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In collaboration with M. Boglione

# Theory of TMD factorization for thrust-dependent observables

OLD DOMINION UNIVERSITY

Jefferson Lab

SAR WORS 2023

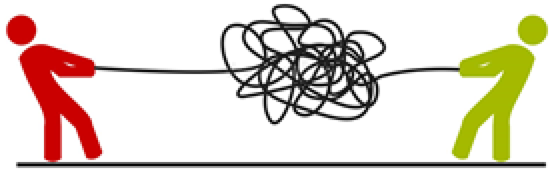
3rd SARDINIAN WORKSHOP ON SPIN

# Extraction of TMD Fragmentation Functions

Access to the 3D-dynamics of confinement


Standard TMD factorization →

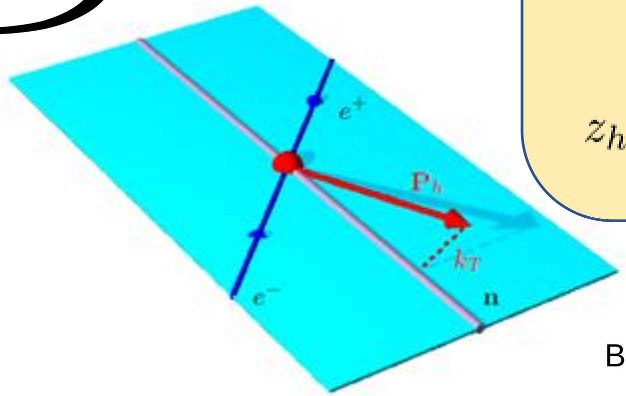
- SIDIS $d\sigma \sim H_{\text{SIDIS}}\, F\, D$
- DIA $d\sigma \sim H_{\text{DIA}}\, D_1\, D_2$



Always two TMDs that have to be extracted *simultaneously*

A process with a single hadron may offer a cleaner access to TMD FFs

$$d\sigma \overset{??}{\propto} D$$




**Single-Inclusive Hadroproduction (SIA)**

$$e^+e^- \rightarrow h\,X$$

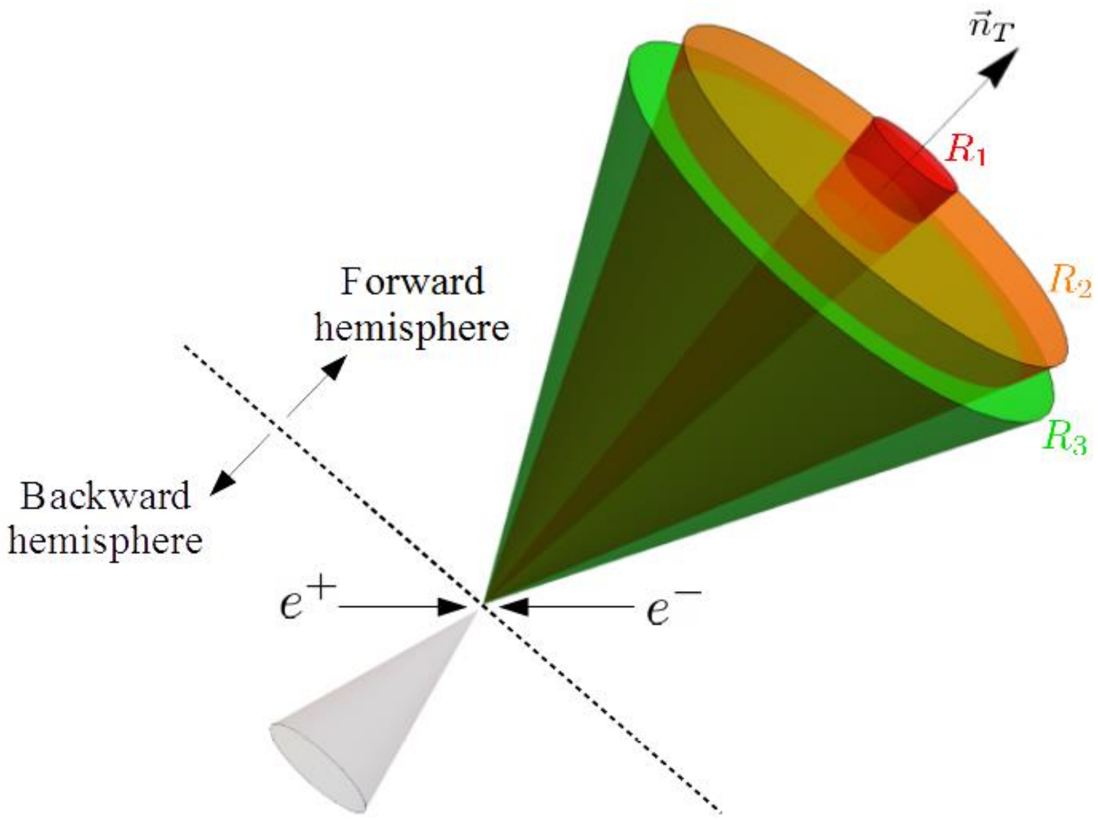
The transverse momentum of the detected hadron is measured w.r.t. the thrust axis

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\text{c.m.}),\,i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\text{c.m.}),\,i}|}, \quad P_T \text{ w.r.t } \vec{n}$$

BELLE collab., *Phys.Rev.D 99 (2019)*

# Three kinematic regions

Depending on where the hadron is located within the jet the underlying kinematics can be remarkably different, resulting in different factorization theorems




The hadron is detected very close to the **axis** of the jet.

- ❑ Extremely small $P_T$
- ❑ Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

The hadron is detected in the **central region** of the jet.

- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

The hadron is detected near the **boundary** of the jet.

- ❑ Moderately small $P_T$
- ❑ The hadron transverse momentum affects the topology of the final state directly

The three regions are uniquely determined by the specific role of **soft** and **soft-collinear** radiation:

$u \to 1 - T$

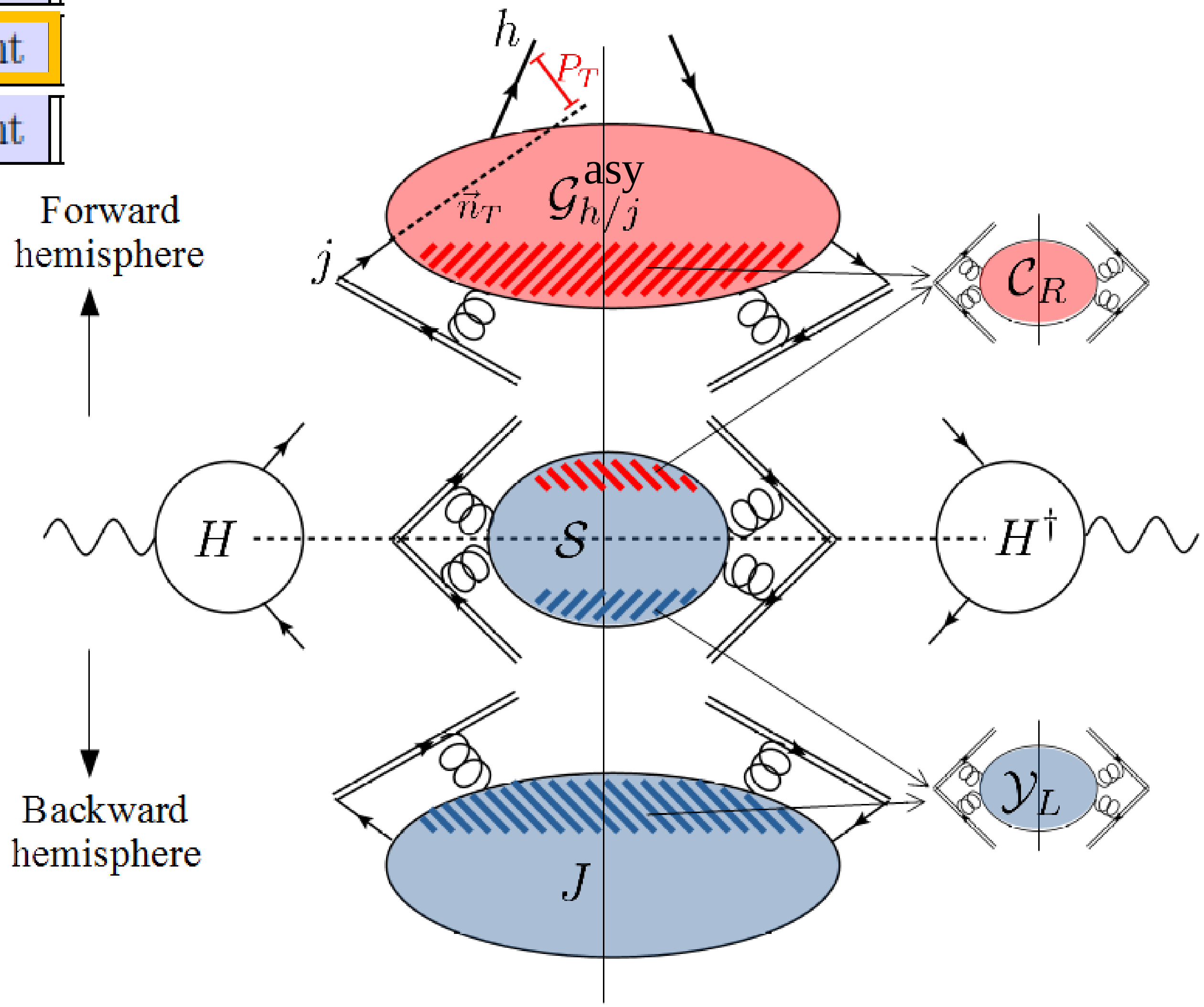
$b_T \to P_T/z$

| | soft | soft-collinear | collinear |
|---|---|---|---|
| $R_1$ | TMD-relevant | TMD-relevant | TMD-relevant |
| $R_2$ | TMD-irrelevant | TMD-relevant | TMD-relevant |
| $R_3$ | TMD-irrelevant | TMD-irrelevant | TMD-relevant |

Red blobs are TMD-relevant

Blue blobs are TMD-irrelevant

$$d\sigma_{R_2} \sim |H|^2 \, J \, \frac{\mathcal{S}}{\mathcal{Y}_L} \, \frac{\mathcal{G}^{\text{asy}}_{h/j}}{\mathcal{C}_R}$$




Different decompositions lead to different factorized cross sections

Factorization works in the same way for all the three regions, but it produces different results depending on the underlying kinematics

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| $R_3$ | TMD-irrelevant | TMD-irrelevant | TMD-relevant |

$R_1$ $$d\sigma \sim H J(u) \Sigma(u, b_T)\, D(z, b_T)$$

TMD FF + NP soft

The hadron is detected very close to the **axis** of the jet.

- Extremely small $P_T$
- Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

$R_2$ $$d\sigma \sim H J(u) S(u)\, D(z, b_T)$$

TMD FF

The hadron is detected in the **central region** of the jet.

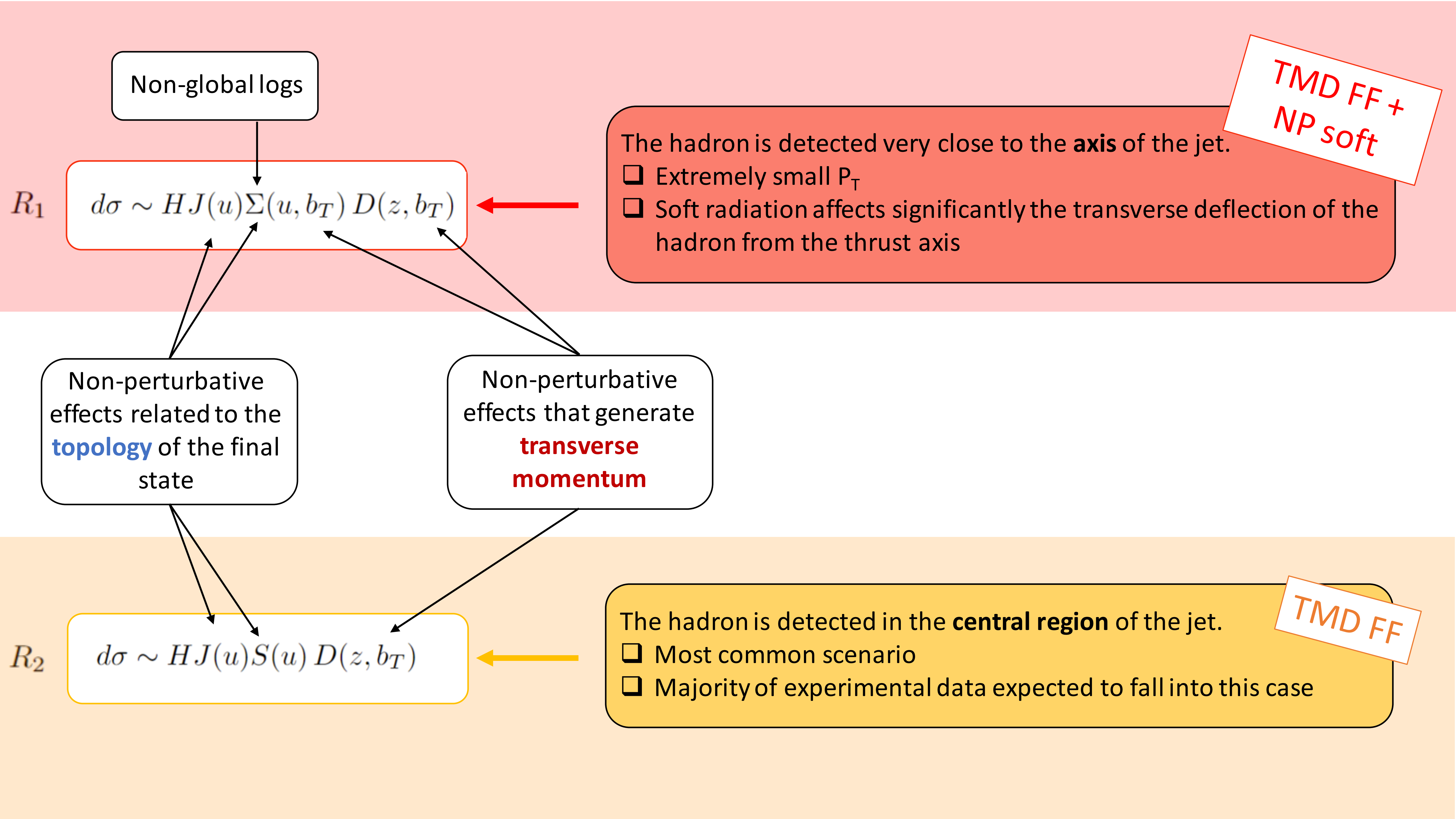
- Most common scenario
- Majority of experimental data expected to fall into this case

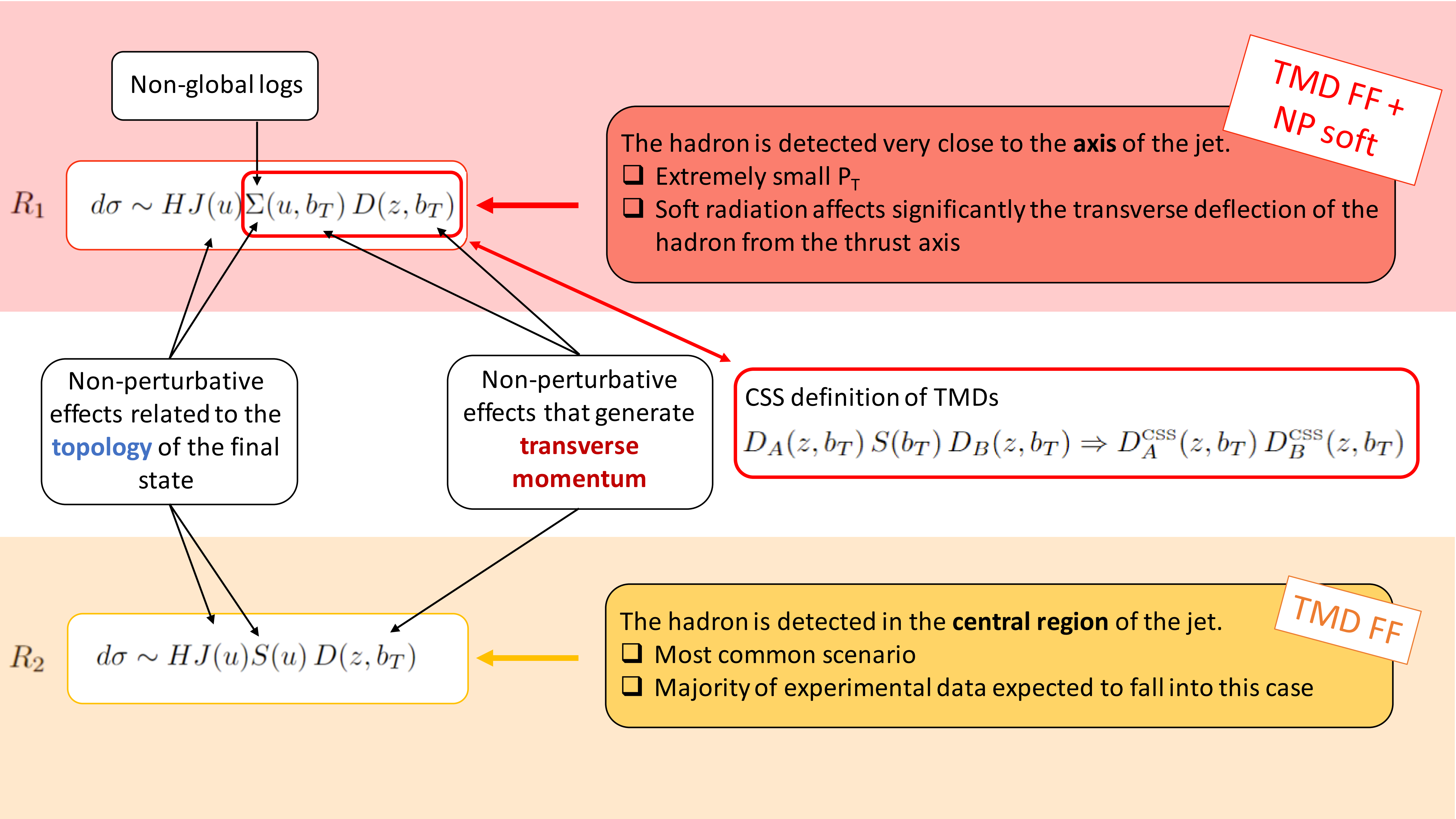
$R_3$ $$d\sigma \sim H J(u) S(u)\, G(z, u, b_T)$$

Generalized FJF

The hadron is detected near the **boundary** of the jet.

- Moderately small $P_T$
- The hadron transverse momentum affects the topology of the final state directly


Non-global logs
$R_1$
$d\sigma \sim H J(u) \Sigma(u, b_T)\, D(z, b_T)$
The hadron is detected very close to the **axis** of the jet.
Extremely small $P_T$
Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis
TMD FF + NP soft
Non-perturbative effects related to the **topology** of the final state
Non-perturbative effects that generate **transverse momentum**
$R_2$
$d\sigma \sim H J(u) S(u)\, D(z, b_T)$
The hadron is detected in the **central region** of the jet.
Most common scenario
Majority of experimental data expected to fall into this case
TMD FF



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Non-perturbative effects related to the **topology** of the final state
Non-perturbative effects that generate **transverse momentum**
CSS definition of TMDs
$D_A(z, b_T)\, S(b_T)\, D_B(z, b_T) \Rightarrow D_A^{\rm CSS}(z, b_T)\, D_B^{\rm CSS}(z, b_T)$
$R_2$
$d\sigma \sim H J(u) S(u)\, D(z, b_T)$
TMD FF
The hadron is detected in the **central region** of the jet.
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Non-global logs

$$R_1 \quad d\sigma \sim H J(u) \Sigma'(u, b_T)\, D^{\rm CSS}(z, b_T)$$

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TMD FF + NP soft

Non-perturbative effects related to the **topology** of the final state

Non-perturbative effects that generate **transverse momentum**

NOT THE SAME TMD FRAGMENTATION FUNCTION!


Is **universality** in danger?

$$R_2 \quad d\sigma \sim H J(u) S(u)\, D(z, b_T)$$

The hadron is detected in the **central region** of the jet.

- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

TMD FF

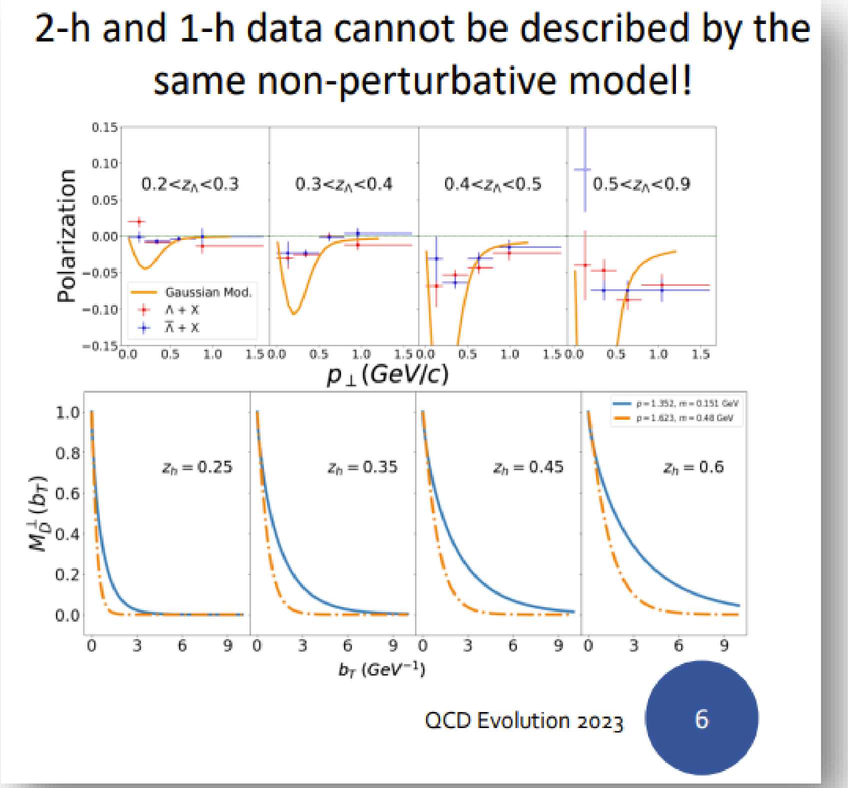
Standard TMD factorization can be extended beyond the standard processes (DY, SIDIS, DIA) at the cost of including a new, independent, non-perturbative function (the **soft model**).

$$D^{\mathrm{CSS}}(z, b_T) = D(z, b_T)\,\sqrt{M_S(b_T)} \longrightarrow$$ Universality is saved!

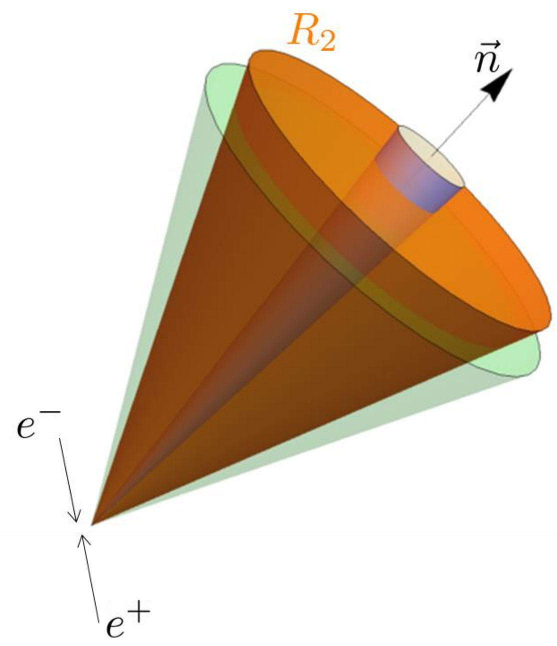
Expected **tension** between 1-hadron and 2-hadron processes

2-h extraction / 1-h extraction → Access to soft physics!

From Zaccheddu's talk at QCD evolution



# Central region…

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TMD FF

$$d\sigma_{R_2} \sim |H|^2 J(u) S(u)\, D(z, b_T)$$

# …peculiar role of rapidities

$$d\sigma_{R_2} \sim |H|^2\, J(u)\, \frac{\mathcal{S}(u; y_1, \cancel{y_2})}{\mathcal{Y}_L(u; \cancel{y_2})} \frac{\mathcal{G}^{\rm asy}_{h/j}(z, b_T, u)}{\mathcal{C}_R(b_T, u; y_1)}$$

$$D(z, b_T; y_1)$$

$$\Longrightarrow \frac{\partial}{\partial y_1} \ldots \mathcal{S}(\tau, y_1, \ldots)\, D(z, b_T, y_1) \neq 0$$

# Rapidity divergences in the central region

$SIA^{thr}$ has a **double nature**:

## Thrust dependent observable

The thrust *naturally* regularizes the rapidity divergences.
The 2-jet limit corresponds to removing the regulator and to exposing the rapidity divergences in fixed order calculations.

So the final result depends on a regulator? Yes, but...

1) The thrust is *measured*.
2) When the regulator is removed the (factorized) cross section vanishes

## TMD observable

The rapidity cut-offs *artificially* regularize the rapidity divergences.
The limits $y_{1,2} \to \pm\infty$ correspond to removing the regulator and to exposing the rapidity divergences in fixed order calculations.

So the final result depends on a regulator? Yes (in principle), but…

1) The rapidity cut-offs are just mathematical tools.
2) In standard TMD factorization they cancel among themselves before the limit is taken and the final cross section is rapidity cut-offs independent.

Both kind of regularization coexists in SIA$^{thr}$.
Therefore, it should not be surprising that the two mechanisms intertwine and that **thrust and rapidity regulators are strictly related**.

This signals a *redundancy* of regulators: one can be expressed in terms of the other.
In particular, the rapidity cut-off $y_1$ should be a function of thrust, such that when it is removed, also $\tau$ is removed. In other words:

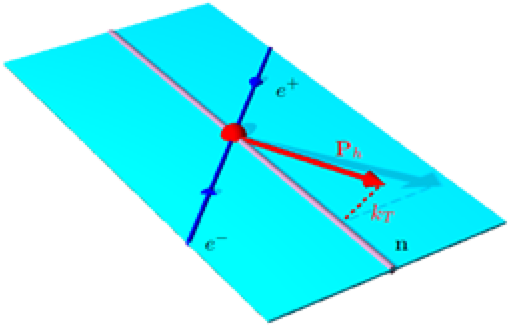
$$\tau \to 0 \Longleftrightarrow y_1 \to +\infty$$

Peculiar and very unique feature of the central region!

Naively from kinematics...

$$y_h \geq -\log\sqrt{\tau} \quad \Rightarrow \quad y_1 \propto -\log\sqrt{\tau}$$

Rapidity of detected hadron




...but also formally:
The double counting due to the overlap between soft and collinear (forward) radiation is cancelled only if the rapidity cut-off is fixed to a function of thrust and transverse momentum

SOFT-COLLINEAR

$$k_T \lesssim c_1/b_T \quad \leftarrow \text{COLLINEAR}$$

$$\text{SOFT} \rightarrow \quad k_T \lesssim Q\, e^{y_1}/u_E$$

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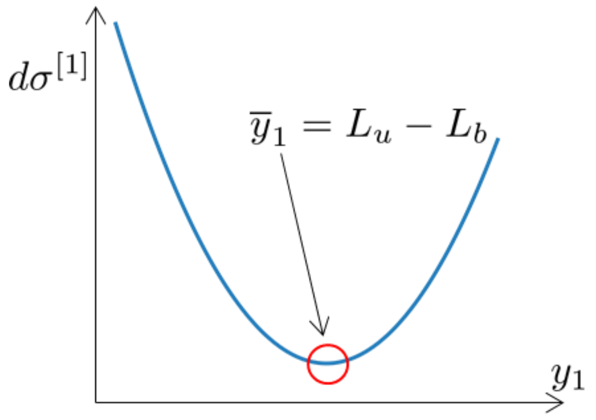
SOFT-COLLINEAR

$$k_T \lesssim c_1/b_T$$ ← COLLINEAR

SOFT → $$k_T \lesssim Q\, e^{y_1}/u_E$$

$$u_E = u\, e^{\gamma_E};\ c_1 = 2e^{-\gamma_E}$$
$$L_u = \log u_E$$
$$L_b = \log\left(b_T Q/c_1\right)$$




$$\overline{y}_1 = L_u - L_b$$

This is also the **minimum** of the factorized cross section as a function of $y_1$ ↔ It is the (unique) solution of the Collins-Soper evolution equation!

$$\frac{\partial}{\partial y_1} d\sigma_{R_2} = 0$$

$$\overline{y}_1 = L_u - L_{b^\star}\left(1 + \frac{1 - e^{\frac{2\beta_0}{\gamma_K^{[1]}}\left(\widetilde{K}_\star(a_S(\mu_b^\star)) - g_K(b_T)\right)}}{2\beta_0\, a_S(\mu_b^\star) L_{b^\star}}\right)$$

- ❑ Large and positive
- ❑ Consistent with pert. solution: $\overline{y}_1 = L_u - L_b$ as $b_T \to 0$
- ❑ Consistent with kinematics: $\widehat{y}_1 = -\log\sqrt{\tau} + b_T\text{-logs}$

…but also formally:
The double counting due to the overlap between soft and collinear (forward) radiation is cancelled only if the rapidity cut-off is fixed to a function of thrust and transverse momentum

SOFT-COLLINEAR

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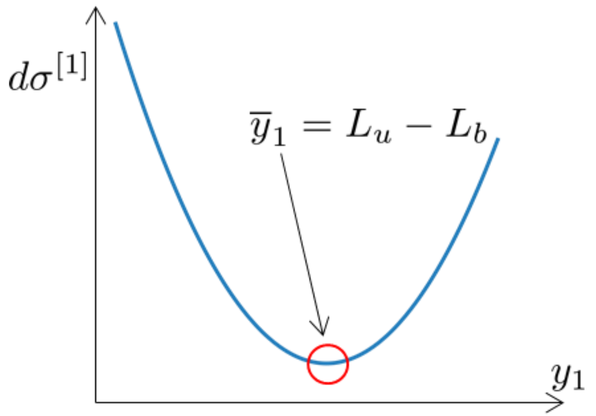


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Factorization errors are affected by the choice of $g_K$.
HINTS FOR PHENOMENOLOGY:
- Monotonic increasing (unique minimum)
- Constant at large distances
  $g_K(b_T) \to g_0$ (const.)

# Factorization theorem in the central region

$$d\sigma_{R_2} \sim H\ J(u)\ \frac{\mathcal{S}(u,\overline{y}_1,y_2)}{\mathcal{Y}_L(u,y_2)}\ \widetilde{D}_{h/j}(z,b_T,\overline{y}_1)$$

$$= H\,J\,\frac{\mathcal{S}}{\mathcal{Y}_L}\bigg|_{\text{ref. scale}} \exp\left\{\int_{\mu_J}^{Q}\frac{d\mu'}{\mu'}\gamma_J + \frac{1}{2}\int_{\mu_S}^{Q}\frac{d\mu'}{\mu'}\gamma_S\right\} \times \widetilde{D}_{h/j}(z,b_T)\bigg|_{y_1=0}$$

$$\times \exp\left\{\frac{1}{2}\int_{\mu_S}^{\mu_S e^{\overline{y}_1}}\frac{d\mu'}{\mu'}\left[\widehat{g}-\gamma_K\log\left(\frac{\mu'}{\mu_S}\right)\right] - \overline{y}_1\ \widetilde{K}\bigg|_{\mu_S}\right\}$$

Genuinely thrust. Exponent is *half* of standard thrust distributionin e+e- annihilation

Genuinely TMD. Reference scales as* in standard TMD factorization

Correlation part. It encodes the correlations between the measured variables


The function $g_K$ does not only appear into the TMD FF!

$$\frac{d\sigma_{R_2}}{dz\,dT\,d^2\vec{P}_T} = -\frac{\sigma_B\,N_C}{1-T}\sum_j e_j^2\left(1+a_S\,H^{[1]}\right)$$

$$\times\int\frac{d^2\vec{b}_T}{(2\pi)^2}e^{i\vec{b}_T\cdot\vec{P}_T/z}\,e^{L_{b^\star}\,n_1+n_2}\,\widetilde{D}^{\text{NLL}}_{h/j}(z,b_T)\bigg|_{\substack{\mu=Q\\ y_1=0}}\left(1+a_S\,C_1\right)\frac{e^{Lf_1+f_2+\frac{1}{L}f_3}}{\Gamma\left(1-g_1\right)}\left(g_1+\frac{1}{L}g_2\right)$$

# Matching $R_2$ with $R_3$

| | soft | soft-collinear | collinear |
|---|---|---|---|
| $R_1$ | TMD-relevant | TMD-relevant | TMD-relevant |
| $R_2$ | TMD-irrelevant | TMD-relevant | TMD-relevant |
| $R_3$ | TMD-irrelevant | TMD-irrelevant | TMD-relevant |

| | soft | soft-collinear | collinear |
|---|---|---|---|
| $R_1$ | TMD-relevant | TMD-relevant | TMD-relevant |
| $R_2$ | TMD-irrelevant | TMD-relevant | TMD-relevant |
| $M_{2,3}$ | TMD-irrelevant | TMD-irrelevant | TMD-relevant |
| $R_3$ | TMD-irrelevant | TMD-irrelevant | TMD-relevant |

Matching region, not an *independent* kinematic region

Remarkably, there is a factorization theorem holding in the matching region!

$$d\sigma_{R_2} \sim |H|^2 \, J \, \frac{\mathcal{S}}{\mathcal{Y}_L} \, \frac{\mathcal{G}^{\text{asy}}_{h/j}}{\mathcal{C}_R} \xrightarrow{Small \; b_T} d\sigma_{M_{2,3}} \sim |H|^2 \, J \, \frac{\mathcal{S}}{\mathcal{Y}_L \, \mathcal{Y}_R} \, \mathcal{G}^{\text{asy}}_{h/j} \xleftarrow{Large \; b_T} d\sigma_{R_3} \sim |H|^2 \, J \, \frac{\mathcal{S}}{\mathcal{Y}_L \, \mathcal{Y}_R} \, \mathcal{G}_{h/j}$$

Remarkably, there is a factorization theorem holding in the matching region!

$$d\sigma_{R_2} \sim |H|^2 \, J \, \frac{\mathcal{S}}{\mathcal{Y}_L} \, \boxed{\frac{\mathcal{G}^{\rm asy}_{h/j}}{\mathcal{C}_R}} \quad \xrightarrow{\text{Small } b_T}$$

$$\boxed{\frac{\mathcal{G}^{\rm asy}_{h/j}}{\mathcal{C}_R}} \to D(z, b_T)$$

$$d\sigma_{M_{2,3}} \sim |H|^2 \, J \, \boxed{\frac{\mathcal{S}}{\mathcal{Y}_L \, \mathcal{Y}_R}} \, \mathcal{C}_R \, \boxed{\frac{\mathcal{G}^{\rm asy}_{h/j}}{\mathcal{C}_R}}$$

$$\boxed{\frac{\mathcal{S}}{\mathcal{Y}_L \, \mathcal{Y}_R}} \to S_{\rm thr}(u), \qquad \boxed{\frac{\mathcal{G}^{\rm asy}_{h/j}}{\mathcal{C}_R}} \to D(z, b_T)$$

$$\xleftarrow{\text{Large } b_T} \quad d\sigma_{R_3} \sim |H|^2 \, J \, \boxed{\frac{\mathcal{S}}{\mathcal{Y}_L \, \mathcal{Y}_R}} \, \mathcal{G}_{h/j}$$

$$\boxed{\frac{\mathcal{S}}{\mathcal{Y}_L \, \mathcal{Y}_R}} \to S_{\rm thr}(u)$$

Remarkably, there is a factorization theorem holding in the matching region!

$$d\sigma_{R_2} \sim |H|^2\, J\, \frac{\mathcal{S}}{\mathcal{Y}_L}\, D \quad \xrightarrow{\text{Small } b_T} \quad d\sigma_{M_{2,3}} \sim |H|^2\, J\, S_{\rm thr}\, \mathcal{C}_R\, D \quad \xleftarrow{\text{Large } b_T} \quad d\sigma_{R_3} \sim |H|^2\, J\, S_{\rm thr}\, \mathcal{G}_{h/j}$$

This is *exactly* the result obtained in SCET for Region 2!

Joint thrust and TMD resummation in electron-positron and electron-proton collisions #2

Yiannis Makris (INFN, Pavia), Felix Ringer (LBNL, NSD), Wouter J. Waalewijn (Delta ITP, Amsterdam and Nikhef, Amsterdam) (Sep 24, 2020)

Published in: *JHEP* 02 (2021) 070 • e-Print: 2009.11871 [hep-ph]

$$\frac{d\sigma_{R_{M_{2,3}}}}{dz\, du\, d^2\vec{b}_T} = \mathcal{R}(u, b_T)\, \frac{d\sigma_{R_2}}{dz\, du\, d^2\vec{b}_T}$$

Totally **non-perturbative** object

The differences with SCET result are at non-perturbative level

Constraining the generalized FJF

$$\mathcal{G}_{h/j} \xrightarrow[b_T]{\text{large}} \mathcal{C}_R\, D_{h/j}$$

Relevant for other processes (hadron in jet)

Relevant for EIC!

# Conclusions

- The factorization theorem(s) for SIA with thrust is now **complete** and **consistent.**

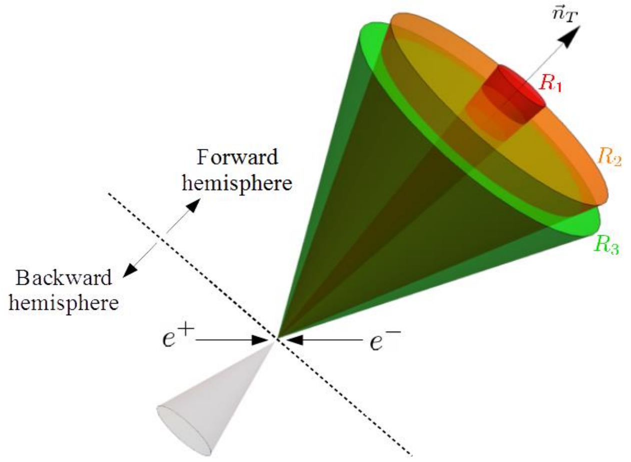
$$
\begin{aligned}
d\sigma_{R_2} &\sim H\, J(u)\, \frac{\mathcal{S}(u,\overline{y}_1,y_2)}{\mathcal{Y}_L(u,y_2)}\, \widetilde{D}_{h/j}(z,b_T,\overline{y}_1) \\
&= H\, J\, \frac{\mathcal{S}}{\mathcal{Y}_L}\bigg|_{\text{ref. scale}} \exp\left\{\int_{\mu_J}^{Q}\frac{d\mu'}{\mu'}\gamma_J + \frac{1}{2}\int_{\mu_S}^{Q}\frac{d\mu'}{\mu'}\gamma_S\right\} \times \widetilde{D}_{h/j}(z,b_T)\Big|_{y_1=0} \\
&\times \exp\left\{\frac{1}{2}\int_{\mu_S}^{\mu_S e^{\overline{y}_1}}\frac{d\mu'}{\mu'}\left[\widehat{g} - \gamma_K \log\left(\frac{\mu'}{\mu_S}\right)\right] - \overline{y}_1\ \widetilde{K}\Big|_{\mu_S}\right\}
\end{aligned}
$$

  - TMD universality
  - Role of rapidity scale

**Thank You!**

- The intertwining between thrust and rapidity regulators offers new perspective on the study of the Collins-Soper kernel

$$
\overline{y}_1 = L_u - L_{b^\star}\left(1 + \frac{1 - e^{\frac{2\beta_0}{\gamma_K^{[1]}}\left(\widetilde{K}_\star(a_S(\mu_b^\star)) - g_K(b_T)\right)}}{2\beta_0\, a_S(\mu_b^\star) L_{b^\star}}\right)
$$




- All the results obtained in the past few years properly fit together

**Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes**

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Jun 5, 2023)

e-Print: 2306.02937 [hep-ph]