THEORY PERSPECTIVES ON ELECTROMAGNETIC HADRON STRUCTURE

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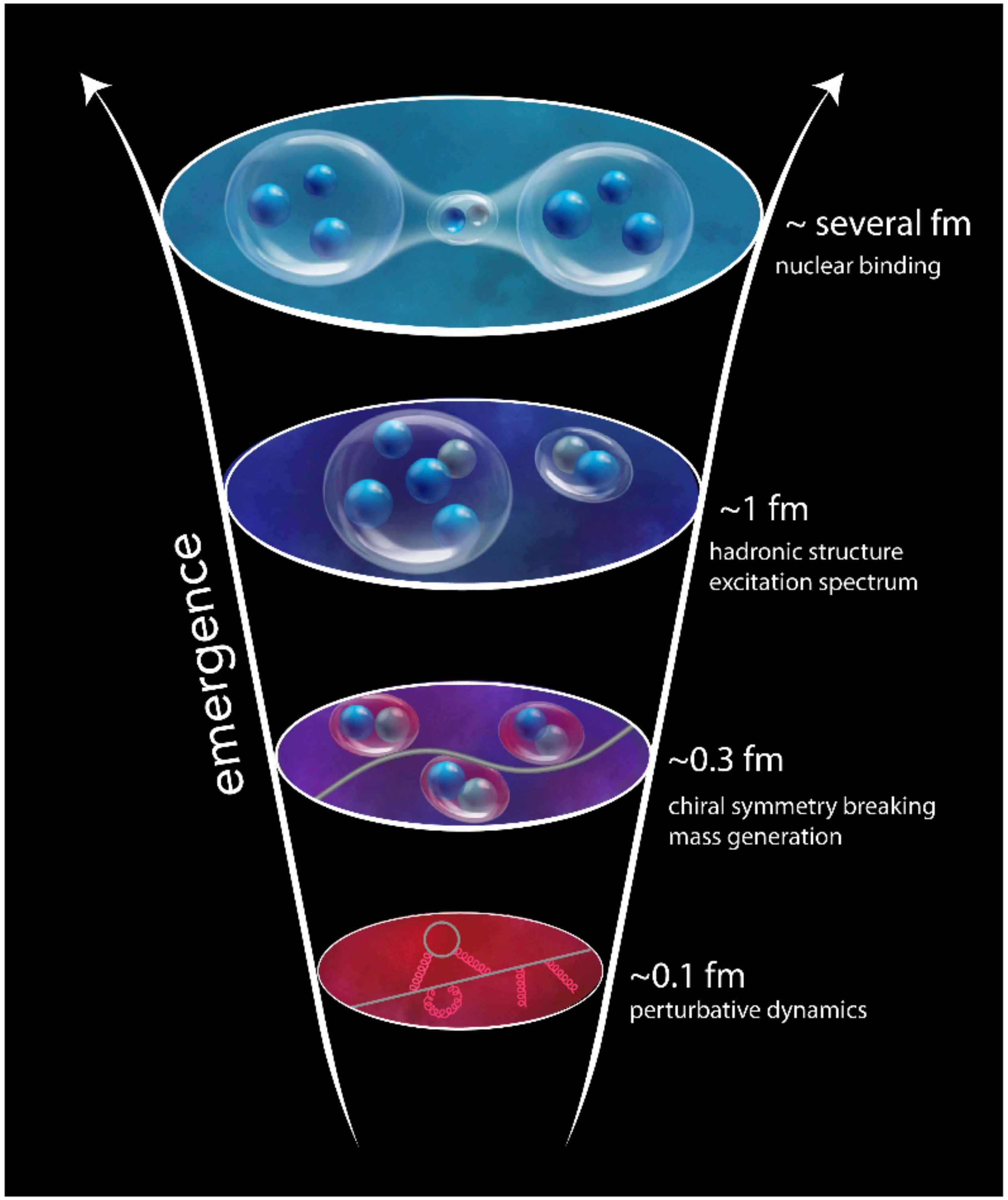
Emergent phenomena in QCD

“the whole is more than the sum of its parts”

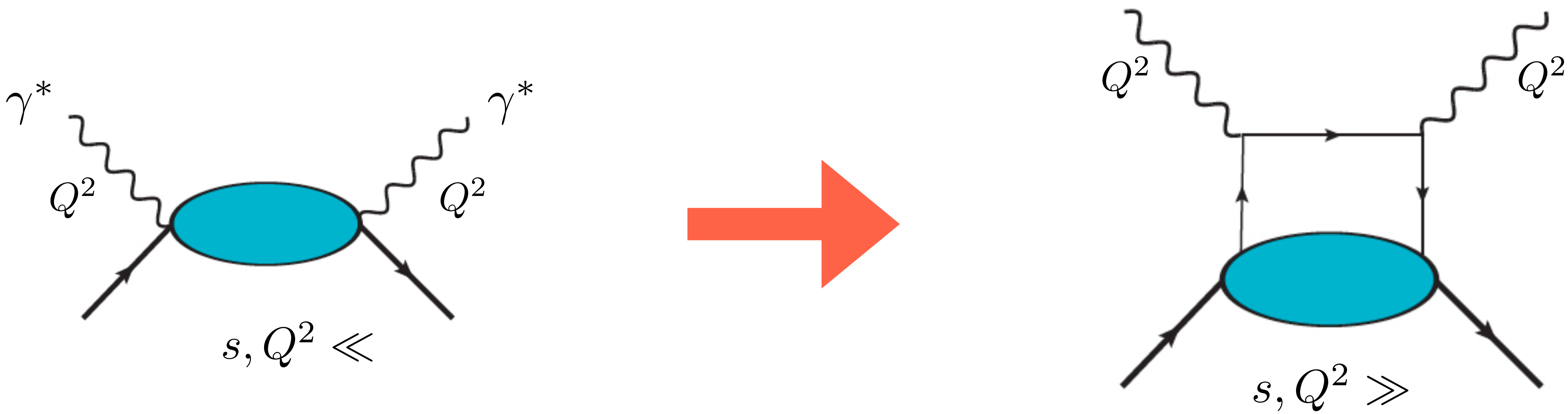
``What proton is depends on how you look at it, or rather on how hard you hit it''

A. Cooper-Sarkar, CERN Courier, June, 2019



Two-scale processes:
length resolution scale
soft momentum scale to probe the emergent regimes at different scales

VVCS generalized pol. DIS parton distributions



Nucleon structure at long distance *Parton structure*

$$\gamma_0(Q^2) = \frac{1}{2\pi^2}\int_{\nu_0}^{\infty} \frac{\sigma_{TT}(\nu, Q^2)}{\nu^3}\, d\nu = \frac{e^2 4M^2}{\pi Q^6}\int_0^{x_0} dx\, x^2 \left[g_1(x, Q^2) - \frac{4M^2}{Q^2}\, x^2\, g_2(x, Q^2)\right]$$

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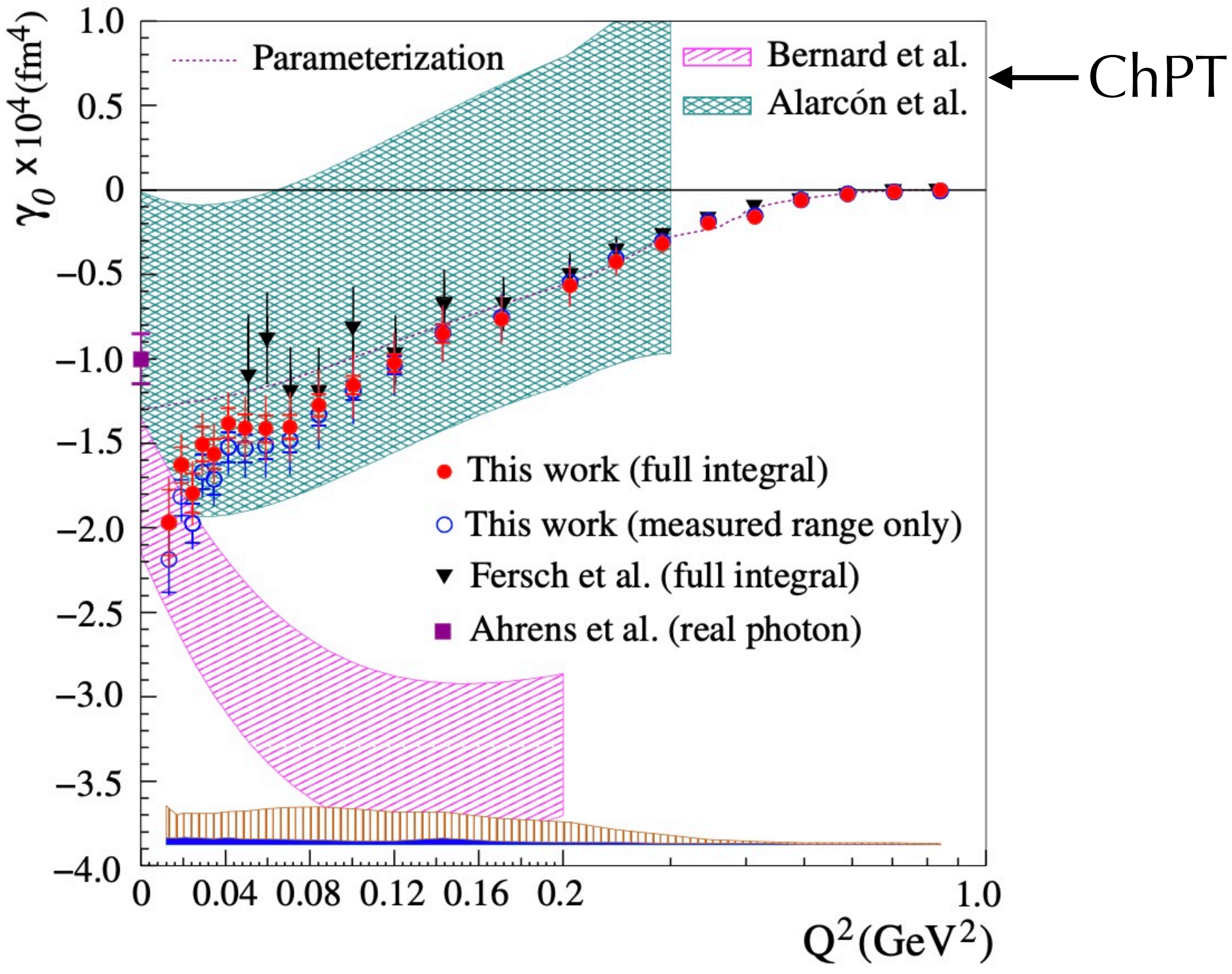

Hall A JLab Coll, Nature Phys. 18 (2022)

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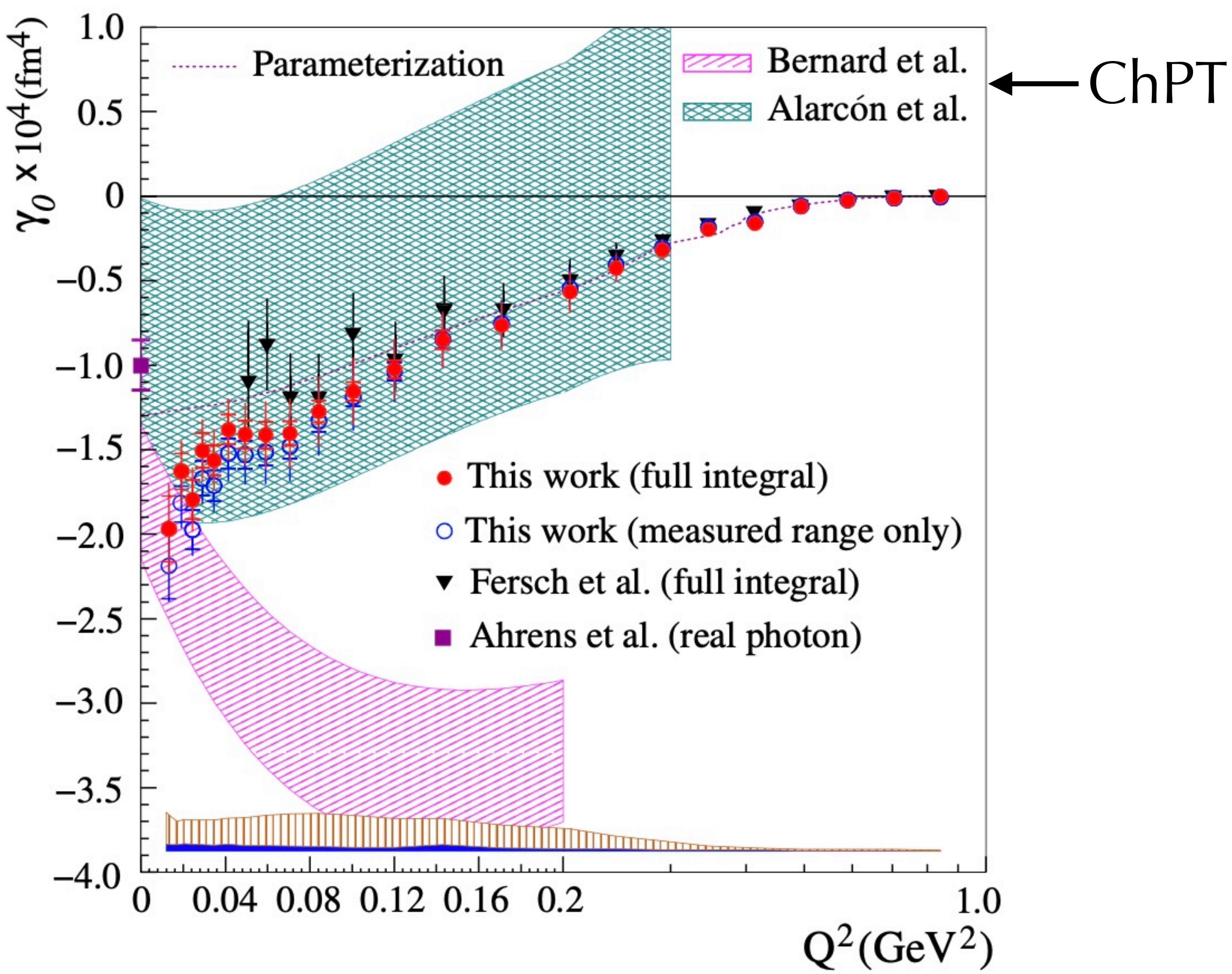
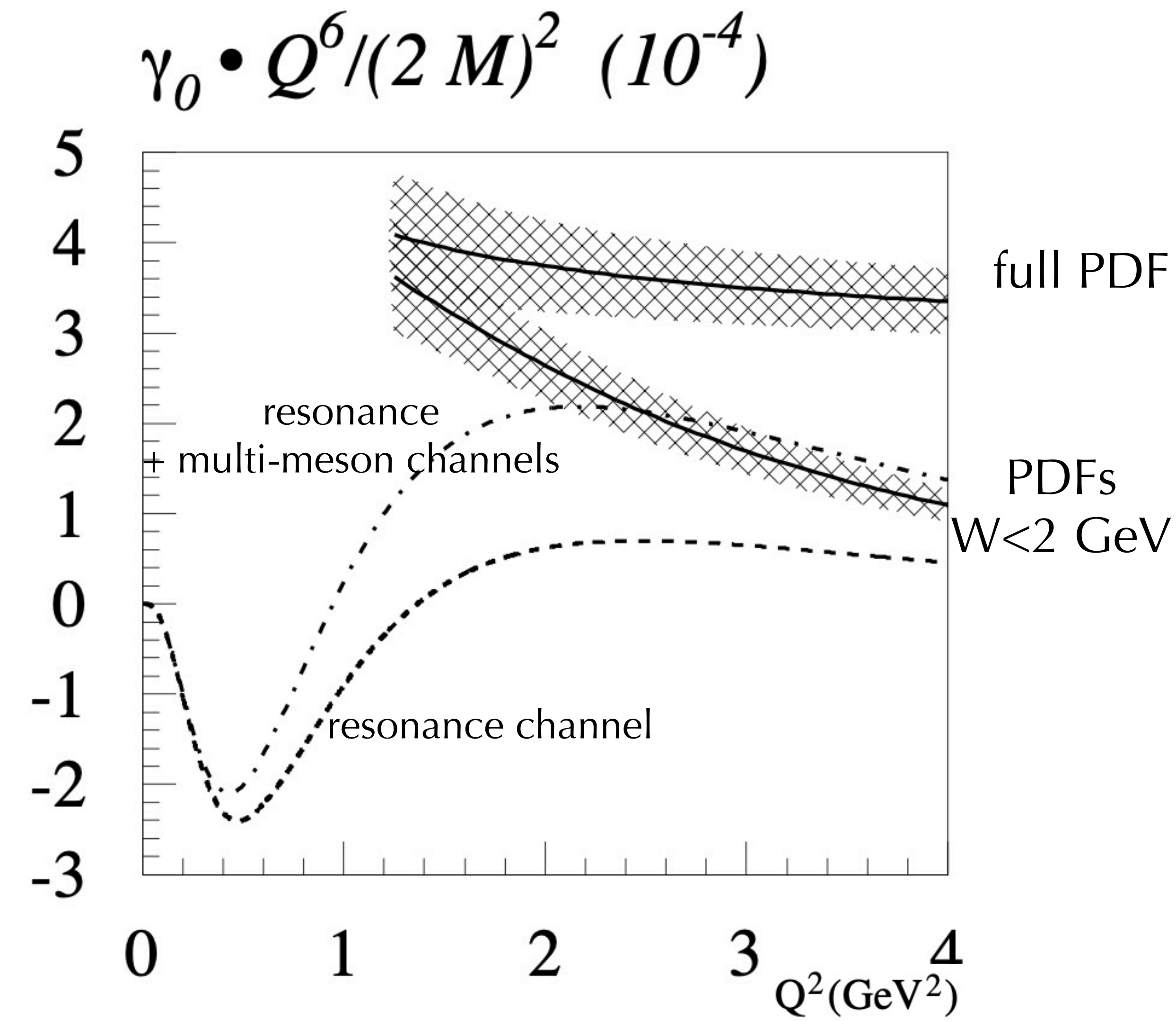
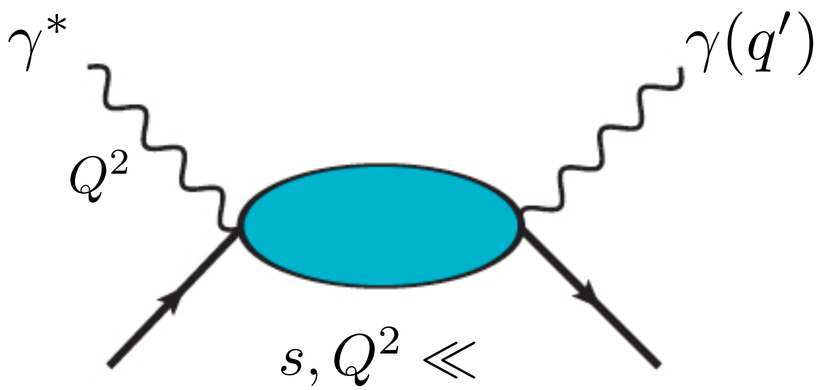


Hall A JLab Coll, Nature Phys. 18 (2022)



Drechsel, BP, Vanderhaeghen, Phys. Rep. 383 (2002)

VCS generalized pol.



$s, Q^2 \ll$

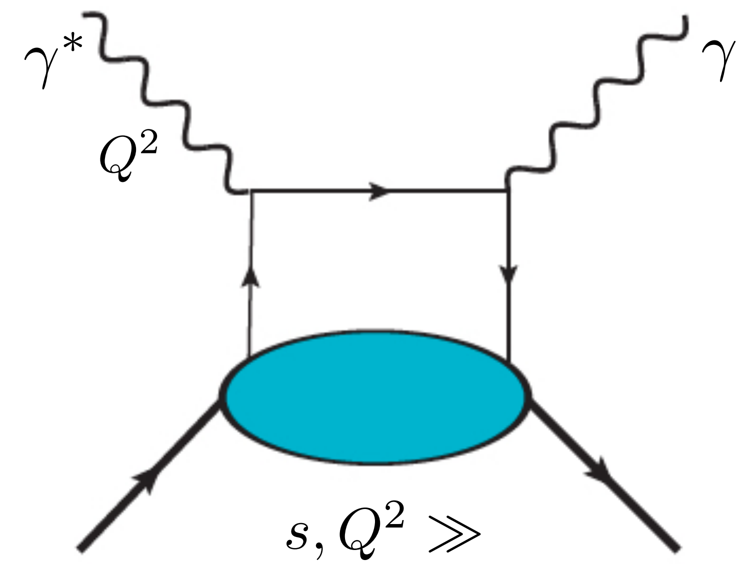
$q' \ll$

electron scattering by a target which is in constant electric and magnetic fields

$\downarrow \; q_\perp \to b_\perp$

Spatial distribution of electric and magnetic polarization density

DVCS
generalized parton distributions



$s, Q^2 \gg$

$-t \ll$

EMT form factors

$\downarrow \; -t \to b_\perp$

Spatial distribution of mechanical properties and gluons fields

Real Compton Scattering at low energies

Measure of the strength of induced polarizations: 2 scalar polarizabilities + 4 spin polarizabilities

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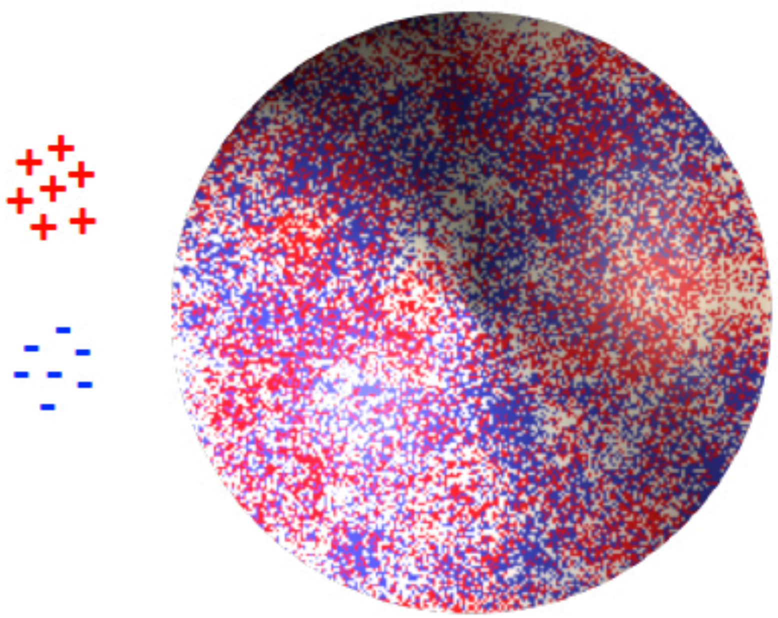

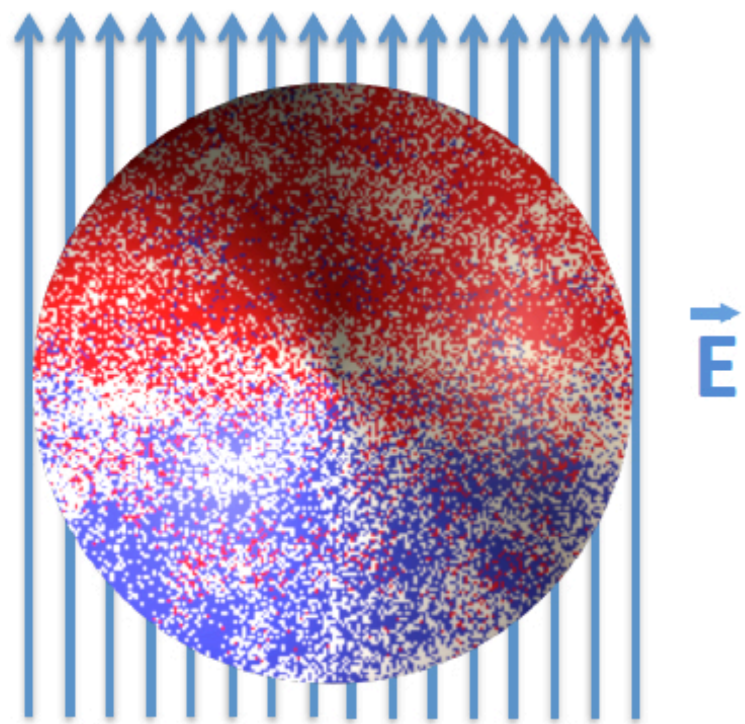

$$\vec{D}_E \sim \alpha_{\mathrm{E1}}\, \vec{E}$$

Unlike atoms,
it is not proportional to volume

$$V \sim \langle r_p \rangle^3 \approx 0.6\,\mathrm{fm}^3$$

$$\alpha_{\mathrm{E1}} \approx 10^{-4}\, V_p$$

much ``stiffer'' than hydrogen!

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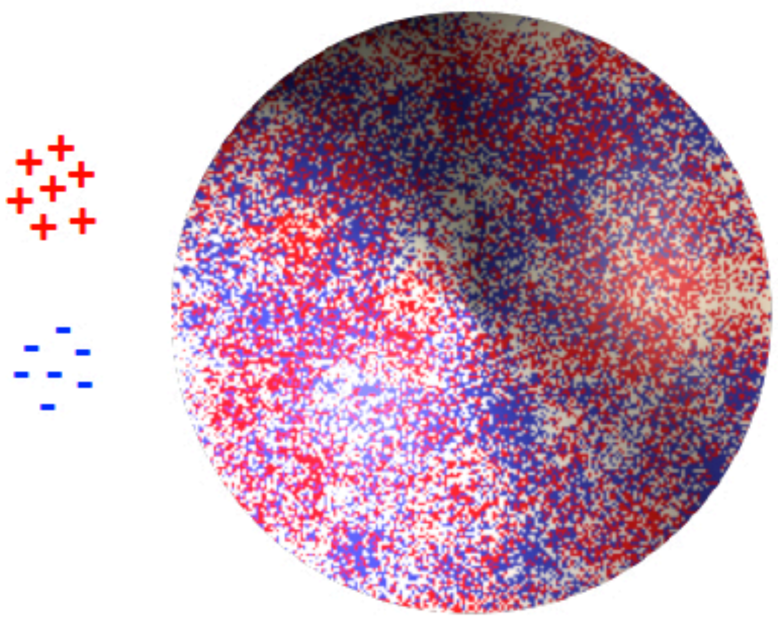
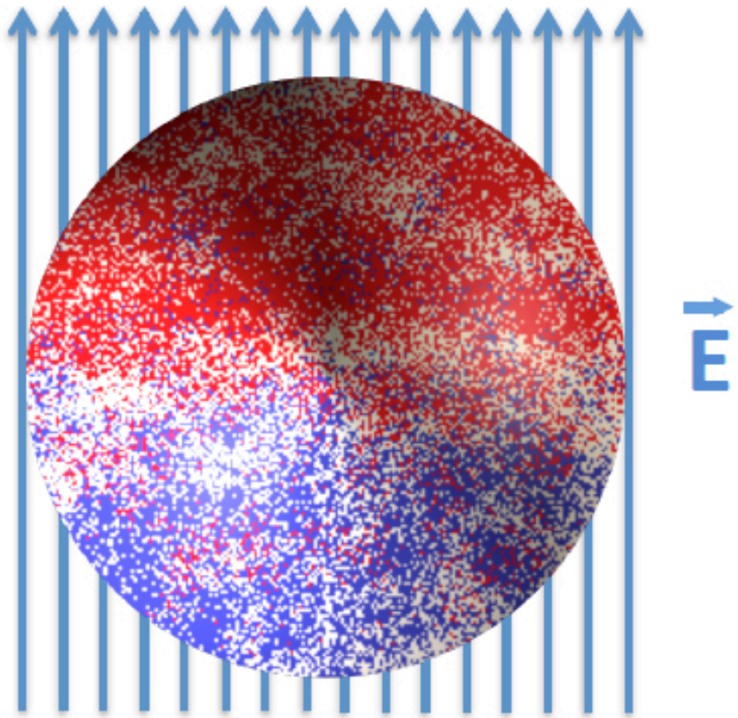
Measure of the strength of induced polarizations: 2 scalar polarizabilities + 4 spin polarizabilities





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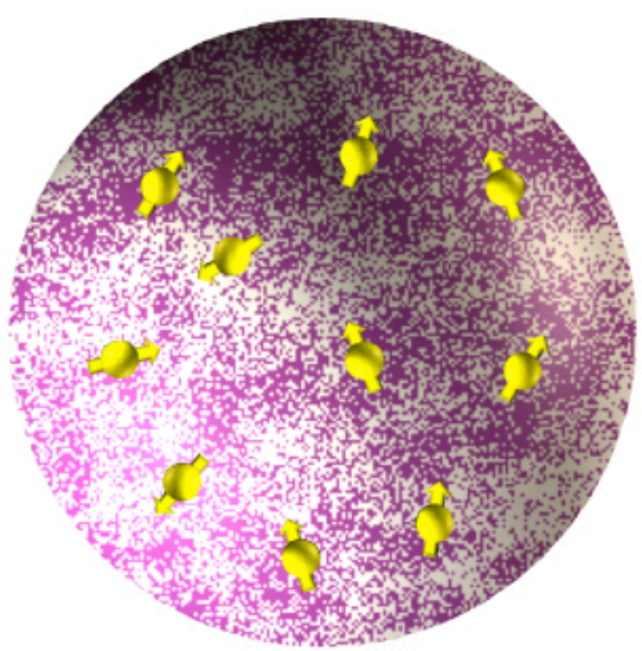
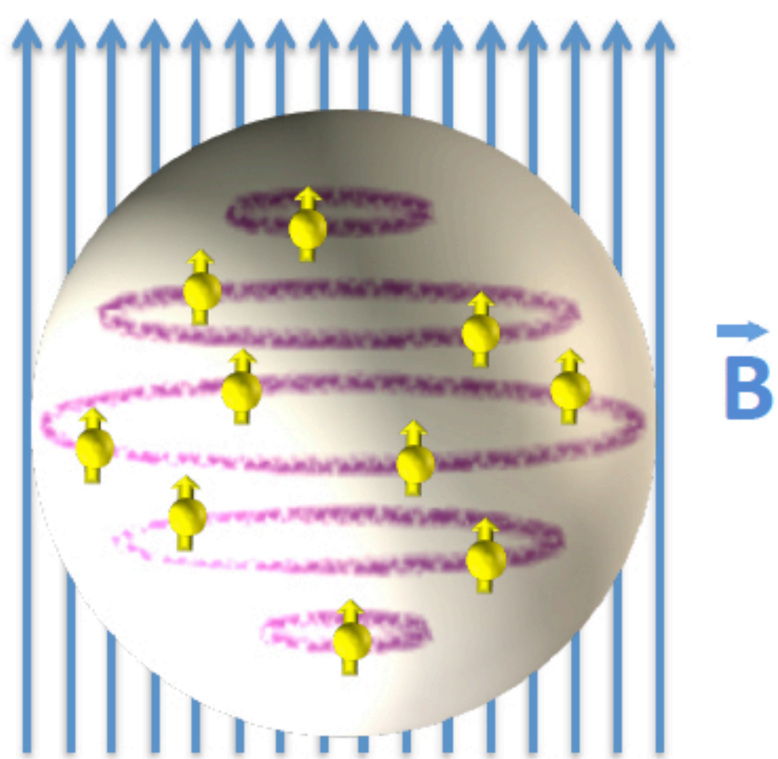


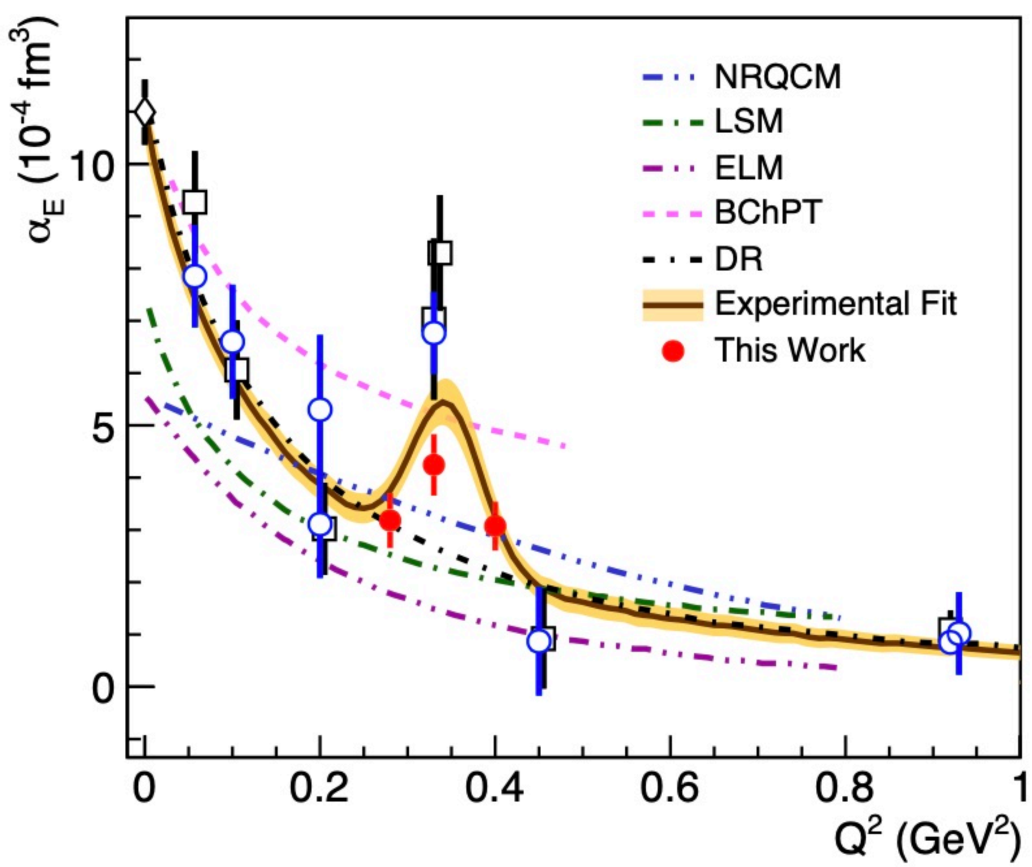


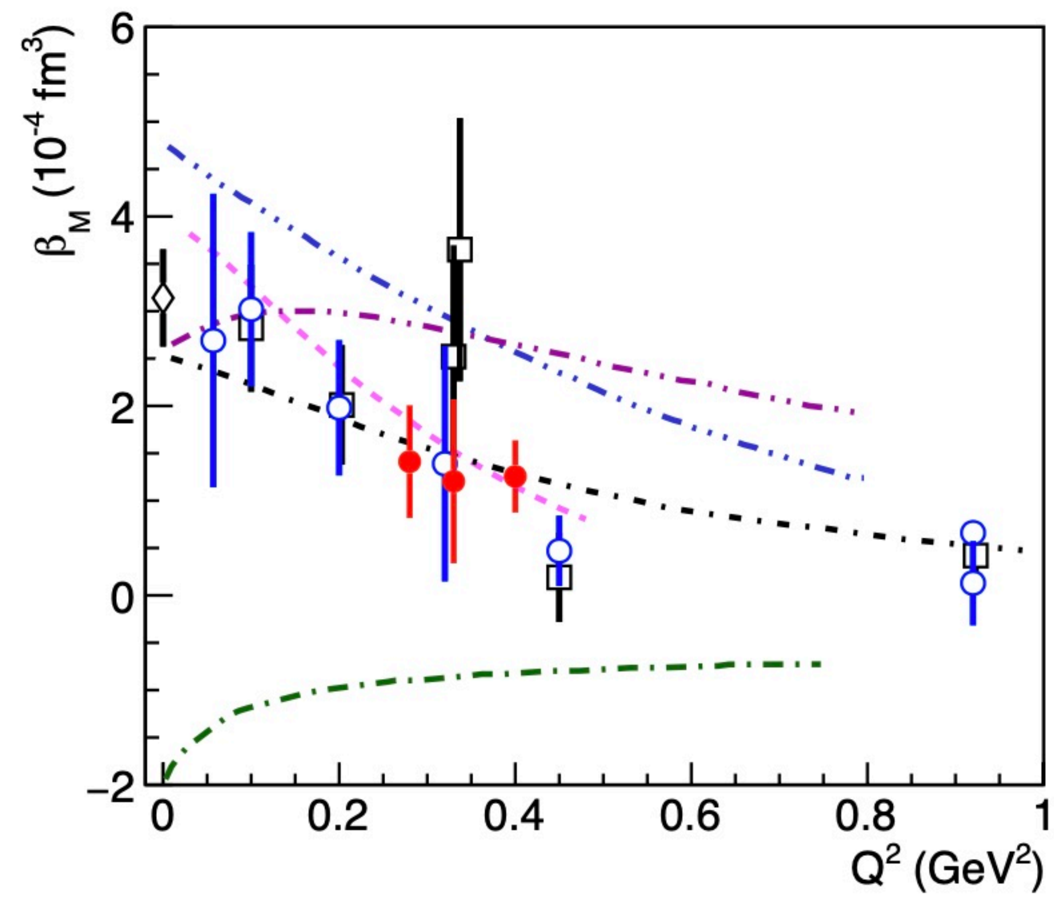
$$\vec{D}_M \sim \beta_{\mathrm{M1}}\,\vec{B}$$

$\beta^{\mathrm{para}}_{\mathrm{M1}} > 0$ proton spin aligns with external field

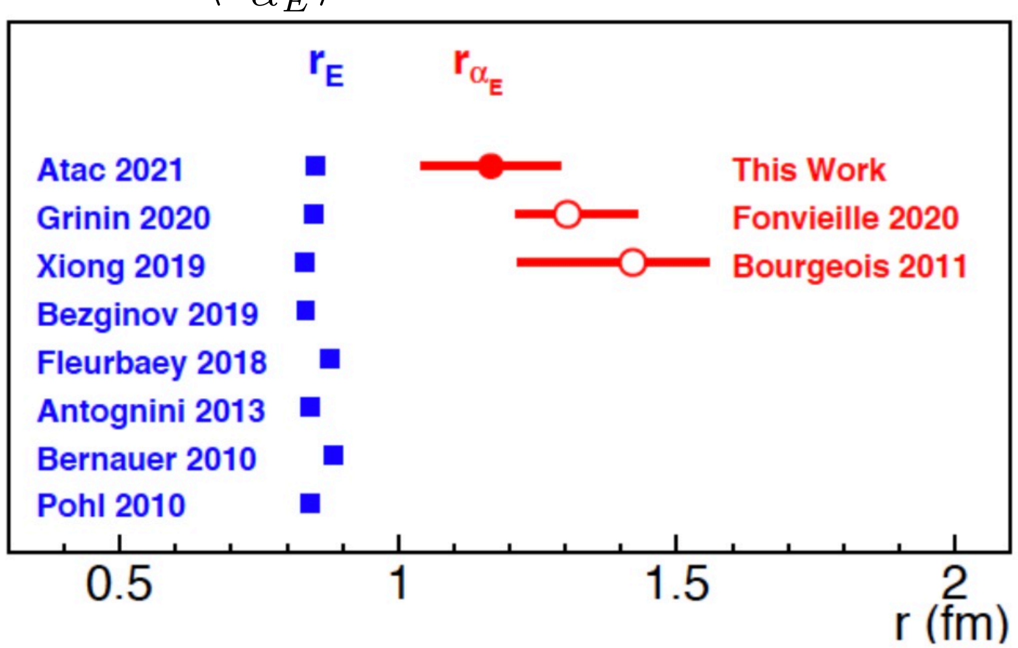
$\beta^{\mathrm{dia}}_{\mathrm{M1}} < 0$ induced current of pion cloud generates field opposite to the external one

Scalar VCS Generalised Polarizabilities

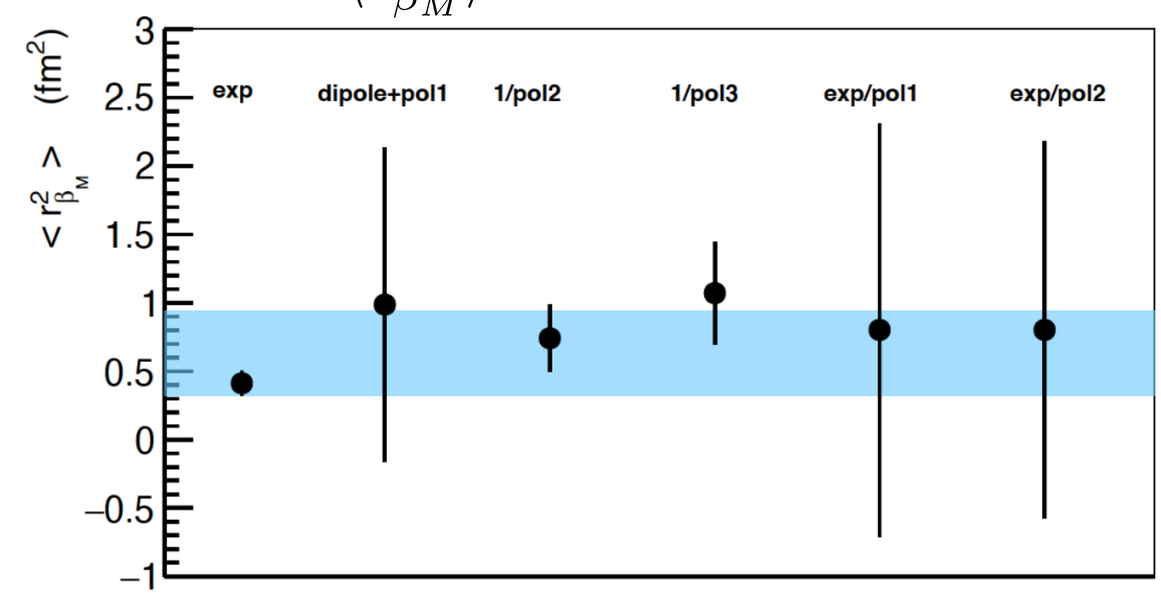




$$\langle r^2_{\alpha_E} \rangle = 1.36 \pm 0.29 \text{ fm}^2$$



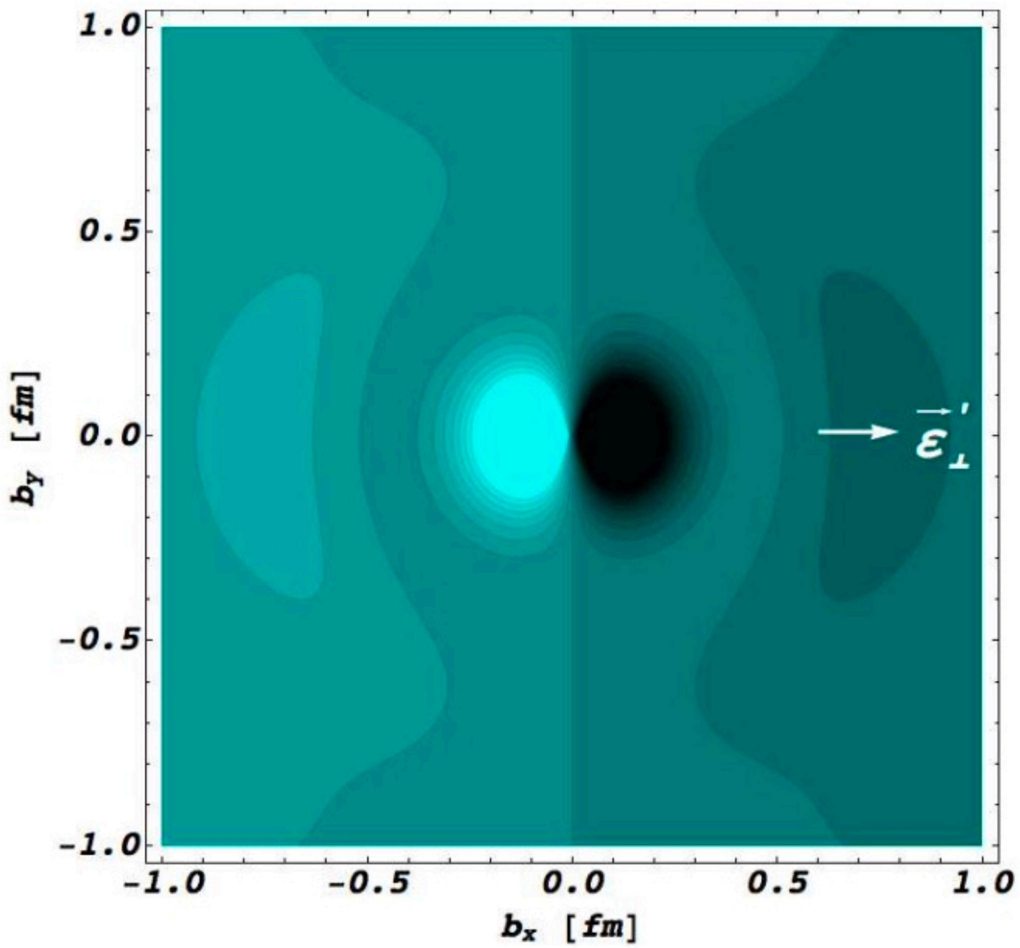
$$\langle r^2_{\beta_M} \rangle = 0.63 \pm 0.31 \text{ fm}^2$$



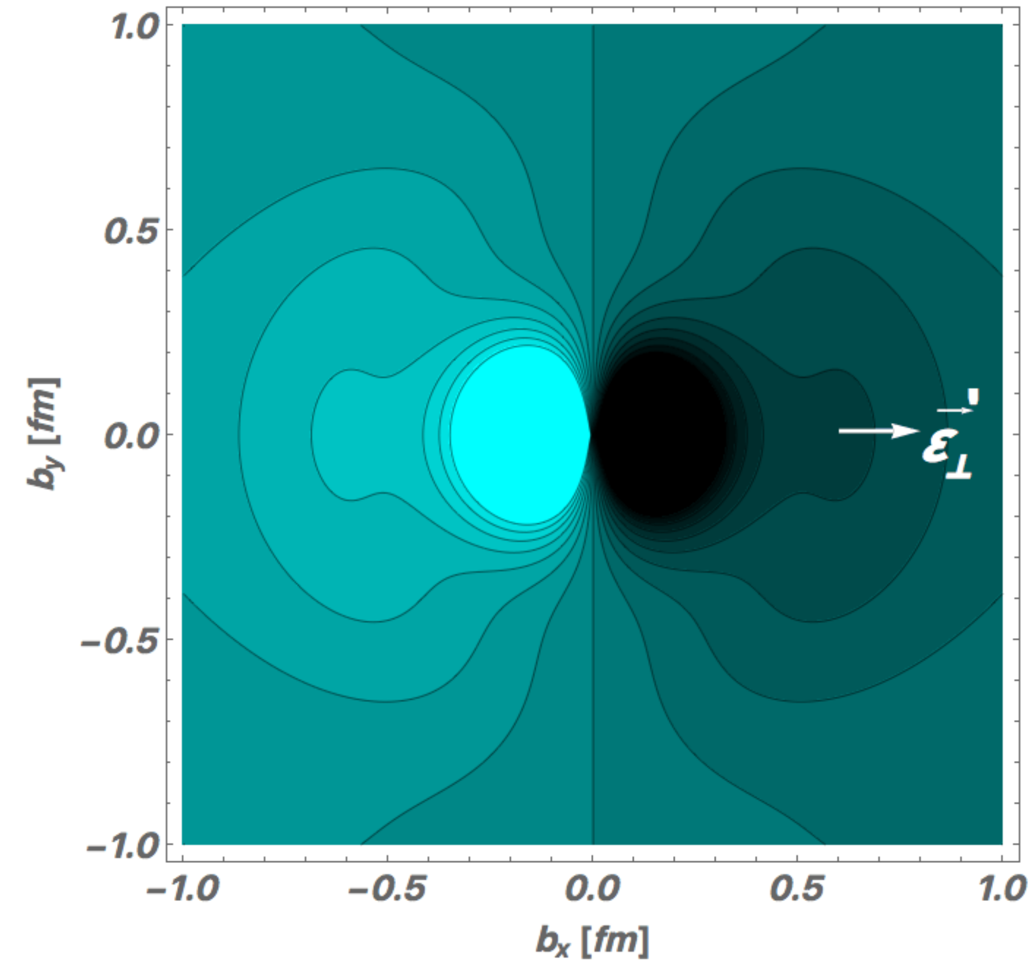
Li, Sparveris, BP, et al, Nature 611 (2022)

Spatial density of induced polarizations

Smooth dipole fit



Including new data



light (dark) regions ⟶ larger (smaller) values

Frame with fast moving proton
in the longitudinal direction and $Q^2 = q_\perp^2$

$\vec{q}_\perp \overset{\text{FT}}{\longleftrightarrow} \vec{b}_\perp$ true probabilistic interpretation!

$\vec{E} \sim i q'^0 \vec{\epsilon}'_\perp$ quasi-static electric field ⟶ induced polarization depending on scalar and spin GPs

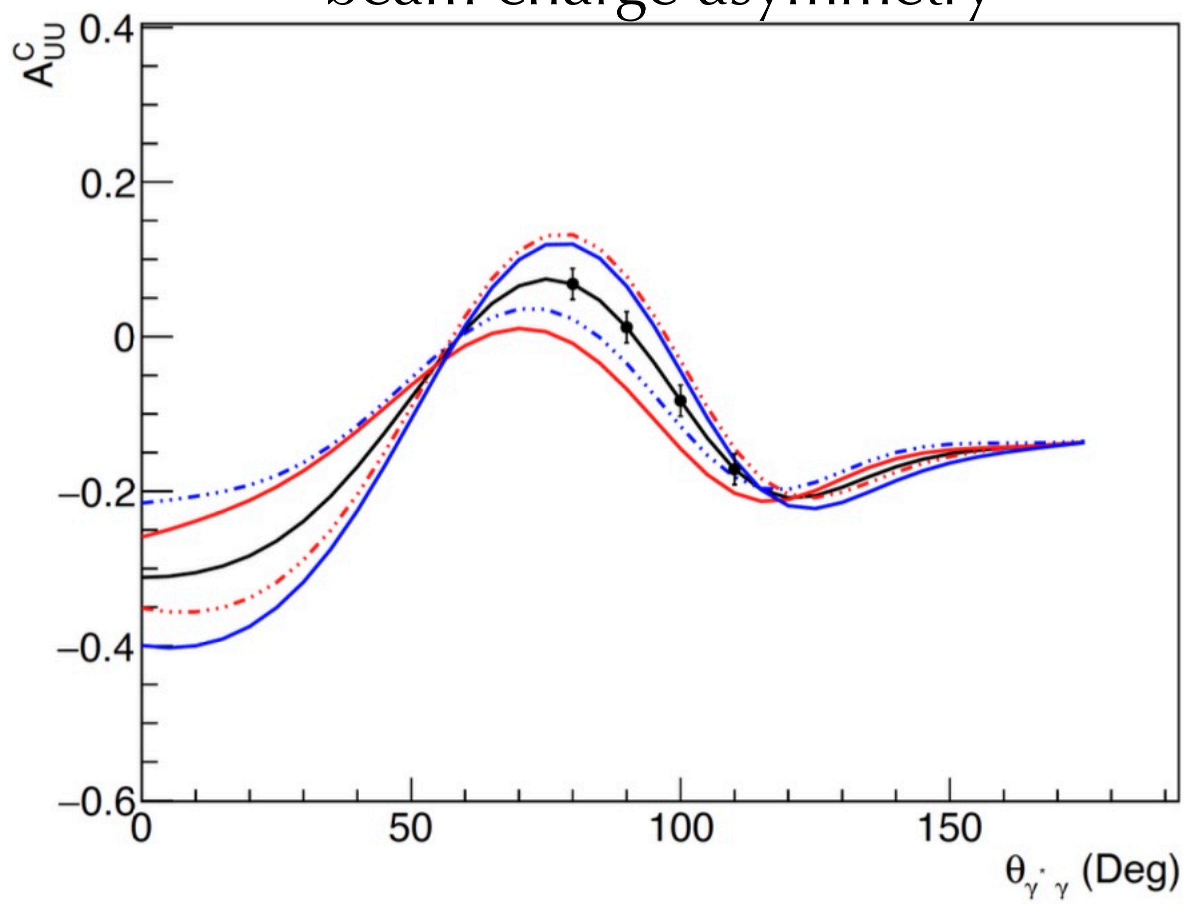
Gorchtein, Lorcé, BP, Vanderhaeghen, PRL104 (2010) 112001

Moving forward….positron beam

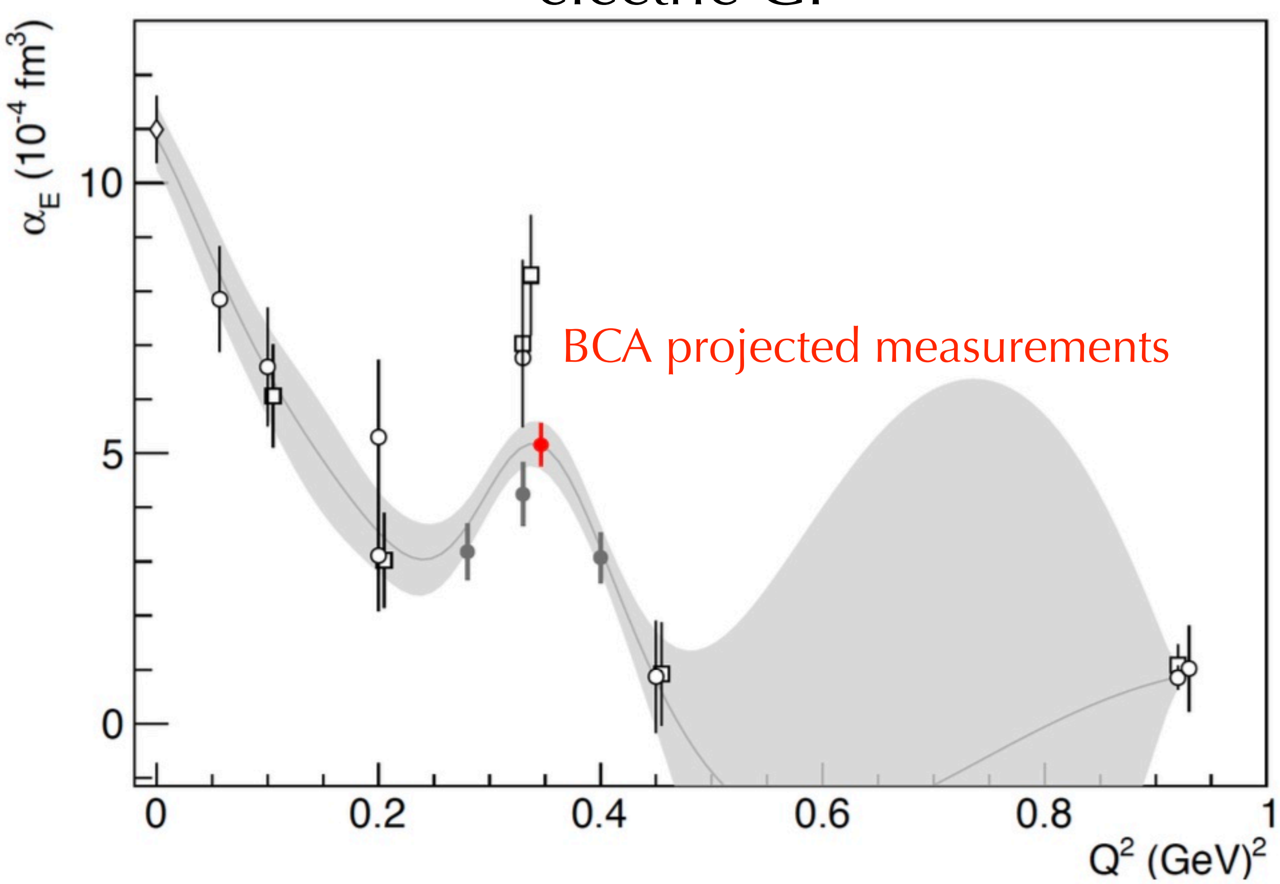
- Positrons allow for an independent path to access experimentally the GPs

-Targeted measurements in the area of interest, higher & lower in Q^2

beam charge asymmetry



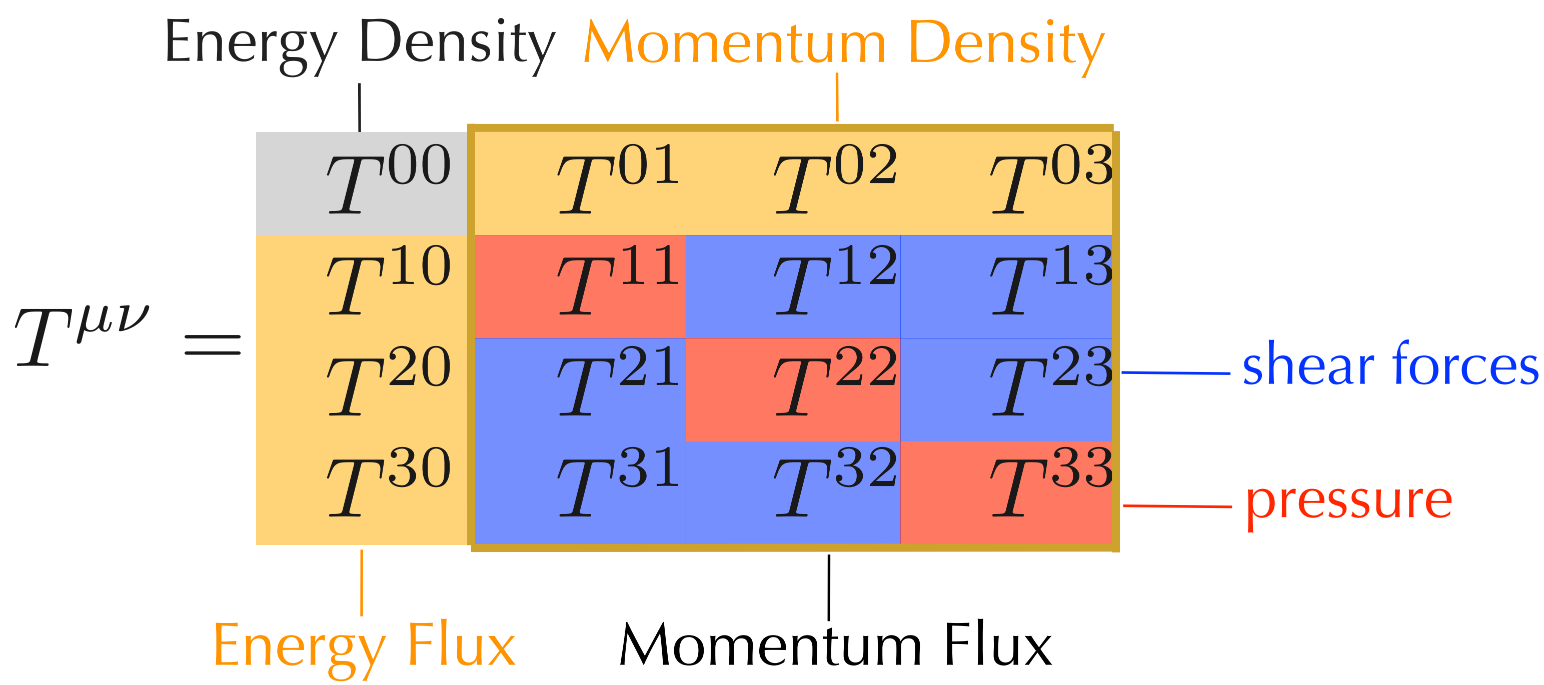
electric GP



LOI JLab PAC 51, "Measurement of the Generalized Polarizabilities of the Proton with positron and polarized electron beams"

Accardi, B.P., Vanderhaeghen, et al., "An experimental program with high duty-cycle polarized and unpolarized positron beam at Jefferson Lab, Eur.Phys.J.A 57 (2021) 8, 26

High Q² and low t: EMT form factors from GPDs



$$\langle p', s' | T_a^{\mu\nu}(0) | p, s \rangle = \bar{u}(p', s')\, \Gamma_a^{\mu\nu}(P, \Delta)\, u(p, s)$$

$$\Gamma^{\mu\nu}(P,\Delta) = \frac{P^\mu P^\nu}{M_N} A_a(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} D_a(t) + M g^{\mu\nu} \bar{C}_a(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\lambda}\Delta_\lambda}{M_N} J_a(t) - \frac{P^{[\mu} i\sigma^{\nu]\lambda}\Delta_\lambda}{M_N} S_a(t)$$

The first 4 FFs associate with the symmetric (Belinfante) part of the EMT

The antisymmetric part (spin contribution) only for quark: $S_G(t) = 0$

Total EMT is not renormalized, quark and gluon contributions require renormalization

$A_a(0)$ longitudinal momentum fraction carried by quarks and gluons

$$A(0) = \sum_q A_q(0) + A_G(0) = 1$$

$$A_a(0) = \int_0^1 x\, f_1^a\, dx \quad \text{input from DIS}$$

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$J_a(0)$ parton angular momentum

$$J(0) = \sum_q J_q(0) + J_G(0) = \frac{1}{2}$$

$$J^{q,g}(0) = \tfrac{1}{2} \int_{-1}^{1} dx\, x\, \left(H^{q,g}(x,\xi,0) + E^{q,g}(x,\xi,0)\right) \qquad \text{Ji's sum rule}$$

$J^{q,g}(t)$ FT in impact parameter space does not give AM density!

Lorcé, BP, Mantovani, PLB 776 (2018)

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$S_q(0)$ quark spin

$$\frac{1}{2}\Delta\Sigma = \sum_q S_q(0) \qquad\qquad S_q(0) = \frac{1}{2}\int_0^1 g_1(x)\, dx \quad \text{input from DIS}$$

$$L_q = J_q(0) - S_q(0) \qquad \text{quark kinetic OAM}$$

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$D(0)$ Druck (D) term $\quad D(0) = \sum_q D_q(0) + D_G(0) \qquad$ pressure distribution

Polyakov, PLB 555 (2003)

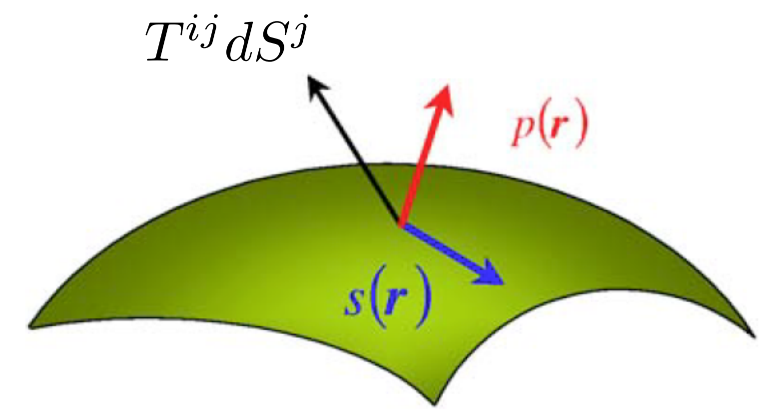
D(t) form factor from data

➡ Fourier transform in coordinate space

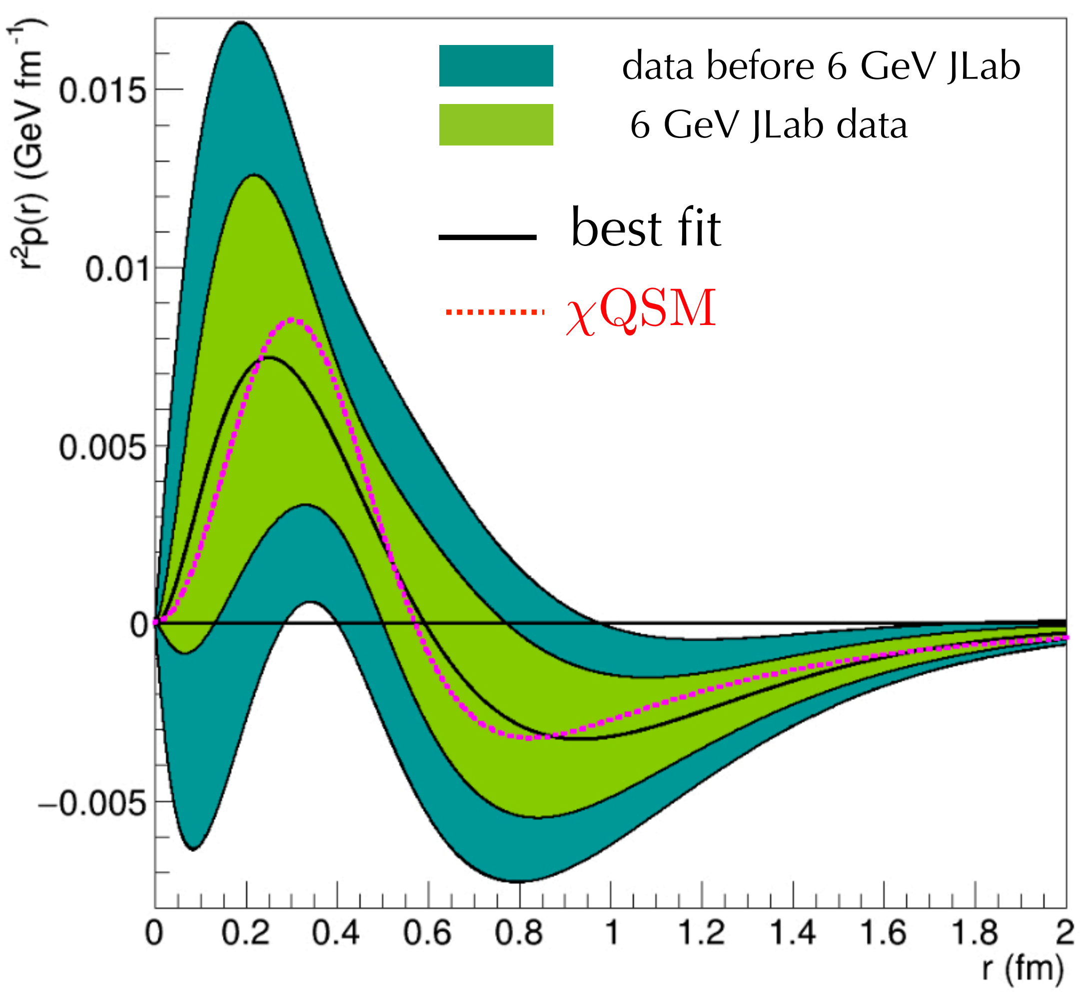
$$T_{ij}^{Q}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\,\delta_{ij}$$

shear forces ($s(\vec{r})$) — pressure ($p(\vec{r})$)

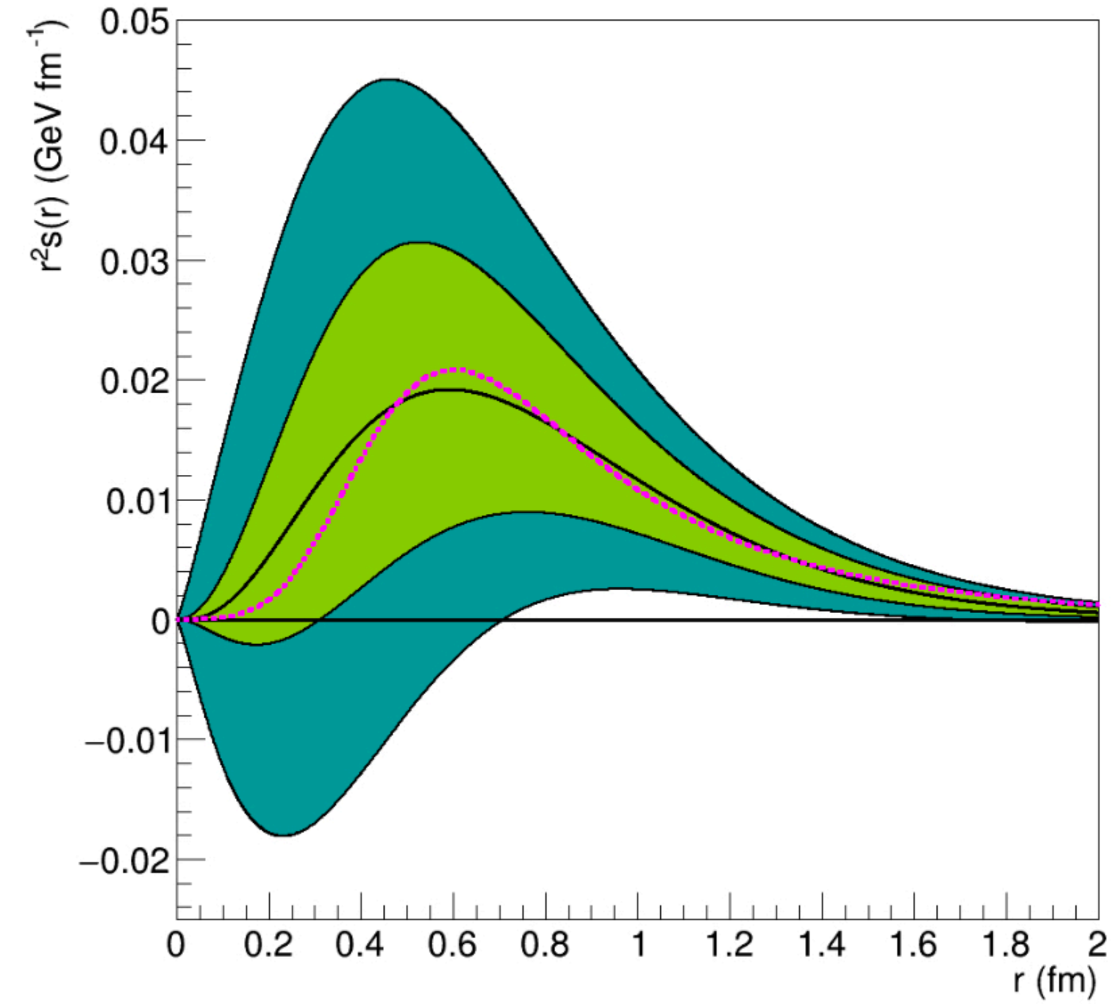
"mechanical properties" of nucleon



$$p(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}D(r)$$



$$s(r) = -\frac{1}{4M_N}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}D(r)$$

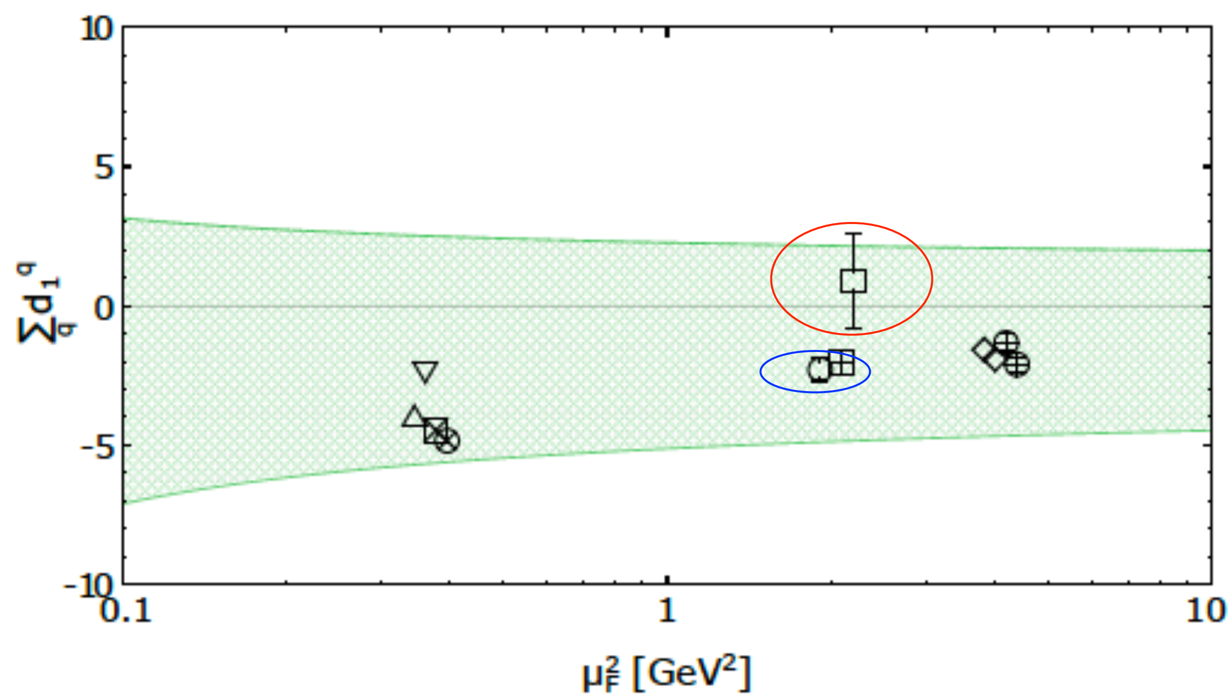


Girod, Elouadrhiri,Burkert, Nature 557 (2018) 7705; Polyakov, Schweitzer, IJMA 33 (2018) 1830025

Necessary to verify model assumptions in the exp extraction with more data coming from JLab, COMPASS and the future EIC, EIcC

Kumericki, Nature 570 (2019) 7759; Dutrieux et al, Eur. Phys. J. C81 (2021) 4

global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param., Girod et al.

CLAS data, with neural networks Kumericki

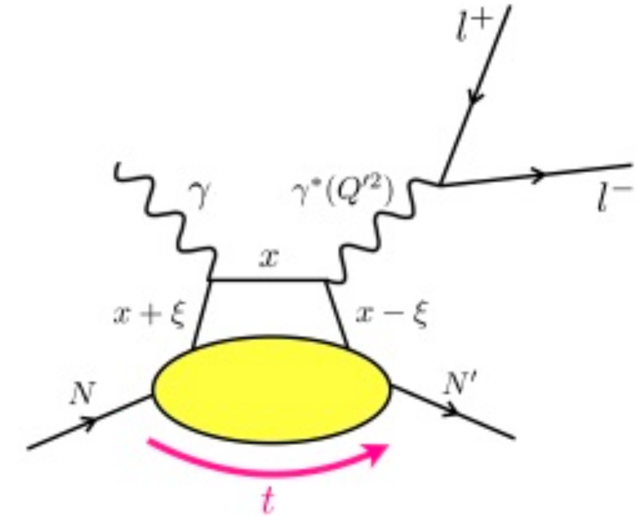
$$\sum_q d_1^q < 0$$

in all model calculations
for a stable proton

Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
□	0.88 ± 1.69	2.2	2	from experimental data
◇	-1.59	4	2	t-channel saturated model
	-1.92	4	2	t-channel saturated model
△	-4	0.36	3	χQSM
▽	-2.35	0.36	2	χQSM
⊠	-4.48	0.36	2	Skyrme model
⊞	-2.02	2	3	LFWF model
⊗	-4.85	0.36	2	χQSM
⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)

Timelike Compton scattering

Chatagnon et al. (CLAS12 Coll.), PRL127, 262501(2021)

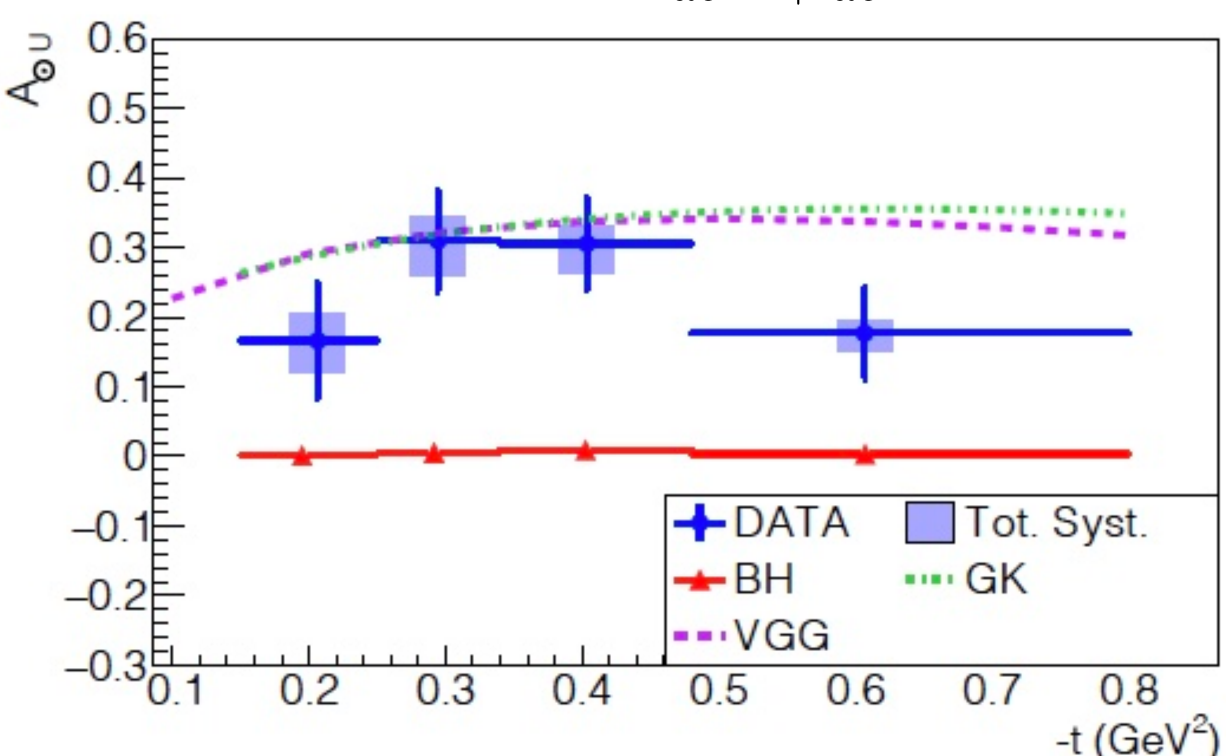


photon polarization asymmetry

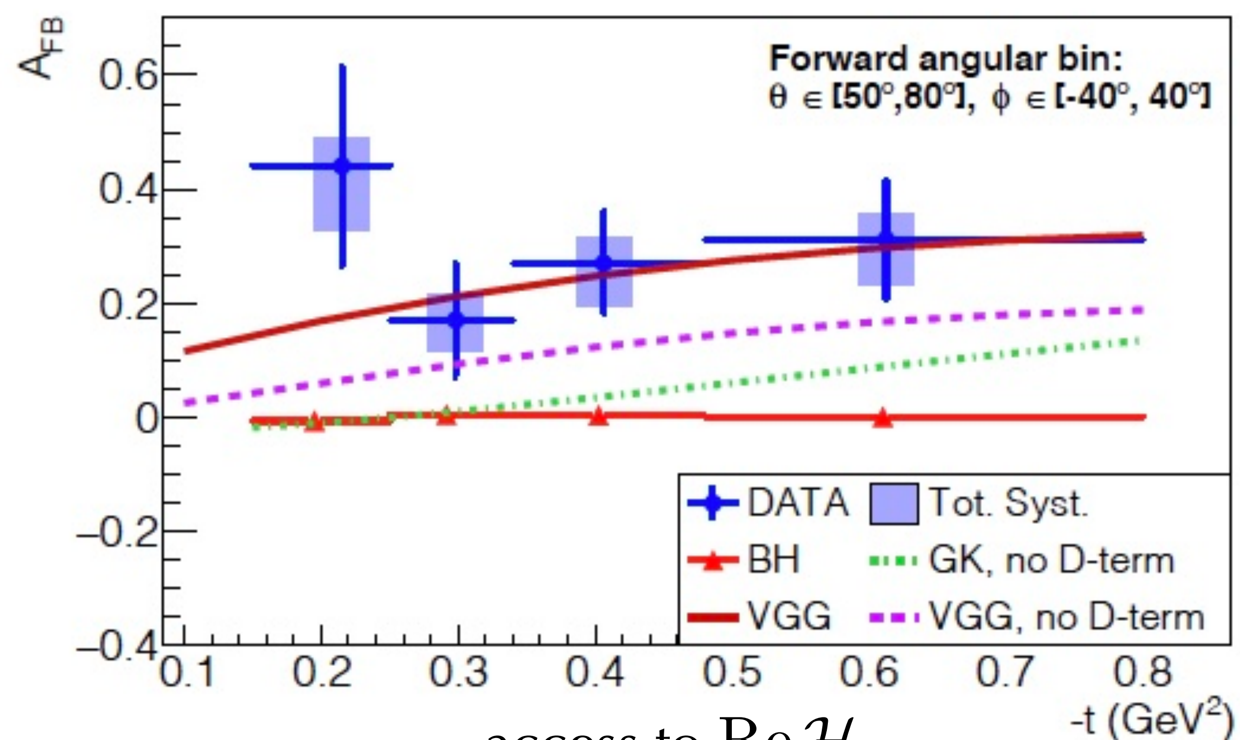
$$A_{\odot U} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$$

forward-backward asymmetry

$$A_{FB} = \frac{d\sigma(\theta,\phi) - d\sigma(180^\circ - \theta, 180^\circ + \phi)}{d\sigma(\theta,\phi) + d\sigma(180^\circ - \theta, 180^\circ + \phi)}$$



access to Im $\mathcal{H}$

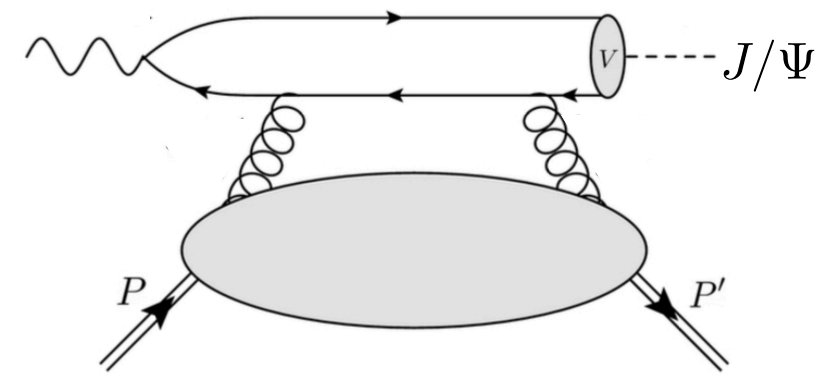


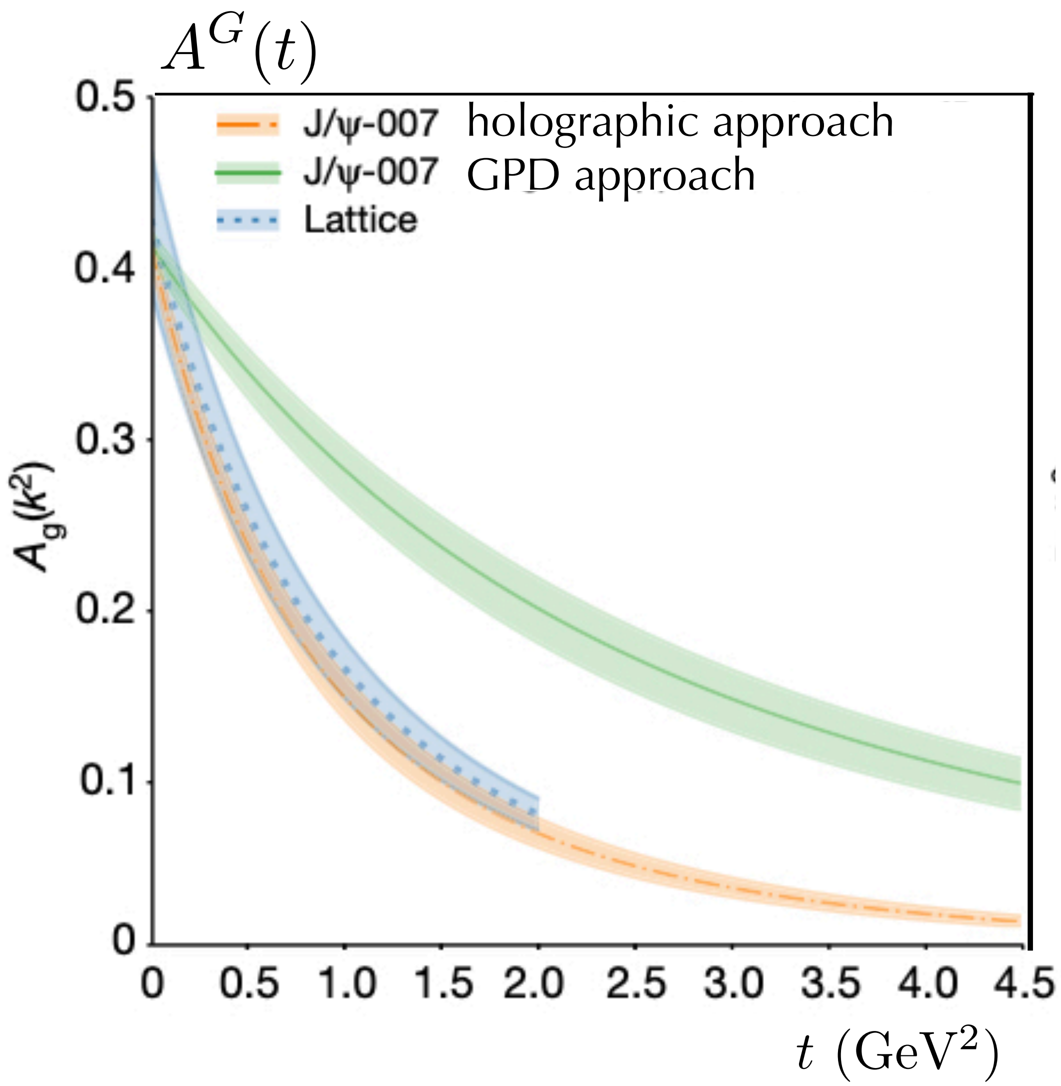
access to Re $\mathcal{H}$

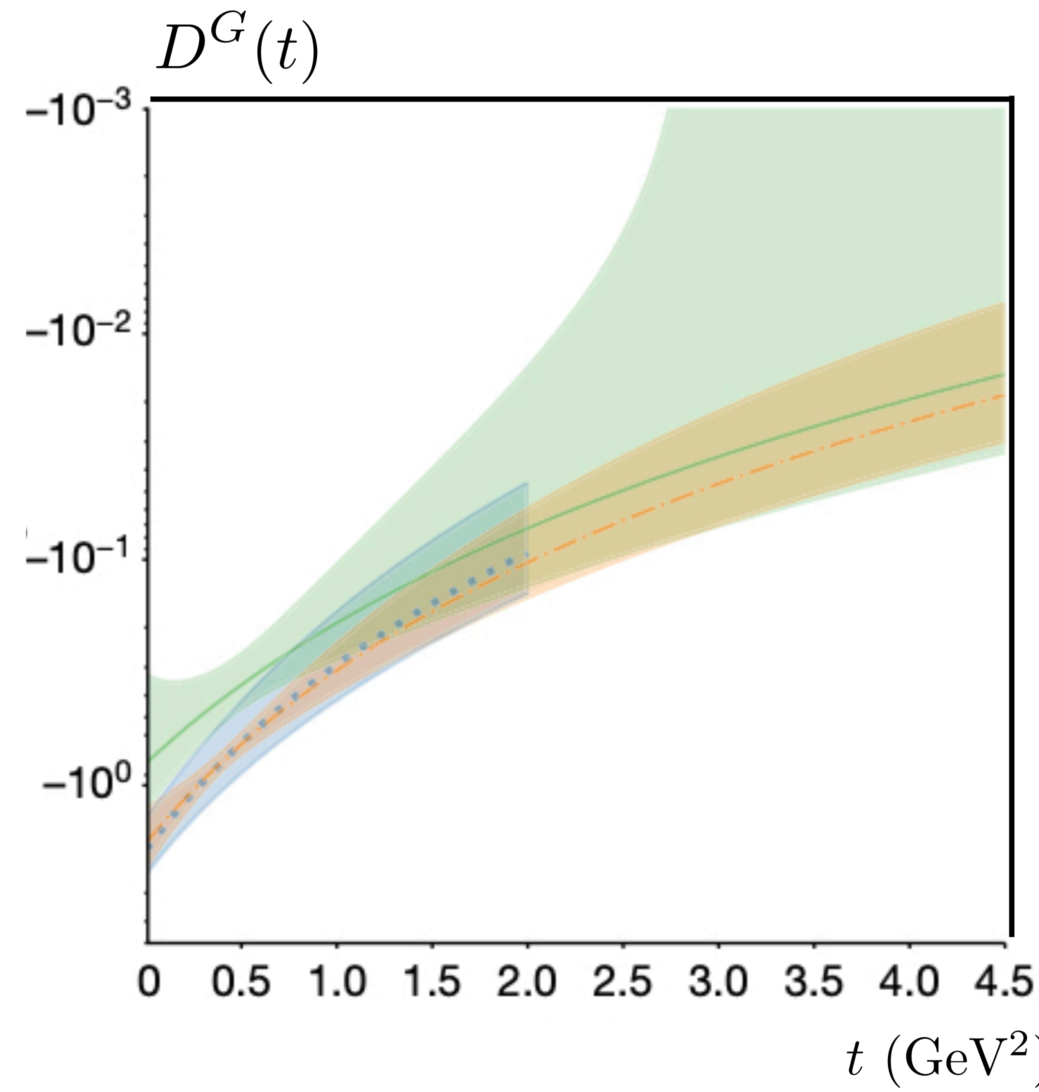
- ✓ Test of the universality of GPDs
- ✓ Further data from JLab12 and future EIC
- ✓ New promising path towards the extraction of Re $\mathcal{H}$ and then the D-term

Gluonic EMT Form Factors

Duran et al., Nature 615 (2023) 7954







- proof of concept of feasibility to extract gluonic structure
- further measurements planned with SOLID at JLab
- JLab22 crucial for these measurements: high luminosity and leverage in t
- EIC: complementary measurements for Υ photo- and electro-production, but require L=100 fb^{-1}

Future from JLab22 upgrade and EIC

EIC and JLab22 complementary to:

- Cover larger energy domain to ensure convergence in dispersion analysis of GPDs

- Span a larger range of t for a meaningful FT

- JLab22 bridges between EIC (gluon components) and JLab12 (valence region)

High luminosity at JLab22 gives unique possibility to measure new processes so-far unexplored

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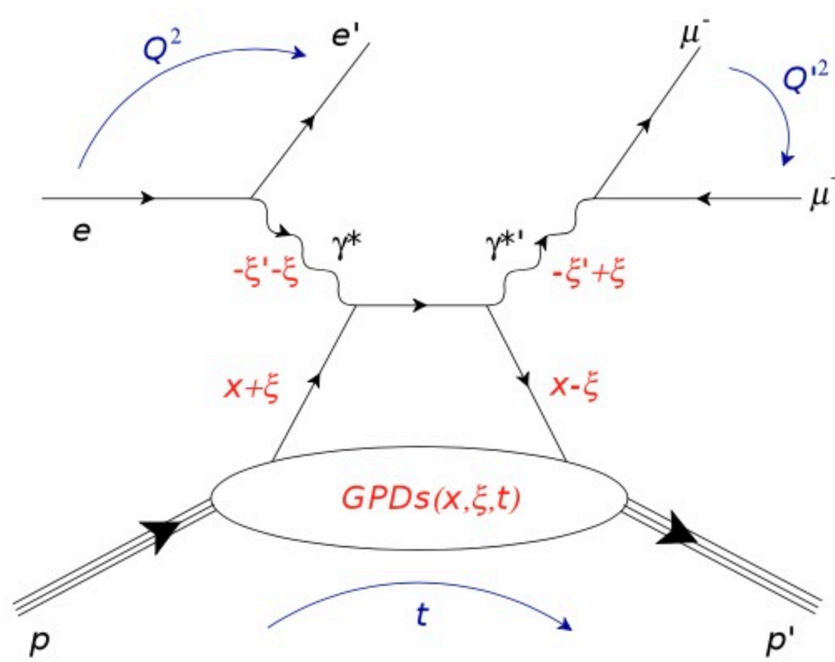
$$e + p \to e' + (l^+ l^-) + p$$



- dilepton electroproduction suppressed by a factor $\alpha_{QED} \sim 10^{-2}$ compared to DVCS
- disentangle the longitudinal momentum variables by varying the dilepton mass

EMT and the proton mass

- Forward matrix element of total EMT

$$\langle T^{\mu\nu}\rangle \equiv \langle p|T^{\mu\nu}|p\rangle = 2p^\mu p^\nu$$

Proton mass

$$n\,\langle T^\mu{}_\mu\rangle = n\,\langle T^{00}\rangle\Big|_{\vec{p}=0} = \frac{\langle H_{\rm QCD}\rangle}{\langle p|p\rangle}\Big|_{\vec{p}=0} = M$$

$\left(n = \dfrac{1}{2M}\right.$ depends on normalization of state)

$$H_{\rm QCD} = \int d^3x \mathcal{H}_{QCD} = \int d^3x\, T^{00}$$

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$$H_{\rm QCD} = \int d^3x \mathcal{H}_{QCD} = \int d^3x\, T^{00}$$

- Forward matrix element quark and gluon contributions

$$\langle T^{\mu\nu}_{i,R}\rangle = 2p^\mu p^\nu\, A_i(0) + 2M^2 g^{\mu\nu} \bar{C}_i(0)$$

Conservation of full EMT:

$$A_q(0) + A_g(0) = 1 \qquad\qquad \bar{C}_q(0) + \bar{C}_g(0) = 0$$

in forward limit, matrix elements of EMT fully determined by two form factors

any mass sum rule for the proton related to at most two independent numbers

Trace decomposition

(Hatta, Rajan, Tanaka, JHEP 12, 008 (2018) / Tanaka, JHEP 01, 120 (2019))

- Trace anomaly of EMT: $T^{\mu}{}_{\mu} = (m\bar{\psi}\psi)_R + \gamma_m (m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R$

(Adler, Collins, Duncan, 1977 / Nielsen, 1977 / Collins, Duncan, Joglekar, 1977 / ...)

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(Adler, Collins, Duncan, 1977 / Nielsen, 1977 / Collins, Duncan, Joglekar, 1977 / ...)

- Total EMT not renormalized, but individual terms $T_i^{\mu\nu}$ require (extra) renormalization

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu} \qquad T_q^{\mu\nu} = \frac{i}{4}\bar{\psi}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi \qquad T_g^{\mu\nu} = -F^{\mu\alpha}F_{\alpha}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$T^{\mu}{}_{\mu} = (T_{q,R})^{\mu}{}_{\mu} + (T_{g,R})^{\mu}{}_{\mu} \quad \begin{cases} (T_{q,R})^{\mu}{}_{\mu} = (1+y)(m\bar{\psi}\psi)_R + x(F^2)_R \\ (T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - x\right)(F^2)_R \end{cases}$$

$$M = \overline{M}_q + \overline{M}_g = n\Big(\langle (T_{q,R})^{\mu}{}_{\mu}\rangle + \langle (T_{g,R})^{\mu}{}_{\mu}\rangle\Big)$$

Trace decomposition

(Hatta, Rajan, Tanaka, JHEP 12, 008 (2018) / Tanaka, JHEP 01, 120 (2019))

- Trace anomaly of EMT: $T^{\mu}{}_{\mu} = (m\bar{\psi}\psi)_R + \gamma_m (m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R$

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- x and y related to finite parts of renormalization constants ⟶ choose a scheme

D1 scheme: $x = 0,\ y = \gamma_m$ $\quad (T_{q,R})^{\mu}{}_{\mu} = (1+\gamma_m)(m\bar{\psi}\psi)_R \quad (T_{g,R})^{\mu}{}_{\mu} = \frac{\beta}{2g}(F^2)_R$

D2 scheme: $x = y = 0$ $\quad (T_{q,R})^{\mu}{}_{\mu} = (m\bar{\psi}\psi)_R \quad (T_{g,R})^{\mu}{}_{\mu} = \gamma_m(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R$

3 term energy decomposition

Rodini, Metz, BP, JHEP 09, 67 (2020); PRD 102, 111 (2020); Lorcé, Metz, BP, Rodini, JHEP 11,121 (2021)

- Sum rule based on decomposition of T^{00}

$$M = M_q + M_m + M_g = n\left(\langle \mathcal{H}_q \rangle + \langle \mathcal{H}_m \rangle + \langle \mathcal{H}_g \rangle\right)$$

- Renormalized operators:

$\mathcal{H}_q = (\psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi)_R$ quark (kinetic plus potential) energy

$\mathcal{H}_m = (\bar{\psi}m\psi)_R$ quark mass term

$\mathcal{H}_g = \frac{1}{2}(\vec{E}^2 + \vec{B}^2)_R$ gluon energy

- Clear interpretation!

4 term sum rule by Ji

Ji, PRL 74, 1071 (1995); PRD 52, 271 (1995); Ji et al., NPB 971 (2021) 115537

- Sum rule based on decomposition of T^{00} into traceless and trace part

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \hat{T}^{\mu\nu}$$

trace part $\hat{T}^{\mu\nu} = \frac{1}{4} g^{\mu\nu} T^{\alpha}{}_{\alpha}$

traceless part $\overline{T}^{\mu\nu} = T^{\mu\nu} - \hat{T}^{\mu\nu}$

- Motivation: trace (scalar) part and traceless (tensor) part do not mix under renormalization

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- Final four term sum rule obtained by

(i) decomposition of $\overline{T}^{\mu\nu}$ and $\hat{T}^{\mu\nu}$ into quark and gluon contributions

(ii) rearranging in the quark sector (reshuffling between traceless and trace part)

$$M = M_{q[\mathrm{Ji}]} + M_m + M_{g[\mathrm{Ji}]} + M_a = n\left(\langle\mathcal{H}_{q[\mathrm{Ji}]}\rangle + \langle\mathcal{H}_m\rangle + \langle\mathcal{H}_{g[\mathrm{Ji}]}\rangle + \langle\mathcal{H}_a\rangle\right)$$

$\mathcal{H}_{q[\mathrm{Ji}]} = (\psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi)_{R[\mathrm{Ji}]}$ (quark kinetic plus potential energy)[Ji]

$\mathcal{H}_m = (\bar{\psi}m\psi)_R$ mass energy

$\mathcal{H}_{g[\mathrm{Ji}]} = \frac{1}{2}(E^2 + B^2)_{R[\mathrm{Ji}]}$ (gluon energy)[Ji]

$\mathcal{H}_a = \frac{1}{4}\left[\frac{\beta}{2g}(F^2)_R + \gamma_m(\bar{\psi}m\psi)_R\right]$ anomaly energy

Comparison with our renormalized operators

$$\mathcal{H}_{g[\mathrm{Ji}]} = \mathcal{H}_g - \frac{1}{4}(T_{g,R})^\mu_\mu$$
$$= \frac{1}{2}(E^2 + B^2)_R + \frac{y-\gamma_m}{4}(m\bar{\psi}\psi)_R - \frac{1}{4}(\frac{\beta}{2g} - x)(F^2)_R$$

- Similar result for $\mathcal{H}_{q[\mathrm{Ji}]}$
- Interpretation of operator of $\mathcal{H}_{q[Ji]}$ and $\mathcal{H}_{g[\mathrm{Ji}]}$?

- More recent result in dimensional regularisation (Ji, Liu, Schäfer, 2021)

$$\mathcal{H}_m = (\bar{\psi} m \psi)_R$$

$$\mathcal{H}_a = \frac{1}{4}\left[\frac{\beta}{2g}(F^2)_R + \gamma_m (\bar{\psi} m \psi)_R\right]$$

$$(\mathcal{H}_q + \mathcal{H}_g)_{[\mathrm{JLS}]} = \left(\psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi + \frac{2-2\epsilon}{4-2\epsilon}E^2 + \frac{2}{4-2\epsilon}B^2\right)_R$$

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- We find exact agreement with our result by using

$$-\frac{\epsilon}{4}(E^2 - B^2) = \frac{\epsilon}{8}F^2 = -\frac{1}{4}\left(\gamma_m(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R\right)$$

$$(\mathcal{H}_q + \mathcal{H}_g)_{[\mathrm{JLS}]} = \mathcal{H}_q + \mathcal{H}_g - \mathcal{H}_a$$

$$\mathcal{H}_m + \mathcal{H}_a + (\mathcal{H}_q + \mathcal{H}_g)_{[\mathrm{JLS}]} = \mathcal{H}_m + \mathcal{H}_q + \mathcal{H}_g$$

Two independent inputs from experiments

- First input: parton momentum fractions a_i, related to traceless parton operators

$$\frac{3}{2}M^2\, a_q = \langle \bar{T}^{00}_{q,R} \rangle \qquad \frac{3}{2}M^2\, a_g = \langle \bar{T}^{00}_{g,R} \rangle \qquad (a_q + a_g = 1)$$

- Second input: quark mass term related to sigma-term

$$2M^2\, b = (1+\gamma_m)\, \langle (m\bar{\psi}\psi)_R \rangle \qquad 2M^2\,(1-b) = \frac{\beta}{2g}\, \langle (F^2)_R \rangle$$

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- For example: 3-term sum rule in terms of a_i and b

$$M_q = \frac{3}{4}M\, a_q + \frac{1}{4}M\left(\frac{(y-3)b}{1+\gamma_m} + x(1-b)\frac{2g}{\beta}\right)$$

$$M_m = M\frac{b}{1+\gamma_m}$$

$$M_g = \frac{3}{4}M\, a_g + \frac{1}{4}M\left[\frac{(\gamma_m - y)b}{1+\gamma_m} + \left(1 - x\frac{2g}{\beta}\right)(1-b)\right]$$

Overview: comparison of sum rules

- 2-term trace decomposition (T^{μ}_{μ}) $M = \overline{M}_q + \overline{M}_g$ 1 indep. term (b)
- 3-term energy decomposition (T^{00}) $M = M_q + M_m + M_g$ 2 indep. terms (a, b)
- 4-term sum rule (T^{00}) $M = M_{q[\mathrm{Ji}]} + M_m + M_{g[\mathrm{Ji}]} + M_a$ 2 indep. terms (a, b)

additional relation (virial theorem) $M_{q[\mathrm{Ji}]} = -\frac{3\gamma_m}{4+\gamma_m} M_m + M_{g[\mathrm{Ji}]} - 3M_a = 0$

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- measuring $\langle F^2 \rangle$ (further constraints on b) is relevant for all sum rules

- Key experiments: heavy quarkonium photo- and electro-production at threshold

- measurements at JLab (GlueX and SoLID) for J/Ψ
- at EIC: J/Ψ and Υ photo- and electro-production

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- All the sum rules are scale and scheme dependent

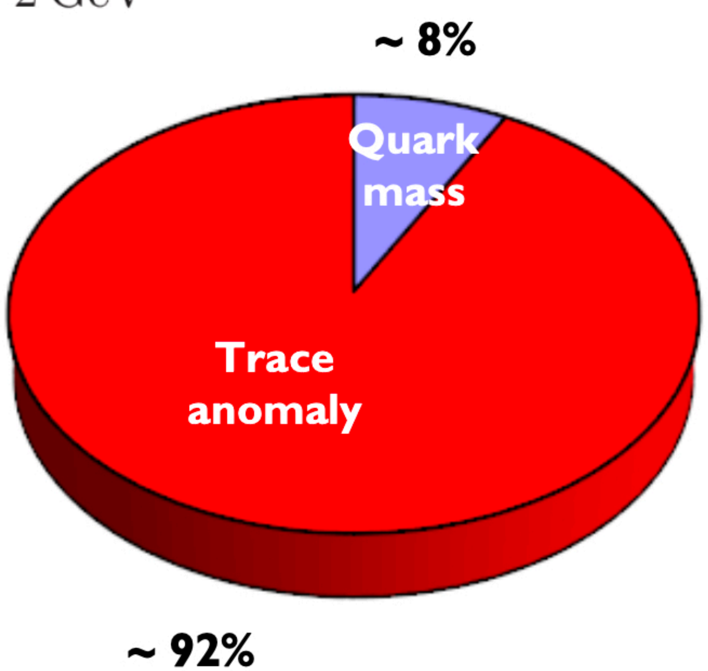
- Closest agreement in D2 scheme (x=y=0)

$$\overline{M}_q^{\mathrm{D2}} = M_m \qquad M_q^{\mathrm{D2}} = M_{q[\mathrm{Ji}]} \qquad M_g^{\mathrm{D2}} = M_{g[\mathrm{Ji}]} + M_a$$

Mass decompositions in D2 scheme

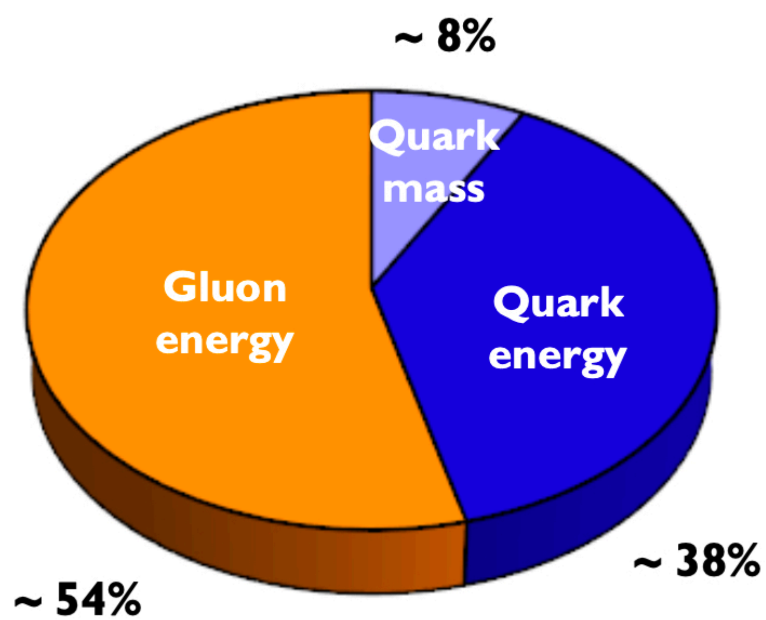
Trace decomposition

$\mu = 2\,\mathrm{GeV}$



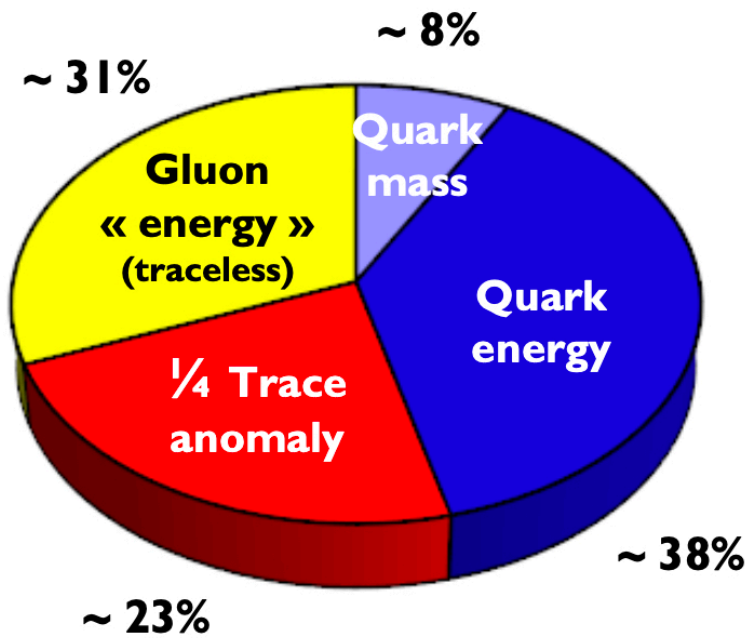
$$\overline{M}_q^{\mathrm{D2}} = M_m$$

3-term energy decomposition



$$M_q^{\mathrm{D2}} = M_{q[\mathrm{Ji}]}$$

4-term sum rule (Ji)



$$M_g^{\mathrm{D2}} = M_{g[\mathrm{Ji}]} + M_a$$

Summary

- Understanding the strong interaction dynamics of non-pQCD and ``how'' hadrons emerge from fundamental QCD principles, is a complex problem which demands different approaches and precise measurements of multiple observables

- Unique insight from JLab exp. program (JLab12, positron beam, JLab22) and EIC

- I have just considered a few examples from two-photon processes:

  - VVCS and VCS polarizabilities to probe the long distance dynamics

  - GPDs/EMT to probe the "mechanical" and mass properties of the proton emerging from parton dynamics

- Following talks: perspectives to unravel the complex properties of hadronization and TMD physics from JLab, EIC, LHCspin with a better understanding theory foundations and new phenomenological tools

- Congratulations to Pavia, Cagliari, Torino for the new approved PRIN project "ProtoTaste: Tasting the flavor of the proton in its full dimensions"!