TMD evolution study of the azimuthal asymmetry in unpolarized J/ψ production at EIC

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3rd SARDINIAN WORKSHOP ON SPIN

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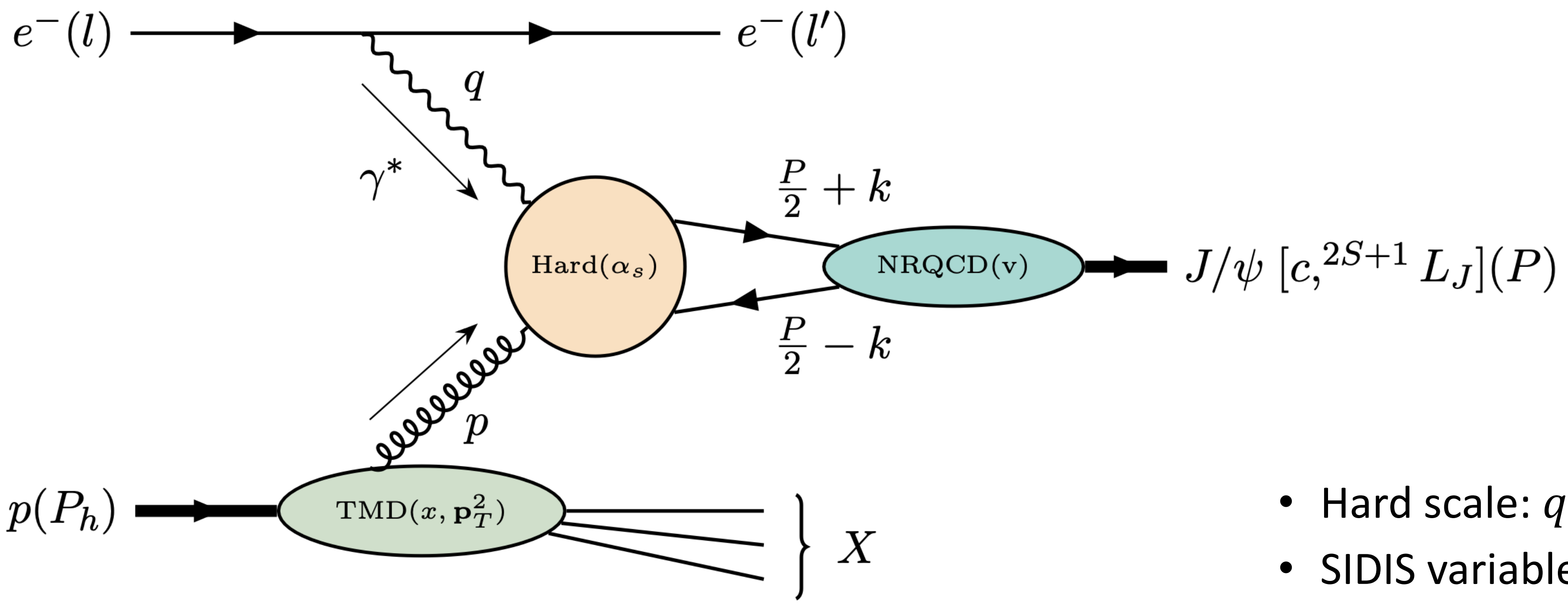
Motivation and overview of the process

- J/ψ production in ep gives via its P_T - spectrum access to nonperturbative quantities: the gluon TMDs and the color-octet NRQCD LDMEs *Bacchetta et al. 2001*



- Hard scale: $q^2 = -Q^2$
- SIDIS variables: x_B, y, z

- Assume TMD factorization to probe the transverse momentum of the partonic gluon via the observed quarkonium: $P_T = p_T$

Expansions and parametrizations

- v expansion related to quantum numbers [Bodwin et al. 1995/ 2005]

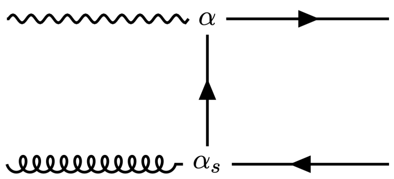
	$1,{}^1S_0$	$1,{}^3S_1$	$8,{}^1S_0$	$8,{}^3S_1$	$1,{}^1P_1$	$1,{}^3P_0$	$1,{}^3P_1$	$1,{}^3P_2$	$8,{}^1P_1$	$8,{}^3P_0$	$8,{}^3P_1$	$8,{}^3P_2$
J/ψ		1	v^3	v^4						v^4	v^4	v^4

$$\sigma[AB \to H + X] = \sum_n C_n^{AB}(\Lambda)\, \langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$$

- $n = {}^{2S+1}L_J$

- Unpolarized proton parameterized by two functions at LO (1/hard scale) [Mulders and Rodrigues 2001]
  - Unpolarized gluon distribution: f_1^g
  - Linearly polarized gluon distribution: $h_1^{\perp g}$

- LO diagram (2x)



$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{x}{2}\Big\{ -g_T^{ij}\, f_1(x, \mathbf{k}_T^2) + \Big(\frac{k_T^i k_T^j}{M_h^2} + g_T^{ij}\frac{\mathbf{k}_T^2}{2M_h^2}\Big) h_1^{\perp}(x, \mathbf{k}_T^2)\Big\}$$

$$d\sigma = \frac{1}{2S}\frac{d^3 l'}{(2\pi)^3 2E_{l'}}\frac{d^3 P}{(2\pi)^3 2E_P}\int dx\, d^2\mathbf{p}_T\, (2\pi)^4 \delta^4(q + p - P) \times \frac{1}{x^2 Q^4}\, L(l, q)_{\nu\sigma}\, \Gamma_{\mu\rho}(x, \mathbf{p}_T)\, \mathcal{M}^{\mu\nu}(q, P)\, \mathcal{M}^{*\rho\sigma}(q, P)\,,$$

The differential cross section

$$\frac{d\sigma(J/\psi)}{dx_B\, dy\, d^2\mathbf{q}_T} \equiv d\sigma^U = \mathcal{N}\left[\mathrm{A}^U\, f_1^g(x,\mathbf{q}_T^2) + \frac{\mathbf{q}_T^2}{M_h^2}\,\mathrm{B}^U\, h_1^{\perp g}(x,\mathbf{q}_T^2)\cos(2\phi_T)\right]$$

$$\mathrm{A}^U = \mathrm{A}^T = [(1-y)^2+1]\mathcal{A}_{U+L}^{\gamma^* g\to J/\psi} - y^2\mathcal{A}_L^{\gamma^* g\to J/\psi}$$

$$\mathrm{B}^U = \mathrm{B}^T = (1-y)\mathcal{B}_T^{\gamma^* g\to J/\psi}$$

$$\mathcal{N} = (2\pi)^2\frac{\alpha^2\alpha_s e_c^2}{yQ^2M(M^2+Q^2)}$$

- $A, B \propto F's$
- U + L, L and T denote the polarization of the virtual photon Pisano et al. 2013

- The heavy-quark spin relations simplify the expressions

  Bodwin et al. 1995

$$\mathcal{A}_{U+L}^{\gamma^* g\to J/\psi} = \langle 0|\mathcal{O}_8^{J/\psi}({}^1S_0)|0\rangle + \frac{12}{N_c}\frac{7M^2+3Q^2}{M^2(M^2+Q^2)}\langle 0|\mathcal{O}_8^{J/\psi}({}^3P_0)|0\rangle\,,$$

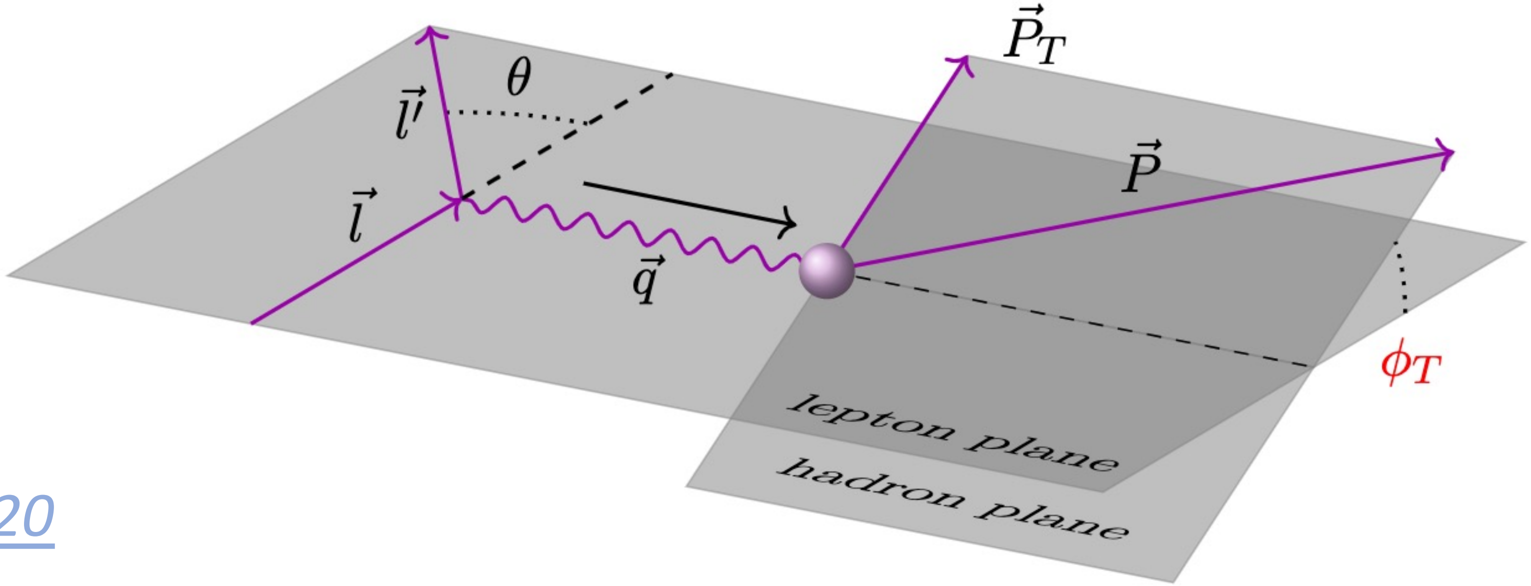
$$\mathcal{A}_{L}^{\gamma^* g\to J/\psi} = \frac{96}{N_c}\frac{Q^2}{(M^2+Q^2)^2}\langle 0|\mathcal{O}_8^{J/\psi}({}^3P_0)|0\rangle\,,$$

$$\mathcal{B}_{T}^{\gamma^* g\to J/\psi} = -\langle 0|\mathcal{O}_8^{J/\psi}({}^1S_0)|0\rangle + \frac{12}{N_c}\frac{3M^2-Q^2}{M^2(M^2+Q^2)}\langle 0|\mathcal{O}_8^{J/\psi}({}^3P_0)|0\rangle\,.$$

Azimuthal asymmetries at the EIC

- Depending on the polarization of the particles one can observe different azimuthal asymmeties in detectors

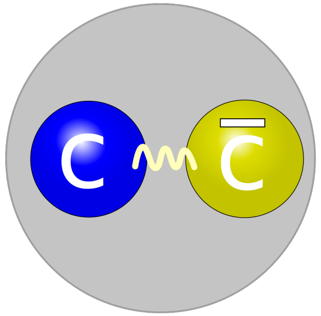
- Specifically, if all particles are unpolarized there is one due to $h_1^{\perp g}$



- More assymetries can be found in [Bacchetta et al. 2020]

The shape function

- Binding of quarks described by NRQCD
- However, NRQCD does **not** incorporate:
  - Final state smearing in hadronization of quarkonium
  - Soft gluon emission to become colorless (in octet)



- Therefore we need to include the TMDShF Δ [Echevarria 2019](#) [&](#) [Fleming et al. 2020](#)

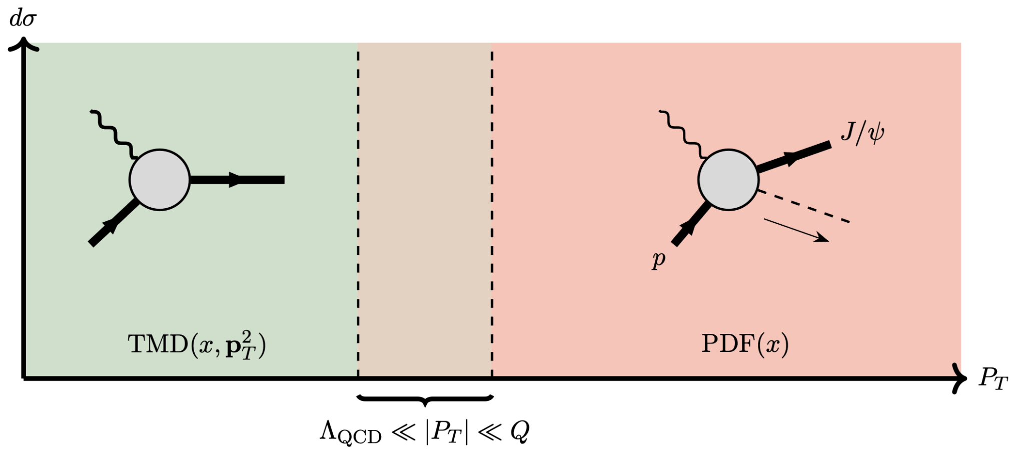
$$\frac{d\sigma(J/\psi)}{dx_B\, dy\, d^2\mathbf{q}_T} = \mathcal{N}\left[\sum_n \mathcal{A}^{[n]}\, \mathcal{C}[f_1^g \Delta^{[n]}] + 2\sum_n \mathcal{B}^{[n]}\, \mathcal{C}[w h_1^{\perp g} \Delta_h^{[n]}] \cos(2\phi_T)\right]$$

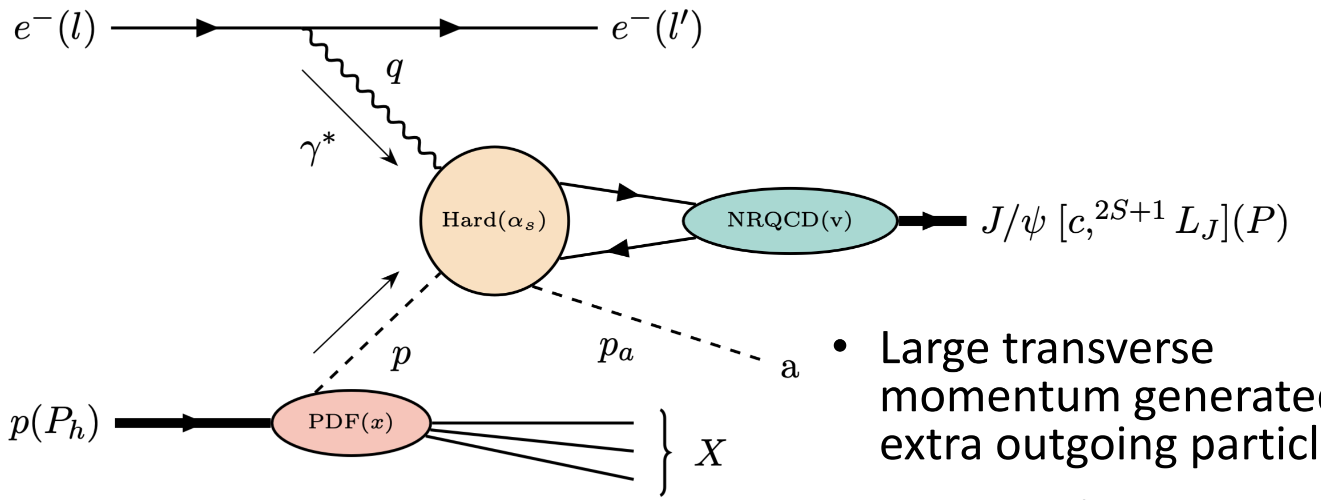
$$\mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2) = \int d^2\mathbf{p}_T \int d^2\mathbf{k}_T\, \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T)\, f_1^g(x, \mathbf{p}_T^2)\, \Delta^{[n]}(\mathbf{k}_T^2)$$

$$\mathcal{C}[w h_1^{\perp g} \Delta_h^{[n]}](x, \mathbf{q}_T^2) = \int d^2\mathbf{p}_T \int d^2\mathbf{k}_T\, \delta^2(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T)\, w(\mathbf{p}_T, \mathbf{k}_T)\, h_1^{\perp g}(x, \mathbf{p}_T^2)\, \Delta_h^{[n]}(\mathbf{k}_T^2)$$

$$w(\mathbf{p}_T, \mathbf{k}_T) = \frac{1}{2M_h^2 \mathbf{q}_T^2}[2(\mathbf{p}_T \cdot \mathbf{q}_T)^2 - \mathbf{p}_T^2 \mathbf{q}_T^2]$$

Matching the two formalisms (revised)

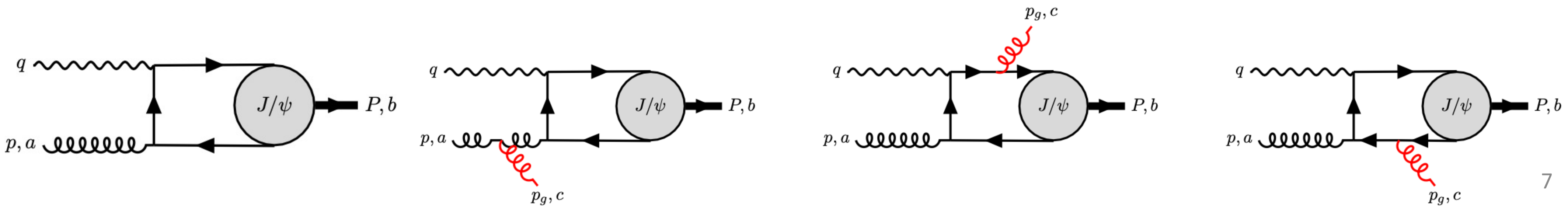




- Large transverse momentum generated by extra outgoing particle
- At LO 12 diagrams
- Parton and outgoing particle are the same (gluons for matching)

- In the overlap region of the two formalisms, the cross sections should match, allows for extration of the TMDShF; poles in gluon diagrams (talk Luca)!

$\Longrightarrow$ Boer, Bor, Maxia, Pisano and Feng [2304.09473]



Introduction of evolution

- Beyond tree level, the TMDs and hard factor become scale dependent *Collins and Soper 1981*
- Implementing evolution is more easily done in impact parameter space, where convolutions become simple products

$$\frac{d\sigma}{d(\text{kinematic variables})\, d^2\mathbf{q}_T} = \int d^2\mathbf{b}_T\, e^{-i\mathbf{b}_T\cdot\mathbf{q}_T}\, \hat{W}(\mathbf{b}_T, \mu_H) + \mathcal{O}(\mathbf{q}_T^2/\mu_H^2)$$

$$\hat{W}(\mathbf{b}_T, \mu_H) = \hat{A}(\mathbf{b}_T; \zeta_A, \mu)\, \hat{B}(\mathbf{b}_T; \zeta_B, \mu)\, \mathcal{H}(\mu_H; \mu)$$

$$\mathcal{C}[f_1^g \Delta^{[n]}](x, \mathbf{q}_T^2) = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \hat{f}_1^g(x, \mathbf{b}_T^2)\, \hat{\Delta}^{[n]}(\mathbf{b}_T^2)$$

$$\mathcal{C}[w h_1^{\perp g} \Delta_h^{[n]}](x, \mathbf{q}_T^2) = -\int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_2(b_T q_T)\, \hat{h}_1^{\perp g}(x, \mathbf{b}_T^2)\, \hat{\Delta}_h^{[n]}(\mathbf{b}_T^2)$$

$$\hat{f}_1^g(x, \mathbf{b}_T^2) \equiv \int d^2\mathbf{p}_T\, e^{i\mathbf{b}_T\cdot\mathbf{p}_T}\, f_1^g(x, \mathbf{p}_T^2)\,,$$

$$\hat{h}_1^{\perp g}(x, \mathbf{b}_T^2) \equiv \int d^2\mathbf{p}_T\, \frac{(\mathbf{b}_T\cdot\mathbf{p}_T)^2 - \frac{1}{2}\mathbf{b}_T^2\mathbf{p}_T^2}{\mathbf{b}_T^2 M_h^2}\, e^{i\mathbf{b}_T\cdot\mathbf{p}_T}\, h_1^{\perp g}(x, \mathbf{p}_T^2)\,,$$

$$\hat{\Delta}_{(h)}^{[n]}(\mathbf{b}_T^2) \equiv \int d^2\mathbf{k}_T\, e^{i\mathbf{b}_T\cdot\mathbf{k}_T}\, \Delta_{(h)}^{[n]}(\mathbf{k}_T^2)\,.$$

The Sudakov factors and scales

- CS Evolution: $\hat{f}(x, \mathbf{b}_T^2; \zeta, \mu) = e^{-S_A(b_T, \zeta, \zeta_0, \mu, \mu_0)}\, \hat{f}(x, \mathbf{b}_T^2; \zeta_0, \mu_0)$

$$S_A(b_T, \zeta, \zeta_0, \mu, \mu_0) = -\frac{1}{2}\hat{K}(b_T, \mu_0) \ln \frac{\zeta}{\zeta_0} - \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'), 1) - \frac{1}{2}\gamma_K(\alpha_s(\mu')) \ln \frac{\zeta}{\mu'^2}\right]$$

$$\hat{K}(b_T, \mu) = -\alpha_s(\mu)\frac{C_A}{\pi}\ln \frac{\mu^2 b_T^2}{b_0^2} + \mathcal{O}(\alpha_s^2)\ , \ \ \gamma_K(\alpha_s(\mu)) = 2\alpha_s(\mu)\frac{C_A}{\pi} + \mathcal{O}(\alpha_s^2)\ , \ \ \gamma(\alpha_s(\mu), \zeta/\mu^2) = \alpha_s(\mu)\frac{C_A}{\pi}\left[\beta_0 - \ln \frac{\zeta}{\mu^2}\right] + \mathcal{O}(\alpha_s^2)$$

- To avoid large logs in the hard factor $\mu \sim \mu_H$
- TMDs should be evaluated at their natural scale $\sqrt{\zeta_0} \sim \mu_0 \ll \sqrt{\zeta} \sim \mu$
- Instead of choosing a low (still perturbative scale), is common to take

$$\sqrt{\zeta_0} \sim \mu_0 \sim \boxed{\mu_b \equiv b_0/b_T} = 2e^{-\gamma_E}/|\mathbf{b}_T| \qquad \boxed{\mu_b \leq \mu_H}$$

$$\boxed{\hat{W}(\mathbf{b}_T, \mu_H) \equiv \hat{W}(b_T^*, \mu_H) e^{-S_{NP}}}$$

- **b_T must be constrained**

$$b_0/\mu_H \leq b_T \leq b_{T,\max}$$

Perturbative tails and TMDShF evolution

$$\hat{f}_1^g(x, \mathbf{b}_T^2; \mu_b) = f_1^g(x; \mu_b) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T\,\Lambda_{\rm QCD})$$

$$\hat{h}_1^{\perp g}(x, \mathbf{b}_T^2; \mu_b) = -\frac{\alpha_s(\mu_b)}{\pi}\int_x^1 \frac{dx'}{x'}\left(\frac{x'}{x}-1\right)\left\{C_A\, f_1^g(x';\mu_b) + C_F \sum_{i=q,\bar{q}} f_1^i(x';\mu_b)\right\} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T\,\Lambda_{\rm QCD})$$

Sun et al. 2011

$$\Delta_{ep}^{[n]}(\mu_H) = \Delta_{\rm ShF}^{[n]}(\mu_H) \times S_{ep}(\mu_H)$$

$$\hat{\Delta}_{\rm SF}^{[n]}(z, \mathbf{b}_T^2; \mu_H, \mu_b) = \langle 0|\mathcal{O}(n)|0\rangle \left(1 + \frac{\alpha_s}{2\pi}C_A\left[1 + \ln\frac{M^2}{\mu_H^2}\right]\ln\frac{\mu_H^2}{\mu_b^2}\right)\delta(1-z)$$

$$S_{ep}(\mathbf{b}_T^2; \mu_H, \mu_b) = 1 + \frac{\alpha_s}{2\pi}C_A\left[2\ln\frac{\mu_H^2}{M^2+Q^2}\right]\ln\frac{\mu_H^2}{\mu_b^2}$$

$$S_A(\mathbf{b}_T^2; \mu_H, \mu_b) = \frac{1}{2}\frac{C_A}{\pi}\int_{\mu_b^2}^{\mu_H^2}\frac{d\mu'^2}{\mu'^2}\,\alpha_s(\mu')\left[\ln\frac{\mu_H^2}{\mu'^2} - \left(\beta_0 + \mathrm{B}_{\rm CO}\right)\right] + \mathcal{O}(\alpha_s^2)$$

$$\mathrm{B}_{\rm CO}(\mu_H) = \left(1 + \ln\frac{\mu_H^2 M^2}{(Q^2+M^2)^2}\right)$$

- $\Delta = \langle O \rangle\, \delta(1-z)$
- $\Delta_h \equiv \Delta$
- α_S 1-loop

b_T Domains and the Convolutions

1) $b_0/\mu_H \le b_T$

$$\mu_b \to \mu_b' = \frac{\mu_H b_0}{\mu_H b_T + b_0}$$

Boer and Den Dunnen 2014

$$\mu_b \to \tilde{\mu}_b' = \frac{b_0}{\sqrt{b^2 + b_0^2/\mu_H^2}}$$

Collins et al. 2016

2) $b_T \le b_{T,\max}$

$$b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\max})^2}}$$

Collins et al. 1984

$$\mu_b \to \mu_b' = \frac{\mu_H b_0}{\mu_H b_T + b_0} \to \mu_{b*}' = \frac{\mu_H b_0}{\mu_H b_T^* + b_0}$$

$$\mathcal{C}[f_1^g \Delta^{[n]}] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, e^{-S_A(b_T^*;\mu_H,\mu_{b*}')} e^{-S_{NP}(b_T;\mu_H)}\, \hat{f}_1^g(x, b_T^*)\, \hat{\Delta}^{[n]}(b_T^*)$$

$$\mathcal{C}[w h_1^{\perp g} \Delta_h^{[n]}] = -\int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_2(b_T q_T)\, e^{-S_A(b_T^*;\mu_H,\mu_{b*}')} e^{-S_{NP}(b_T;\mu_H)}\, \hat{h}_1^{\perp g}(x, b_T^*)\, \hat{\Delta}_h^{[n]}(b_T^*)$$

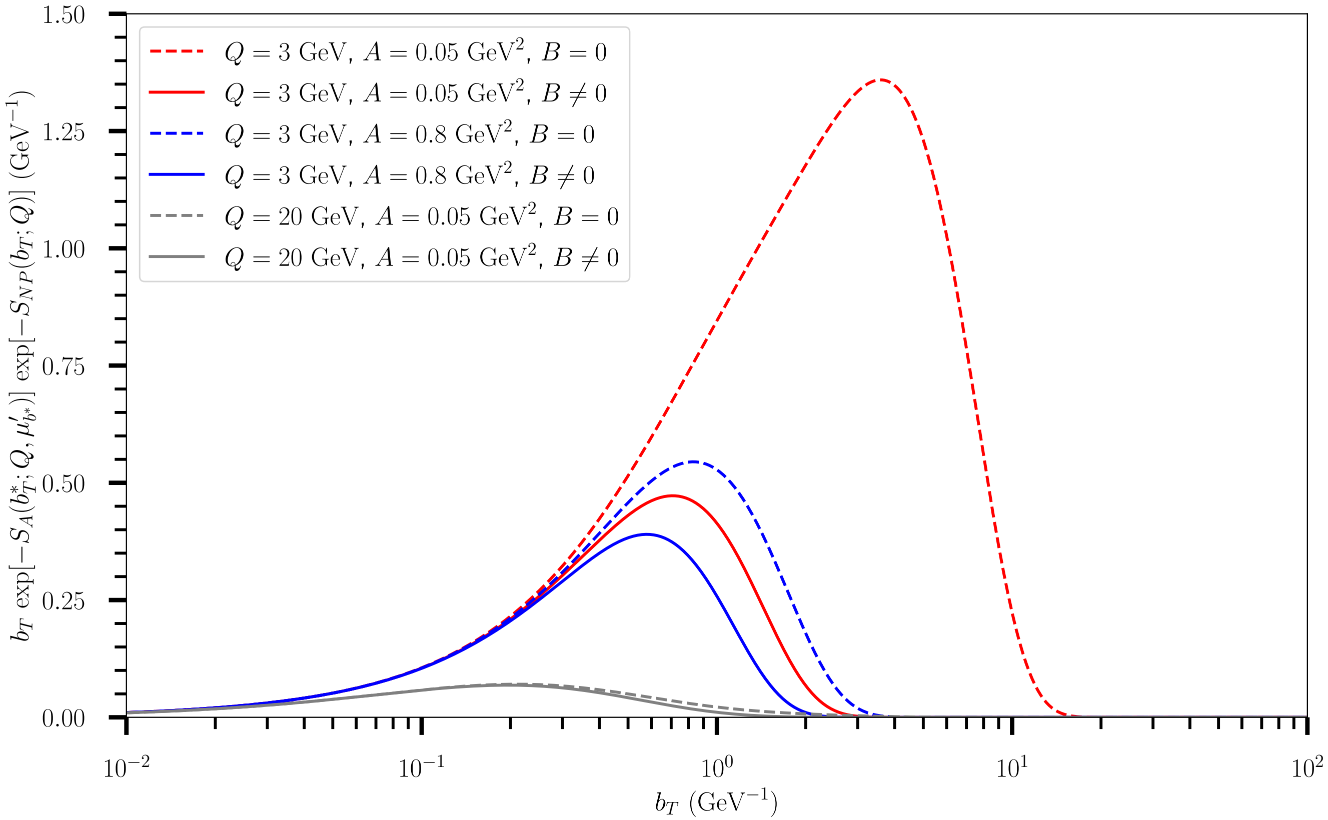
$$\mu_H = Q$$

The Non-perturbative Sudakov factor

$$\mathcal{C}[wh_1^{\perp g}\Delta_h^{[n]}] \leq \mathcal{C}[f_1^g \Delta^{[n]}]$$

$$S_{NP}(b_T;Q) = \left[g_1 \ln\frac{Q}{2\mu_{NP}} + g_2\left(1 + 2g_3 \ln\frac{10 x x_0}{x_0 + x}\right)\right] b_T^2$$

Aybat and Rogers 2011



$$S_{NP}(b_T;Q) = \left[A \ln\frac{Q}{\mu_{NP}} + B(x)\right] b_T^2$$

$$A = \frac{C_A}{C_F} g_1 = 0.414\ \mathrm{GeV}^2$$

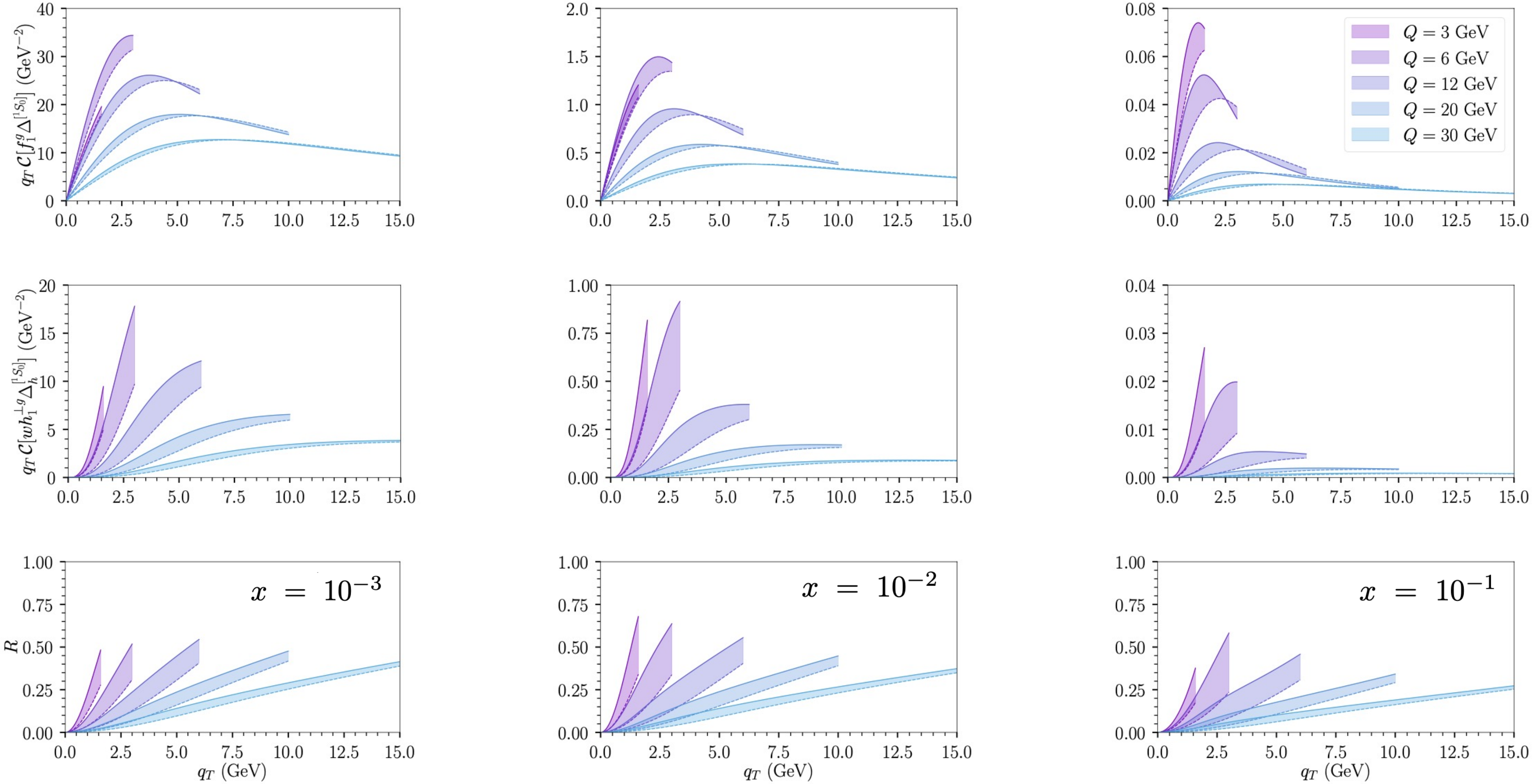
Scarpa 2020

$b_{T,\mathrm{lim}}$ (GeV^{-1})	r (fm $\sim 1/(0.2$ GeV$)$)	A (GeV^2)
2	0.2	0.80
4	0.4	0.20
8	0.8	0.05

x	B (GeV^2)
10^{-1}	0.456
10^{-2}	0.521
10^{-3}	0.715

Bor and Boer 2022

A study of convolutions



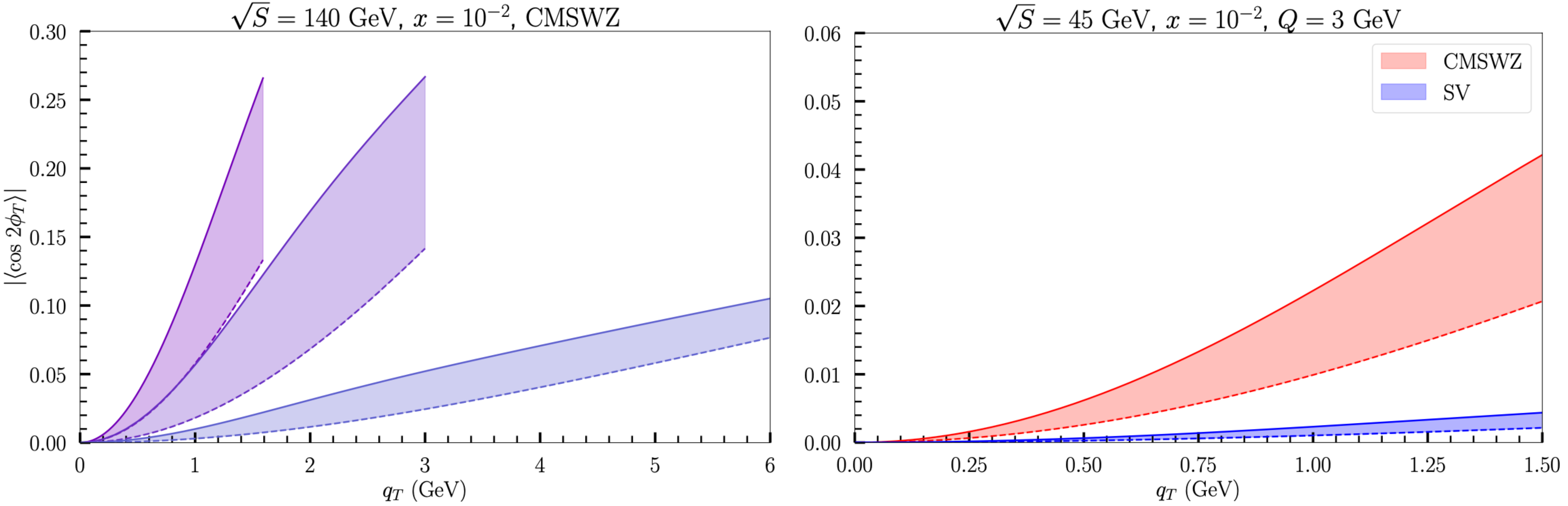
$$R = \frac{\mathcal{C}[wh_1^{\perp g}\Delta_h]}{\mathcal{C}[f_1^g\Delta]}$$

Bor and Boer 2022

Predictions of the azimuthal asymmetry

$$\langle \cos 2\phi_T \rangle = \frac{\int d\phi_T \cos 2\phi_T \, d\sigma}{\int d\phi_T \, d\sigma} = \frac{\sum_n \mathcal{B}^{[n]} \, \mathcal{C}[w h_1^{\perp g} \Delta_h^{[n]}]}{\sum_n \mathcal{A}^{[n]} \, \mathcal{C}[f_1^g \Delta^{[n]}]} \overset{!}{=} \frac{\sum_n \mathcal{B}^{[n]} \, \langle 0|\mathcal{O}(n)|0\rangle}{\sum_n \mathcal{A}^{[n]} \, \langle 0|\mathcal{O}(n)|0\rangle} \cdot R = \frac{\mathrm{B}}{\mathrm{A}} \cdot R$$

$$S = Q^2/x_B y \quad x = x_B + \frac{M^2}{yS} = \frac{M^2+Q^2}{yS} = x_B \frac{M^2+Q^2}{Q^2}$$

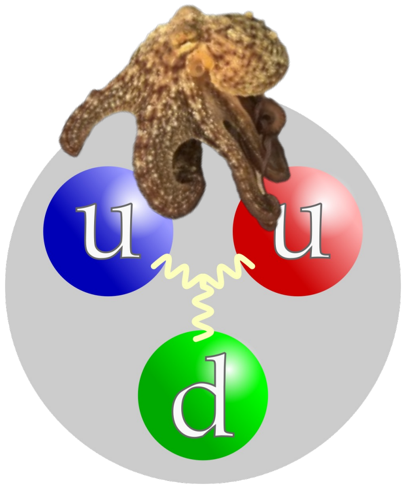


- Prefactor strongly depedent on LDMEs
  Bacchetta et al. 2020

Bor and Boer 2022

Findings

- The azimuthal asymmetry grows monotonically in the TMD regime
- The asymmetry cannot grow beyond 1, therefore, the expectation is that a maximum will be reached outside the TMD region
- We find decreasing asymmetries with decreasing x values
- The SV LDMEs at $\mathrm{Q} = 3$GeV does not follow the observed trend and is exceptionally small due to a cancellation of the S- and P-wave LDMEs at Q^2 near M^2
- The CO LDMEs forms the dominant source of uncertainty in our computations. Therefore, going to the next order in $\alpha_S$will not lead to much more precise predictions at the current stage
- The asymmetry is expected to be measurably large, especially at the larger center of mass energy of 140 GeV



Outlook

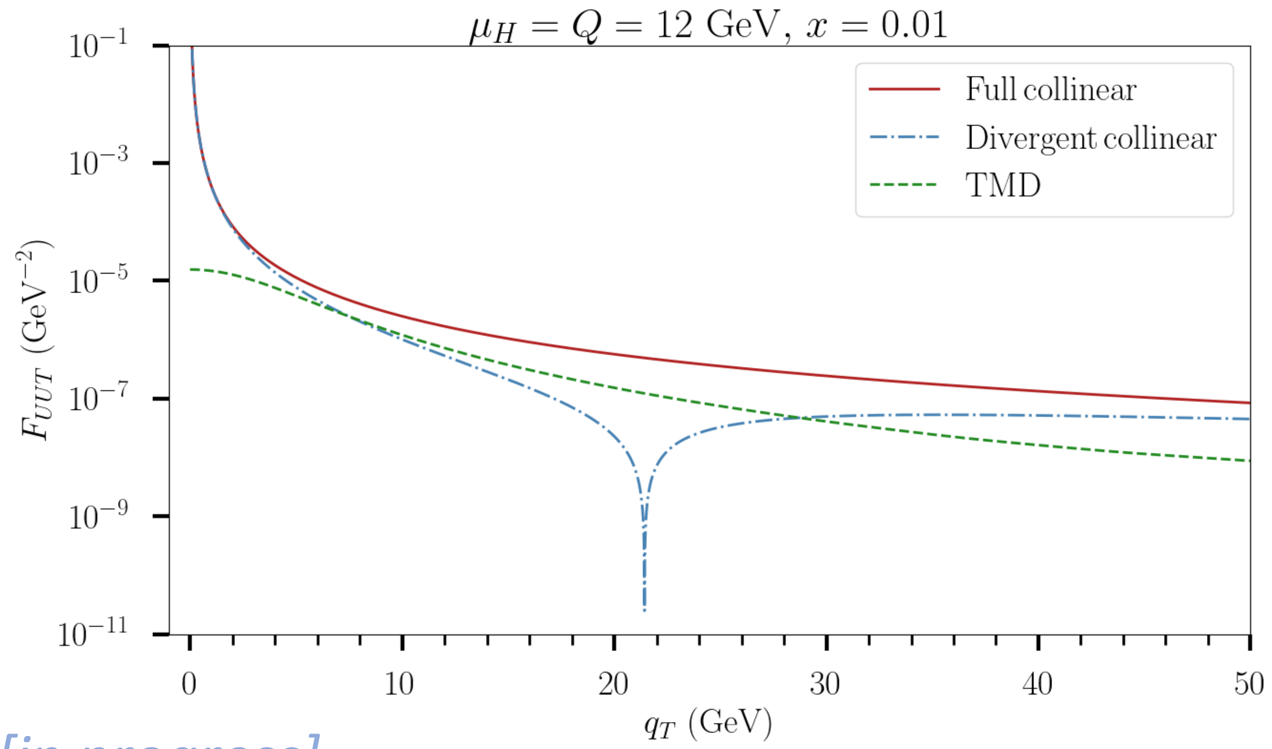
- A full theoretical P_T spectrum that can be compared with future experiments
- TMD (W) **vs** Divergent collinear **vs** Collinear

Ideas:

- Fit Tsallis or weight function [Echevarria et al. 2018](#) ;

 - by varying SNP and/or valid factorization regions

- Implement $F_{UU\,\cos 2\phi}$: check no TMDShF
- Error analysis from TMD regime



[Boer, Bor, Lansberg and Maxia [in progress]](#)