m_2\rangle \;\longleftrightarrow\; \alpha^{i_1}_{-\frac{n_1}{w}} \cdots \alpha^{i_l}_{-\frac{n_l}{w}} \, |\mathrm{BPS}\rangle_{h=\frac{w+1}{2}}$$

Analysis for fermions is similar, and we thus get exactly the (single-particle) spectrum of

$$\mathrm{Sym}_N(\mathbb{T}^4)$$

in the large N limit.

[Eberhardt, MRG, Gopakumar ’18]

Symmetric orbifold basics

Recall basic structure of symmetric orbifold

$$(\mathbb{T}^4)^N / S_N$$

untwisted sector: permutation invariant combinations

twisted sectors: associated to conjugacy classes of S_N

labelled by cycle shapes, i.e. partitions of N

concentrate on **single cycle sectors** $\longleftrightarrow$ analogue of single trace

Symmetric orbifold basics

In the w-cycle twisted sector the operators that transform diagonally under the mondromy are

$$S^{[p]}(z) = \sum_{\ell=1}^{w} e^{2\pi i \frac{p\ell}{w}} S^{\ell}(z)$$

with eigenvalue

$$(1 \cdots w)\, S^{[p]} = e^{-2\pi i \frac{p}{w}}\, S^{[p]}$$

Symmetric orbifold basics

As a consequence they have the mode expansion

$$S^{[p]}(z) = \sum_{\ell=1}^{w} e^{2\pi i \frac{p\ell}{w}} S^{\ell}(z) = \sum_{n+h\in\mathbb{Z}} S^{[p]}_{n+\frac{p}{w}} z^{-n-\frac{p}{w}-h}$$

with fractional mode numbers of the form

$$n + \frac{p}{w} \qquad (n + h \in \mathbb{Z})$$

Symmetric orbifold basics

Their excitation spectrum is hence

$$h^{\mathrm{ST}} = \frac{N}{w} + \frac{w+1}{2}$$

fractional modes

conf dim of BPS state

which matches worldsheet spectrum from above:

$$\boxed{J_0^3 = \tilde{J}_0^3 + \frac{w}{2} = \frac{N}{w} + \frac{w+1}{2}}$$

Plan of talk

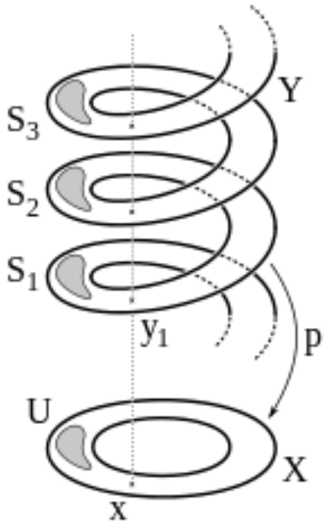
1. Introduction and Motivation
2. Matching the spectrum in AdS3/CFT2
3. **Matching correlators in AdS3/CFT2**
4. Conclusions

Correlators in sym orbifold

An efficient method to actually calculate correlation functions of (twist) fields is in terms of covering maps.

[Lunin, Mathur ’00]
[Pakman, Rastelli, Razamat ’09]

z



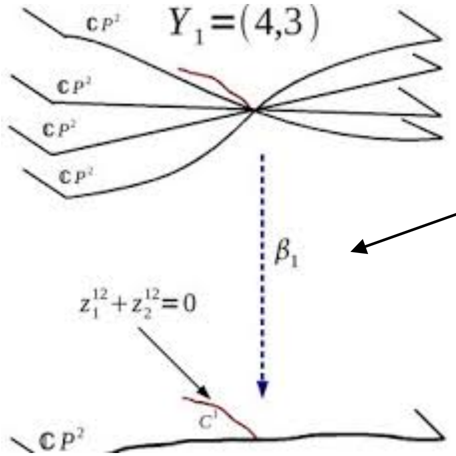
$x = \Gamma(z)$

Locally, the effect of a w-cycle twist field is to introduce a w-fold covering.

$$z \mapsto \Gamma(z) = x_0 + a(z - z_0)^w + \cdots$$

Correlators in sym orbifold

To describe the full correlator combine these local coverings into a global covering surface.



Then use the **covering map** to lift correlator to covering surface.

covering map: conformal map from covering surface to base surface.

Correlators in sym orbifold

The situation is particularly simple if the w-cycle twist field describes the ground state of the twisted sector: then the correlator on the covering surface is just the vacuum correlator.

In this case, the correlator is completely determined by the conformal factor that comes from the covering map. In turn this is described by a Liouville term [Lunin, Mathur ’00]

$$S_L[\phi] = \frac{c}{48\pi} \int d^2z \sqrt{g} \big(2\partial\phi\bar{\partial}\phi + R\phi\big) \quad \text{with} \quad e^{\phi} = |\partial_z \Gamma|^2$$

covering map

Correlators

The symmetric orbifold sphere correlators (of the ground states) thus have the schematic form

[Lunin, Mathur '00]

$$\langle \prod_{i=1}^{n} \mathcal{O}_{h_i}^{w_i}(x_i) \rangle = \sum_{\Gamma} \Big(\text{conf. factor assoc. to } \Gamma \Big)$$

Here $\Gamma(z)$ is a holomorphic covering map $\Gamma : \mathcal{C} \to \mathbb{C}_\infty$ with the property that

Riemann
surface of
genus g

$$\Gamma(z) = x_i + a_i (z - z_i)^{w_i} + \cdots \qquad (z \sim z_i)$$

Correlators in sym orbifold

The covering surface has, in general, a non-trivial genus, i.e. it is not necessarily a sphere. The genus captures the 1/N corrections of the symmetric orbifold correlators since

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle \sim N^{1-g-\frac{n}{2}}$$

[Lunin, Mathur '00]
[Pakman, Rastelli, Razamat '09]

Since string coupling is related to 1/N via

$$g_s \sim \frac{1}{\sqrt{N}}$$

this suggests that **covering surface = world-sheet**.

[Pakman, Rastelli, Razamat '09]

Covering maps

In order for the world-sheet to play the role of the covering surface, a surprising localisation property must hold.

This is consequence of the fact that for a specified configuration of

$$z_i \ , \qquad x_i \ , \qquad w_i \ , \quad i = 1, \ldots, n \ ,$$

a covering map typically does not exist (for $n \geq 4$).

Covering maps

For example, for the case of a 4-point function with

$$w_i = 1 \ , \quad i = 1, 2, 3, 4 \ ,$$

the covering map is just a Moebius transformation. However, this only maps the 4 points to each other provided that the cross-ratios agree

$$\frac{(x_1 - x_2)(x_3 - x_4)}{(x_1 - x_3)(x_2 - x_4)} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_3)(z_2 - z_4)}$$

Covering maps

Thus the world-sheet correlator should be of the form

$$\left\langle \prod_{i=1}^{4} V^{1}_{h_i}(x_i; z_i) \right\rangle \sim \delta\left(\frac{(x_1-x_2)(x_3-x_4)}{(x_1-x_3)(x_2-x_4)} - \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_3)(z_2-z_4)} \right) .$$

so that

$$\int dz_4 \, \langle \prod_{i=1}^{4} V^{1}_{h_i}(x_i; z_i) \rangle = \text{contr. of Moebius covering}$$

Covering maps

More generally, worldsheet correlator should satisfy

$$\langle\prod_{i=1}^{n} V_{h_i}^{w_i}(x_i;z_i)\rangle_g = \sum_{\Gamma_g} c(\Gamma_g)\,\delta\big(z_i \text{ compatible with cov. map } \Gamma_g\big)$$

so that worldsheet modular integral leads to the sum

$$\int_{\mathcal{M}_{g,n}} d\mu(z_i)\,\langle\prod_{i=1}^{n} V_{h_i}^{w_i}(x_i;z_i)\rangle_g = \sum_{\Gamma_g} \tilde{c}(\Gamma_g) \cong \langle\prod_{i=1}^{n} \mathcal{O}_{h_i}^{w_i}(x_i)\rangle\Big|_{N^{1-g-\frac{n}{2}}}$$

Correlators from worldsheet

We can now test this idea from worldsheet perspective

$$\langle \prod_{i=1}^{n} \mathcal{O}^{w_i}_{h_i}(x_i) \rangle = \int_{\mathcal{M}} d\mu(z_i) \, \langle \prod_{i=1}^{n} V^{w_i}_{h_i}(x_i; z_i) \rangle$$

The relevant vertex operators on the world-sheet have the general form

see also [Kutasov, Seiberg '99]

$$V^{w}_{h}(x; z) = e^{x J^{+}_{0}} \, e^{z L_{-1}} \, V(|j, m\rangle^{(w)}, 0; 0) e^{-z L_{-1}} \, e^{-x J^{+}_{0}}$$

spacetime CFT coordinate

worldsheet coordinate

affine primary in w-flowed sector (ground state)

identify fields and states

$h = m + \frac{wk}{2}$

spacetime conformal dimension

[Eberhardt, MRG, Gopakumar, '19]

Covering maps

Their correlation functions are constrained by the Ward identities, i.e. by considering the correlators

$$\left\langle J^a(z) \prod_{i=1}^{n} V_{h_i}^{w_i}(x_i; z_i) \right\rangle$$

say on the sphere.

The analysis is a bit subtle since the vertex operators do not describe highest weight states (because of spectral flow).

OPE behaviour

In fact, because of the x-dependence of the vertex operators, the OPEs are of the form

$$J^+(z)V_h^w(x;\zeta) \sim (h - \tfrac{kw}{2} + j)\frac{V_{h+1}^w(x;\zeta)}{(z-\zeta)^{w+1}} + \sum_{l=1}^{w-1}\frac{J_l^+ V_h^w(x;\zeta)}{(z-\zeta)^{l+1}} + \frac{\partial_x V_h^w(x;\zeta)}{(z-\zeta)}$$

$$J^3(z)V_h^w(x;\zeta) \sim x\,(h - \tfrac{kw}{2} + j)\frac{V_{h+1}^w(x;\zeta)}{(z-\zeta)^{w+1}} + x\sum_{l=1}^{w-1}\frac{J_l^+ V_h^w(x;\zeta)}{(z-\zeta)^{l+1}} + \frac{(h + x\partial_x)V_h^w(x;\zeta)}{(z-\zeta)}\,,$$

$$J^-(z)V_h^w(x;\zeta) \sim x^2(h - \tfrac{kw}{2} + j)\frac{V_{h+1}^w(x;\zeta)}{(z-\zeta)^{w+1}} + x^2\sum_{l=1}^{w-1}\frac{J_l^+ V_h^w(x;\zeta)}{(z-\zeta)^{l+1}}$$

$$+ \frac{(2hx + x^2\partial_x)V_h^w(x;\zeta)}{(z-\zeta)}\,,$$

usual action of zero mode

E.g., all singular terms in the OPE with J^- are proportional to x since usual OPE is regular because of $J_n^- = \tilde{J}_{n+w}^-$.

Correlation functions

However, can still deduce recursion relations from these OPEs. The resulting equations are quite complicated, but very remarkably, the ansatz $(x_1 = z_1 = 0,\, x_2 = z_2 = 1,\, x_3 = z_3 = \infty)$

$$\langle \prod_{i=1}^{n} V^{w_i}_{h_i}(x_i; z_i) \rangle = \sum_{\Gamma} \prod_{i=1}^{n} (a_i^{\Gamma})^{-h_i} \prod_{i=4}^{n} \delta(x_i - \Gamma(z_i))\, W_{\Gamma}(z_4, \ldots, z_n)$$

that reproduces the Lunin-Mathur answer and that involves delta-functions for the relevant cross-ratios, solves all of these relations provided that **k=1** and all $j_i = \frac{1}{2}$.

[Eberhardt, MRG, Gopakumar, '19]

Correlation functions

Using the free field realisation of

$$\mathfrak{psu}(1,1|2)_1 \;\cong\; \left\{ \begin{array}{l} 4 \text{ symplectic bosons } \xi^{\pm} + \text{conj} \\ 4 \text{ complex fermions } \psi^{\pm} + \text{conj} \end{array} \right.$$

we have also shown that this delta-function localised solution is in fact the only solution at k=1; this can be deduced from

$$\Big\langle \big(\xi^{-}(z) + \underset{\substack{\uparrow \\ \text{covering map}}}{\Gamma(z)}\, \xi^{+}(z)\big) \prod_{i=1}^{n} V_{h_i}^{w_i}(x_i; z_i) \Big\rangle_{\text{phys}} = 0 \ .$$

[Dei, MRG, Gopakumar, Knighton ’20]

Plan of talk

1. Introduction and Motivation
2. Matching the spectrum in AdS3/CFT2
3. Matching correlators in AdS3/CFT2
4. **Conclusions**

Conclusions and Outlook

We have given strong evidence that the symmetric orbifold theory is exactly dual to string theory with one unit of NS-NS flux (k=1):

$$\mathrm{Sym}_N(\mathbb{T}^4) \quad = \quad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

1 unit of NS-NS flux

In particular, the **spectrum** agrees precisely, and the structure of the symmetric orbifold **correlators** is reproduced from the worldsheet.

Conclusions and Outlook

$$\mathrm{Sym}_N(\mathbb{T}^4) \quad = \quad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

1 unit of NS-NS flux

The worldsheet plays the role of the covering surface of the symmetric orbifold, thus making the duality manifest in this way.

In particular, this relation implies that the **duality actually works to all orders in 1/N**!

see also [Eberhardt, '20]
[Knighton, '21]

Conclusions and Outlook

$$\mathrm{Sym}_N(\mathbb{T}^4) \quad = \quad \mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

1 unit of NS-NS flux

The world-sheet theory exhibits signs of a topological string theory:

- only short representations of psu(1,1|2) appear
- correlation functions localise to isolated points

cf [Aharony, David, Gopakumar, Komargodski, Razamat '07]
[Razamat '08], [Gopakumar '11], [Gopakumar, Pius '12]
[Gopakumar Mazenc '22]

Outlook

It would be interesting to understand how much of this structure generalises to $\mathrm{AdS}_5 \times \mathrm{S}^5$.

In particular, there is a natural generalisation of the free field description to the higher dimensional setting, and it leads to the correct spectrum of free N=4 SYM in D=4, using a natural postulate for which degrees of freedom survive the physical state condition.

[MRG, Gopakumar, '21]

[However, we have so far not managed to `derive' this physical state condition from first principles. Also, the structure of the resulting correlators does not quite match up with N=4 SYM yet.]

Outlook

Part of the difficulty is that the superconformal symmetry of N=4 SYM is $\mathfrak{psu}(2,2|4)$, whereas for AdS_3 it is

$$\underbrace{\mathfrak{psu}(1,1|2)}_{\text{Left-movers both on ws \& in target space}} \oplus \underbrace{\mathfrak{psu}(1,1|2)}_{\text{Right-movers both on ws \& in target space}}$$

Suggests that for string theory dual to free N=4 SYM in 4D, the left- and right-movers on the worldsheet must be somehow coupled.

Outlook

[MRG, Gopakumar, Nairz, in progress]

We suspect that this is a consequence of presence of RR-flux (vs pure NS-NS flux) for AdS_3 .

In order to understand this aspect in more detail have recently started to study RR-flux deformation of AdS_3 from dual symmetric orbifold perspective.

↑

2-cycle twisted
sector perturbation

It seems that this perturbation can be described in terms of a `dynamical spin chain’, very similar to what happened for N=4 SYM.

cf. [Beisert ’05]

Outlook

[MRG, Gopakumar, Nairz, in progress]

We suspect that this is a consequence of presence of RR-flux (vs pure NS-NS flux) for AdS_3 .

It seems that this perturbation can be described in terms of a `dynamical spin chain', very similar to what happened for N=4 SYM.

cf. [Beisert '05]

In fact, the perturbation also induces a non-trivial anti-commutator between left- and right-moving supercharges….

cf. [Gava Narain '02]

Future directions

Finally, from the CFT perspective it would be very interesting to understand the structure of the `x-dependent’ fusion rules:

- what is appropriate math description?
- Verlinde formula?
- ….

Thank you!