Towards collinear helicity parton distribution functions at next-to-next-to-leading order accuracy

Sar WorS 2023 — 3rd Sardinian Workshop on Spin

Emanuele R. Nocera

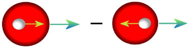
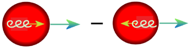
Università degli Studi di Torino and INFN — Torino

Hotel Baia di Nora – 7$^{\text{th}}$ June 2023

Helicity-dependent PDFs and the proton spin

The densities of partons with spin $(^{\uparrow})$ or $(^{\downarrow})$ *w.r.t.* the parent nucleon

$$\Delta f(x) \equiv f^{\uparrow}(x) - f^{\downarrow}(x)\,, \qquad f = u, \bar{u}, d, \bar{d}, s, \bar{s}, g$$

$\Delta q(x) =$  $\qquad \Delta g(x) =$ 

$$\Delta q(x) = \frac{1}{4\pi}\int dy^- e^{-ixP^+y^-}\langle h(P,S)|\bar{\psi}(0,y^-,\mathbf{0}_\perp)\gamma^+\gamma^5\psi(0)|h(P,S)\rangle$$

$$\Delta g(x) = \frac{1}{4\pi x P^+}\int dy^- e^{-ixP^+y^-}\langle h(P,S)|G^{+\alpha}(0,y^-,\mathbf{0}_\perp)\tilde{G}^{+}_{\alpha}(0)|h(P,S)\rangle$$

$$G^{\alpha}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$$

A realisation of the total proton angular momentum decomposition

$$\mathcal{S}(\mu^2) = \sum_f \left\langle P;S|\hat{J}^z_f(\mu^2)|P;S\right\rangle = \frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu^2) + \Delta G(\mu^2) + \mathcal{L}_q(\mu^2) + \mathcal{L}_g(\mu^2)$$

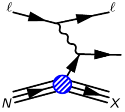
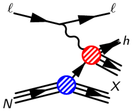
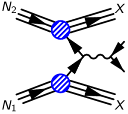
$$\Delta\Sigma(\mu^2) = \sum_{q=u,d,s}\int_0^1 [\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)] \qquad \Delta G(\mu^2) = \int_0^1 dx\Delta g(x,\mu^2)$$

$$a_0 = \left\langle P;S|\hat{J}^z_\Sigma(\mu^2)|P;S\right\rangle \xrightarrow{\text{naive p.m.}} 2\langle S_z^{q+\bar{q}}\rangle \simeq 1 \qquad \text{EMC 1988 } \mathrm{a}_0 = 0.098 \pm 0.076 \pm 0.113$$

$$a_0 = \left\langle P;S|\hat{J}^z_\Sigma(\mu^2)|P;S\right\rangle \overset{\overline{\text{MS}}}{=} \Delta\Sigma(\mu^2) - n_f\frac{\alpha_s(\mu^2)}{2\pi}\Delta G(\mu^2) \qquad \Delta G(\mu^2) \propto \left[\alpha_s(\mu^2)\right]^{-1}$$

1. Helicity PDFs from a global QCD analysis

Spin asymmetries

PROCESS	MEASURED ASYMMETRIES	SUBPROCESSES	PROBED PDFS
 DIS $\ell^{\pm} + N \to \ell^{\pm} + X$	$A_1 \approx \frac{\sum_q \Delta q(x) + \Delta\bar{q}(x)}{\sum_{q'} q'(x) + \bar{q}'(x)}$	$\gamma^* q \to q$	$\Delta q + \Delta\bar{q}$ Δg (NLO)
 SIDIS $\ell^{\pm} + N \to \ell^{\pm} h + X$	$A_1^h \approx \frac{\sum_q \Delta q(x) \otimes D_q^h(z)}{\sum_{q'} q'(x) \otimes D_{q'}^h(z)}$	$\gamma^* q \to q$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$ Δg (NLO)
	$A_{LL}^{\gamma N \to D_0 X} \approx \frac{\Delta g \otimes D_c^{D^0}(z)}{g(x) \otimes D_c^{D^0}(z)}$	$\gamma^* g \to c\bar{c}$	Δg
 *pp* $N_1 + N_2 \to \{j, W^{\pm}, h\} + X$	$A_{LL}^{jet} \approx \frac{\sum_{a,b=q,\bar{q},g} \Delta f_a(x_1) \otimes \Delta f_b(x_2)}{\sum_{a,b,c=q,\bar{q},g} f_a(x_1) \otimes f_b(x_2)}$	$gg \to qg$ $qg \to qg$	Δg
	$A_L^{W^+} \approx \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$	$u_L \bar{d}_R \to W^+$ $d_L \bar{u}_R \to W^+$	Δu $\Delta\bar{u}$ Δd $\Delta\bar{d}$
	$A_{LL}^h \approx \frac{\sum_{a,b,c=q,\bar{q},g} \Delta f_a(x_1) \otimes \Delta f_b(x_2) \otimes D_c^h(z)}{\sum_{a,b,c=q,\bar{q},g} f_a(x_1) \otimes f_b(x_2) \otimes D_c^h(z)}$	$gg \to qg$ $qg \to qg$	Δg

Factorisation and evolution

1. Collinear factorisation of physical observables $\mathcal{O}_I$
   - a convolution between coefficient functions $\mathcal{C}_{If}(x,\alpha_s(\mu^2))$ and PDFs $f(x,\mu^2)$

$$\mathcal{O}_I = \sum_{f=q,\bar{q},g} \mathcal{C}_{If}(y,\alpha_s(\mu^2)) \otimes f(y,\mu^2) + \text{p.s. corrections} \qquad f\otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$

   - coefficient functions allow for a perturbative expansion in terms of $a_s = \alpha_s/(4\pi)$

$$\mathcal{C}_{If}(y,\alpha_s) = \sum_{k=0} a_s^k C_{If}^{(k)}(y) \quad \begin{cases} \text{DIS (up to NNLO)} & \text{[NPB 417 (1994) 61]} \\ \text{SIDIS (up to NNLO)} & \text{[PRD 104 (2021) 094046]} \\ pp \text{ (up to (N)NLO)} & \begin{cases} \text{[NPB 539 (1999) 455, PRD 70 (2004) 034010]} \\ \text{[PLB 817 (2021) 136333]} \\ \text{[PRD 67 (2003) 054004, ibidem 054005]} \end{cases} \end{cases}$$

2. Evolution of parton distributions
   - a set of $(2n_f+1)$ integro-differential equations, n_f is the number of active flavors

$$\frac{\partial}{\partial \ln\mu^2} f_i(x,\mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} \mathcal{P}_{ji}\left(z,\alpha_s(\mu^2)\right) f_j\left(\frac{x}{z},\mu^2\right)$$

   - with perturbative computable splitting functions

$$\mathcal{P}_{ji}(z,\alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(z) \quad \begin{cases} \text{LO} & \text{[NP B126 (1977) 298]} \\ \text{NLO} & \text{[ZP C70 (1996) 637, PR D54 (1996) 2023]} \\ \text{NNLO} & \text{[NP B889 (2014) 351]} \end{cases}$$

Theoretical constraints

1. Polarized PDFs must lead to positive cross sections
   - at LO, polarized PDFs are bounded by their unpolarized counterparts

$$|\Delta f(x,\mu^2)| \leq f(x,\mu^2)$$

   - beyond LO, other relations hold, but are of limited effect [NP B534 (1998) 277]

2. Polarized PDFs must be integrable
   - *i.e.* require that the axial matrix elements of the nucleon are finite

$$\langle P,S|\bar{\psi}_q\gamma^\mu\gamma_5\psi_q|P,S\rangle \longrightarrow \text{finite for each flavor } q$$

3. Assume SU(2) and SU(3) symmetry
   - relate the octet of axial-vector currents to β-decay of spin-$1/2$ hyperons

$$a_3 = \int_0^1 dx\,\Delta T_3 = 1.2701\pm 0.0025 \qquad a_8 = \int_0^1 dx\,\Delta T_8 = 0.585\pm 0.025$$ [PDG 2014]

$$\Delta T_3 = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d}) \qquad \Delta T_8 = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) - 2(\Delta s + \Delta\bar{s})$$

   - note: violations of SU(3) symmetry are advocated in the literature [ARNPS 53 (2003) 39]

Why helicity PDFs at NNLO?

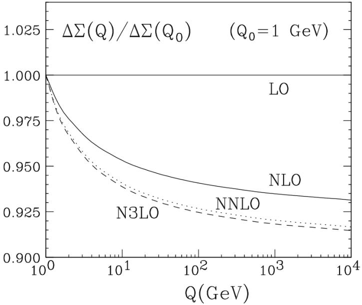
[PRD 99 (2019) 054001]

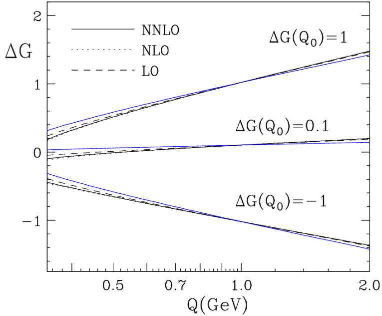
Splitting functions known at NNLO

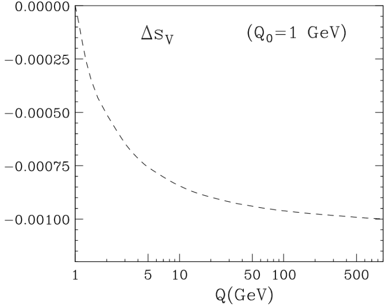
Computation of the evolution of $\Delta\Sigma$ at N^{3}LO and of ΔG at NNLO

"Static" solutions for ΔG at every perturbative order that tend to an asymptotic limit ($\simeq 0.1$)

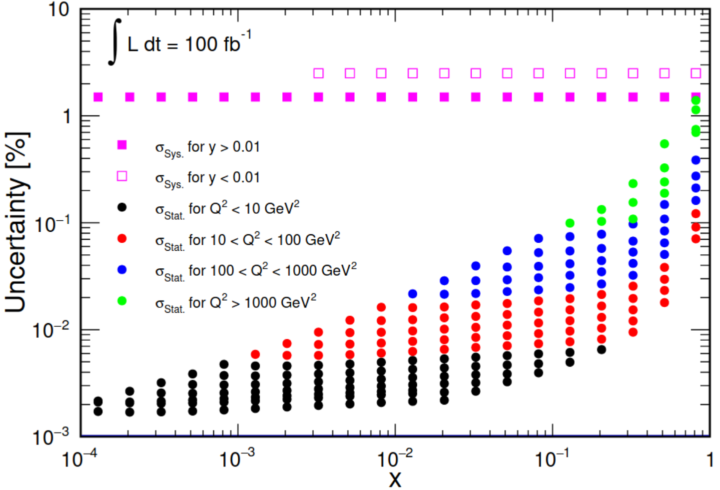
NNLO perturbative evolution predicts $\Delta s - \Delta\bar{s} < 0$, 1% of the total $\Delta s + \Delta\bar{s}$







Why helicity PDFs at NNLO?

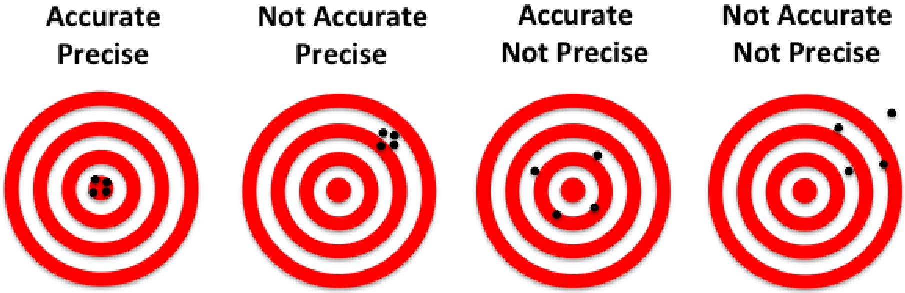


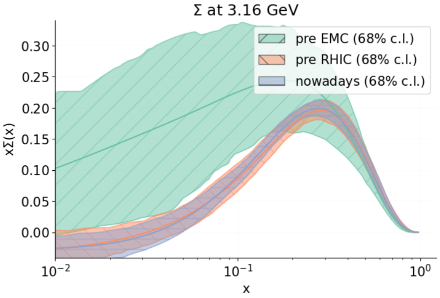
[Figure from the EIC Yellow Report arXiv:2103.05419]

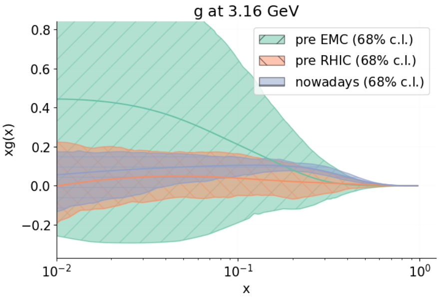
At the EIC, cross sections are expected to be measured with a precision of 1%.

Theoretical predictions must match that precision.

Why helicity PDFs at NNLO?





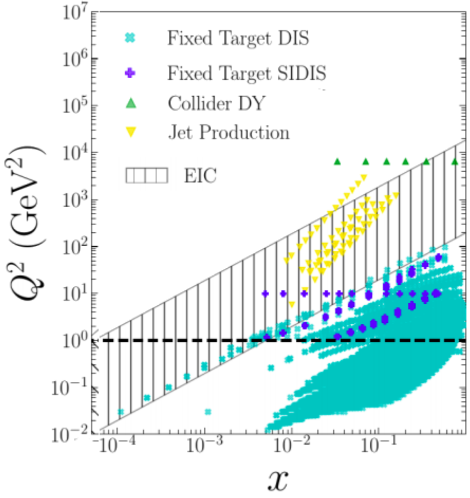


2. MAPpolPDF1.0

In preparation with V. Bertone and A. Chiefa

DISCLAIMER: ALL RESULTS ARE PRELIMINARY

The data set



kinematic cuts:

$Q^2 > 1$ GeV^2 $\quad$ $W^2 > 6.25$ GeV^2

Data set	Obs.	$N_{\rm dat}$	Reference
EMC	g_1^p	10	[NPB 328 (1989) 1]
SMC	g_1^p	12	[PRD D58 (1998) 112001]
	g_1^d	12	[PRD 58 (1998) 112001]
E142	g_1^n	7	[PRD 54, (1996) 6620]
E143	g_1^p	25	[PRD 58 (1998) 112003]
	g_1^d	25	[PRD 58 (1998) 112003]
E154	g_1^n	11	[PRL 79 (1997) 26]
E155	g_1^p	22	[PLB 493 (2000) 19]
	g_1^n	22	[PLB 493 (2000) 19]
COMPASS	g_1^p	15	[PLB 753 (2016) 18]
	g_1^d	17	[PLB 769 (2017) 34]
HERMES	g_1^n	8	[PLB 404 (1997) 383]
	g_1^p	14	[PRD 75 (2007) 012007]
	g_1^d	14	[PRD 75 (2007) 012007]
COMPASS	$A_{1,p,d}^{\pi^+}$	22	[PLB 680 (2009) 217]
	$A_{1,p,d}^{\pi^-}$	22	[PLB 680 (2009) 217]
	$A_{1,p,d}^{K^+}$	22	[PLB 680 (2009) 217]
	$A_{1,p,d}^{K^-}$	22	[PLB 680 (2009) 217]
HERMES	$A_{1,p,d}^{\pi^+}$	18	[PRD 99 (2019) 112001]
	$A_{1,p,d}^{\pi^-}$	18	[PRD 99 (2019) 112001]
	$A_{1,d}^{K^+}$	9	[PRD 99 (2019) 112001]
	$A_{1,d}^{K^-}$	9	[PRD 99 (2019) 112001]
Total		**356**	

The methodology

Monte Carlo representation of data uncertainties into PDF uncertainties ($N_{\rm rep}=150$)

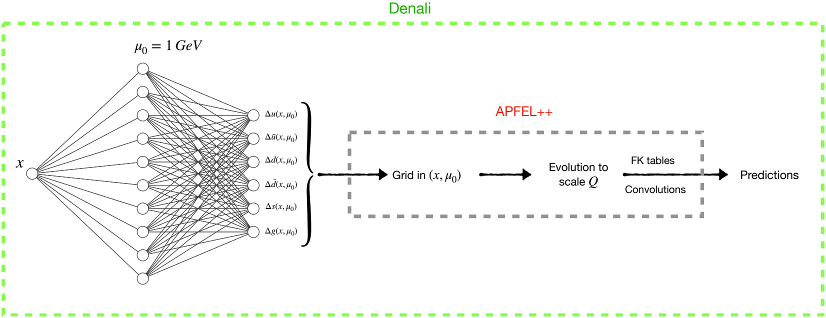
Neural Network parametrisation with form $x\Delta f(x,\mu_0)=\mathcal{N}_i(x;\boldsymbol{\theta})-\mathcal{N}_i(1;\boldsymbol{\theta})$

Parametrisation basis at $\mu_0=1$ GeV: $\{\Delta u,\Delta\bar{u},\Delta d,\Delta\bar{d},\Delta s=\Delta\bar{s},g\}$

For each helicity PDF replica, input unpolarised PDFs and FFs are random replicas from NNPDF3.1 [EPJ C77 (2017) 663] and MAPFF1.0 [PL8 34 (2022) 137456]

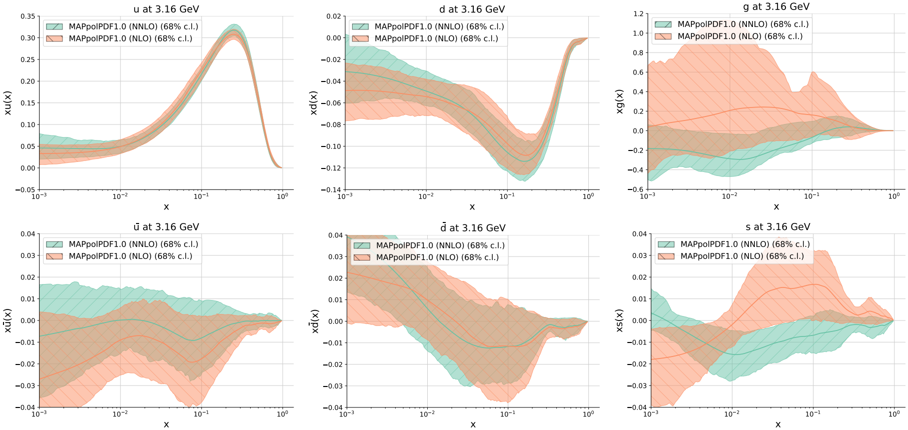
Constraints on a_3 and a_8 implemented through pseudodata

The last layer is bound to the positivity inequality by construction



[Image credit: A. Chiefa]

Impact of perturbative corrections



$\chi^2/N_{\rm dat} = 0.79$ (NNLO) $\chi^2/N_{\rm dat} = 0.67$ (NLO)

Deterioration of fit quality at NNLO — under investigation

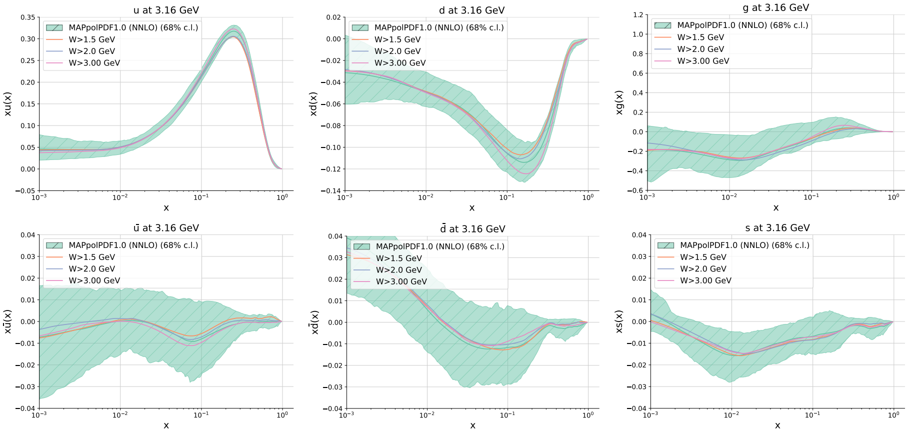
(improvement seen for DIS and SIDIS data separately)

(dependence on kinematic cuts?)

(similar situation encountered in MAPFF1.0 for $\pi^\pm$)

Impact of perturbative corrections generally moderate, except for Δg and Δs

Impact of W cut

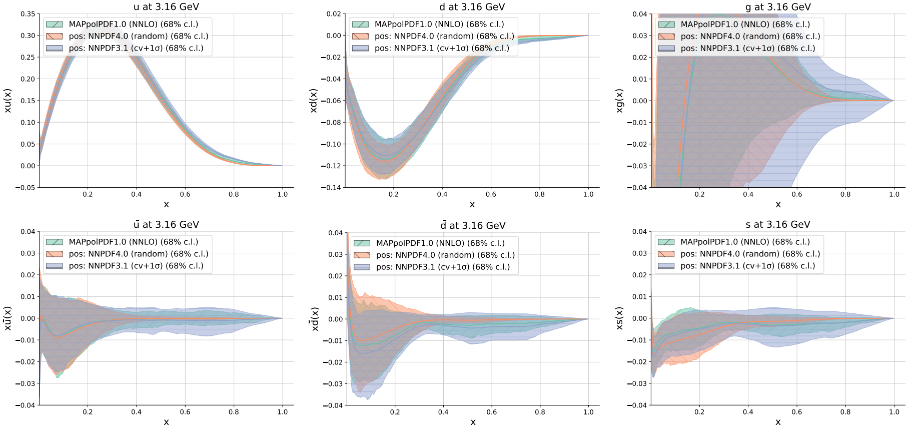


$$\chi^2/N_{\rm dat} = 0.79\ (W > 2.5\ \text{GeV}) \qquad \chi^2/N_{\rm dat} = 0.97\ (W > 1.0\ \text{GeV})$$
$$\chi^2/N_{\rm dat} = 0.89\ (W > 2.0\ \text{GeV}) \qquad \chi^2/N_{\rm dat} = 0.76\ (W > 3.0\ \text{GeV})$$

Improvement of fit quality as the cut on W is raised
(check what happens at NLO)

The impact of the W kinematic cut on helicity PDFs remains however moderate

Impact of positivity PDF set



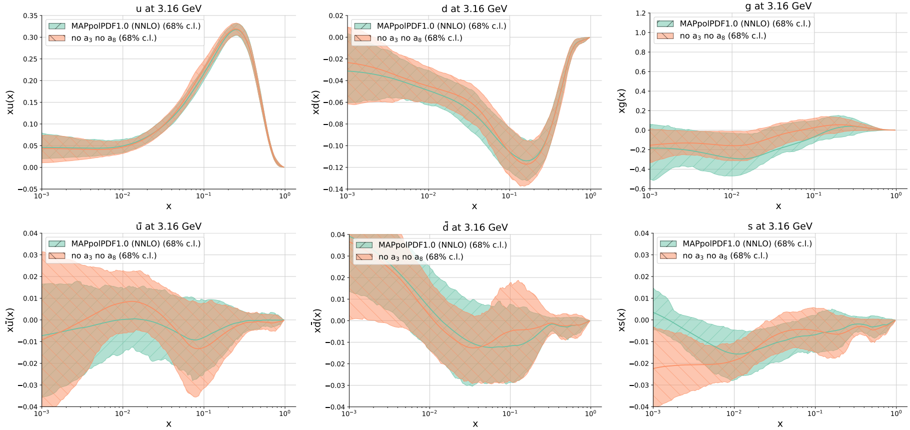
$\chi^2/N_{\rm dat} = 0.79$ (rand NNPDF3.1 PDFs) $\chi^2/N_{\rm dat} = 0.81$ (rand NNPDF4.0 PDFs)

$\chi^2/N_{\rm dat} = 0.80$ (fixed NNPDF3.1 PDFs cv$+1\sigma$)

Moderate impact from unpolarised PDF set used to enforce positivity on helicity PDFs
(the more precise the unpolarised PDF set, the larger the impact)

Significant impact from taking unpolarised PDF replicas at random or not
(the unpolarised PDF uncertainty is counteracted by correlations)

Impact of constraints on a_3 and a_8



$\chi^2/N_{\rm dat} = 0.79$ (w/ a_3 and a_8) $\qquad \chi^2/N_{\rm dat} = 0.77$ (w/o a_3 and a_8)

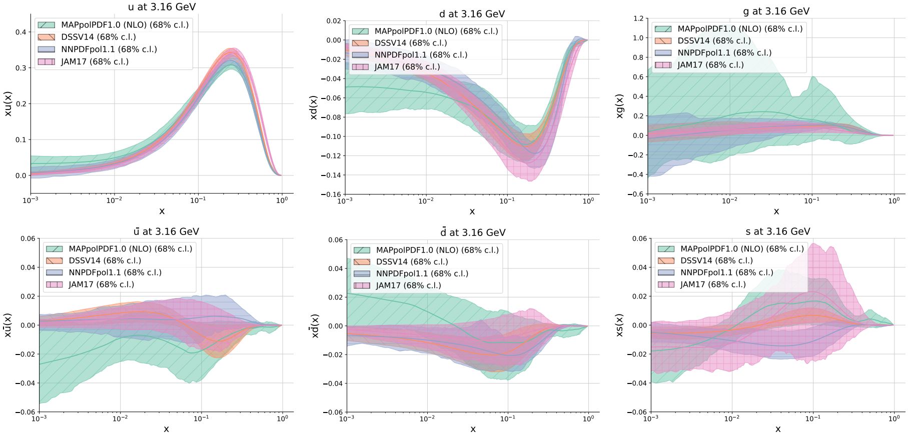
Impact on helicity PDFs from constraining a_3 and a_8 is small at NNLO

The moments of T_3 and T_8 distributions are recovered with large uncertainties

$x_{\rm min} = 10^{-5}$: $a_3 = 1.197 \pm 0.415$ (1.268 ± 0.004) $\qquad a_8 = 1.824 \pm 1.041$ (0.594 ± 0.035)

The same may not hold at NLO (check in progress)

Comparison with other helicity PDF sets



DSSV14 [PRD 100 (2019) 114027] NNPDFpol1.1 [NPB 887 (2014) 276] JAM17 [PRL 119 (2017) 132001]

DSSV14 and NNPDFpol1.1 include jet production data

DSSV14, JAM17 and MAPpolPDF1.0 include SIDIS data

JAM17 simultaneously determine helicity PDFs and FFs

MAPpolPDF1.0 is in broad agreement with all the other PDF sets

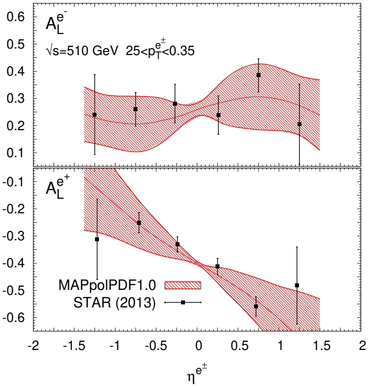
Some phenomenological considerations

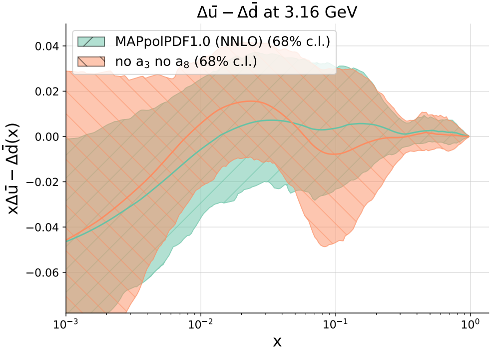
The proton spin content

$\Delta\Sigma = 0.322 \pm 0.076$ $\Delta G = -0.960 \pm 1.607$ NNLO, Q^2=10 GeV^2

$\Delta\Sigma = 0.352 \pm 0.099$ $\Delta G = +1.416 \pm 2.293$ NLO, Q^2=10 GeV^2

The polarised sea quark asymmetry and comparison with polarised Drell-Yan production





cfr. [PRD 99 (2019) 051102]

3. To conclude

Summary and outlook

MAPpolPDF1.0 is going to be the first ever NNLO global helicity PDF determination

The data set is made of inclusive and semi-inclusive DIS so far

We plan to extend it to Drell-Yan production in polarised proton-proton collisions

Approximate inclusion of jet production data is under consideration

Perturbative corrections are generally small, except for Δg and Δs

Imposing (or not) constraints from positivity and/or decay of baryons plays some role

MAPpolPDF1.0 is in broad agreement with other helicity PDF sets

Summary and outlook

MAPpolPDF1.0 is going to be the first ever NNLO global helicity PDF determination

The data set is made of inclusive and semi-inclusive DIS so far

We plan to extend it to Drell-Yan production in polarised proton-proton collisions

Approximate inclusion of jet production data is under consideration

Perturbative corrections are generally small, except for Δg and Δs

Imposing (or not) constraints from positivity and/or decay of baryons plays some role

MAPpolPDF1.0 is in broad agreement with other helicity PDF sets

Thank you