Sar WorS 2023, June 7, 2023, Pula, Italy

Transverse SSA in inclusive eN collisions at the EIC

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in collaboration with Daniel Rein, Patrick Tollkühn, Werner Vogelsang

What is the final goal?

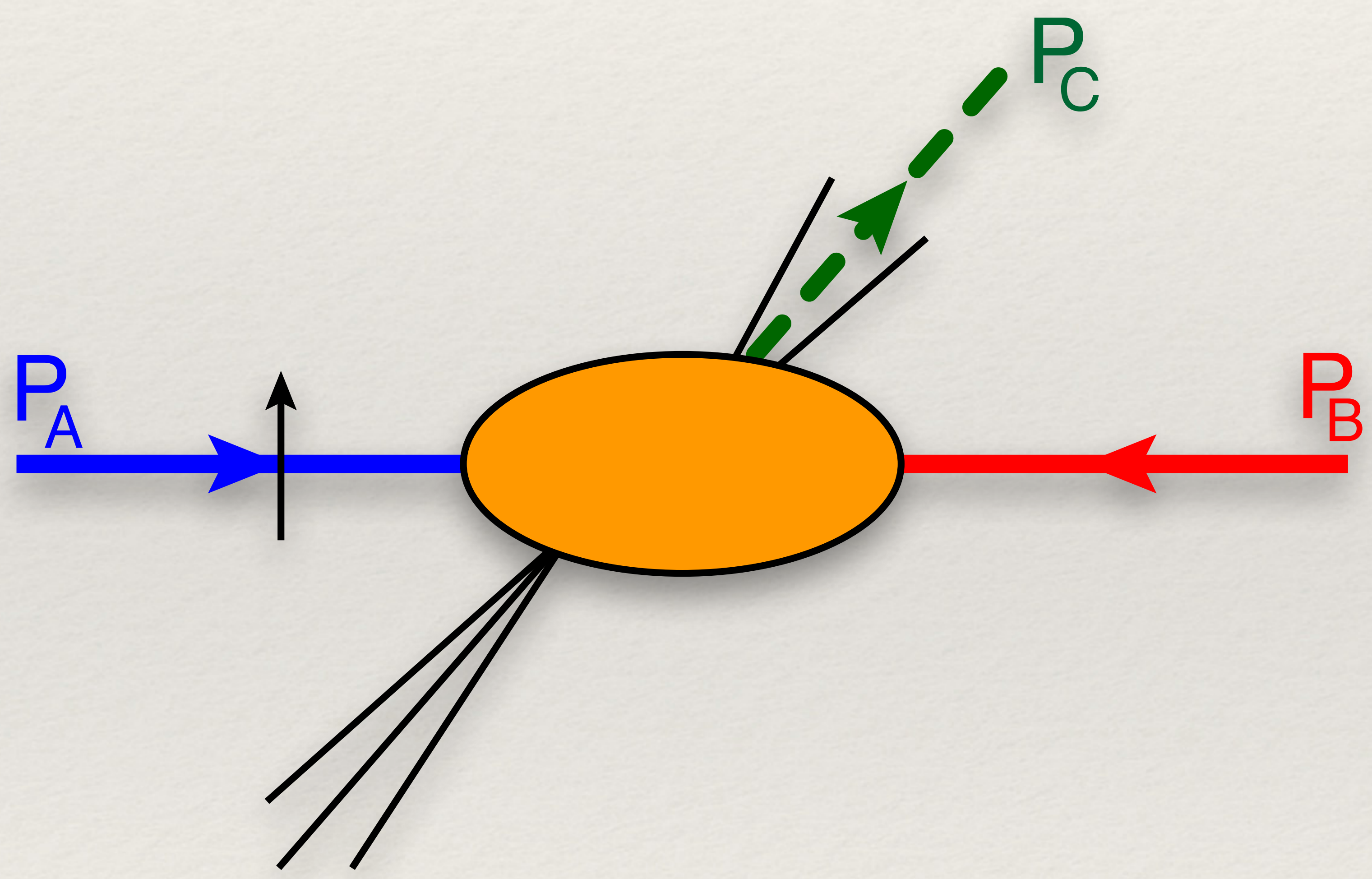
Transverse Single (Double) Spin Asymmetries
in single-inclusive processes in collinear pQCD

$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

or

$$\Delta\sigma_T = \frac{\sigma^{\rightarrow\uparrow} - \sigma^{\rightarrow\downarrow}}{\sigma^{\rightarrow\uparrow} + \sigma^{\rightarrow\downarrow}}$$

$$P_A^{\uparrow} + P_B \to P_C + X$$



Experimental data

Polarized proton collisions at RHIC (STAR, PHENIX, BRAHMS)

$$p^{\uparrow}p \to (\pi,\ \text{jet}\,, \gamma, l, \Lambda, J/\psi, \ldots)X$$

Theory: LO in pQCD

[Koike, Yoshida, Qiu, Metz, Pitonyak, Kang, …]

Polarized eN collisions (HERMES, JLab, COMPASS, EIC)

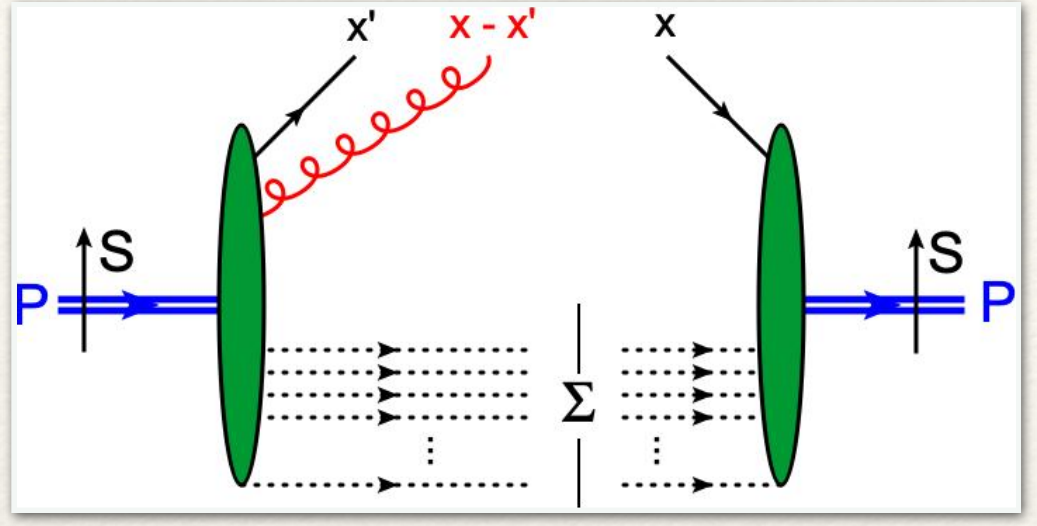
$$l^{(\rightarrow)}N^{\uparrow} \to (l, \pi,\ \text{jet}\,, \gamma, \Lambda, J/\psi, \ldots)X$$

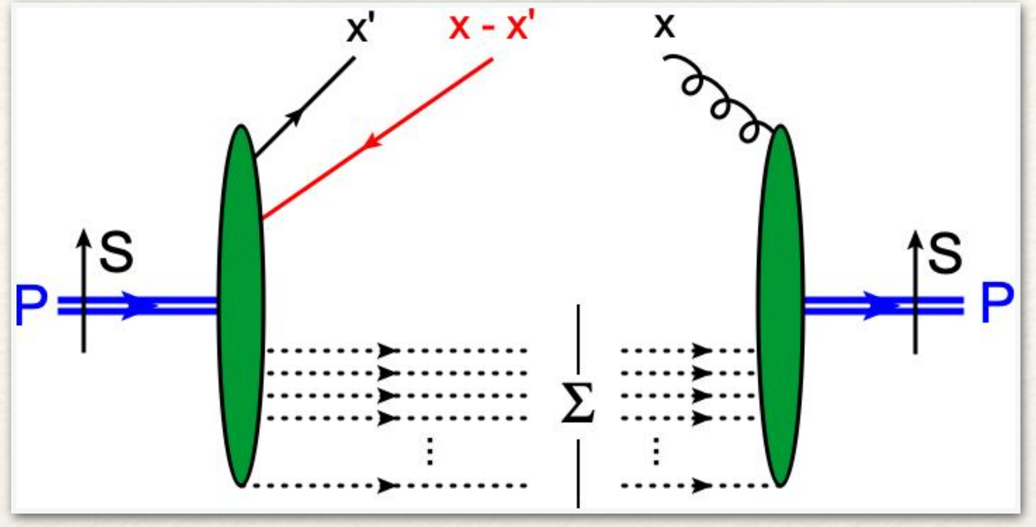
Theory: LO , some NLO

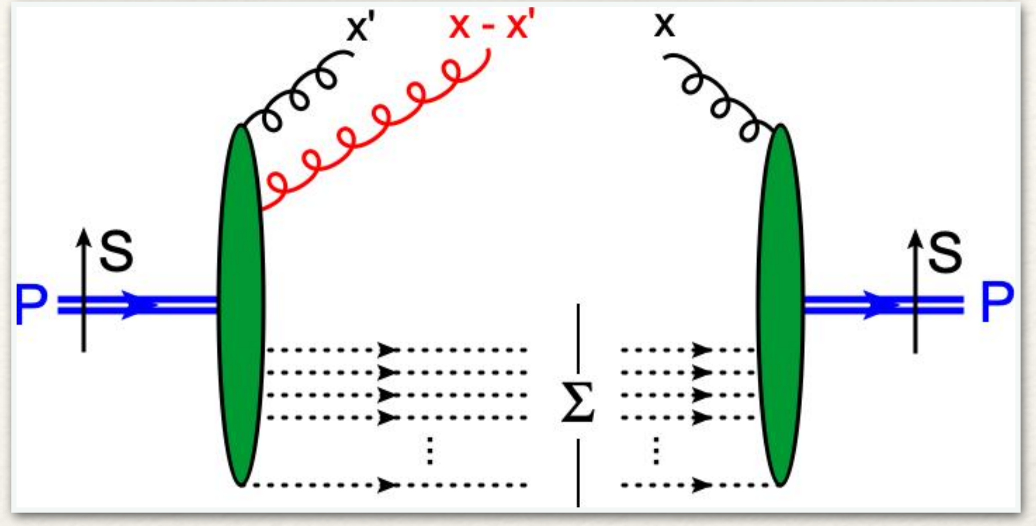
Goal: global analysis (at NLO) (one day…)

What do we expect to learn (about the nucleon)?

Interference effect of non-valence nucleon LF wave functions







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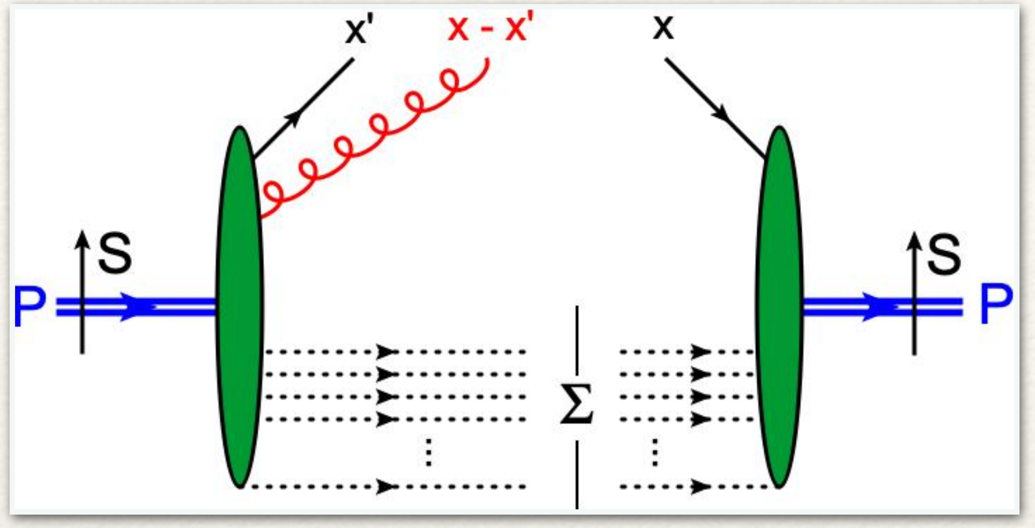

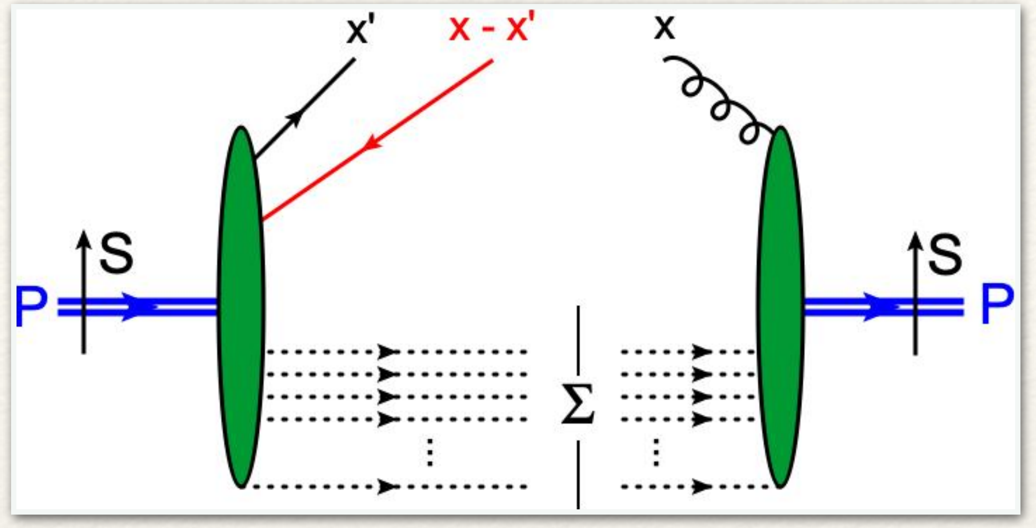

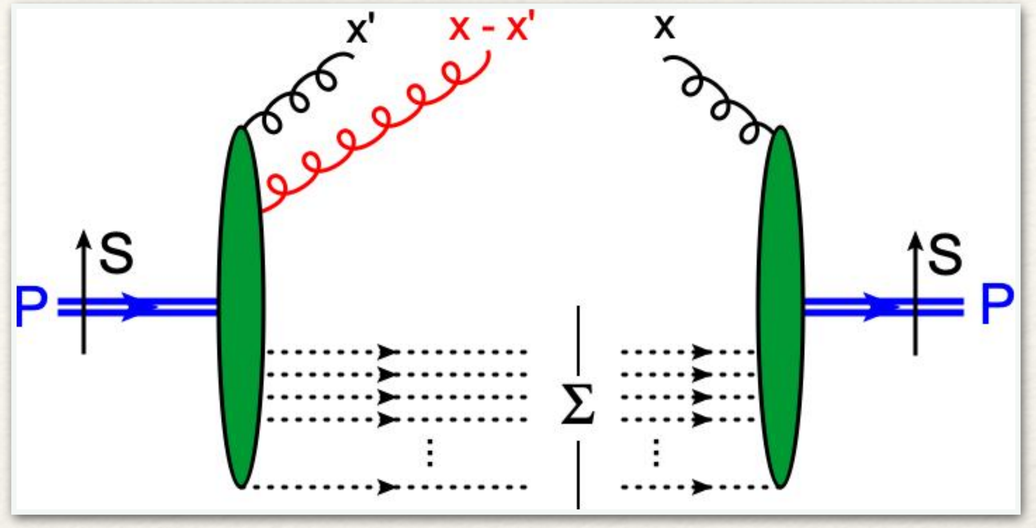

(Chiral-even) Quark - Gluon - Quark correlation functions

$$2M\, i\epsilon^{Pn\rho S} F^q_{FT}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} \mathrm{e}^{i\lambda x'} \mathrm{e}^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

$$2M\, S_T^\rho G^q_{FT}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} \mathrm{e}^{i\lambda x'} \mathrm{e}^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\gamma_5\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

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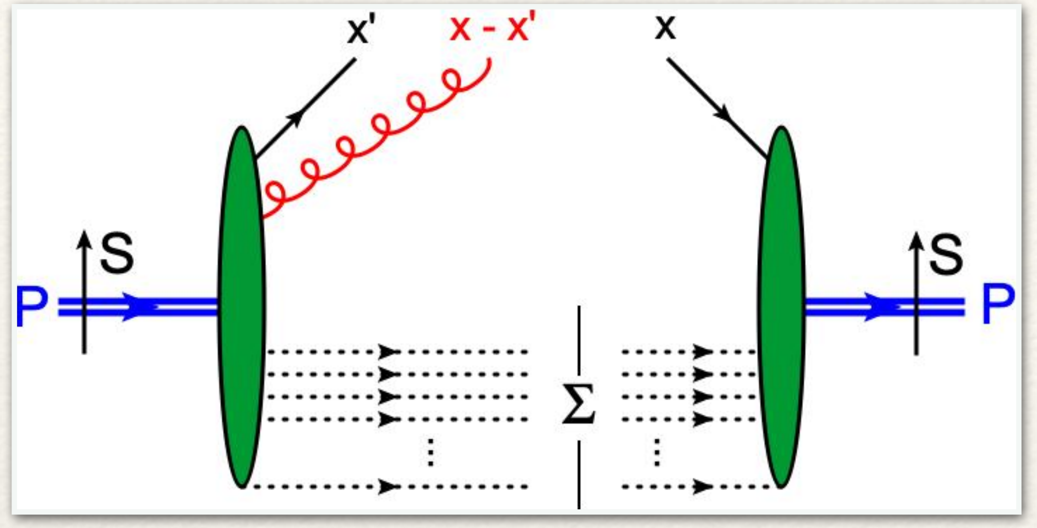

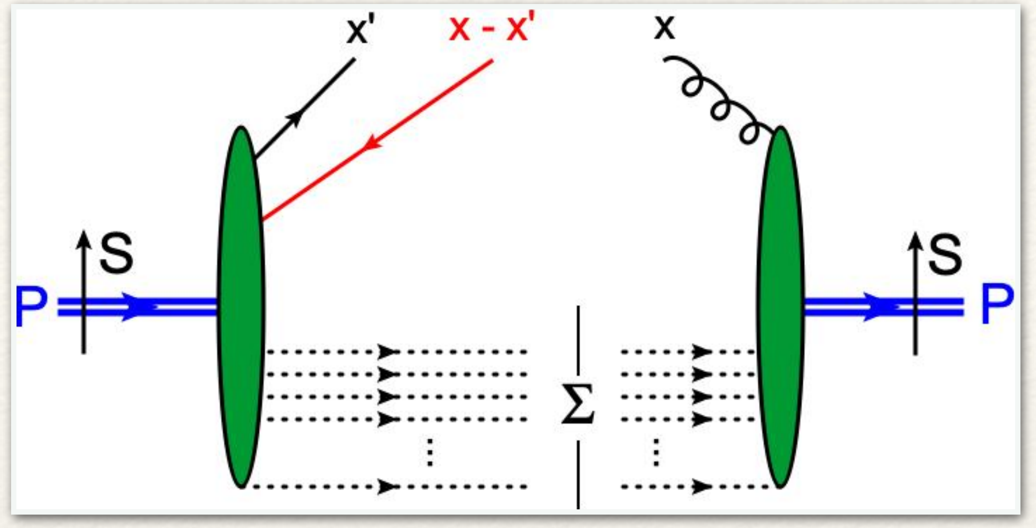

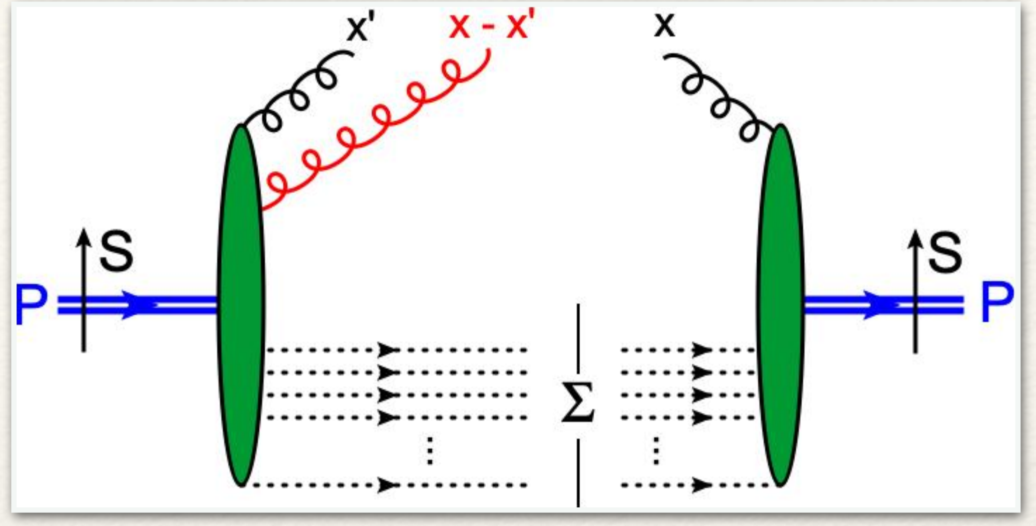

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Why relevant?

→ collinear pQCD: they generate transverse SSA

→ dynamical information: color Lorentz force [M. Burkardt]

→ TMD physics: large transverse momentum behavior of Sivers, Boer-Mulder, worm gear function & their evolution

What do we know about QGQ correlation functions?

Support properties $-1 \leq x, x' \leq 1 \quad |x - x'| \leq 1$ and possibly continuous

Symmetry
$$F^q_{FT}(x, x') = +F^q_{FT}(x', x)$$
$$G^q_{FT}(x, x') = -G^q_{FT}(x', x)$$

charge conjugation
$$F^{\bar{q}}_{FT}(x, x') = +F^q_{FT}(-x, -x')$$
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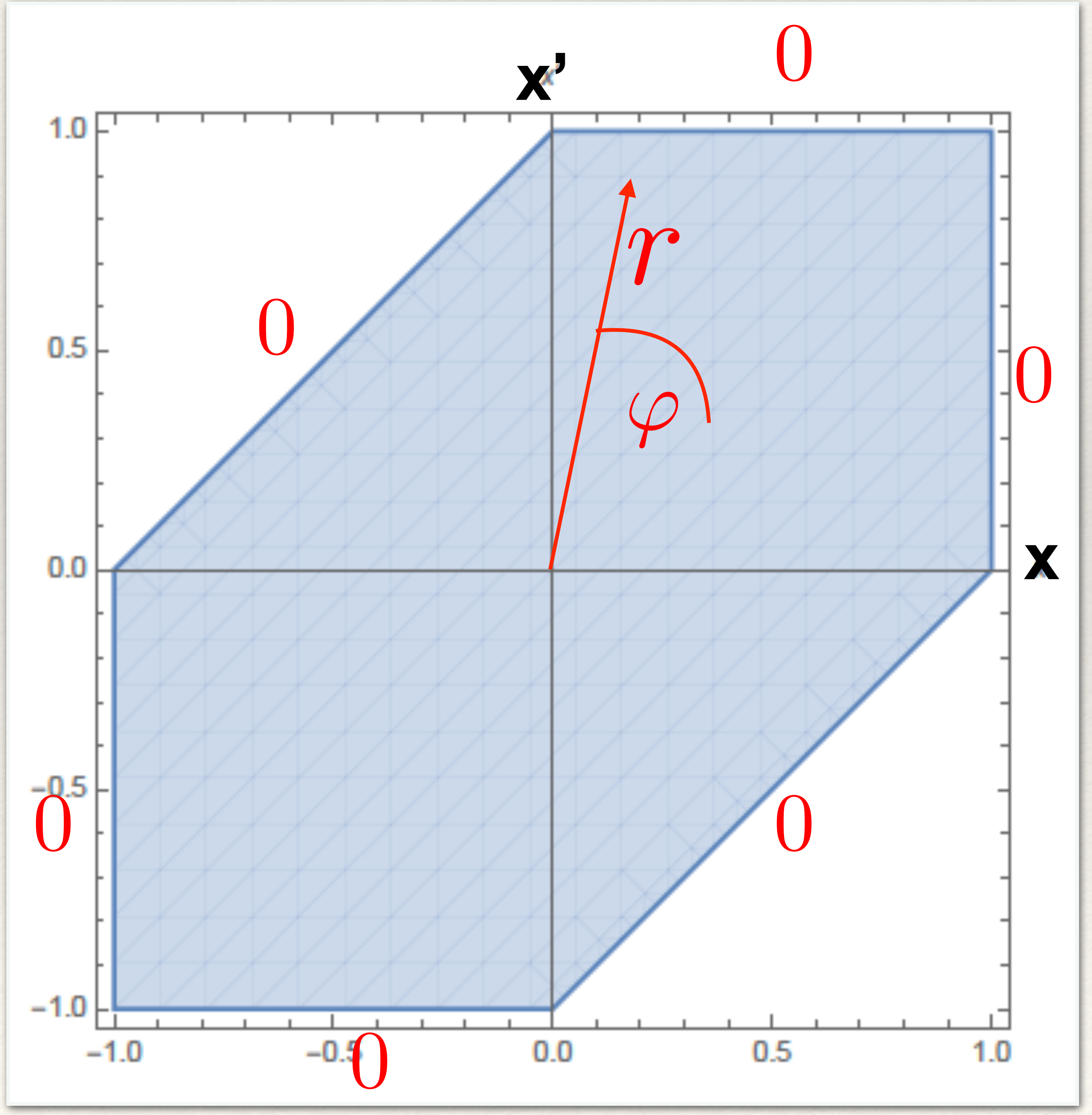

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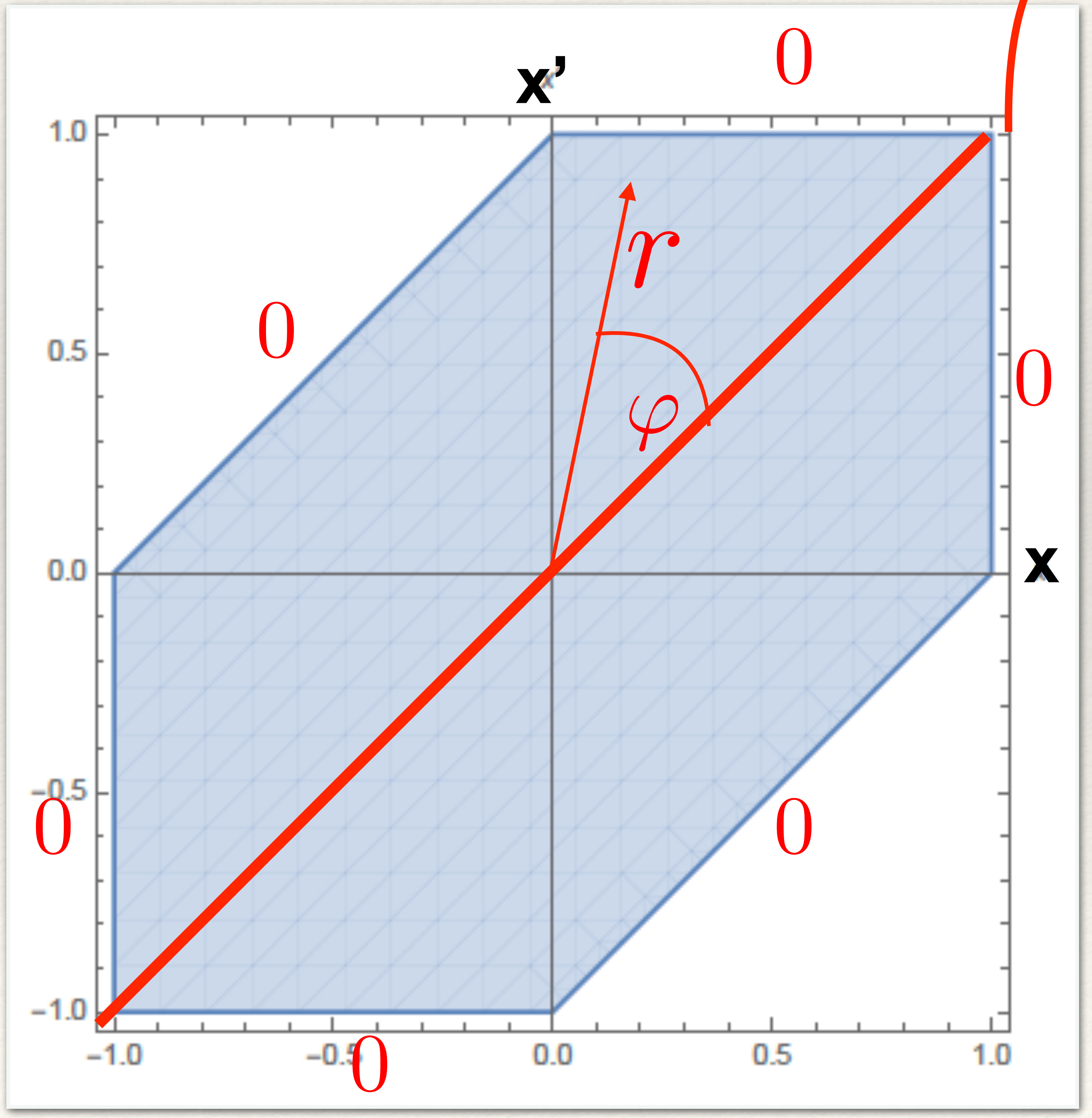
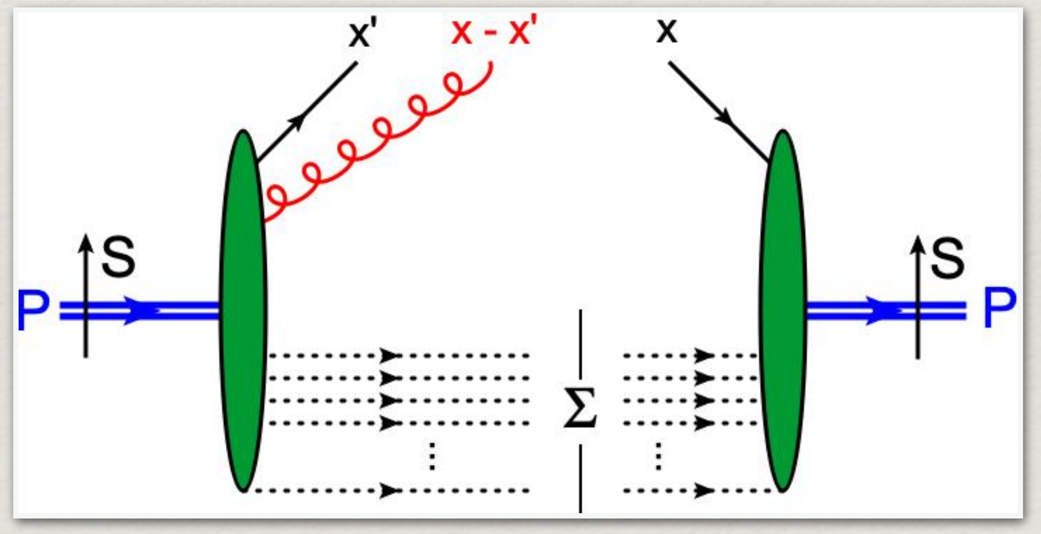
"Soft gluon pole" (SGP)

$$\pi F^q_{FT}(x,x) = f_{1T}^{\perp(1),q}(x)$$
$$G^q_{FT}(x,x) = 0$$





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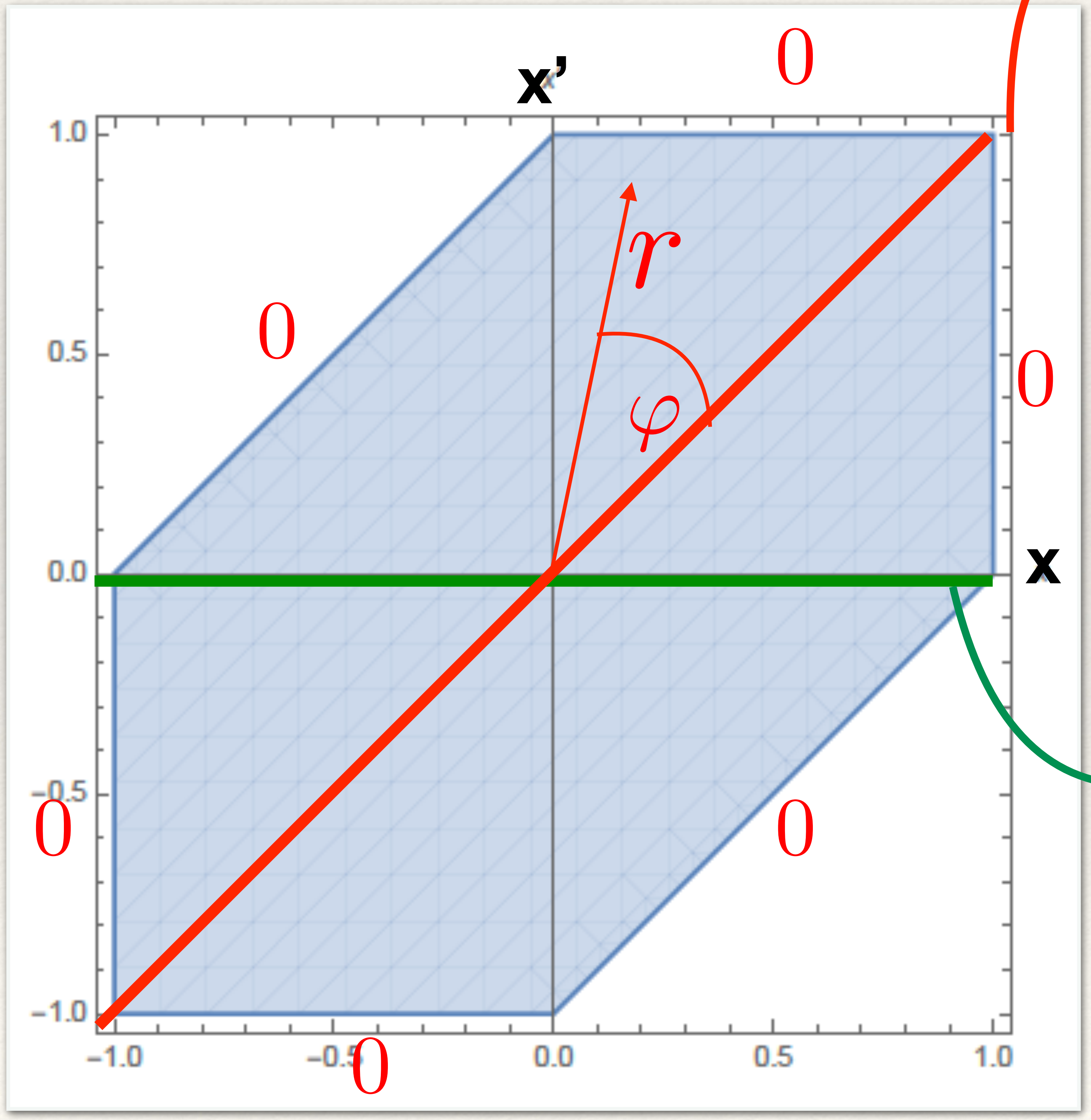

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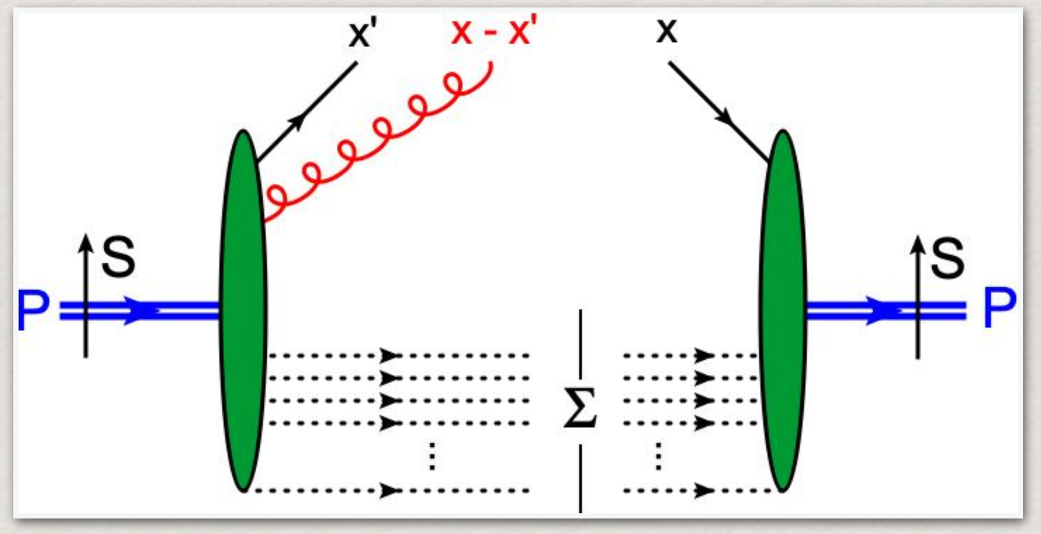

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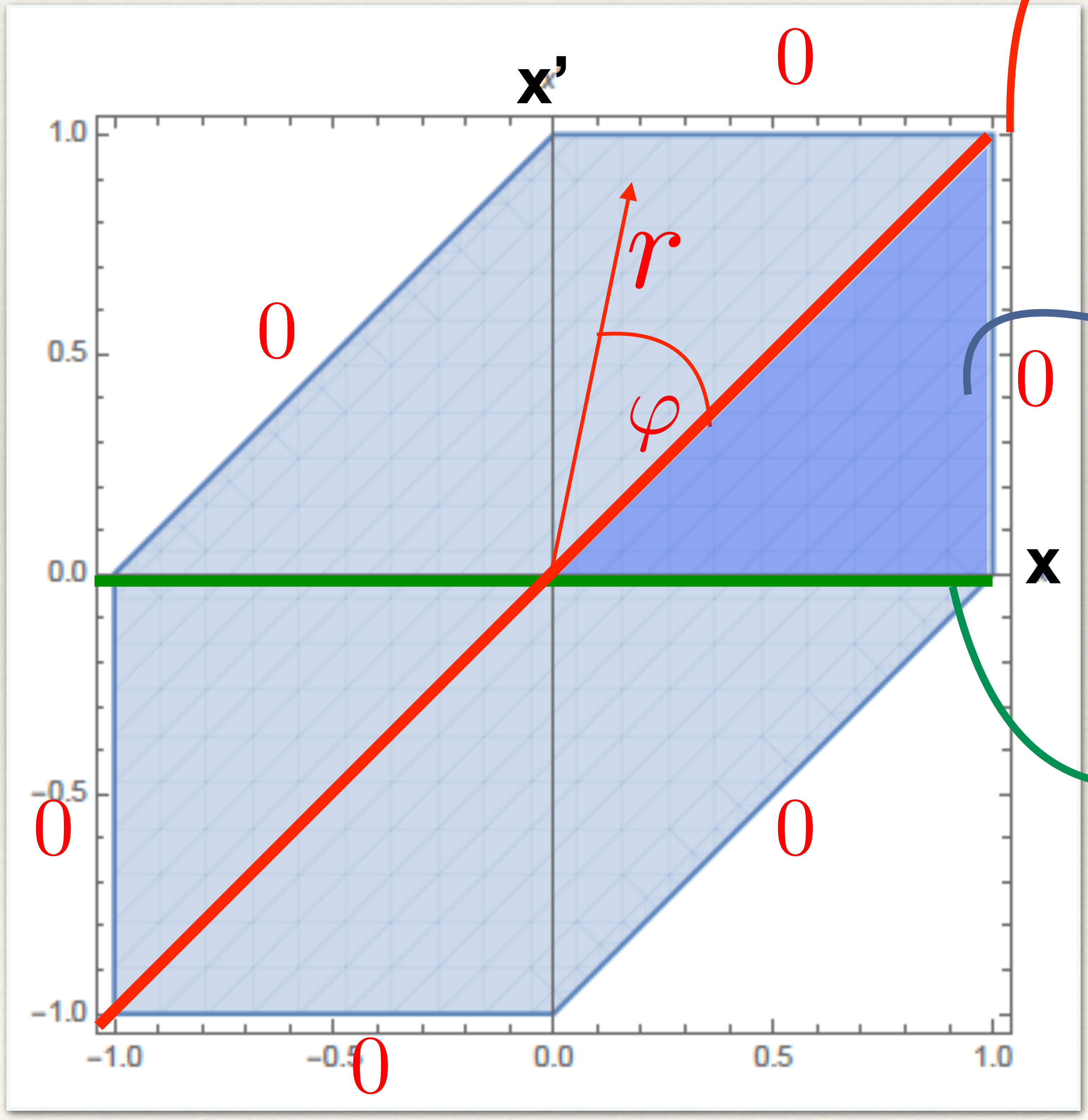

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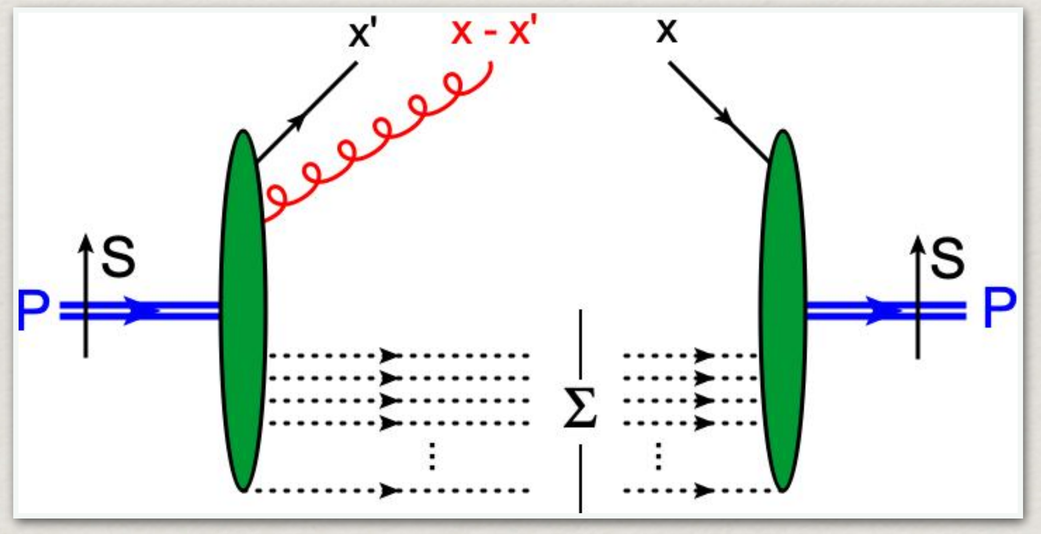

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Antiquark-Quark -Gluon

$$F_{FT}(-x', x - x') = ?$$
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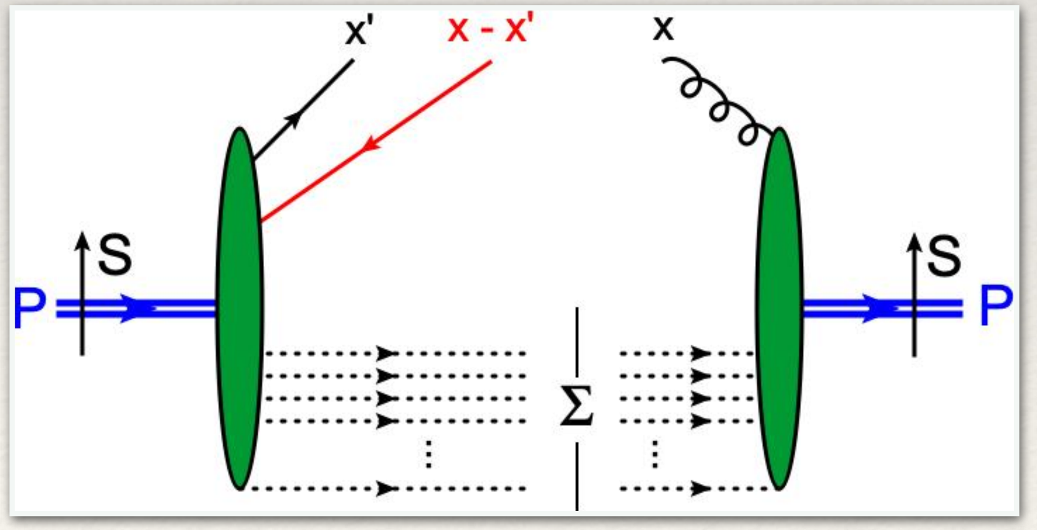

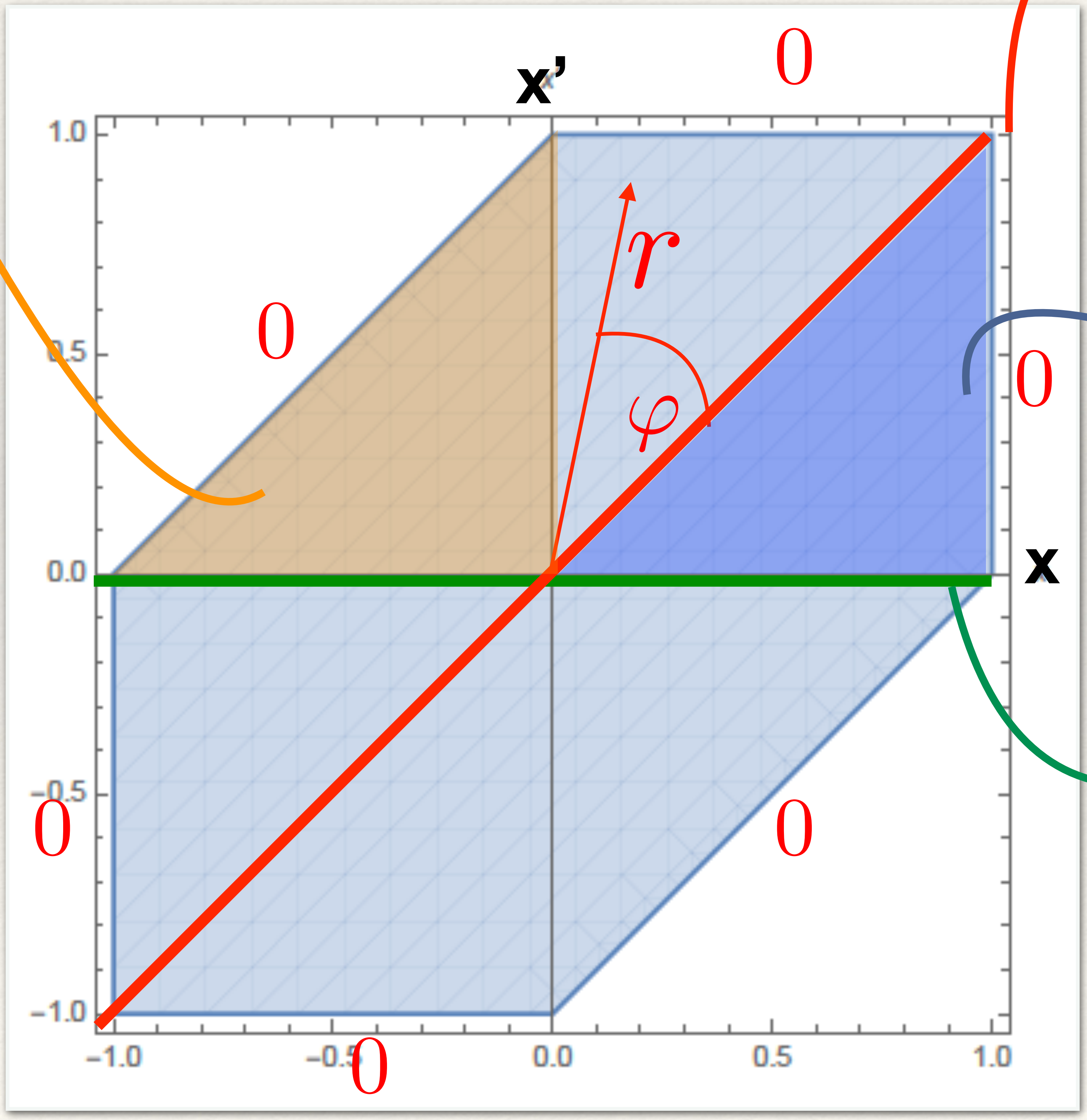

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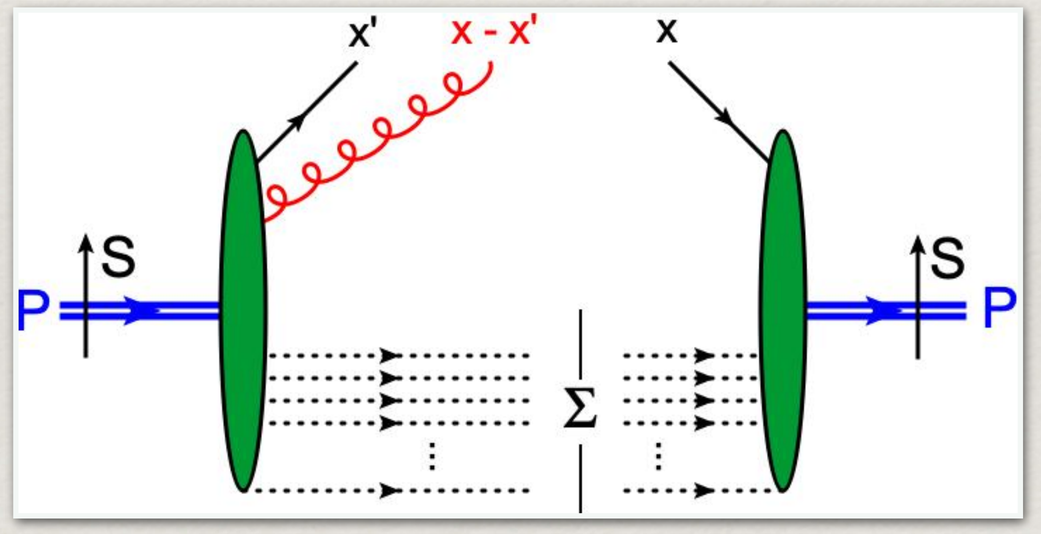

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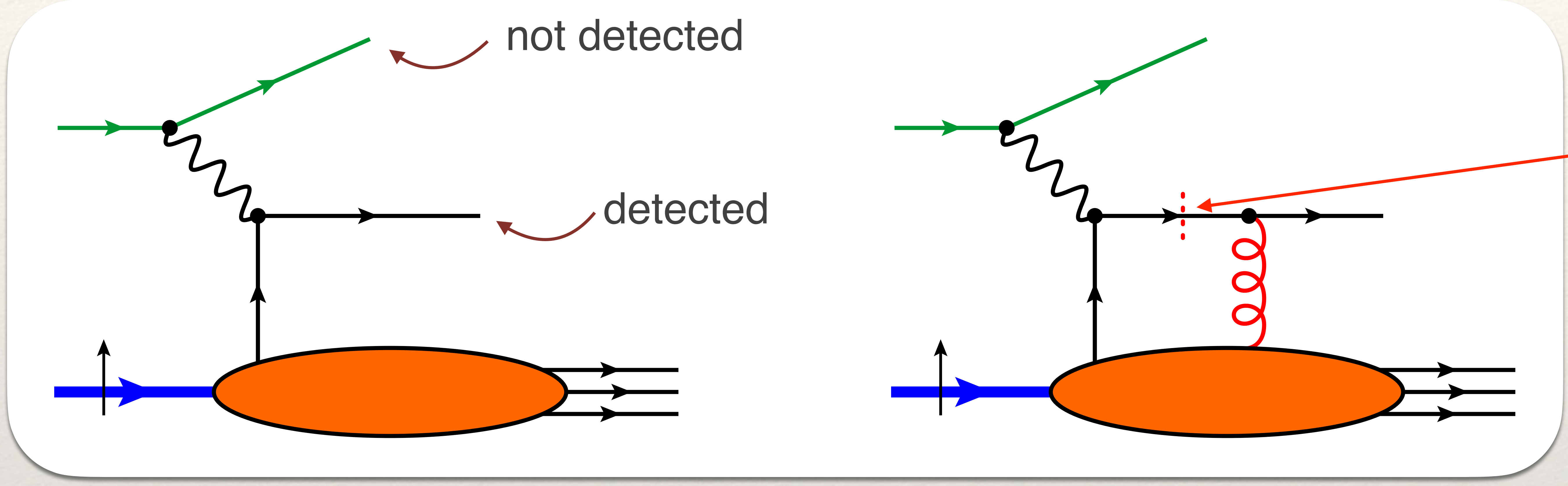
Inclusive π (or jet) production in l+N collisions

How do QGQ correlations generate an SSA?

Example: Single-inclusive jet production e $N^{\uparrow}$ → jet X

[Gamberg, Kang, Metz, Pitonyak, Prokudin; Kanazawa, Koike, Metz, Pitonyak, MS]

Simple LO diagrams



Soft gluon pole

$$\frac{1}{x - x' + i\epsilon} = \frac{\mathcal{P}}{x - x'} - i\pi\delta(x - x')$$

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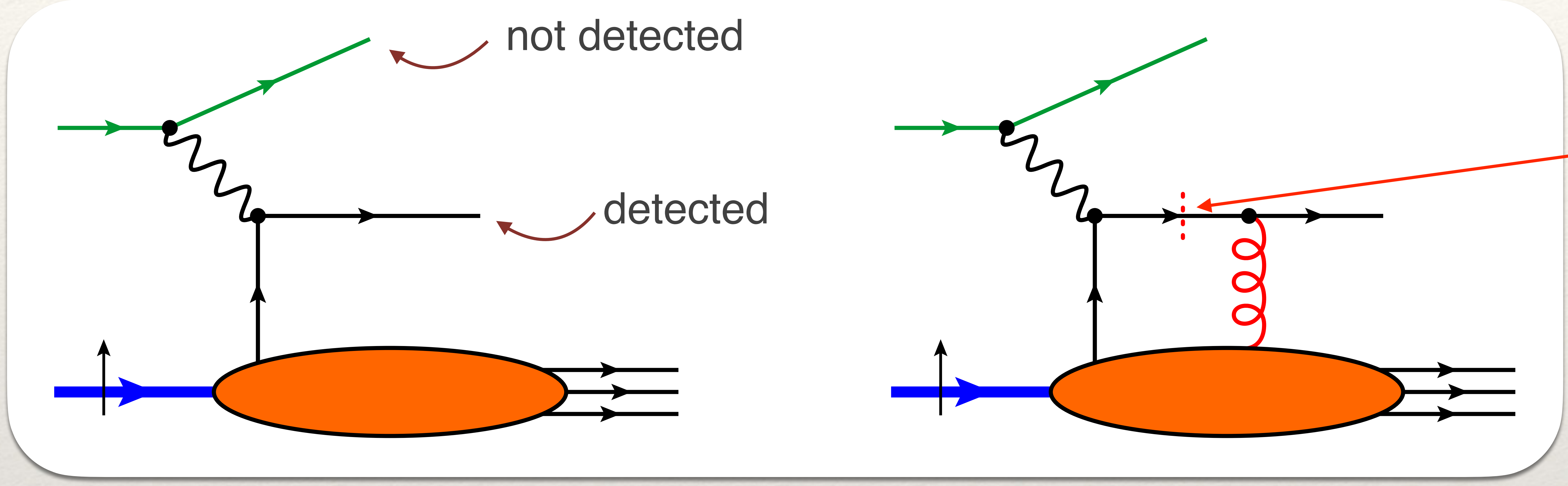


Soft gluon pole

$$\frac{1}{x - x' + i\epsilon} = \frac{\mathcal{P}}{x - x'} - i\pi\delta(x - x')$$

$$A_N \propto \left(1 - x\frac{d}{dx}\right) F^q_{FT}(x,x)$$

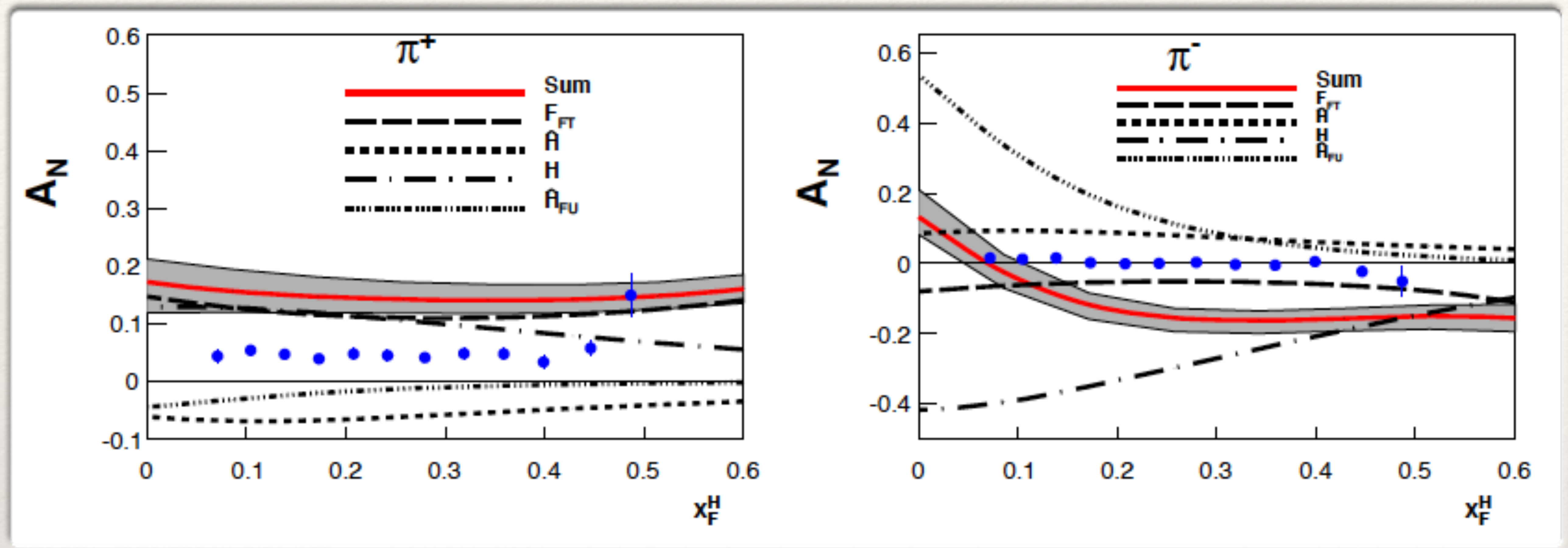
SSA generated by soft-gluon pole only

Feasible at a future EIC, NLO corrections might be large

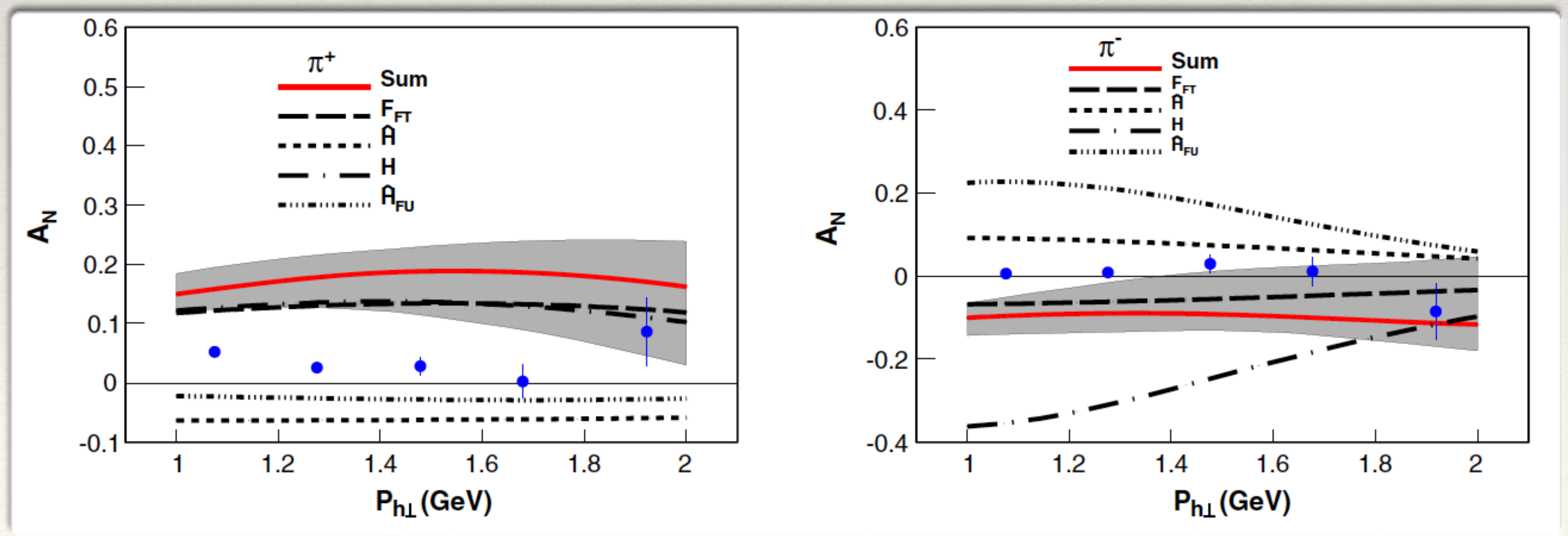
Numerical estimate of the transverse SSA at LO:

[Gamberg, Kang, Metz, Pitonyak, Prokudin; 2014]

x_F - dependence:



P_{hT}- dependence:



Experimental Input:

- Precise data from HERMES & JLab12 for π - production

Theoretical Input:

- Sivers funct. from SIDIS
- Transversity from SIDIS
- Collins funct. from e^+e^- & SIDIS
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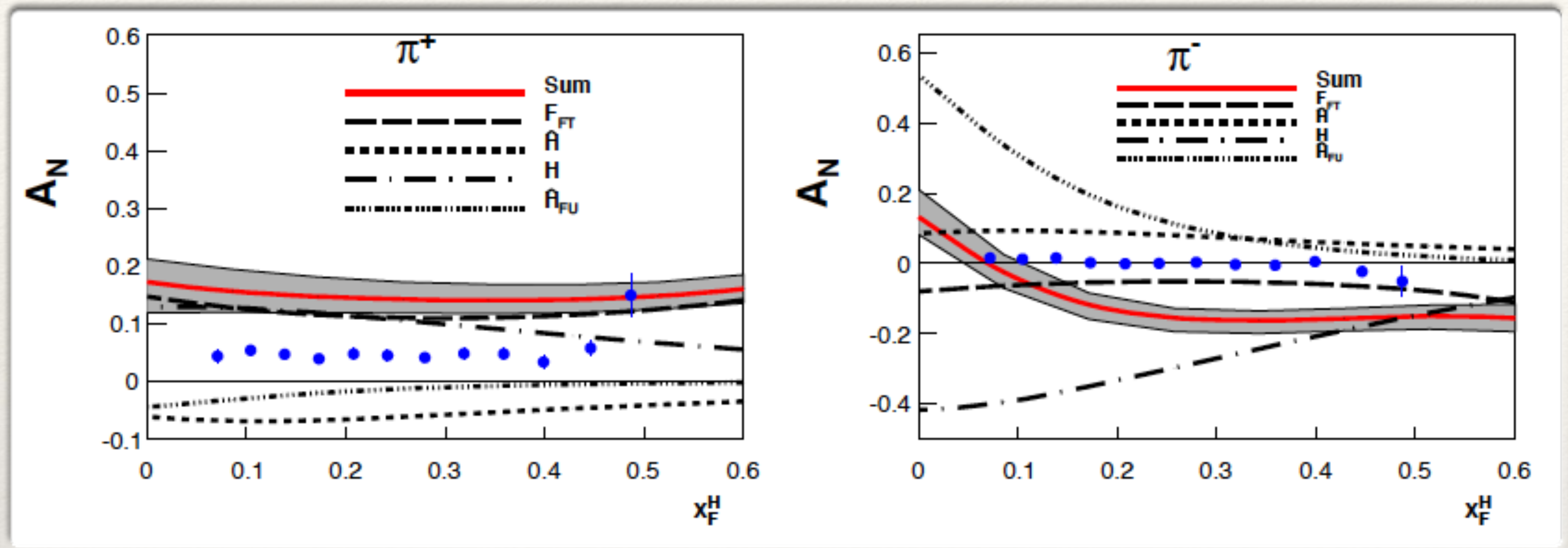

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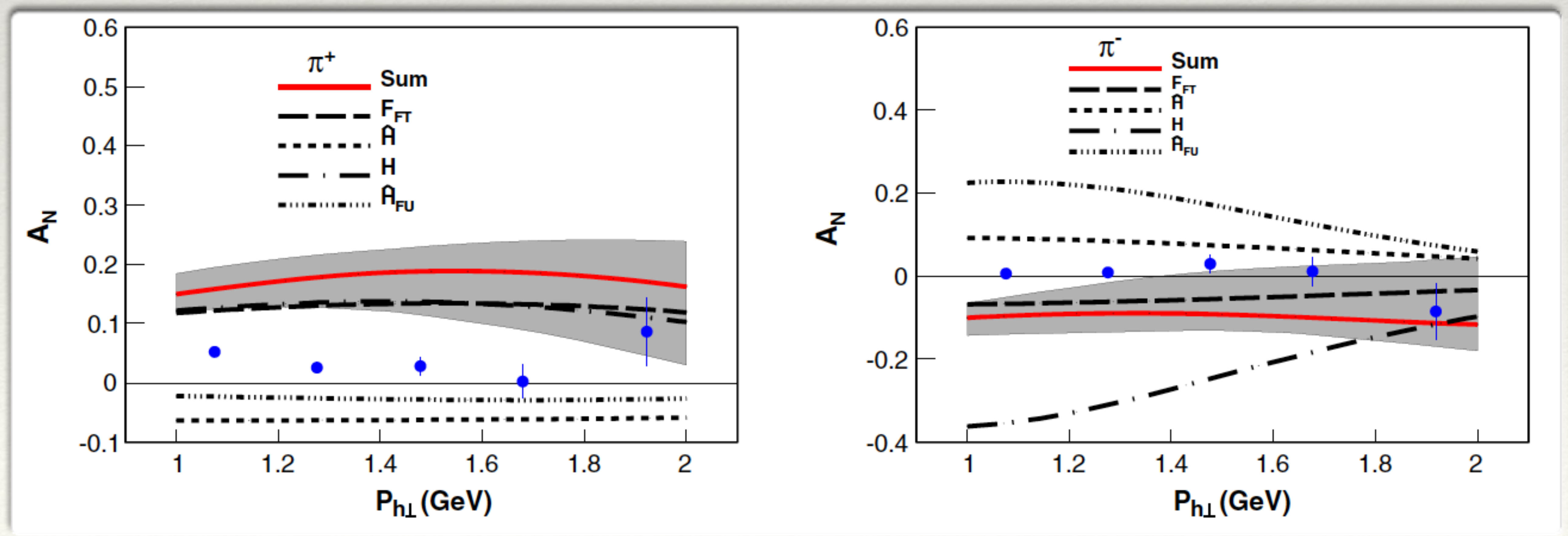

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LO input typically overshoots the data → NLO?

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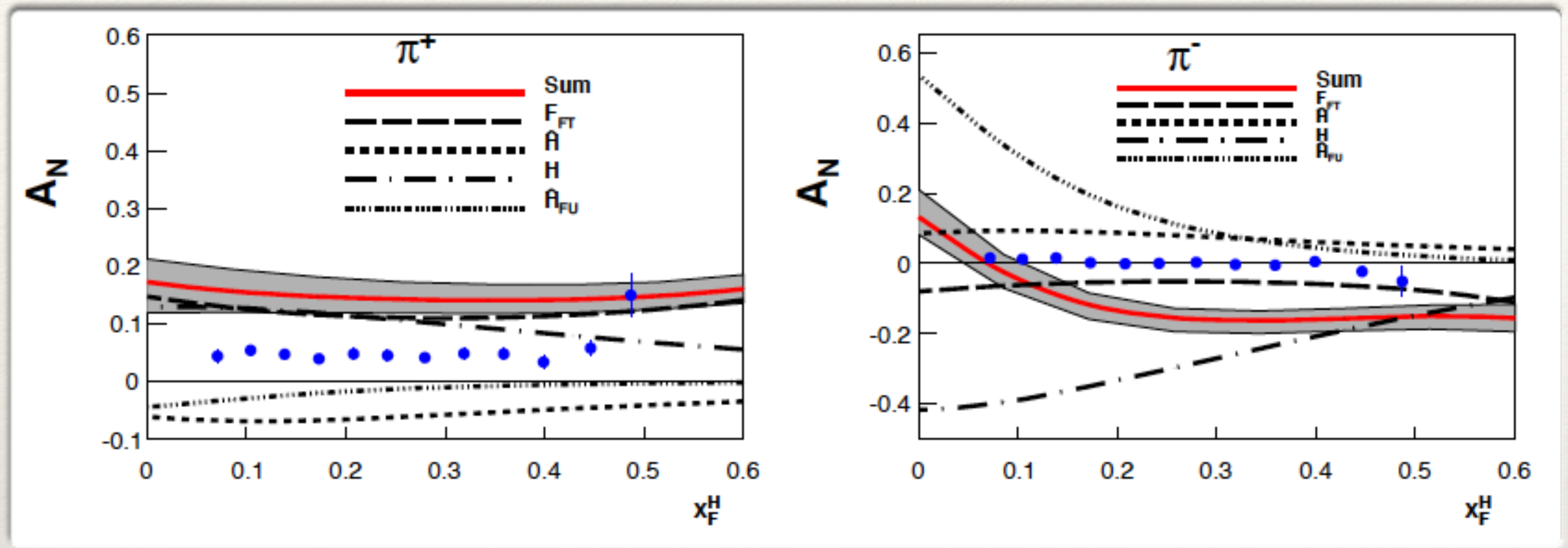

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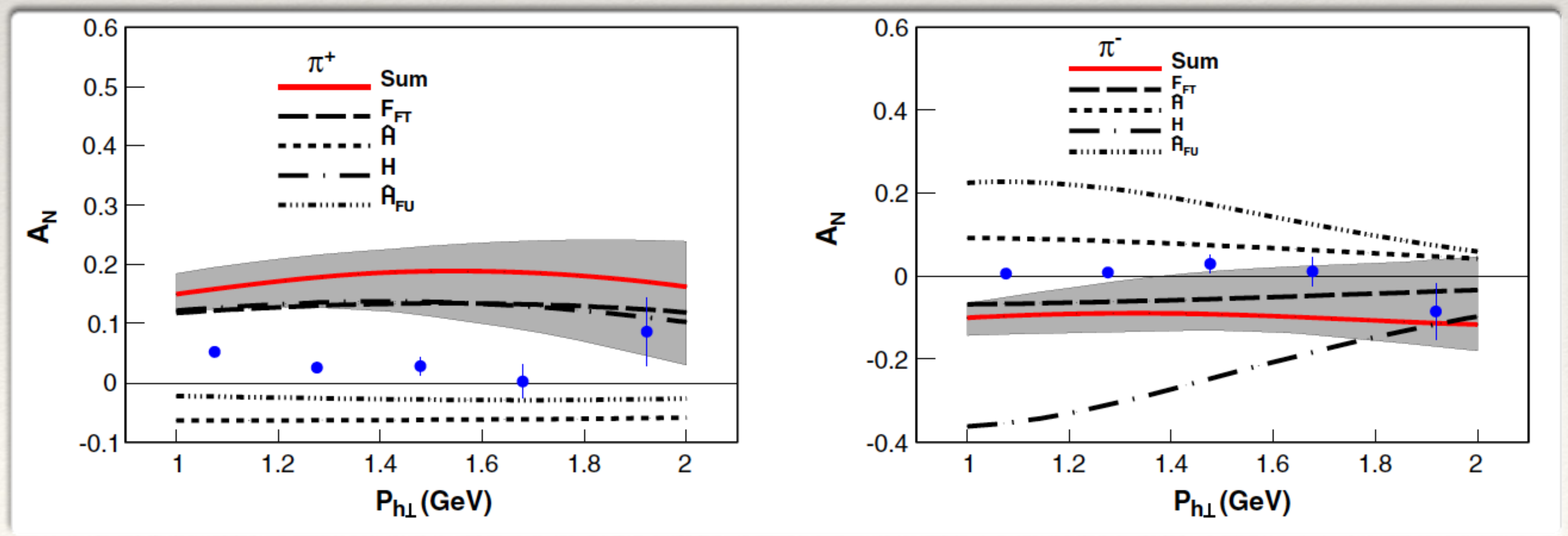

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→ NLO?

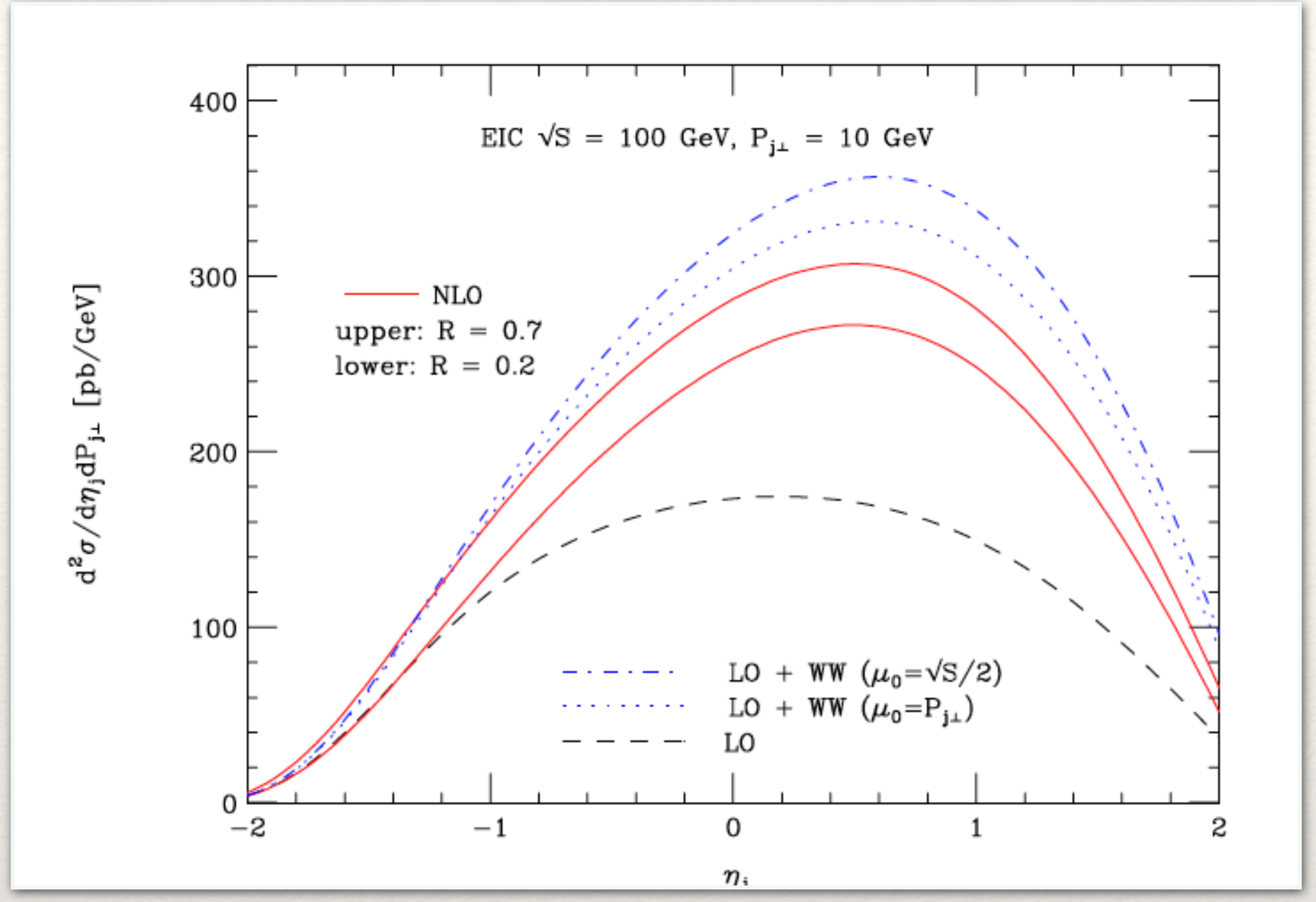
$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

unpolarized cross section

Jet Production at EIC (no fragmentation)

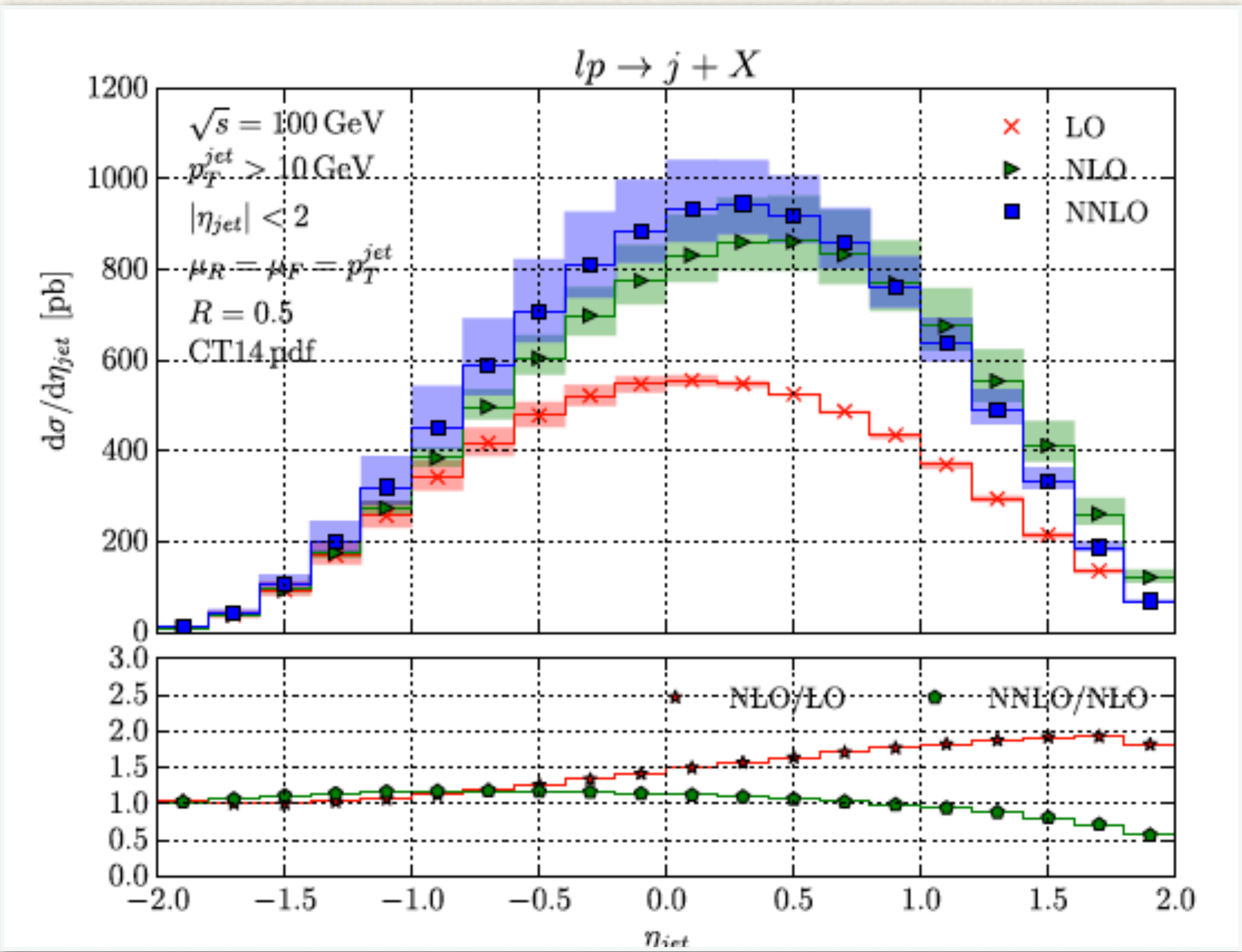
NLO: K ~ 1 - 2

[Hinderer, MS, Vogelsang, PRD, 2015, 2017]



NNLO

[Abelof et al., PLB 763, 52 (2016)]



→ perturbative series converges at NNLO

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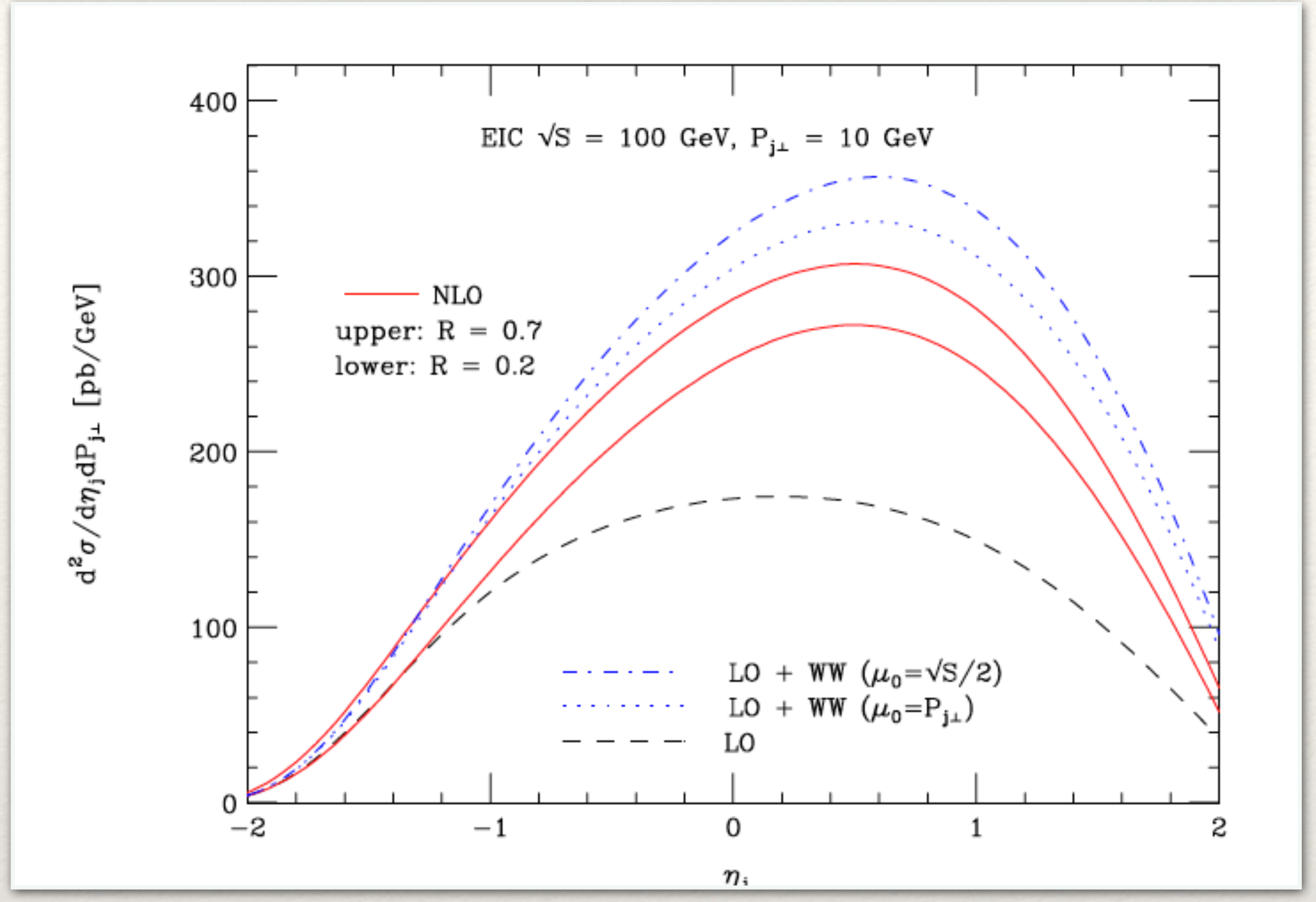

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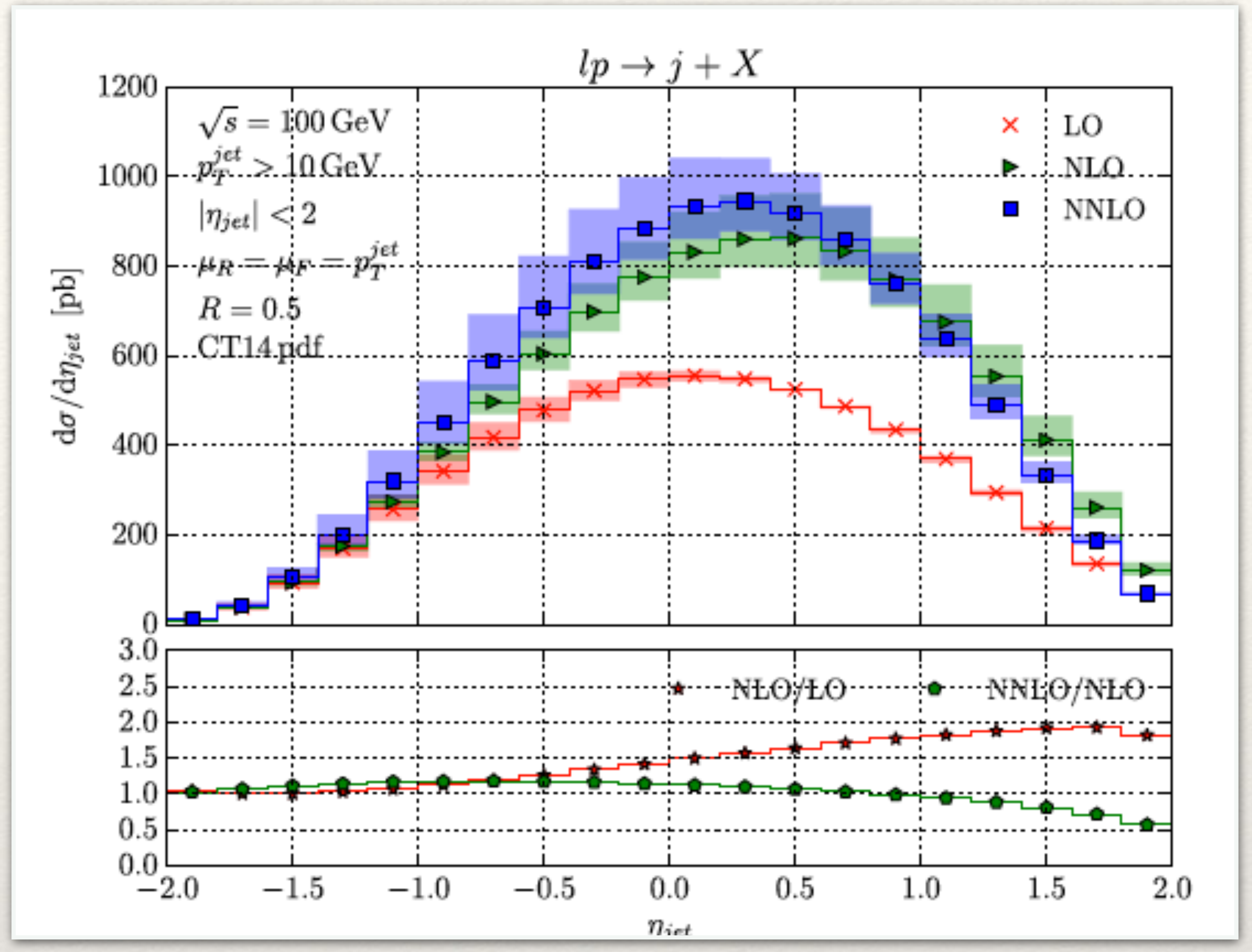


→ perturbative series converges at NNLO

Transverse Nucleon SSA at NLO:

[Tollkühn, MS, Vogelsang, ongoing work]

NLO calculation is quite complicated, several interdependent effects (SGP, SFP, HP), divergencies

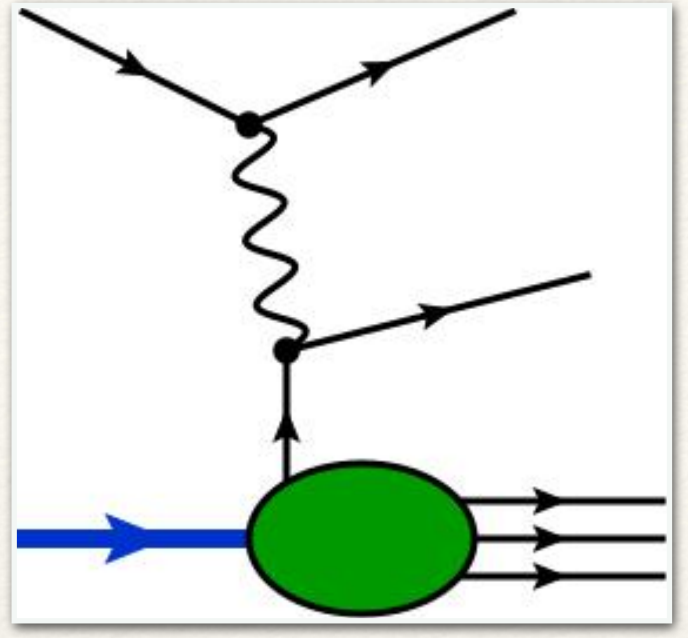
not yet finished, but some progress:

quark-gluon → quark channel: virtual diagrams, handling the hard poles

quark-gluon → gluon channel: reformulated hard pole contributions

Semi-inclusive γ production in l+N collisions

Deep-inelastic scattering: e(l) + N(P) $\longrightarrow$ e(l') + X



hard scale: $\tilde{Q}^2 = -(l - l')^2$ scaling variable: $\tilde{x}_B = \frac{Q^2}{2P \cdot (l - l')}$

Unpolarized cross section (LO parton model): $E' \frac{d\sigma}{d^3 l'} \propto \sum_q e_q^2 \, f_1^q(\tilde{x}_B)$

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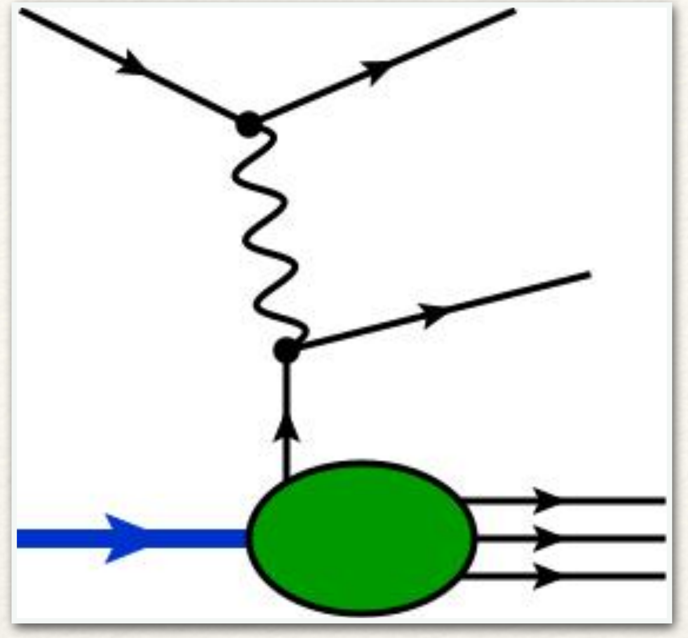

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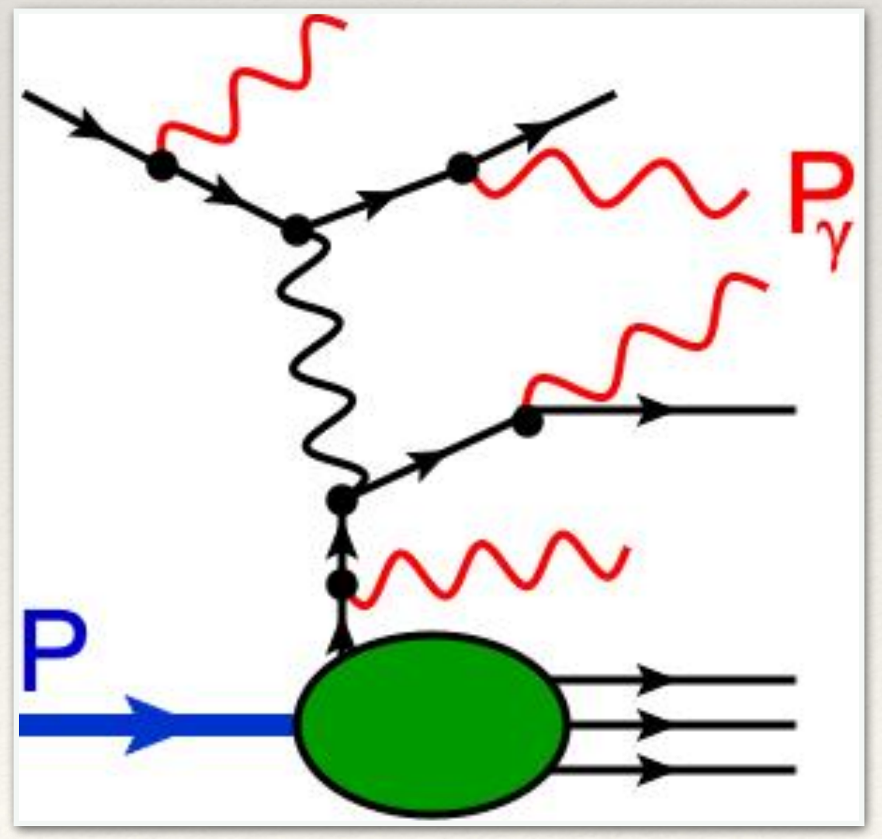
Unpolarized cross section (LO parton model): $E'\frac{d\sigma}{d^3l'} \propto \sum_q e_q^2\, f_1^q(\tilde{x}_B)$

Semi-inclusive γ production in DIS: e(l) + N(P) $\longrightarrow$ e(l') + γ(P_γ)+ X

[Albaltan, Prokudin, M.S., PLB 804 (2020), 135367]

unpolarized cross section in the parton model

[Brodsky, Gunion, Jaffe, PRD 1972; see also works by Metz et al; Pisano, Mukherjee; de Rujula, Vogelsang; …]



- avoid photon fragmentation: isolated photons
- collinear factorization: information on final quark is integrated out
  - two scales: $Q^2 = -(l-l'-P_\gamma)^2$ $\quad \tilde{Q}^2 = -(l-l')^2$
  - two scaling variables: $x_B = \frac{Q^2}{2P\cdot(l-l'-P_\gamma)}$ $\quad \tilde{x}_B = \frac{\tilde{Q}^2}{2P\cdot(l-l')}$

$$E'E_\gamma\frac{d\sigma}{d^3\vec{l}'\,d^3\vec{P}_\gamma} = \hat{\sigma}_{BH}\, f^{BH}(x_B) + \hat{\sigma}_C\, f^C(x_B) + \hat{\sigma}_I\, f^I(x_B)$$

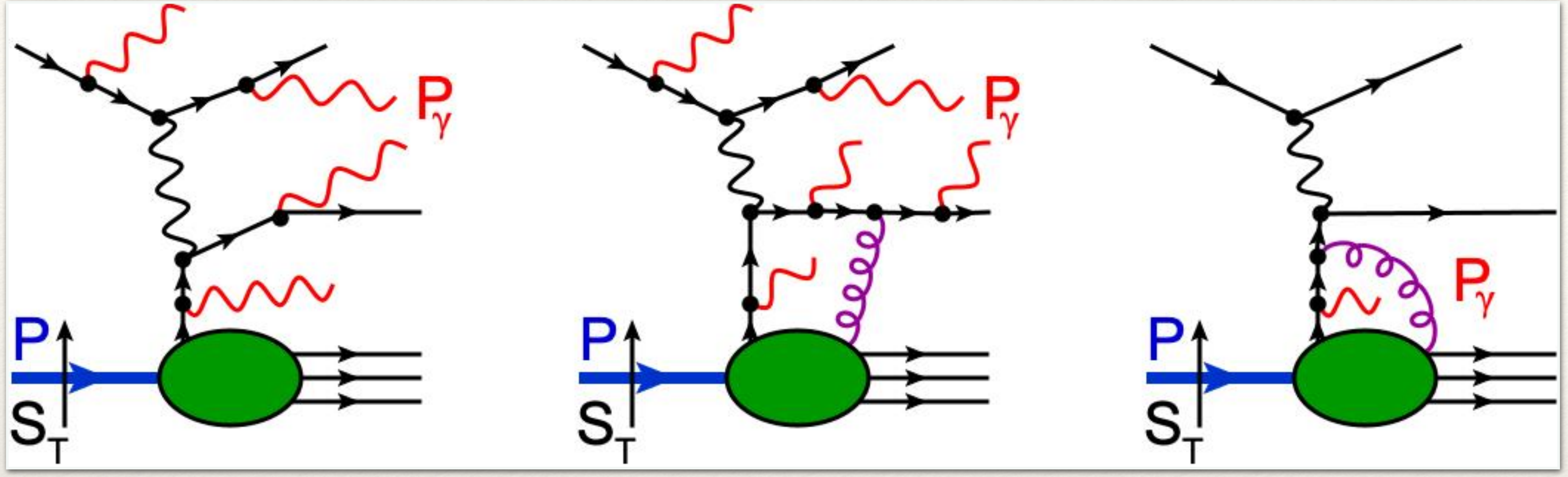
$$f^{BH} = \sum_{q=u,d,\dots} e_q^2(f^q + f^{\bar{q}})$$

$$f^C = \sum_{q=u,d,\dots} e_q^4(f^q + f^{\bar{q}})$$

$$f^I = \sum_{q=u,d,\dots} e_q^3(f^q - f^{\bar{q}})$$

Transverse spin effects in photon SIDIS

Include *intrinsic*, *kinematical* & *dynamical* twist - 3 contributions



SSA: need a phase (propagator poles)

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2) Soft Fermion Poles: $F_{FT}(x_B, 0)$
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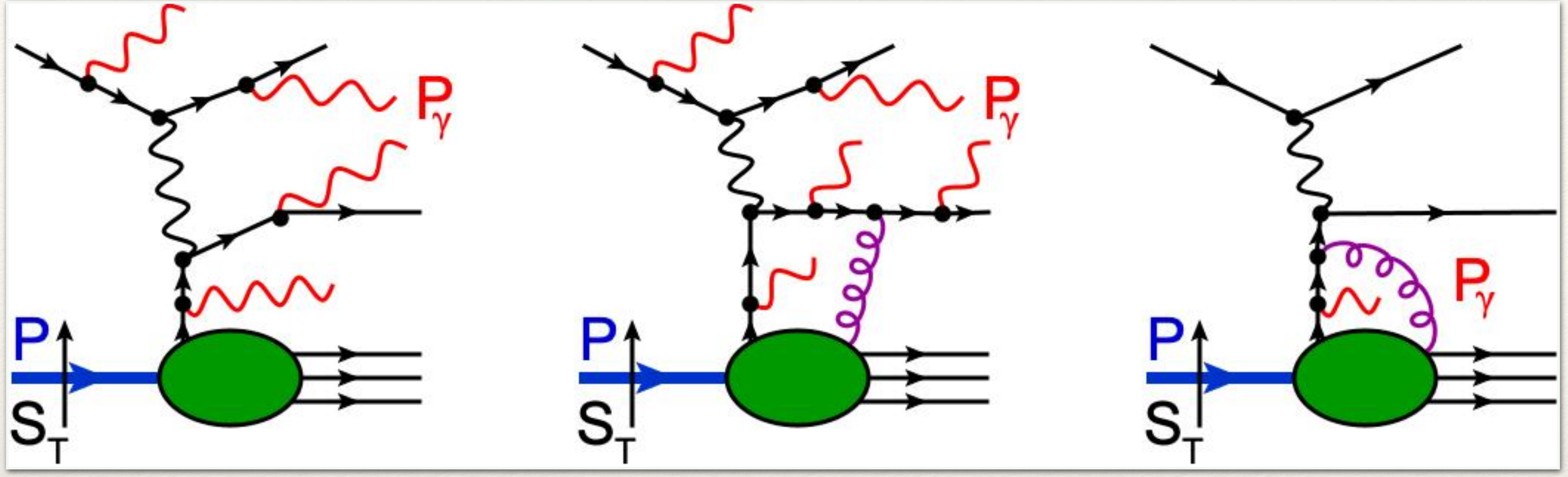

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3) Hard Poles: $F_{FT}(x_B,\tilde{x}_B)$

Single-Spin Asymmetry:

$$A_{UT} \propto \frac{M}{Q}\left[\epsilon^{Pll'S}\hat{\sigma}^1_{UT} + \epsilon^{PlP_\gamma S}\hat{\sigma}^2_{UT}\right] \propto \left[\sin(\phi_S-\phi_{l'})\hat{\sigma}^1_{UT} + \sin(\phi_S-\phi_\gamma)\hat{\sigma}^2_{UT}\right]$$

$$\hat{\sigma}^{i=1,2}_{UT} = \sum_{n=C,I}\left[\hat{\sigma}^{n;i=1,2}_{HP,F}\,F^n_{FT}(x_B,\tilde{x}_B) + \hat{\sigma}^{n;i=1,2}_{HP,G}\,G^n_{FT}(x_B,\tilde{x}_B) + \hat{\sigma}^{n;i=1,2}_{SFP,F}\,F^n_{FT}(x_B,0) + \hat{\sigma}^{n;i=1,2}_{SFP,G}\,G^n_{FT}(x_B,0)\right]$$

- no BH contributions
- SSA entirely generated by Hard & Soft Fermion Poles
- Allows to scan the support of quark-gluon-quark functions

- Theory: As good as it gets…
- Experiment (EIC): ???

Inclusive γ production in l+N collisions

(EIC version of pp $\rightarrow$ γX)

Inclusive (high-p_T) γ production in DIS: $e(l) + N(P) \longrightarrow \gamma(P_\gamma) + X$

[D. Rein, M.S., W. Vogelsang, arVix:23XX.XXXX (unpolarized), work in progress (transverse polarization)]

Conceptually & Experimentally: simpler observable: single-inclusive $E_\gamma \frac{d\sigma}{d^3\vec{P}_\gamma}$ $\longrightarrow$ "Left - Right" Asymmetry

Mandelstam variables $s = (P + l)^2,\ t = (P - P_\gamma)^2,\ u = (l - P_\gamma)^2$ scaling fraction $x_0 = \frac{-u}{s+t} \qquad v = 1 + s/t$

Theoretically: Phase-space integration on lepton non-trivial: even LO parton model has elements of NLO calculation!

$\longrightarrow$ interesting on its own…

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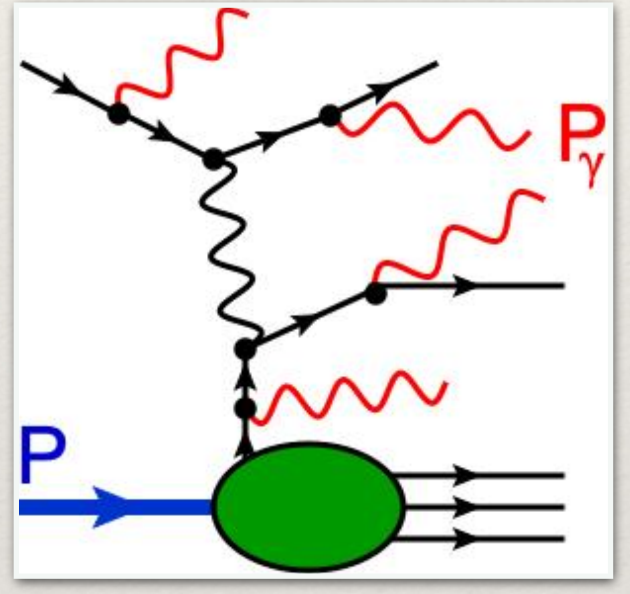

unpolarized cross section in LO parton model

Bethe-Heitler Terms

$$E_\gamma \frac{d\sigma_{BH}^{LO}}{d^3\vec{P}_\gamma} = \frac{\alpha_{\rm em}^3}{\pi s u} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_{BH}(v, w, \mu^2)\, f_1^{BH}\left(\frac{x_0}{w}, \mu\right)$$

$$f^{BH} = \sum_{q=u,d,\ldots} e_q^2 (f^q + f^{\bar{q}})$$

collinear divergence in e^- phase space

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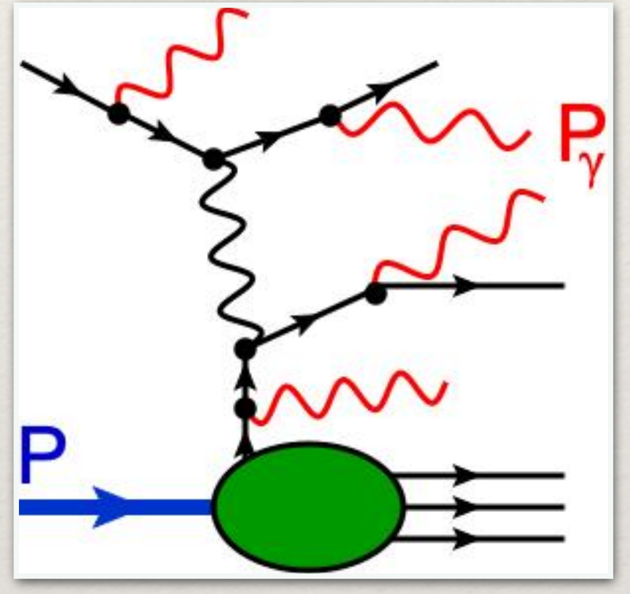

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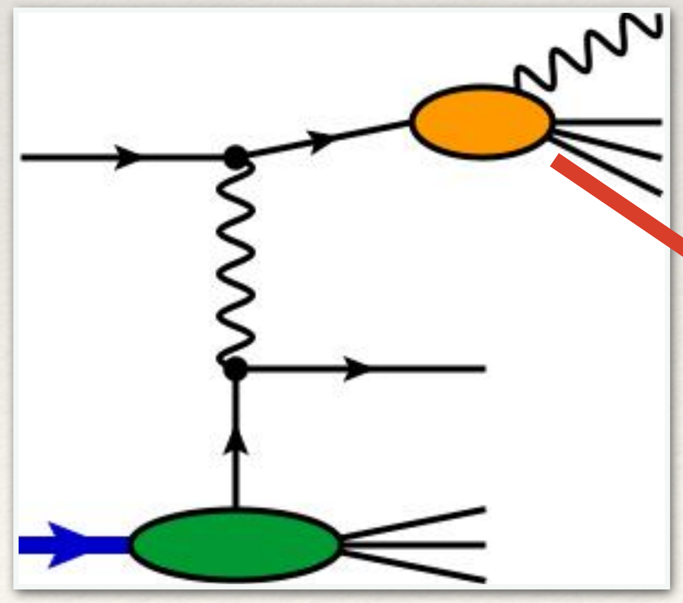

$$+\frac{\alpha_{\rm em}^2}{su}\int_{x_0}^1 \frac{dw}{w}\,\hat{\sigma}^{WW}_{BH}(v,w)\, D_1^{l/\gamma}(1-v(1-w),\mu)\, f_1^{BH}\left(\frac{x_0}{w},\mu\right)$$

"Weizsäcker-Williams" fragmentation function, perturbative in QED

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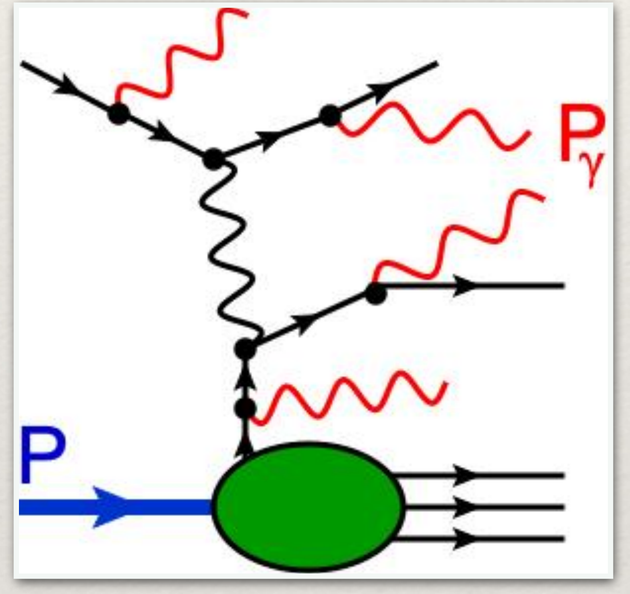

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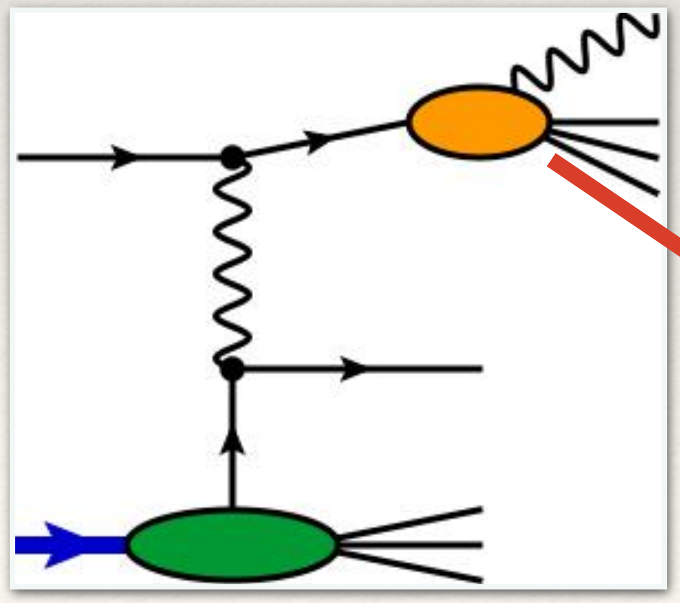
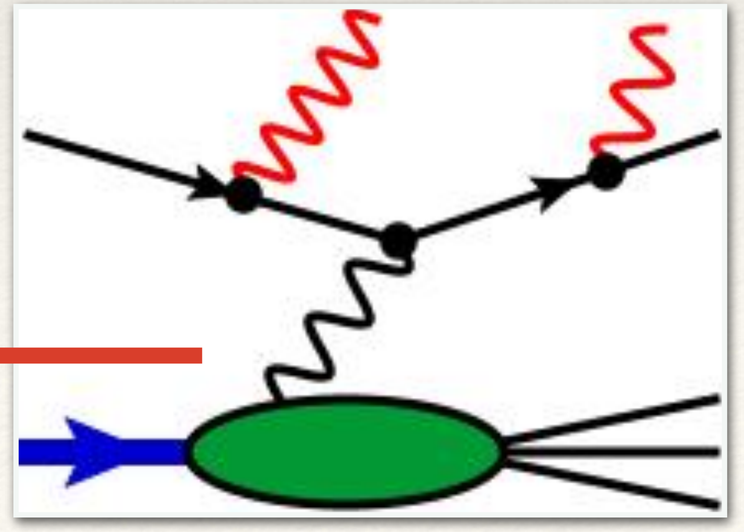
collinear divergence in e⁻ phase space



$$+ \frac{\alpha^2_{\rm em}}{su} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}^{WW}_{BH}(v, w)\, D_1^{l/\gamma}(1 - v(1-w), \mu)\, f_1^{BH}\left(\frac{x_0}{w}, \mu\right)$$

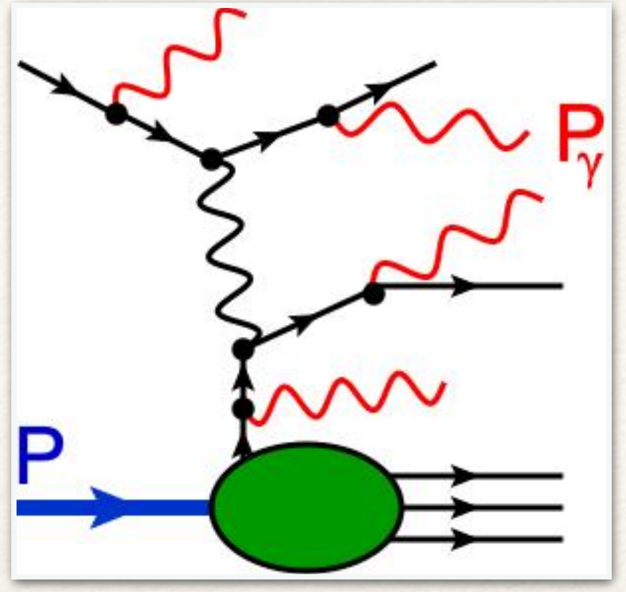
“Weizsäcker-Williams” fragmentation function, perturbative in QED

$$+ \frac{\alpha^2_{\rm em}}{su} \hat{\sigma}_{BH}(v)\, f_1^{\gamma/N}(x_0, \mu)$$



γ/N PDF

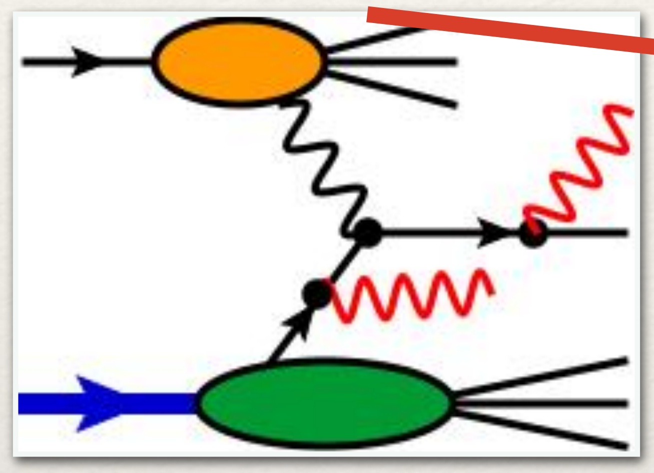
Compton Terms



$$E_\gamma \frac{d\sigma_C^{LO}}{d^3\vec{P}_\gamma} = \frac{\alpha_{\rm em}^3}{\pi s u} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_C(v, w, \mu^2)\, f_1^C(\tfrac{x_0}{w}, \mu)$$

$$f^C = \sum_{q=u,d,\ldots} e_q^4 (f^q + f^{\bar{q}})$$

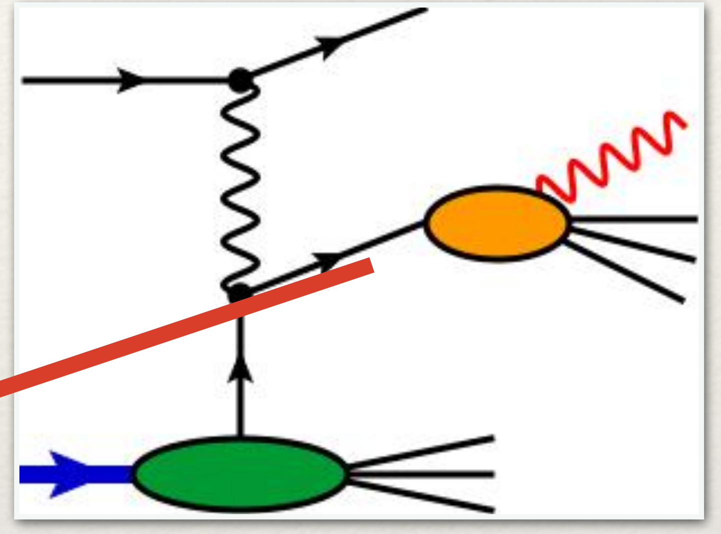
collinear divergence in e- phase space



$$+ \frac{\alpha_{\rm em}^2}{su} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_C^{WW}(v, w)\, f_1^{\gamma/l}(\tfrac{1-v}{1-vw}, \mu)\, f_1^C(\tfrac{x_0}{w}, \mu)$$

“Weizsäcker-Williams” distribution function, perturbative in QED

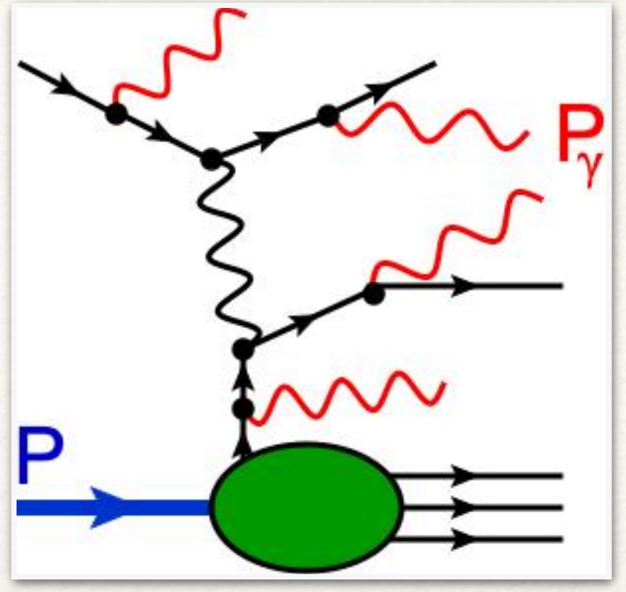
$$+ \frac{\alpha_{\rm em}^2}{su} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_C^{\gamma FF}(v, w) \sum_{f=u,d,\ldots} Q_f^2 D_1^{f/\gamma}(1 - v(1-w), \mu)\, (f_1^{f/N} + f_1^{\bar{f}/N})(\tfrac{x_0}{w}, \mu)$$



q ⟶ γ Fragmentation function

could be eliminated by “isolation”

Compton Terms



$$E_\gamma \frac{d\sigma_C^{LO}}{d^3\vec{P}_\gamma} = \frac{\alpha_{em}^3}{\pi s u} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_C(v,w,\mu^2)\, f_1^C(\tfrac{x_0}{w},\mu)$$

$$f^C = \sum_{q=u,d,\dots} e_q^4 (f^q + f^{\bar{q}})$$

collinear divergence in e- phase space

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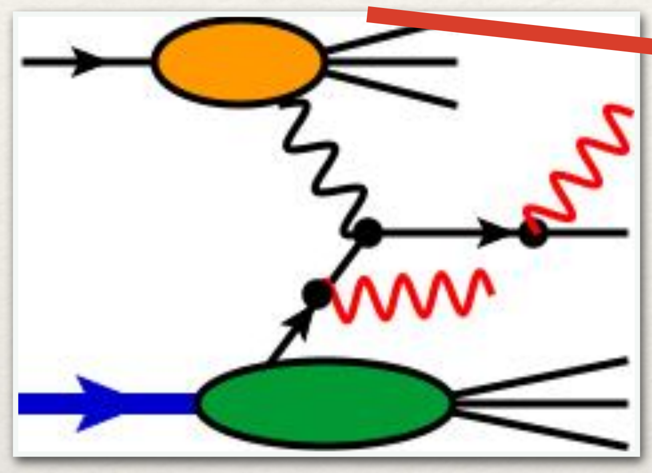
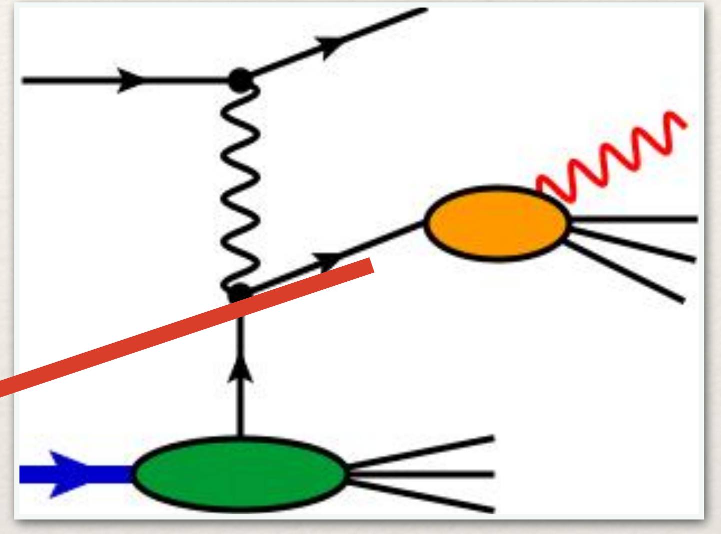


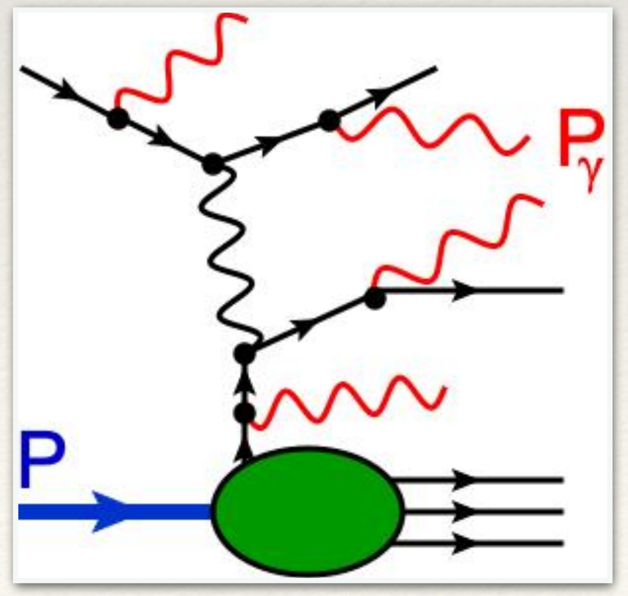
"Weizsäcker-Williams" distribution function, perturbative in QED

$$+\frac{\alpha_{em}^2}{su} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_C^{\gamma FF}(v,w) \sum_{f=u,d,\dots} Q_f^2 D_1^{f/\gamma}(1-v(1-w),\mu)\,(f_1^{f/N} + f_1^{\bar{f}/N})(\tfrac{x_0}{w},\mu)$$



q ⟶ γ Fragmentation function
could be eliminated by "isolation"

Interference Terms



$$E_\gamma \frac{d\sigma_I^{LO}}{d^3\vec{P}_\gamma} = \frac{\alpha_{em}^3}{\pi s u} \int_{x_0}^1 \frac{dw}{w}\, \hat{\sigma}_I(v,w)\, f_1^I(\tfrac{x_0}{w},\mu)$$

$$f^I = \sum_{q=u,d,\dots} e_q^3 (f^q - f^{\bar{q}})$$

No collinear divergence for interference contributions,
but phase space more difficult…

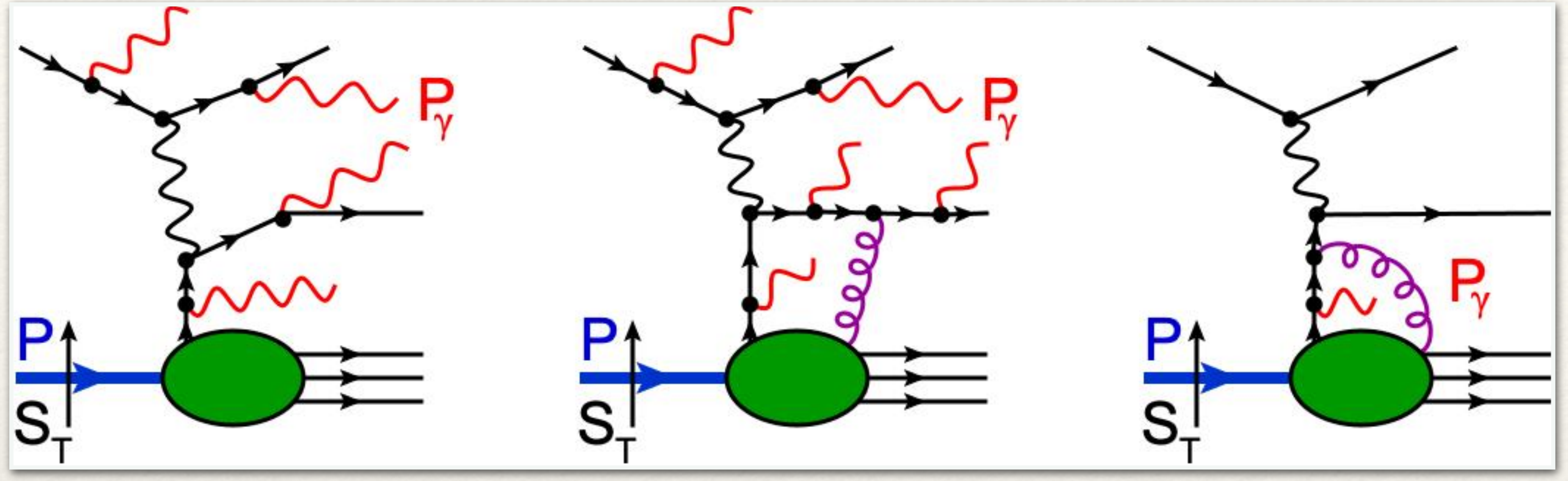
EIC c.m. frame (forward direction = proton direction)

- Clear hierarchy of channels by orders of magnitude!
  γPDF/BH dominates in (proton) backward region & larger transverse γ momentum
  EIC data may help to further constrain γPDF
- Compton / γFF (NLO in α_s!) dominates in (proton) forward region & smaller transverse γ momenta, we may learn about γFF
- Photon Isolation: Small cone approximation (R=0.7), accept hadronic activity inside cone for $E_X < 0.1\, E_\gamma$!

Transverse nucleon SSA

[D. Rein, MS, W. Vogelsang; work in progress]

$$eN^\uparrow \to \gamma X$$



$$\hat{\sigma}_{UT}^{i=1,2} = \sum_{n=C,I} \left[\hat{\sigma}_{HP,F}^{n;i=1,2} F_{FT}^{n}(x_B, \tilde{x}_B) + \hat{\sigma}_{HP,G}^{n;i=1,2} G_{FT}^{n}(x_B, \tilde{x}_B) + \hat{\sigma}_{SFP,F}^{n;i=1,2} F_{FT}^{n}(x_B, 0) + \hat{\sigma}_{SFP,G}^{n;i=1,2} G_{FT}^{n}(x_B, 0) \right]$$

Simply integrating over lepton momentum doesn't work!

it's complicated…

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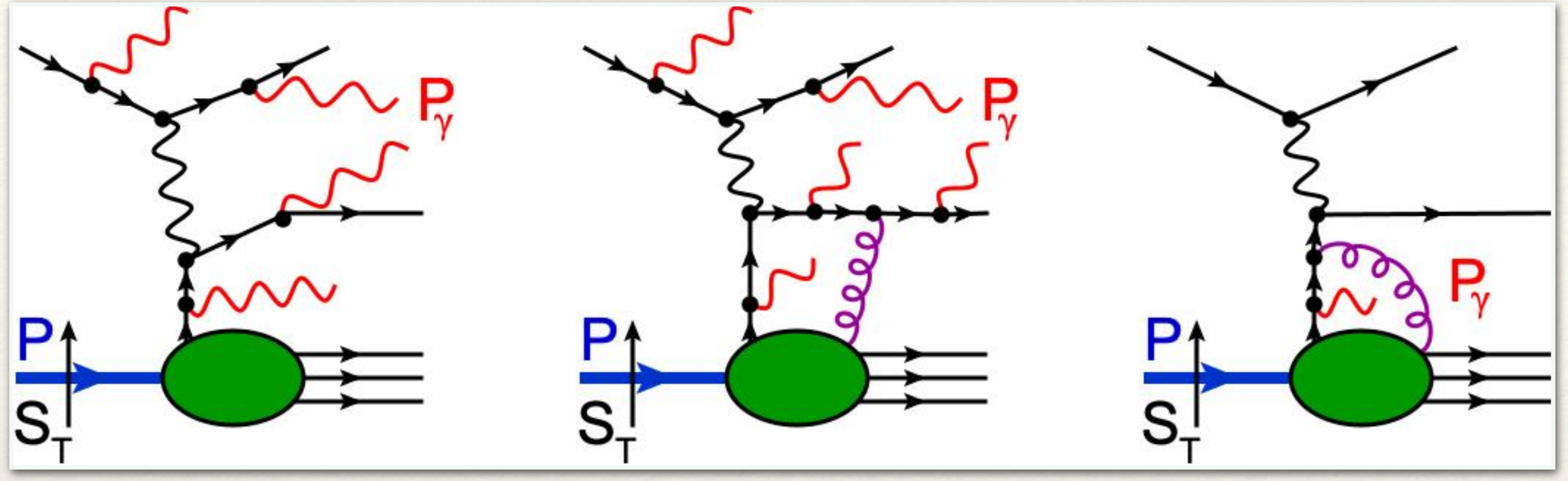

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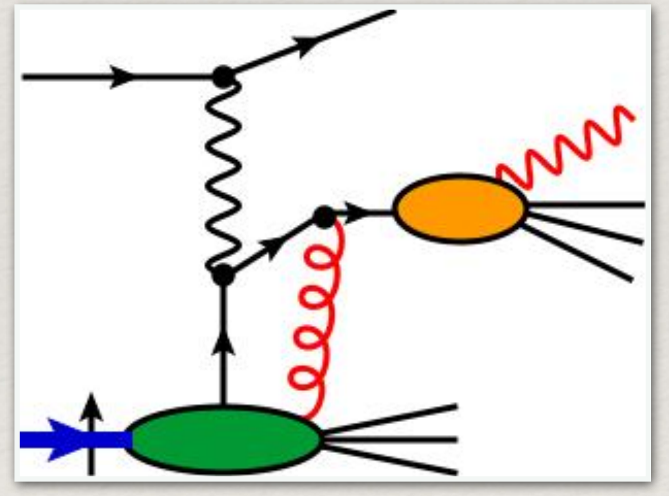
Simply integrating over lepton momentum doesn't work!

it's complicated…

Compton channel observations:

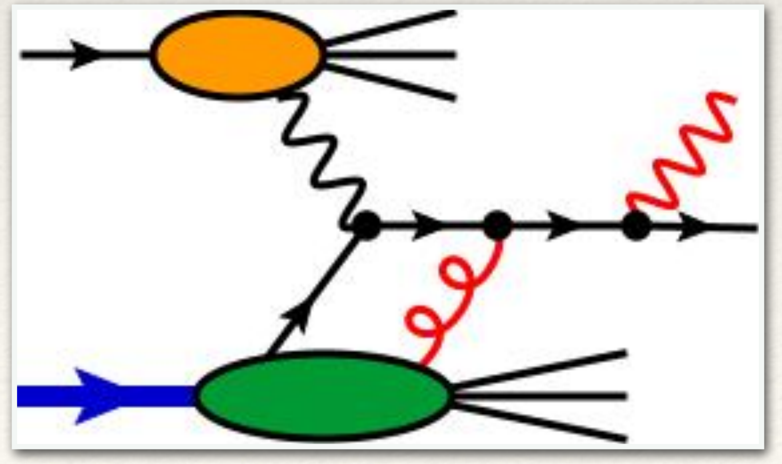
1) Super leading order



$$d\sigma(S_T) \propto \left(1 - x\frac{d}{dx}\right) F_{FT}^{q}(x,x)\, D_1^{\gamma/q}(z)$$

SGP cancels in Compton channel, how to combine?

2) WW contribution to soft fermion pole



BUT: WW contribution + SFP in Compton channel doesn't cancel 1/ε divergency (!?!)

How to solve these issues? Reexamine hard pole contributions

$$d\sigma^{HP}(S_T) \propto \int_{x_0}^{1} \frac{dw}{w} \int_{0}^{x} dx' \, \hat{\sigma}^{HP}(x', w) \, F_{FT}(x, x') \Big|_{x=\frac{x_0}{w}}$$

reverse order of phase space & x' integration:
No 1/ε poles,
BUT: endpoint singularities at x' = x & x' = 0

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regular partonic factor, but irregular soft function

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regular partonic factor, but irregular soft function

$$\frac{F(x,x')}{x'(x'-x)^2} = \frac{1}{x'}\left[\frac{F(x,x') - F(x,x) - (x'-x)\frac{1}{2}\frac{d}{dx}F(x,x)}{(x'-x)^2} - \frac{F(x,0) - F(x,x) + x\frac{1}{2}\frac{d}{dx}F(x,x)}{x^2}\right]$$

regular hard pole contribution, but…

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regular hard pole contribution, but…

$$+ \int_{x_0}^{1} \frac{dw}{w} \, \hat{\sigma}^{HP\to SGP}(w) \, F(x,x) \qquad + \int_{x_0}^{1} \frac{dw}{w} \, \hat{\sigma}^{HP\to dSGP}(w) \, x \frac{d}{dx} F(x,x) \qquad + \int_{x_0}^{1} \frac{dw}{w} \, \hat{\sigma}^{HP\to SFP}(w) \, F(x,0)$$

… additional contributions to soft gluon poles and soft fermion poles, with 1/ε singularities!

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regular hard pole contribution, but…

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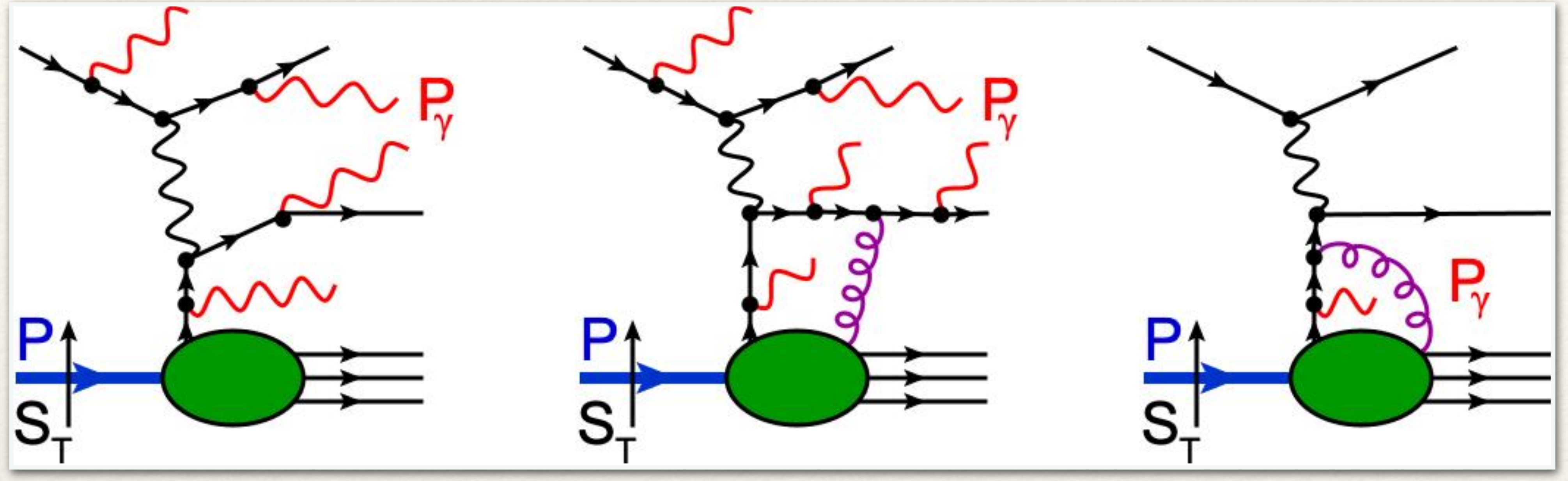
… additional contributions to soft gluon poles and soft fermion poles, with 1/ε singularities!

Conjecture:
combine all SGP contributions, SFP contributions
→ eventually all 1/ε singularities cancel!

Interference channel observations:



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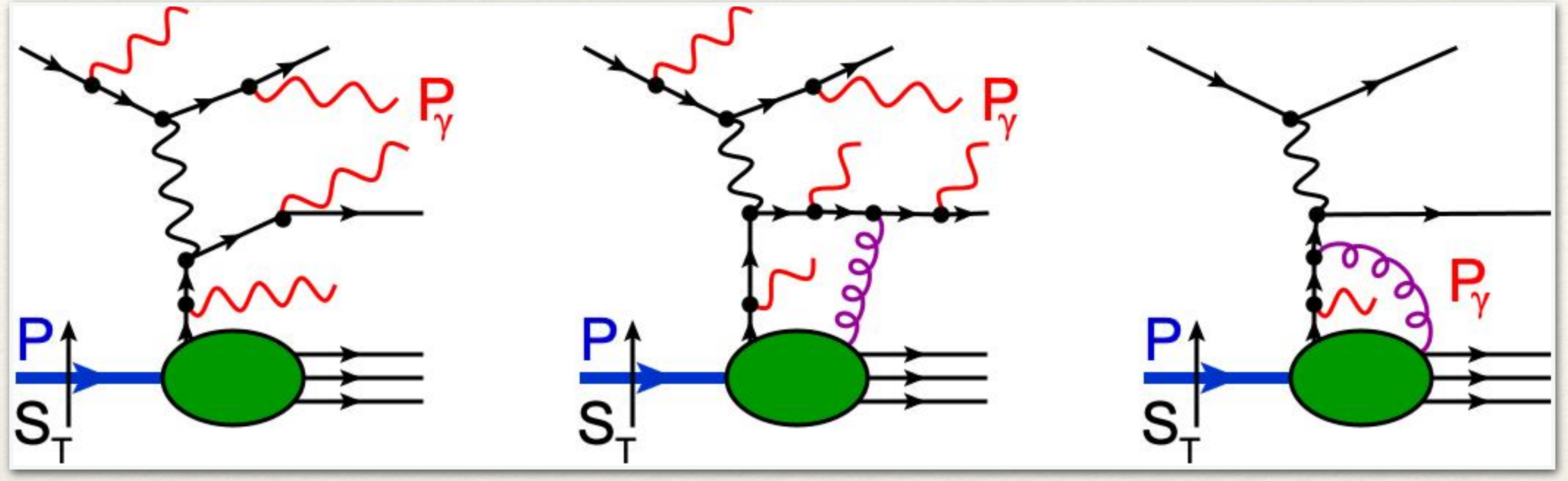
1) hard pole contributions: completely regular, no 1/ε poles, no endpoint singularities!

2) no soft gluon pole contributions!

3) soft fermion pole contributions → 1/ε singularity remains (!?!)

Interference channel observations:



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Conjecture:

source of SFP 1/ε pole: collinear singularity of exchanged photon

→ need to calculate with non-zero lepton mass

→ Interference channel may play a more prominent role for SSA!

Outlook & Summary

- Inclusive π production in eN: HERMES, JLab data on SSA exist, EIC will be helpful, NLO calculation is needed → Jet production on the way
- Next logical step: twist-3 fragmentation contribution for π production (whole different story…)
- Semi-inclusive γs in eN: (hopefully) feasible at the EIC, super helpful to learn about QGQ correlation functions
- Inclusive γs in eN: simpler observables, but theoretically more challenging
- Unpolarized cross section may be interesting to learn about photonics distributions
- Double longitudinal spin asymmetry: constrain photon-in-nucleon helicity distribution
- LO formula for Transverse SSA on the way

Back-up slides

How could the QGQ correlation functions look like?

Polar coordinates $(x, x') = r(\cos(\varphi + \frac{\pi}{4}), \sin(\varphi + \frac{\pi}{4}))$

Symmetry

$$F^q_{FT}(x, x') = +F^q_{FT}(x', x)$$

$$G^q_{FT}(x, x') = -G^q_{FT}(x', x)$$

$$F^q_{FT}(r, \varphi) = \sum_{n=0}^{\infty} a_n(r)\ \cos(n\varphi)$$

$$G^q_{FT}(r, \varphi) = \sum_{n=0}^{\infty} a_n(r)\ \sin(n\varphi)$$

Fourier series

How could the QGQ correlation functions look like?

Polar coordinates $(x, x') = r(\cos(\varphi + \frac{\pi}{4}), \sin(\varphi + \frac{\pi}{4}))$

Symmetry

$$F^q_{FT}(x, x') = +F^q_{FT}(x', x)$$

$$G^q_{FT}(x, x') = -G^q_{FT}(x', x)$$

Fourier series

$$F^q_{FT}(r, \varphi) = \sum_{n=0}^{\infty} a_n(r)\ \cos(n\varphi)$$

$$G^q_{FT}(r, \varphi) = \sum_{n=0}^{\infty} a_n(r)\ \sin(n\varphi)$$

charge conjugation

$$F^{\bar{q}}_{FT}(x, x') = +F^q_{FT}(-x, -x')$$

$$G^{\bar{q}}_{FT}(x, x') = +G^q_{FT}(-x, -x')$$

$$\sum_{q=u,d,s} e_q^2 \left(F^q_{FT} + F^{\bar{q}}_{FT}\right)(r, \varphi)$$

$$= \sum_q e_q^2 \frac{1}{\pi} \left(f_{1T}^{\perp(1),q} + f_{1T}^{\perp(1),\bar{q}}\right) \left(\frac{r}{\sqrt{2}}\right) \left[1 + \sum_{n=1}^{\infty} \tilde{a}^q_{2n}(r)\ (\cos(2n\varphi) - 1)\right]$$

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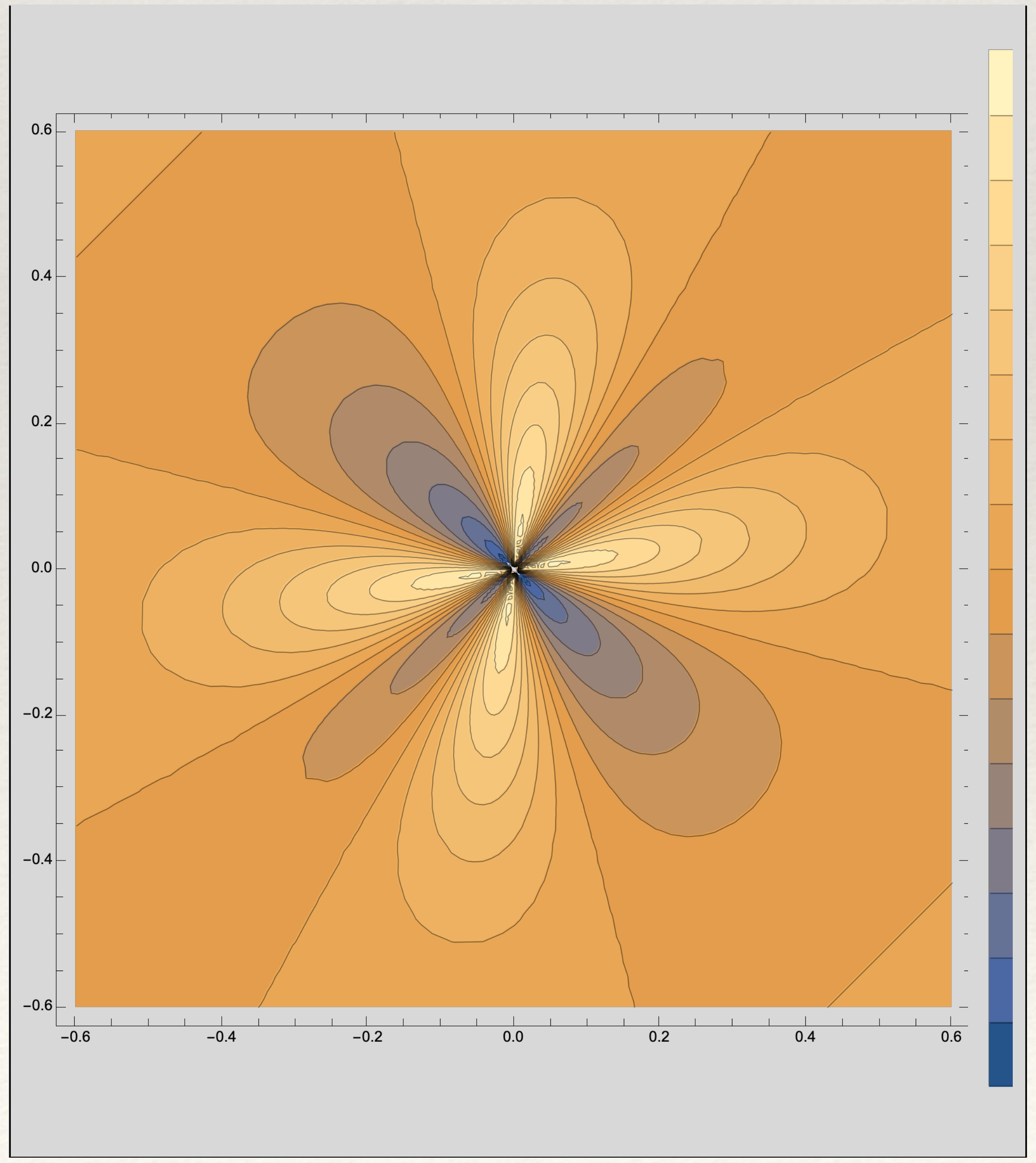


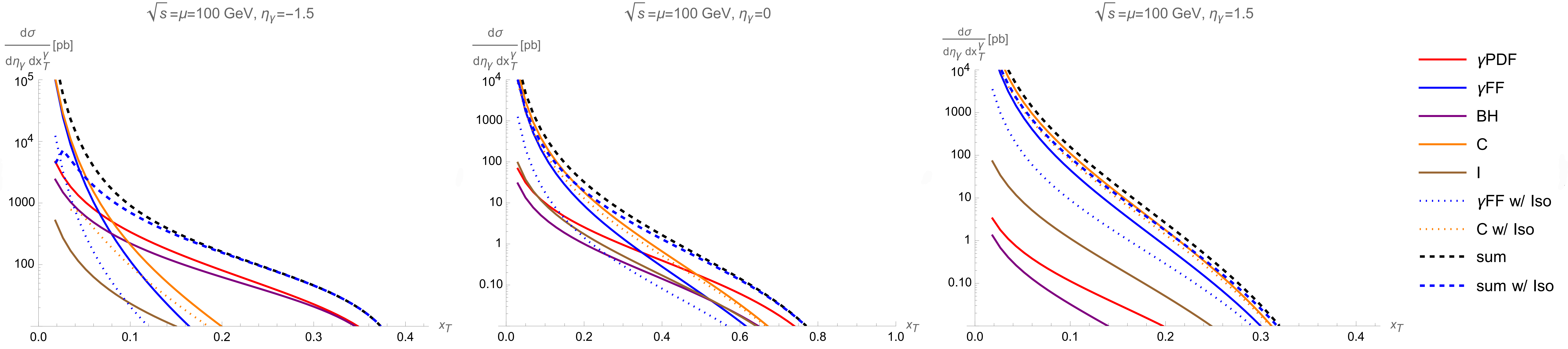
Example for F_{FT} (random choice)

$$\tilde{a}^u_2 = -\frac{1}{2},\ \tilde{a}^u_4 = 1,\ \tilde{a}^u_6 = \frac{1}{3}$$

Ideally: Fit values from experimental data

→ EIC!

Transverse Momentum photon distribution at EIC



- Clear hierarchy of channels in backward - forward region
  γPDF/BH dominates in backward region (medium - large transverse γ momentum)
- Compton dominates in (proton) forward region
- JLab 24 GeV: Similar behavior (without isolation)

Double Longitudinal Spin Asymmetry

$$e^{\rightarrow} N^{\rightarrow} \rightarrow \gamma X$$

$$A_{LL} = \frac{\sigma^{\rightarrow/\rightarrow} - \sigma^{\leftarrow/\rightarrow} - \sigma^{\rightarrow/\leftarrow} + \sigma^{\leftarrow/\leftarrow}}{\sigma^{\rightarrow/\rightarrow} + \sigma^{\leftarrow/\rightarrow} + \sigma^{\rightarrow/\leftarrow} + \sigma^{\leftarrow/\leftarrow}}$$

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Replace quark - photon unpolarized PDFs by polarized helicity PDFs

$\not{l} \rightarrow \not{l}\gamma_5 \qquad \not{k} \rightarrow \not{k}\gamma_5$

$f_1^{BH,C,I}(x) \rightarrow g_1^{BH,C,I}(x)$ DSSV

$f_1^{\gamma/N}(x) \rightarrow g_1^{\gamma/N}(x)$???

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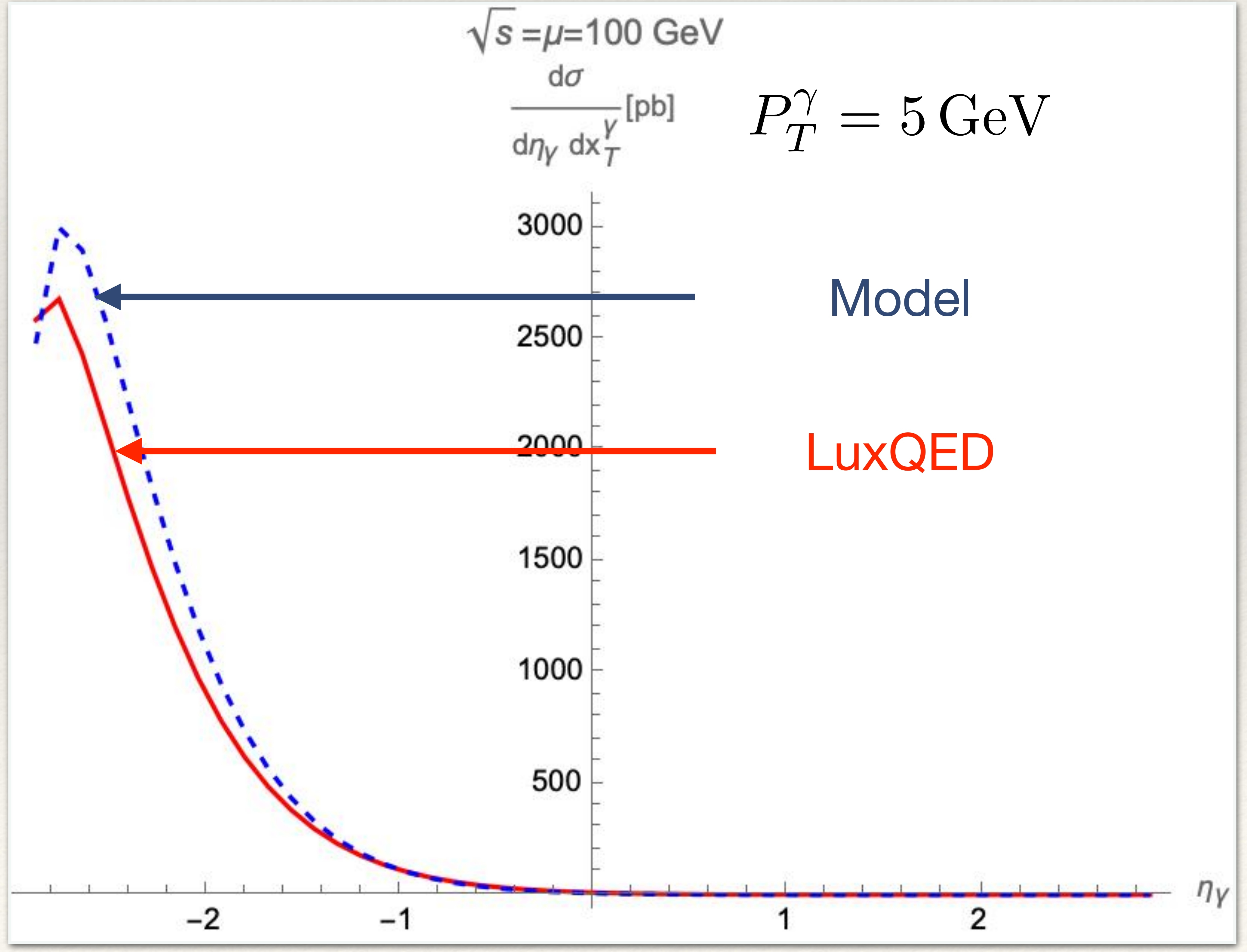
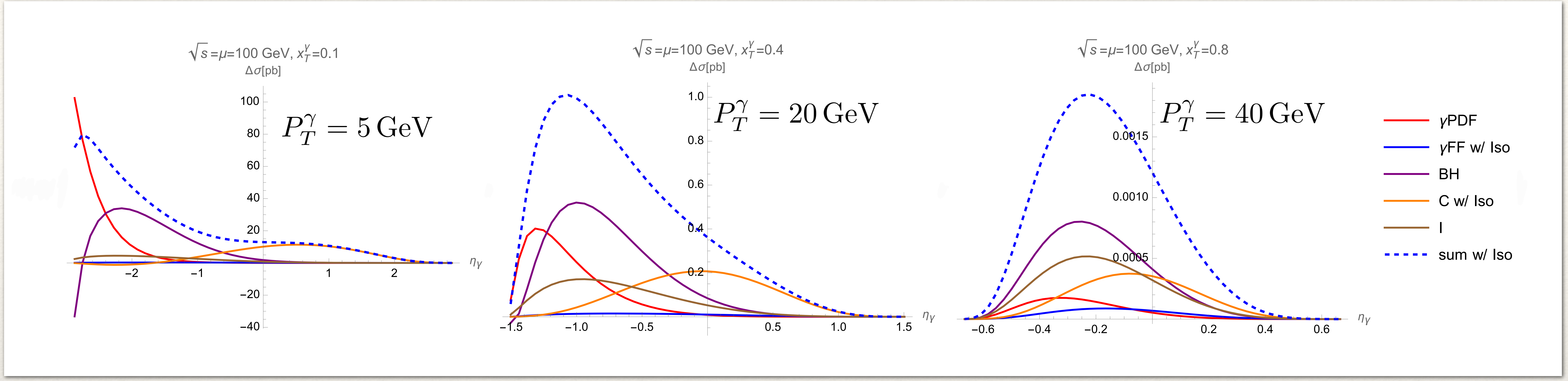
Model for polarized photon-in-nucleon PDF

$$g_1^{\gamma/N}(x) = \alpha_{\mathrm{em}}\, \Delta G(x)$$

not too bad for unpolarized γPDF contribution…



Polarized cross section at EIC vs. pseudorapidity of isolated photon



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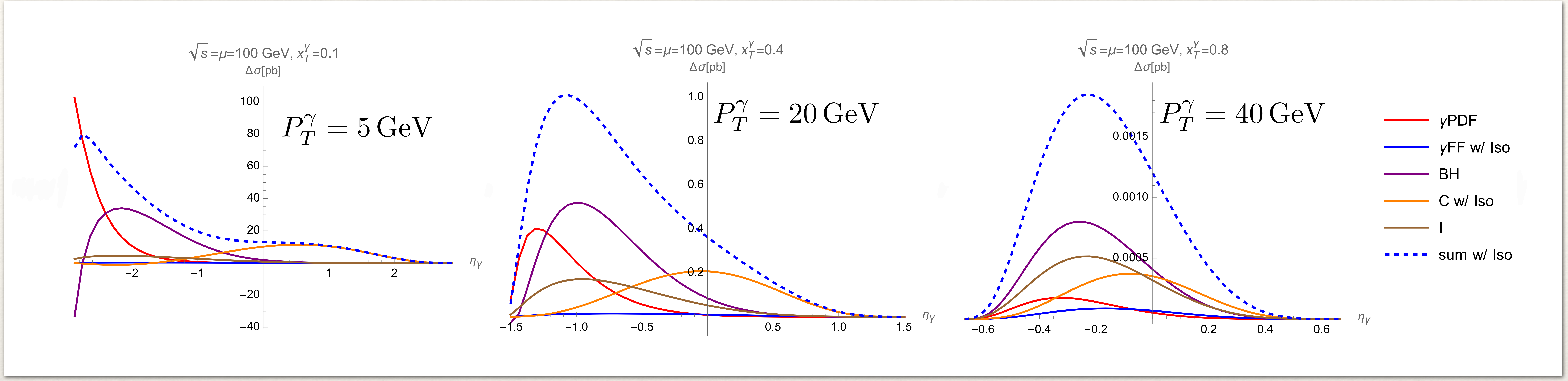
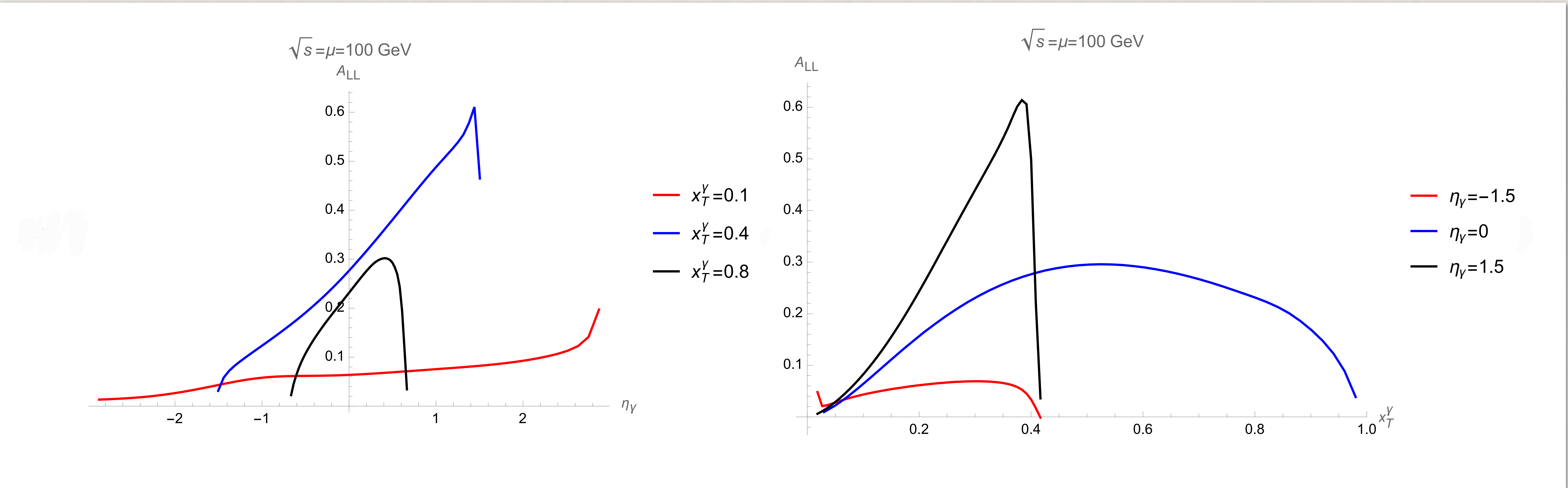


Double Spin Asymmetry



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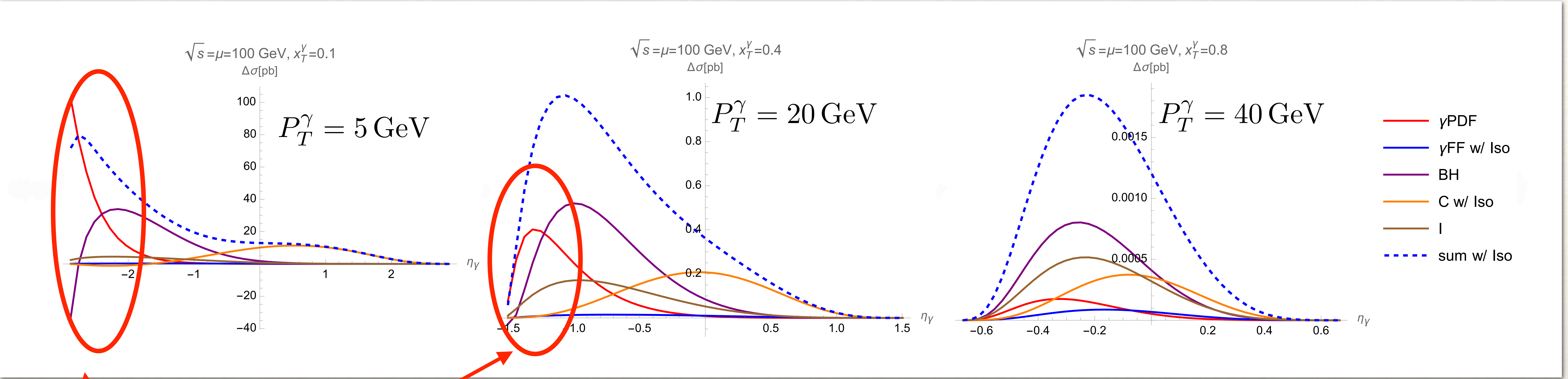

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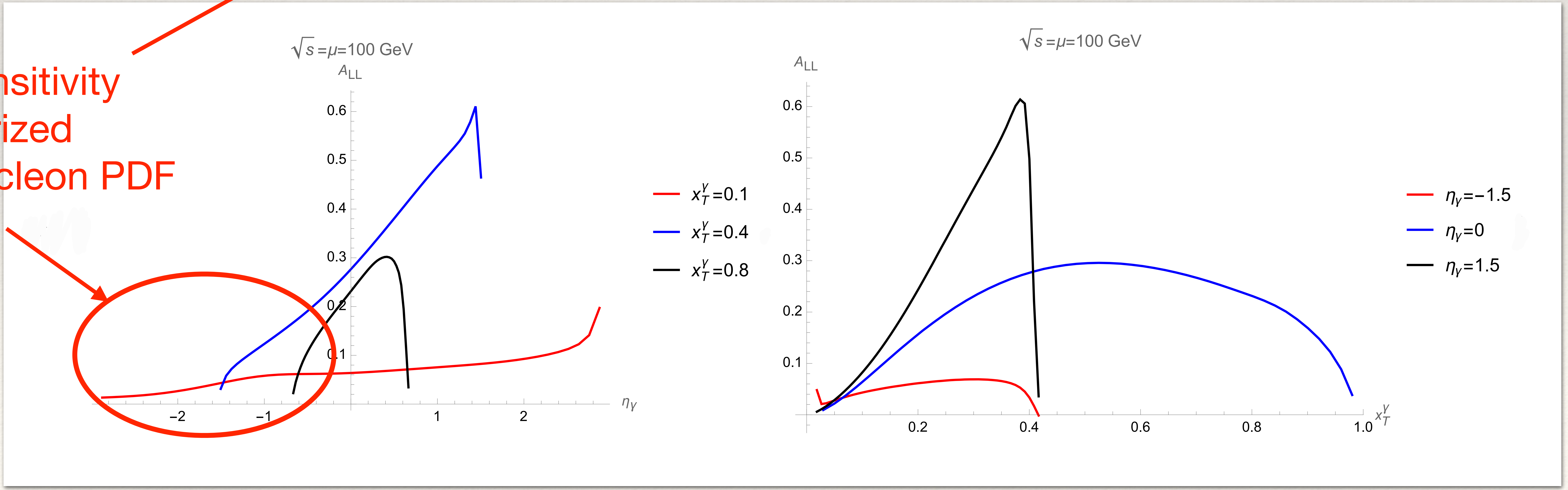

largest sensitivity to polarized photon-in-nucleon PDF