Phenomenological analysis of $e^+e^- \rightarrow hX$ data and extraction of TMD Fragmentation Functions from thrust dependent observables

SAR WORS 2023
3rd SARDINIAN WORKSHOP ON SPIN

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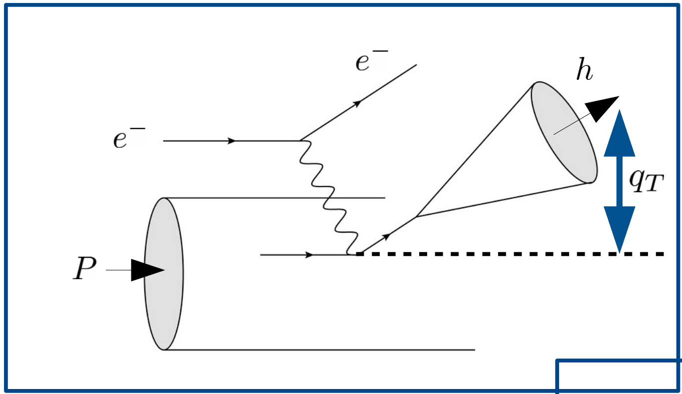
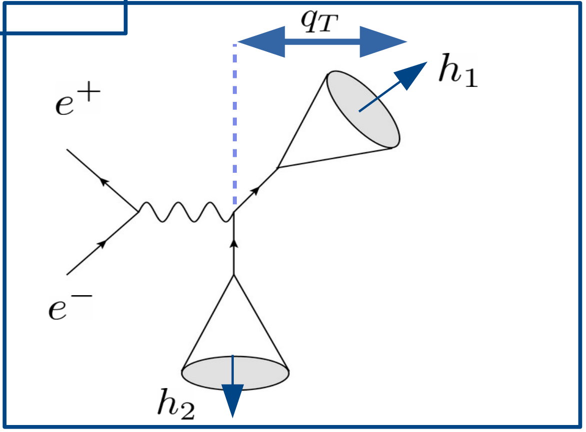
In collaboration with A. Simonelli

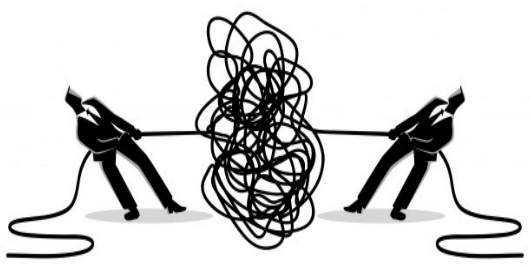
UNIVERSITÀ DI TORINO

SIDIS and e^+e^- annihilations in two hadrons





In SIDIS and e^+e^- cross sections, distribution and fragmentation TMDs are convoluted.
How can they be disentangled?

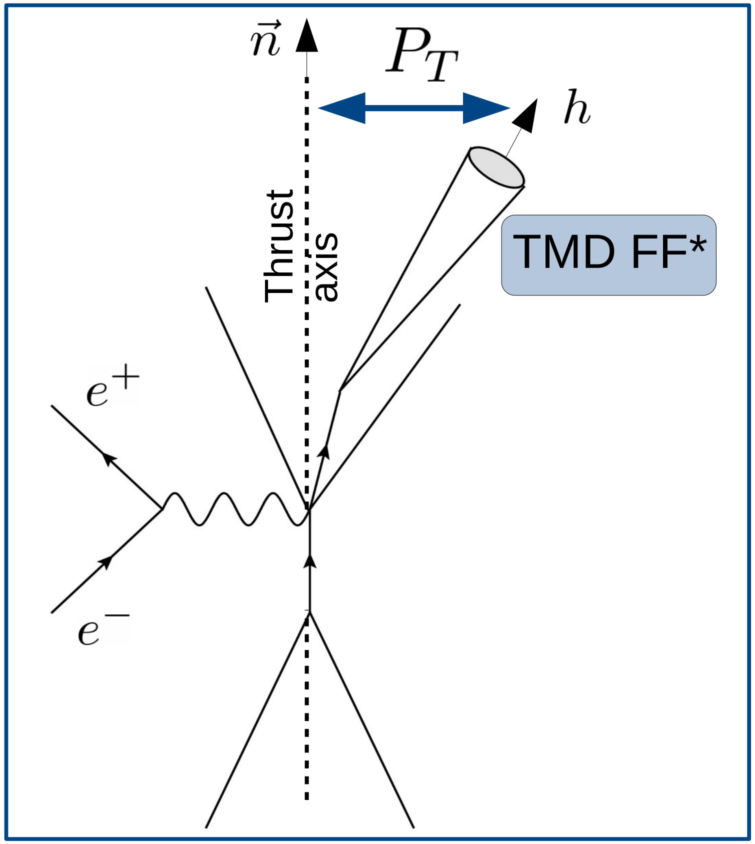


$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\rm sidis} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\, F(b_T)\, D(b_T)$$

3D-picture of partons inside the target hadron

3D-picture of partons hadronizing into the detected hadron

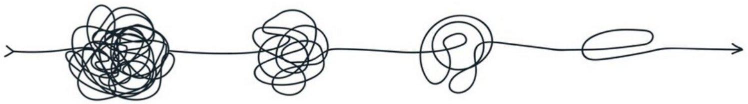
e^+e^- annihilations in one hadron: $e^+ e^- \to h\, X$



$$\frac{d\sigma}{dP_T} = d\hat{\sigma} \otimes D^\star(P_T)$$

3D-picture of the **hadronization** of partons into hadrons

In $e^+ e^- \to h\, X$ cross sections, only one fragmentation TMD appears



One of the cleanest ways to access TMD Fragmentation Functions***...**

BUT

$D^*(P_T)$ is not the same as $D(P_T)$!!!

Relation between FF and FF*

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$D = D^{\star} \sqrt{M_S}$$

SQUARE ROOT DEFINITION

Usual definition of TMDs.
Soft Gluon Factor contributing to the cross section are included in the two TMDS and equally shared between them.

FACTORIZATION DEFINITION

Purely collinear TMD, totally free from any soft gluon contribution.

SOFT MODEL

The Soft Gluon Factor appearing in the cross section (process dependent) is **not** included in the TMD

- Same for Drell-Yan, SIDIS and 2-hadron production. (2-h class universality).
- Non-perturbative function (phenomenology).

$e^+e^- \to hX$ cross section

M. Boglione, A. Simonelli, 2306.02937 [hep-ph]

The hadronic cross section is not a convolution of a partonic cross section with a TMD FF

$$d\sigma_{R_2} \sim H\; J(u)\, \frac{\mathcal{S}(u,\overline{y}_1,y_2)}{\mathcal{Y}_L(u,y_2)}\, \widetilde{D}_{h/j}(z,b_T,\overline{y}_1)$$

$$= H\,J\,\frac{\mathcal{S}}{\mathcal{Y}_L}\Big|_{\text{ref. scale}} \exp\left\{\int_{\mu_J}^{Q}\frac{d\mu'}{\mu'}\gamma_J + \frac{1}{2}\int_{\mu_S}^{Q}\frac{d\mu'}{\mu'}\gamma_S\right\} \times \widetilde{D}_{h/j}(z,b_T)\Big|_{y_1=0}$$

$$\times \exp\left\{\frac{1}{2}\int_{\mu_S}^{\mu_S e^{\overline{y}_1}}\frac{d\mu'}{\mu'}\left[\widehat{g} - \gamma_K \,\log\left(\frac{\mu'}{\mu_S}\right)\right] - \overline{y}_1\; \widetilde{K}\Big|_{\mu_S}\right\}$$

Genuinely thrust. Exponent is *half* of standard thrust distributionin e+e- annihilation

Genuinely TMD. Reference scales as* in standard TMD factorization

Correlation part. It encodes the correlations between the measured variables

The function g_K does not only appear into the TMD FF!

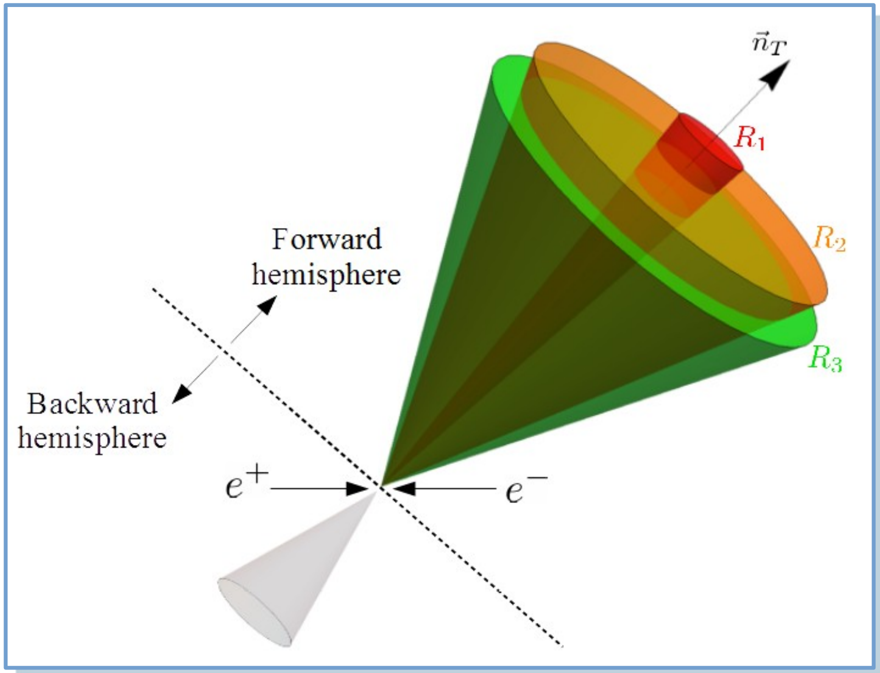
$$\frac{d\sigma_{R_2}}{dz\,dT\,d^2\vec{P}_T} = -\frac{\sigma_B\,N_C}{1-T}\sum_j e_j^2\left(1 + a_S\,H^{[1]}\right)$$

$$\times \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{b}_T\cdot\vec{P}_T/z}\, e^{L_{b^\star}\,n_1+n_2}\, \widetilde{D}^{\text{NLL}}_{h/j}(z,b_T)\Big|_{\substack{\mu=Q\\ y_1=0}} (1 + a_S\,C_1)\,\frac{e^{Lf_1+f_2+\frac{1}{L}f_3}}{\Gamma(1-g_1)}\left(g_1 + \frac{1}{L}g_2\right)$$

Slide credit: A. Simonelli

Kinematic Regions

M. Boglione, A. Simonell, JHEP 02 (2022)



	soft	soft-collinear	collinear
R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

	soft	soft-collinear	collinear
R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

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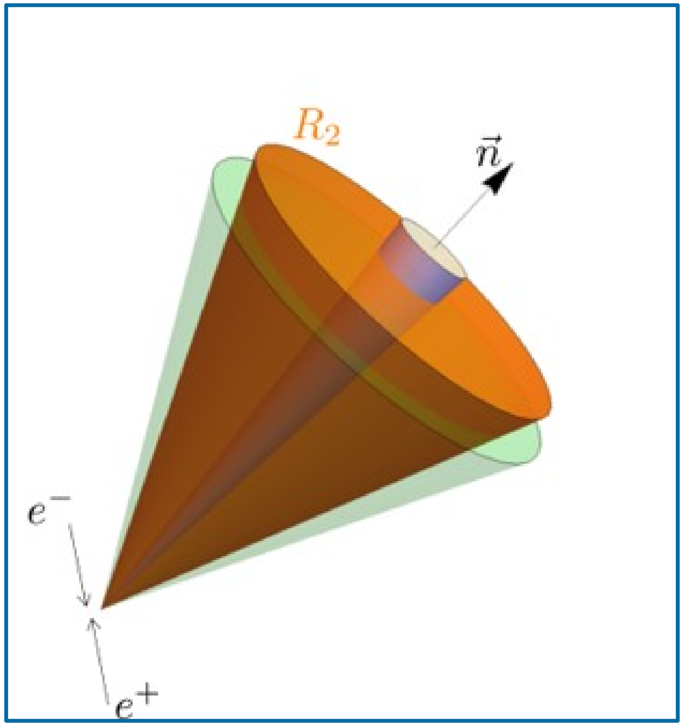

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R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

Phenomenology of $e^+e^- \to hX$ processes

M. Boglione, A. Simonelli, 2306. 02937 [hep-ph]

T	z												P_T/z max	N
	0.20 – 0.25	0.25 – 0.30	0.30 – 0.35	0.35 – 0.40	0.40 – 0.45	0.45 – 0.50	0.50 – 0.55	0.55 – 0.60	0.60 – 0.65	0.65 – 0.70	0.70 – 0.75	0.75 – 0.80		
0.80 – 0.85													$0.16\,Q$	57
0.85 – 0.90													$0.15\,Q$	60
0.90 – 0.95													$0.14\,Q$	61
0.95 – 1.00													$0.13\,Q$	52

Avoiding Region 3

Avoiding Region 1

Selection of 230 data points, binned in z, P_T and T

Phenomenology

M. Boglione, A. Simonelli, 2306. 02937 [hep-ph]

T	z												P_T/z max	N
	$0.20-0.25$	$0.25-0.30$	$0.30-0.35$	$0.35-0.40$	$0.40-0.45$	$0.45-0.50$	$0.50-0.55$	$0.55-0.60$	$0.60-0.65$	$0.65-0.70$	$0.70-0.75$	[illegible]	[illegible]	[illegible]
$0.80-0.85$												[illegible]	[illegible]	[illegible]
$0.85-0.90$												[illegible]	[illegible]	[illegible]
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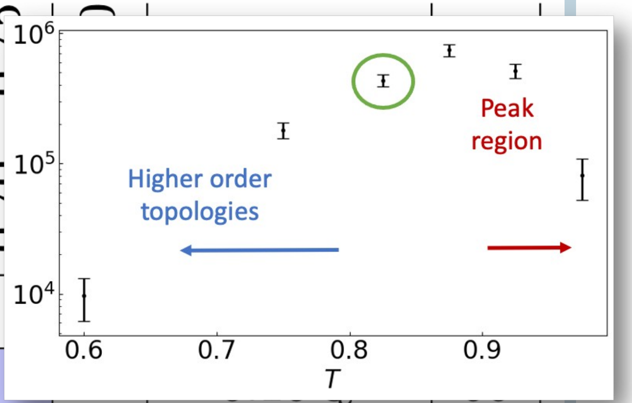


BELLE Phys. Rev. D99 (2019) 11 112006

Step 1: preliminary fit to pin down the profile of the of the TMD

Selection of 57 data points, binned in z and P_T at fixed T (0.80<T<0.85)

TMD Fragmentation Function

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_f \int_{z_h}^{1} \frac{dz}{z} \frac{d\widehat{\sigma}_f}{dz_h/z\, dT} D_{1,\pi^\pm/f}(z,\, P_T,\, Q,\, (1-T)\,Q^2)$$

Fourier Transform of:

Collinear FFs

$$\widetilde{D}_{h/j}(z,\, a_S(\mu), \mathcal{L}_b, \log\frac{\sqrt{\zeta}}{\mu};\, b_T) = \widetilde{D}_{h/j,\,\star}(z,\, a_S(\mu_b^\star)) \times$$

$$\times\, \exp\left\{\frac{1}{2}\,\widetilde{K}_\star\,(a_S(\mu_b^\star))\,\log\frac{\sqrt{\zeta}}{\mu_b^\star} + \int_{\mu_b^\star}^{\mu}\frac{d\mu'}{\mu'}\gamma_D\left(a_S(\mu'),\,\log\left(\frac{\sqrt{\zeta}}{\mu'}\right)\right)\right\} \times$$

Perturbative part (NLL)

$$\times\, M_D(z, b_T;\, j, h)\exp\left\{-\frac{1}{2}\,g_K(b_T)\,\log\frac{\sqrt{\zeta}}{M_h}\right\}$$

Non-Perturbative part Phenomenological Model

M_D embeds the non-perturbative, long-range behavior of the TMD FF

g_K is universal, independent of the TMD definition used

Phenomenological parametrization: M_D

M. Boglione, A. Simonelli, 2306.02937 [hep-ph]

$$M_D(z, b_T) = \frac{2}{\Gamma(p(z)-1)} \left(\frac{b_T\, m(z)}{2}\right)^{p(z)-1} K_{p(z)-1}\left(b_T\, m(z)\right)$$

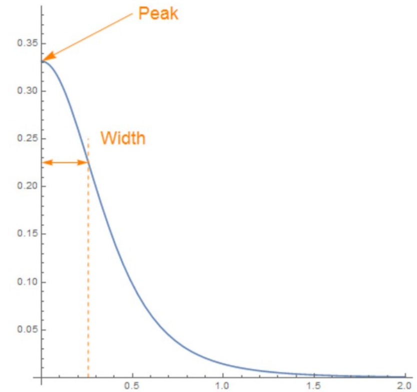
Gaussian behaviour at small b_T
Exponential fall off at large b_T

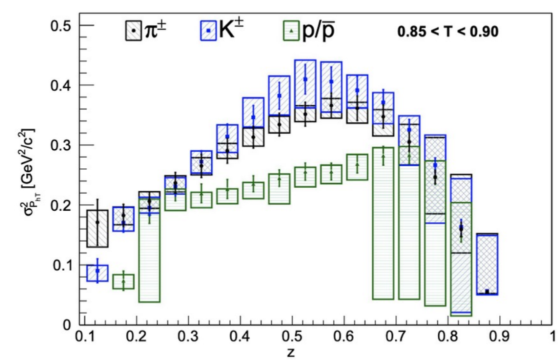
BK parameters depend on z

$$p(z) = \frac{1}{2}\left(\frac{3}{1-R(z)} - 1\right), \qquad m(z) = \frac{W(z)}{z}\sqrt{\frac{3}{1-R(z)}}$$

$$R(z) = 1 - \alpha\,\frac{f(z)}{f(z_0)} \text{ with } f(z) = z\,(1-z)^{\frac{1-z_0}{z_0}}, \qquad W(z) = \frac{m_\pi}{R(z)^2}$$

The z behaviour of M_D is model in such a way that it offers a direct check that the theory lines appropriately reproduce the width of the measured cross sections, at each value of z (z_0 ~ 0.5 - 0.6)





BELLE Phys. Rev. D99 (2019) 11 112006

Phenomenological parametrization: g_K

M. Boglione, A. Simonelli, 2306.02937 [hep-ph]

In this analysis we consider 2 different hypothesis for g_K for which, asymptotically, we have g_K = const.

J. Collins, T. Rogers, Phys. Rev. D 91, 074020 (2015)
C. Aidala et al., Phys.Rev. D89, 094002 (2014)
A.. Vladimirov Phys. Rev. Lett. 125, 192002 (2020).

Quadratic at small bT $\quad g_K \sim g_2 b_T^2 + \ldots \text{ for } b_T \to 0.$

Constant at large bT $\quad g_K \to g_0 \text{ for } b_T \to \infty.$

$$g_K^A(b_T) = g_0 \tanh\left(\beta^2 \frac{b_T^2}{b_{MAX}^2}\right),$$
$$g_K^B(b_T) = g_0 \tanh\left(\beta^2 \, b_T^\star \, b_T\right).$$

Testing different b_T behaviors of g_K allows us to give a reliable estimate of the uncertainties affecting our analysis

Phenomenological results – Preliminary fit

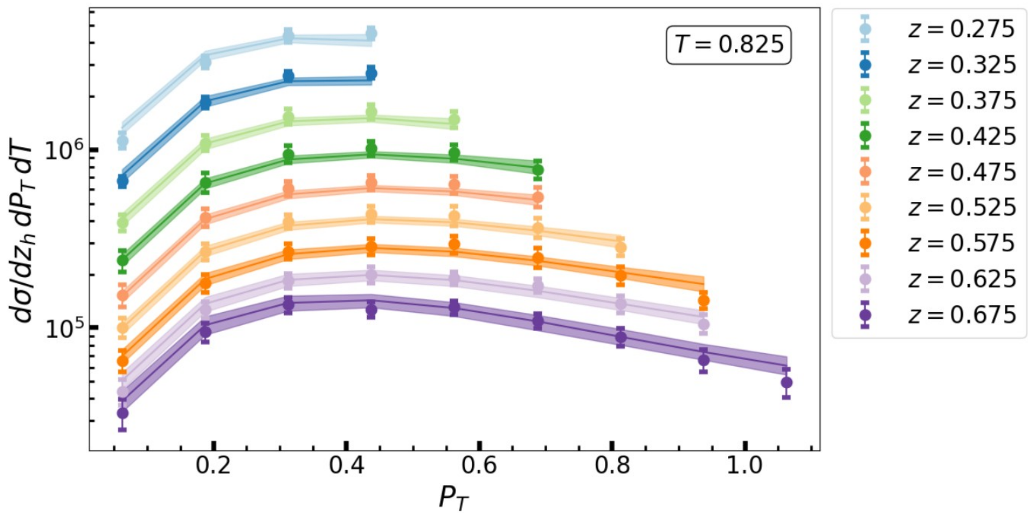
M. Boglione, A. Simonelli, 2306.02937 [hep-ph]

Preliminary Fit

4 free parameters

Data selection:
57 data points
0.275 < z < 0.675
$q_T < 0.16\, Q$
T=0.825

$\chi^2/\text{d.o.f.}$	0.6183
z_0	$0.5521^{+0.0415}_{-0.0398}$
α	$0.3644^{+0.0250}_{-0.0282}$
g_0	$0.2943^{+0.0329}_{-0.0261}$
β	$4.7100^{+1.9856}_{-1.9856}$



BELLE Collaboration, R. Seidl et al., Phys. Rev. D99 (2019), no. 11 112006

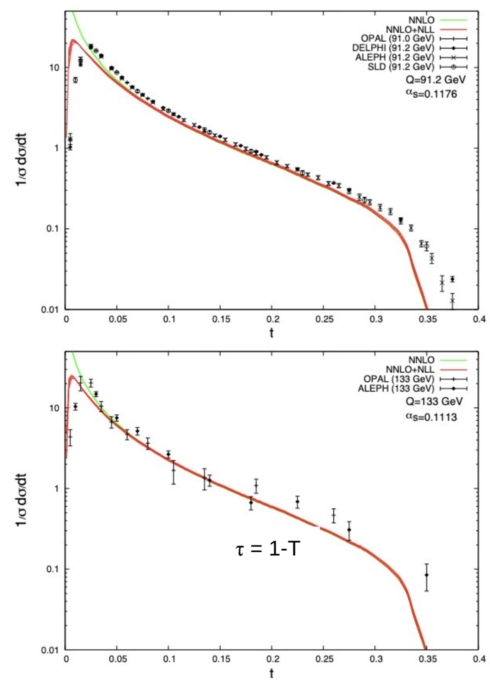
Thrust non-pertubative effects

R.A. Davison, B.R. Webber, Eur.Phys.J.C59:13-25, 2009

Non-perturbative contributions affect thrust distributions
(well known from fully inclusive e^+e^- annihilation data)

Non perturbative contributions
to thrust distributions
are very large
at small Q.

A simple shift is not enough to reach agreement with experimental data



Non perturbative contributions
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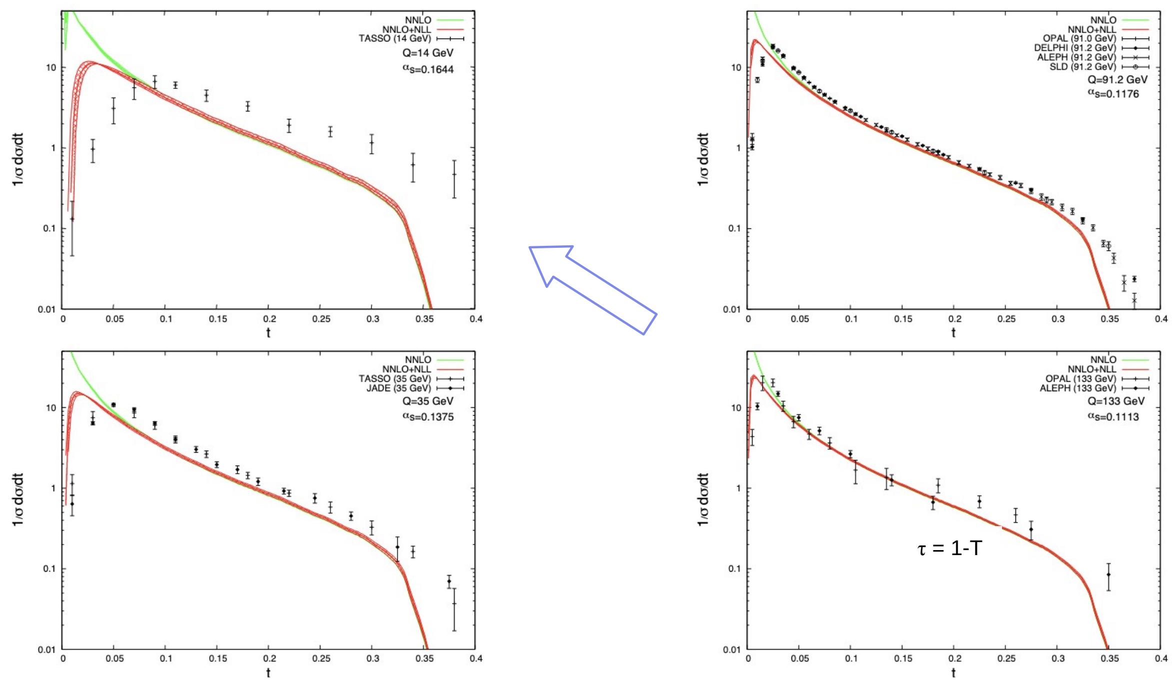

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Phenomenological results – T dependence

M. Boglione, A. Simonelli, 2306.02937 [hep-ph]

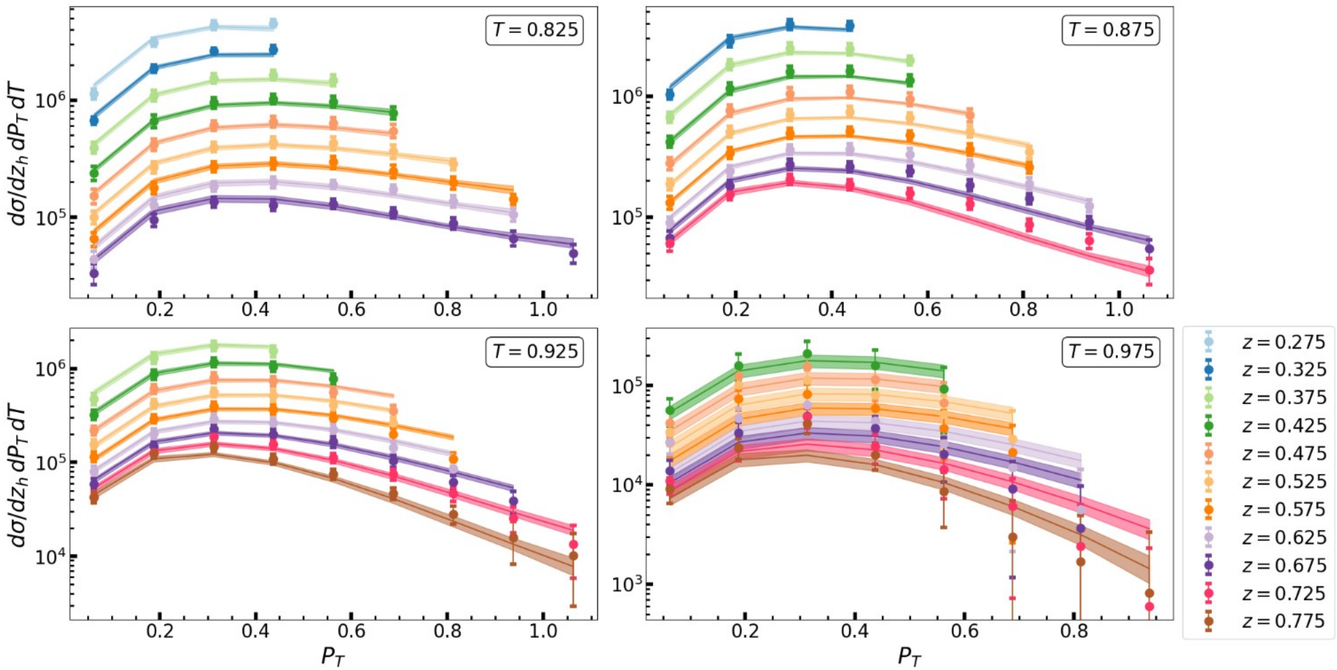
$$\frac{d\sigma}{dz\,dT\,d^2\vec{P}_T} = \left.\frac{d\sigma^{\text{res.}}}{dz\,dT\,d^2\vec{P}_T}\right|_{T-T_0} f_{\text{NP}}(1-T)$$

Shift the resummed factorized cross section

Multiply by shaping function $f_{NP}(1-T)$

$f_{NP}(1-T) \to 1$ when $T \to 0$

$f_{NP}(1-T) \to 0$ when $T \to 1$



BELLE Collaboration, R. Seidl et al.,Phys. Rev. D99 (2019), no. 11 112006

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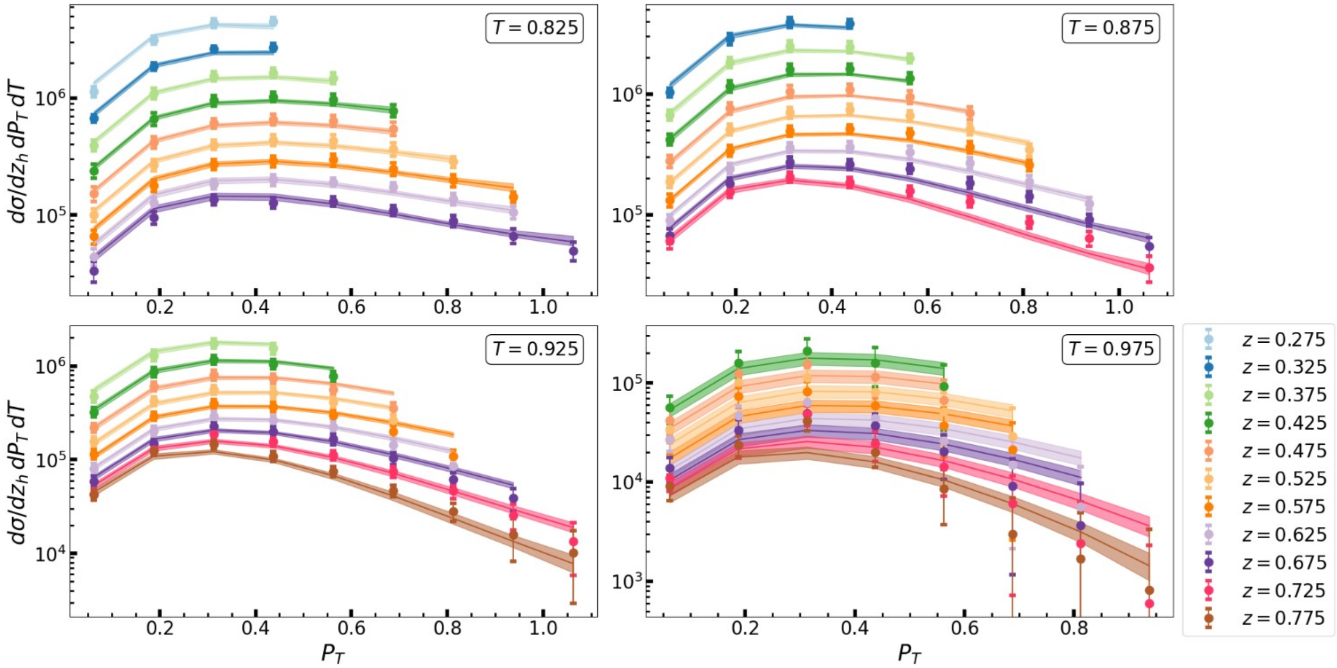
Fit A

χ^2/d.o.f.	1.0749
z_0	$0.5335^{+0.0194}_{-0.0180}$
α	$0.3403^{+0.0114}_{-0.0122}$
g_0	$0.1044^{+0.0446}_{-0.0742}$
β	$1.6765^{+0.8150}_{-0.8150}$
T_0	$0.0617^{+0.0295}_{-0.0134}$
ρ	$7.7205^{+0.2834}_{-0.2099}$

Fit B

χ^2/d.o.f.	1.3421
z_0	$0.5334^{+0.0192}_{-0.0189}$
α	$0.3394^{+0.0127}_{-0.0134}$
g_0	$0.1205^{+0.0305}_{-0.0367}$
β	$2.0610^{+2.1042}_{-0.5193}$
T_0	$0.0467^{+0.0117}_{-0.0077}$
ρ	$8.1643^{+0.3053}_{-0.3011}$

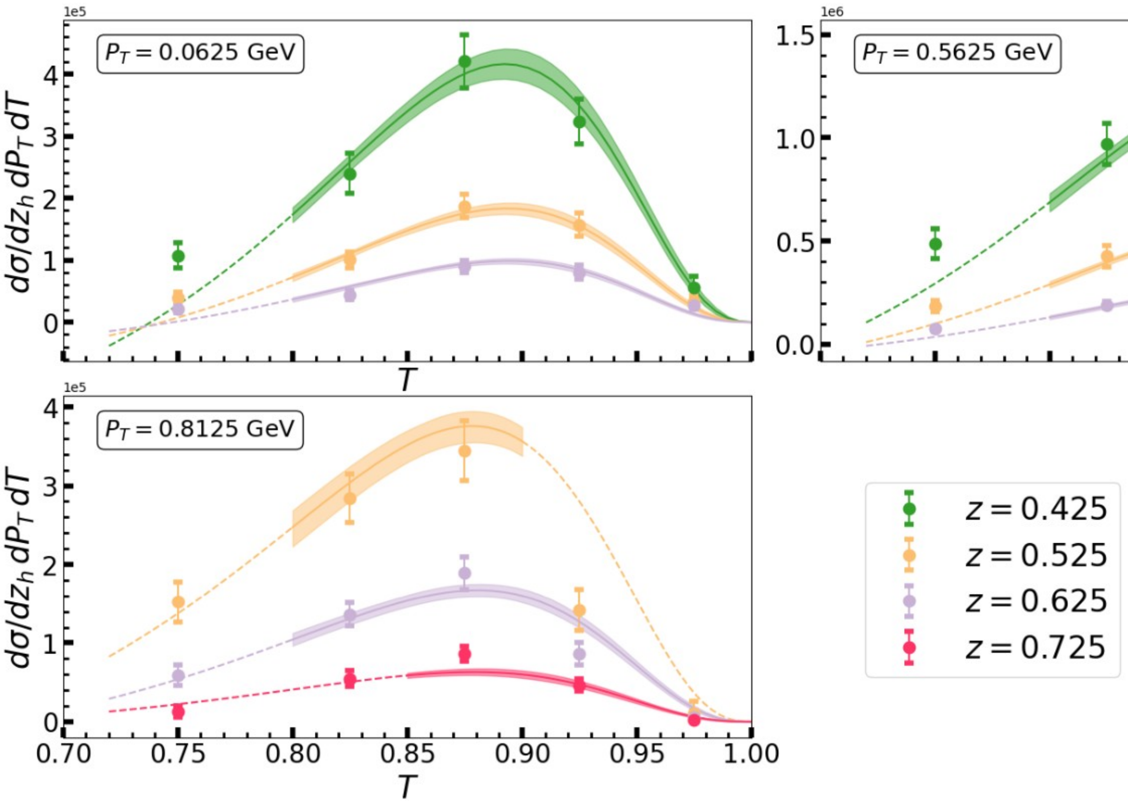


BELLE Collaboration, R. Seidl et al.,Phys. Rev. D99 (2019), no. 11 112006

Phenomenological results –T dependence

M. Boglione, A. Simonelli, 2306.02937 [hep-ph]

Thrust distributions at fixed values of z and P_T



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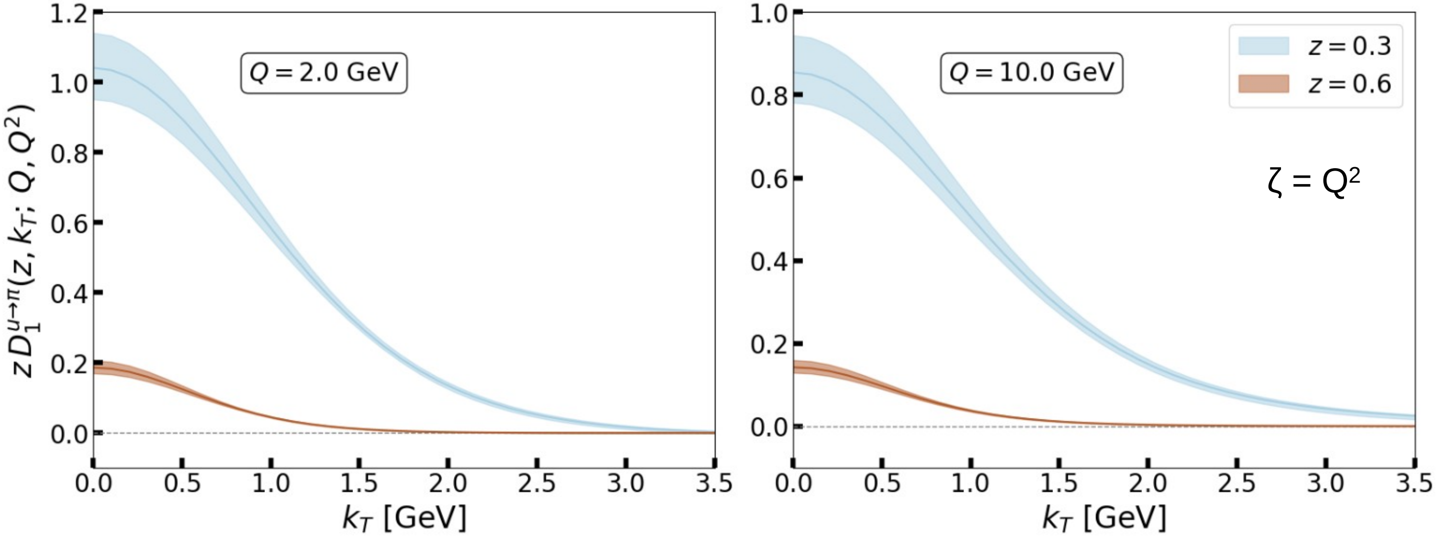
BELLE Collaboration, R. Seidl et al.,Phys. Rev. D99 (2019), no. 11 112006

Phenomenological results

M. Boglione, A. Simonelli, 2306.02937 [hep-ph]

Extraction of the unpolarized TMD FF for different values of z and Q

up/down quark into a charged pion



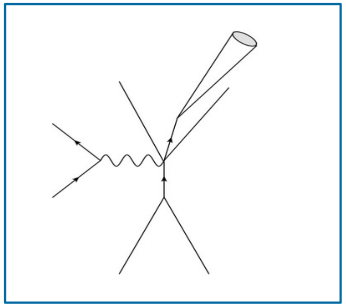
$\zeta = Q^2$

The TMD FF becomes broader with a less pronounced peak
For increasing values of z, as indicated by experimental data

Conclusions

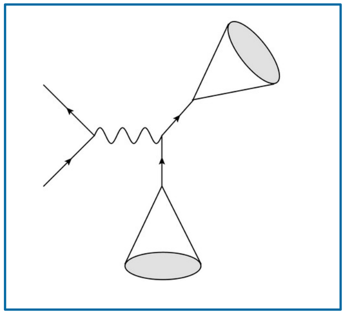
- We have achieved the first rigorous (and successful) description of the **thrust dependence** of the $e^+e^- \to hX$ cross section, accompanied by an optimal description of the z and P_T profiles.
- The riddle about different factorization theorems obtained in SCET studies has been solved.
- Models to account for the non-perturbative z and P_T behaviours are devised with special attention to experimental data
- The non perturbative parametrization of the Collins-Soper kernel are modeled according to recent theoretical studies (for example, g_K behaves as a constant at large b_T)
- Future studies will address the relation between the TMD fragmentation function extracted in $e^+e^- \to hX$ and the analogous function emerging from SIDIS and $e^+e^- \to h_2\, h_1\, X$ (study of the soft model M_S)

Outlook



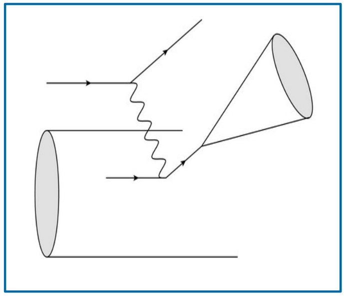
1. $e^+ e^- \to h\, X$

   Extraction of the unpolarized TMD FF, D*, for charged pions from BELLE data (using factorization definition)



2. $e^+ e^- \to h_1 h_2\, X$

   Two non-perturbative functions:
   D*, known from step 1
   Soft Model M_s, obtained as ratio: $M_S = D/D^\star$

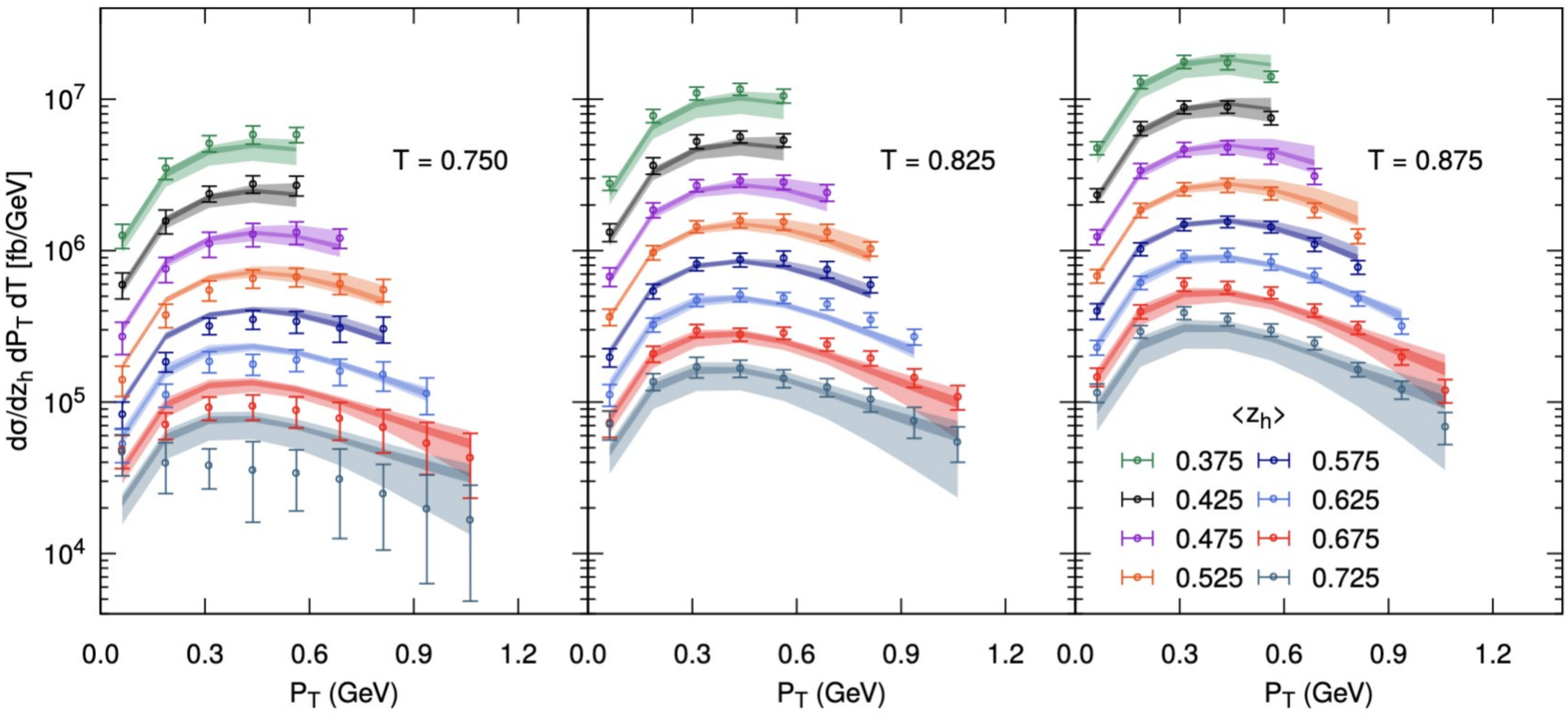


3. $SIDIS$

   Three non-perturbative functions in the cross section
   D*, known from step 1.
   Soft Model M_s, known from step 2.
   Extraction of the TMD PDF, F* (in the factorization definition, F*≠F).

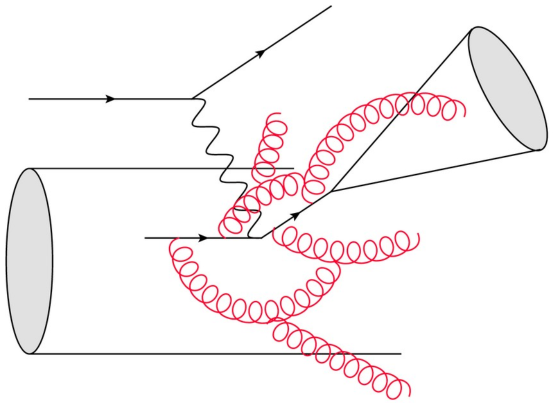
Back up slides

Old pheno results



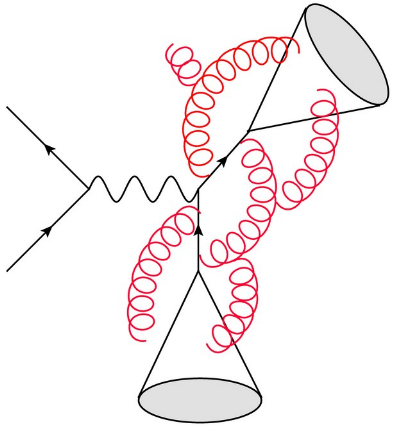
Soft Gluon contribution

SIDIS



$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{sidis}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \, F(b_T)\, D(b_T)$$

Double hadron production



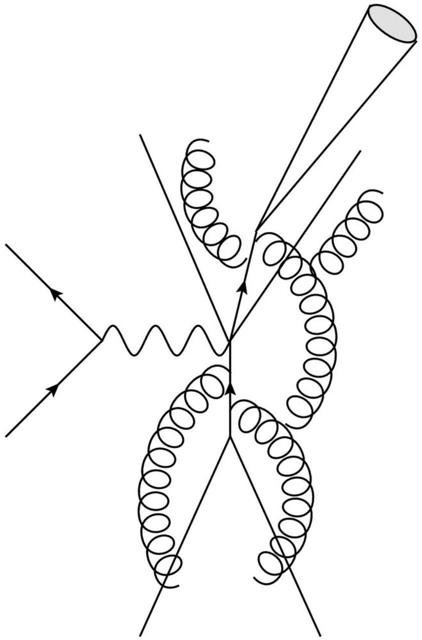
$$\frac{d\sigma}{dq_T} = \mathcal{H}_{\text{2-h}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \, D_1(b_T)\, D_2(b_T)$$

Soft Gluon Factor:
- Non-Perturbative contribution
- Evenly shared by the TMDs

Soft Gluons

M. Boglione, A. Simonelli, Eur. Phys. J. C 81 (2021)

$$e^+e^- \to hX$$



$$\frac{d\sigma}{dP_T} = d\widehat{\sigma} \otimes D^\star(P_T)$$

Soft Gluon Factor:

- Perturbative contribution
- The TMD FF***** is **free** from any soft gluon contributions

$D(P_T)$ and $D^*(P_T)$ are different,
BUT
the relation between D and D* is known!

We can perform combined analyses and disentangle non-perturbative terms.

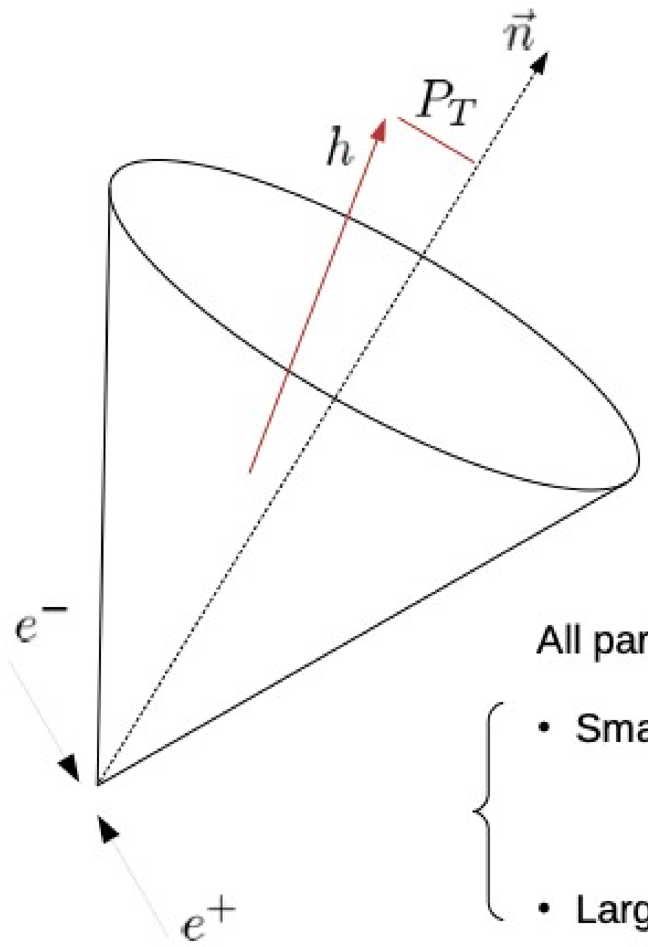
Rapidity divergencies and thrust in Region 2

ISSUES FROM TREATMENT OF RAPIDITY DIVERGENCES

- Peculiar interplay between soft and collinear contributions ⇒ some of the rapidity divergences are naturally regulated by the thrust, T, but those associated to strictly TMD parts of the cross section need an extra artificial regulator, which is a rapidity cut- off.
- This induces a redundancy, which generates an additional relation between the regulator, the transverse momentum and thrust.
- This relation inevitably spoils the picture in which the cross section factorizes into the convolution of a partonic cross section (encoding the whole T dependence) with a TMD FF (which encapsulates the whole P_T dependence).
- Thrust resummation is intertwined with the transverse momentum dependence, making the treatment of the large T behavior highly non-trivial.
- A proper phenomenological analysis of Region 2 must rely on a factorized cross section where the regularization of rapidity divergences is properly taken into ac- count. All difficulties encountered in the theoretical treatment get magnified in the phenomenological applications.
- In our previous analysis we adopted some approximations, in order to simplify the structure of the factorization theorem without altering its main architecture.
- In this new analysis we relax these approximations and achieve a full resummation of the thrust dependence of the SIA cross section

The $e^+e^- \to hX$ process

The cross section is differential in: $z_h = \frac{E}{Q/2}$, $T = \frac{\sum_i |\vec{P}_{(\text{c.m.}),\,i} \cdot \hat{n}|}{\sum_i |\vec{P}_{(\text{c.m.}),\,i}|}$, P_T w.r.t $\vec{n}$



$0.5 \leq T \leq 1$

Spherical distribution ◄ ► 2-jet limit

2-jet final state is the most probable topology configuration

All particles inside the jet in which h is detected must have:

- Small transverse momentum $P_T \ll P^+ = z_h \frac{Q}{\sqrt{2}}$
- Large rapidity $y_P = \frac{1}{2} \log \frac{2(P^+)^2}{P_T^2 + M_h^2} \gg 0$