Transversity and tensor charge: role of the Soffer Bound

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U. D'Alesio, CF, A. Prokudin, Phys. Lett. B 803 (2020) 135347

Introduction - Transversity function

- collinear transversity function $h_1^q(x)$ describes the collinear structure of spin-$\frac{1}{2}$ hadrons at leading twist
- chiral-odd quantity $\Rightarrow$ not accessible in inclusive DIS
- extracted in SIDIS (TMD framework) or in two-hadron production with polarized dihadron FF (collinear pQCD)
- Soffer Bound [J. Soffer, Phys. Rev. Lett. 74 (1995) 1292–1294]

$$|h_1^q(x,Q^2)| \leq \frac{1}{2}\left[f_{q/p}(x,Q^2) + g_{1L}^q(x,Q^2)\right] \equiv SB^q(x,Q^2)$$

- bound preserved by Q^2 evolution up to NLO in QCD

[V. Barone, Phys. Lett. B 409 (1997) 499–502; W. Vogelsang, Phys. Rev. D 57 (1998) 1886–1894]

Introduction - tensor charges (I)

- quarks contribute to nucleon tensor charge via the first Mellin moment of the non-singlet quark combination, δq

$$\delta q = \int_0^1 \left[h_1^q(x) - h_1^{\bar{q}}(x) \right] dx$$

- isovector combination of tensor charges, g_T

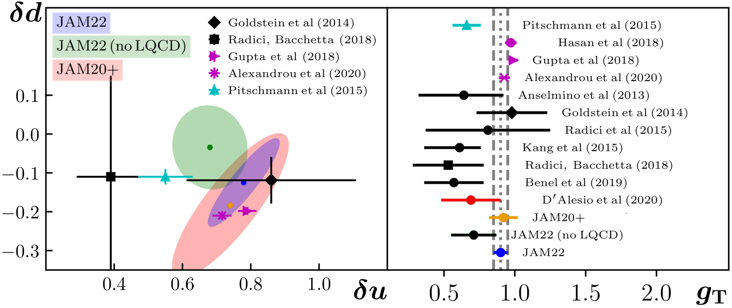
$$g_T = \delta u - \delta d$$

- δq and g_T relatively easy to compute in lattice QCD, also estimated starting from phenomenological extractions

- g_T is related to BSM effects: a bridge between QCD phenomenology, lattice QCD and BSM physics

Introduction - tensor charges (II)

Current situation for tensor charges:



Adapted from Fig. 3 of **[L. Gamberg *et al.*, Phys. Rev. D 106 (2022) 3, 034014]**

Caveats:

- different parametrizations for different phenomenological analyses
- experimental data not available for the full x-range $\Rightarrow$ extrapolation
- lattice QCD estimates done with different settings, computed as matrix element over $0 < x < 1$

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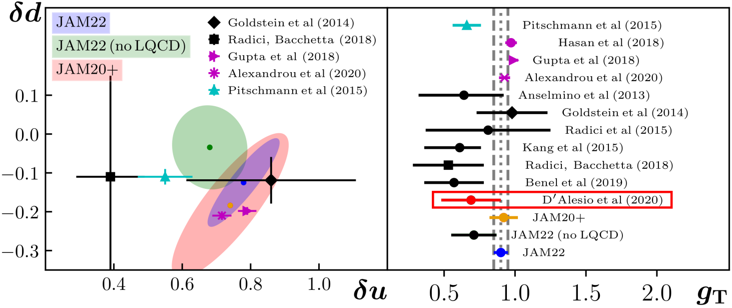

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Soffer Bound & phenomenological fits

- adopting

$$h_1^q(x, Q_0^2) \propto SB^q(x, Q_0^2)$$

is very common in phenomenological fits, both in collinear QCD and TMD physics

[M. Anselmino *et al.*, Phys. Rev. D 75 (2007) 054032 & Phys. Rev. D 92 (11) (2015) 114023]
[A. Bacchetta, A. Courtoy, M. Radici, JHEP03 (2013) 119; M. Radici, A. Bacchetta, Phys. Rev. Lett. 120 (19) (2018) 192001]

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- new approach: no automatic fulfillment of the SB in the parametrization, but application of the SB a posteriori

  [U. D'Alesio, CF, A. Prokudin, Phys. Lett. B 803 (2020) 135347]

TMD transversity & Collins global fit (I)

- global fit of TMD transversity and Collins functions from SIDIS and e^+e^- data
- $h_1^q(x, k_\perp^2)$ accessible through SIDIS azimuthal asymmetries:

$$A_{UT}^{\sin(\phi_h+\phi_S)} = \frac{2(1-y)}{1+(1-y)^2} \frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}}$$

  where $F_{UU} = \mathcal{C}[f_1 D_1]$ and $F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}[h_1 H_1^\perp]$
- Collins function also accessible from $\cos(2\phi_0)$ modulation of $e^+e^- \to h_1 h_2 X$ cross sections via $A_0^{UL(C)} \propto \mathcal{C}[\bar{H}_1^\perp H_1^\perp]$
- baseline fit: [M. Anselmino *et al.*, Phys. Rev. D 92 (2015) 11, 114023]
- dataset:
  (a) $A_{UT}^{\sin(\phi_h+\phi_S)}$ data from HERMES and COMPASS
  (b) $A_0^{UL(C)}$ measurements from BELLE, BABAR and BESIII
- $N_{\rm pts} = 278$

TMD transversity & Collins global fit (II)

- Gaussian parametrization:

$$h_1^q(x, k_\perp^2) = h_1^q(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}\,, \qquad h_1^q(x, Q_0^2) = \mathcal{N}_q^T(x)\, SB^q(x, Q_0^2)$$

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta\,\frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}, \quad (q = u_v, d_v)$$

- upon constraining

$$|N_q^T| \le 1$$

SB is automatically fulfilled

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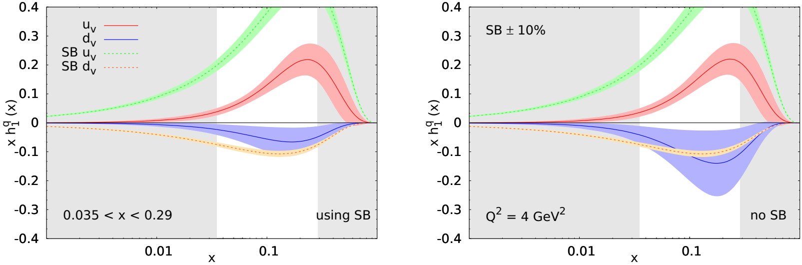
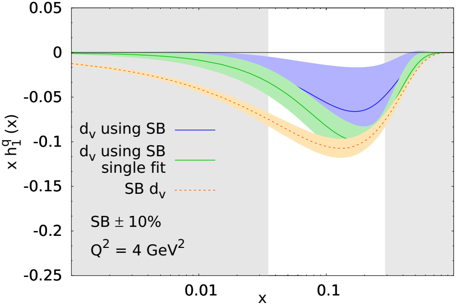
- two cases: "using SB" ($|N_q^T| \leq 1$ a posteriori) or "no SB" ($\cancel{|N_q^T| \leq 1}$)

Fit results (I)



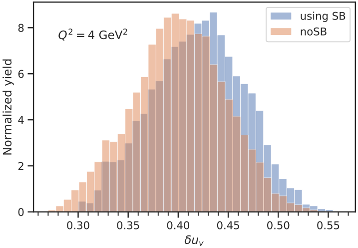
- shaded grey areas correspond to regions where data is not available
- almost same $\chi^2_{\text{dof}} \approx 0.93$
- out of 10^5 MC sets produced for the "noSB" case, $\approx$ 16% fulfill $|N_q^T| \leq 1 \Rightarrow$ sets for "using SB" case
- $h_1^{u_v}(x)$ does not change while relaxing the SB constraint, $h_1^{d_v}(x)$ apparently violates SB
- violation is less than 1σ statistically significant where data is available

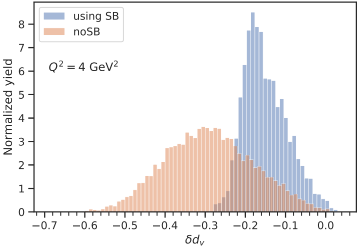
Fit results (II)



- "using SB single fit": apply SB a priori – automatic fulfillment of the SB throughout the fit
  $\Rightarrow$ $N^T_{d_v}$ saturates at its lower value, MINUIT underestimates the uncertainty on $N^T_{d_v}$ $\Rightarrow$ uncertainty for $h_1^{d_v}$ underestimated
- "using SB": apply SB a posteriori $\Rightarrow$ minimizator explores other configurations in the parameter space, compatible with the SB, that were not seen due to the bias introduced in the parametrization

Fit results - tensor charges (I)





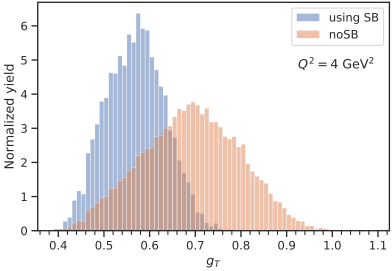
- δu_v does not change very much

  reflects the marginal difference on the fitted functions

- broader distribution for δd_v

  removing the SB constraint allows to properly explore the parameter space

Fit results - tensor charges (II)



- peak of the "no SB" distribution moves towards lattice estimates
- tail overlaps with the lattice QCD range $0.9 \lesssim g_T \lesssim 1.1$

	δu_v	δd_v	g_T
		$Q^2 = 4$ GeV^2	
using SB	0.42 ± 0.09	-0.15 ± 0.11	0.57 ± 0.13
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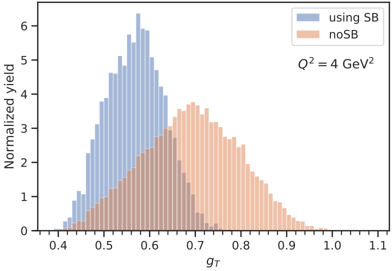

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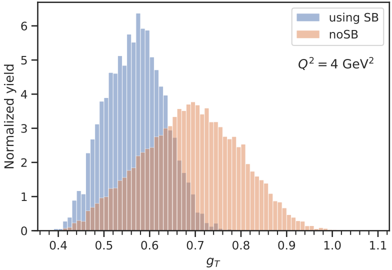

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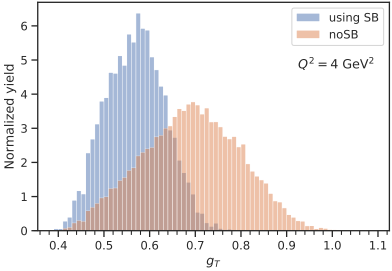

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Conclusions and outlook

- we have studied the role of the SB in the determination of transversity and the tensor charges
- we proposed a new approach for the application of positivity bounds in phenomenological fits
- relaxing the constraint on the SB in the parametrization allows to:
  - properly explore the parameter space
  - ease the tension between phenomenological analyses and lattice QCD computation for g_T
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Thank you

Backup

Fit results - settings

- Collinear PDFs: CTEQ66
- Collinear helicity: DSSV
- Collinear FF: DEHSS 2014 (π)/2017 (K)
- central value and uncertainties:

$$\mathrm{E}[\mathcal{O}] = \int d^n a\, \mathcal{P}(\boldsymbol{a}|\mathrm{data})\, \mathcal{O}(\boldsymbol{a}) \simeq \sum_k w_k\, \mathcal{O}(\boldsymbol{a}_k)$$

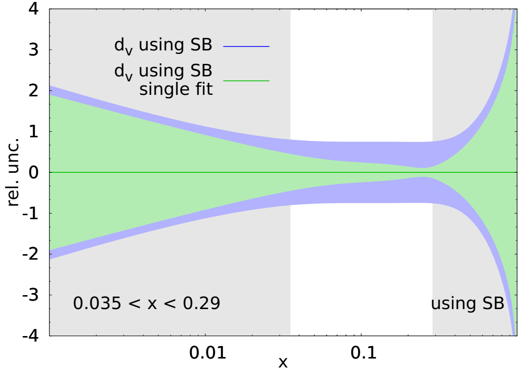
$$\mathrm{V}[\mathcal{O}] = \int d^n a\, \mathcal{P}(\boldsymbol{a}|\mathrm{data})\, (\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}])^2$$

$$\simeq \sum_k w_k\, (\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}])^2$$

- Giele-Keller weights:

$$w_k = \frac{\exp\left[-\frac{1}{2}\,\chi^2(\boldsymbol{a}_k)\right]}{\sum_i w_i}$$

Fit results - using SB



- auomatic fulfillment of the SB brings to underestimate the uncertainty
- underestimation is more severe in the region of fitted data

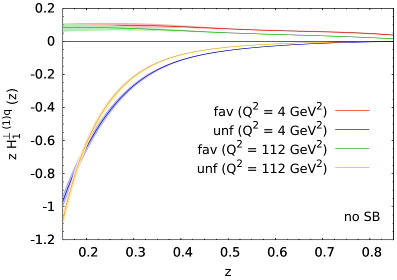
Fit results - Collins function

- parametrization:

$$H_1^{\perp q}(z,p_\perp^2) = \mathcal{N}_q^C(z)\,\frac{z m_h}{M_C}\,\sqrt{2e}\,e^{-p_\perp^2/M_C^2}\,D_{h/q}(z,p_\perp^2)\,,\ (q = \mathrm{fav},\mathrm{unf})$$

$$\mathcal{N}_{\mathrm{fav}}^C(z) = N_{\mathrm{fav}}^C\,z^\gamma(1-z)^\delta\,\frac{(\gamma+\delta)^{\gamma+\delta}}{\gamma^\gamma\delta^\delta}\,, \qquad \mathcal{N}_{\mathrm{unf}}^C(z) = N_{\mathrm{unf}}^C$$

- Collins function mostly constrained by e^+e^- data
  essentially no change between "using SB" and "no SB" cases



$$H_1^{\perp(1)\,q}(z) = z^2\int d^2\boldsymbol{p}_\perp\,\frac{p_\perp^2}{2m_h^2}\,H_1^{\perp q}(z,z^2p_\perp^2)$$

$$= \sqrt{\frac{e}{2}}\,\frac{1}{z m_h}\,\frac{M_C^3\langle p_\perp^2\rangle}{(\langle p_\perp^2\rangle + M_C^2)^2}\,\mathcal{N}_q^C(z)D_{h/q}(z)$$