A light-front model for pion parton distribution functions

SAR WORS

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Based on: B. Pasquini, S. Rodini, S. Venturini, 2303.01789

HAS QCD
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

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Outline

- Light-front formalism
- Our model
- Fit results
- Outlook

Light-front formalism

$$|\psi\rangle = \sum_{n} \sum_{c_i} \sum_{\mu_i} \int [Dx]_n \, \psi_n^{\Lambda}(r) |n, w_1, w_2, ..., w_n\rangle$$

$$w_i = (p_i^{+}, \vec{p}_{\perp i}, \mu_i, c_i)$$

$$|n, w_1, w_2, ..., w_n\rangle = \prod_{j=1}^{n_q} b^{\dagger}_{q_j} \left(w_j\right) \prod_{l=1}^{n_{\bar{q}}} d^{\dagger}_{q_l} \left(w_l\right) \prod_{m=1}^{n_{gl}} a^{\dagger} \left(w_m\right) |0\rangle$$

$$\psi_n^{\Lambda}(r) = \langle \psi | n, w_1^{c_1}, w_2^{c_2}, \ldots, w_n^{c_n} \rangle$$

↕

Light-Front Wave Functions

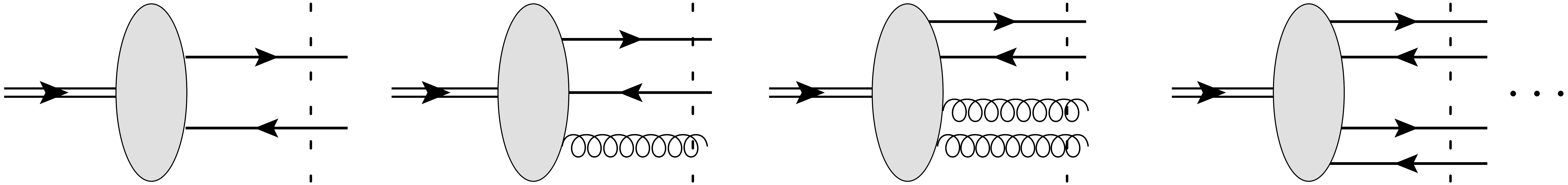
Light-front formalism

LFWF overlap representation

$$\Phi^{[\gamma^+]}(x;p) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ikz} \langle\pi(p)|\bar{q}(0)\mathcal{U}_{(0,z)}\gamma^+ q(z)|\pi(p)\rangle\Big|_{z^+=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$\Phi^{[\gamma^+]}(\Delta;p) = \frac{1}{2p^+}\langle\pi(p')|\bar{q}(0)\mathcal{U}_{(0,z)}\gamma^+ q(z)|\pi(p)\rangle$$

$$|\pi(p)\rangle = \underline{\psi_{q\bar{q}}}|\pi(p)_{q\bar{q}}\rangle + \underline{\psi_{q\bar{q}g}}|\pi(p)_{q\bar{q}g}\rangle + \underline{\psi_{q\bar{q}gg}}|\pi(p)_{q\bar{q}gg}\rangle + \underline{\psi_{q\bar{q}s\bar{s}}}|\pi(p)_{q\bar{q}s\bar{s}}\rangle + \ldots$$



Light-front formalism

LFWF overlap representation

Pion PDF

$$f_{1,\pi}(x) = \sum_{\substack{N=q\bar{q},\\ q\bar{q}g,\dots}} \int [dx]_N \, [d^2\vec{k}_\perp]_N \left|\psi_N\left(\{x_i\}\,,\left\{\vec{k}_{\perp i}\right\}\right)\right|^2 \delta(x-x_i)\delta_{iq}$$

Pion e.m. FF

$$F_1(\Delta) = \sum_{\substack{N=q\bar{q},\\ q\bar{q}g,\,\dots}} \sum_{a} \sum_{\underset{\displaystyle\updownarrow}{j}} \sum_{\beta=\beta'} e_a \delta_{s_j a} \int [dx]_N [d^2\vec{k}_\perp]_N \;\psi^*_{N,\beta'}(r')\psi_{N,\beta}(r)$$

active parton

Our model

LFWF overlap representation

$$\underline{\psi_{q\bar{q}}\left(1,2\right)} = \phi_{q\bar{q}}\left(x_1,x_2\right)\Omega_2\left(x_1,x_2,\vec{k}_{\perp 1},\vec{k}_{\perp 2}\right)$$

$$\underline{\psi_{q\bar{q}g}\left(1,2,3\right)} = \phi_{q\bar{q}g}\left(x_1,x_2,x_3\right)\Omega_3\left(x_1,x_2,x_3,\vec{k}_{\perp 1},\vec{k}_{\perp 2},\vec{k}_{\perp 3}\right)$$

$$\underline{\psi_{q\bar{q}gg}\left(1,2,3,4\right)} = \phi_{q\bar{q}gg}\left(x_1,x_2,x_3,x_4\right)\Omega_4\left(x_1,x_2,x_3,x_4,\vec{k}_{\perp 1},\vec{k}_{\perp 2},\vec{k}_{\perp 3},\vec{k}_{\perp 4}\right)$$

$$\underline{\psi_{q\bar{q}s\bar{s}}\left(1,2,3,4\right)} = \phi_{q\bar{q}s\bar{s}}\left(x_1,x_2,x_3,x_4\right)\Omega_4\left(x_1,x_2,x_3,x_4,\vec{k}_{\perp 1},\vec{k}_{\perp 2},\vec{k}_{\perp 3},\vec{k}_{\perp 4}\right)$$

$\phi_{q\bar{q}}, \phi_{q\bar{q}g}, \phi_{q\bar{q}gg}, \phi_{q\bar{q}s\bar{s}}$ ⟷ **Pion Distribution Amplitudes**

$$\langle 0|\bar{u}(z)\;\Gamma\;\mathcal{U}_{(z,-z)}\;d(-z)|\pi^-(p)\rangle$$

Our model

Pion Distribution Amplitudes

$$\prod_{i=1}^{N} x_i^{2j_i-1}, \qquad j_i = \begin{cases} 1\,, & i = q, \bar{q} \\ 3/2\,, & i = g \end{cases}$$

V. M. Braun and I. E. Filyanov, *Z. Phys. C 48,239 (1990)*

$$\phi_{q\bar{q}}(x_1, x_2) = \mathcal{N}_{q\bar{q}}(x_1\ x_2)^{\gamma_q} \sum_n C_{q_n} \left(\mathcal{G}_n^{\gamma_q + \frac{1}{2}}\,(2x_1 - 1) + \mathcal{G}_n^{\gamma_q + \frac{1}{2}}\,(2x_2 - 1) \right)$$

2 (+1) parameters

$$\phi_{q\bar{q}g}(x_1, x_2, x_3) = \mathcal{N}_{q\bar{q}g}\ x_1 x_2\ x_3^2 \sum_N \sum_{n \leq N} C_n^N \Bigg((1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}(\frac{x_2 - x_1}{1 - x_3})$$
$$+ (1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}(\frac{x_1 - x_2}{1 - x_3}) \Bigg)$$

1 + (1) parameters

$$\phi_{q\bar{q}s\bar{s}}\,(x_1, x_2, x_3, x_4) = \mathcal{N}_{q\bar{q}s\bar{s}}\ x_1 x_2 x_3 x_4$$

$$\phi_{q\bar{q}gg}\,(x_1, x_2, x_3, x_4) = \mathcal{N}_{q\bar{q}gg}\ x_1 x_2\,(x_3 x_4)^2$$

(1) parameter

Our model

Ω-functions

S. J. Brodsky, T. Huang, P. Lepage (1983)

$$\Omega_{N,\beta}\left(x_1, \boldsymbol{k}_{\perp 1}, x_2, \boldsymbol{k}_{\perp 2}, \ldots, x_N, \boldsymbol{k}_{\perp N}\right) = \frac{(16\pi^2 a_\beta^2)^{N-1}}{\prod_{i=1}^{N} x_i} \exp\left(-a_\beta^2 \sum_{i=1}^{N} \frac{\boldsymbol{k}_{\perp i}^2}{x_i}\right)$$

DAs ⟷ $\int \left[d^2\boldsymbol{k}_\perp\right]_N \Omega_{N,\beta} = 1$

PDF ⟷ $\int \left[d^2\boldsymbol{k}_\perp\right]_N \Omega^2_{N,\beta} = \dfrac{(8\pi^2 a_\beta^2)^{N-1}}{\prod_{i=1}^{N} x_i}$

DAs ⟷ $\int [d^2\boldsymbol{k}_\perp]_N \Omega_{N,\beta} = \dfrac{1}{\left(2\sqrt{2}\pi a_\beta\right)^{N-1}}$

PDF ⟷ $\int [d^2\boldsymbol{k}_\perp]_N \Omega^2_{N,\beta} = \dfrac{1}{\prod_{i=1}^{N} x_i}$

Fit results

PDF fit

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
260	**6**	**0.884**

NA10	E615	WA70
70	91	99

Phys.Rev.D 102 (2020) 1, 014040

FF fit

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
100	**3** (4 - 1)	**1.194**

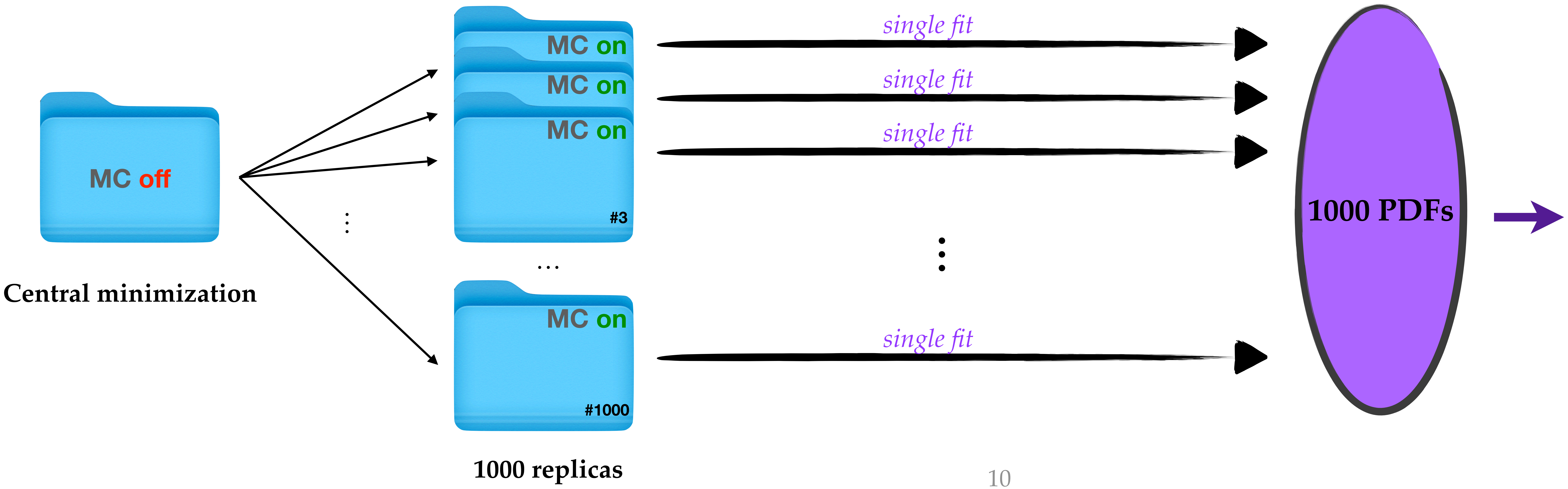
$$\text{CI } 68\% = [0.890, 1.204]$$

Fit results

PDF fit - error bands

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
260	**6**	**0.884**

Replica method



MC off

Central minimization

MC on

MC on

MC on

#3

…

MC on

#1000

1000 replicas

single fit

single fit

single fit

single fit

1000 PDFs

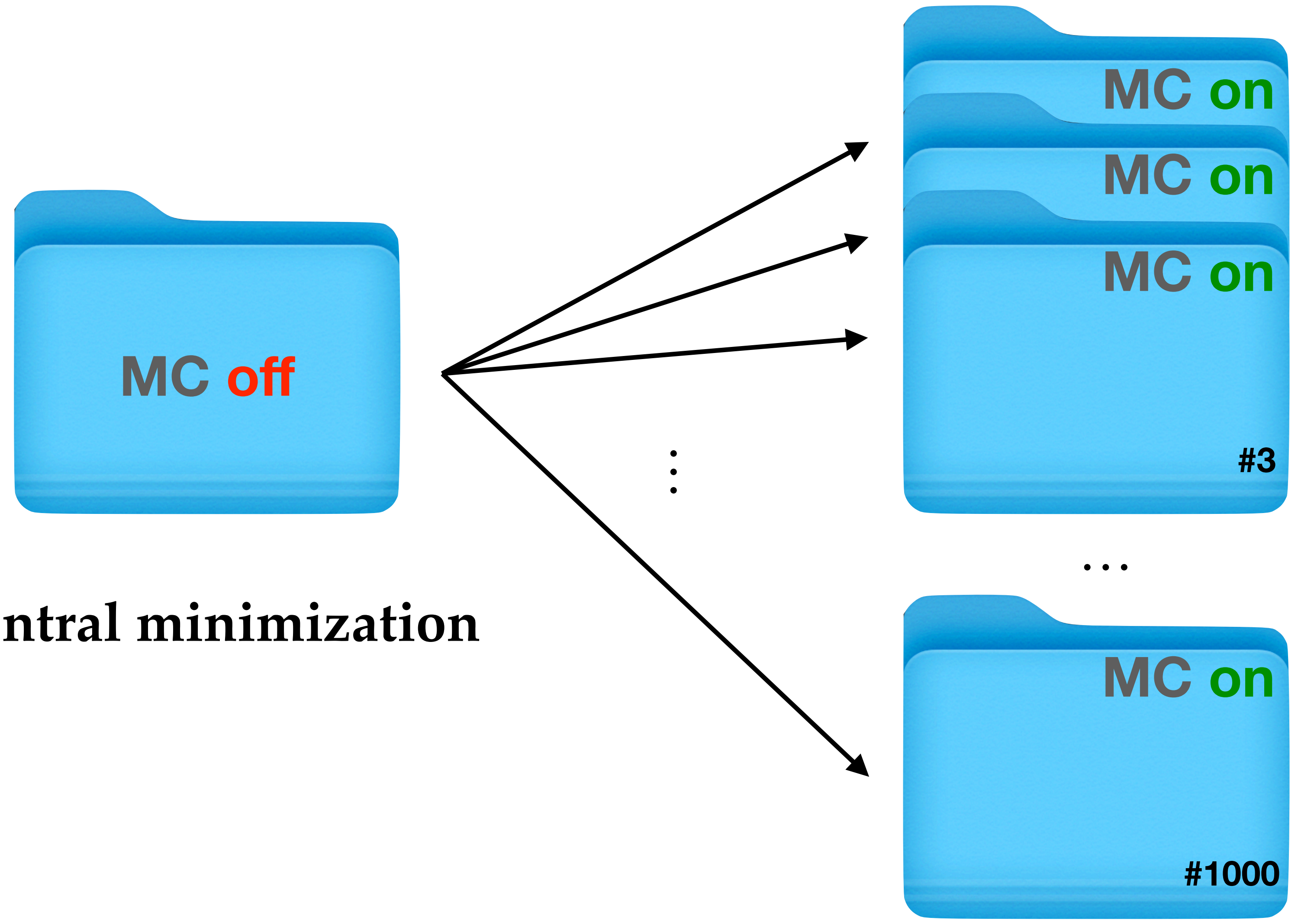
Plot the bands

- Consider many (~1000) values of x in [0,1].
- For each x, compute the mean r(x) and the standard deviation s(x).
- Build the band r(x) ± s(x).

Fit results

PDF fit - scale variation

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
260	**6**	**0.884**



Central minimization

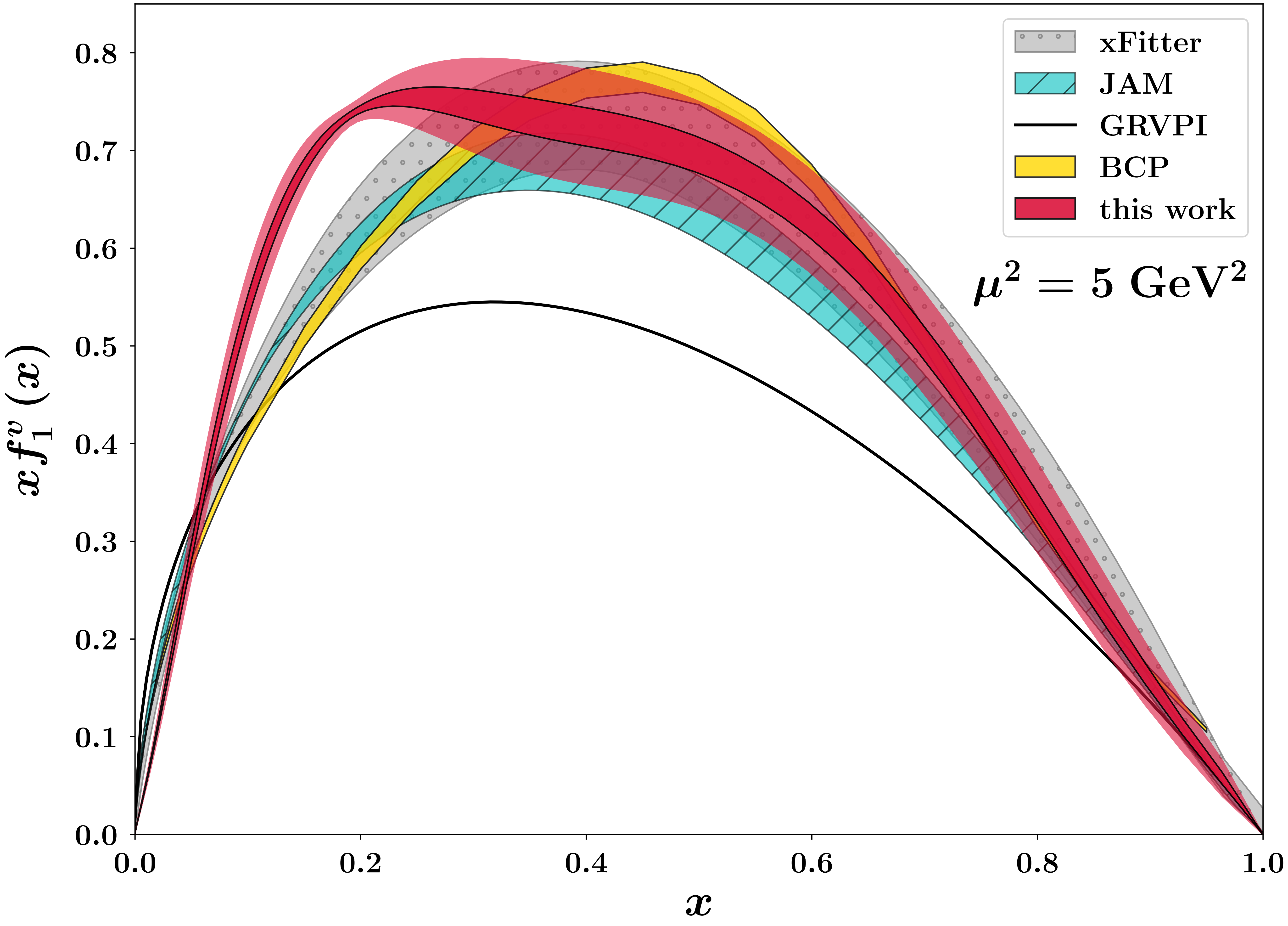
1000 replicas

for each single replica:

- *generate* $\frac{\mu_0}{2} < \mu_F < \mu_0$
- *generate* $\mu_F \leq \mu_R < 2\mu_0$

Fit results

Pion PDFs



I. Novikov et al., *Phys.Rev.D102,014040* (2020)

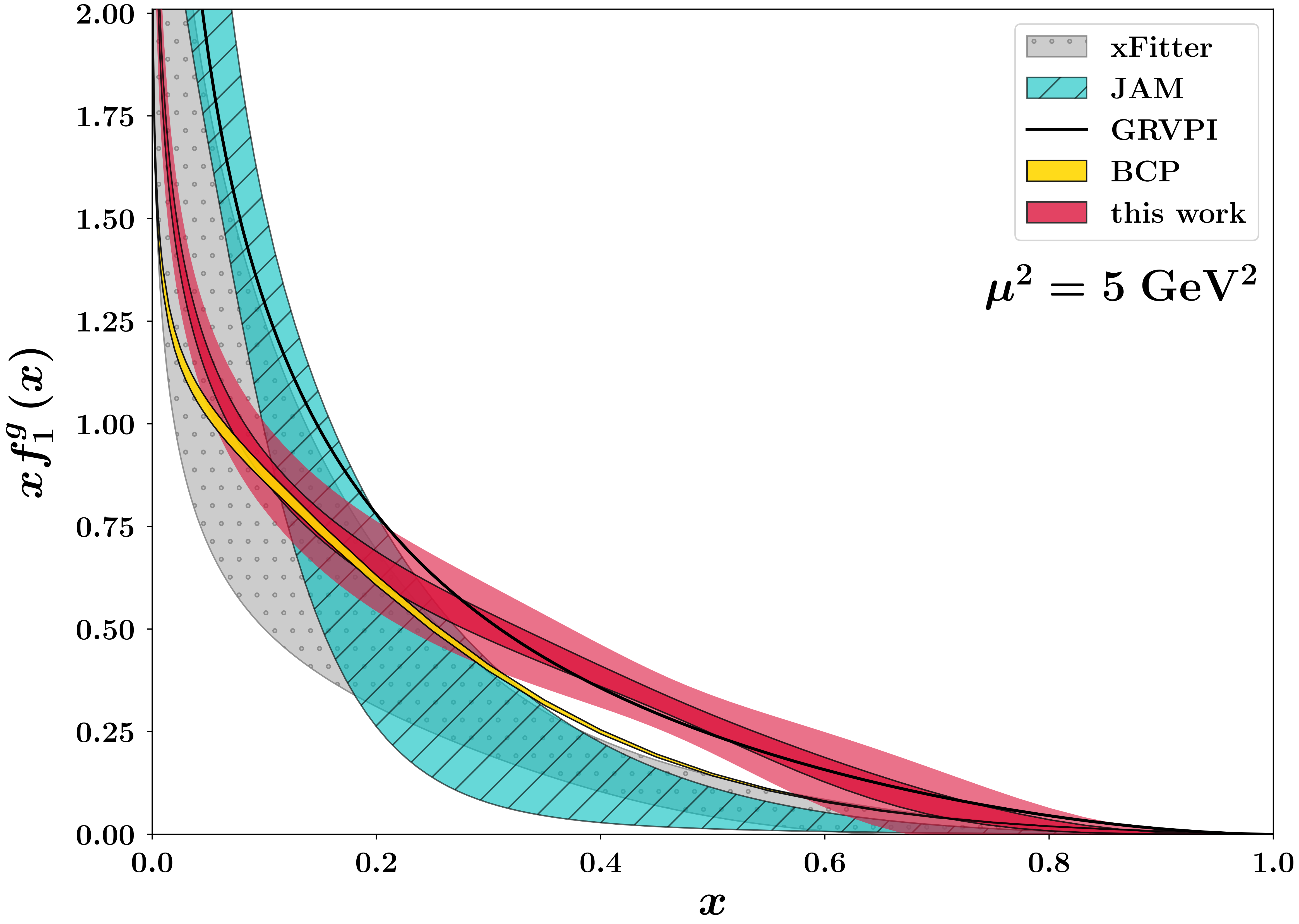
P. C. Barry, C.-R. Ji, N. Sato, W. Melnitchok (JAM), *Phys.Rev.Lett.127, 232001* (2021)

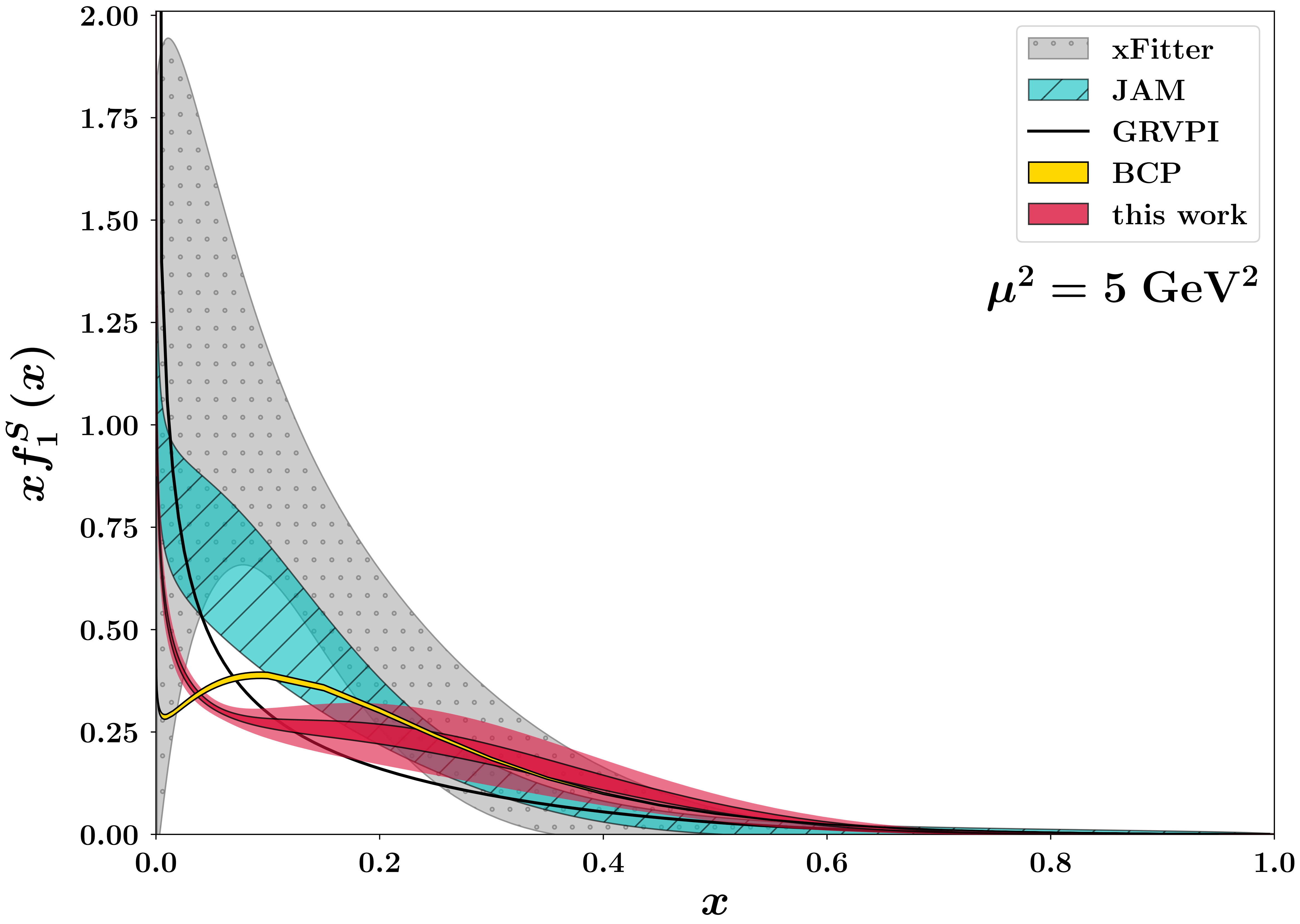
M. Gluck, E. Reya, A. Vogt, *Z.Phys.C 53, 651* (1992)

C. Bourrelly, W.-C. Chang, J.-C. Peng, *Phys.Rev.D 105,076018* (2022)

Fit results

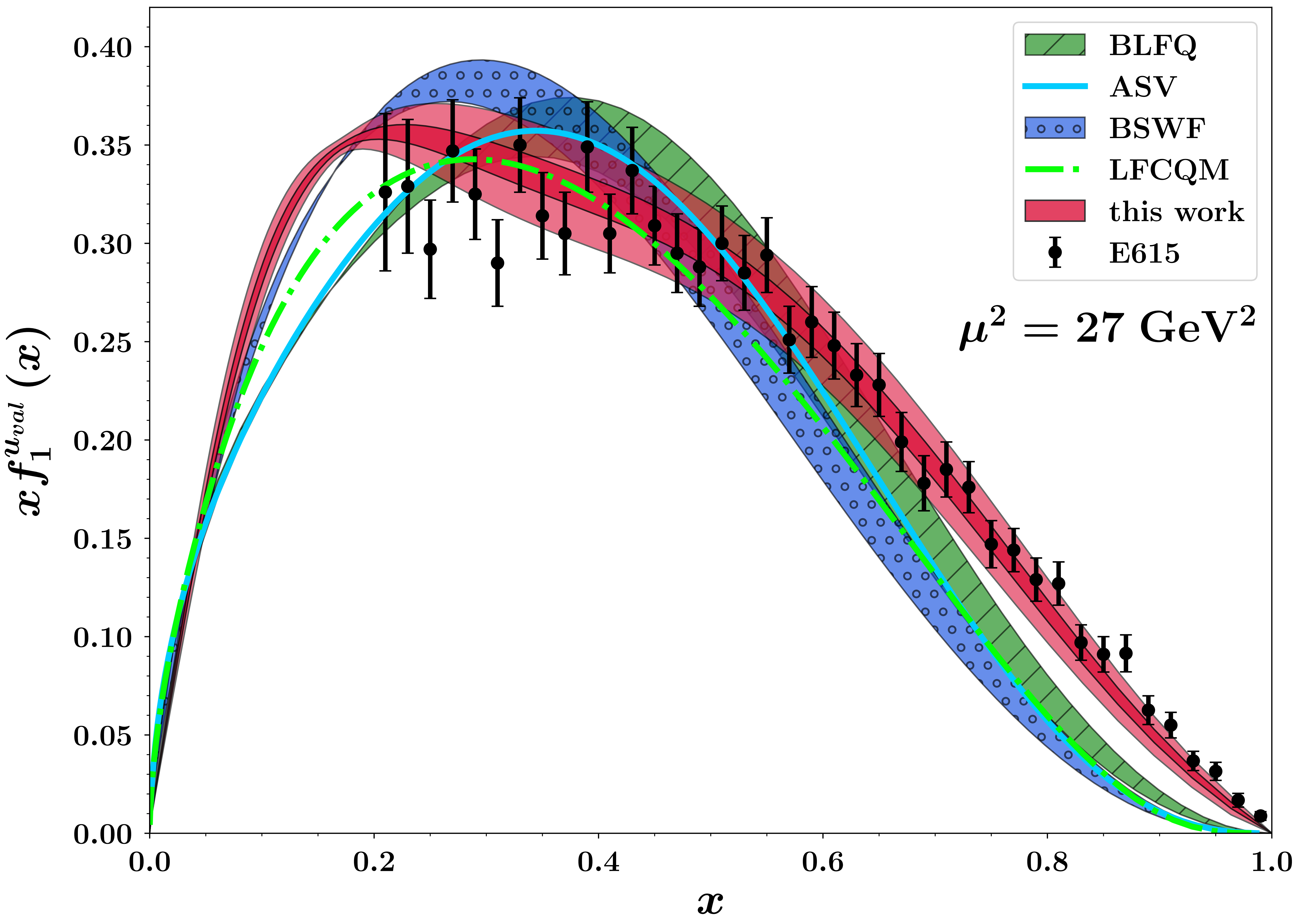
Pion PDFs





Fit results

Pion PDFs



J. Lan, K. Fu, C. Mondal, X. Zhab, P. Vary (BFLQ), *Phys.Lett.B 825, 136890*

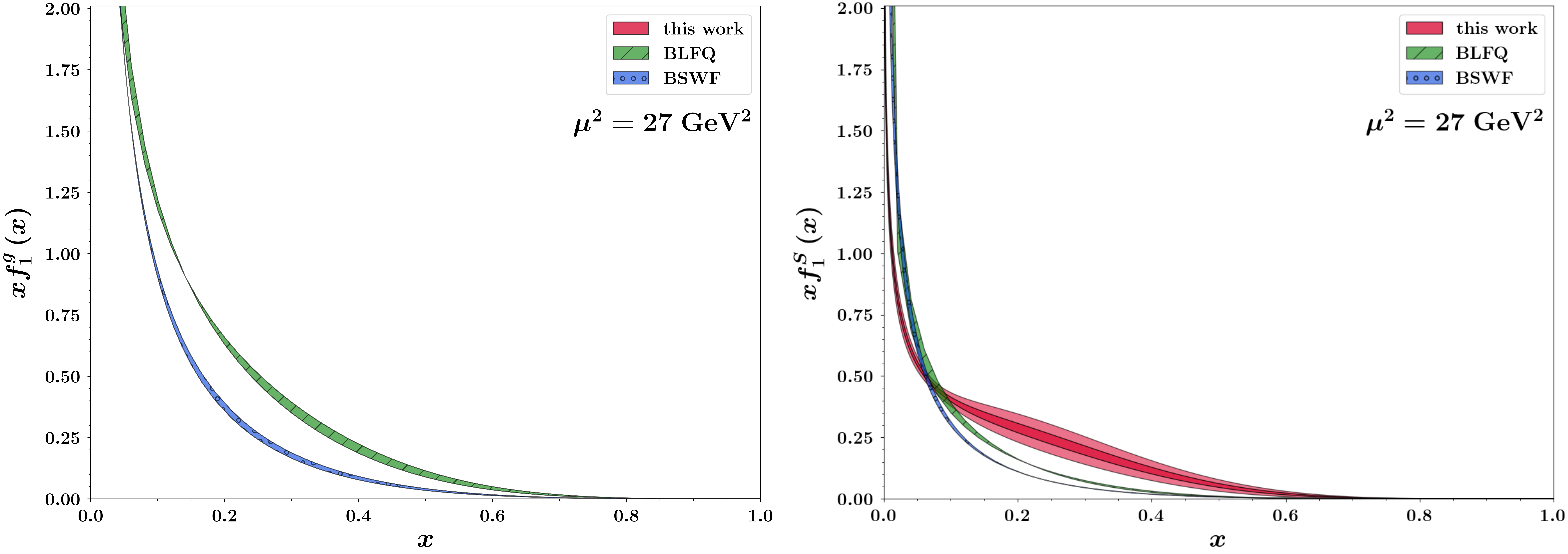
M. Aicher, A. Schafer, W. Vogelsang, *Phys.Rev.Lett. 105, 252003* (2010)

Z.-F. Cui, M. Ding, F. Gao, K. Raya, D. Binosi, L. Chang, C. D. Roberts, J. Rodríguez-Quintero, S.M. Schmidt, *Rur.Phys.J.C 8'0, 1064* (2020)

B. Pasquini, P. Schweitzer, *Phys.Rev.D 90, 014050* (2014)

Fit results

Pion PDFs



Fit results

FF fit - error bands

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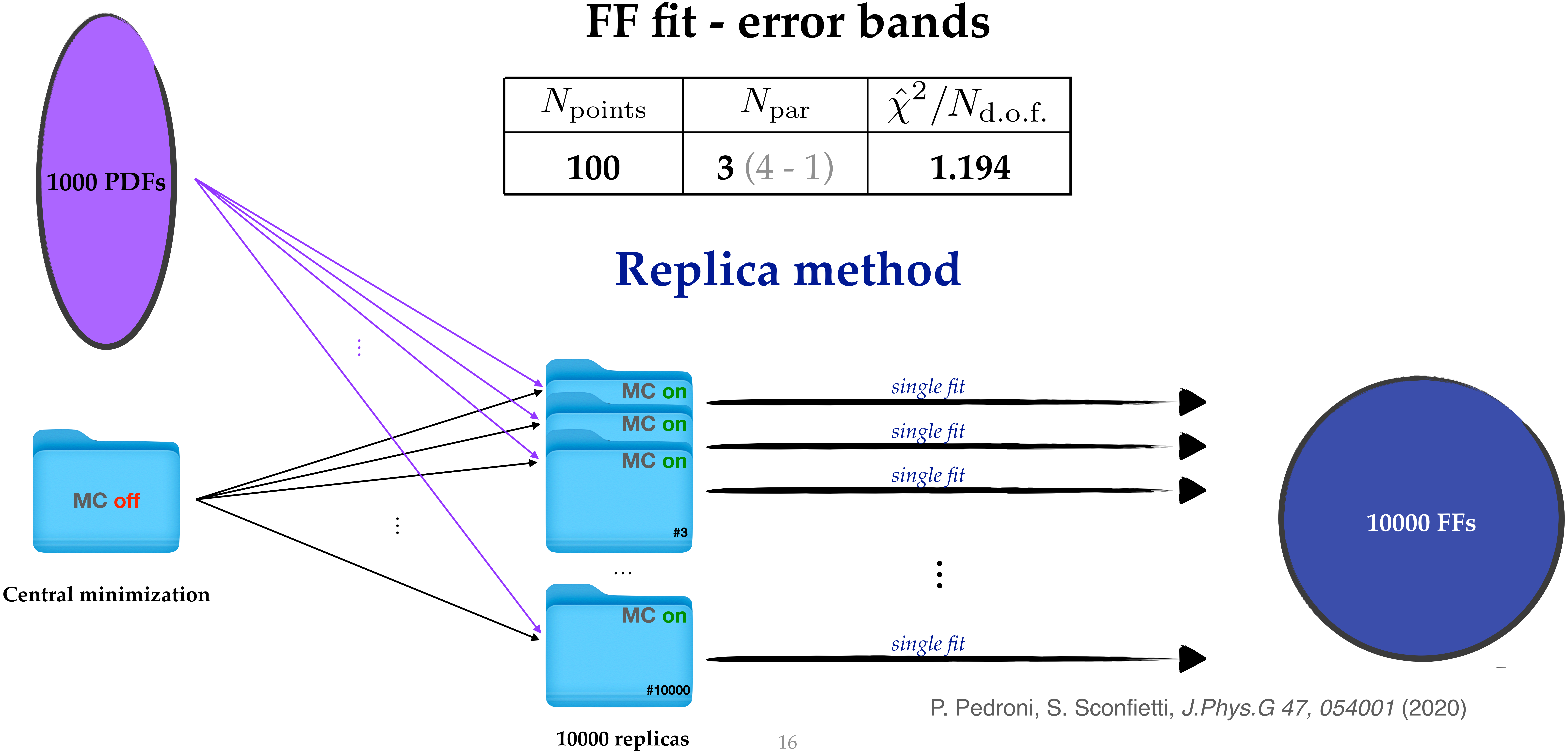
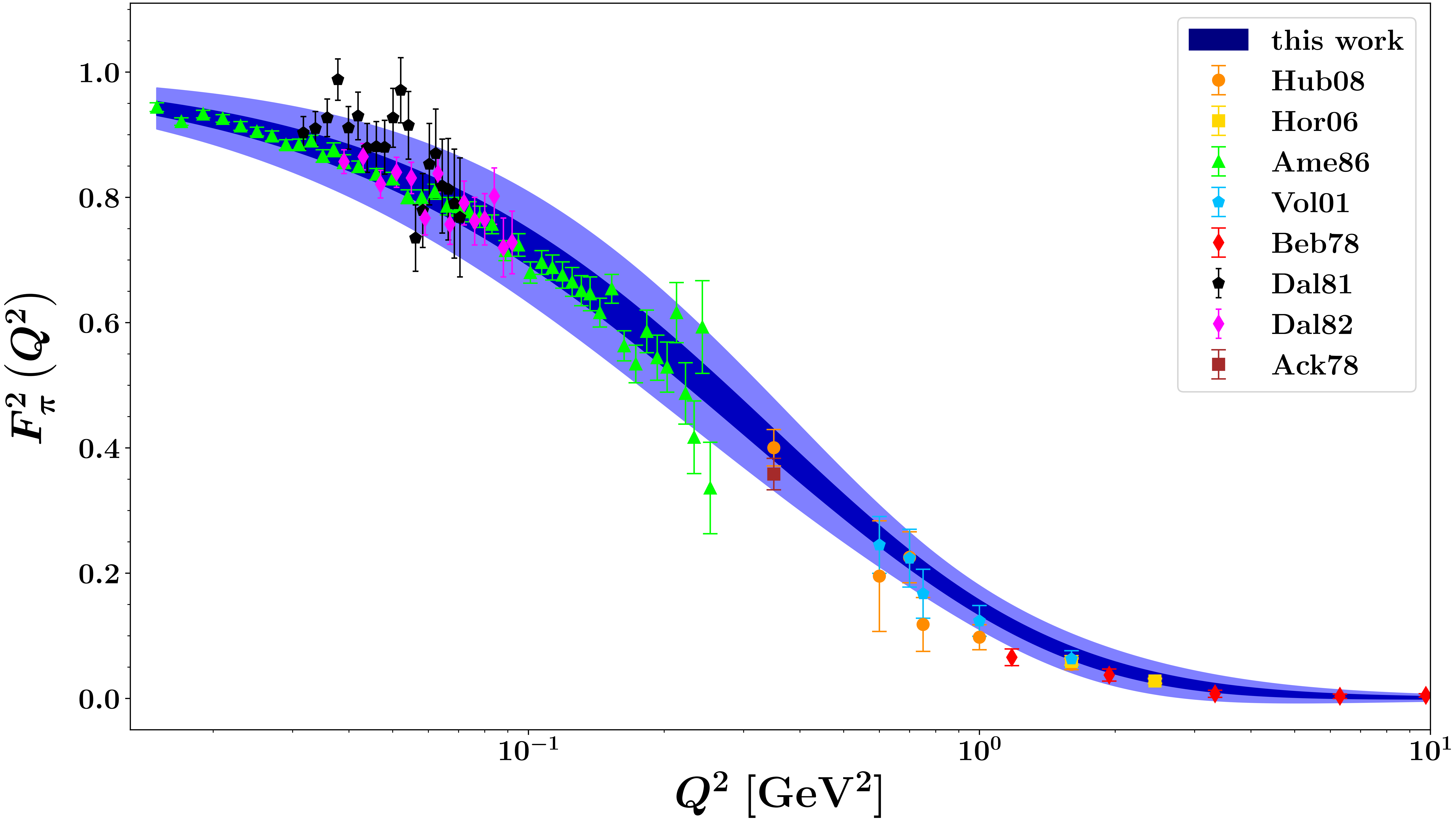


P. Pedroni, S. Sconfietti, *J.Phys.G 47, 054001* (2020)

Fit results

Pion FF fit



Outlook

“State of the art of the model”

Collinear PDF fitted

$$f_{1,\pi}\left(x\right)$$

Electromagnetic Form Factor fitted

$$F_1\left(\Delta\right)$$

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Outlook

"State of the art of the model"

Collinear PDF fitted

$$f_{1,\pi}\left(x\right)$$

Electromagnetic Form Factor fitted

$$F_1\left(\Delta\right)$$

$$f_{1,\pi}(x, \vec{k}_\perp)$$

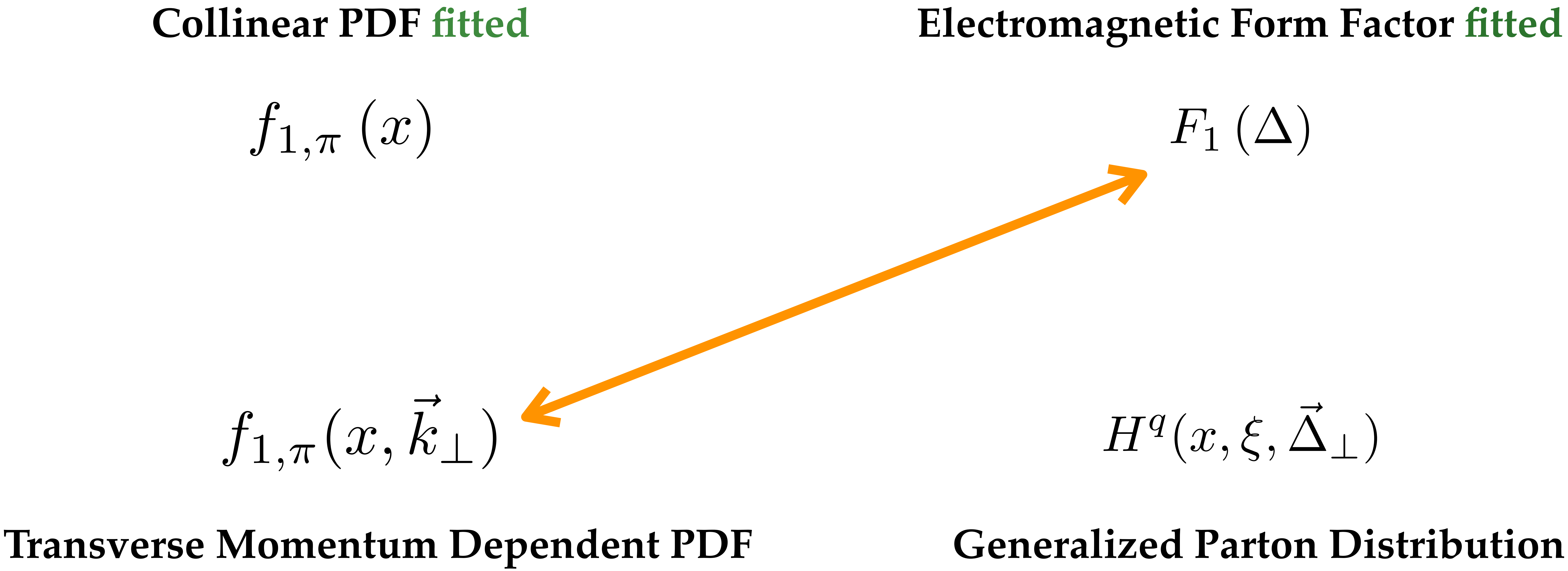
Transverse Momentum Dependent PDF

$$H^q(x, \xi, \vec{\Delta}_\perp)$$

Generalized Parton Distribution

Outlook

"State of the art of the model"



Collinear PDF fitted

$$f_{1,\pi}\left(x\right)$$

Electromagnetic Form Factor fitted

$$F_1\left(\Delta\right)$$

$$f_{1,\pi}(x,\vec{k}_\perp)$$

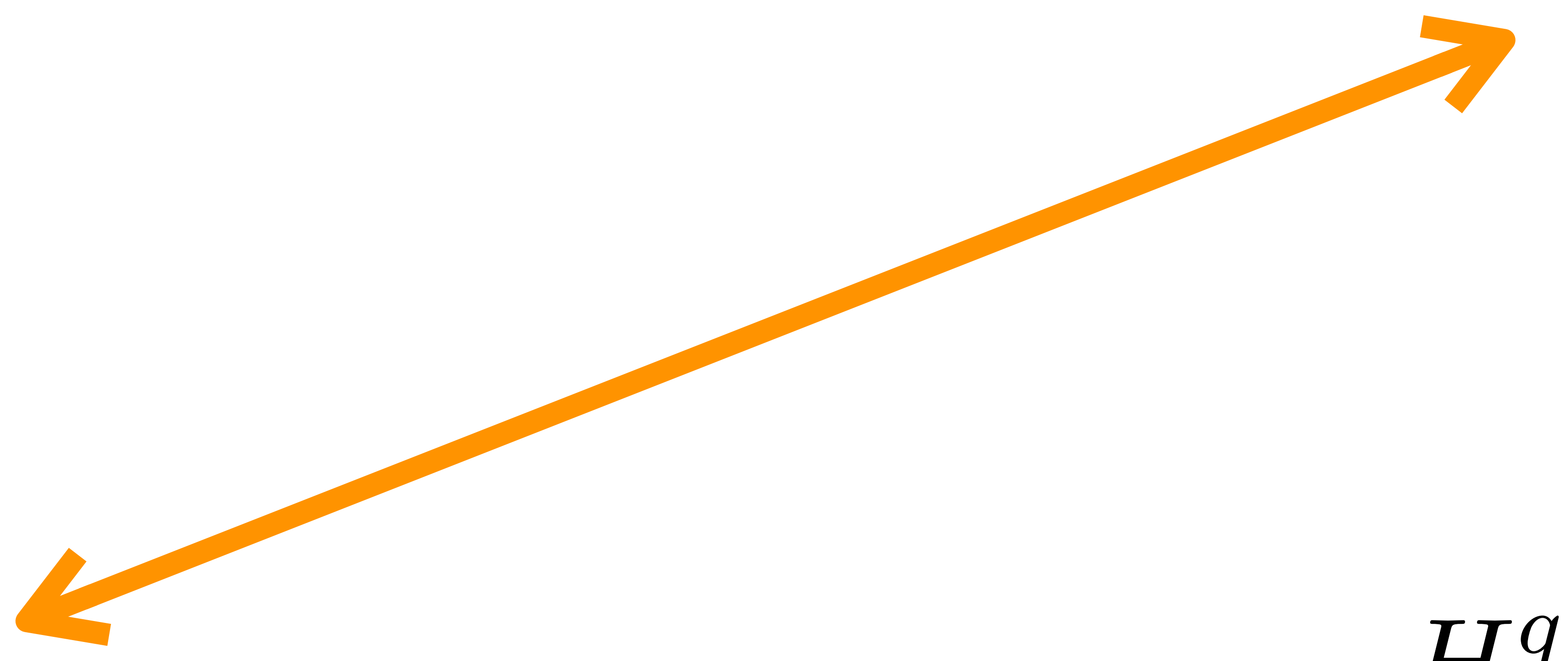
Transverse Momentum Dependent PDF

$$H^q(x,\xi,\vec{\Delta}_\perp)$$

Generalized Parton Distribution

Outlook

“State of the art of the model”



Collinear PDF fitted

$$f_{1,\pi}\left(x\right)$$

Nanga Parbat $f_{1,\pi}(x,\vec{k}_\perp)$

Phys. Rev. D 107(2023) 1, 014014

Transverse Momentum Dependent PDF

Electromagnetic Form Factor fitted

$$F_1\left(\Delta\right)$$

$$H^q(x,\xi,\vec{\Delta}_\perp)$$

Generalized Parton Distribution

Outlook

“State of the art of the model”

Collinear PDF fitted

Electromagnetic Form Factor fitted

$$\begin{tikzcd}
f_{1,\pi}\left(x\right) \arrow[dr] & F_1\left(\Delta\right) \arrow[d] \\
f_{1,\pi}(x, \vec{k}_\perp) \arrow[r] & H^q(x, \xi, \vec{\Delta}_\perp)
\end{tikzcd}$$

Transverse Momentum Dependent PDF

Generalized Parton Distribution

Outlook

"State of the art of the model"

Collinear PDF fitted

$f_{1,\pi}\left(x\right)$

Electromagnetic Form Factor fitted

$F_1\left(\Delta\right)$

$f_{1,\pi}(x,\vec{k}_\perp)$

Transverse Momentum Dependent PDF

$H^q(x,\xi,\vec{\Delta}_\perp)$

Generalized Parton Distribution