3D PROTON IMAGING VIA TIME-REVERSAL ODD

TMD GLUON DISTRIBUTIONS

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Madrid UAH

talento cm Programa de atracción de talento investigador Comunidad de Madrid

ANIVERSARIO PATRIMONIO MUNDIAL

1. Hors d’œuvre

An incomplete family tree

Wigner distributions $\rho\left(x, \mathbf{k}_T, \mathbf{b}_T\right)$ **5D**

$\int d^2\mathbf{k}_T$

$\int d^2\mathbf{b}_T$

Generalized Parton Distributions **GPDs** **3D**

Transverse Momentum Distributions **TMDs** **3D**

$\int d^2\mathbf{k}_T$

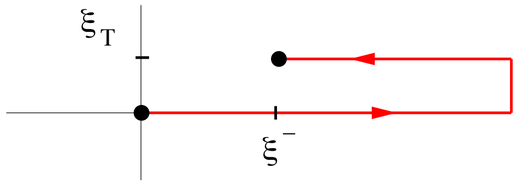
Parton Distribution Functions **PDFs** **1D**

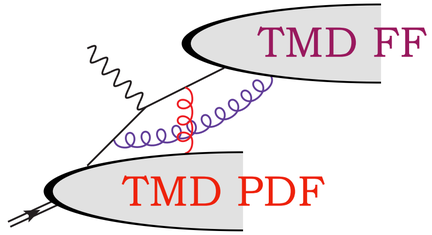
slide adapted from C. Bissolotti

Process dependence of quark TMD PDFs & FFs

SIDIS

[+] staple link





* PDF → color flow annihilated within final state
* FF → color flow from final to initial state

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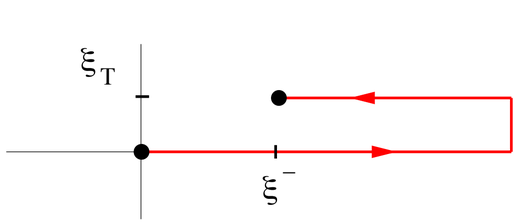

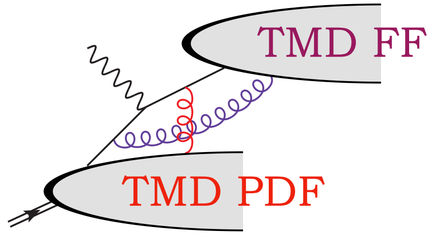

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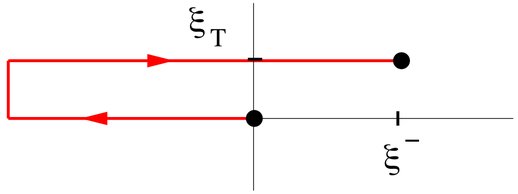
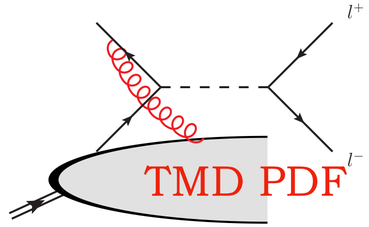
Drell-Yan

[–] staple link





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Process dependence of quark TMD PDFs & FFs

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Drell-Yan

[−] staple link

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$e^+ + e^- \rightarrow$ hadrons

[+] staple link

* FF → color flow annihilated within final state

[J. C. Collins, A. Metz (2004)]

Process dependence of quark TMD PDFs & FFs

SIDIS

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Drell-Yan

[−] staple link

* PDF → color flow from final to initial state

Modified universality

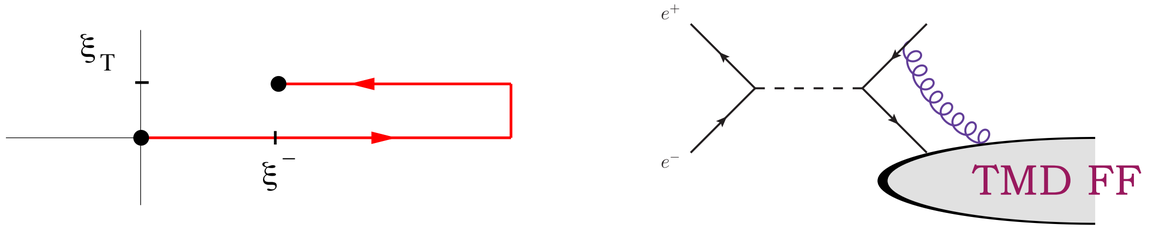
* PDFs → change of sign in T-odd densities

$$f_{1T}^{\perp\,[\mathrm{SIDIS}]} \equiv f_{1T}^{\perp\,[+]} = -f_{1T}^{\perp\,[-]} \equiv -f_{1T}^{\perp\,[\mathrm{DY}]}$$

* FFs → standard universality preserved

$e^+ + e^- \rightarrow$ hadrons

[+] staple link



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[J. C. Collins, A. Metz (2004)]

Gluon TMD PDFs: Gauge links & modified universality

* Gluon TMDs → more complicated structure with respect to quark staple links
* Factorization-preserving processes → two main kinds of modified universality
* Different classes of processes → distinct gluon TMDs, not related to each other

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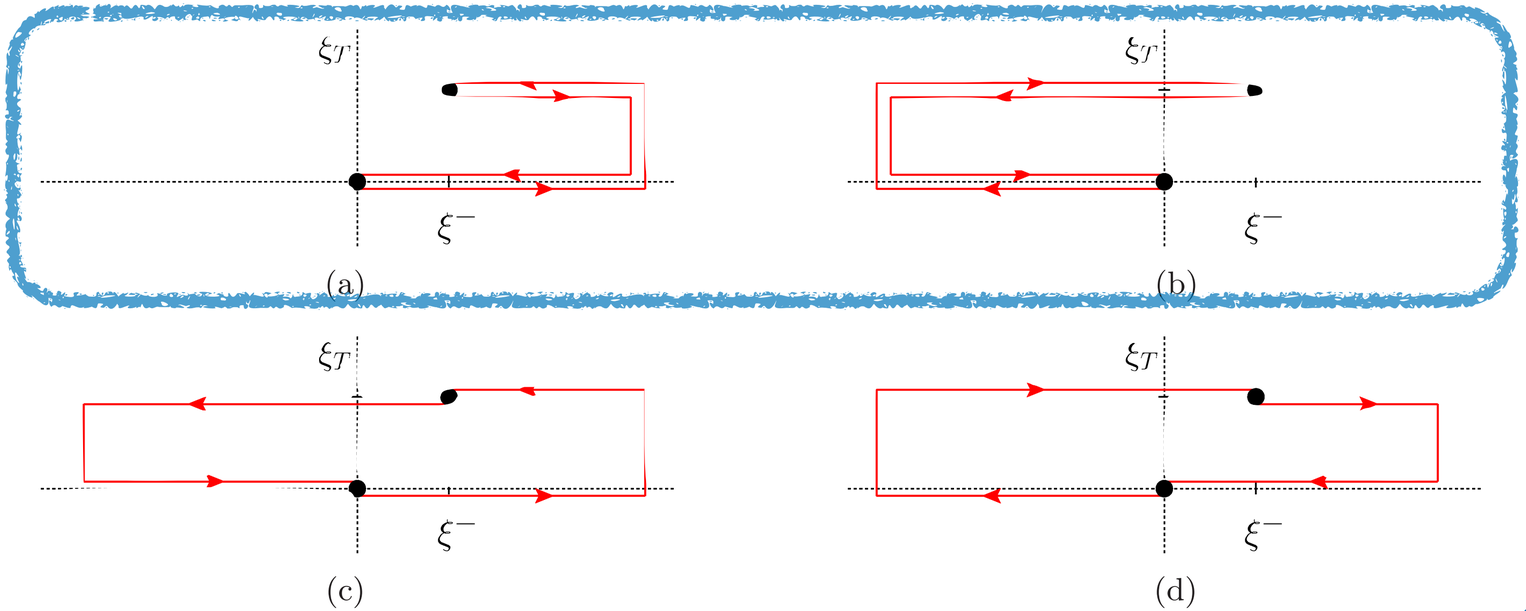
f-type (WW)

(a) $[+,+]$ or (b) $[-,-]$



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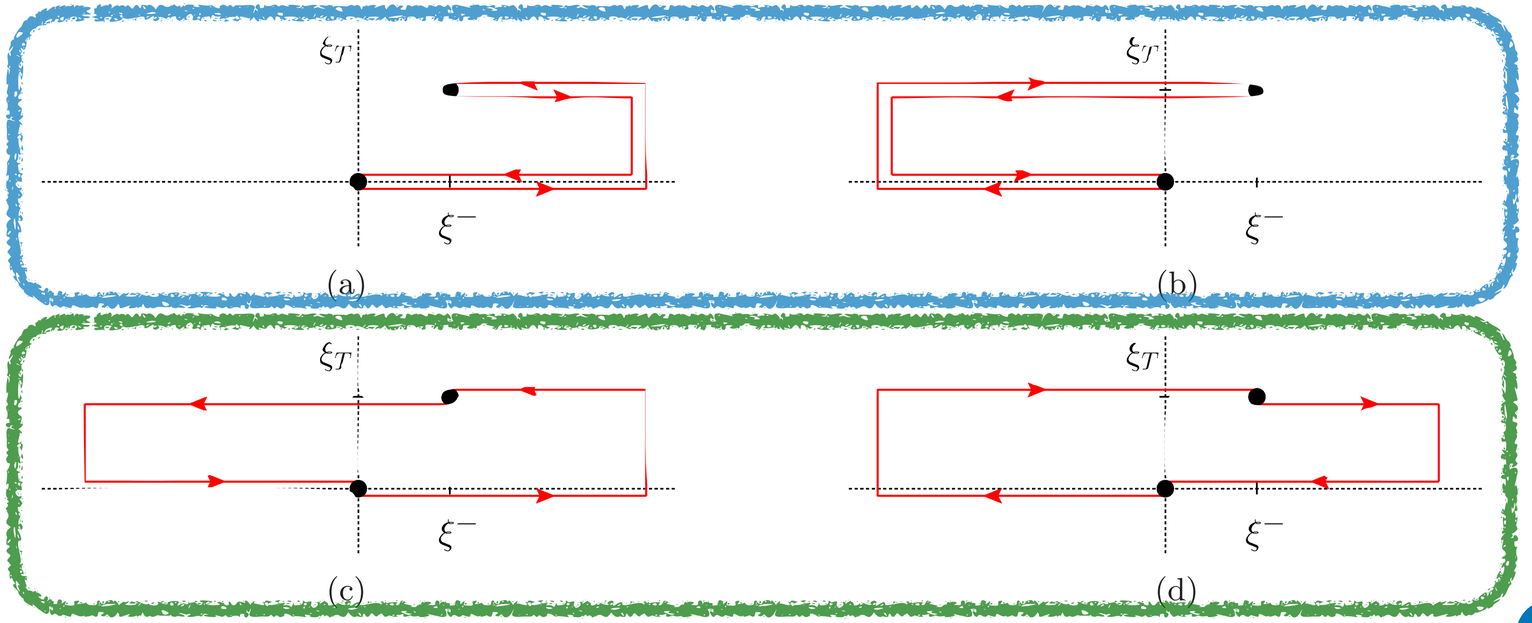
d-type (dipole)

(c) $[+,-]$ or (d) $[-,+]$



Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

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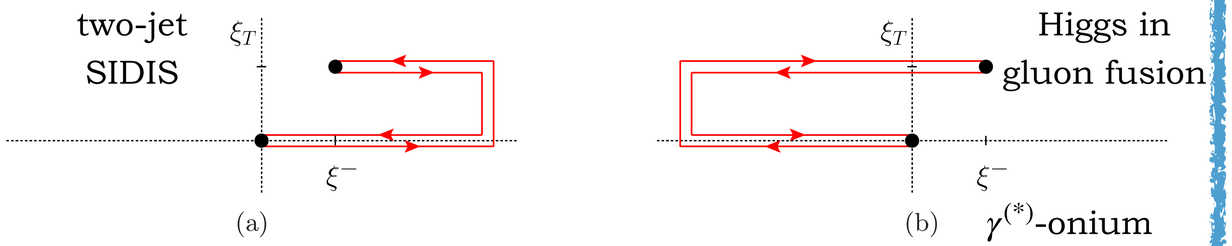

Higgs in gluon fusion

$\gamma^{(*)}$-onium hadroproduction

Accessing f-type and d-type gluon TMD PDFs

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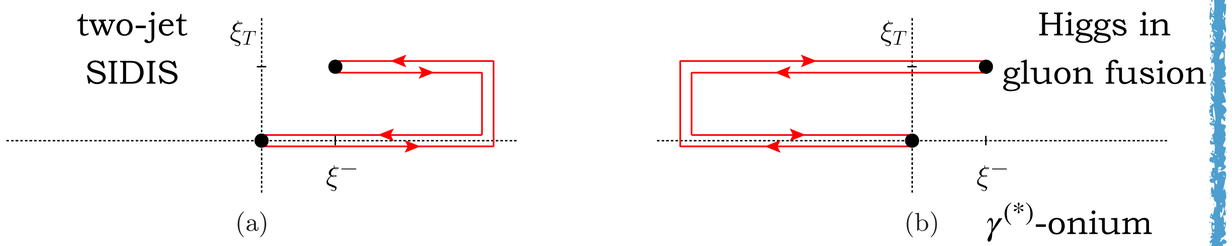

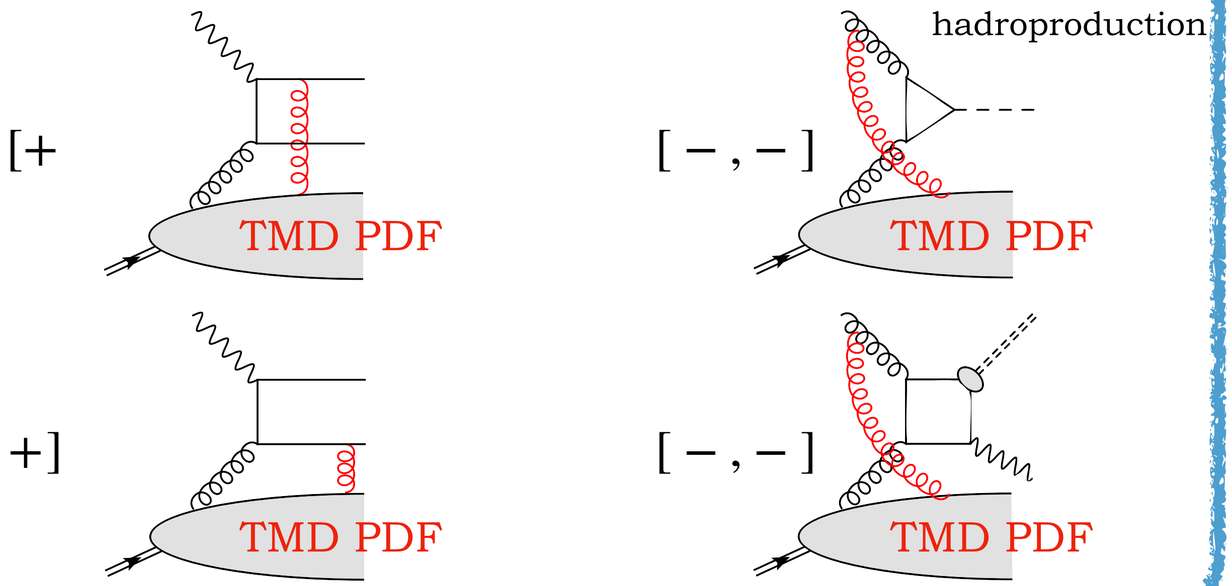

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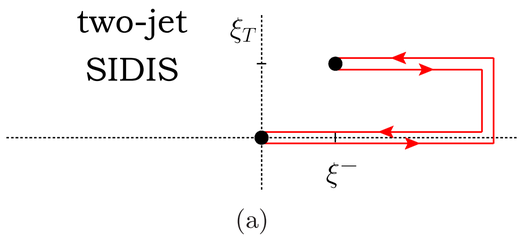
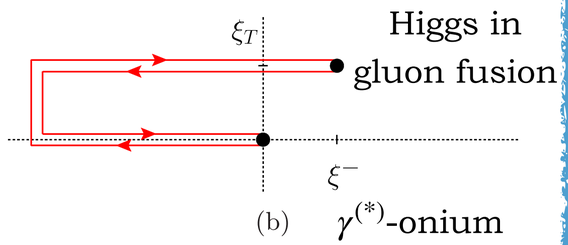
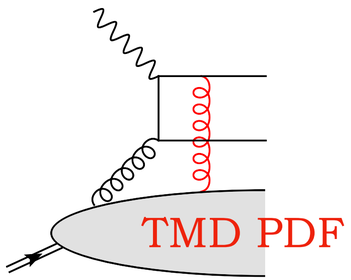




[+



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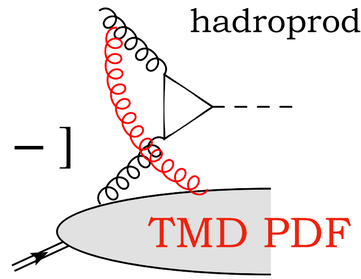
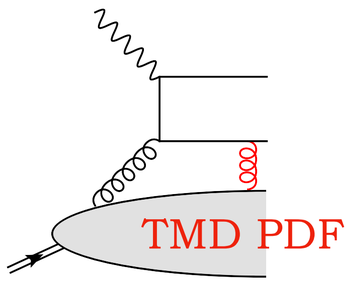


+]



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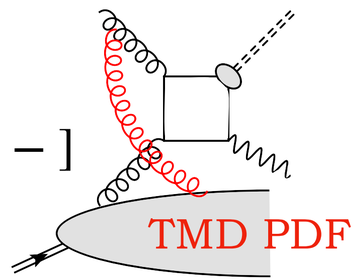
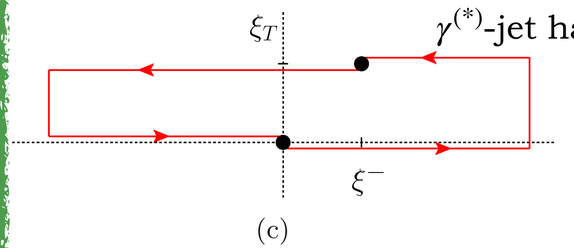
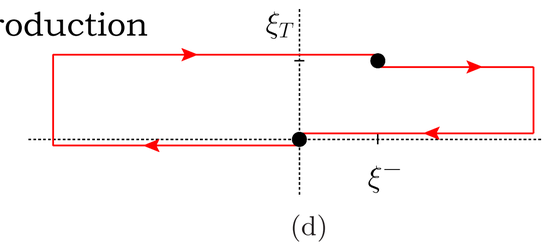


d-type (DP)

(c) [+,−] or (d) [−,+]





Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

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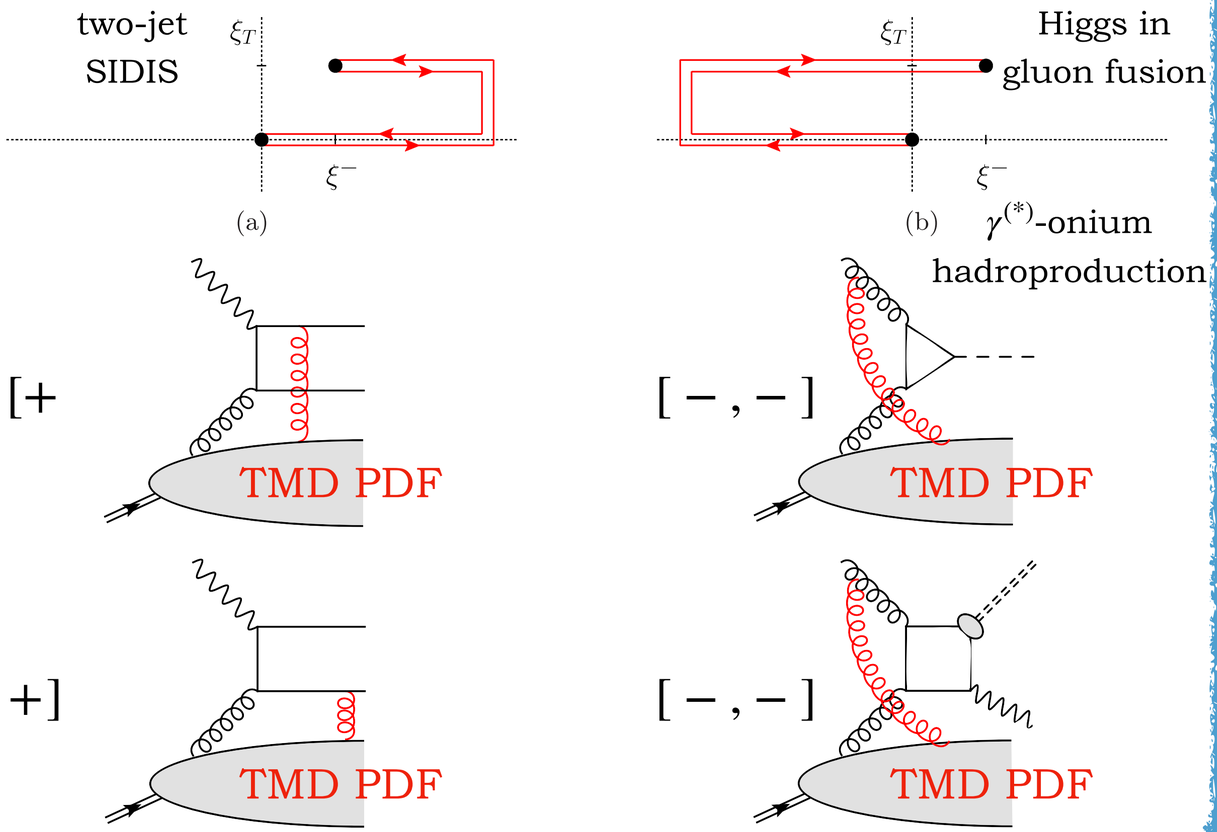
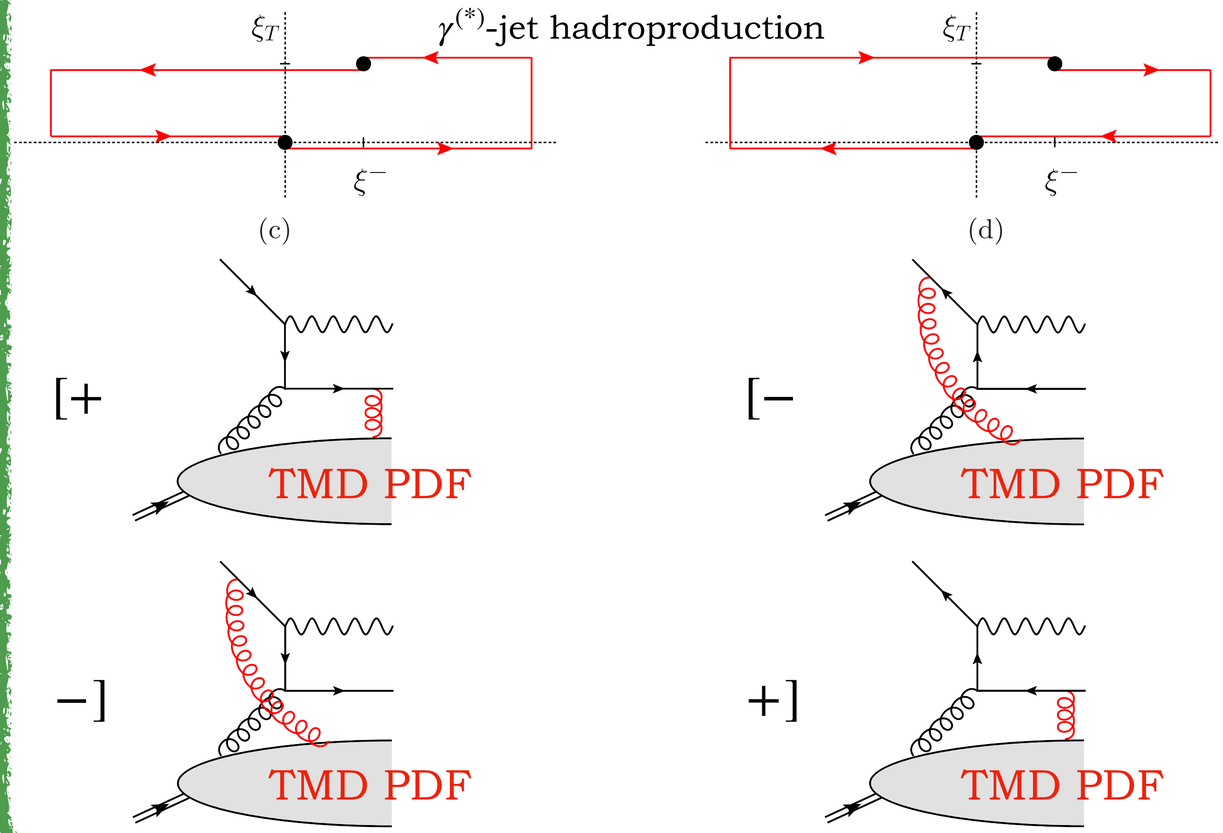


d-type (DP)

(c) [+ , −] or (d) [− , +]



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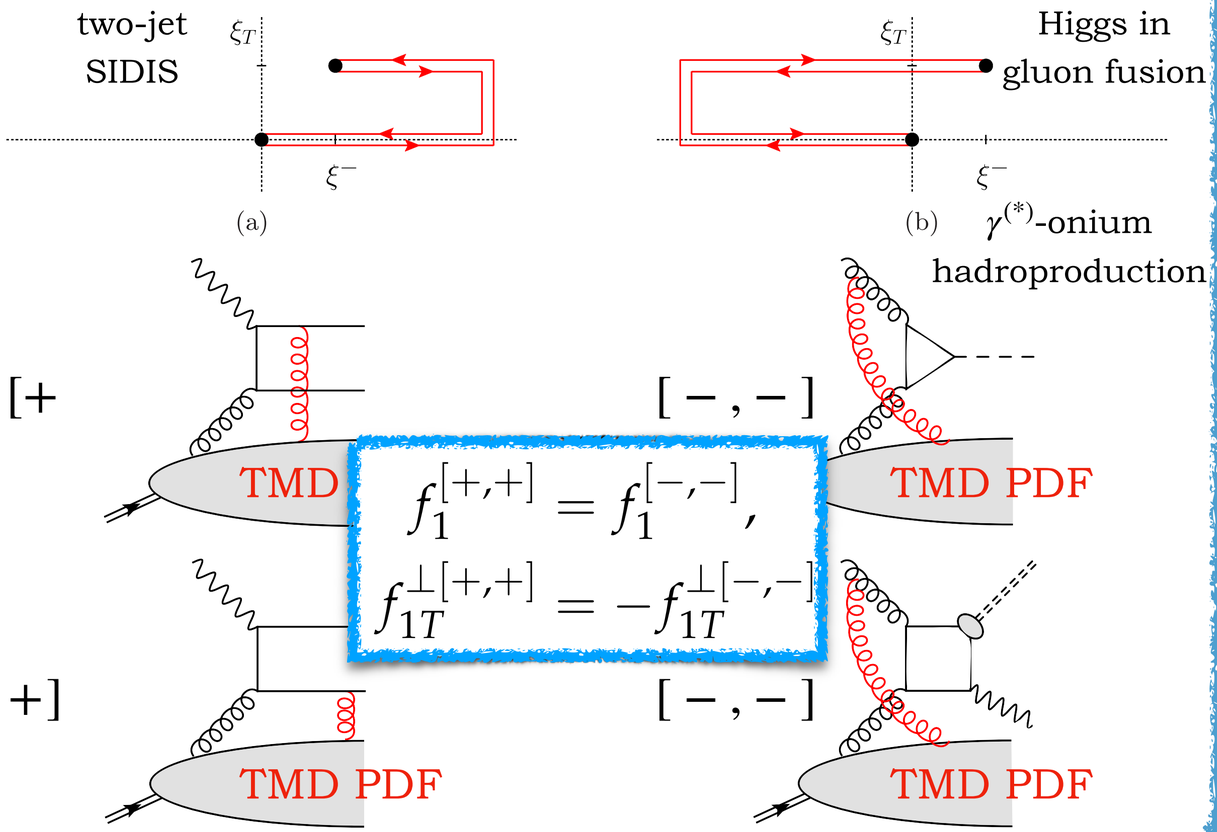

$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

d-type (DP)

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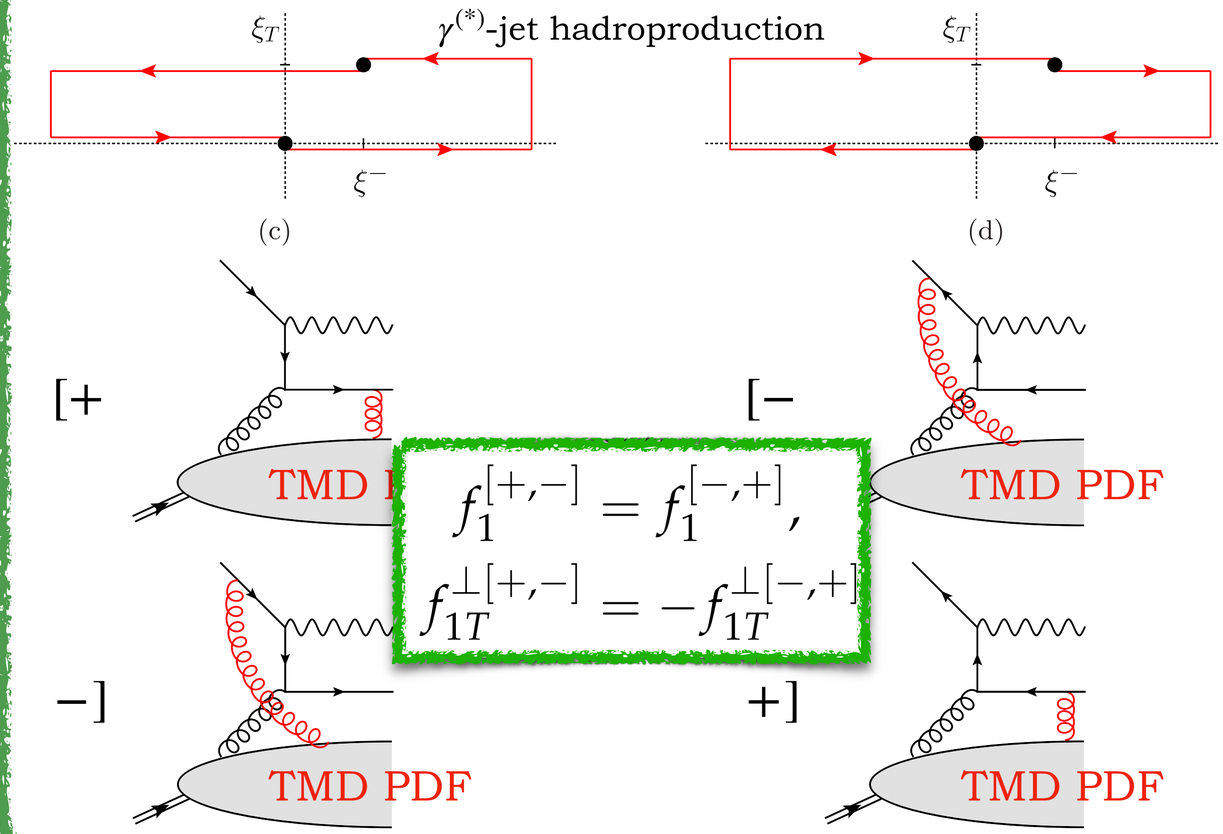

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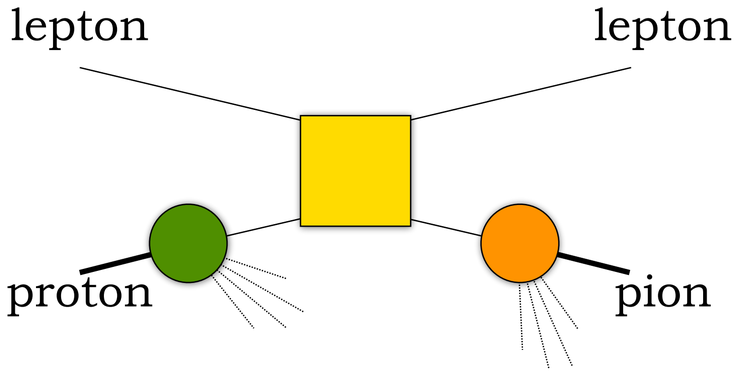
$$f_{1T}^{\perp[+,-]} = -f_{1T}^{\perp[-,+]}$$

¡ Gauge link → two main independent sets of TMDs, **not related** to each other !

Factorization and universality

SIDIS



Factorization and universality

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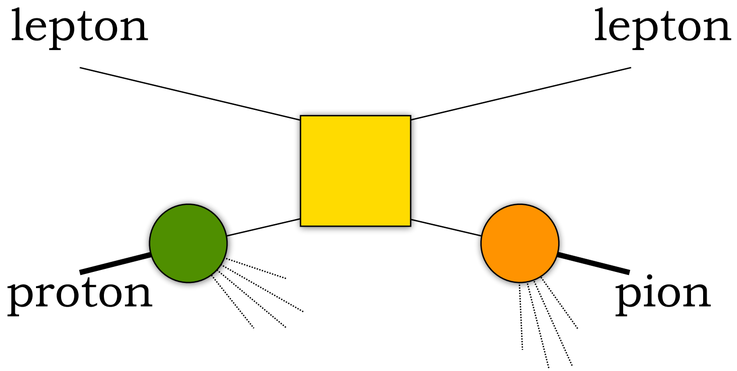

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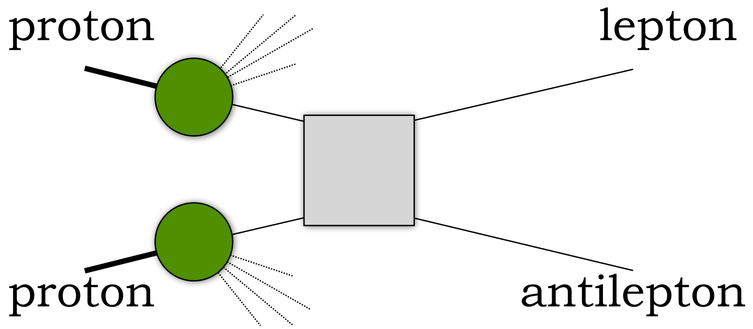

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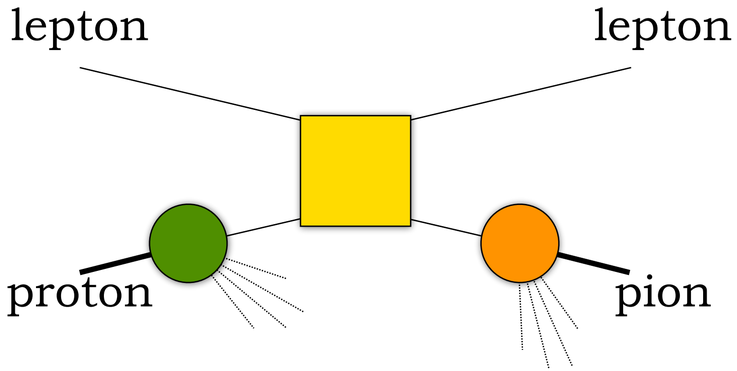

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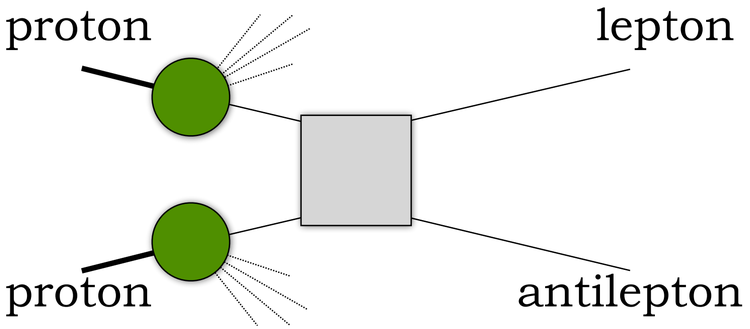
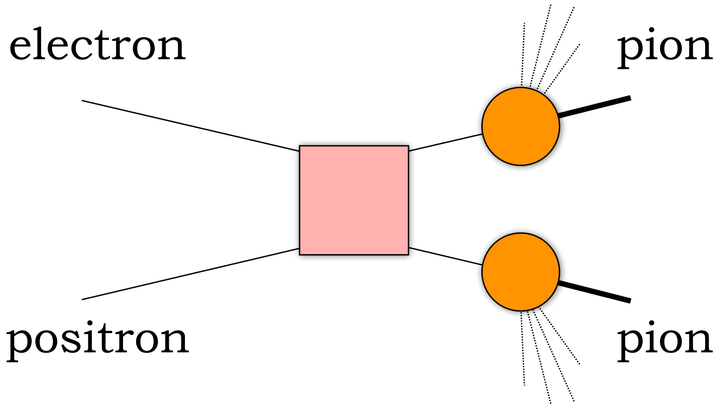
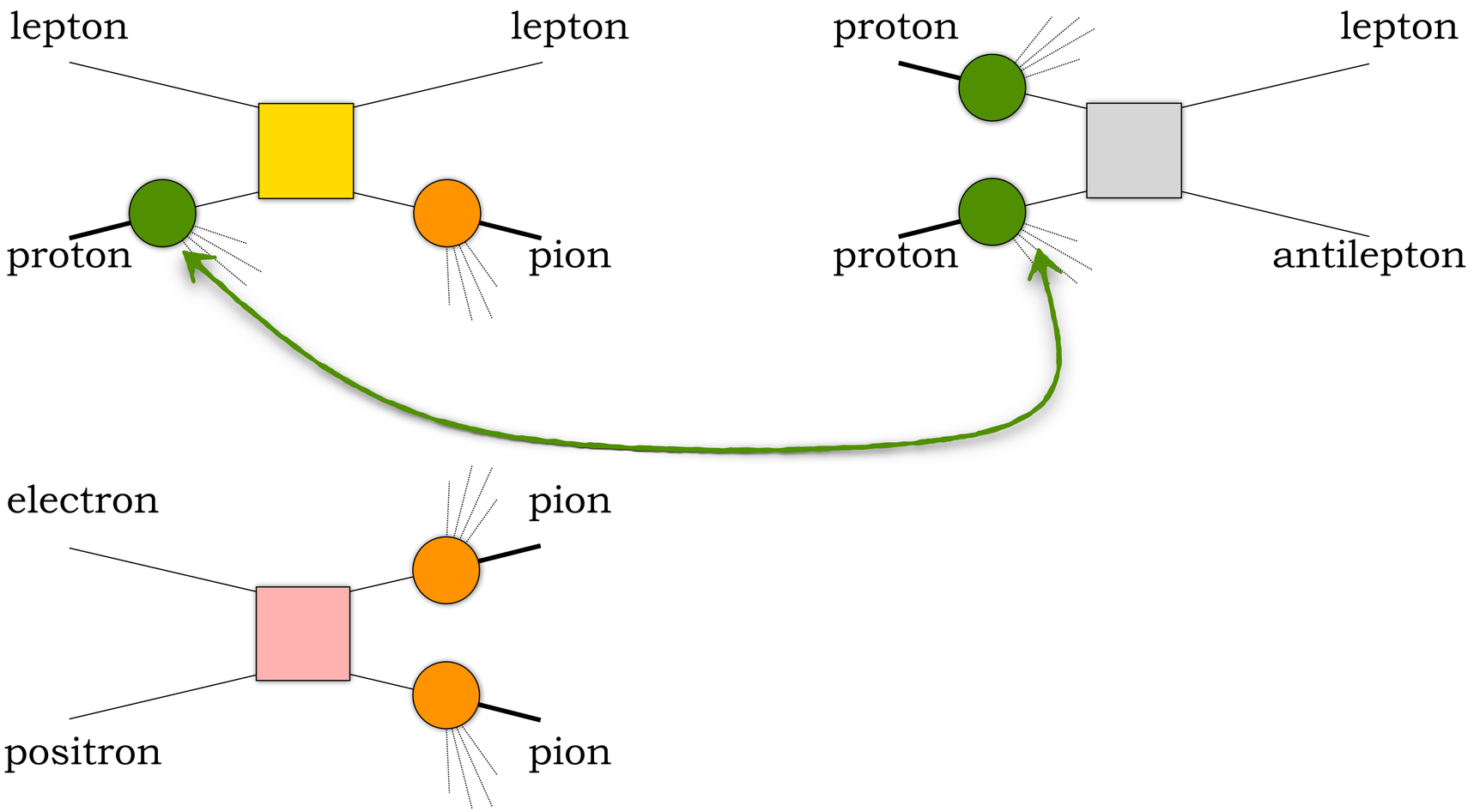




$e^+ + e^- \rightarrow$ **hadrons**

Factorization and universality

SIDIS **Drell-Yan**



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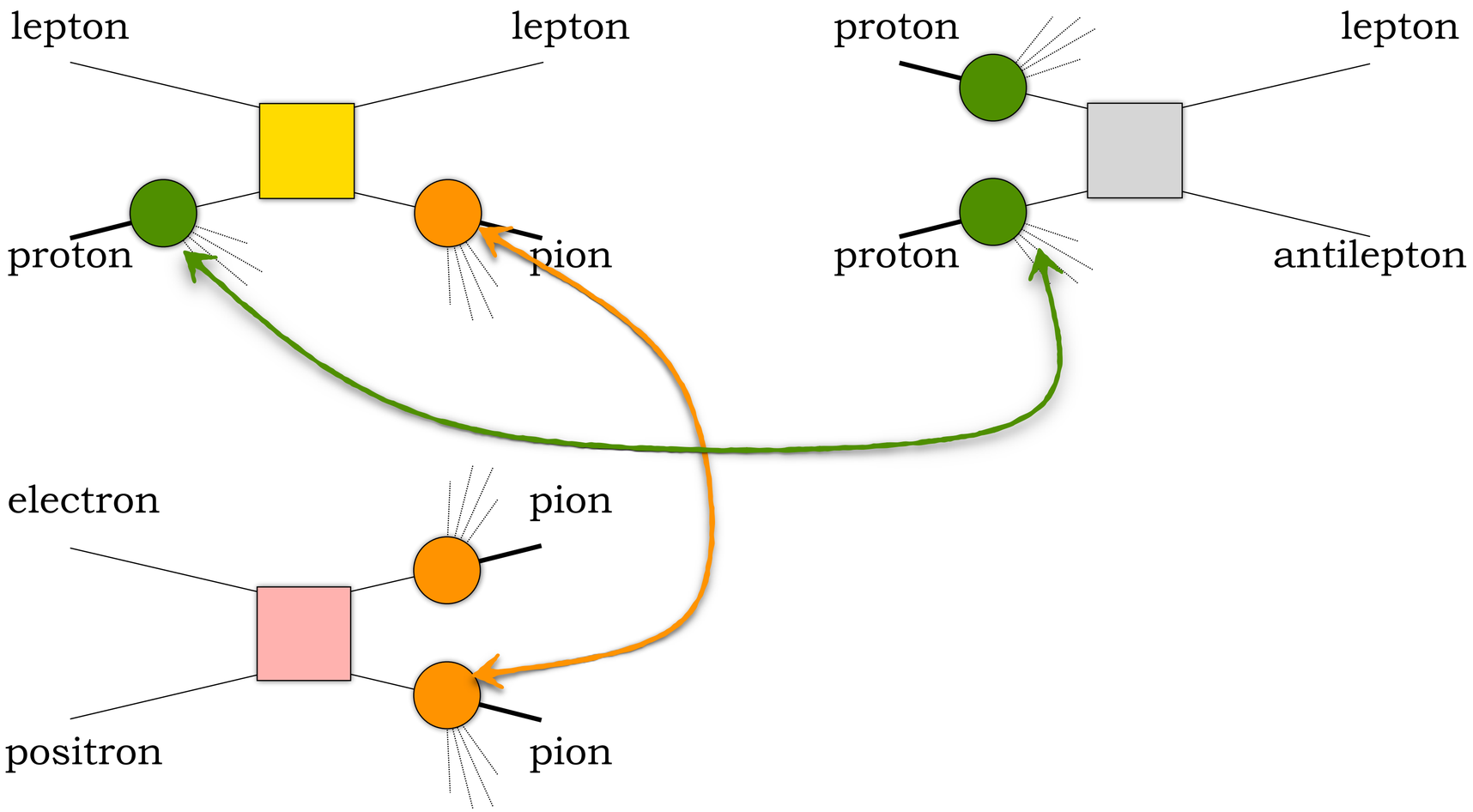

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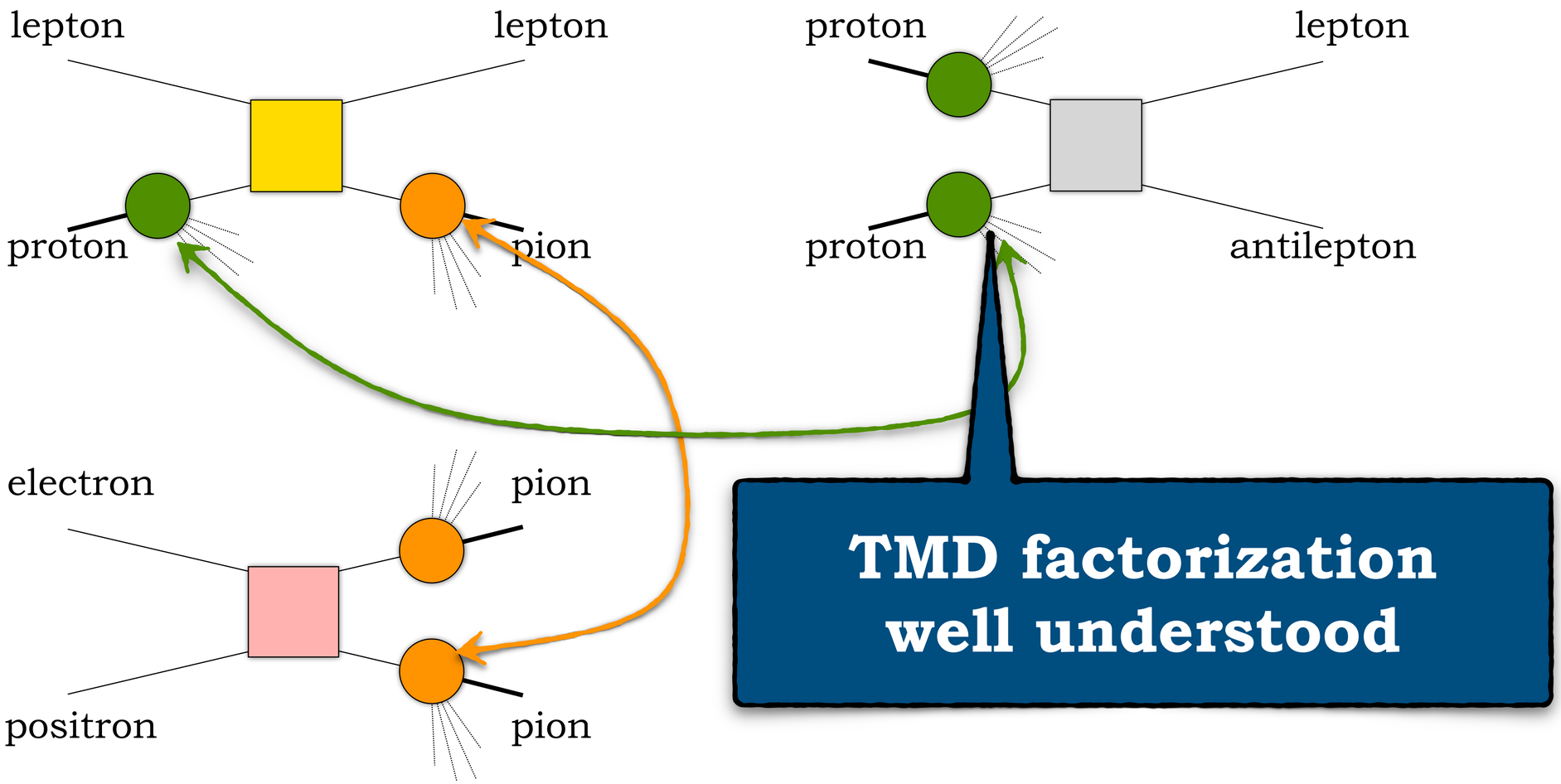

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Factorization and universality

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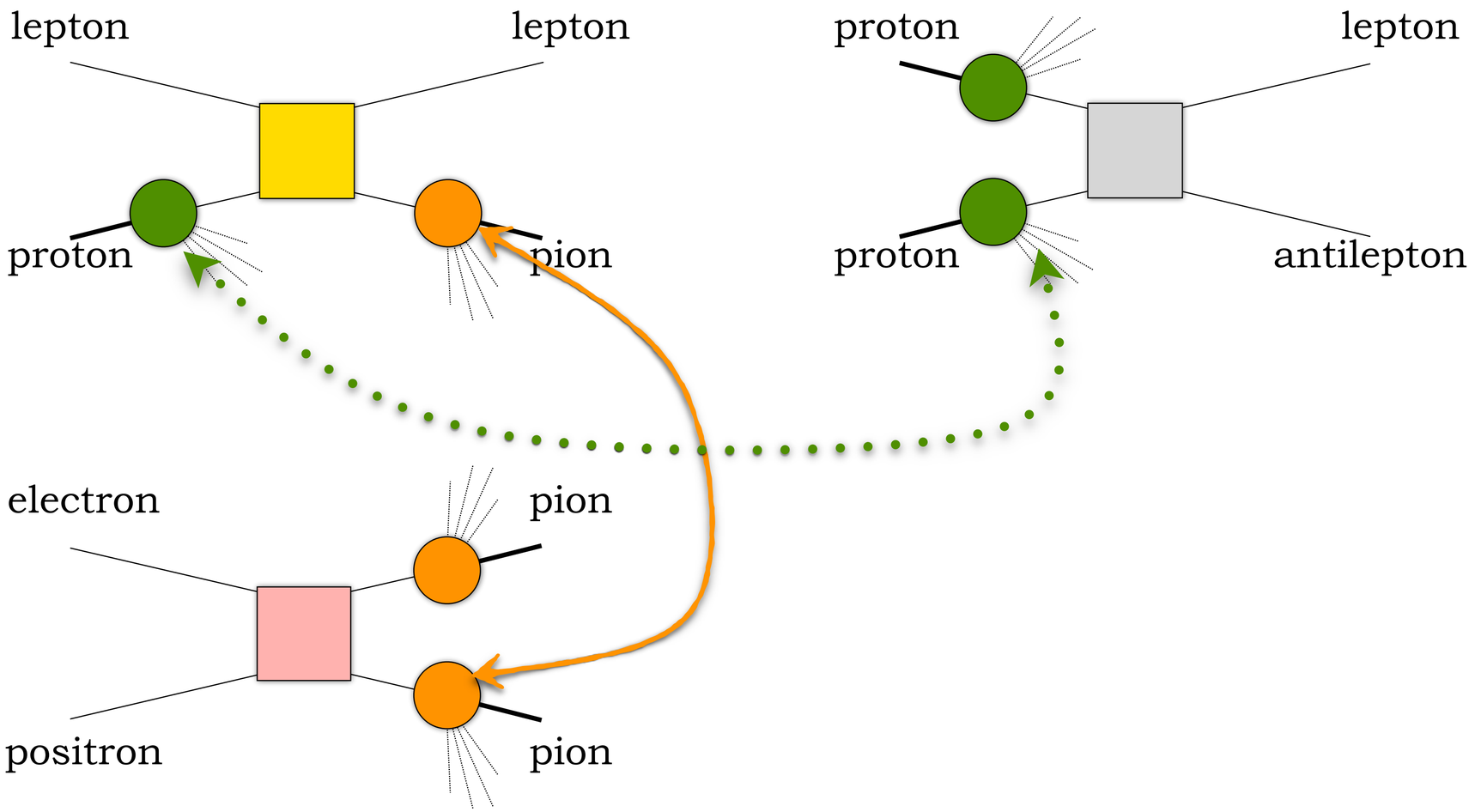

$e^+ + e^- \rightarrow$ **hadrons**

Factorization and universality

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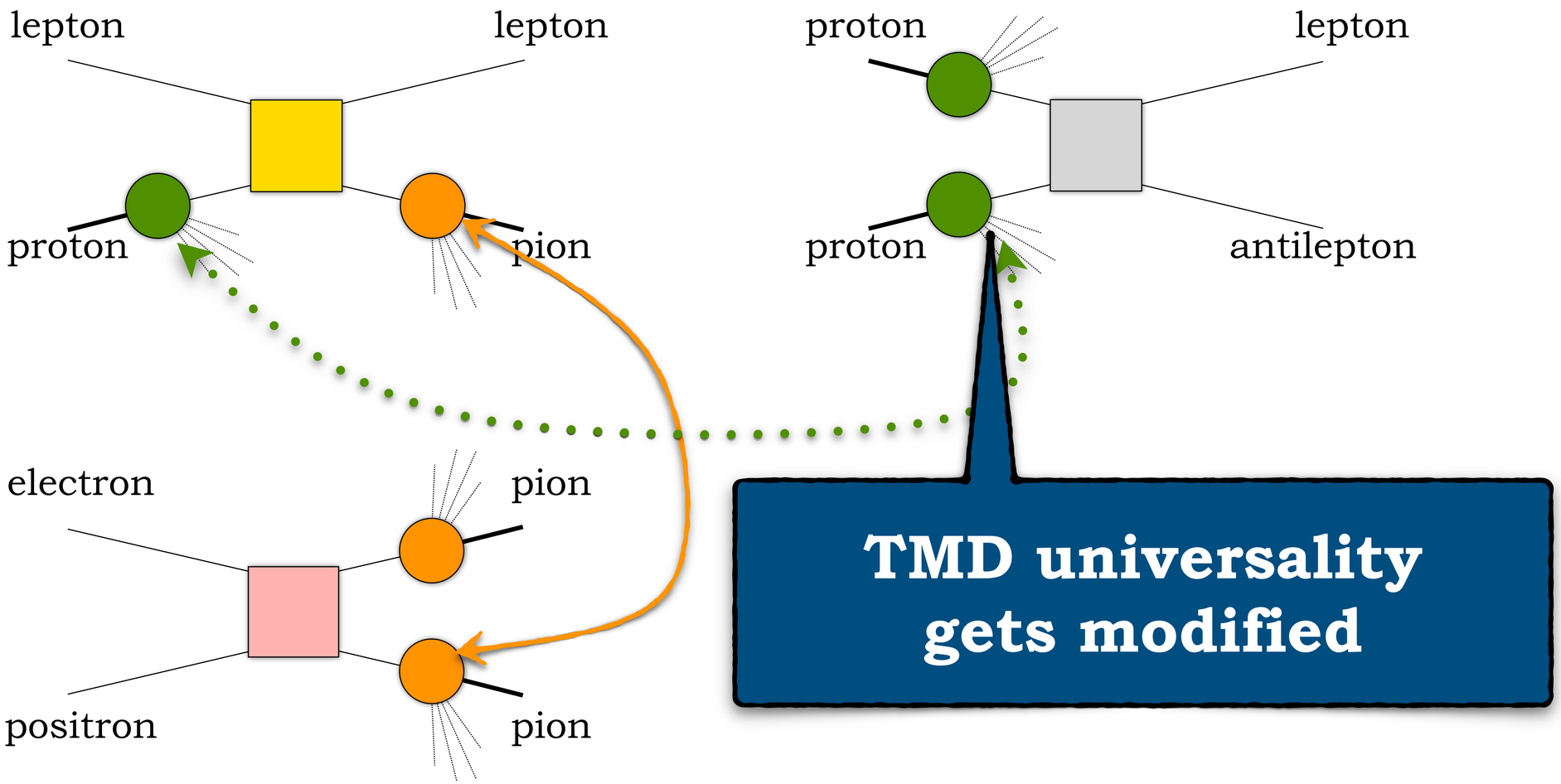
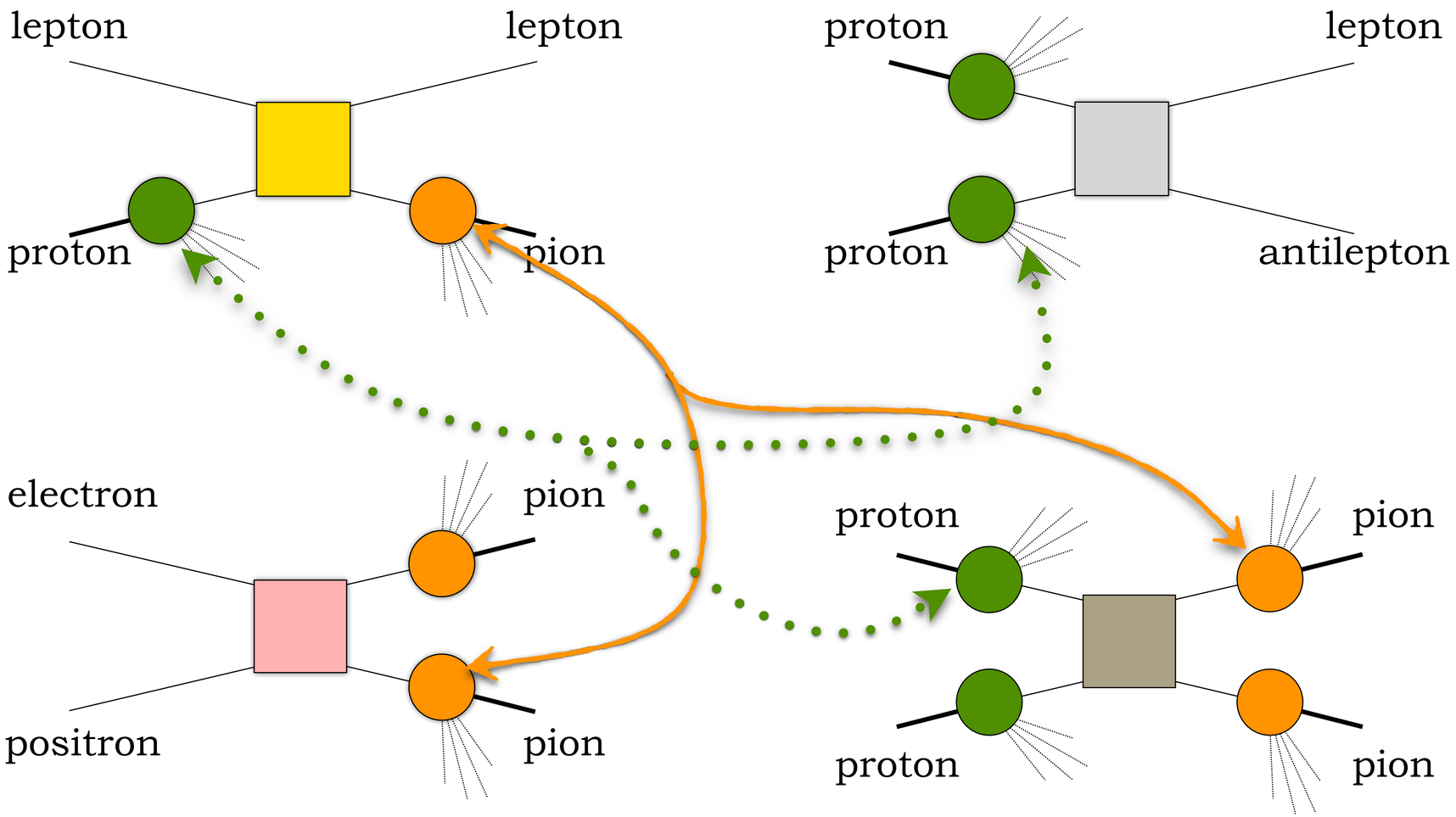


$e^+ + e^- \rightarrow$ **hadrons**

Factorization and universality

SIDIS **Drell-Yan**



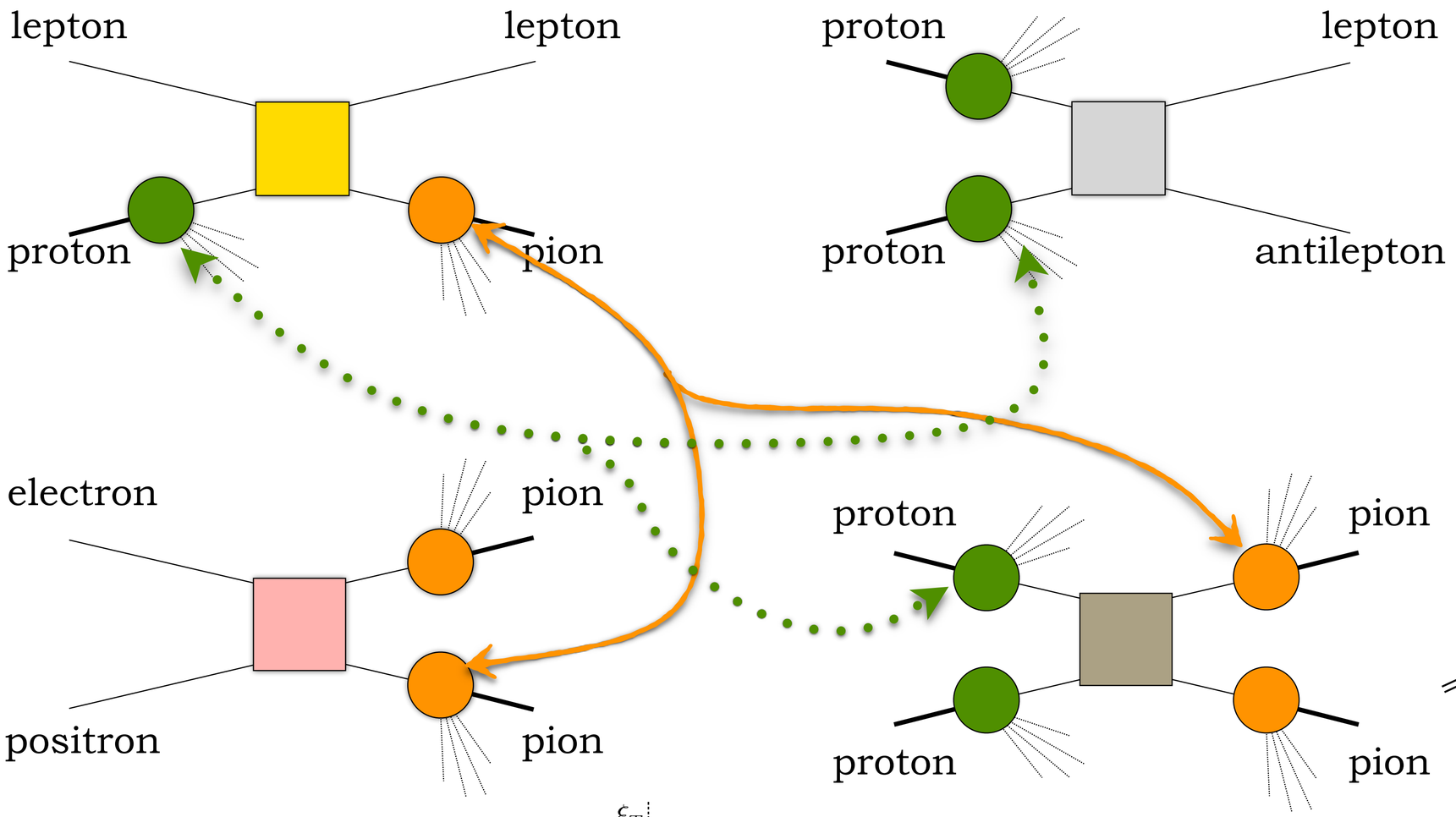
$e^+ + e^- \rightarrow$ **hadrons**

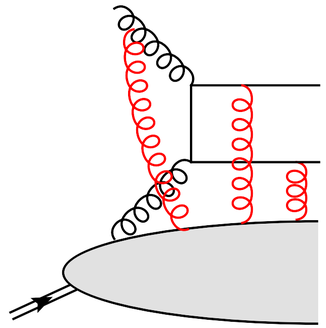
$p + p \rightarrow$ **hadrons**

Factorization and universality

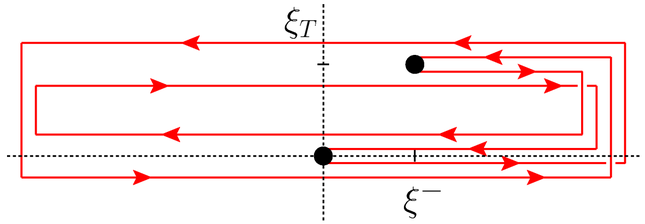
SIDIS

Drell-Yan





$e^+ + e^- \rightarrow$ **hadrons**

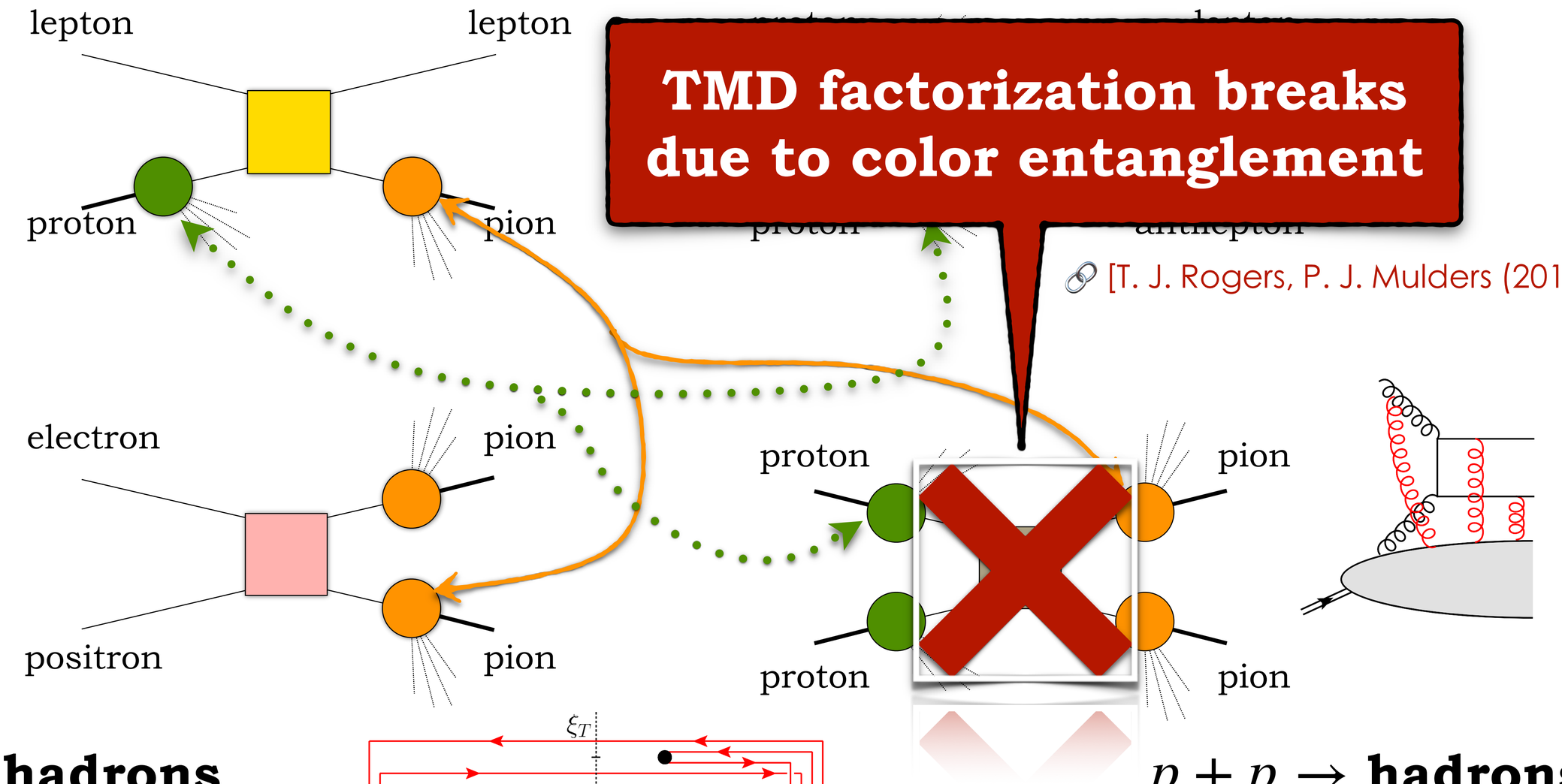


$p + p \rightarrow$ **hadrons**

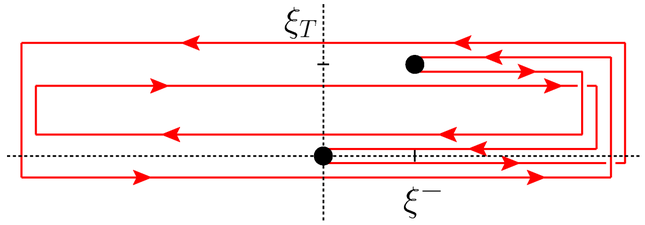
(also) gluon-induced channel

Factorization and universality

SIDIS **Drell-Yan**



$e^+ + e^- \to$ **hadrons**



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Factorization and universality

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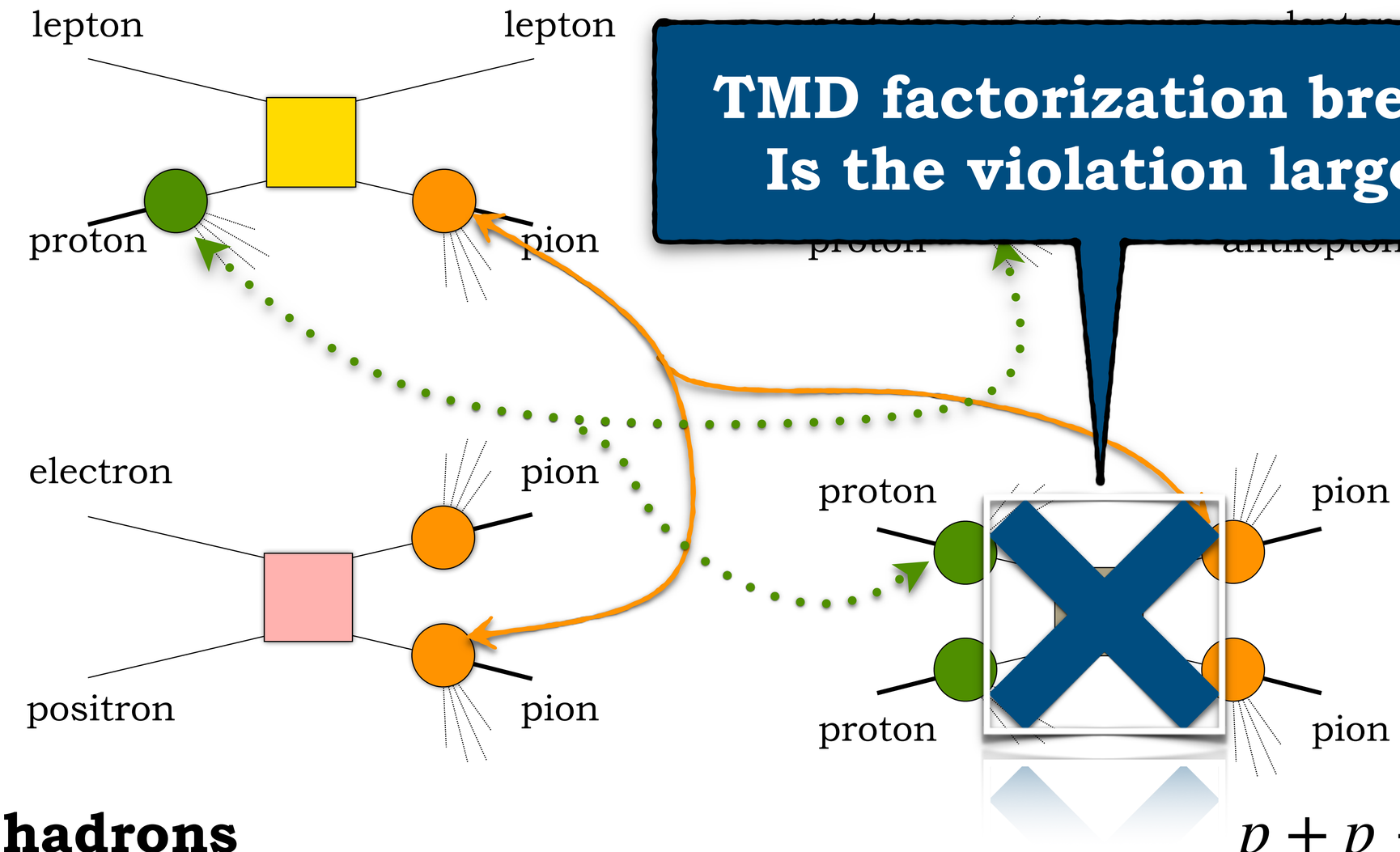


$e^+ + e^- \rightarrow$ **hadrons**

$p + p \rightarrow$ **hadrons**

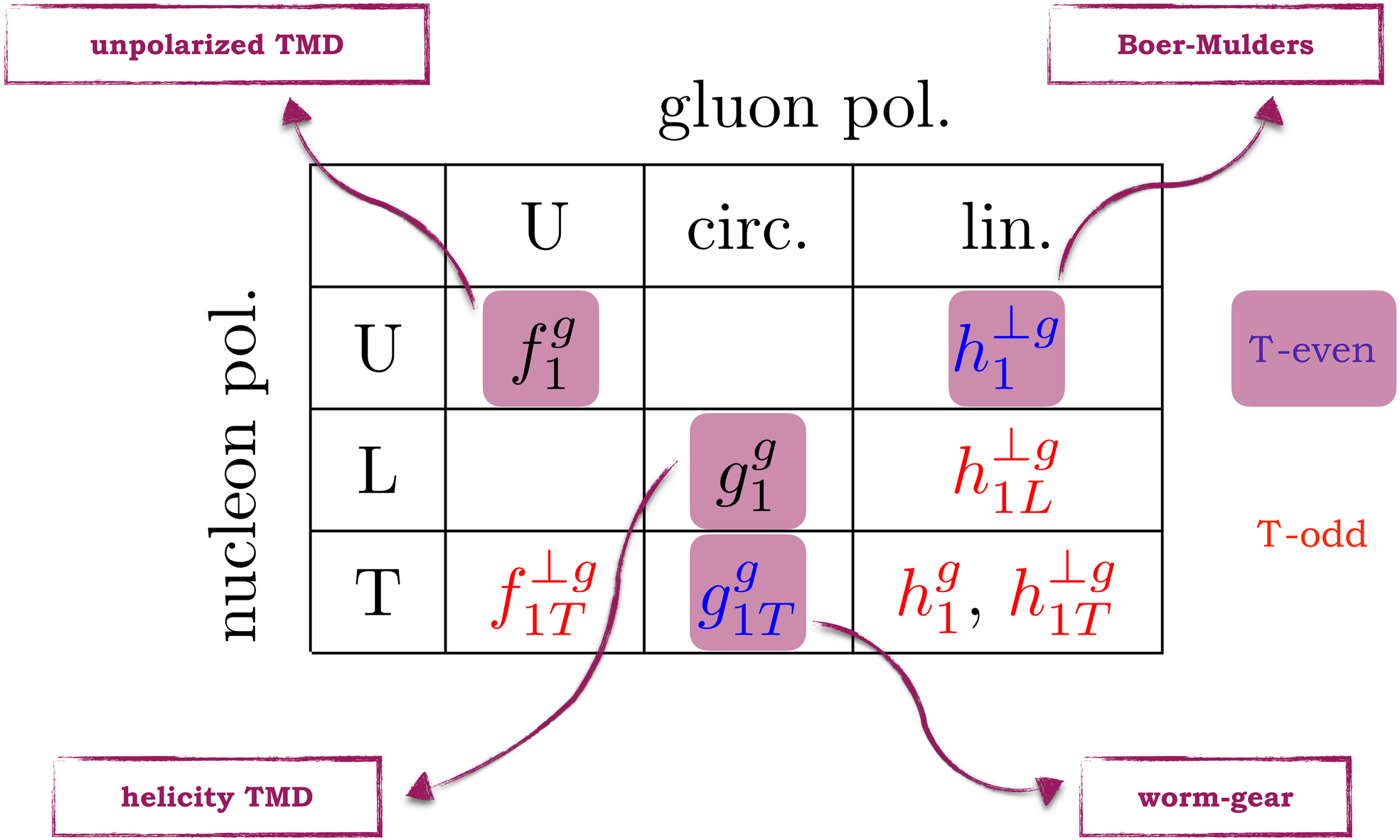
Gluon TMD PDFs at leading twist

nucleon pol. \ gluon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

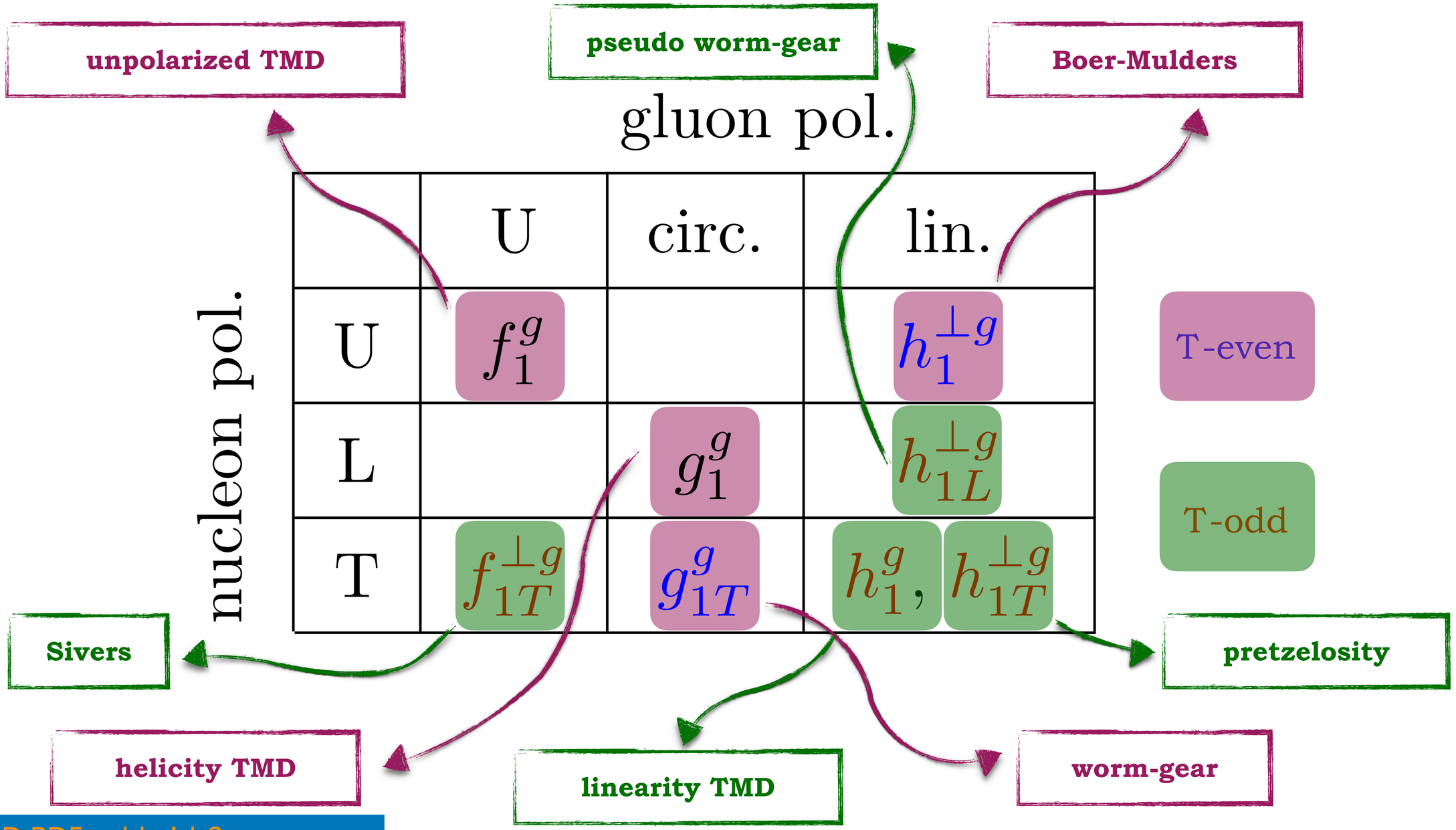
T-even

T-odd

Gluon TMD PDFs at leading twist



Gluon TMD PDFs at leading twist

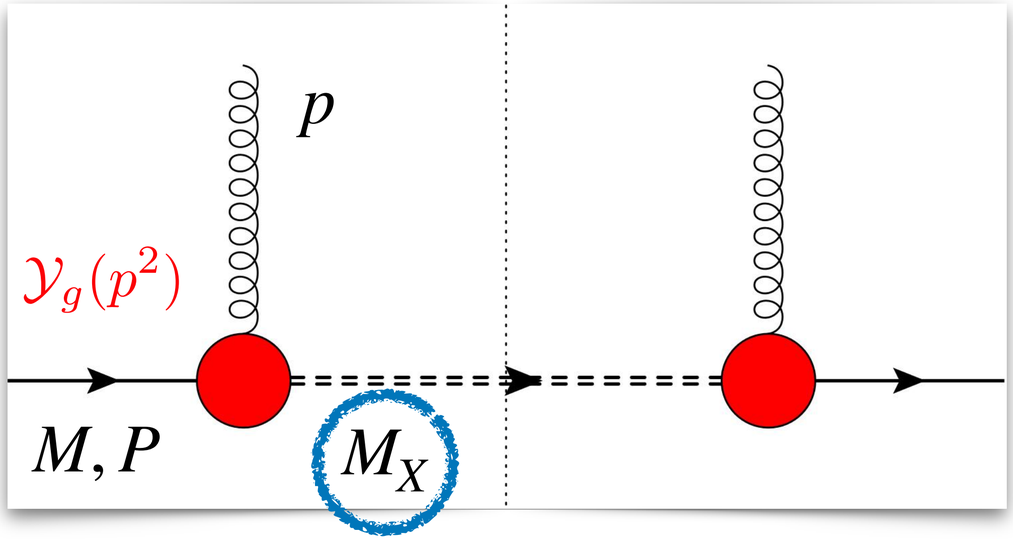


2. Modeling gluon TMD PDFs

Spectator-model gluon TMDs

Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
on-shell and
with mass M_X



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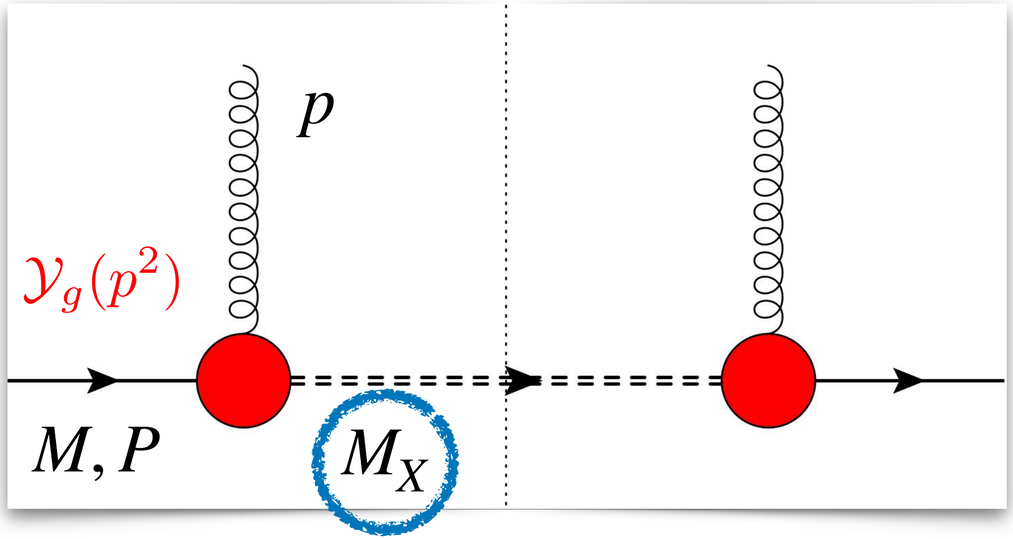


Nucleon-gluon-spectator vertex

$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr\left[(\not{P}+M)\frac{1+\gamma^5\not{S}}{2}G^*_{\mu\rho}(p)G^{\nu\sigma}(p)\mathcal{Y}_g^{\rho *}\mathcal{Y}_{g\sigma}(\not{P}-\not{p}+M)\right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2)\gamma^\mu + i\frac{g_2(p^2)}{2M}\sigma^{\mu\nu}p_\nu$$

mimics proton form factors
(conserved EM current
of a free nucleon)

Spectator-model gluon TMDs

Link with collinear factorization

1. p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

Spectator-model gluon TMDs

Link with collinear factorization

1. p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2}\,\frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

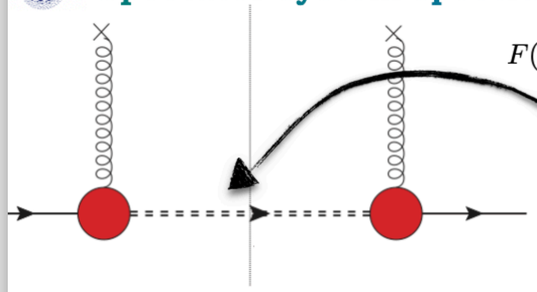
1. Cancels singularity of gluon propagator
2. Suppresses effects of high p_T
3. Compensates log divergences arising from p_T-integration
4. Adds three more parameters: $\kappa_{1,2}$ and Λ_X

Spectator-model gluon TMDs

Our model at a glance

Spectator-system spectral-mass function

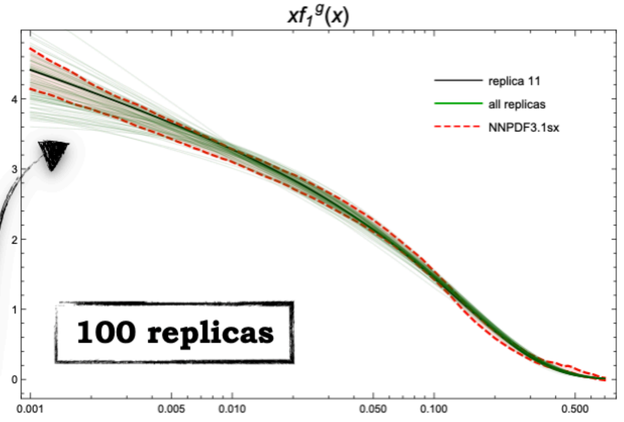
$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$



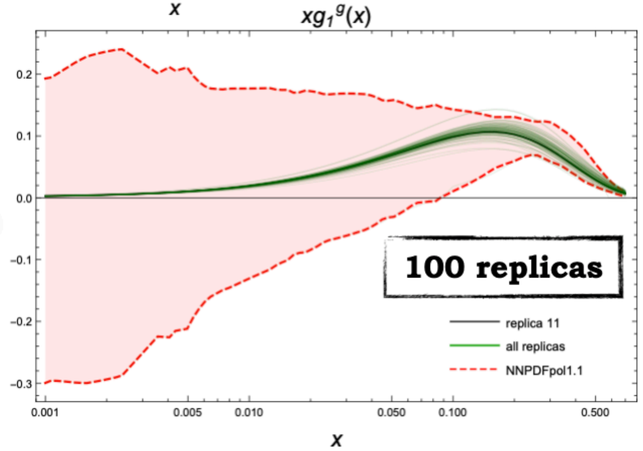
Instead of a single on-shell spectator, a continuum of spectators

Link with collinear factorization

p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution



Spectral function **learns** small- and moderate-x info encoded in **NNPDF** collinear parametrizations (`NNPDF3.1sx + NNPDFpol1.1`)



- ☑ **Simultaneous fit** of f_1 and g_1 PDFs
- ☑ Inclusion of small-x resummation effects (**BFKL**)
- ☑ Calculation of all leading-twist T-even gluon TMDs

2.3 Modeling gluon TMDs 🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]

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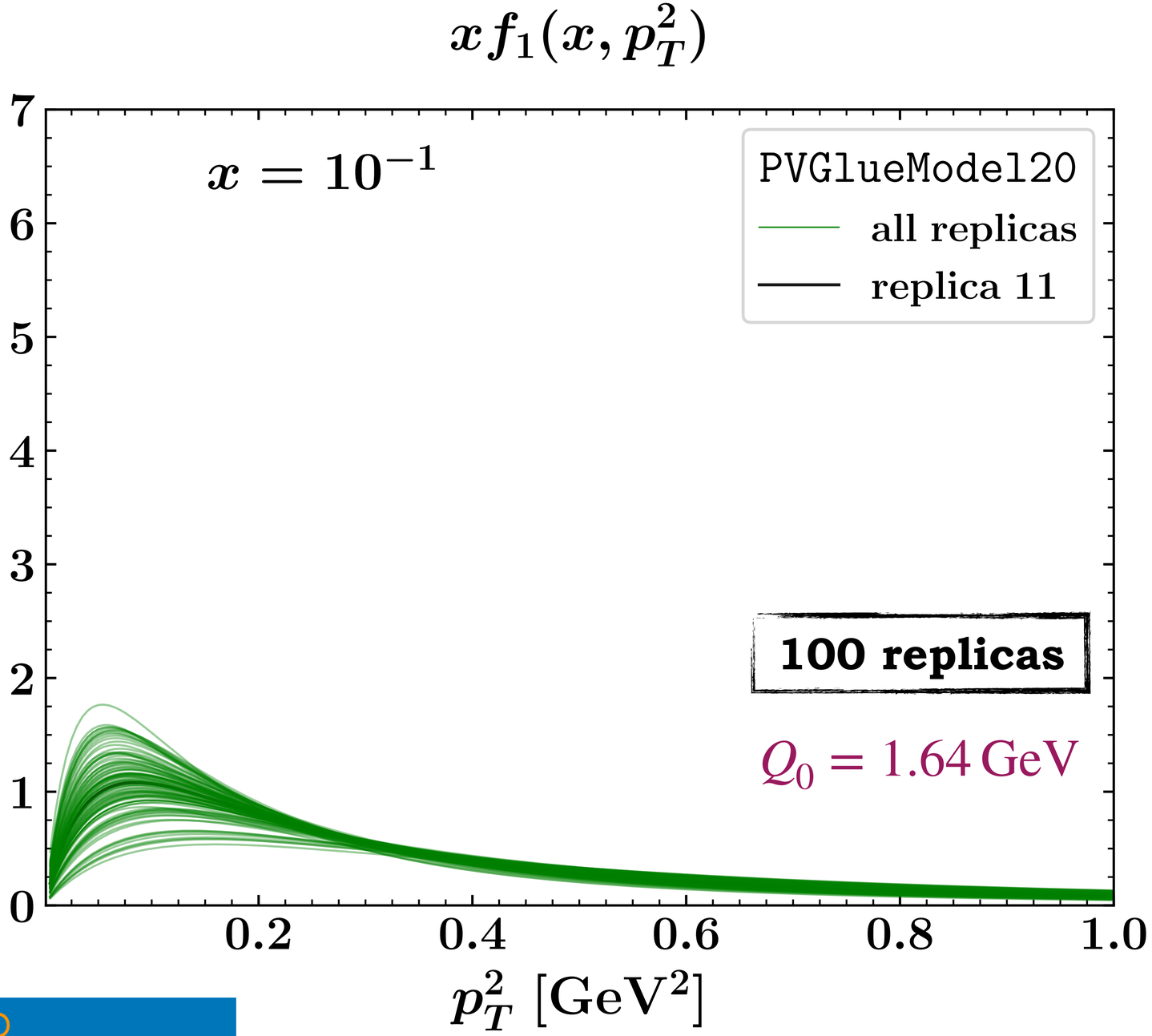
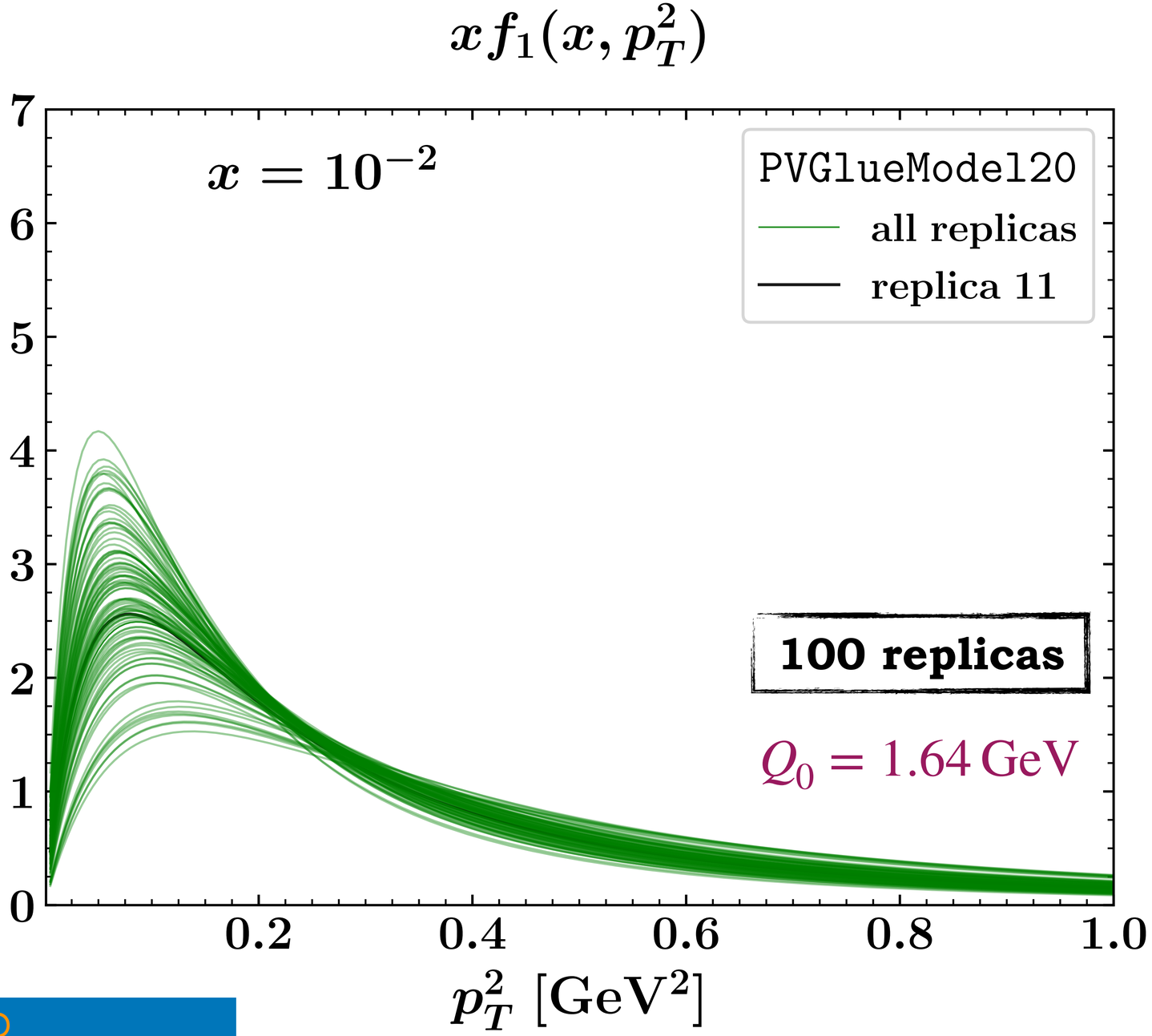
Unpolarized gluon TMD

[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]



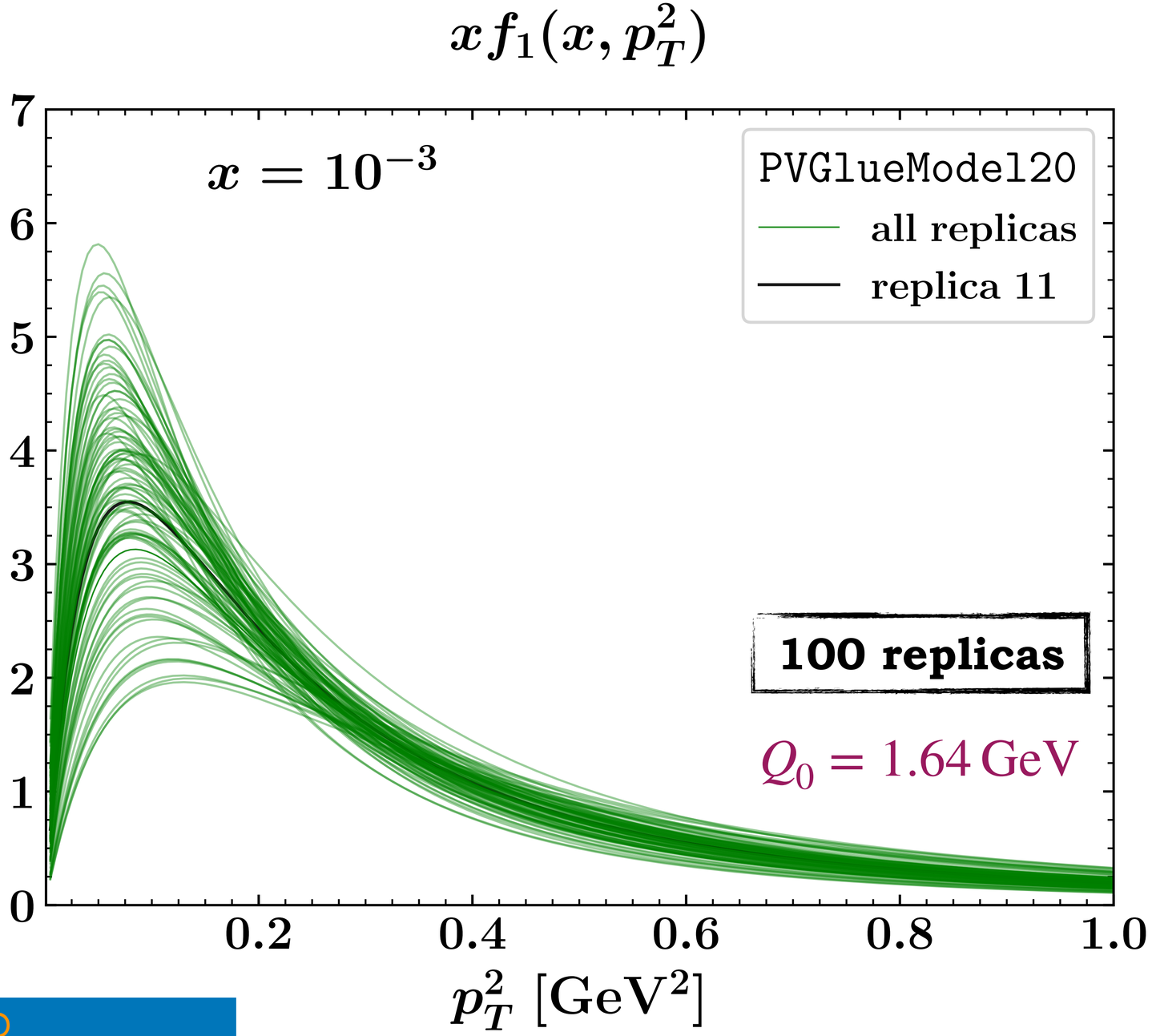
Unpolarized gluon TMD

[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]



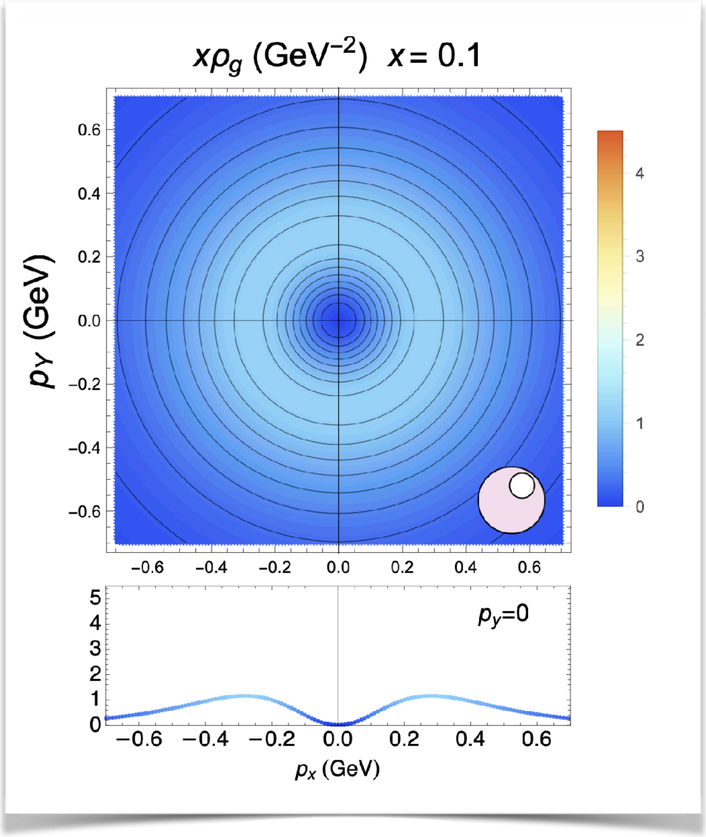
Unpolarized gluon TMD

[A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)]



3D proton imaging: Tomographic reconstruction & TMDs

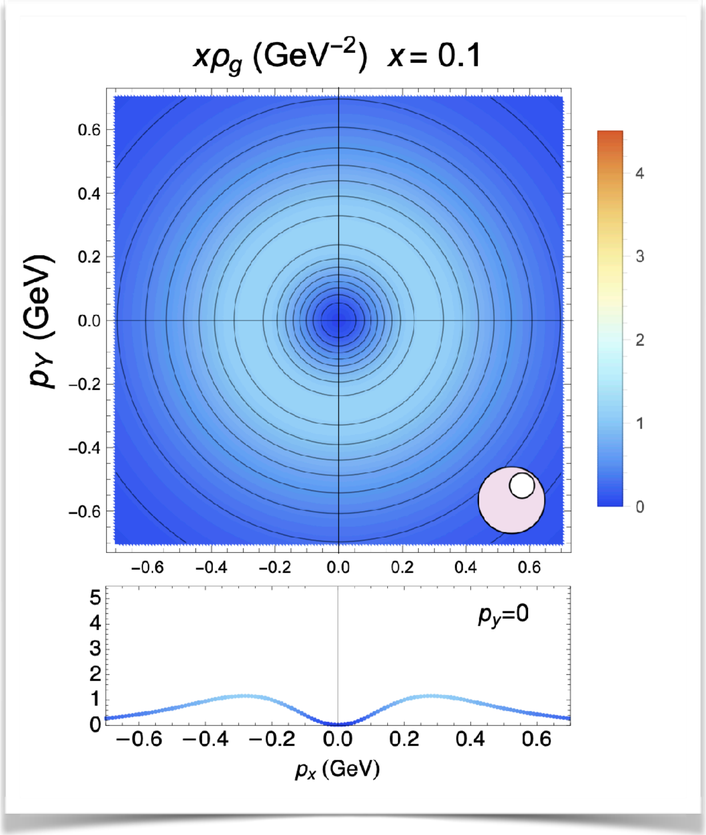
Unpolarized TMD



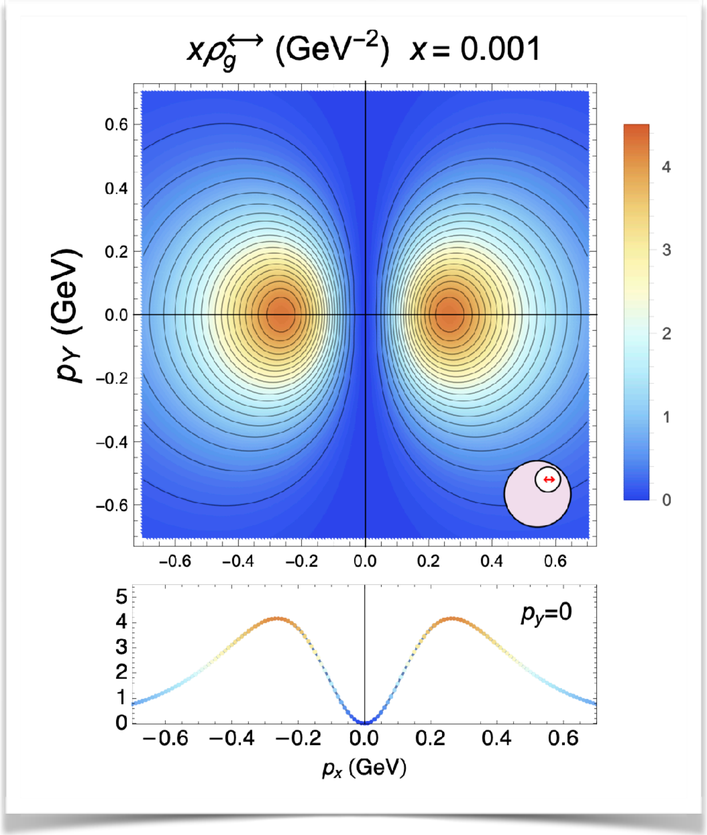
[A. Bacchetta, F.G.C., M. Radici, P. Taels, Eur. Phys. J. C 80 (2020) no.8]

3D proton imaging: Tomographic reconstruction & TMDs

Unpolarized TMD



Boer-Mulders



🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels, Eur. Phys. J. C 80 (2020) no.8]

3D proton imaging: Tomographic reconstruction & TMDs

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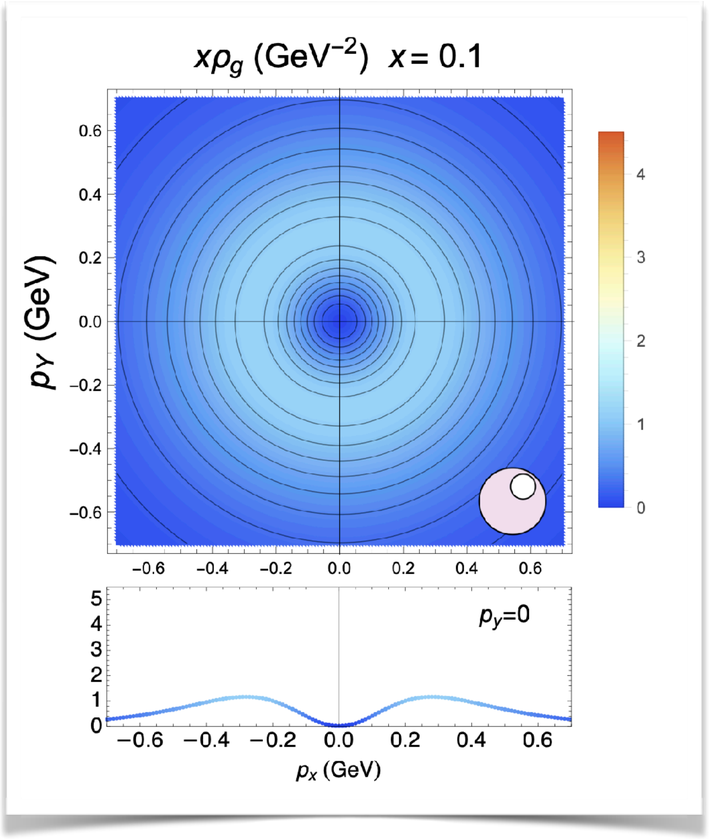
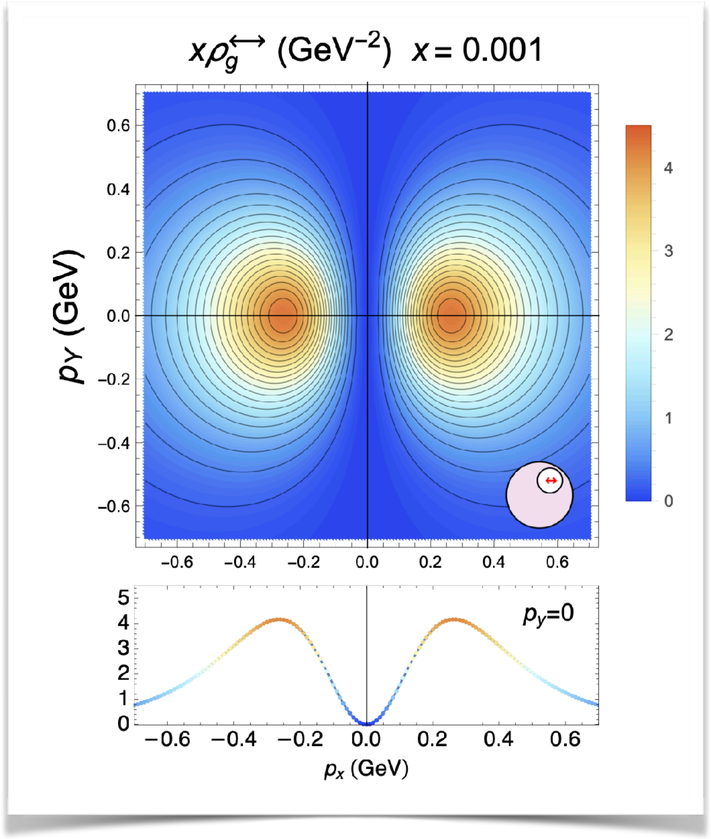
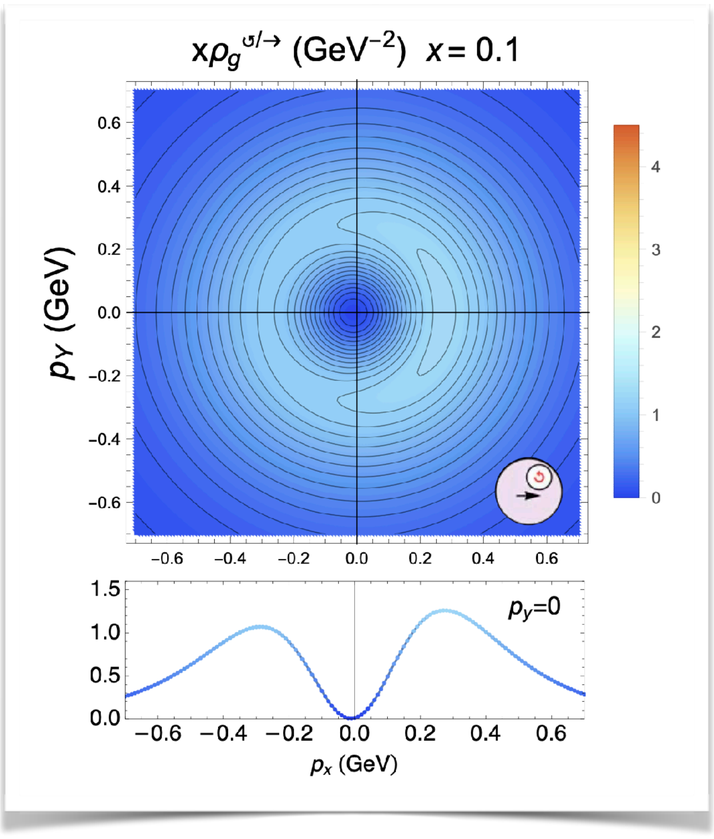


Boer-Mulders



Worm-gear



🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels, Eur. Phys. J. C 80 (2020) no.8]

Anatomy of gluon TMDs

matching coefficients

collinear PDF

nonperturbative Sudakov

nonperturbative TMD function

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right)(x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\rm NP}(b)} F_{\rm NP}(x, b)$$

perturbative Sudakov

perturbative expansion in $\alpha_s(\mu)$

resummation of

$$L = \ln \frac{Q^2}{\mu_b^2}$$

define logarithmic ordering

slide adapted from C. Bissolotti

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Model input

define logarithmic ordering

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resummation of $L = \ln \dfrac{Q^2}{\mu_b^2}$

Model input

define logarithmic ordering

slide adapted from C. Bissolotti

$$f_1(x,\mathbf{b},\mu,\zeta) \;\rightarrow\; C_j^{(f_1)} \otimes f_1^j = \left[1 + \mathcal{O}(\alpha_s)\right]_j \otimes f_1^j$$

Anatomy of gluon TMDs

matching coefficients

collinear PDF

nonperturbative Sudakov

nonperturbative TMD function

$$F(x,\mathbf{b};\mu,\zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right)(x,b_*;\mu_b)\, e^{S(b_*;\mu_b,\mu,\zeta)}\, e^{S_{\rm NP}(b)}\, F_{\rm NP}(x,b)$$

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resummation of $L = \ln \dfrac{Q^2}{\mu_b^2}$

define logarithmic ordering

Model input

slide adapted from C. Bissolotti

$$f_1(x,\mathbf{b},\mu,\zeta) \;\to\; C_j^{(f_1)} \otimes f_1^j = \left[1+\mathcal{O}(\alpha_s)\right]_j \otimes f_1^j$$

$$h_1^\perp(x,\mathbf{b},\mu,\zeta) \;\to\; C_j^{(h_1^\perp)} \otimes f_1^j = \left[\mathcal{O}(\alpha_s)\right]_j \otimes f_1^j$$

¿ Suppression of genuine NP effects ?

3. Building T-odd gluon TMDs

T-odd gluon TMDs in a spectator model

- No residual gluon-spectator interaction at tree level
- Interference with one-gluon exchange (eikonal)

T-odd gluon TMDs in a spectator model

- No residual gluon-spectator interaction at tree level
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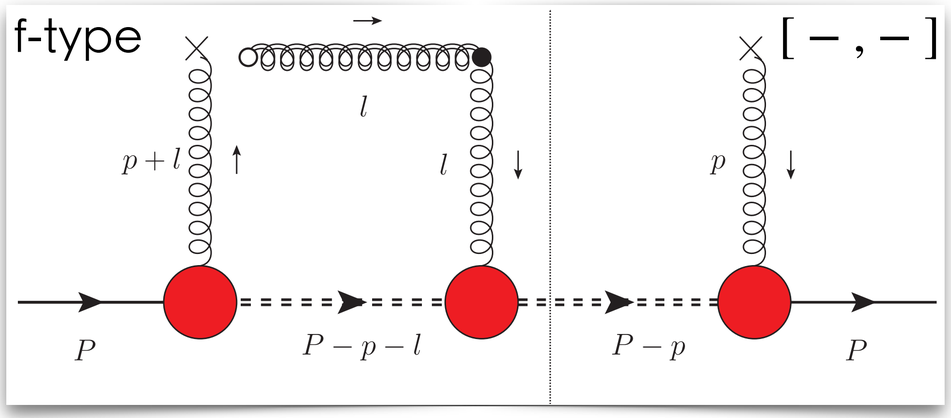
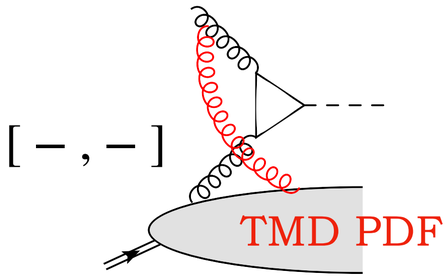
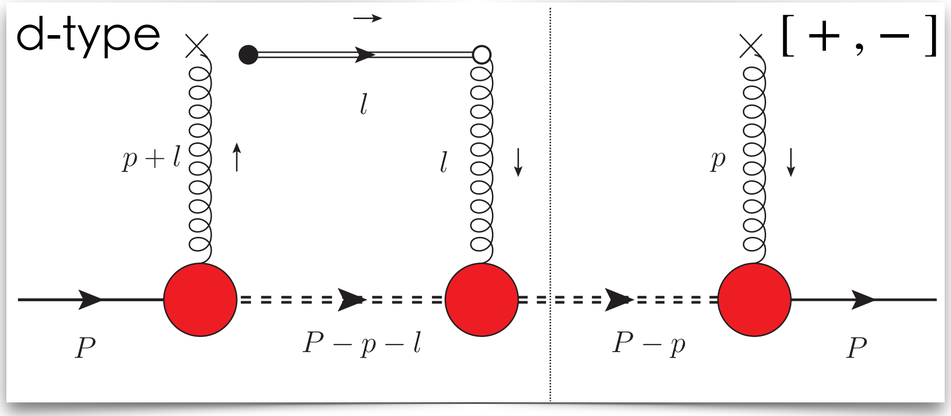
$gg \to H$





f-type (WW) structure

$qg \to \gamma^{(*)}$-jet

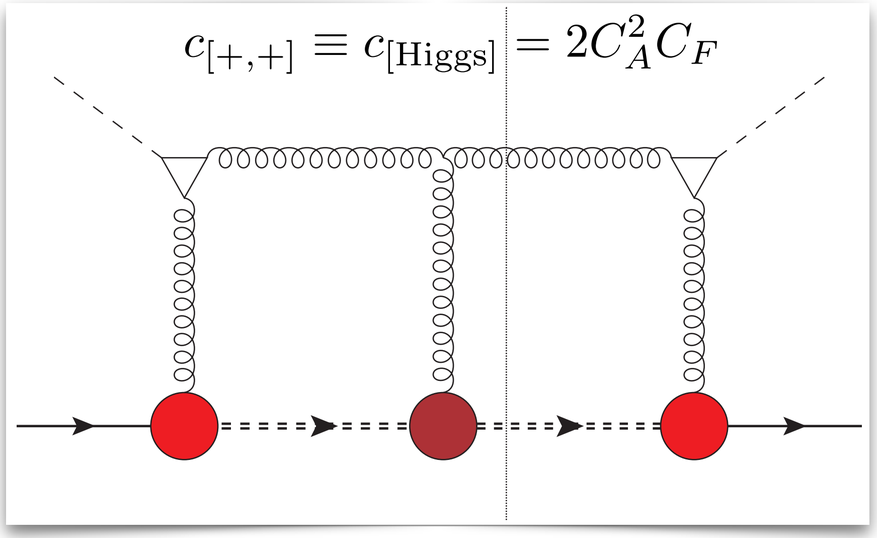


d-type (dipole) structure

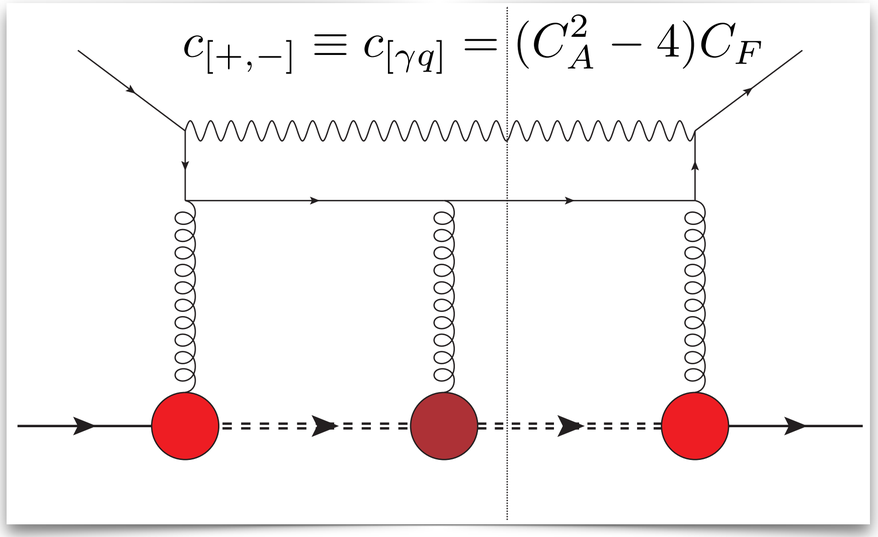
(Sivers & linearity) [A. Bacchetta, F.G.C., M. Radici, EPS-HEP, PANIC (2021)]

T-odd gluon TMDs in a spectator model

Higgs-gluon fusion $\Rightarrow$ f-type [+ , +]

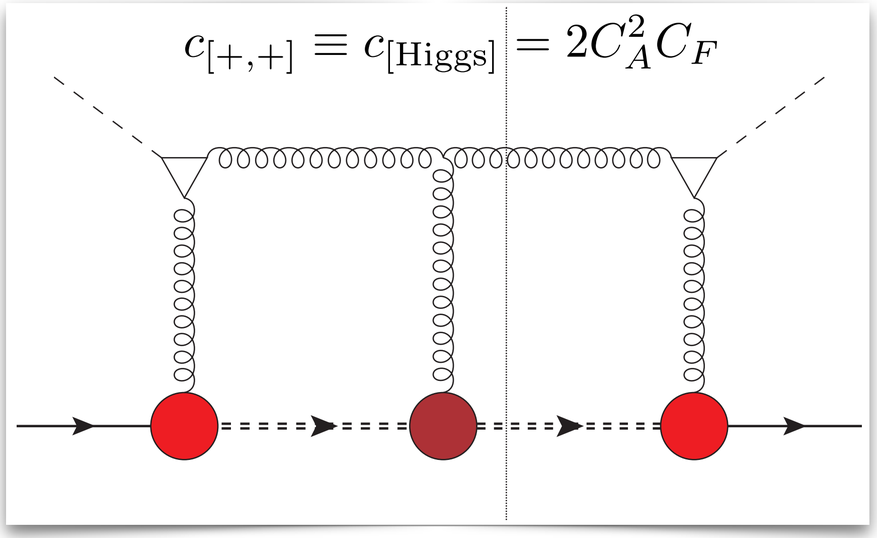


Photon-jet emission $\Rightarrow$ d-type [+ , –]

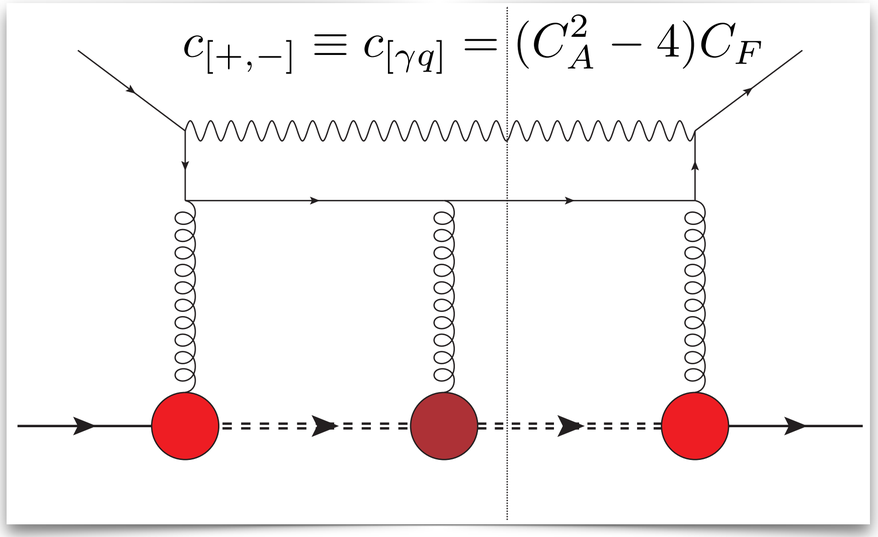


T-odd gluon TMDs in a spectator model

Higgs-gluon fusion ⇒ f-type [+ , +]



Photon-jet emission ⇒ d-type [+ , –]



nucleon-gluon-spectator

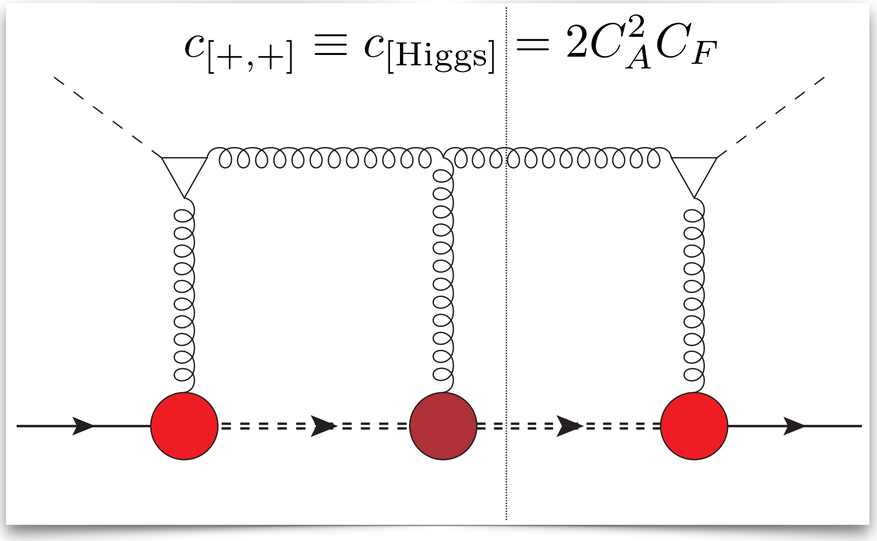
spectator-gluon-spectator

$$\mathcal{Y}^{\mu}_{bc}(p^2) = \delta_{bc}\left[g_1(p^2)\,\gamma_\mu + g_2(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right]$$

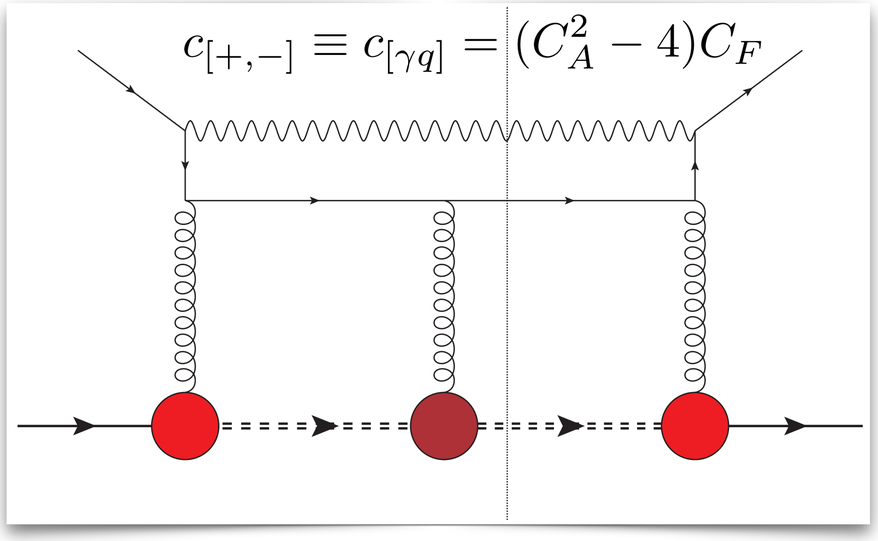
$$\mathcal{X}^{\mu}_{abc}(p^2) = f^{abc}\left[g_1^f(p^2)\,\gamma^\mu + g_2^f(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right] - i\,d^{abc}\left[g_1^d(p^2)\,\gamma^\mu + g_2^d(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right]$$

T-odd gluon TMDs in a spectator model

Higgs-gluon fusion $\Rightarrow$ f-type $[+,+]$



Photon-jet emission $\Rightarrow$ d-type $[+,-]$



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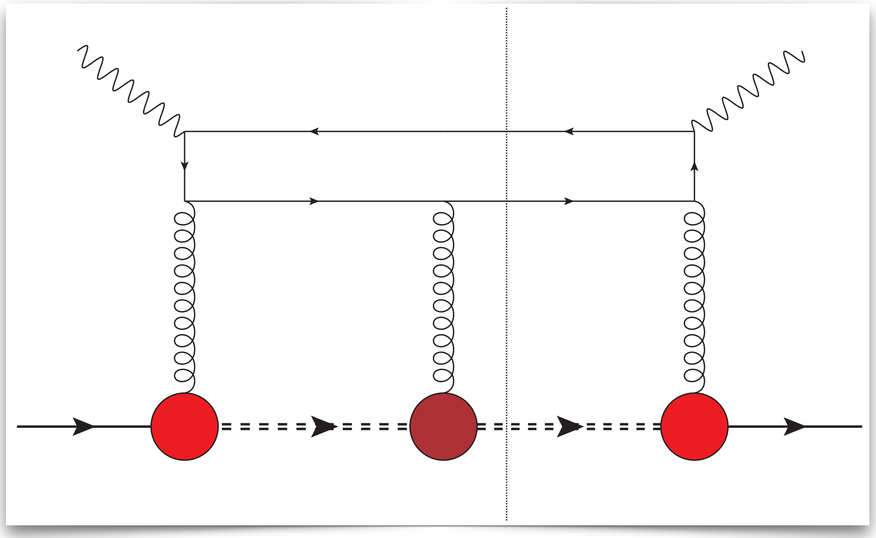
$$\mathcal{X}^{\mu}_{abc}(p^2) = f^{abc}\left[g_1^f(p^2)\,\gamma^\mu + g_2^f(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right] - i\,d^{abc}\left[g_1^d(p^2)\,\gamma^\mu + g_2^d(p^2)\,\frac{i}{2M}\,\sigma^{\mu\nu}\,p_\nu\right]$$

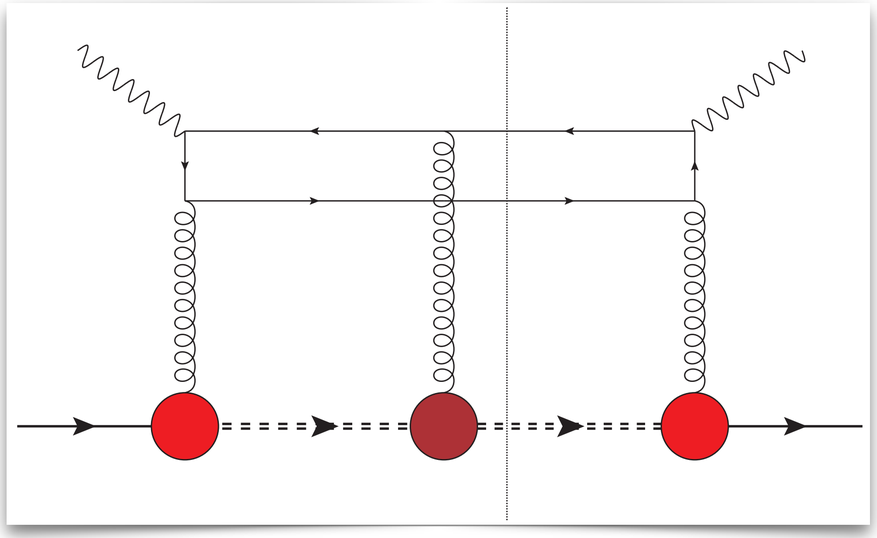
Assumption: $g^d_{1,2}(p^2) = g^f_{1,2}(p^2) \equiv g_{1,2}(p^2)$ $\Leftrightarrow$ $f_{1T}^{\perp[+,-]} = \dfrac{c_{[+,-]}}{c_{[+,+]}} f_{1T}^{\perp[+,+]} \equiv \dfrac{10}{18} f_{1T}^{\perp[+,+]}$

(Sivers & linearity) [A. Bacchetta, F.G.C., M. Radici, EPS-HEP, PANIC (2021)]

Analytic structure of T-odd gluon TMDs

Two-jet SIDIS $\Rightarrow$ f-type [+ , +]





nucleon-gluon-spectator

spectator-gluon-spectator

[A. Bacchetta, F.G.C., M. Radici (in preparation)]

Analytic structure of T-odd gluon TMDs

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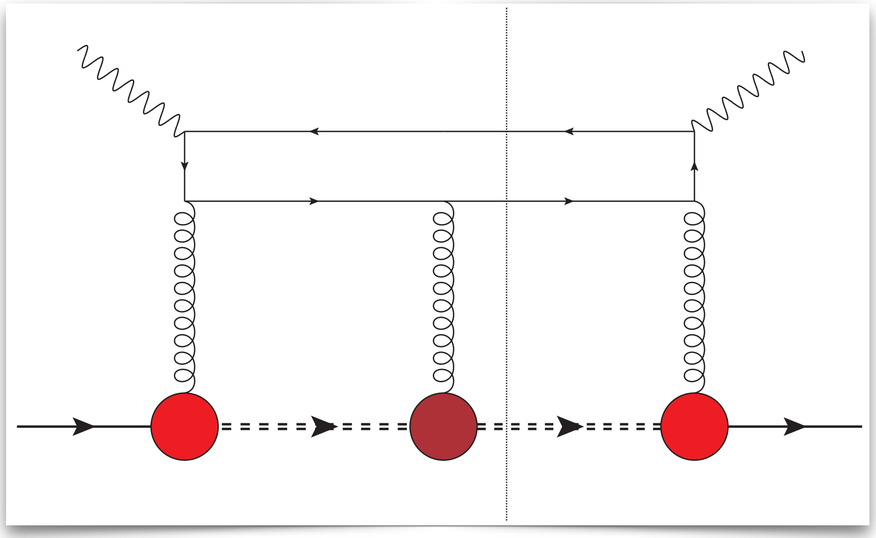
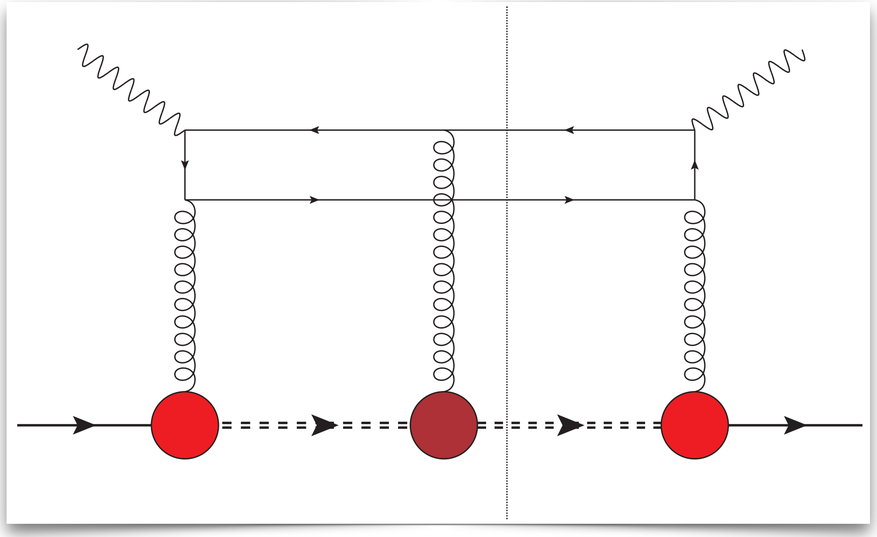




nucleon-gluon-spectator

spectator-gluon-spectator

$8 \times 8 \times 4$

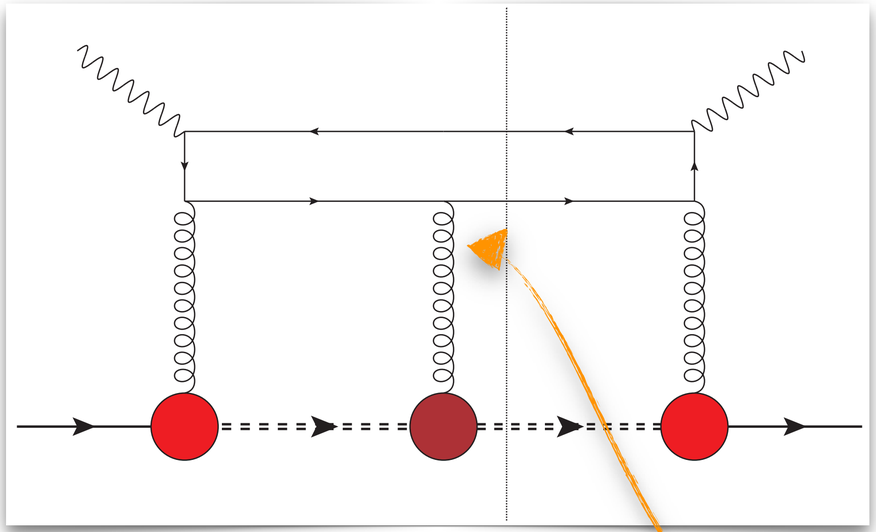
$$F(x, \boldsymbol{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \boldsymbol{p}_T^2)\, g_i(\boldsymbol{p}_T^2)\, g_j(\boldsymbol{p}_T^2)\, g_k(\boldsymbol{p}_T^2)$$

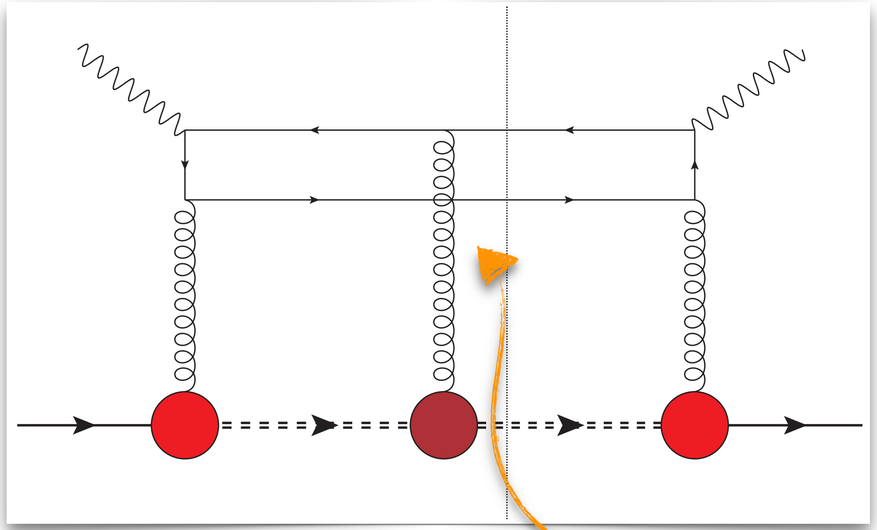
$$C_{ijk}^{(F)}(x, \boldsymbol{p}_T^2) = \sum_{l=1}^{7} \mathcal{C}_{ijk}^{(F),l}(x, \boldsymbol{p}_T^2)\, \mathcal{D}_l(x, \boldsymbol{p}_T^2)$$

[A. Bacchetta, F.G.C., M. Radici (in preparation)]

Analytic structure of T-odd gluon TMDs

Two-jet SIDIS $\Rightarrow$ f-type [+ , +]





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spectator-gluon-spectator

$$F(x,\boldsymbol{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x,\boldsymbol{p}_T^2)\, g_i(\boldsymbol{p}_T^2)\, g_j(\boldsymbol{p}_T^2)\, g_k(\boldsymbol{p}_T^2)$$

$8 \times 8 \times 4$

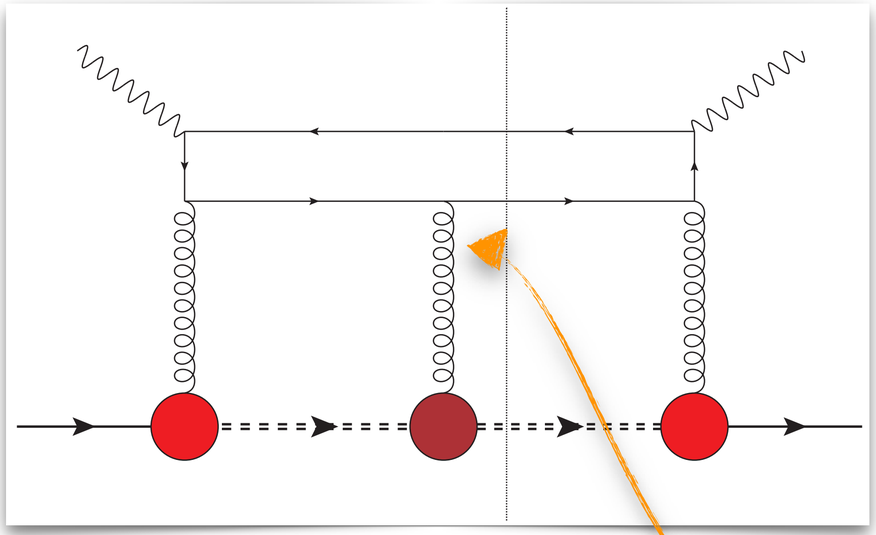
$$C_{ijk}^{(F)}(x,\boldsymbol{p}_T^2) = \sum_{l=1}^{7} \mathcal{C}_{ijk}^{(F),l}(x,\boldsymbol{p}_T^2)\, \mathcal{D}_l(x,\boldsymbol{p}_T^2)$$

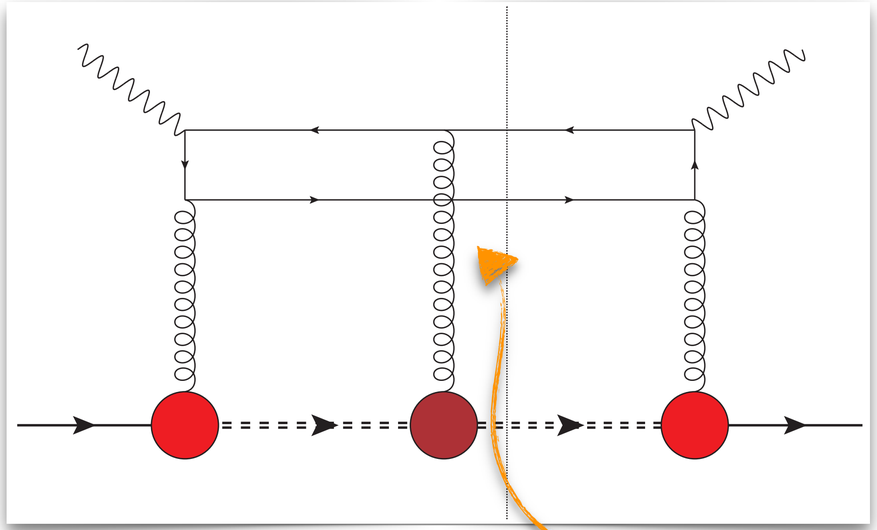
3.1 T-odd gluon TMDs at twist-2

[A. Bacchetta, F.G.C., M. Radici (in preparation)]

Analytic structure of T-odd gluon TMDs

Two-jet SIDIS $\Rightarrow$ f-type [+ , +]





nucleon-gluon-spectator

spectator-gluon-spectator

$$F(x,\boldsymbol{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x,\boldsymbol{p}_T^2)\, g_i(\boldsymbol{p}_T^2)\, g_j(\boldsymbol{p}_T^2)\, g_k(\boldsymbol{p}_T^2)$$

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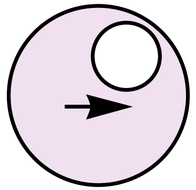
$8 \times 8 \times 4$

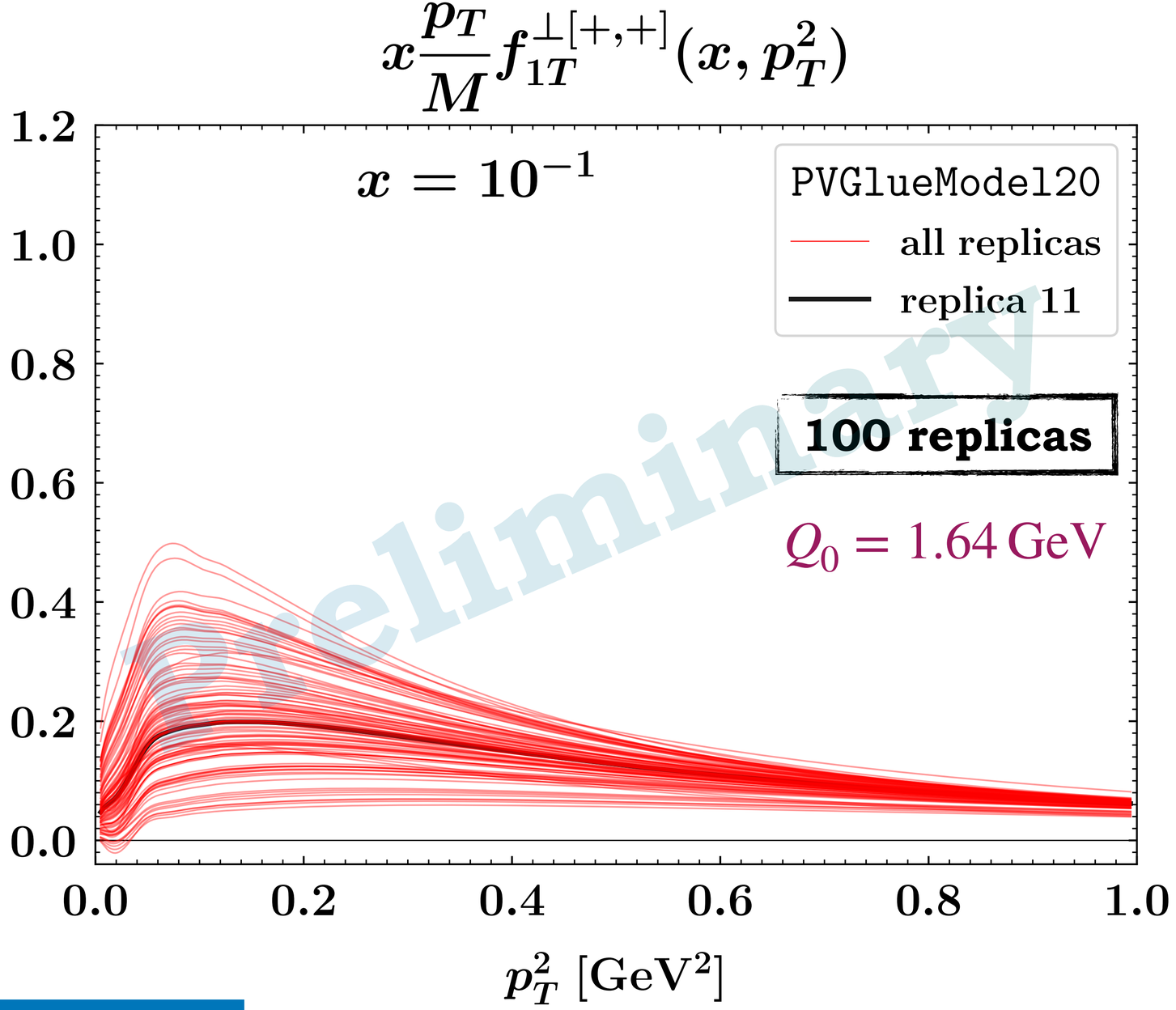
256 coeff. functions

[A. Bacchetta, F.G.C., M. Radici (in preparation)]

Preliminary results for Sivers

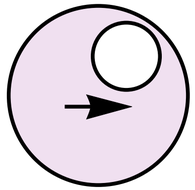
[A. Bacchetta, F.G.C., M. Radici (in preparation)]

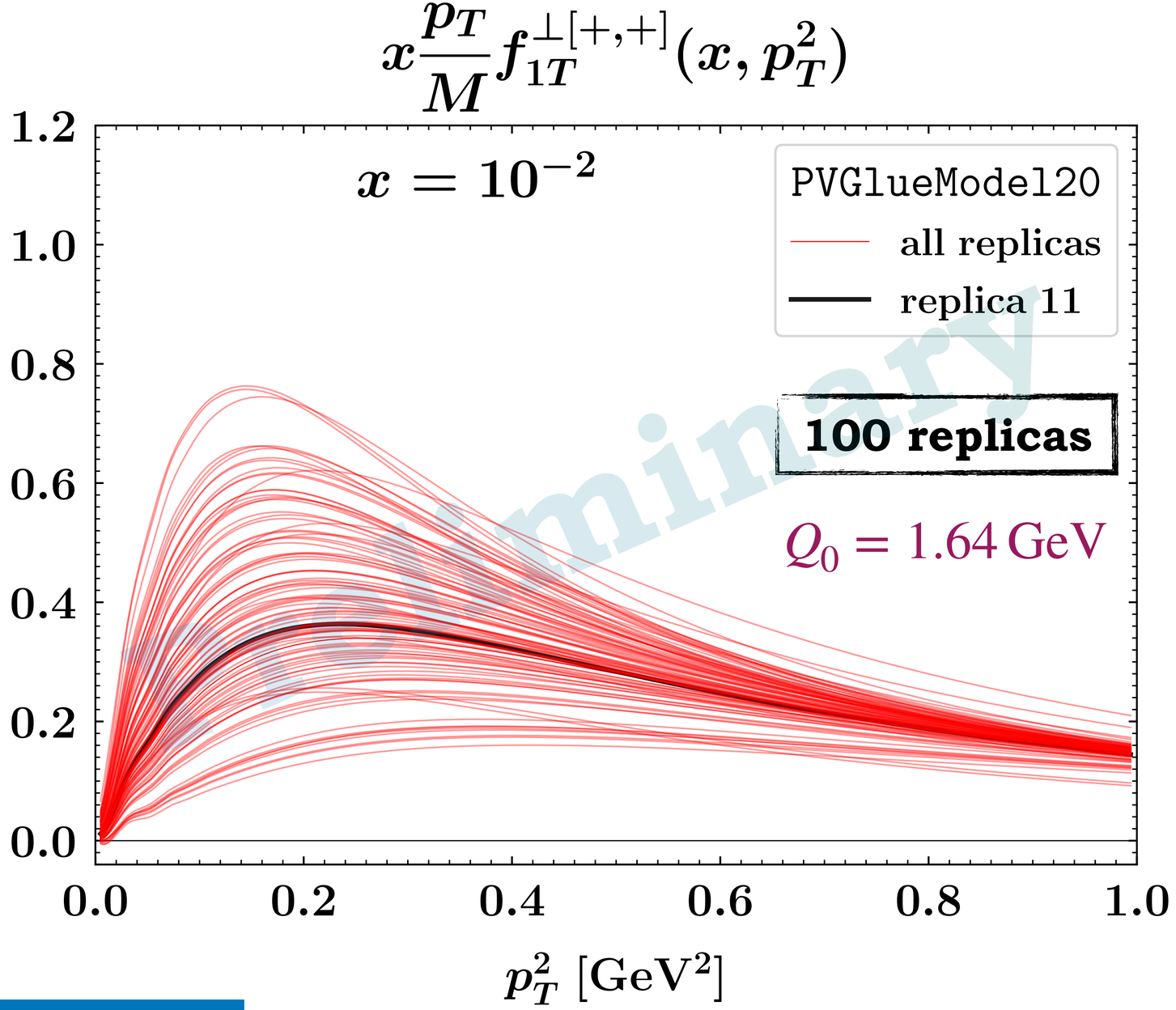




Preliminary results for Sivers

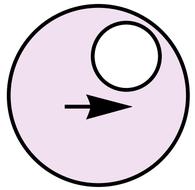
[A. Bacchetta, F.G.C., M. Radici (in preparation)]

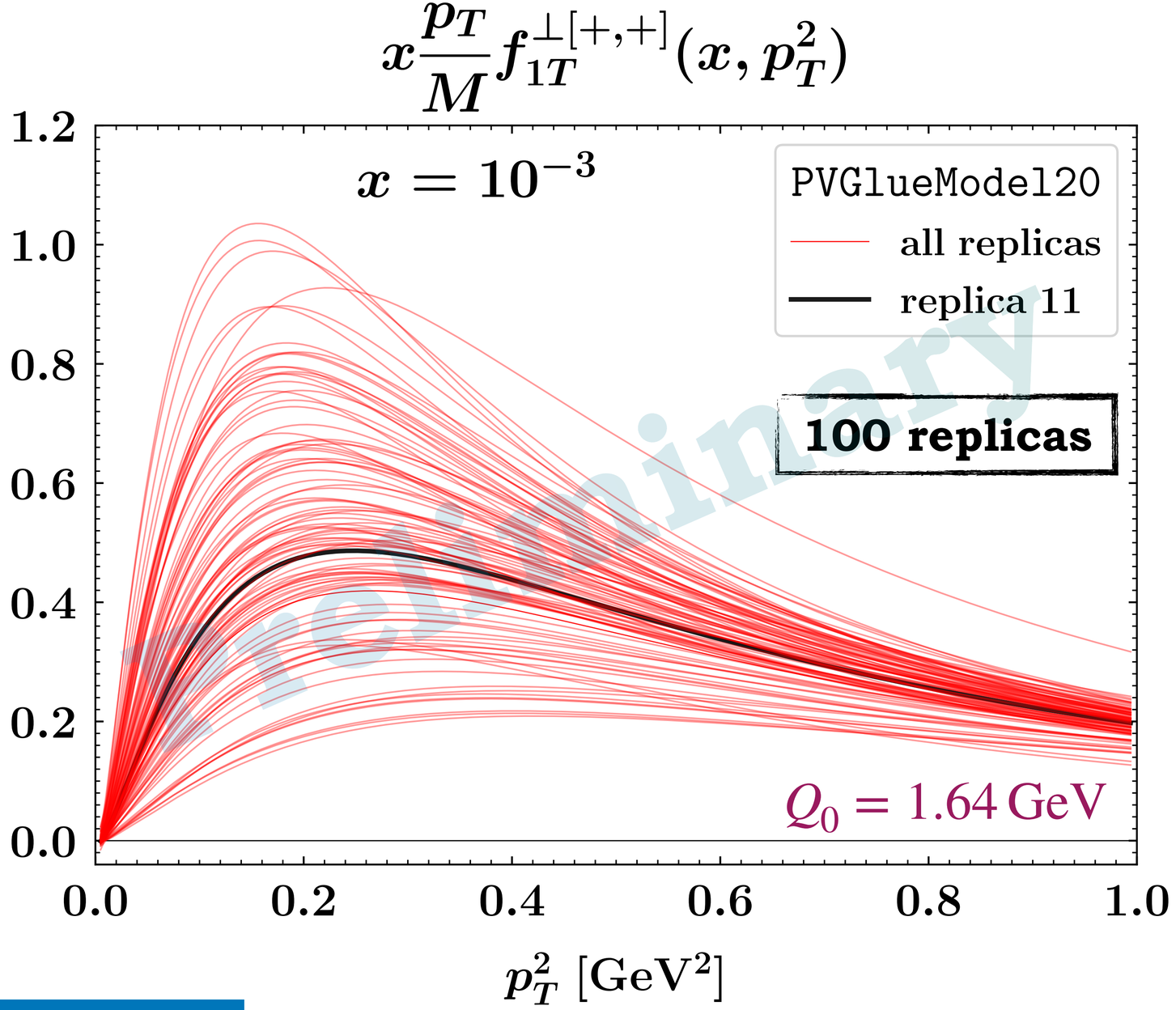




Preliminary results for Sivers

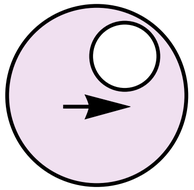
[A. Bacchetta, F.G.C., M. Radici (in preparation)]

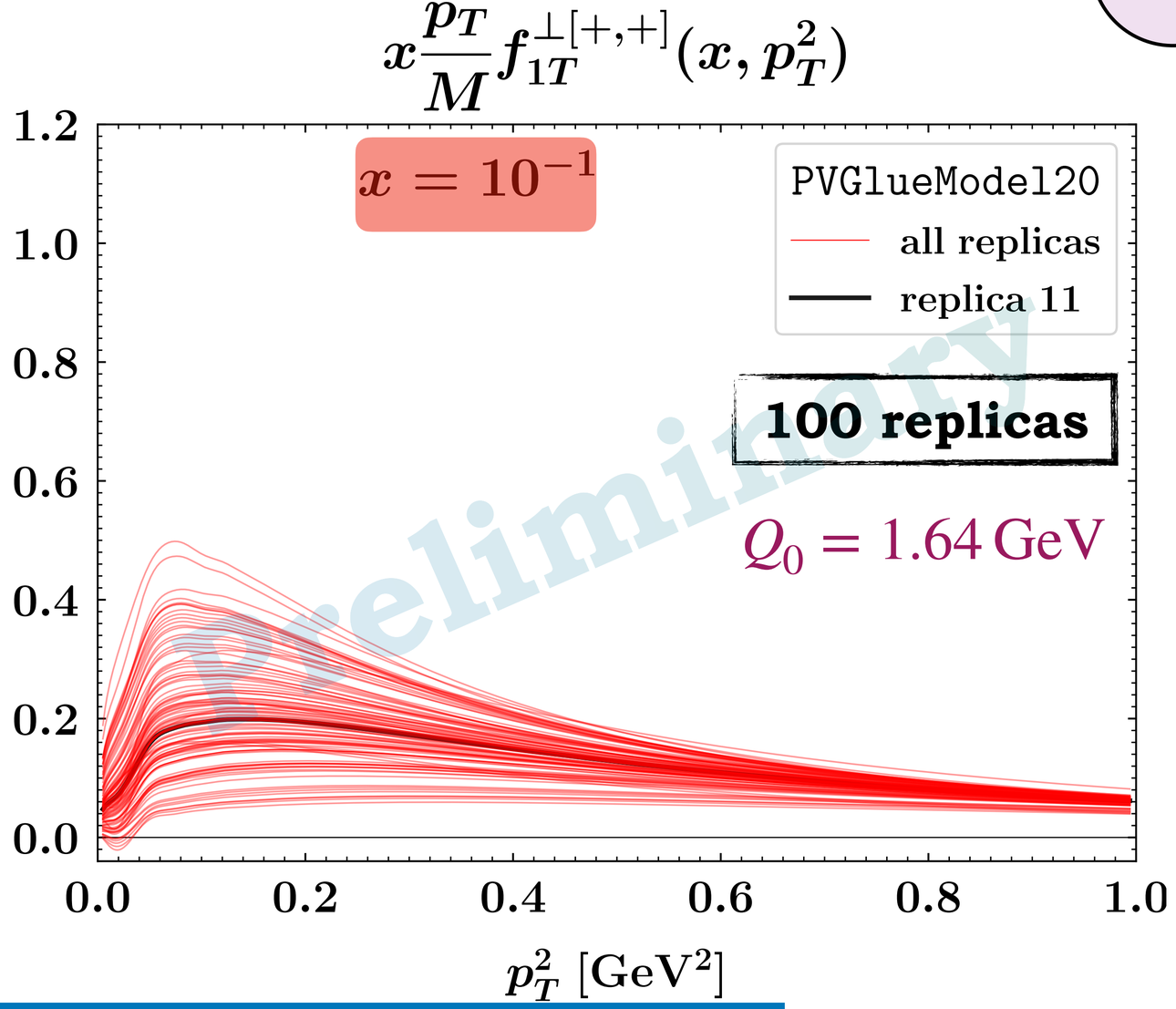


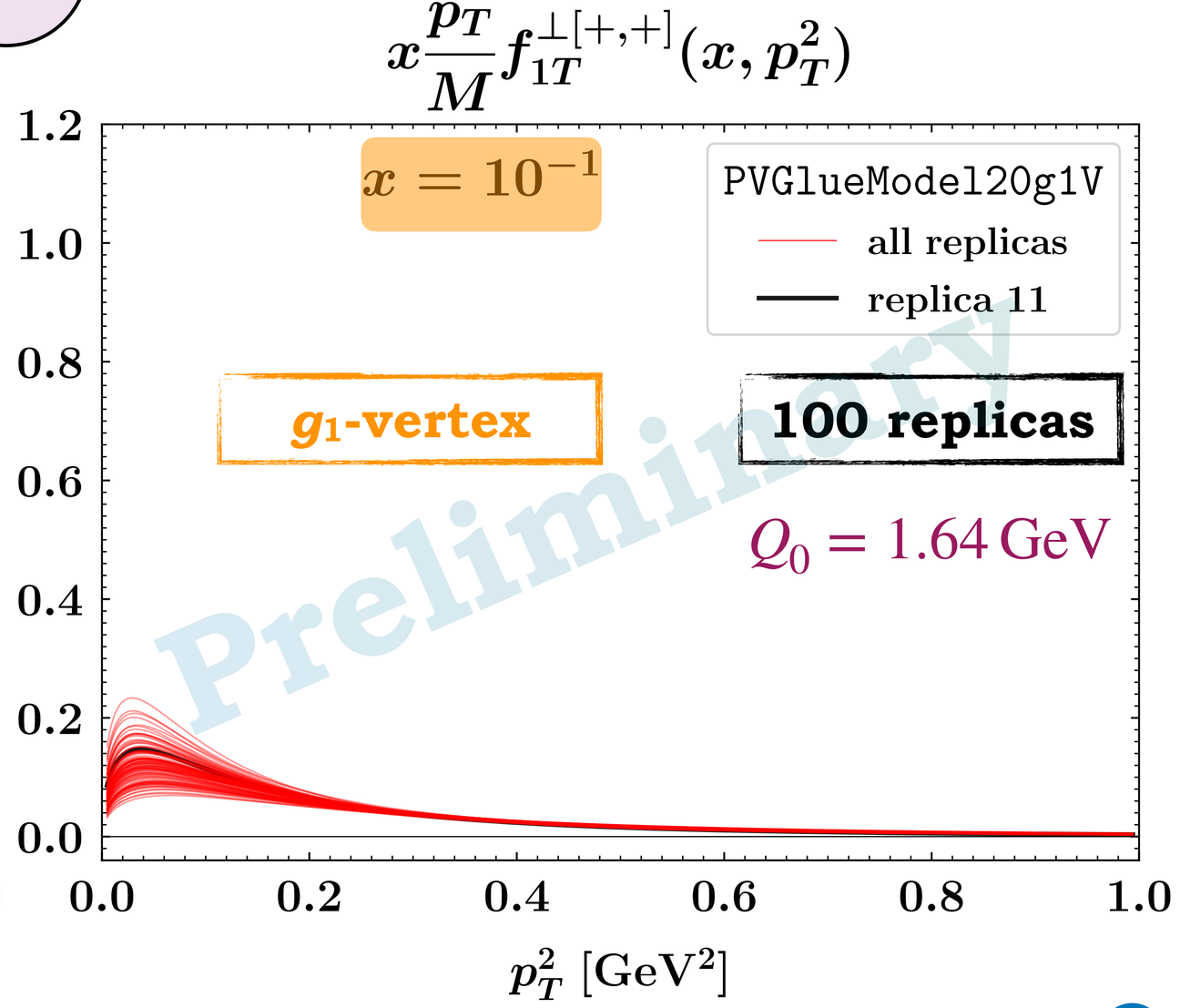


Preliminary results for Sivers

[A. Bacchetta, F.G.C., M. Radici (in preparation)]







Preliminary results for Sivers

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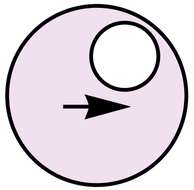
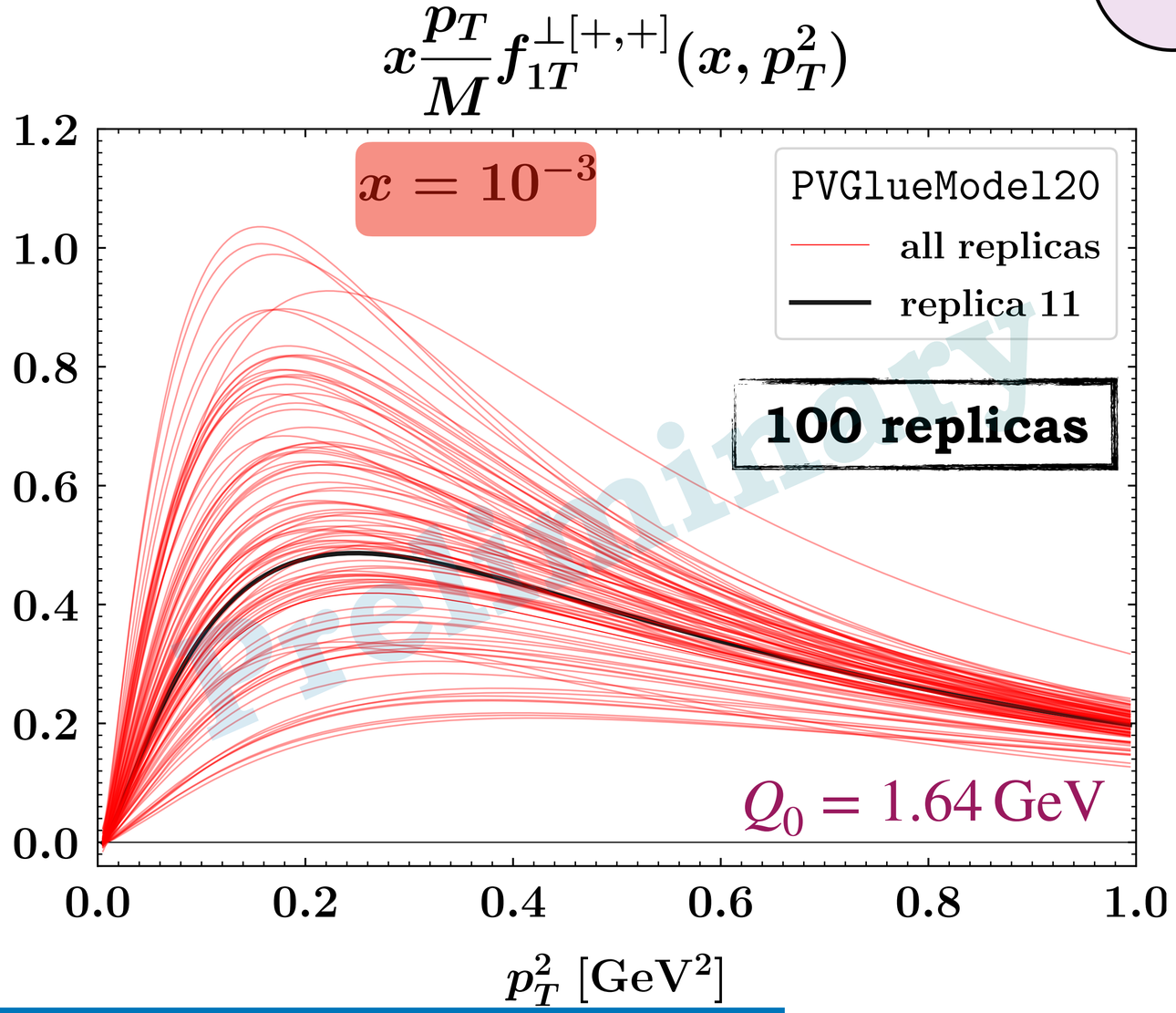
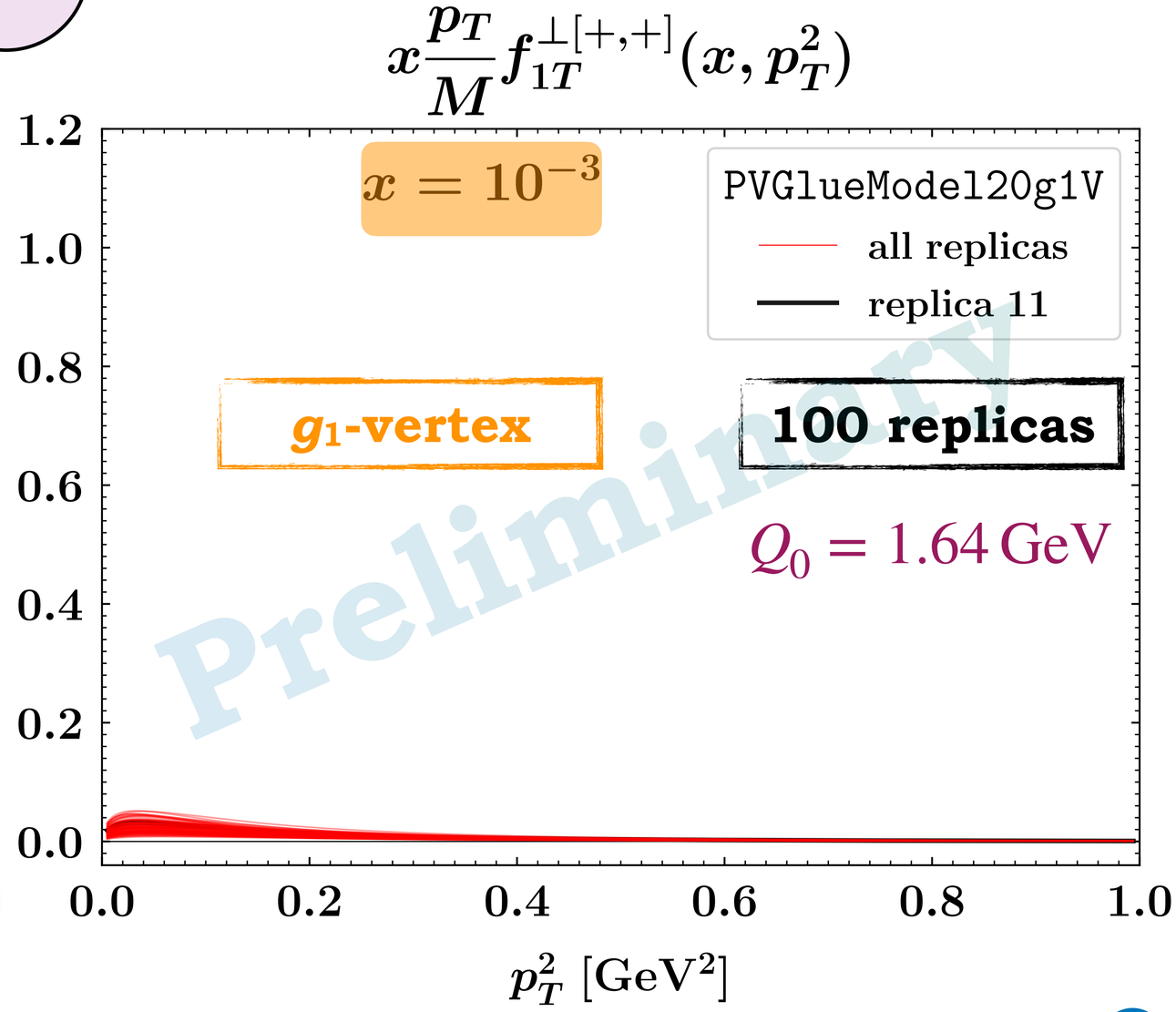
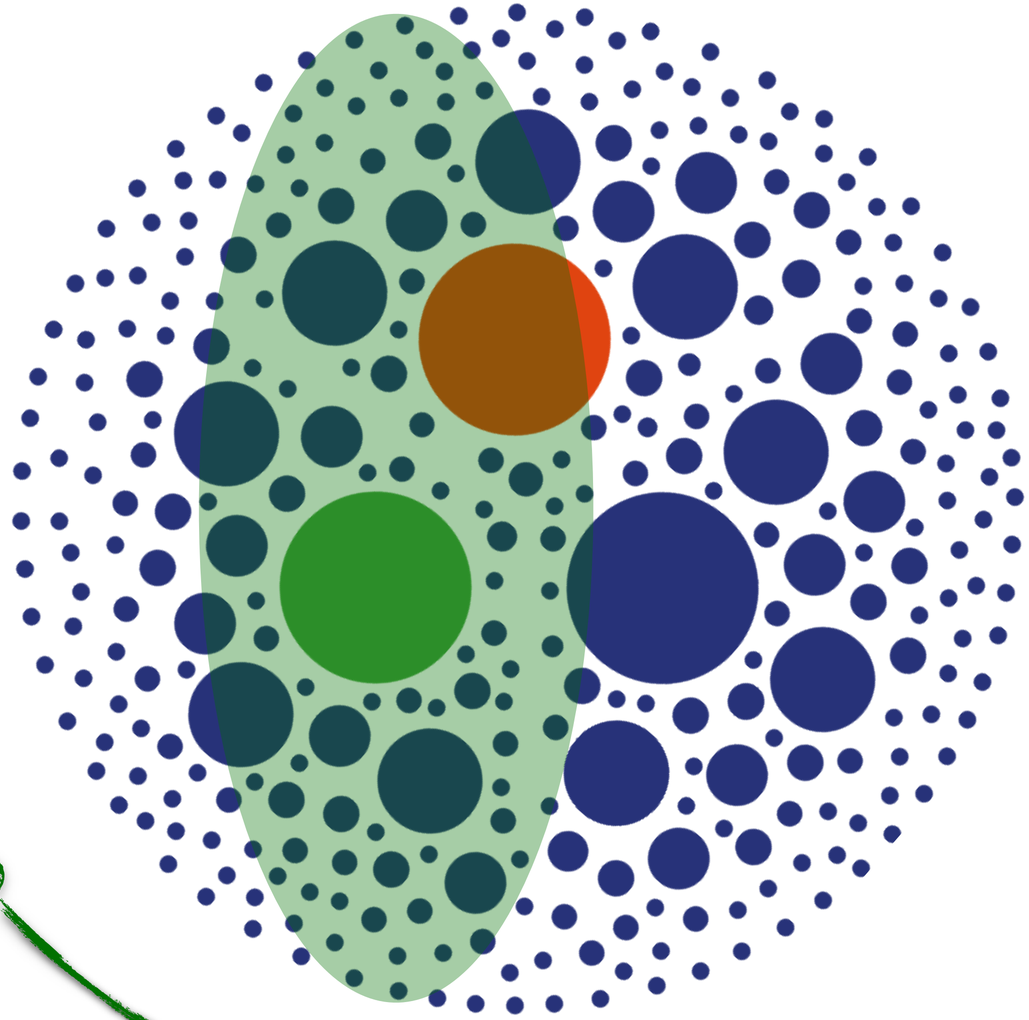






Gluon TMDs @new-gen colliders: A win-win strategy

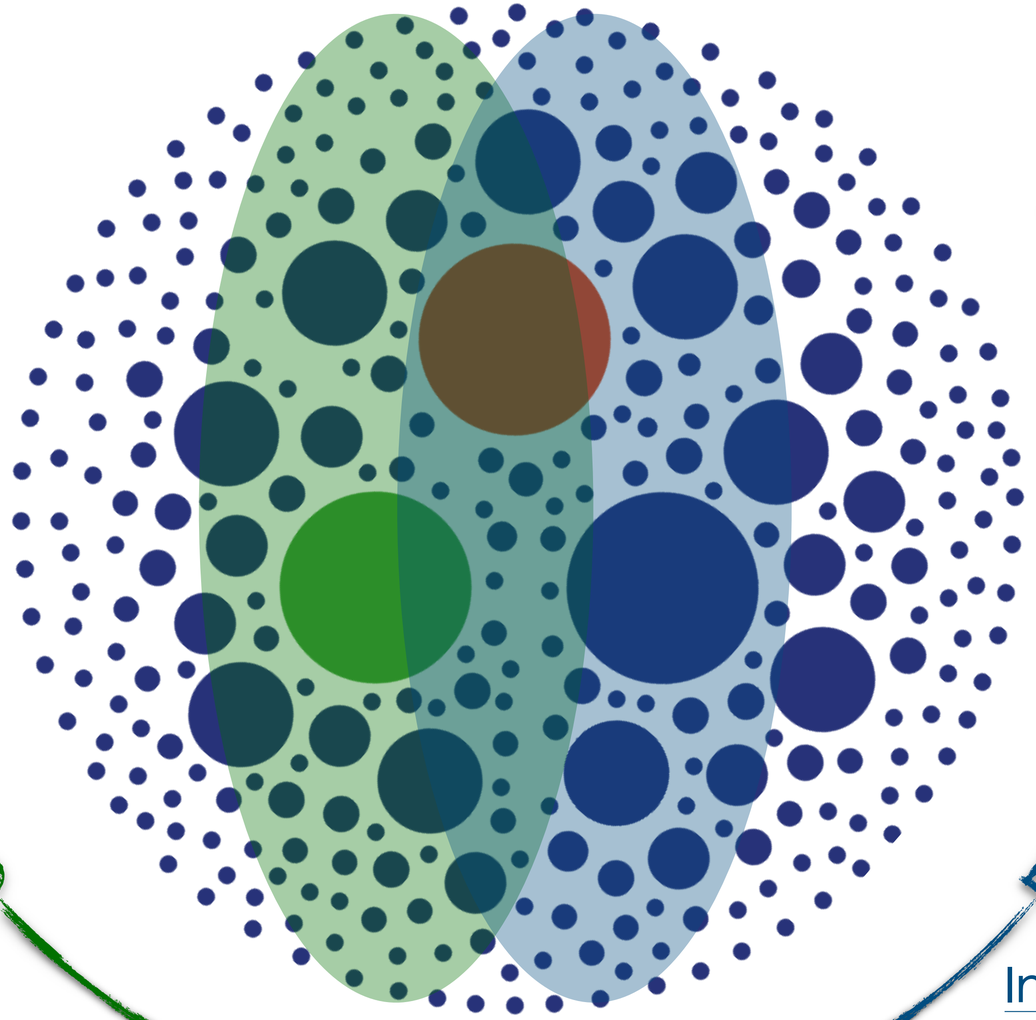


Gluon TMD PDFs

Models & Evolution

Gege links & polarization

Gluon TMDs @new-gen colliders: A win-win strategy



Gluon TMD PDFs

Models & Evolution

Gege links & polarization

Onium studies

Production mechanisms

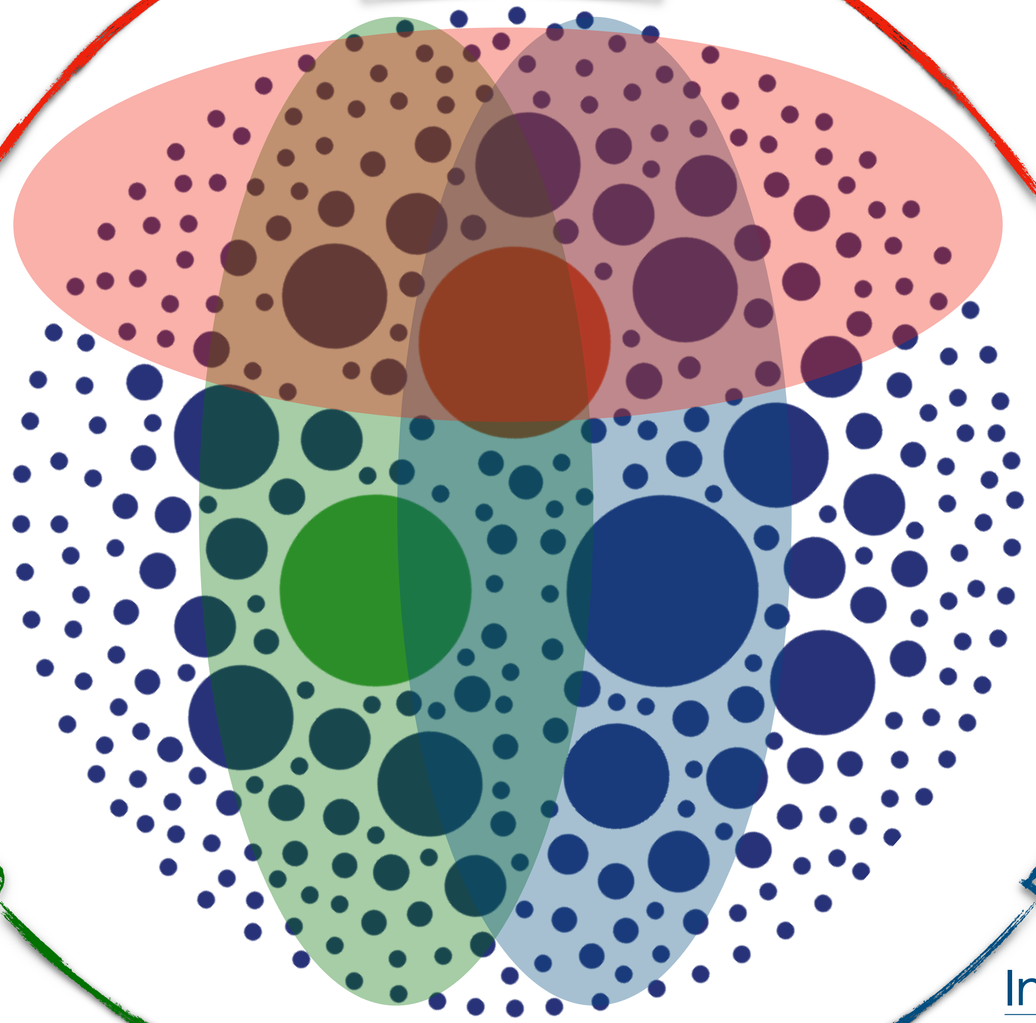
Intrinsic motion & Resummation

Gluon TMDs @new-gen colliders: A win-win strategy

Sivers effect
Boer-Muders effect

EIC
HL & FT

Effects on p_T spectrum
Spin asymmetries



Gluon
TMD PDFs

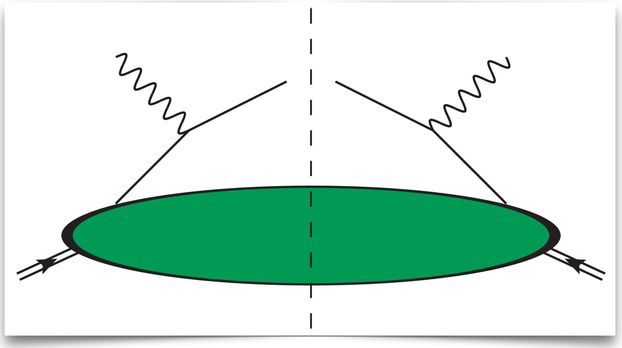
Models & Evolution
Gege links & polarization

Onium
studies

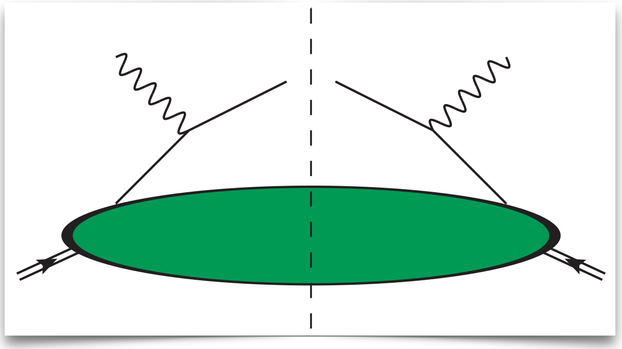
Production mechanisms
Intrinsic motion & Resummation

Extras

Gauge links and processes dependence



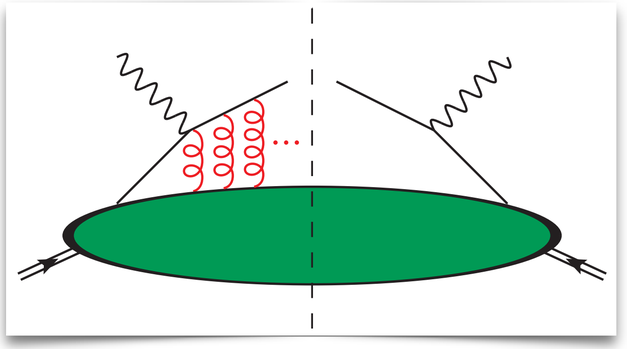
Gauge links and processes dependence



$$\bar{\psi}_j(0)\ U(0,\xi)\ \psi_i(\xi)$$

Gauge link (Wilson line)

Resummation of (calculable)
infinite gluon emissions



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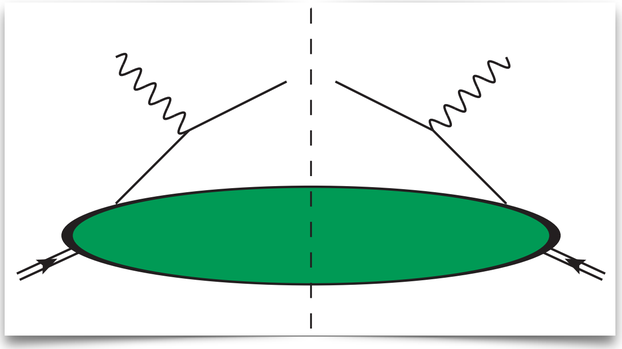

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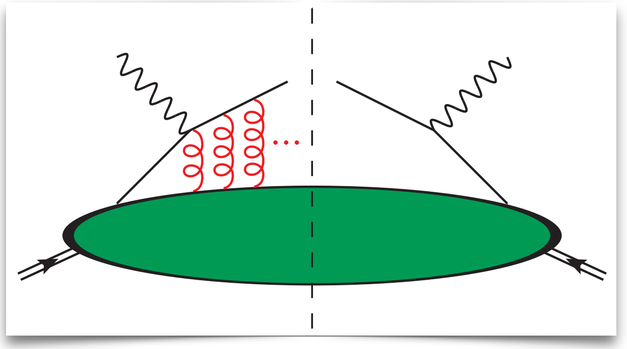
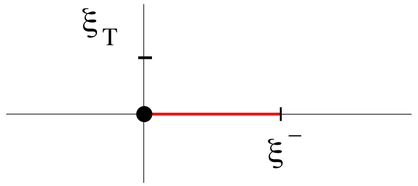


Collinear PDFs

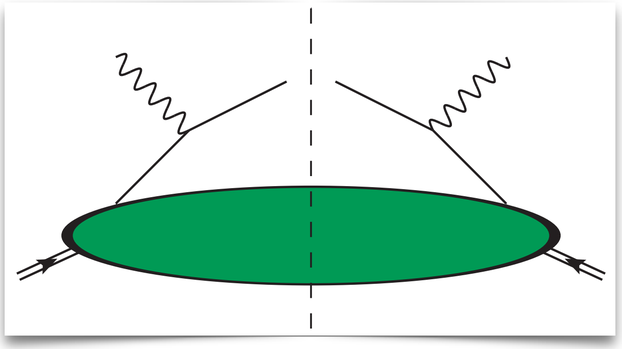
$$\Phi_{ij}(x) \doteq \int d^2\boldsymbol{p}_T \Phi_{ij}(x,\boldsymbol{p}_T) = \int \frac{d\xi^-}{2\pi} e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\psi_i(\xi)|P\rangle_{|\xi^+=0,\xi_T=0}$$



- Light-cone: $\xi^+ = 0$, $\boldsymbol{\xi} = 0$
- **Straight** gauge link (unique!)
- $(A^+ = 0)$ light-cone: WL $= \hat{1}$

☑ **Universality warranted**

Gauge links and processes dependence



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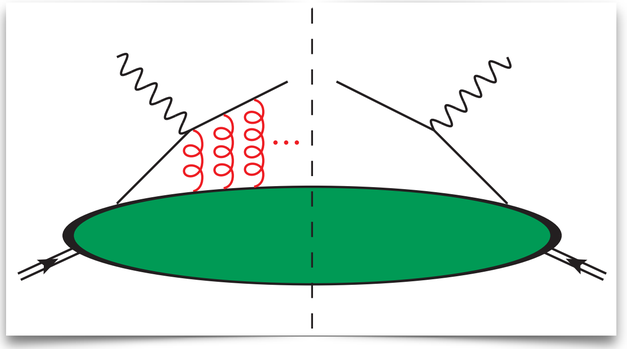

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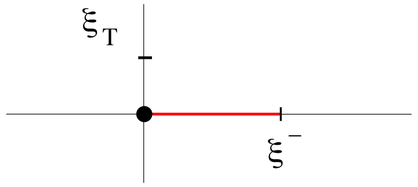

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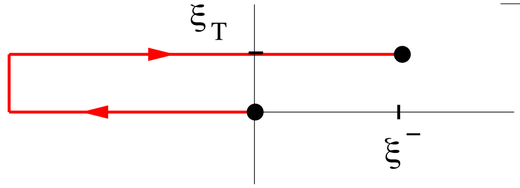
- **Straight** gauge link (unique!)
- $(A^+ = 0)$ light-cone: WL $= \hat{1}$

☑ **Universality warranted**

TMD PDFs

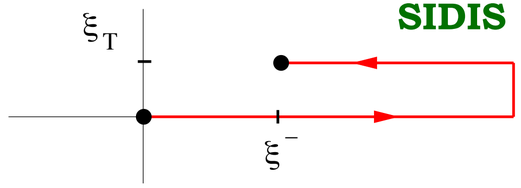
- *Transverse* gauge link not eliminated by gauge choice
- **Staple-like** gauge link (not unique!)

¡! **Process dependence**



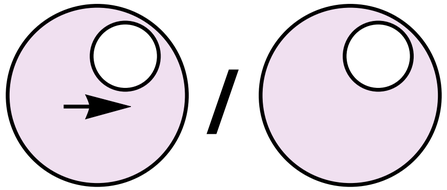
Drell-Yan

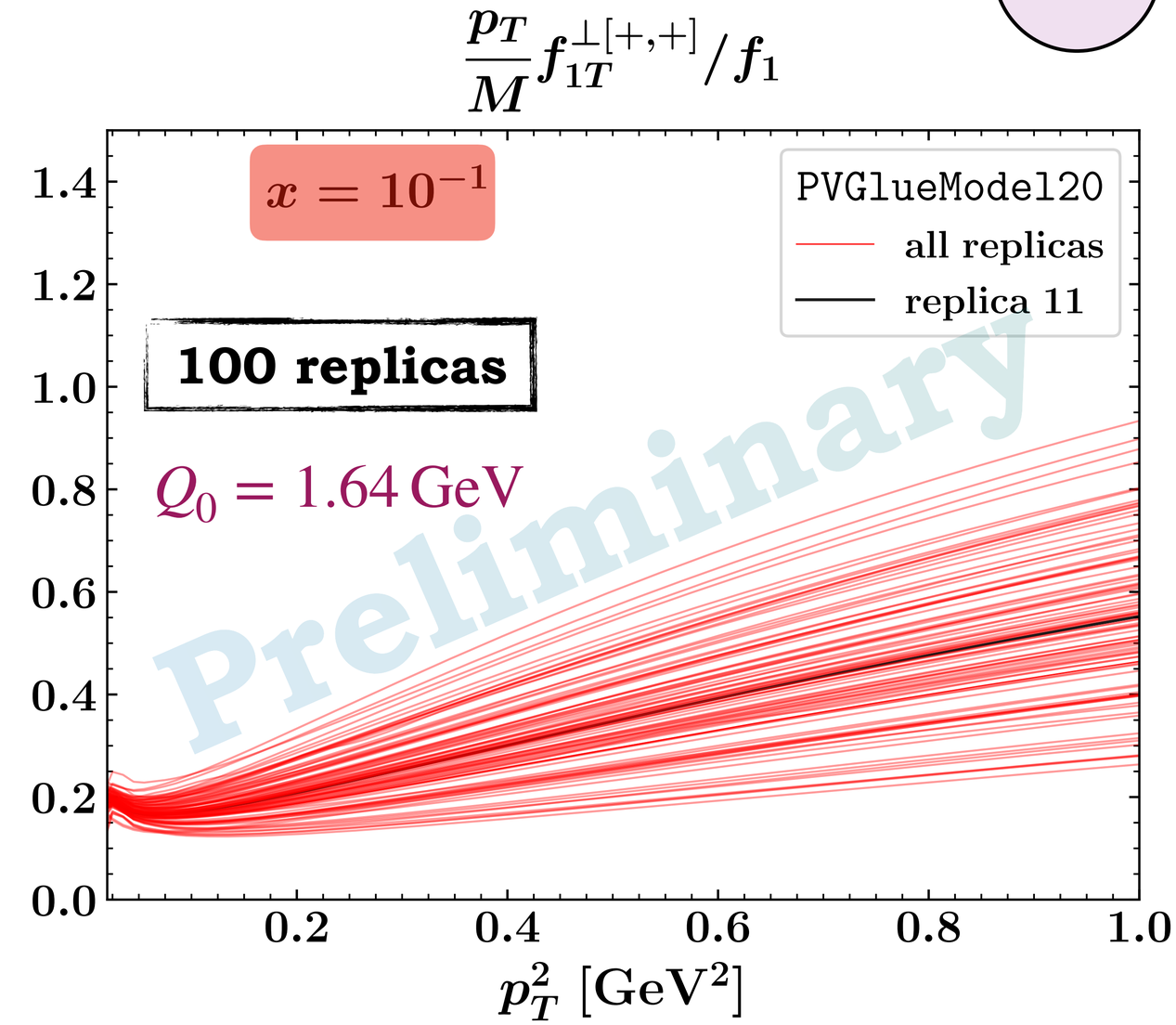
SIDIS

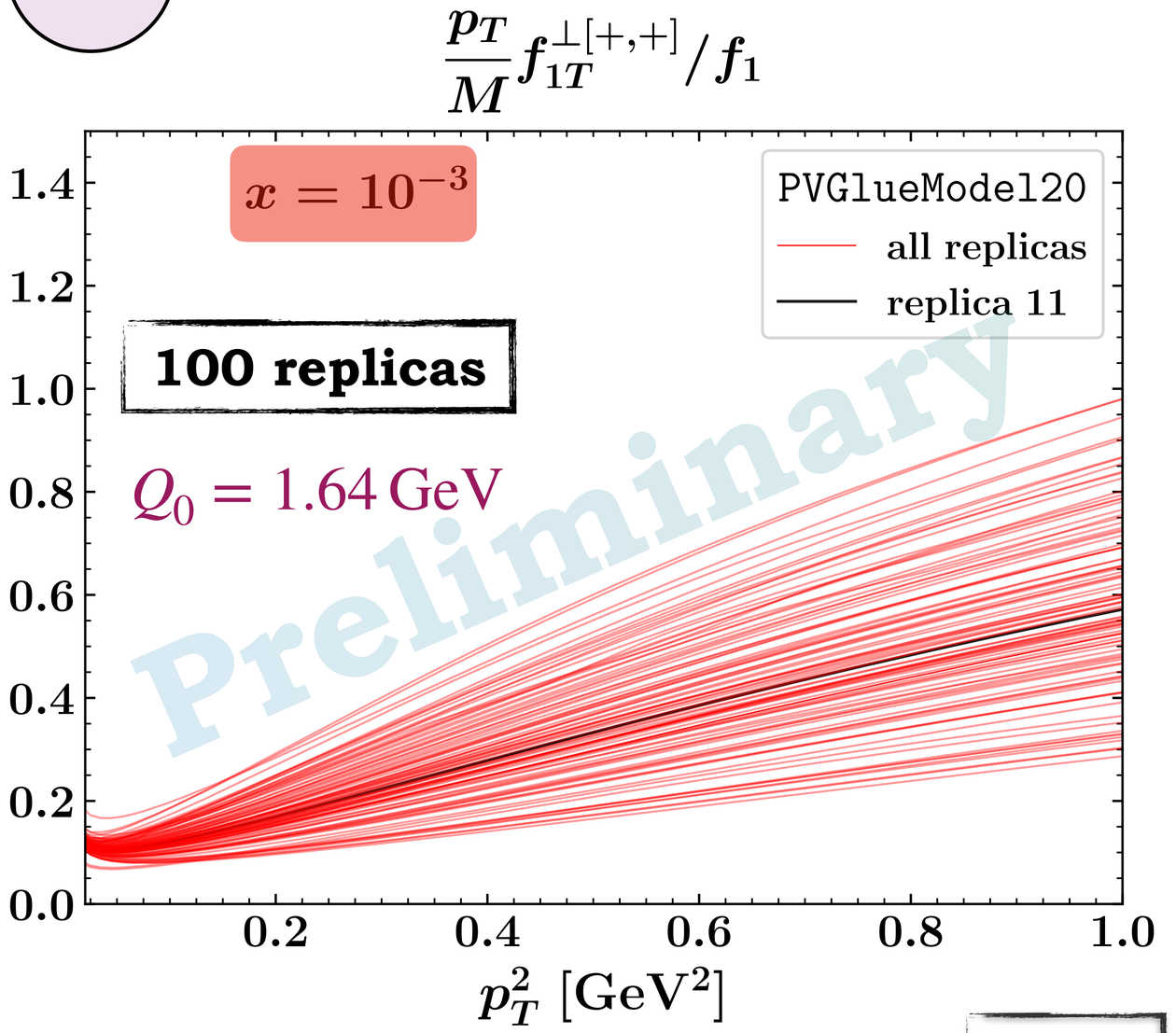


f-type Sivers/unpol.

A. Bacchetta, F.G. C., M. Radici (in preparation)]





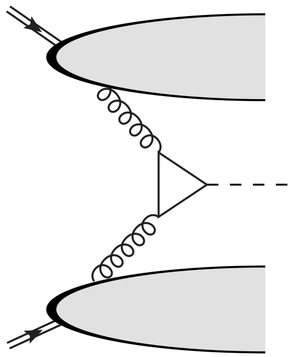


Backup

Accessing gluon TMDs @LHC

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion



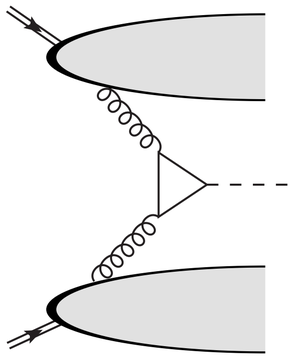
✓ No color entanglement
TMD factorization

¡! 🔗 Large low-p_T bin @CMS
More data @HL-LHC

Backup

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion



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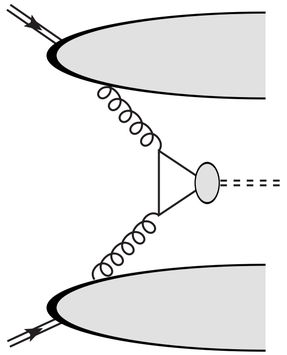
Single quarkonium



✓ $\eta_{c,b}$ ✗ $J/\psi, \Upsilon$

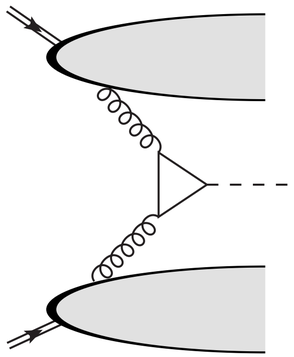
TMD factorization
C-parity selection rules

¡! 🔗 Large-p_T data @LHCb
More data @FT-LHC

Backup

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion



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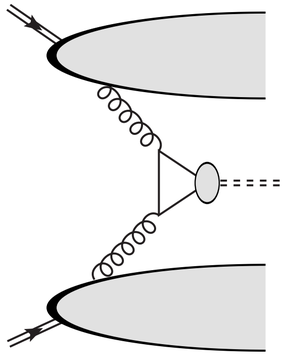

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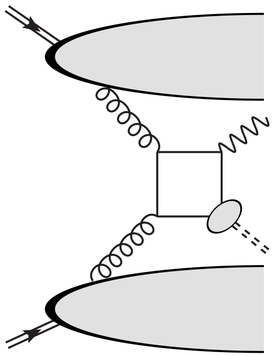
More data @FT-LHC

$J/\psi + \gamma^{(*)}$



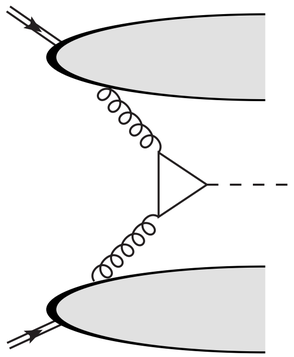
¡✓ Color entanglement
Potential TMD violation (CO)
Back-to-back suppresses CO

✓! Possible studies @HL-LHC
Currently no low-p_T data

Backup

Golden channels for gluon TMD PDFs @LHC

Higgs in gluon fusion



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TMD factorization

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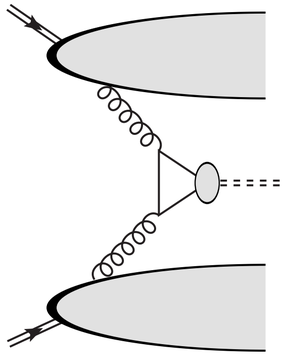
Single quarkonium



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More data @FT-LHC

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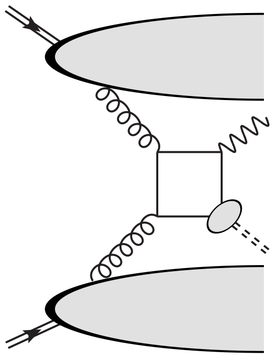

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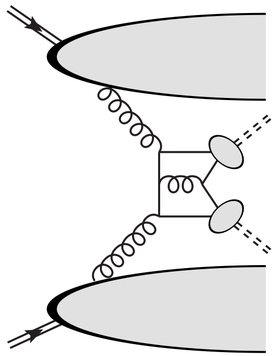
$J/\psi + J/\psi$



✓ No color entanglement
TMD factorization (CSM)

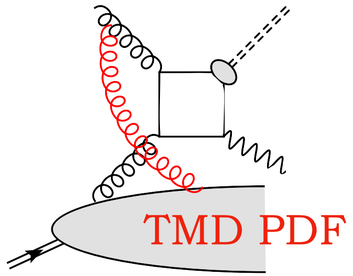
✓ 🔗 Low-p_T data @LHCb
Opportunities @HL- & @FT-LHC

Backup

Quarkonia: Assets & challenges

Assets

Onia ⇒ clean channels of f-type gluon TMDs



Initial-state color flow ⇒ [– , –] gauge link

Sivers	$e\,p^{\uparrow} \to e'\,Q\,\overline{Q}\,X$ $e\,p^{\uparrow} \to e'\,j_1\,j_2\,X$
$f_{1T}^{\perp\,g\,[-,-]}$	√
$f_{1T}^{\perp\,g\,[+,-]}$	×

(overview) [D. Boer (2017)]

Boer-Mulders	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$
$h_1^{\perp\,g\,[-,-]}$(WW)	√
$h_1^{\perp\,g\,[+,-]}$ (DP)	×

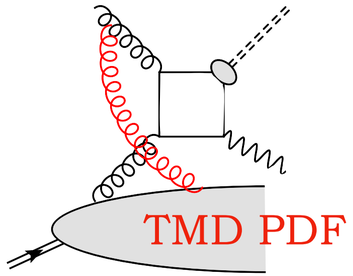
Challenges

Backup

Quarkonia: Assets & challenges

Assets

Onia ⇒ clean channels of f-type gluon TMDs



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$h_1^{\perp g\,[-,-]}$(WW)	√
$h_1^{\perp g\,[+,-]}$ (DP)	×

$\eta_{c,b}$ ⇒ LHC complementarity, TMD factorization

$$\frac{d\sigma}{dq_T} \sim \quad \text{(at low transverse momentum for (pseudo)scalar state)}$$

$$\sim \mathcal{C}[\, f_1^{g/A}\ f_1^{g/B}\,] \pm \mathcal{C}[\, h_1^{\perp g/A}\ h_1^{\perp g/B}\,]$$

unpolarized gluons — **lin. polarized** gluons

(factorization) [M. García Echevarría (2019)]
(pheno) [A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. (in progress)]

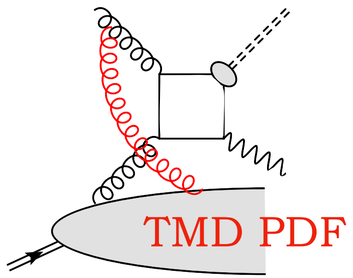
Challenges

Backup

Quarkonia: Assets & challenges

Assets

Onia ⇒ clean channels of f-type gluon TMDs



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unpolarized gluons — **lin. polarized** gluons

(factorization) [M. García Echevarría (2019)]

(pheno) [A. Bacchetta, F.G.C., J.-P. Lansberg, M. Radici, et al. (in progress)]

Challenges

Precision TMD ⇔ production mechanism(s)

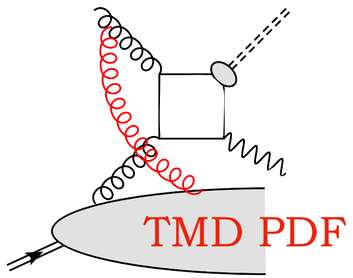
(production mechanisms, LHC) [J.-P. Lansberg (2020)]

Backup

Quarkonia: Assets & challenges

Assets

Onia ⇒ clean channels of f-type gluon TMDs



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$f_{1T}^{\perp g\,[+,-]}$	×

Boer-Mulders	$ep \to e' Q \overline{Q} X$ $ep \to e' j_1 j_2 X$
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Challenges

Precision TMD ⇔ production mechanism(s)

(production mechanisms, LHC) [J.-P. Lansberg (2020)]

Color Evaporation Model

$(Q\bar{Q})$ decorrelated from onium, semi-soft gluon emissions

Overshoots data at large p_T

Color Singlet Model

$(Q\bar{Q})$ to onium, no gluon emissions

Fails at large p_T, improves at NLO

NRQCD and Color Octet

Higher Fock states, soft gluon emissions

Problems at low p_T, fails on polarization

Backup

Quarkonia: A path toward precision

TMD & shape functions

NRQCD ⇒ double expansion: $\alpha_s \oplus v$

NRQCD ⇒ $\mathrm{d}\sigma(|\mathcal{Q}\rangle) \propto \mathscr{H} \otimes \mathrm{LDME}$

$$|\mathcal{Q}\rangle = \mathcal{O}(1)\,|Q\bar{Q}[^3S_1^{(1)}]\rangle + \mathcal{O}(v)\,|Q\bar{Q}[^3P_J^{(8)}g]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^1S_0^{(8)}g]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^3S_1^{(1,8)}gg]\rangle + \mathcal{O}(v^2)\,|Q\bar{Q}[^3D_J^{(1,8)}gg]\rangle + \dots.$$

S-wave quarkonium wave function

Backup

Quarkonia: A path toward precision

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S-wave quarkonium wave function

- TMD ⇒ from LDMEs to shape functions (ShFs)
- 2 mechanisms: bound state + soft-gluon

(factorization) [M. Garcia Echevarria (2019)]
(SCET) [S. Fleming, Y. Makris, T. Mehen (2020)]
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(unpol. J/ψ) [D. Boer, J. Bor, L. Maxia, C. Pisano (in preparation)]

Backup

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Revised TMD shape function in SIDIS

$$\Delta^{[n]}(\boldsymbol{\kappa_T}^2) \propto \frac{\alpha_s}{2\pi^2 \boldsymbol{\kappa_T}^2} C_A \left(1 + \ln \frac{M_{\mathbb{Q}}^2}{M_{\mathbb{Q}}^2 + Q^2}\right) \langle O[n] \rangle$$

Backup

Quarkonia: A path toward precision

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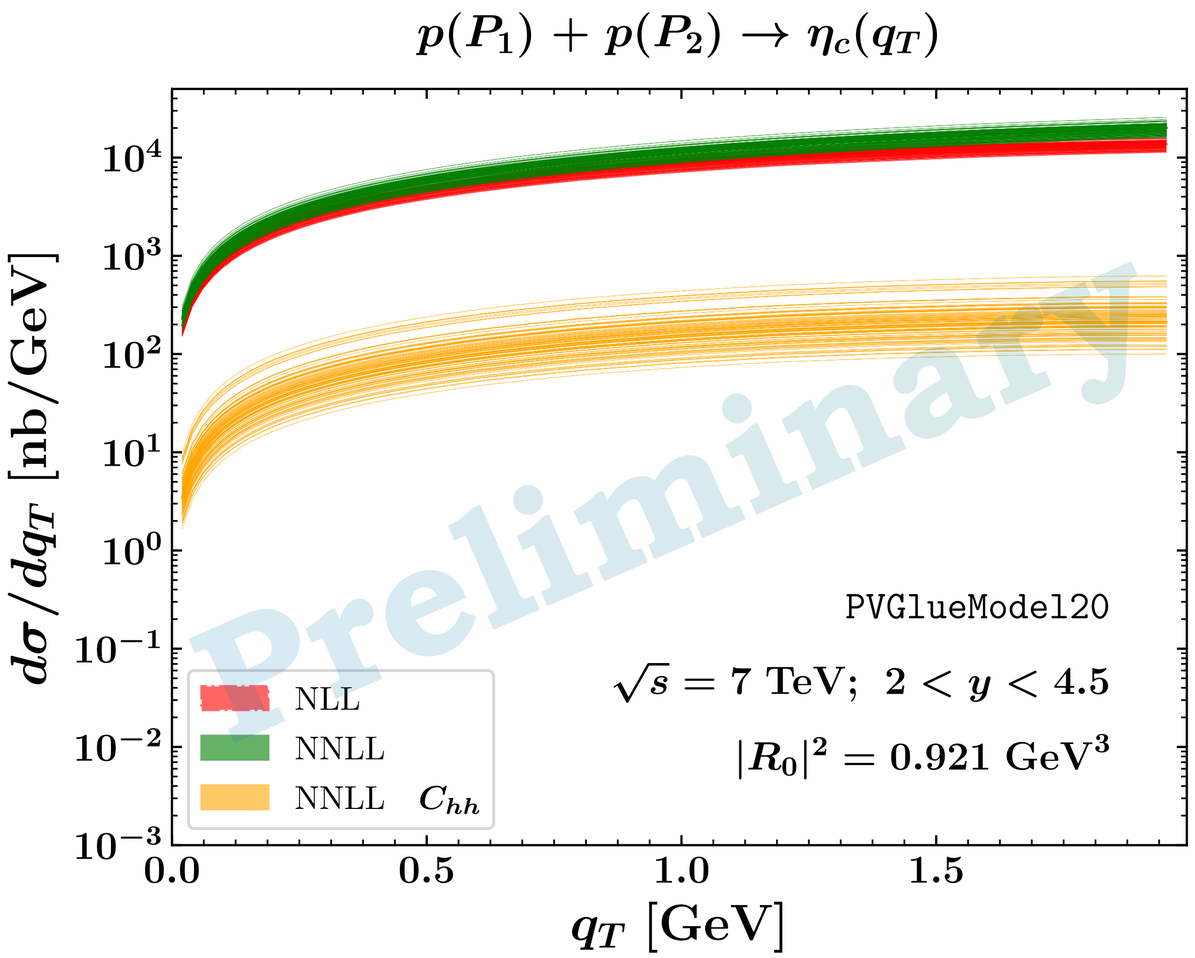
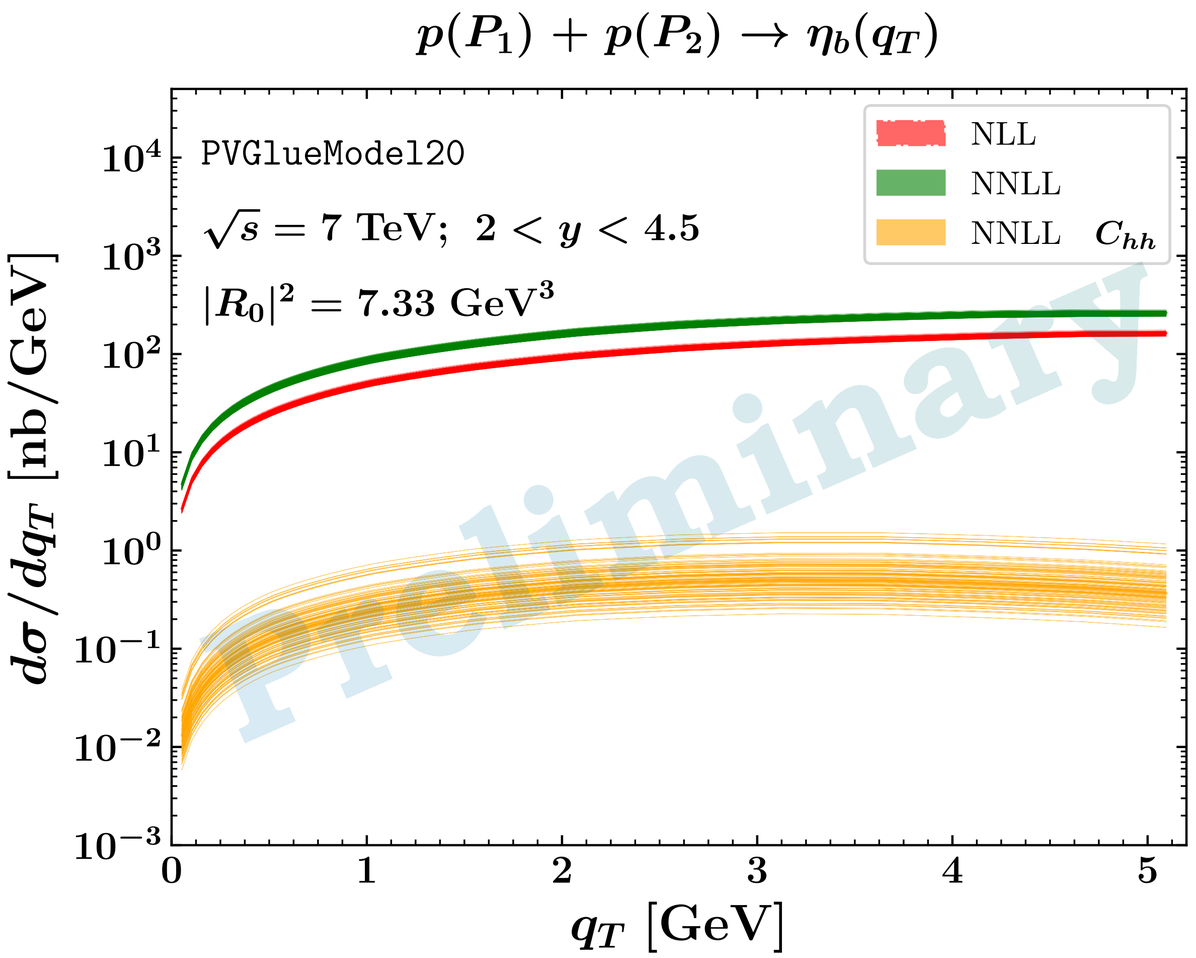
- 2 mechanisms: bound state + soft-gluon
- Perturbative tail ⊗ LDME

Backup

$\eta_{c,b}$ production @ 7TeV LHCb

Perturbative scales fixed, NP-evolution parameters fixed, TMD 100-replica analysis, NRQCD w/o ShFs





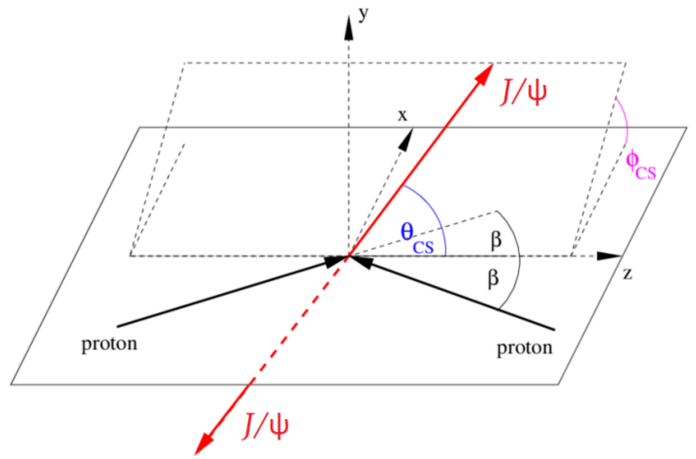
[A. Bacchetta, F.G.C., M.G. Echevarria, J.-P. Lansberg, M. Ozcelik, M. Radici, A. Signori (in preparation)]

Backup

Double J/ψ production @(HL-)LHC

- More spin asymmetries, measurable @HL-LHC

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{QQ}\mathrm{d}Y_{QQ}\mathrm{d}^2\boldsymbol{P}_{QQ_T}\mathrm{d}\Omega} = \frac{\sqrt{Q^2-4M_Q^2}}{(2\pi)^2 8s\,Q^2}\Big\{F_1\,C\Big[f_1^g f_1^g\Big] + F_2\,C\Big[w_2 h_1^{\perp g} h_1^{\perp g}\Big] + \cos 2\phi_{\mathrm{CS}}\Big(F_3 C\Big[w_3 f_1^g h_1^{\perp g}\Big] + F_3' C\Big[w_3' h_1^{\perp g} f_1^g\Big]\Big) + \cos 4\phi_{\mathrm{CS}} F_4 C\Big[w_4 h_1^{\perp g} h_1^{\perp g}\Big]\Big\},$$



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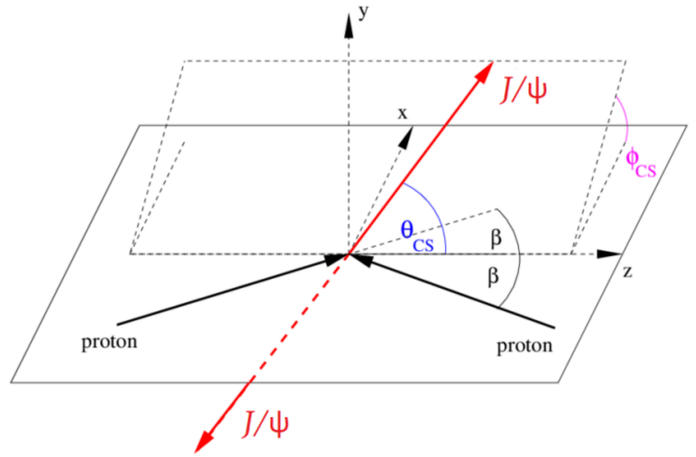

TMD Models

[J.-P. Lansberg et al. (2018)]

$$f_1^g(x,\boldsymbol{k}_T^2,\mu) = \frac{g(x,\mu)}{\pi\langle k_T^2\rangle}\exp\Big(-\frac{\boldsymbol{k}_T^2}{\langle k_T^2\rangle}\Big)$$

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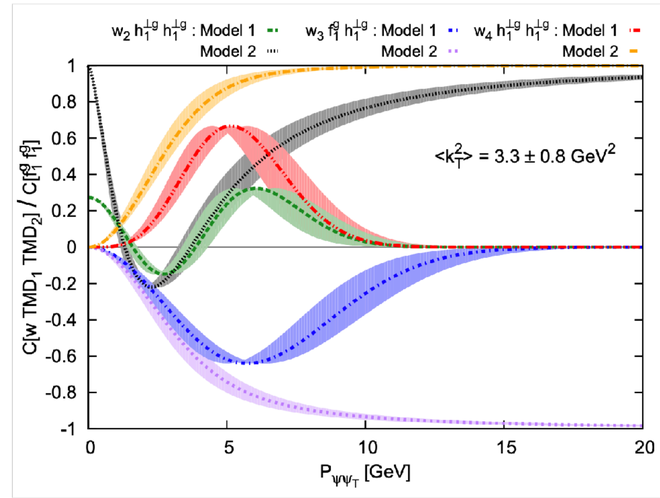

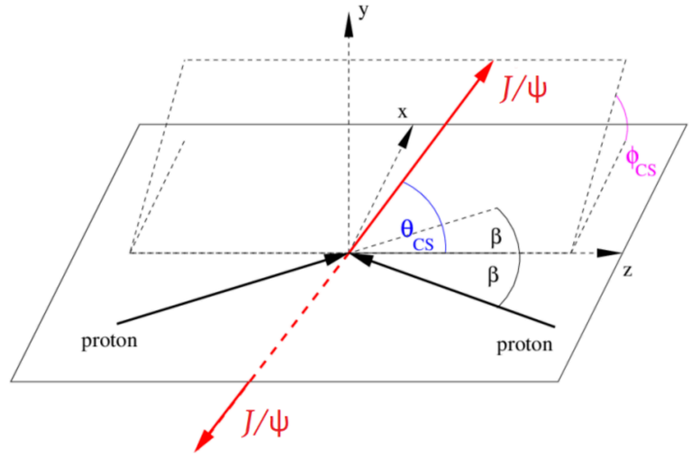

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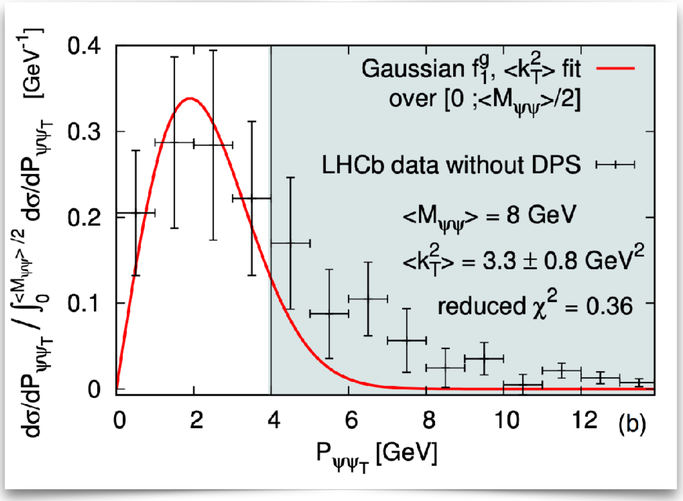

[Model-dependent fit on 13 TeV LHCb data]

Backup

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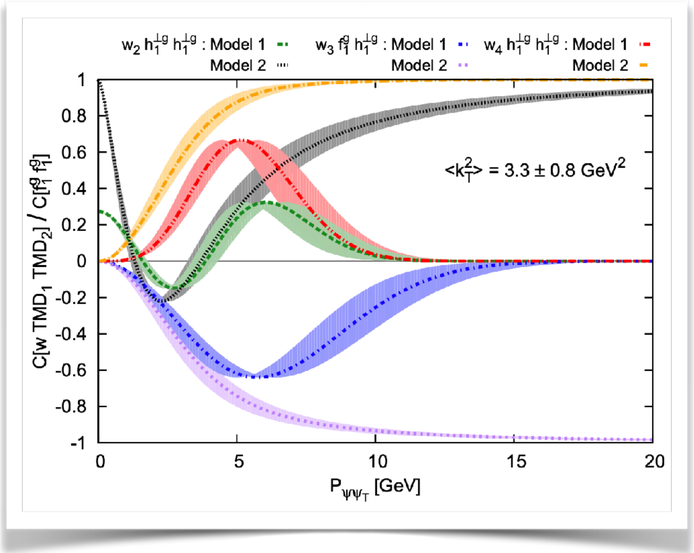

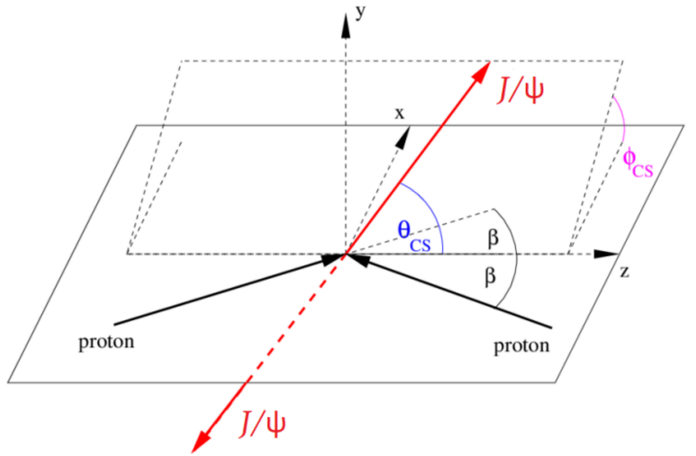

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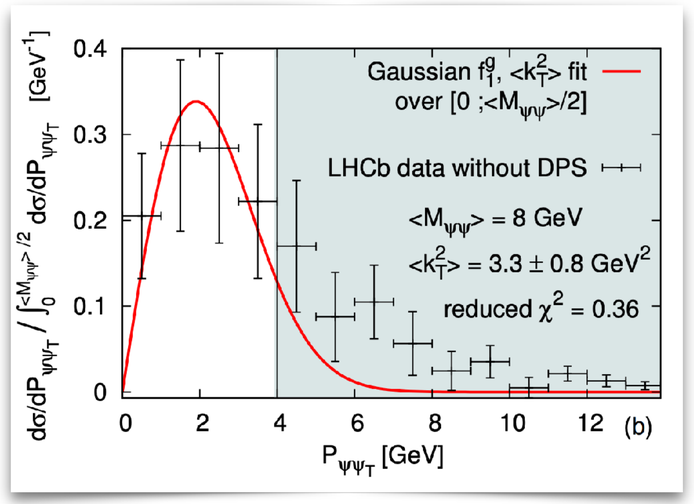
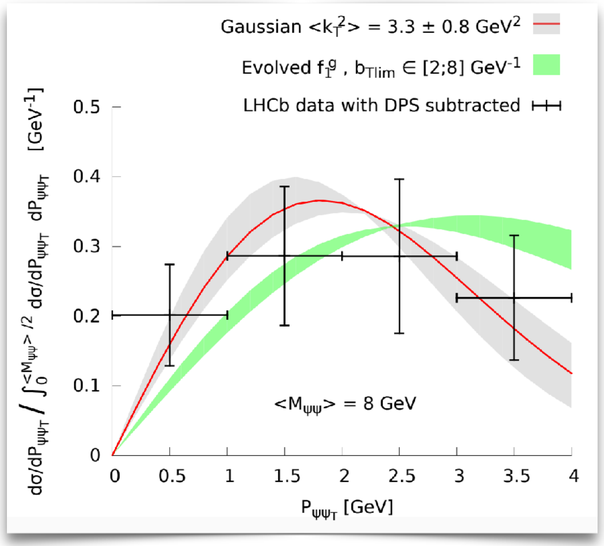


[Model-dependent fit on 13 TeV LHCb data]

NP + TM resummation



[F. Scarpa et al. (2020)]

Backup

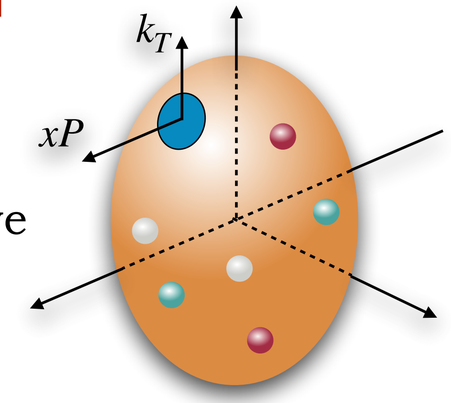
TMD factorization

Parton densities: an incomplete family tree

3D

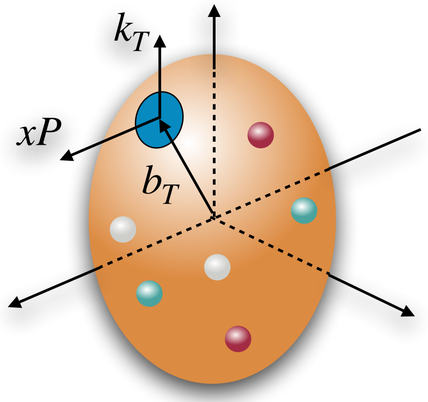
TMDs

(semi-)inclusive



5D

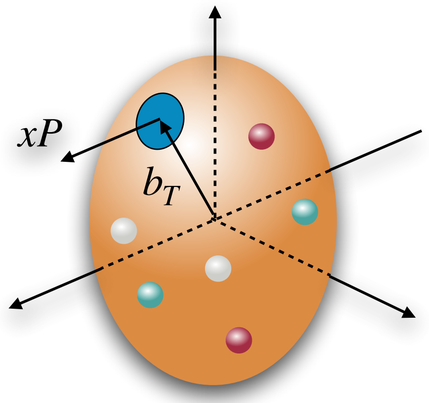
Wigner distributions



3D

FT of GPDs

exclusive



PDFs

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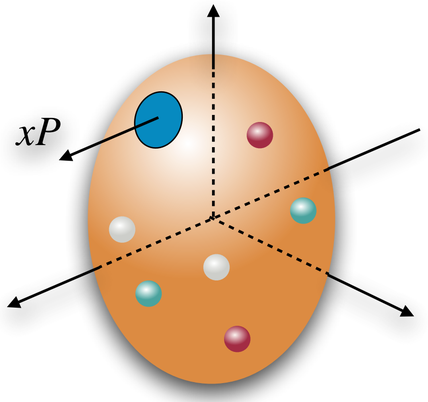
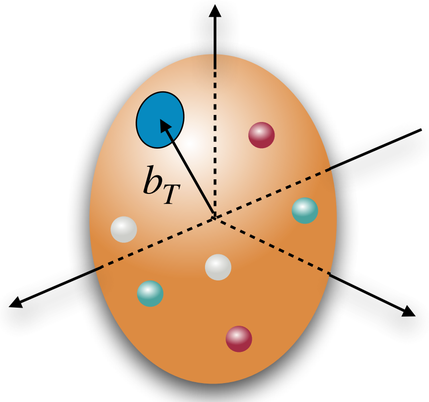


FT of Form Factors



1D

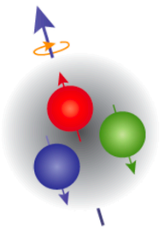
2D

$\vec{b}_\perp$ dependence

$\vec{k}_\perp$ dependence

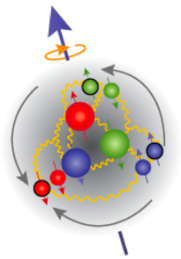
these two variables are NOT Fourier conjugate

The proton spin crisis



quarks helicity

~ 30 %



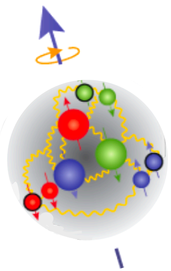
orbital angular momentum

unknown!

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

gluons helicity

~ 20% – 50 % **?**



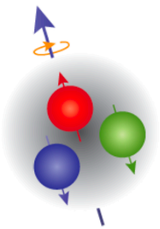
Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description…

🔗 (proton spin crisis) [EMC Collaboration, CERN (1987)]

slide adapted from C. Bissolotti

Backup

The proton spin crisis



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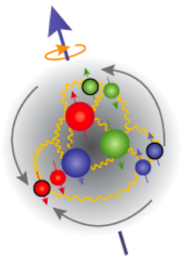

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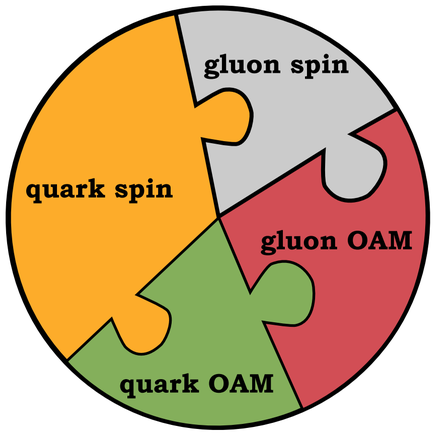

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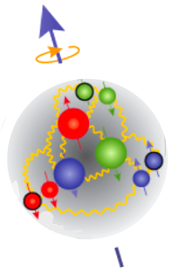

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Backup

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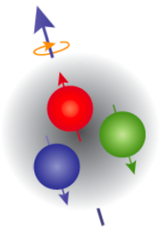

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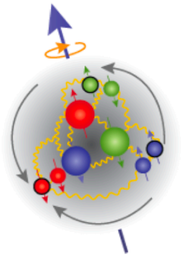

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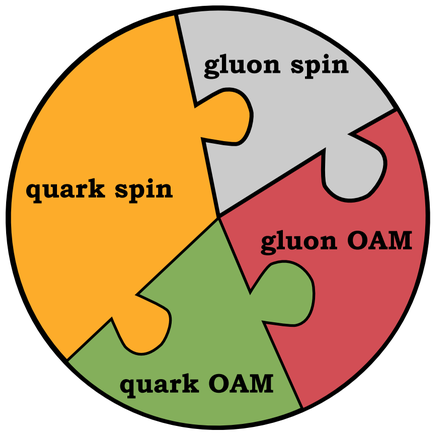

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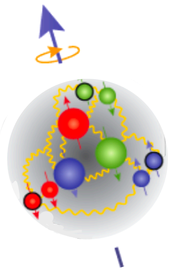

…many other effects in hadronic interactions cannot be understood in the purely collinear approach

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Backup

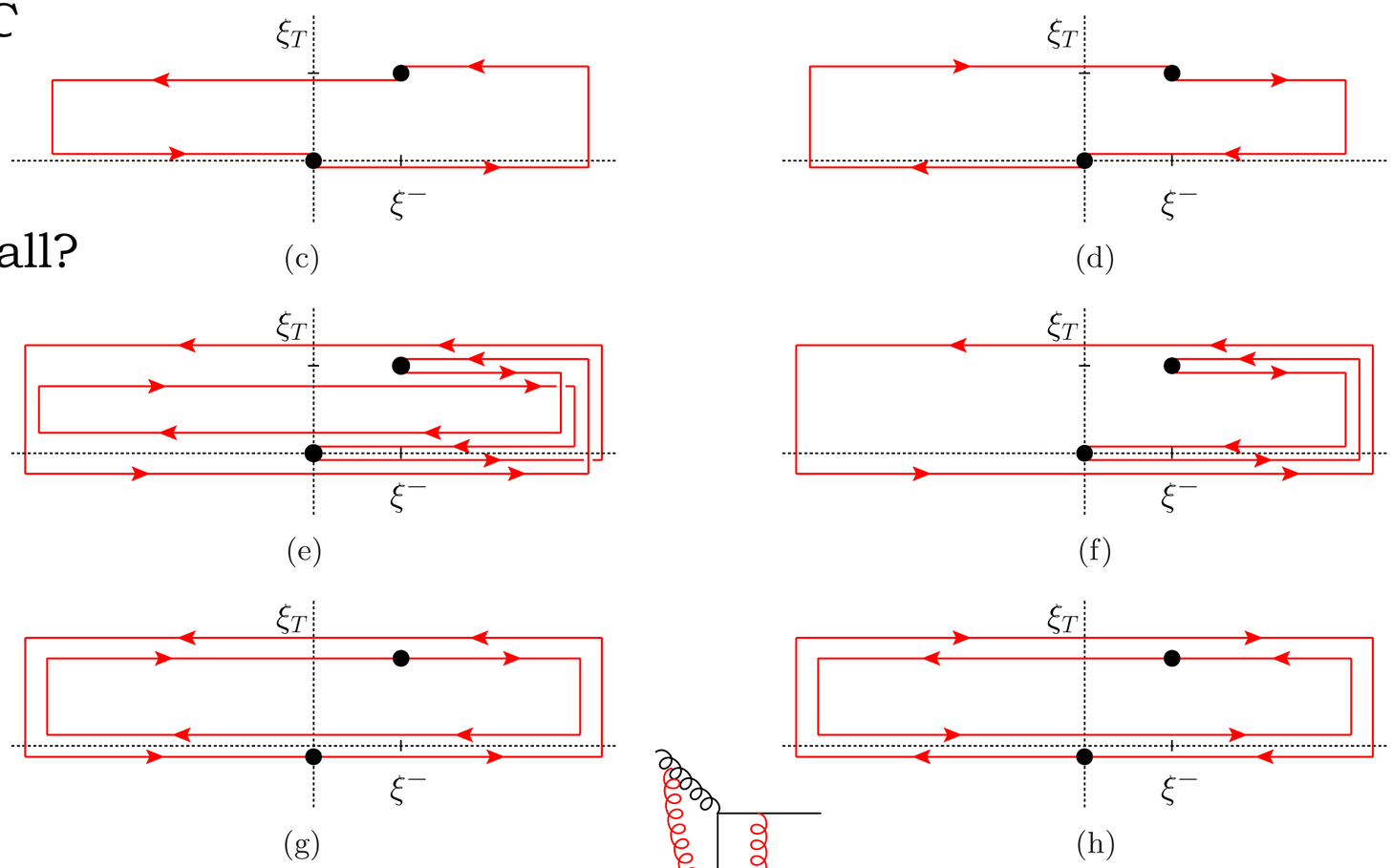
Dihadron hadroproduction and factorization breaking

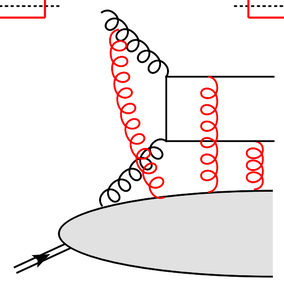
* Proof of factorization violation 🔗 [T. J. Rogers, P. J. Mulders (2010)]
* Assumed factorization in SCET and CGC
* Significance of low-x studies
* Size of factorization-breaking effects small?
* DP TMDs:

  (c) $[+,-]$ and (d) $[-,+]$
* Appearance of new gauge **loop links**:

  (e) $[+\square,+\square]$, (f) $[+,+\square]$,

  (g) $[\square,\square]$, and (h) $[\square,\square]$





Backup

Spectator-model gluon TMDs

Assumptions of the model

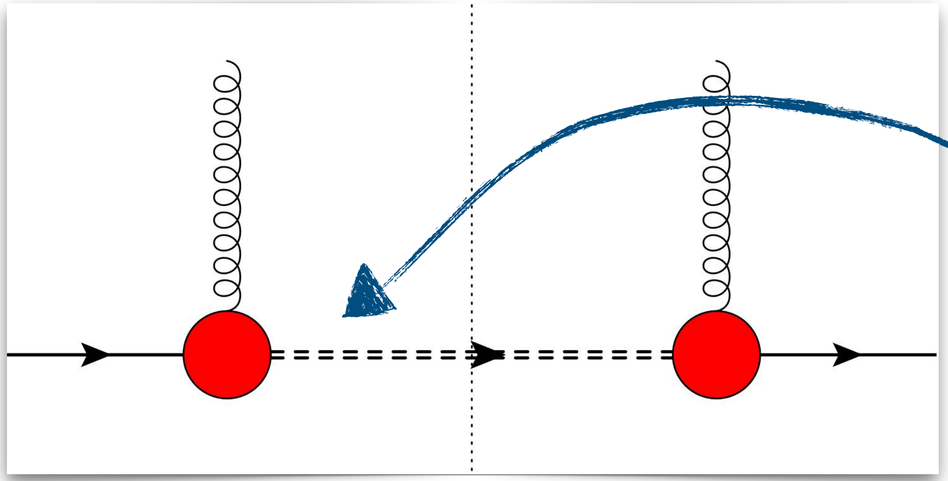
Spectator-system spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

spectral-mass function

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



$\mathcal{Y}_g(p^2)$

Instead of a single on-shell spectator, a continuum of spectators

Backup

Assumptions of the model

Spectator-system spectral-mass function

spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

spectator-model TMD

🔗 [Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X\left(M_X; \{X^{(\mathrm{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}\right) = \mu^{2a}\left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}}\right]$$

low-x (high-μ^2) tail $\propto (a - b)$

$q\bar{q}$ contributions energetically available at large M_X

$$\mu^2 = M_X^2 - M^2$$

moderate-x trend

pure tri-quark contribution at low M_X

Backup

Gluon TMD correlator and T-odd gluon densities

$$\Gamma_U^{ij}(x,\boldsymbol{k}) = x\left[\delta_T^{ij} f_1(x,\boldsymbol{k}^2) + \frac{k_T^{ij}}{M^2} h_1^{\perp}(x,\boldsymbol{k}^2)\right]$$

$$\Gamma_L^{ij}(x,\boldsymbol{k}) = x\left[i\epsilon_T^{ij} S_L\, g_1(x,\boldsymbol{k}^2) + \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^{\perp}(x,\boldsymbol{k}^2)\right]$$

$$\Gamma_T^{ij}(x,\boldsymbol{k}) = x\left[\frac{\delta_T^{ij}\,\epsilon_T^{S_T k_T}}{M} f_{1T}^{\perp}(x,\boldsymbol{k}^2) + \frac{i\epsilon_T^{ij}\,\boldsymbol{k}\cdot\boldsymbol{S}_T}{M} g_{1T}(x,\boldsymbol{k}^2)\right.$$
$$\left. - \frac{\epsilon_T^{k_T\{i} S_T^{j\}} + \epsilon_T^{S_T\{i} k_T^{j\}}}{4M} h_1(x,\boldsymbol{k}^2) - \frac{\epsilon_T^{\{i}{}_{\alpha} k_T^{j\}\alpha S_T}}{2M^3} h_{1T}^{\perp}(x,\boldsymbol{k}^2)\right]$$

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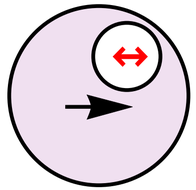
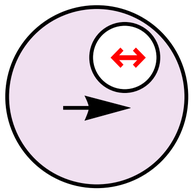
pseudo worm-gear



linearity TMD

pretzelosity



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pseudo worm-gear

linearity TMD

pretzelosity

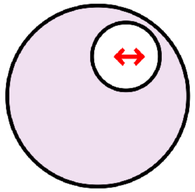
Sivers

$$\frac{\epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x,\boldsymbol{k}^2) = \frac{1}{2}\delta_{Tij}\Gamma_T^{ij}(x,\boldsymbol{k})$$

Backup

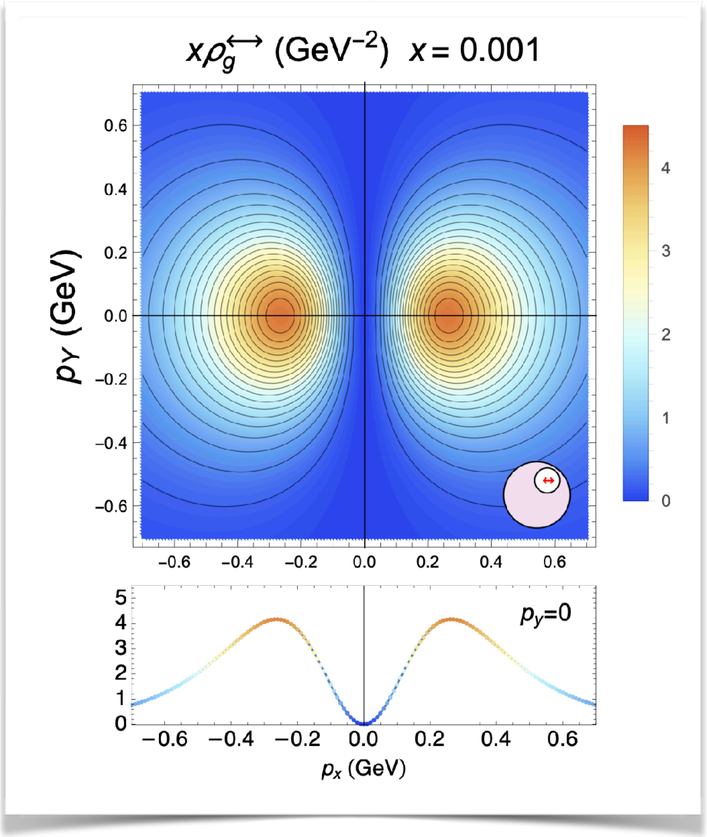
Gluon TMD phenomenology

Boer-Mulders effect in unpolarized pp collisions



$$[\leftrightarrow/\mathbf{u}] \qquad f_1(x,p_x,p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^{\perp}(x,p_x,p_y)$$

unpol. Boer-Mulders



🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

Backup

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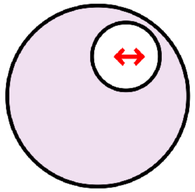

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unpol. — Boer-Mulders

(Pseudo)scalar p_T-distributions: Higgs, $\eta_{c,b}$

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \, \Phi_B^U \, |\mathcal{M}|^2 \quad \text{at low transverse momentum for (pseudo)scalar state}$$

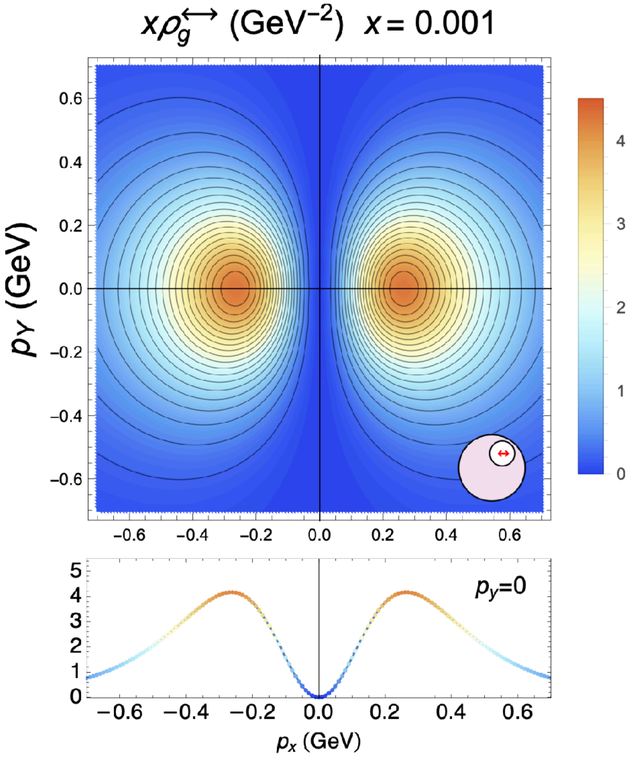
$$\sim \mathcal{C}[\, f_1^{g/A} \, f_1^{g/B} \,] \pm \mathcal{C}[\, h_1^{\perp g/A} \, h_1^{\perp g/B} \,]$$

$\mathcal{C}[\, f_1^{g/A} \, f_1^{g/B} \,]$: **unpolarized** gluons

$\mathcal{C}[\, h_1^{\perp g/A} \, h_1^{\perp g/B} \,]$: **lin. polarized** gluons

NRQCD

$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{v^4}$$

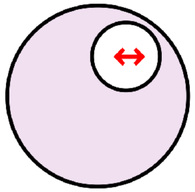


[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
(Higgs+jet angular distributions) [D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

Backup

Boer-Mulders effect in unpolarized pp collisions



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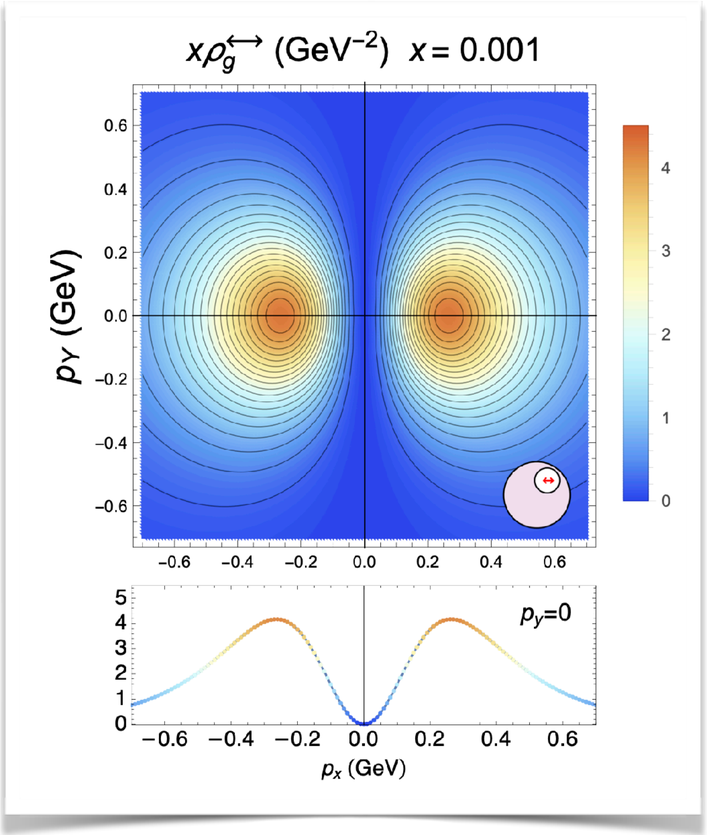


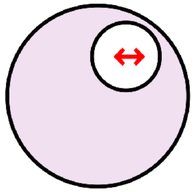
🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

Model prediction at low x

$$\frac{f_1^g(x,p_T^2)}{h_1^{\perp g}(x,p_T^2)} \underset{x\to 0^+}{\sim} \text{constant}$$

Backup

Boer-Mulders effect in unpolarized pp collisions



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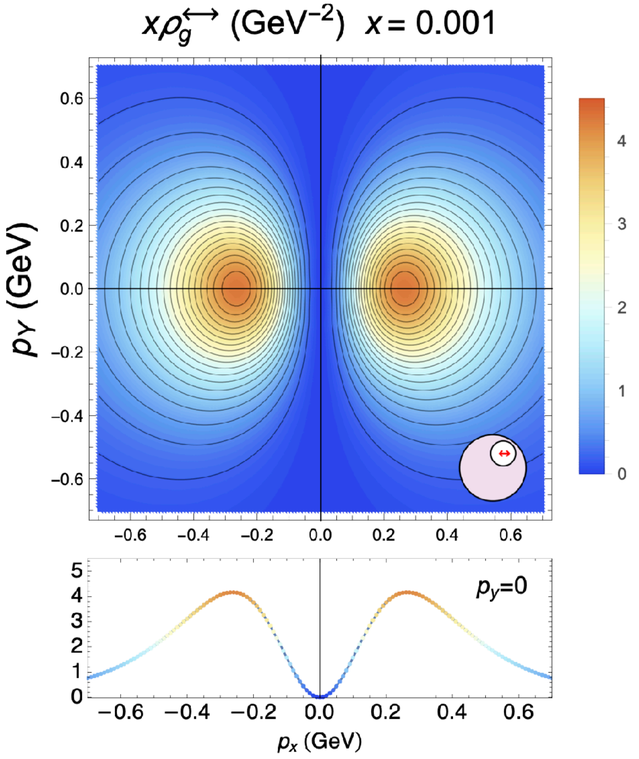

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BFKL regime (linear low-x evolution)

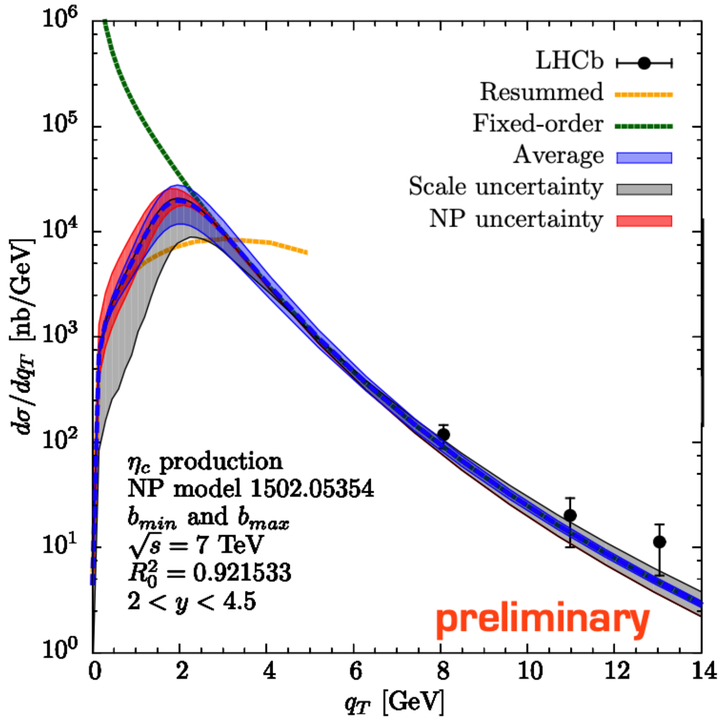
$$f_1^g(x,p_T^2) = h_1^{\perp g}(x,p_T^2) + \text{higher twist}$$

Backup

$\eta_{c,b}$ production @ 7TeV LHCb

η_c production at LHC

full transverse momentum spectrum:
low qT matched with high qT region



blue band: uncertainty from matching

grey band: scale uncertainty

red band: nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = -\left[\frac{a_1}{2} + \frac{a_2}{2}\ \ln Q^2\right] \bar{b}_T^2$$

a_i = 0.5 GeV², var. 50%, envelope
both for unpolarized and
linearly polarized distributions

the formalism is in good shape!
we need the data at low qT

Jefferson Lab

25

slide by Andrea Signori (INT 2017)

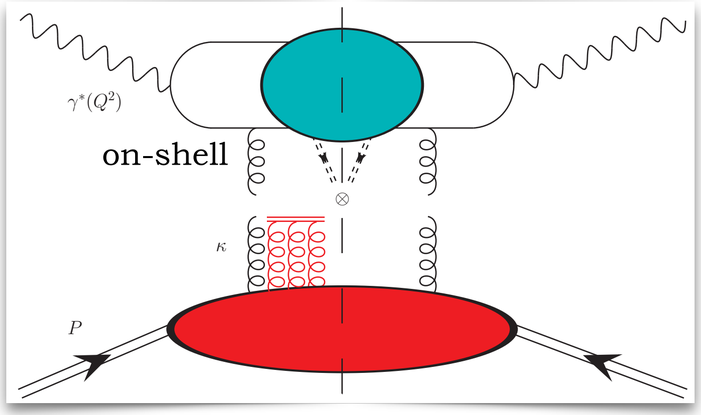
Backup

HEF and the UGD

TMD versus high-energy factorization

TMD

* Semi-inclusive processes
* $\kappa_T \ll$ hardest scale
* Language of parton correlators
* Diagram: SIDIS onium



TMD
PDF

Backup

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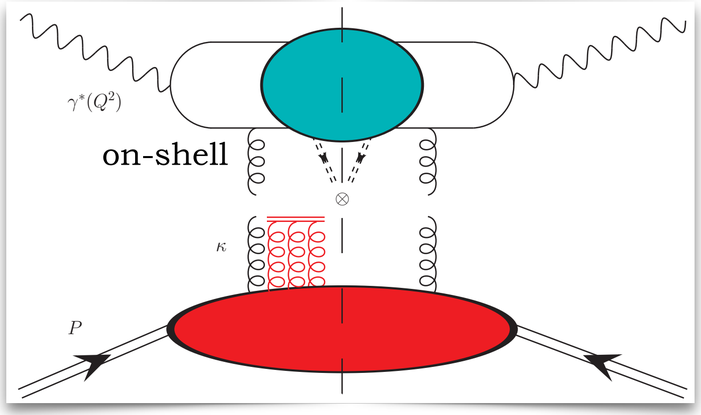
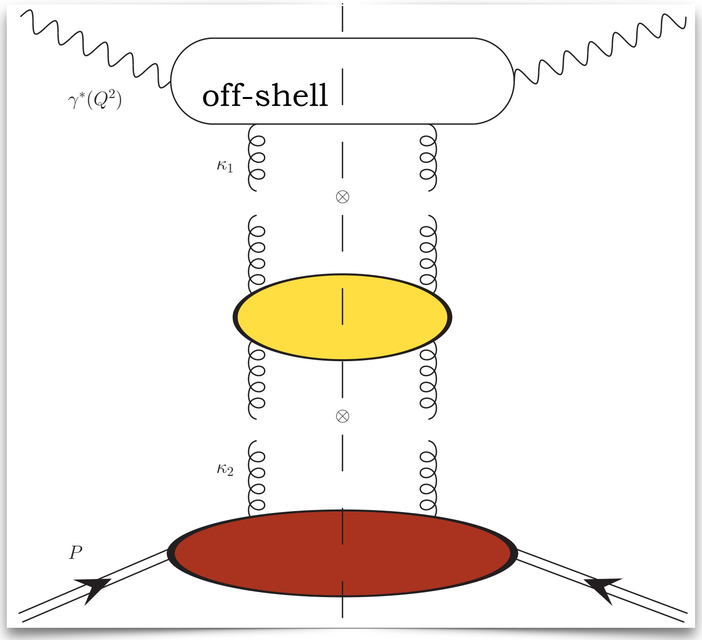


TMD PDF

HEF

* Inclusive or exclusive processes (!)
* Small x, large κ_T
* Language of Reggeized gluons
* Diagram: DIS



$$\Phi^{\gamma^*\to\gamma^*} \otimes \mathcal{G}_{\mathrm{BFKL}} \otimes \Phi^{P}_{[\mathrm{NP}]}$$

Backup

TMD versus high-energy factorization

TMD

IR-safe colorless $\{\Phi^{i\to o}\}$

(Fadin-Martin theorem)

[V.S. Fadin, A.D. Martin (1999)]

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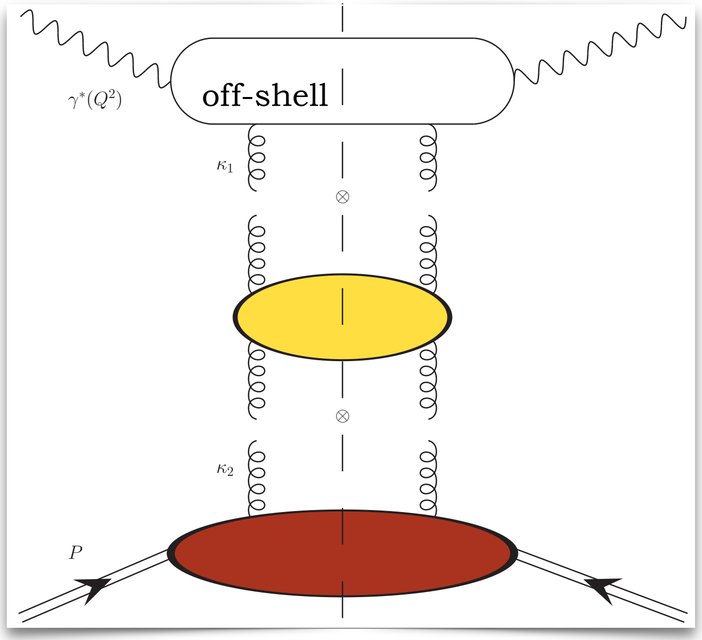

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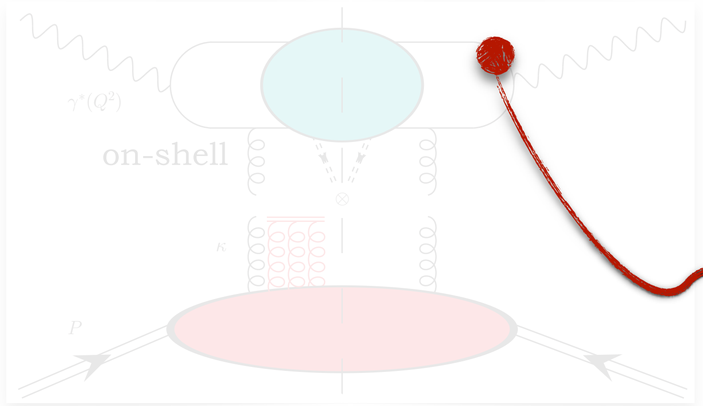
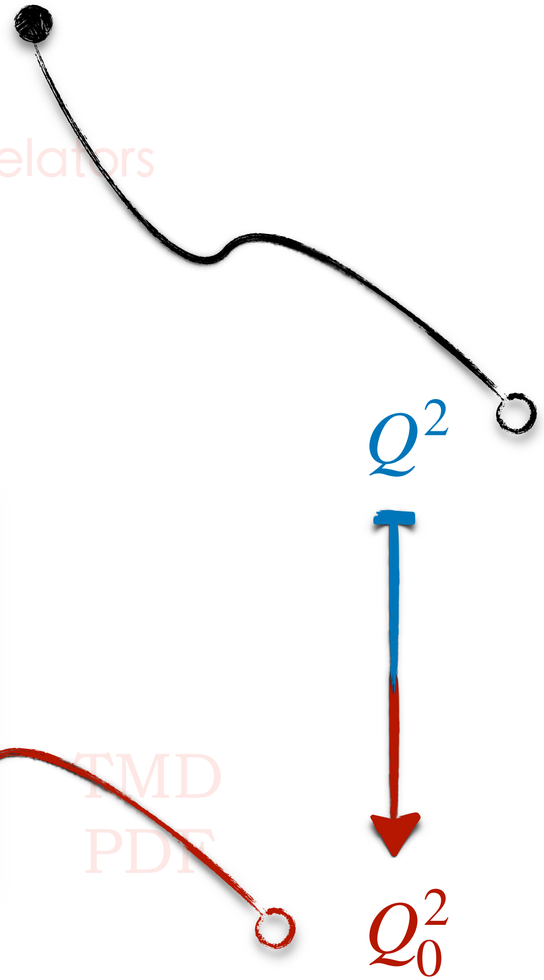
IR diffusion pattern

(Bartels' cigar)

[J. Bartels, H. Lotter (1993)]





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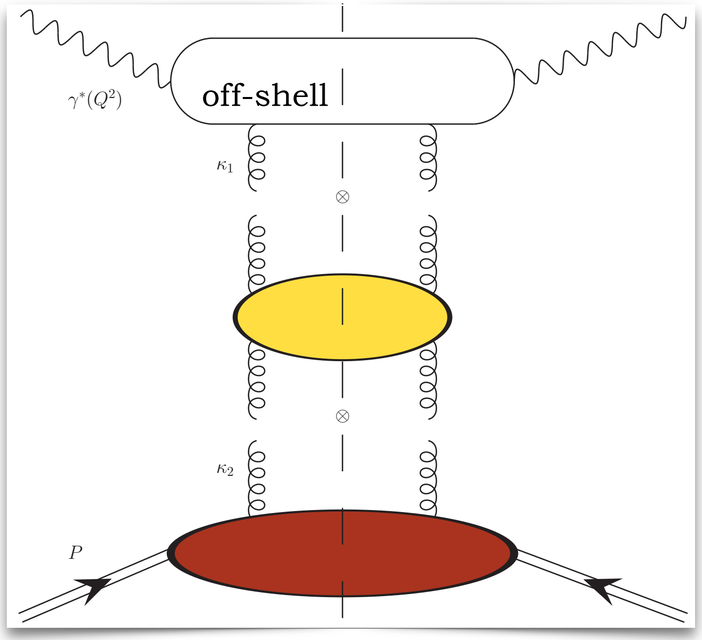


$$\Phi^{\gamma^*\to\gamma^*} \otimes \mathcal{G}_{\text{BFKL}} \otimes \Phi^{P}_{[\text{NP}]}$$

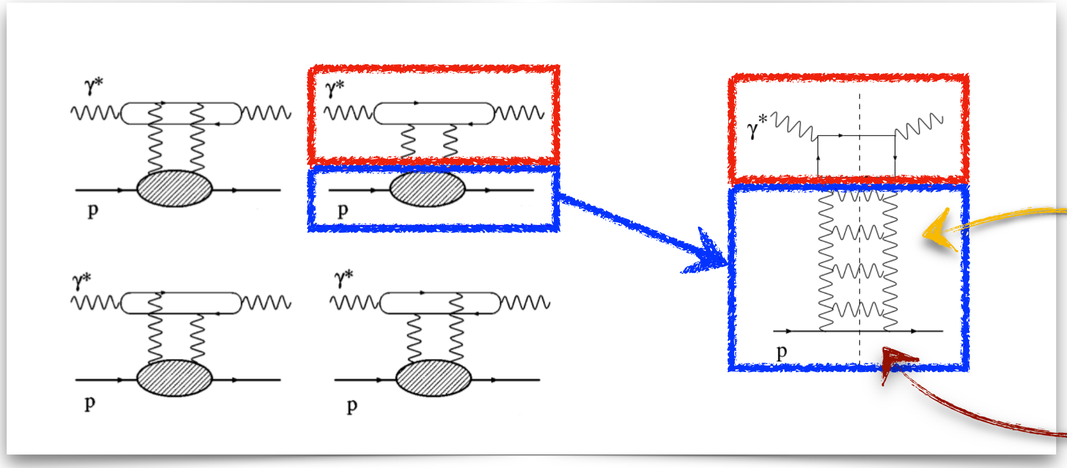
Backup

High-energy factorization and the UGD

- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\text{tot}}(\gamma^* p \to X) \propto \mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \gamma^* p)\} \equiv \Phi_{\gamma^* \to \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

◇ $\mathcal{F}(x, \kappa^2)$ is the **unintegrated gluon distribution** (**UGD**) in the proton



▶ Small-x limit: **UGD** = [**BFKL gluon ladder**] $\circledast$ [**proton impact factor**]

◇ Takes into account the resummation of high-energy logs

◇ Describes the coupling of the gluon Green's function to the proton

▶ Proton impact factor is non-perturbative $\Longrightarrow$ UGD needs to be modeled!

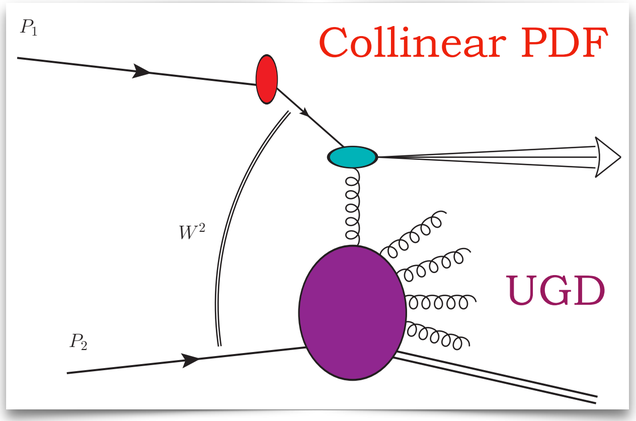
Backup

Hybrid or pure factorization?

Forward emissions

* *Asymmetric* config. ↔ fast parton + small-x gluon

* Hybrid **high-energy**/**collinear** factorization



* *Distinctive signals* of small-x dynamics **expected**

* Phenomenology:
  forward jet, Drell-Yan, Higgs or vector meson

Backup

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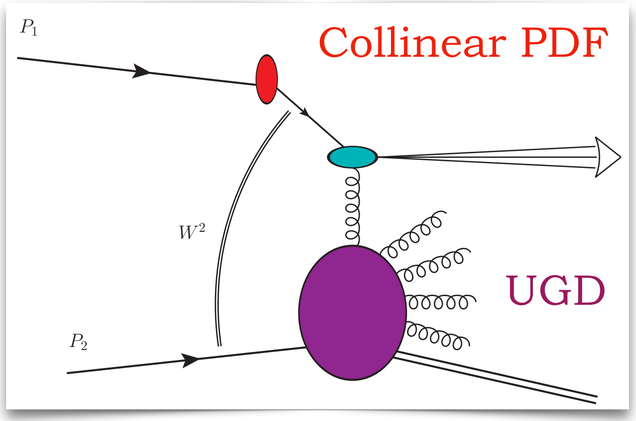

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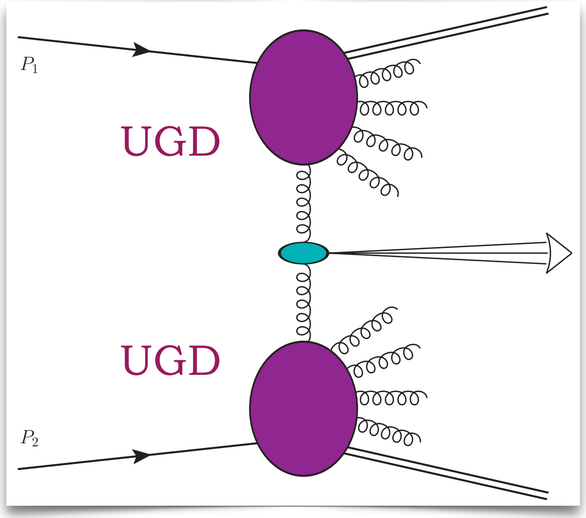
Central emissions

* *Gluon induced* ↔ small-x gluons
* Pure **high-energy** factorization



* Small-x dynamics to **enhance** f.o. description
* Phenomenology:
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Backup

Hybrid or pure factorization?

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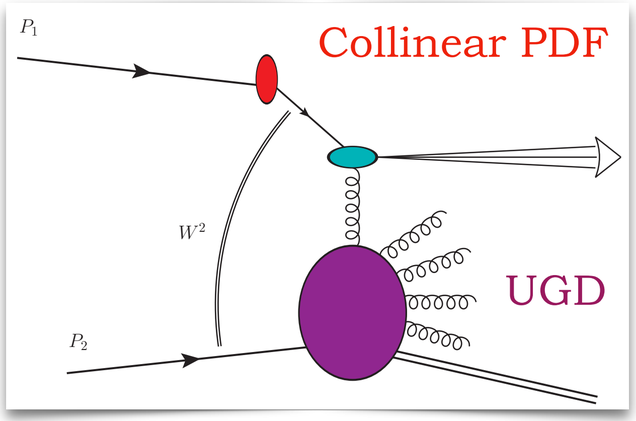

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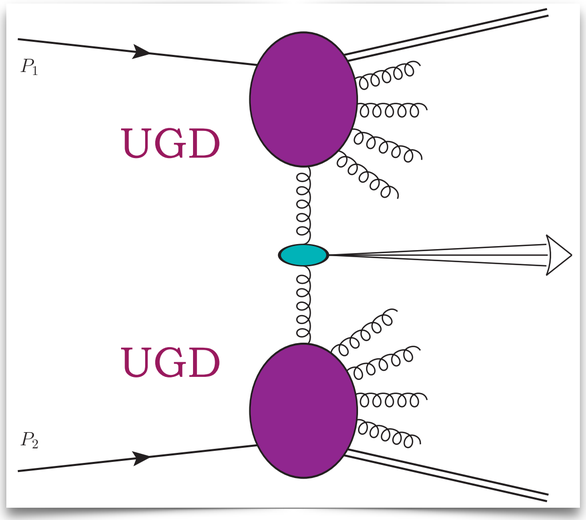

* Small-x dynamics to **enhance** f.o. description
* Phenomenology:
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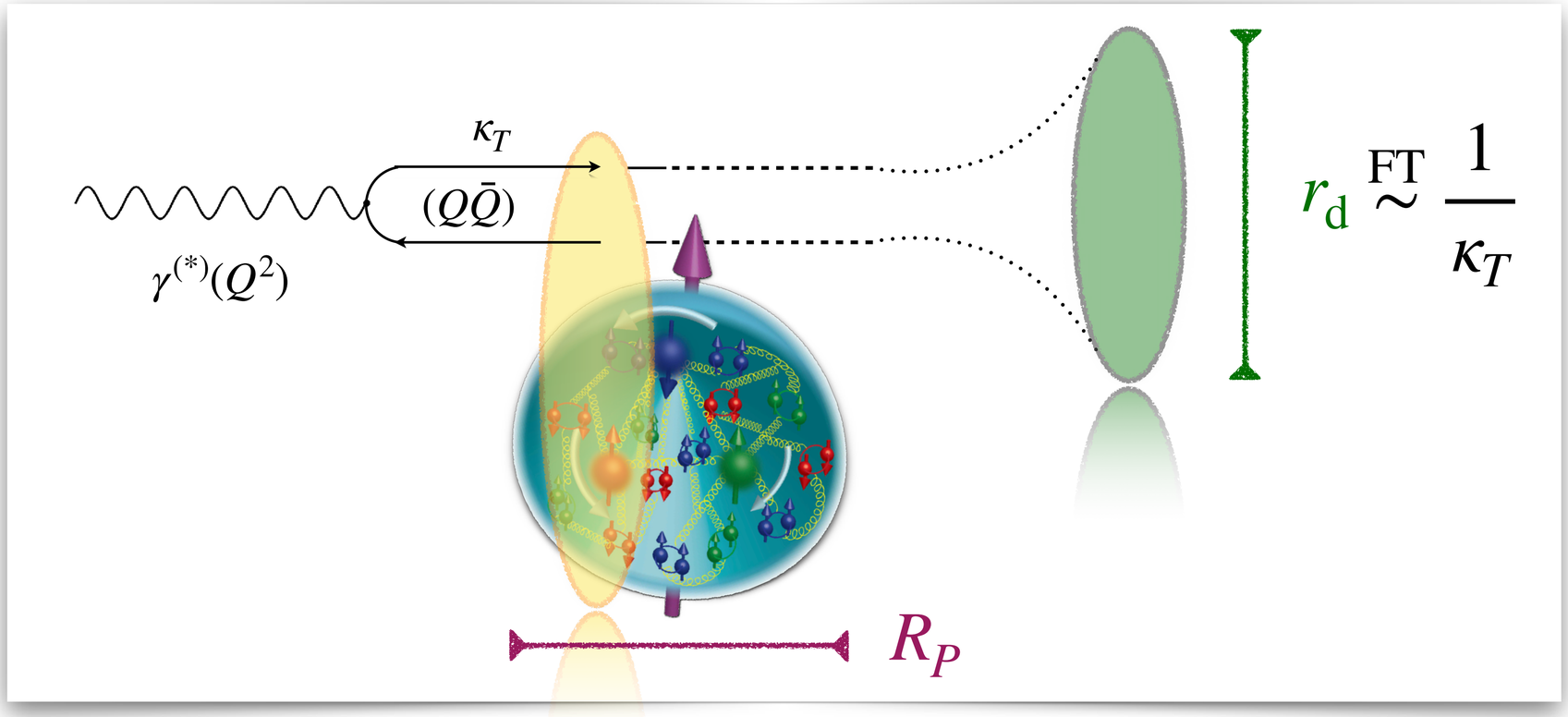
Table complemented by *exclusive* counterparts and *lepto-hadronic* channels

Backup

Diffractive $\gamma^* P$ scatterings and color dipoles



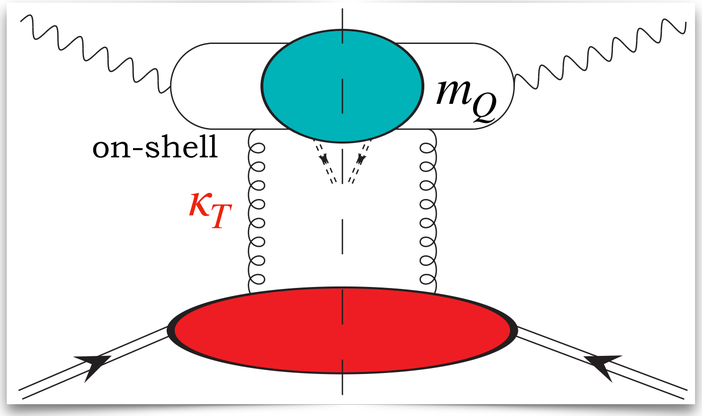
$$W_{\mu\nu} \propto \mathrm{Im}\left\{ i \int \mathrm{d}^4x\, e^{i\,q\cdot x} \langle P | \mathrm{T}\,[J_\mu(x)\, J_\nu(0)] | P \rangle \right\}$$

* Small-x $\Rightarrow$ Ioffe time $\gg R_P$
* At least one J_μ outside proton…
* …color dipole picture!

Backup

Inclusive quarkonium production mechanisms

$$\kappa_T \ll Q$$



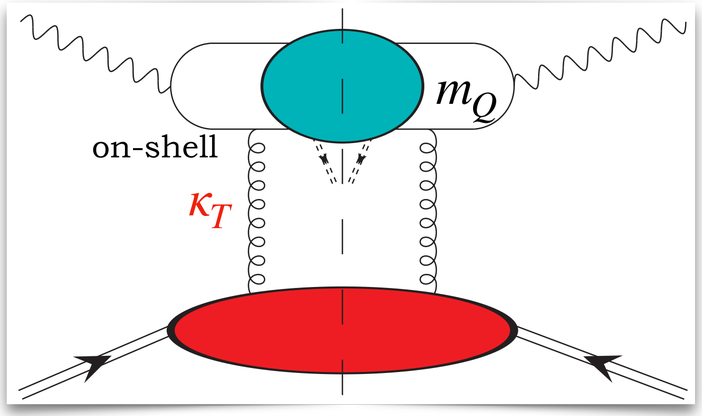
* Gluon TMD PDF
* Short-distance $(Q\bar{Q})$ + ShFs

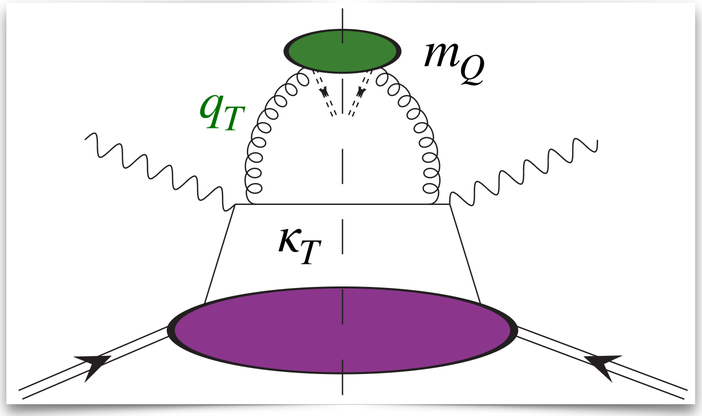
Backup

Inclusive quarkonium production mechanisms

$\kappa_T \ll Q$

$\kappa_T \gg m_Q$





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* Short-distance $(Q\bar{Q})$ + ShFs

* Quark collinear PDF
* Onium in jet
* Single-quark TMD FF

Backup

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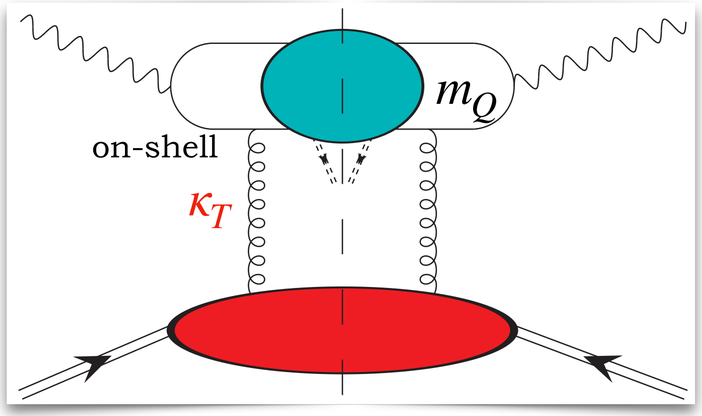

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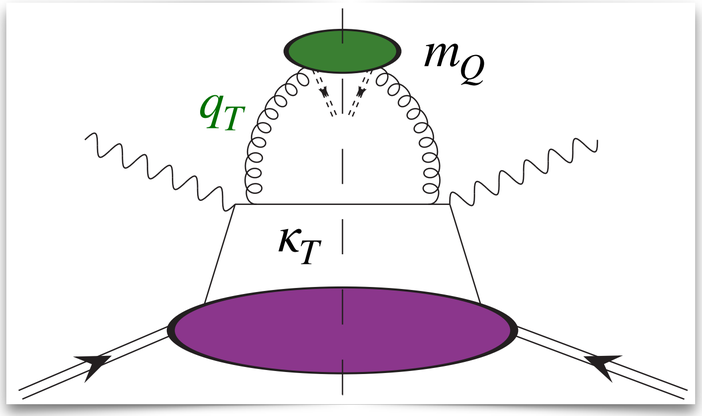

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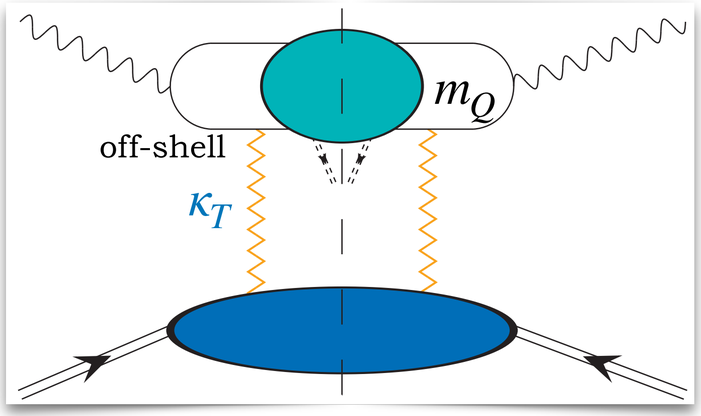
HEF



* BFKL UGD
* Reggeized gluons
* Dipole mechanism

Backup

Exclusive forward ρ meson leptoproduction

Single forward emissions

Exclusive light VM: ρ^0, ω, ϕ

* *Small-size* dipoles $\Rightarrow$ large κ_T
* Collinear description: twist-2/-3 LVM NP **DAs**

$$\Phi^{\gamma^* \to \rho} \propto \int_0^1 \mathrm{d}z \, T_H^{\gamma^* \to \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \, \phi^{\lambda_\rho}(z, \mu_F)$$

* Significance of small κ_T under investigation…
* HERA indication: no large-r_d dynamics
* LVMs as tools: discrimination among UGD models
* LVMs as tools: UGD extraction $\Leftarrow$ HERA + EIC fits

Backup

Single forward emissions

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* LVMs as tools: discrimination among UGD models
* LVMs as tools: UGD extraction ⇐ HERA + EIC fits

Quarkonia

* Size of dipoles ⇒ wide range of κ_T
* Description: **NRQCD** (combined with LFWFs)

$$\left[\mathrm{LFWF} \otimes \mathscr{A}_{\mathrm{dip.}} \right] \overset{\text{dilute}}{\longleftrightarrow} \left[\Phi^{\gamma^* \to J/\Psi} \otimes \mathrm{UGD} \right]$$

* Validity of *small-size* dipoles questionable…
* NRQCD: large-r_d dynamics for $\Psi(2s)$ ($\Upsilon(2s)$?)

🔗 [K. Suzuki *et al.* (2000)]; 🔗 [J. Cepila *et al.* (2019)]; 🔗 [M. Hentschinski *et al.* (2020)]

* Onia as tools: scan of TMD/HEF intersection range

Backup