



TMDs global fits by the MAP Collaboration



Lorenzo Rossi

MAP Collaboration

June 7th



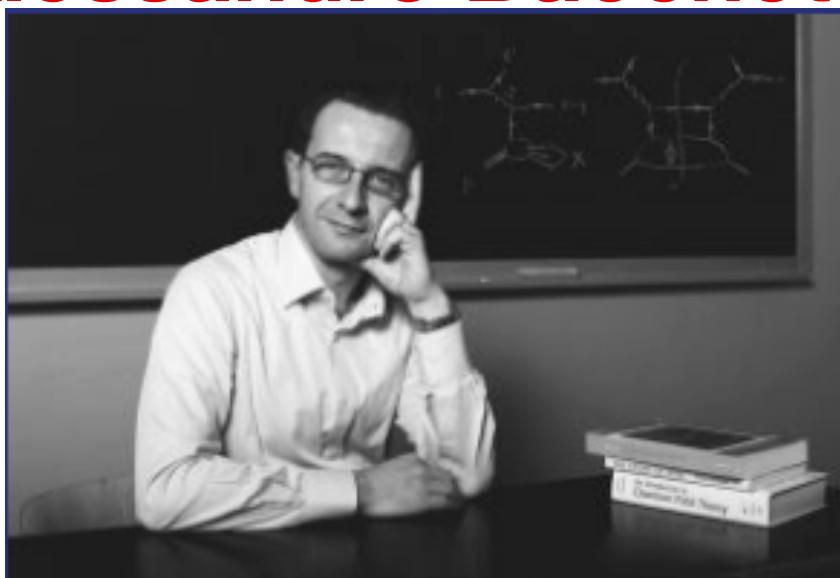
Istituto Nazionale di Fisica Nucleare



**UNIVERSITÀ
DI PAVIA**

Results obtained with contribution from:

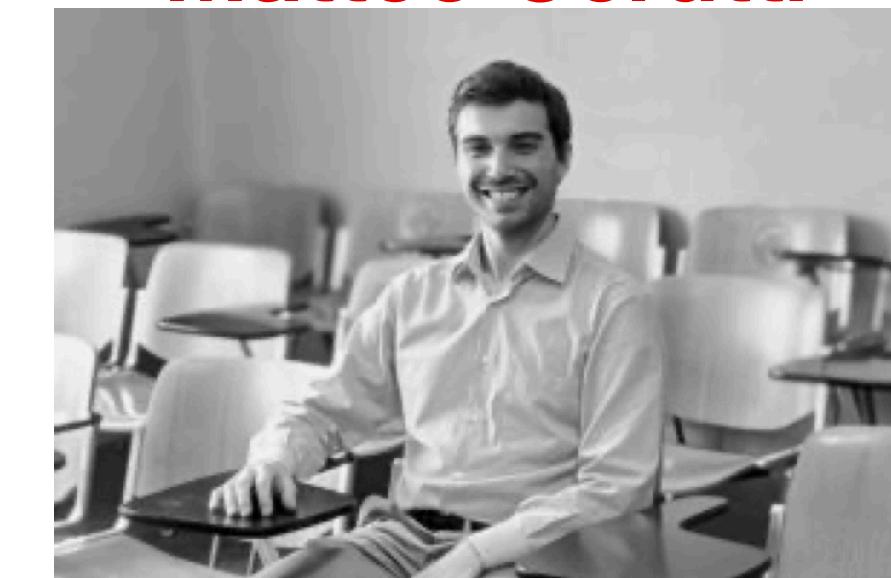
Alessandro Bacchetta



Marco Radici



Matteo Cerutti



Andrea Signori



Valerio Bertone



Chiara Bissolotti



Giuseppe Bozzi



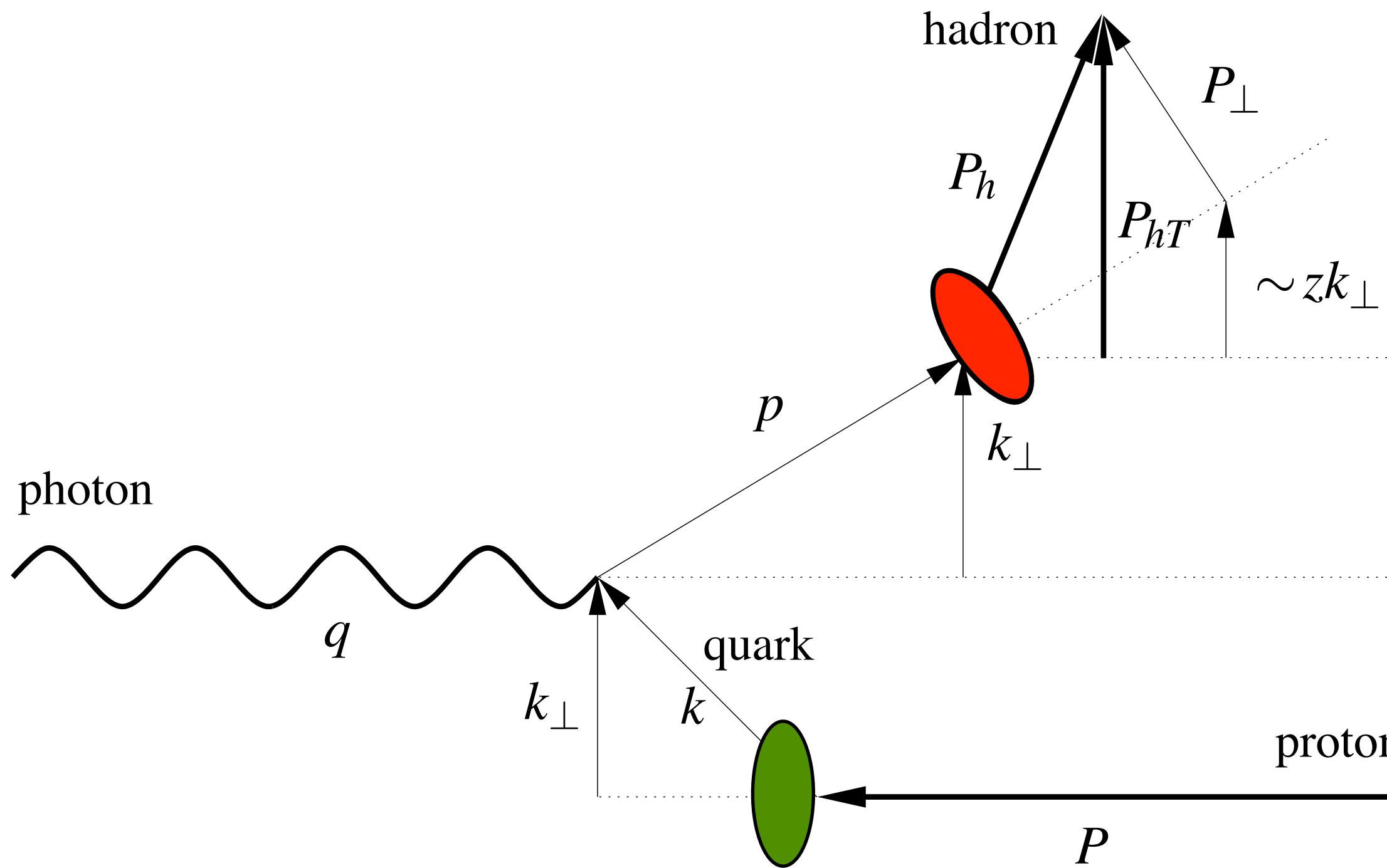
Fulvio Piacenza



Simone Venturini

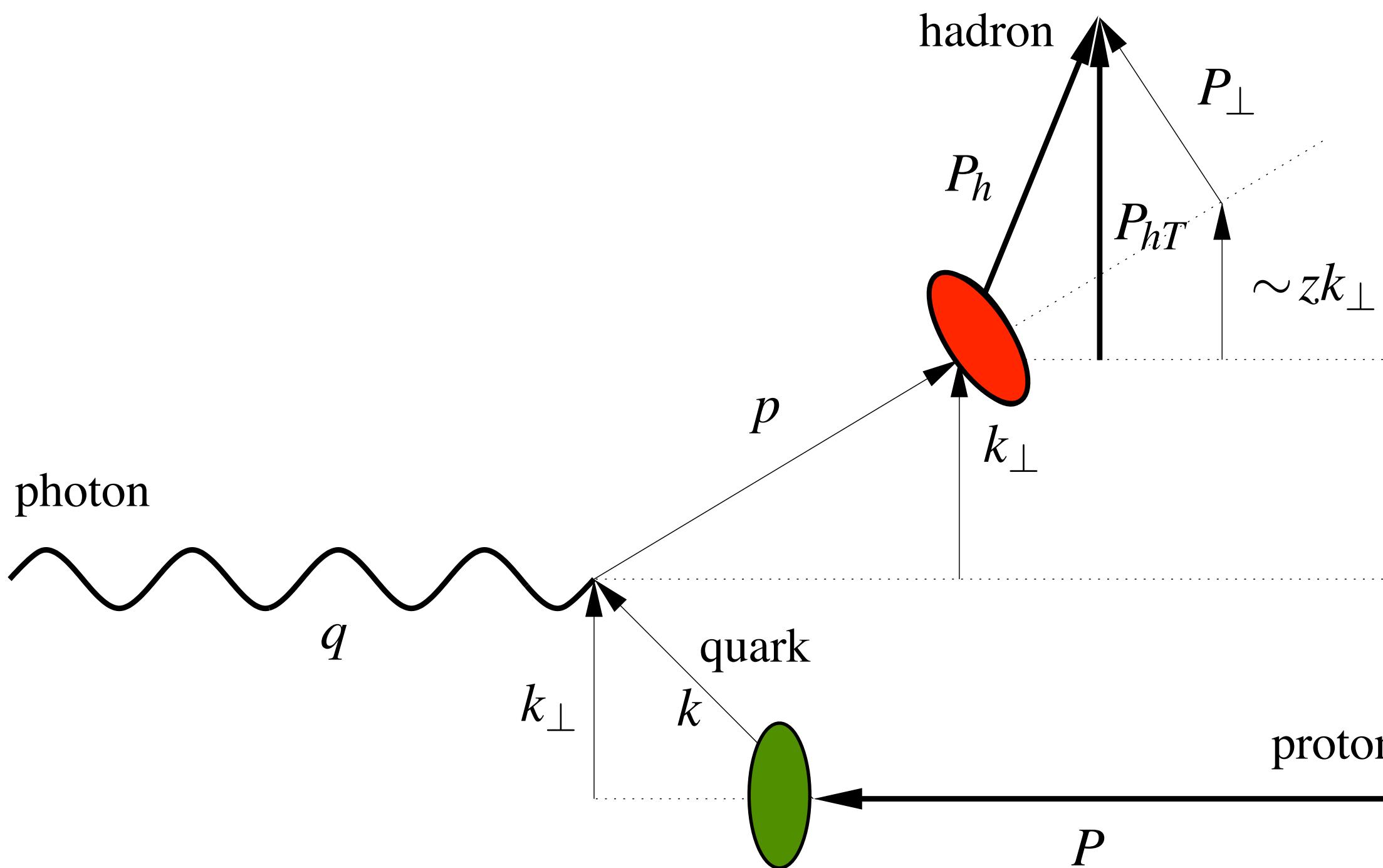


TMD Factorization - SIDIS process



$$\begin{aligned}
 F_{UU,T}(x.z; \mu_F, \mathbf{P}_{hT}^2, Q^2) = & x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^{\mathbf{a}}(x, \mathbf{k}_\perp^2; \mu^2) D_1^{\mathbf{a} \rightarrow \mathbf{h}}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) \\
 & + Y_{UU,T}(Q^2, \mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)
 \end{aligned}$$

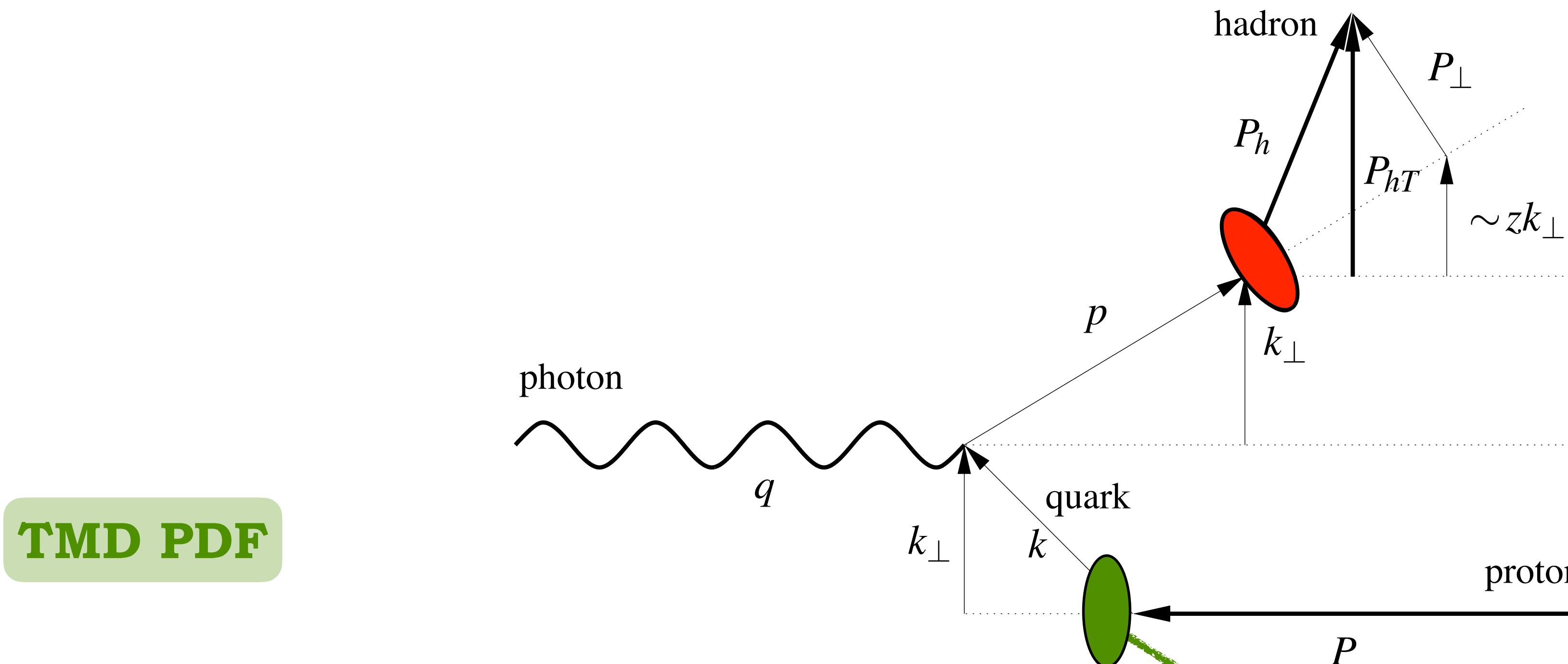
TMD Factorization - SIDIS process



$$F_{UU,T}(x \cdot z; \mu_F, \mathbf{P}_{hT}^2, Q^2) = \boxed{x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{\mathbf{a} \rightarrow \mathbf{h}}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)} \\ + Y_{UU,T}(Q^2, \mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

W Term

TMD Factorization - SIDIS process



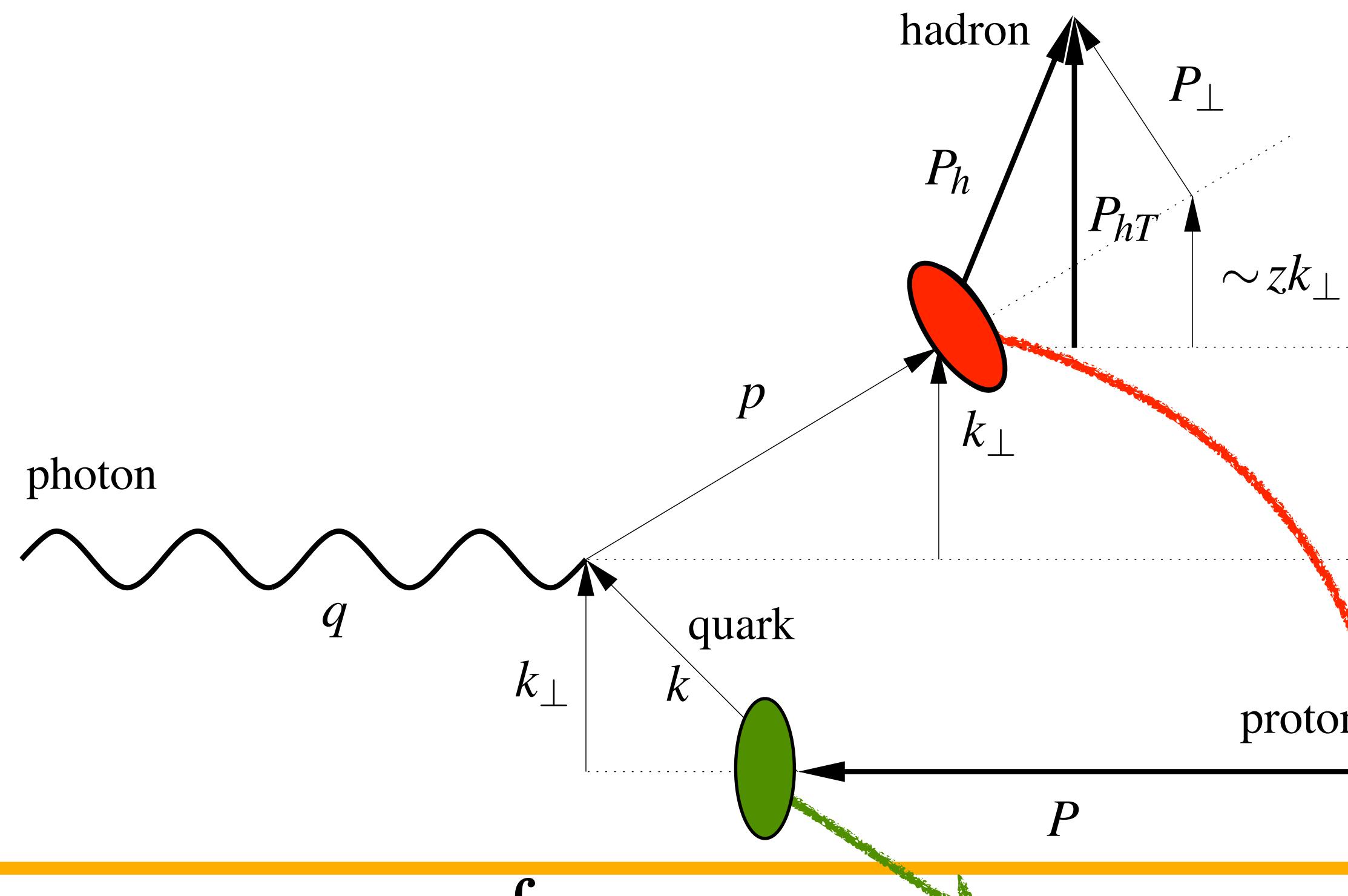
$$\begin{aligned}
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W Term

TMD Factorization - SIDIS process

TMD FF

TMD PDF



$$F_{UU,T}(x \cdot z; \mu_F, \mathbf{P}_{hT}^2, Q^2) = x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

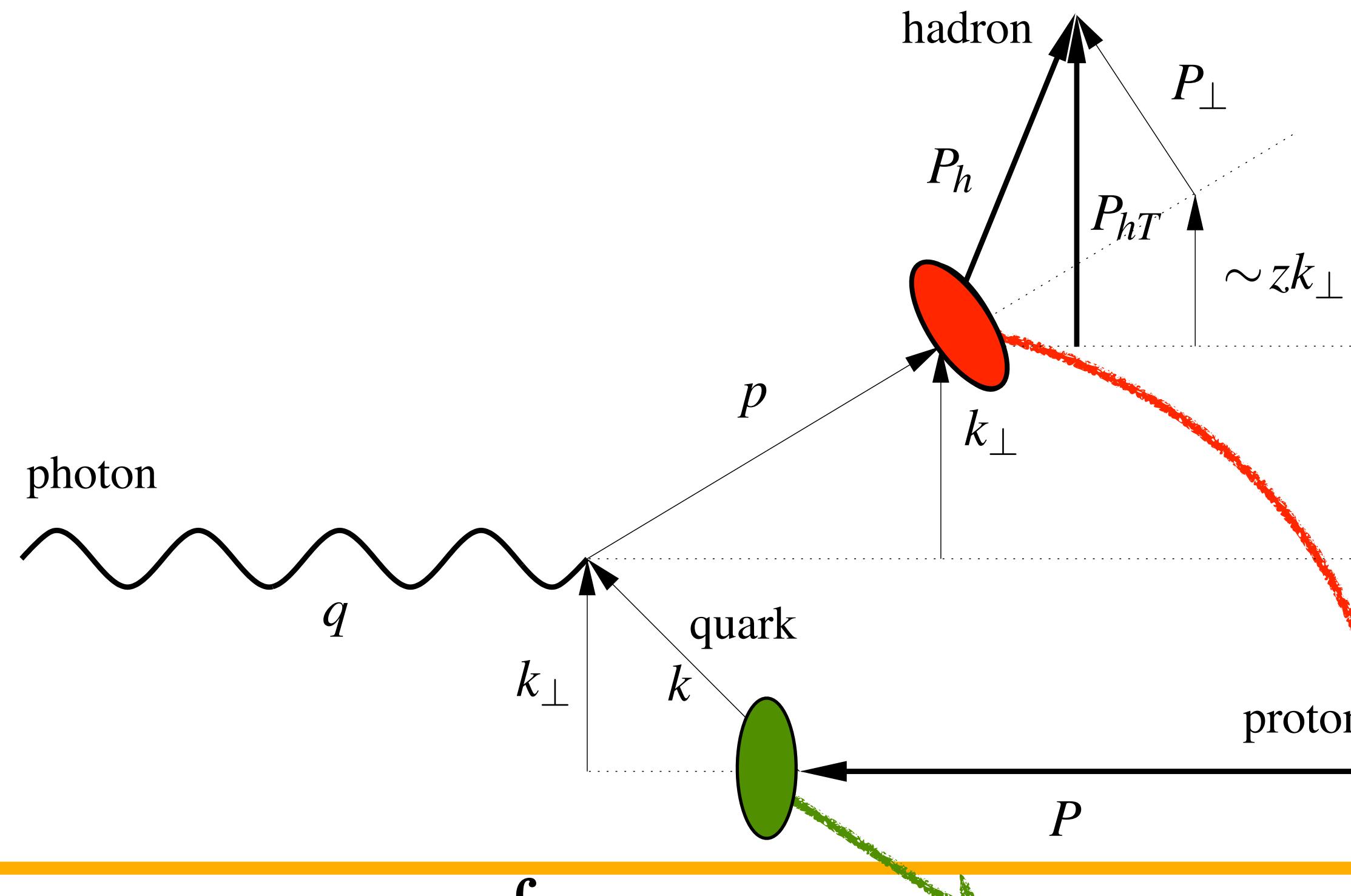
$$+ Y_{UU,T}(Q^2, \mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

W Term

TMD Factorization - SIDIS process

TMD FF

TMD PDF



$$F_{UU,T}(x \cdot z; \mu_F, \mathbf{P}_{hT}^2, Q^2) = x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

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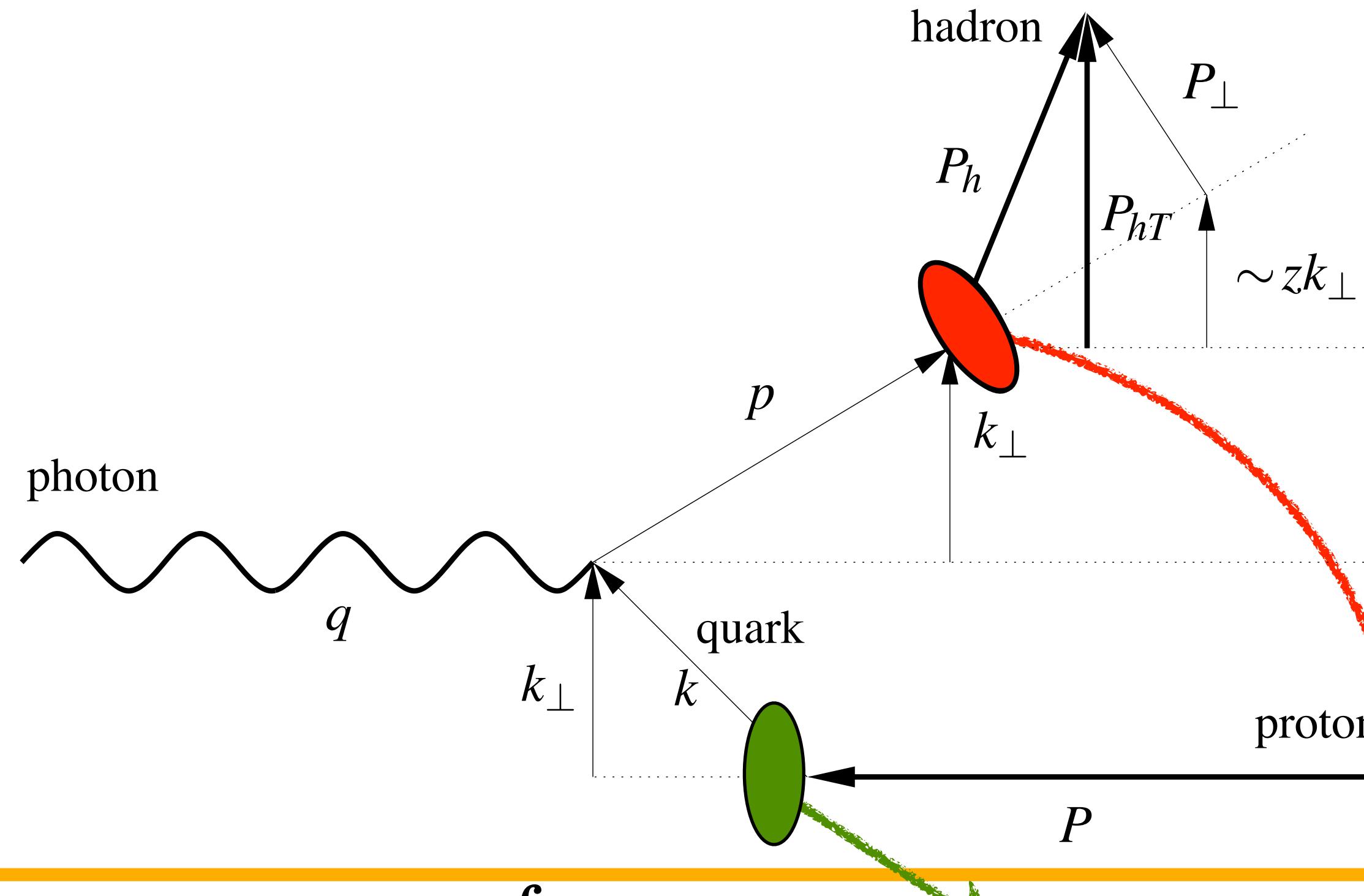
W Term

- The W term dominates in the region where $\mathbf{q}_T \ll \mathbf{Q}$

TMD Factorization - SIDIS process

TMD FF

TMD PDF



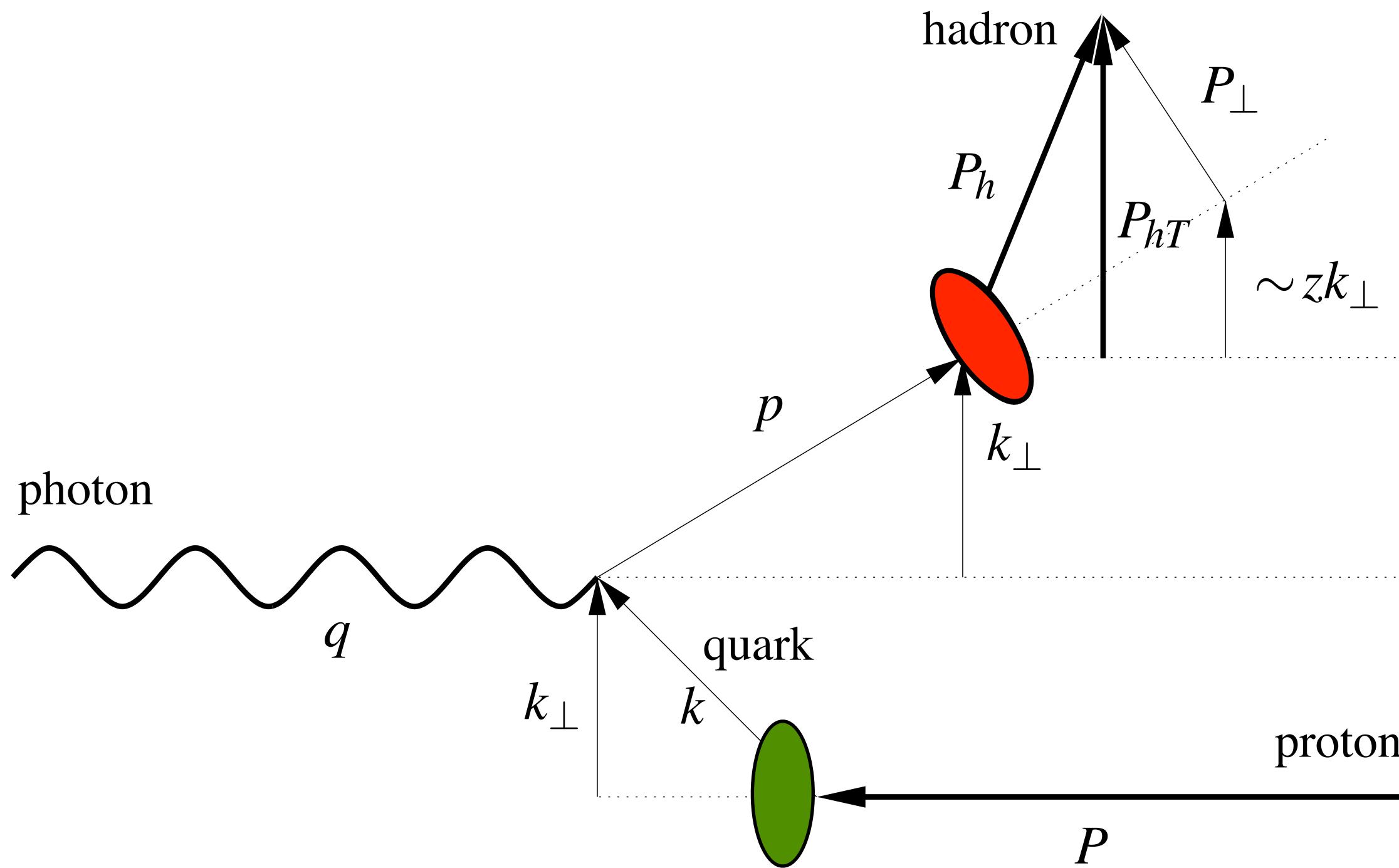
$$F_{UU,T}(x \cdot z; \mu_F, \mathbf{P}_{hT}^2, Q^2) = x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{a \rightarrow h}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

$$+ Y_{UU,T}(Q^2, \mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

W Term

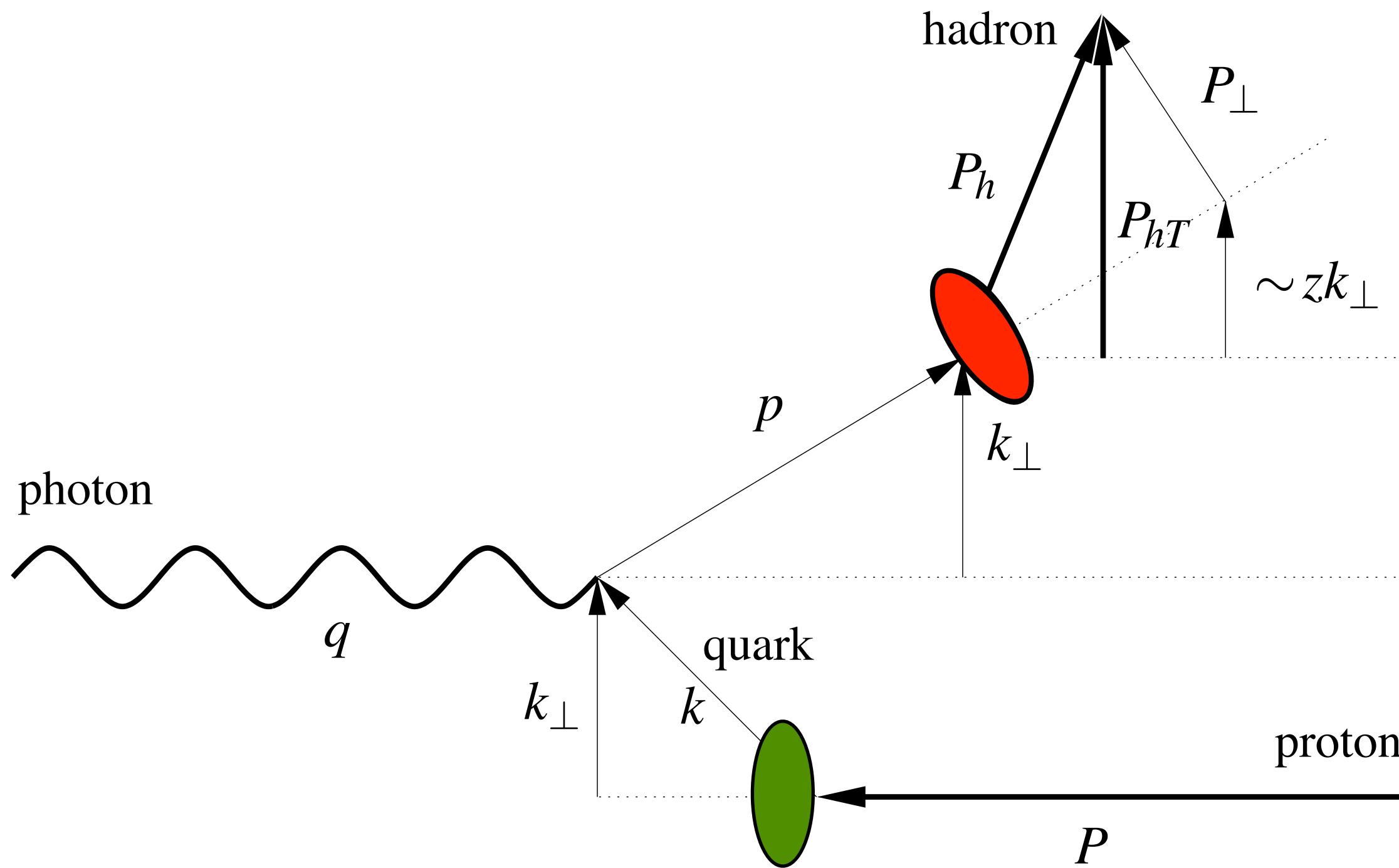
- The W term dominates in the region where $q_T \ll Q$
- The Y term has been excluded in the MAP analysis

TMD Factorization - SIDIS process



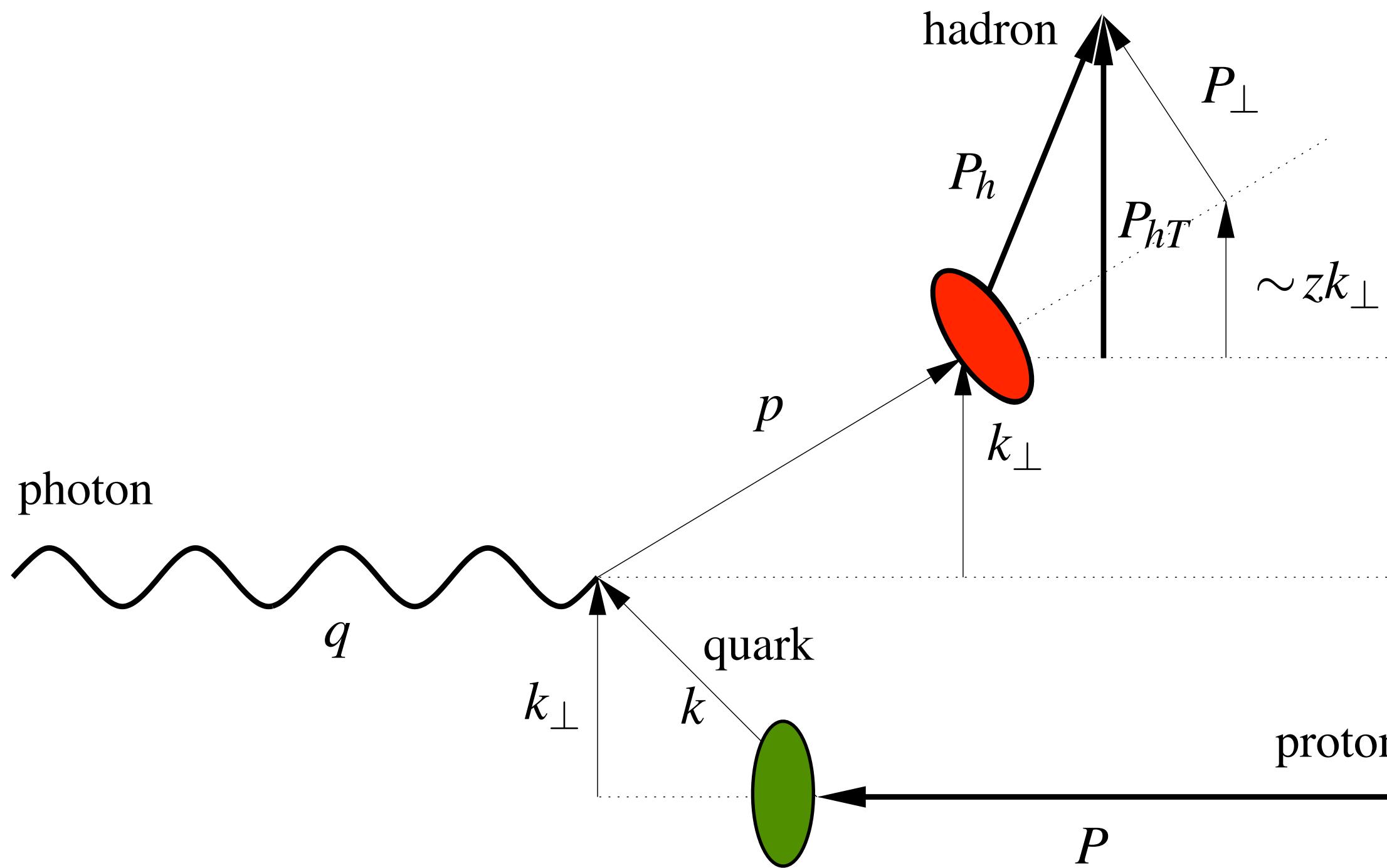
$$F_{UU,T}(x \cdot z; \mu_F, \mathbf{P}_{hT}^2, Q^2) = x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{\mathbf{a} \rightarrow \mathbf{h}}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

TMD Factorization - SIDIS process



$$\begin{aligned}
 F_{UU,T}(x, z; \mu_F, \mathbf{P}_{hT}^2, Q^2) &= x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{\mathbf{a} \rightarrow \mathbf{h}}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) \\
 &= x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int db_T b_T J_0(b_T | \mathbf{P}_{hT} |) f_1^a(x, z^2 b_T^2; \mu^2) D_1^{a \rightarrow h}(z, b_T^2; \mu^2)
 \end{aligned}$$

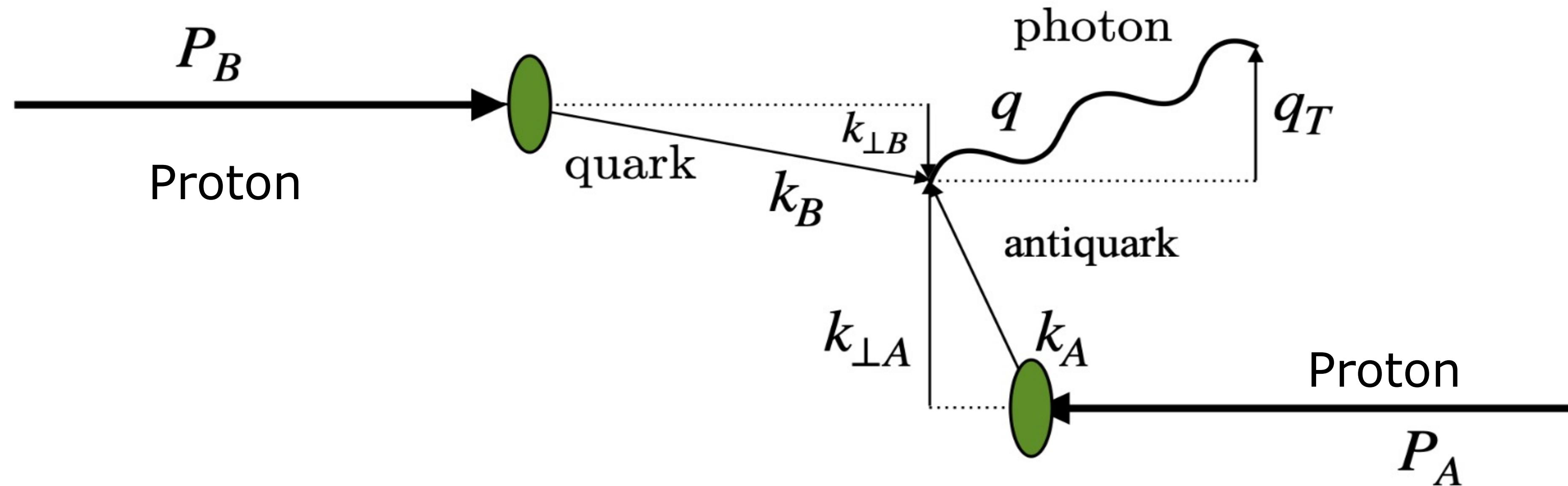
TMD Factorization - SIDIS process



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 F_{UU,T}(x, z; \mu_F, \mathbf{P}_{hT}^2, Q^2) &= x \sum_a H_{UU,T}^a(Q^2, \mu^2) \int d^2 \mathbf{k}_\perp d^2 \mathbf{P}_\perp f_1^a(x, \mathbf{k}_\perp^2; \mu^2) D_1^{\mathbf{a} \rightarrow \mathbf{h}}(z, \mathbf{P}_\perp^2; \mu^2) \delta^{(2)}(z \mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp) \\
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 \end{aligned}$$

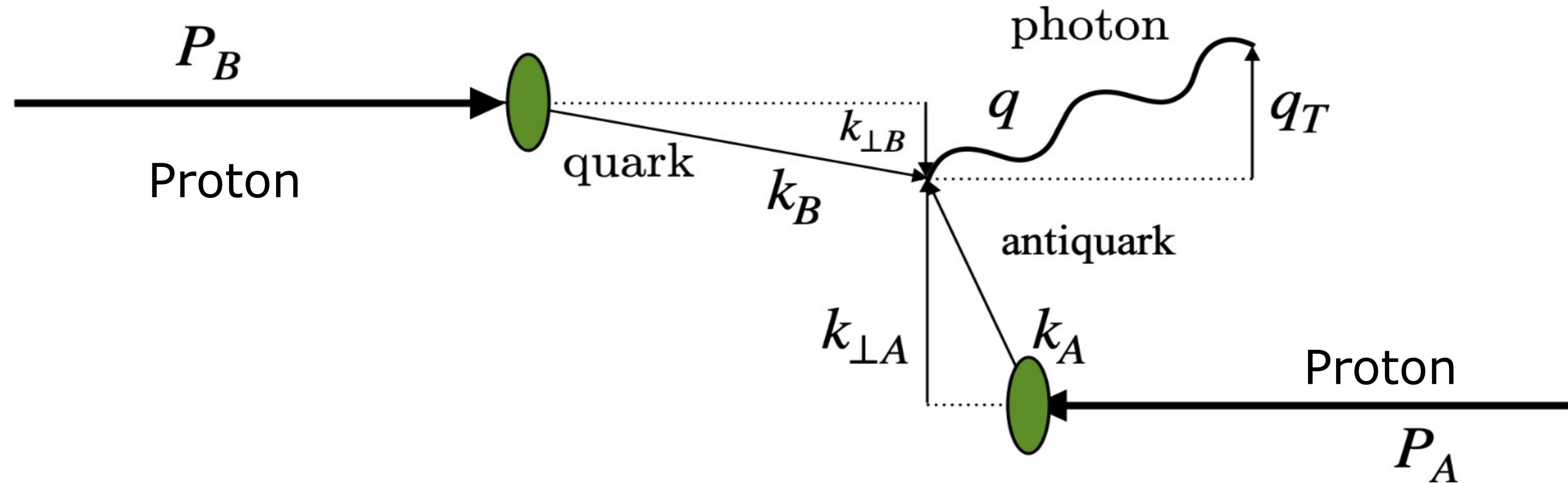
TMDs in impact space to avoid convolutions

TMD Factorization - Drell Yan process



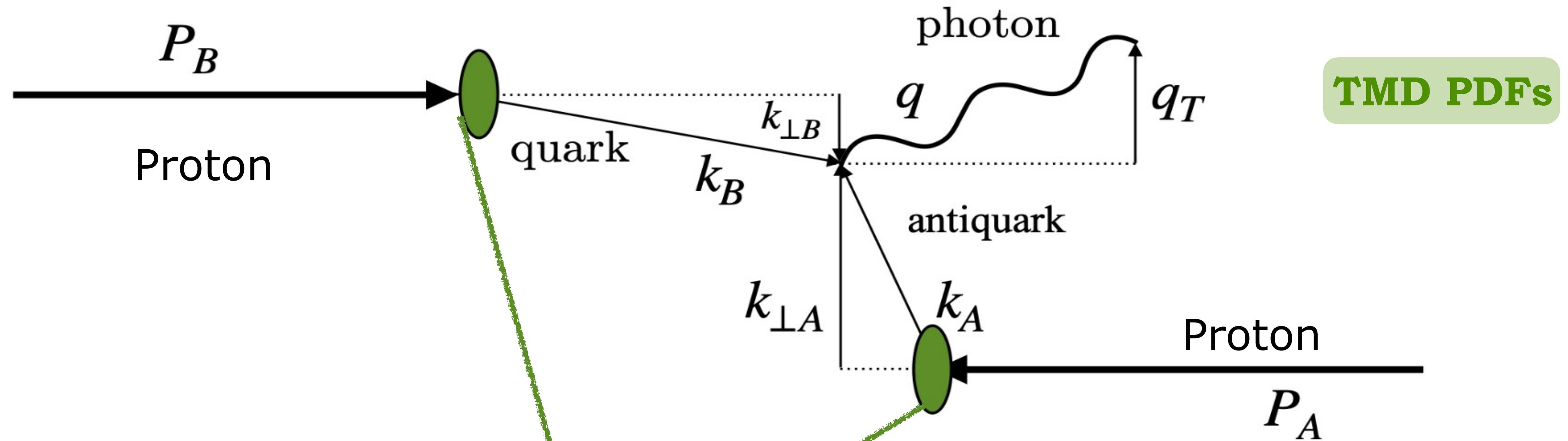
$$F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2) = \sum_a H_{UU}^{1a}(Q^2, \mu^2) \int d^2 \mathbf{k}_{\perp A} d^2 \mathbf{k}_{\perp B} f_1^{\bar{a}}(x_A, \mathbf{k}_{\perp A}, \mu^2) f_1^a(x_B, \mathbf{k}_{\perp B}, \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} + \mathbf{k}_{\perp B} - \mathbf{q}_T)$$

TMD Factorization - Drell Yan process



$$\begin{aligned}
 F_{UU}^1(x_A, x_B, \mathbf{q}_T^2, Q^2) &= \sum_a H_{UU}^{1a}(Q^2, \mu^2) \int d^2\mathbf{k}_{\perp A} d^2\mathbf{k}_{\perp B} f_1^{\bar{a}}(x_A, \mathbf{k}_{\perp A}, \mu^2) f_1^a(x_B, \mathbf{k}_{\perp B}, \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} + \mathbf{k}_{\perp B} - \mathbf{q}_T) \\
 &= \sum_a H_{UU}^{1a}(Q^2, \mu^2) \int db_T f_1^{\bar{a}}(x_A, \mathbf{k}_{\perp A}, \mu^2) f_1^a(x_B, \mathbf{k}_{\perp B}, \mu^2) \delta^{(2)}(\mathbf{k}_{\perp A} + \mathbf{k}_{\perp B} - \mathbf{q}_T)
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TMD Factorization - Drell Yan process



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 \end{aligned}$$

TMD Factorization - structure of TMDs

$$\begin{aligned}\hat{f}_1^q(x_B, \mathbf{b}_T; \mu_F, \zeta_F) &= [C \otimes f_1](x_B, b_\star; \mu_{b_\star}, \mu_{b_\star}^2) \exp \left\{ \int_{\mu_{b_\star}}^{\mu_F} \frac{d\mu'}{\mu'} \gamma(\mu', \zeta_F) \right\} \\ &\times \left(\frac{\zeta}{\mu_{b_\star}^2} \right)^{K(b_\star, \mu_{b_\star})/2} \left[\frac{\zeta}{Q_0} \right]^{-g_K(\mathbf{b}_T)/2} f_1^{NP}(x, \mathbf{b}_T; \zeta, Q_0)\end{aligned}$$

TMD Factorization - structure of TMDs

Matching coeff.
(perturbative calculable)

$$\hat{f}_1^q(x_B, \mathbf{b}_T; \mu_F, \zeta_F) = [C \otimes f_1](x_B, b_\star; \mu_{b_\star}, \mu_{b_\star}^2) \exp \left\{ \int_{\mu_{b_\star}}^{\mu_F} \frac{d\mu'}{\mu'} \gamma(\mu', \zeta_F) \right\}$$
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Collinear PDFs
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TMD Factorization - structure of TMDs

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Perturbative Sudakov
evolution factor

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Collins-Soper
kernel

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Collins-Soper
kernel

NP part of
Collins-Soper Kernel

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NP part of
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Non perturbative part
of TMDs

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Collins-Soper
kernel

NP part of
Collins-Soper Kernel

Non perturbative part
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Fit extraction

Perturbative accuracy

-

Perturbative accuracy

Orders in powers of α_S

Accuracy	H and C	K and γ_F	γ_K	PDFs/FFs and a_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NNLO + NLO
N^3LL	2	3	4	NNLO
N^3LL'	3	3	4	N^3LO

Perturbative accuracy

Orders in powers of α_S

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N^3LL	2	3	4	NNLO
N^3LL'	3	3	4	N^3LO

Collinear fragmentation functions available beyond NLO only recently

A new global fit: MAPTMD22

	Accuracy	SIDIS	DY	Z production	N of points	χ^2/N_{data}
Pavia 2017 arXiv:1703.10157	NLL -	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL	✓	✓	✓	1039	1.06
MAPTMD22	N^3LL-	✓	✓	✓	2031	1.06

A new extraction of proton quark unpolarized TMDs

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- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points

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A new extraction of proton quark unpolarized TMDs

- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points
- Perturbative accuracy: **N^3LL^-**
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- Number of fitted parameters: **21**

A new extraction of proton quark unpolarized TMDs

- Global analysis of Drell-Yan and Semi-Inclusive DIS data sets: **2031** data points
- Perturbative accuracy: **N^3LL^-**
- ***Normalization*** of SIDIS multiplicities beyond NLL
- Number of fitted parameters: **21**
- Extremely good description: **$\chi^2/N_{\text{data}} = 1.06$**

MAPTMD22: datasets included

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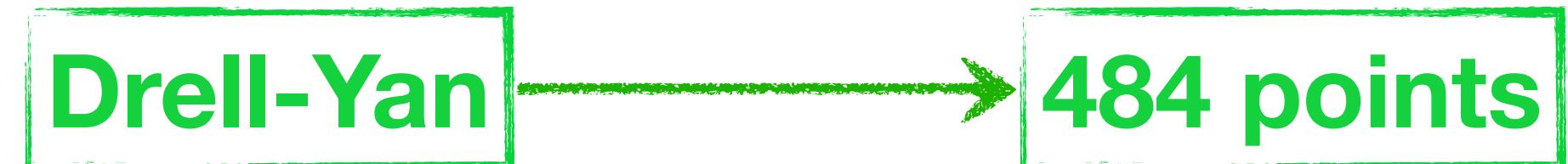
Drell-Yan

Fixed-target low-energy DY

RHIC data

LHC and Tevatron data

MAPTMD22: datasets included

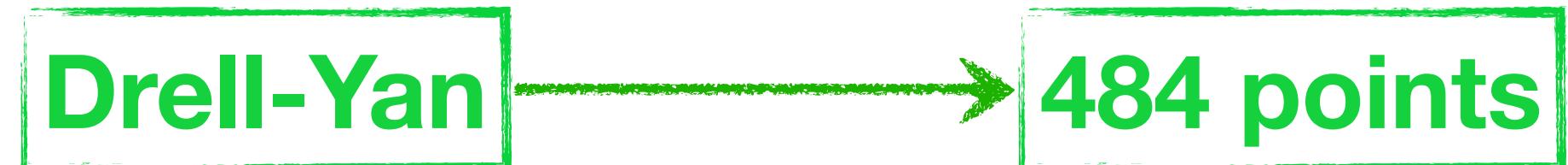


Fixed-target low-energy DY

RHIC data

LHC and Tevatron data

MAPTMD22: datasets included



Fixed-target low-energy DY

RHIC data

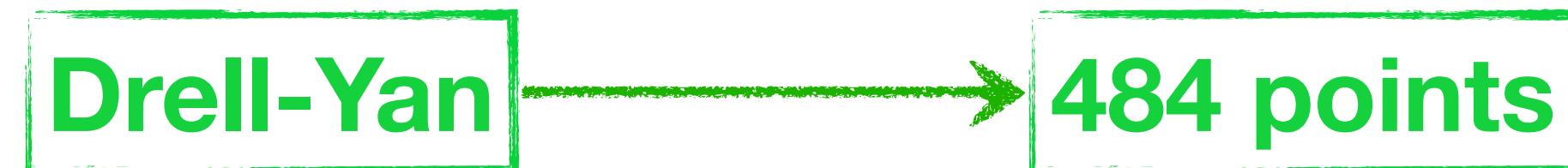
LHC and Tevatron data

SIDIS

HERMES data

COMPASS data

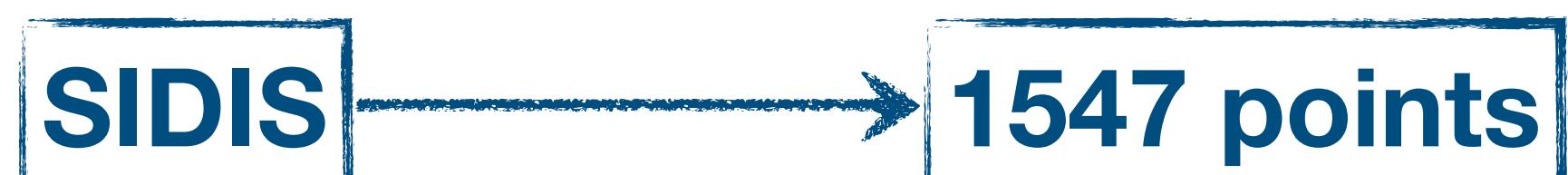
MAPTMD22: datasets included



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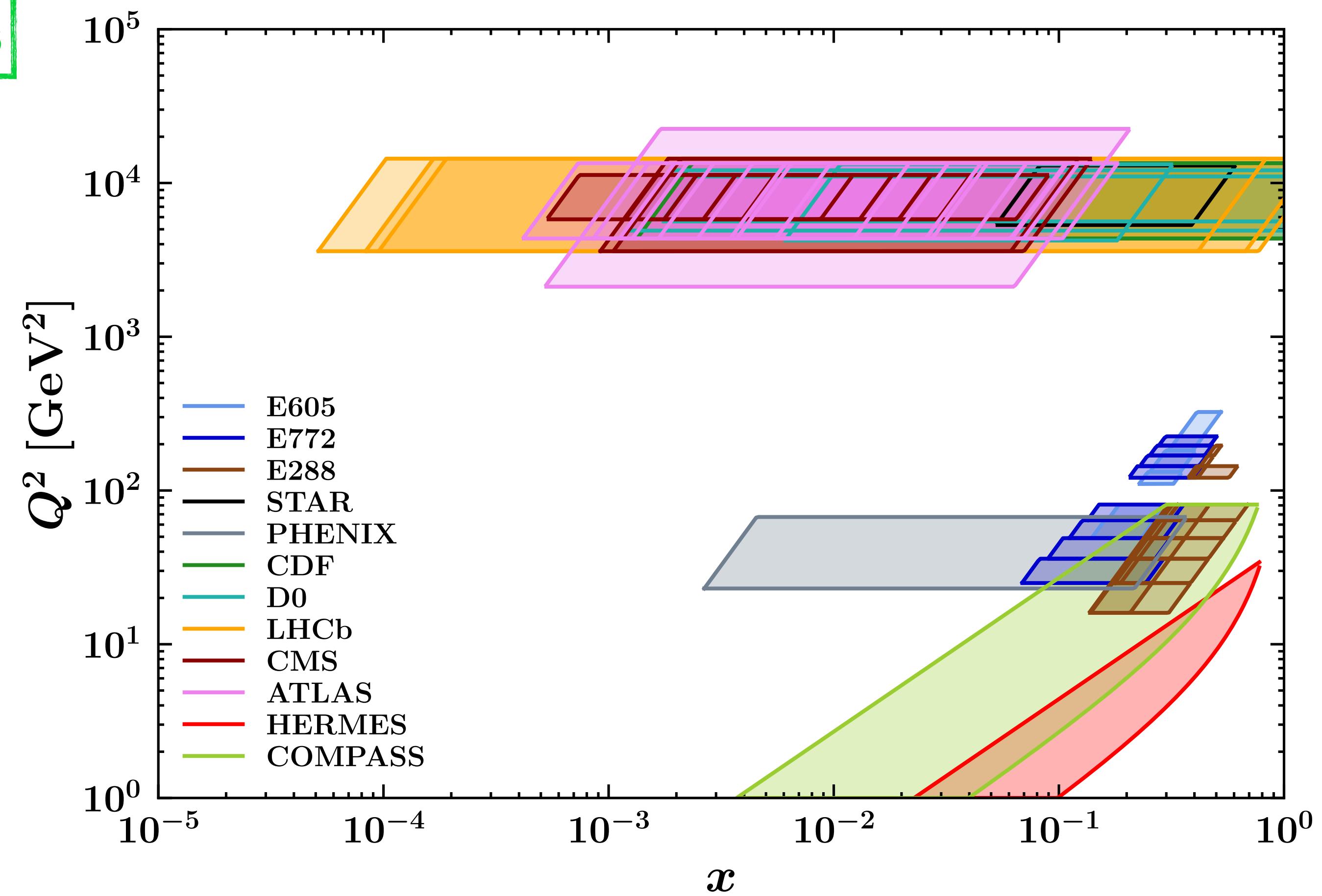
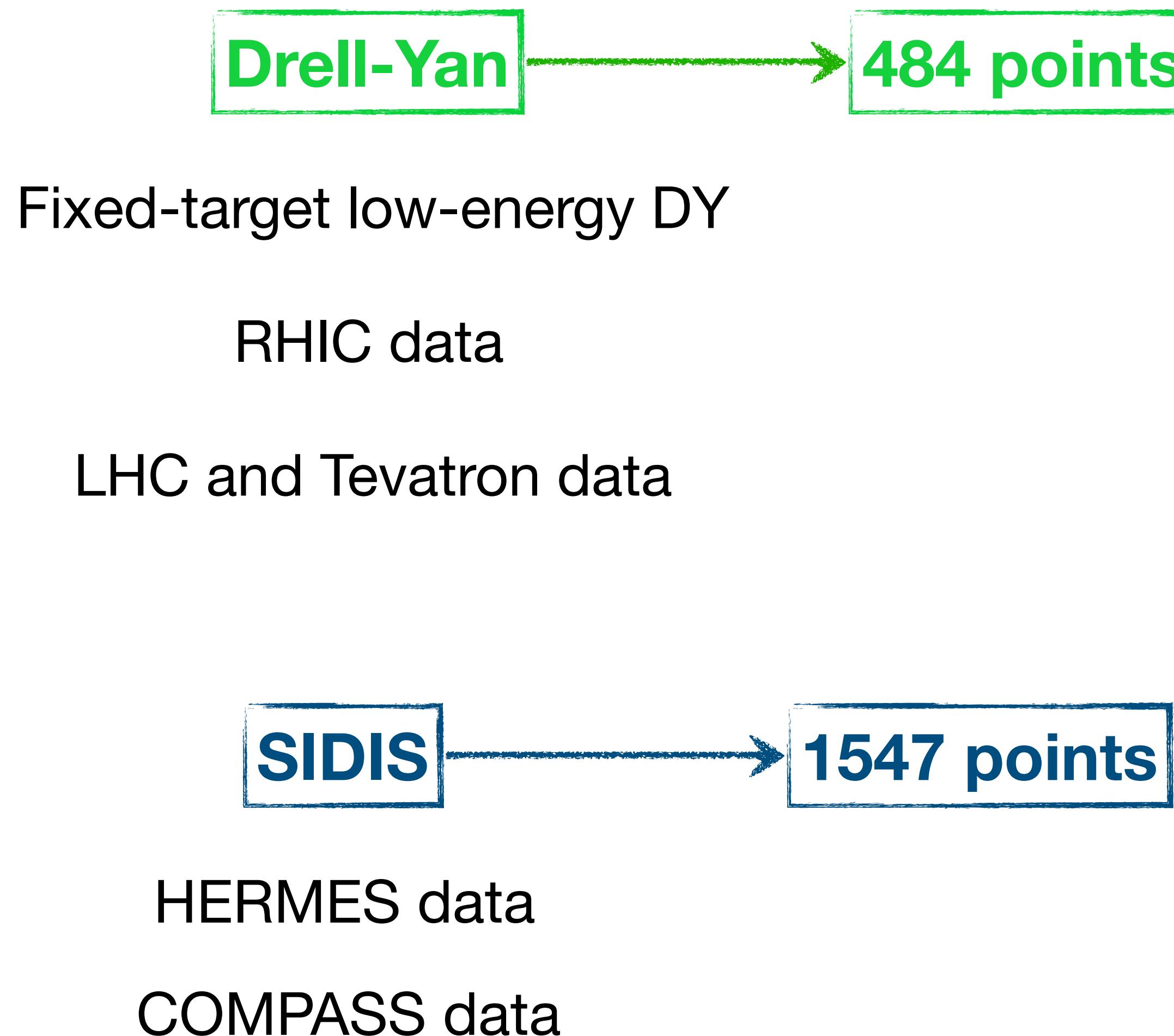
LHC and Tevatron data



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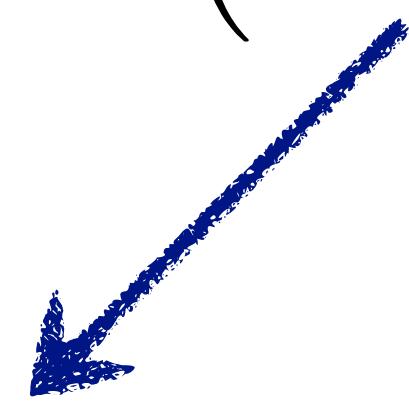
Total: 2031 fitted points

MAPTMD22: Non perturbative part

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

MAPTMD22: Non perturbative part

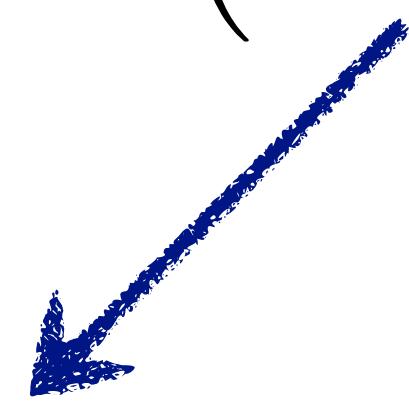
$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$



$$g_1(x) = N_1 \frac{(1-x)^\alpha}{(1-\hat{x})^\alpha} \frac{x^\sigma}{\hat{x}^\sigma}$$

MAPTMD22: Non perturbative part

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$



$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha}{(1-\hat{x})^\alpha} \frac{x^\sigma}{\hat{x}^\sigma}$$

MAPTMD22: Non perturbative part

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g^{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g^{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g^{1C}}} \right)$$

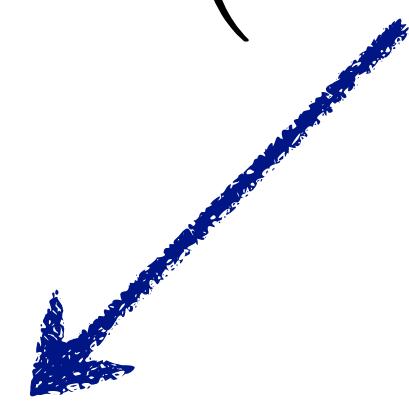
$$g_1(x) = N_1 \frac{(1-x)^\alpha}{(1-\hat{x})^\alpha} \frac{x^\sigma}{\hat{x}^\sigma}$$

$$D_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

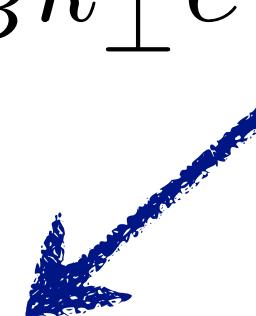
MAPTMD22: Non perturbative part

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$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

MAPTMD22: Non perturbative part

$$f_{1\text{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

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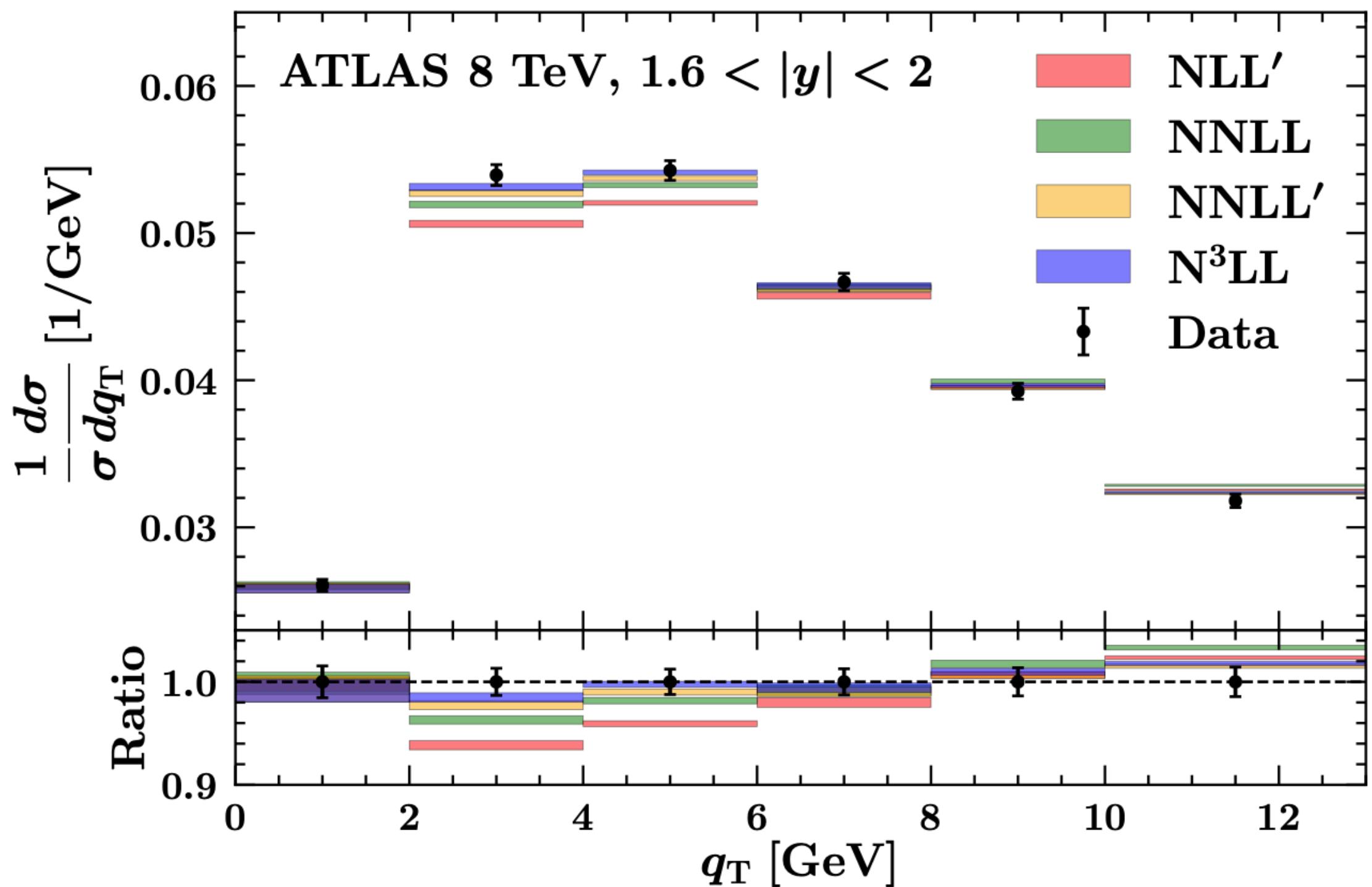
$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters

MAPTMD22: Normalization of SIDIS

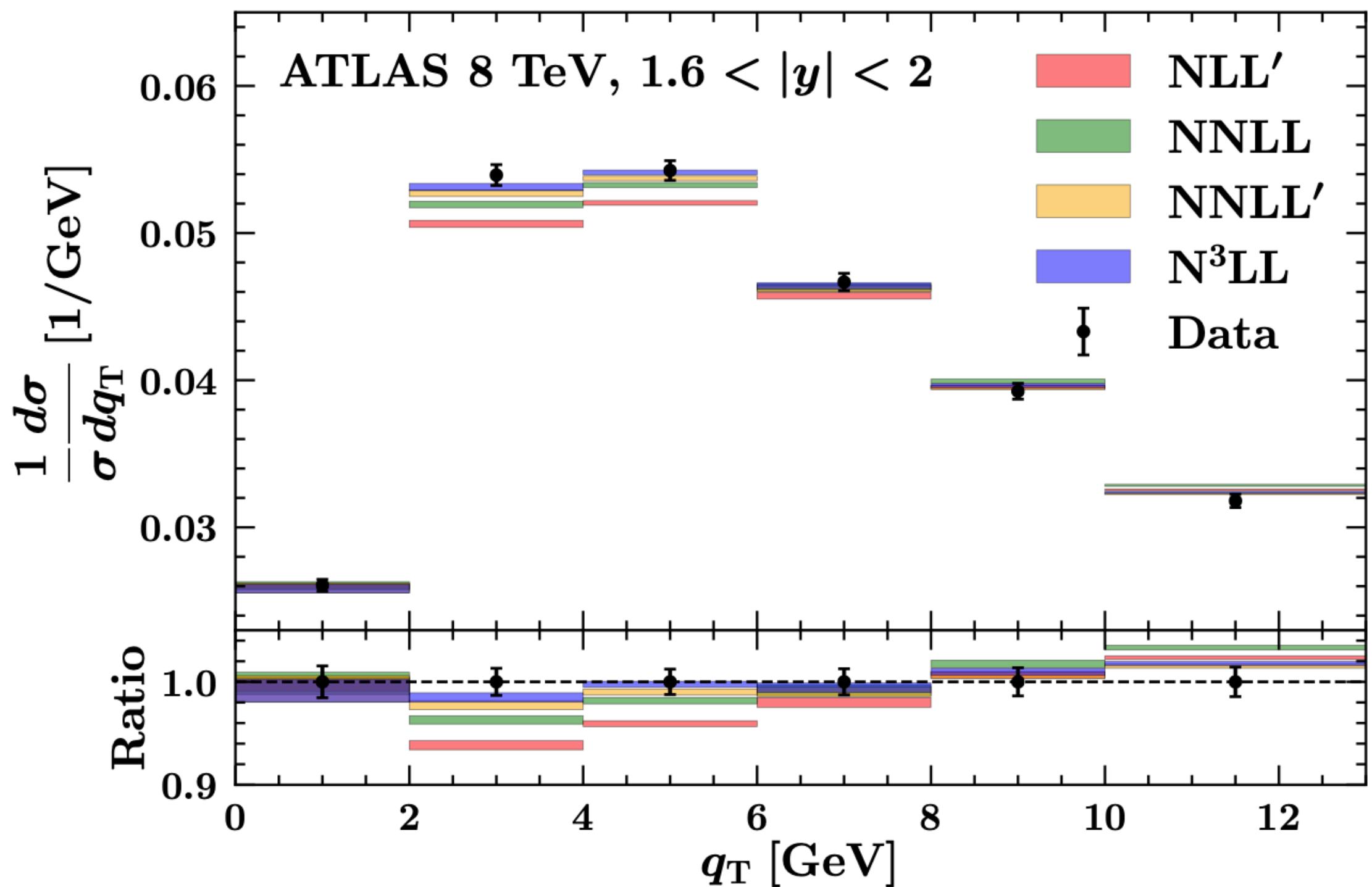
MAPTMD22: Normalization of SIDIS

High Energy Drell-Yan



MAPTMD22: Normalization of SIDIS

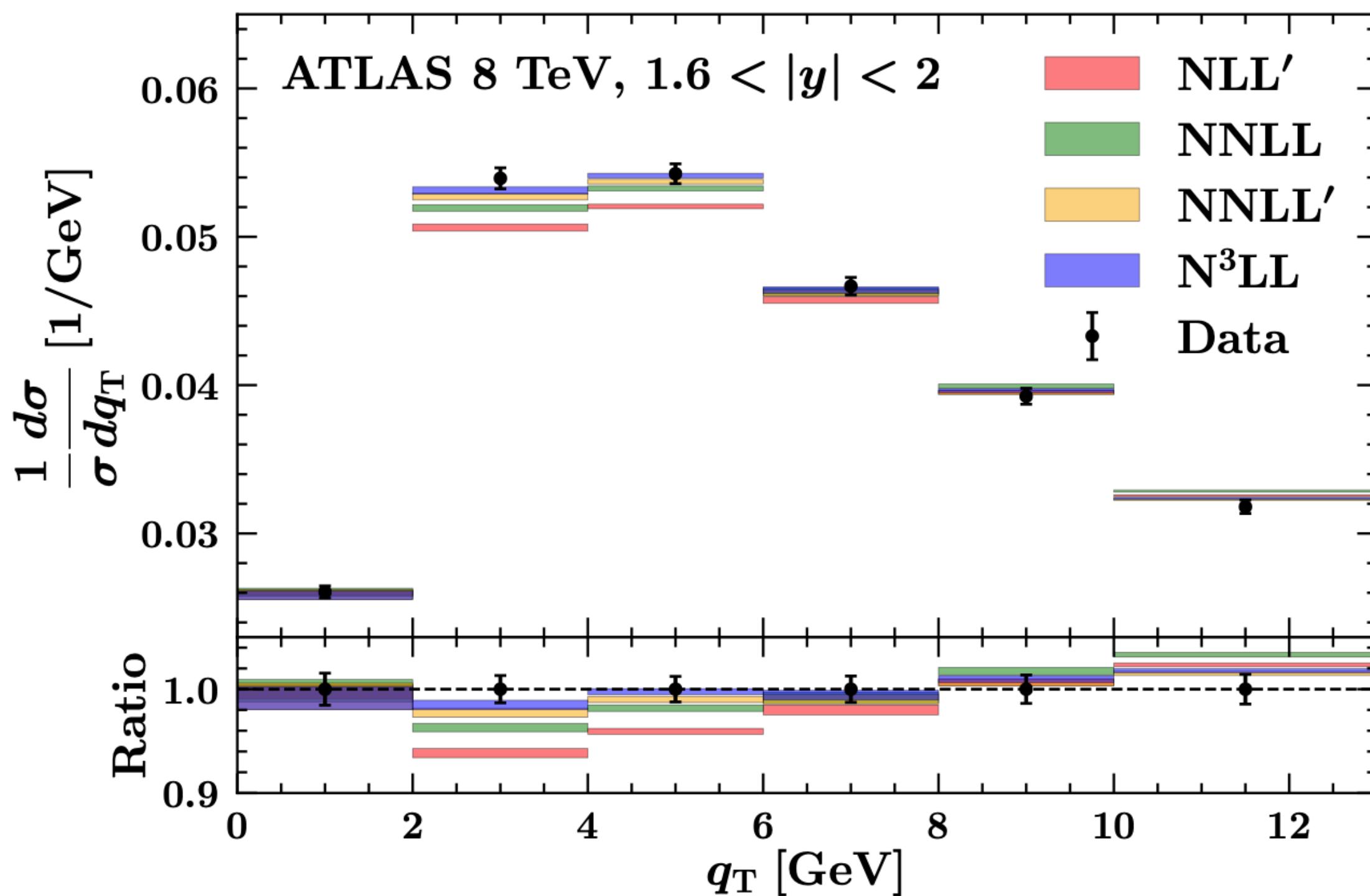
High Energy Drell-Yan



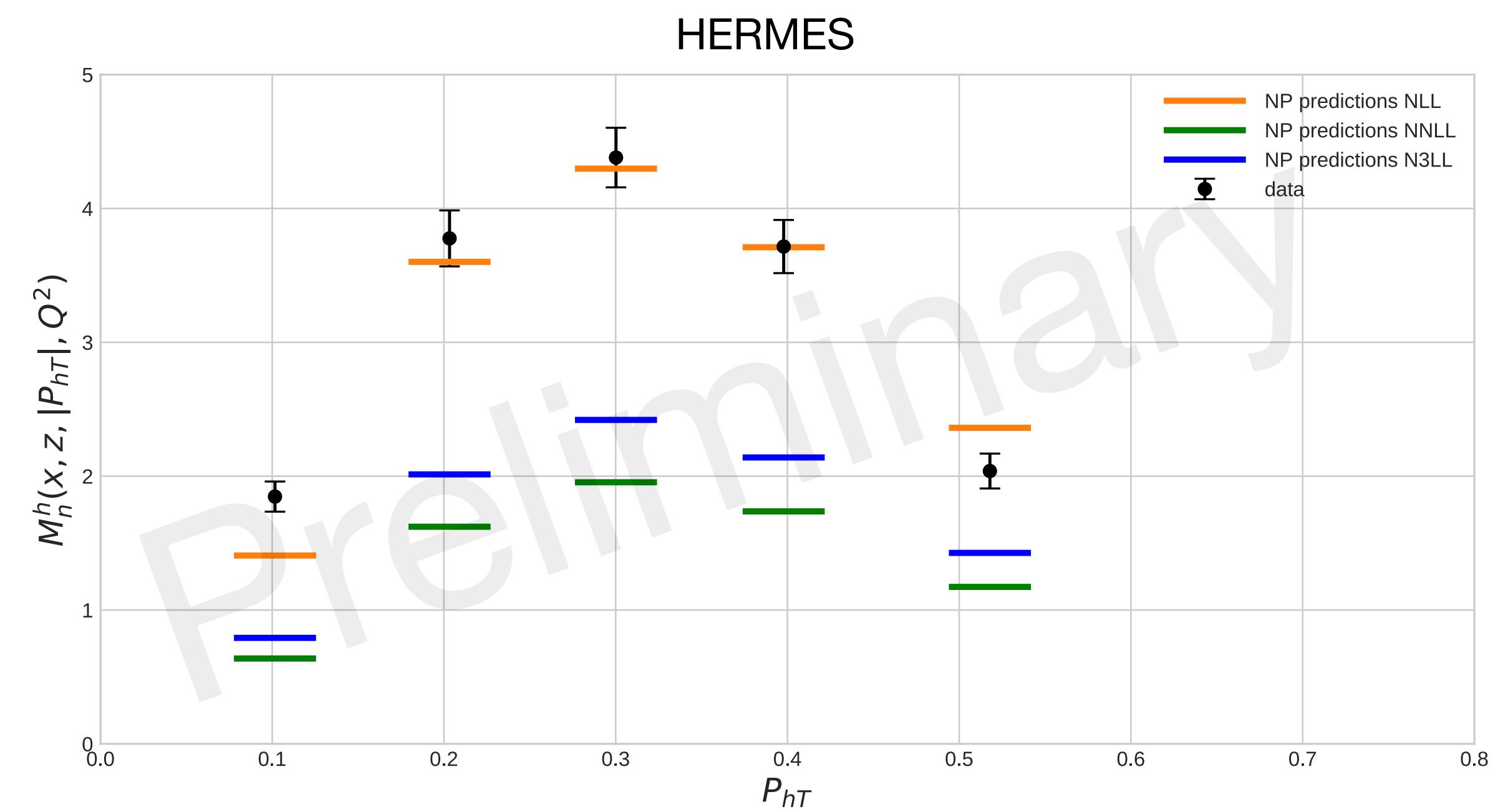
The description improves at high orders

MAPTMD22: Normalization of SIDIS

High Energy Drell-Yan



SIDIS

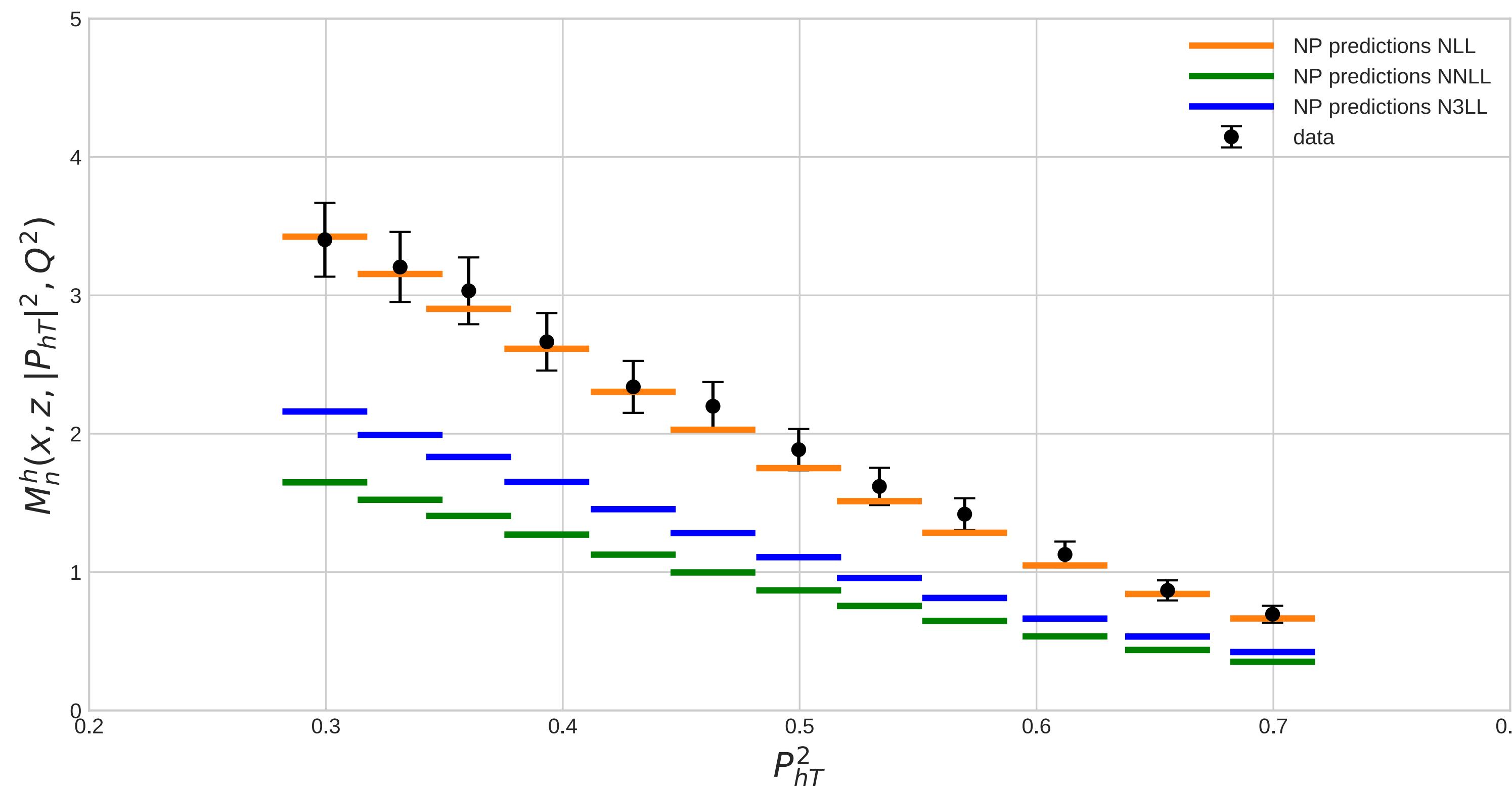


The description improves at high orders

Strange behaviors at higher orders

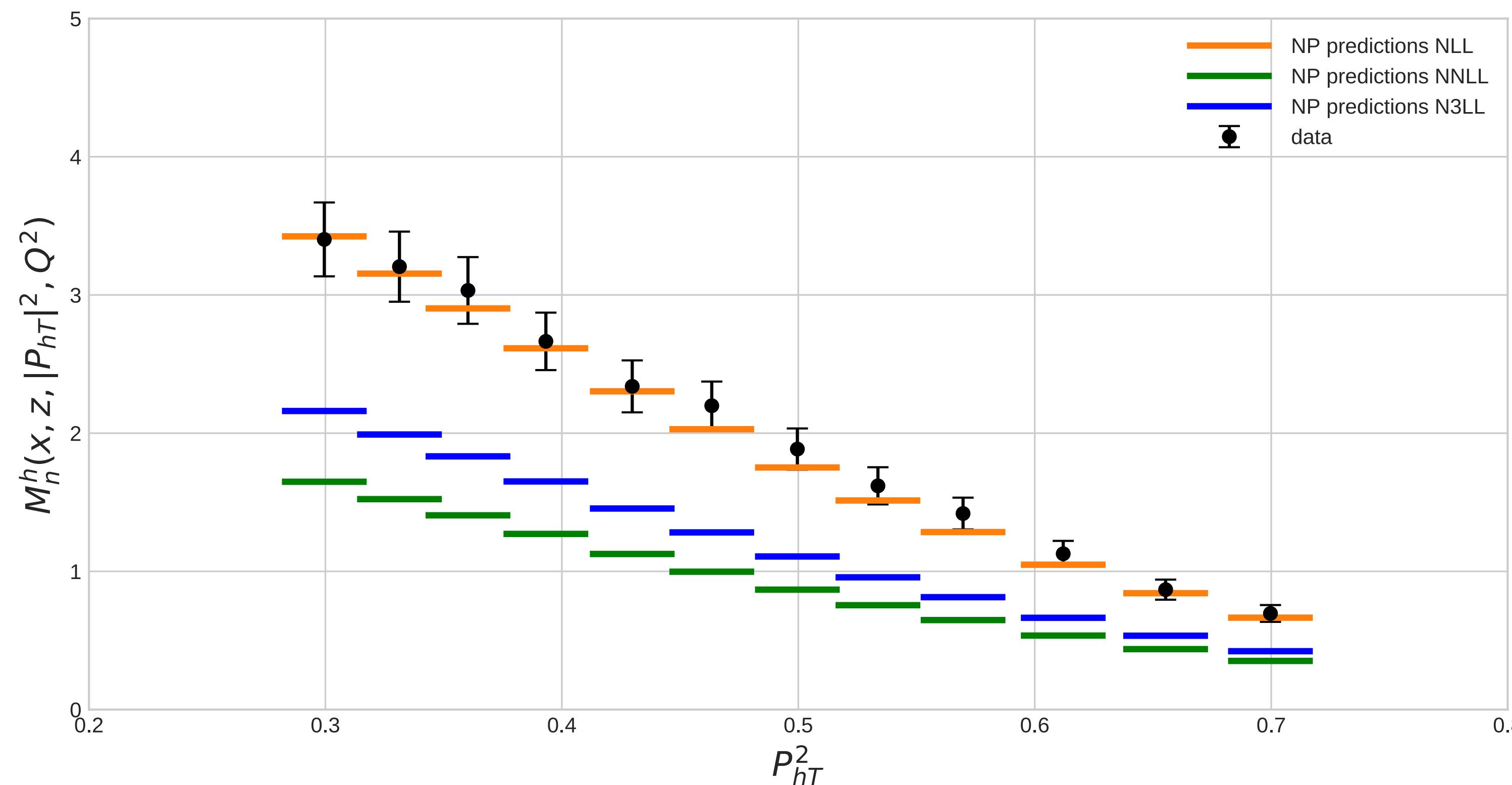
MAPTMD22: Normalization of SIDIS

COMPASS multiplicities (one of many bins)



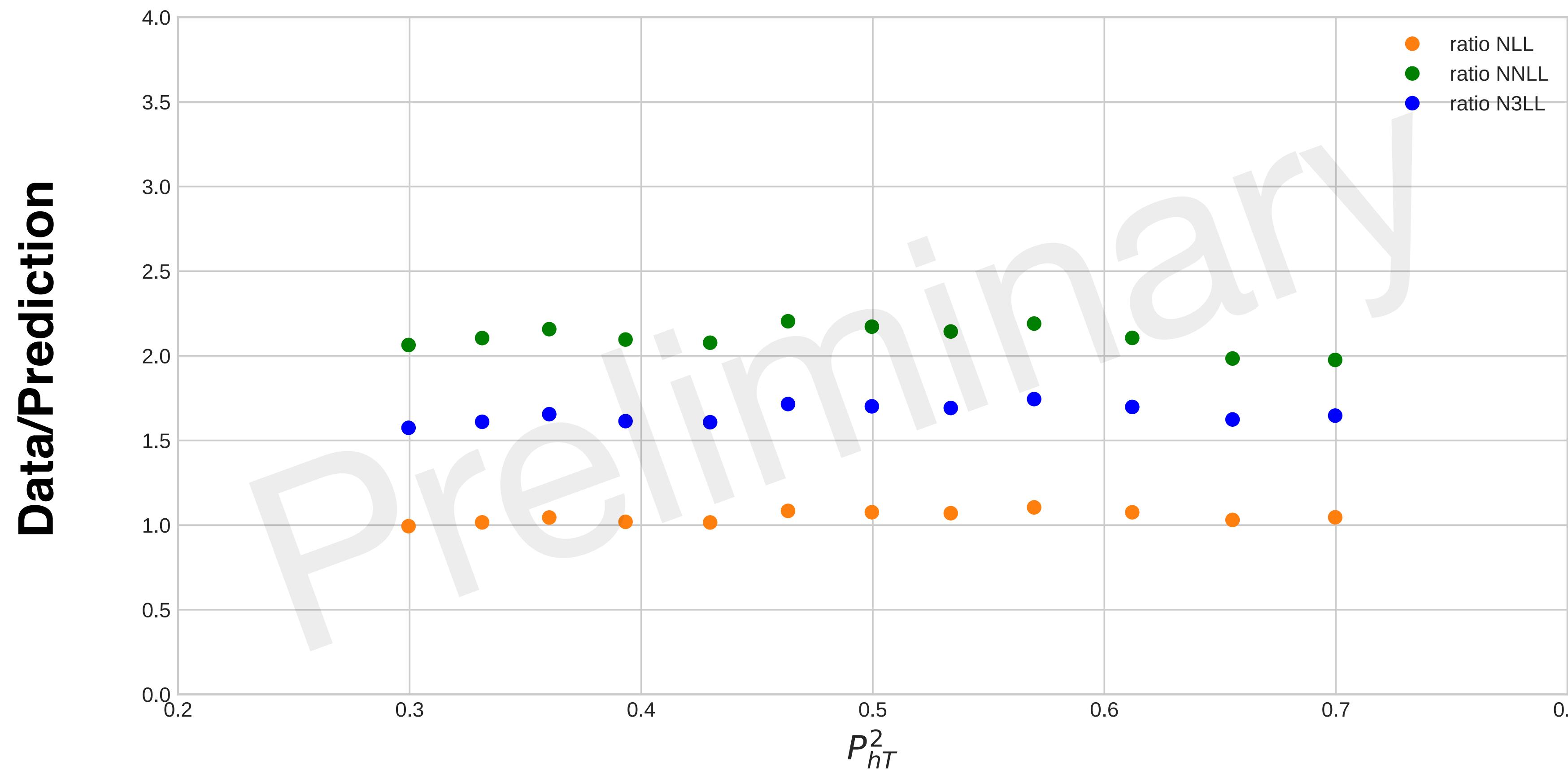
MAPTMD22: Normalization of SIDIS

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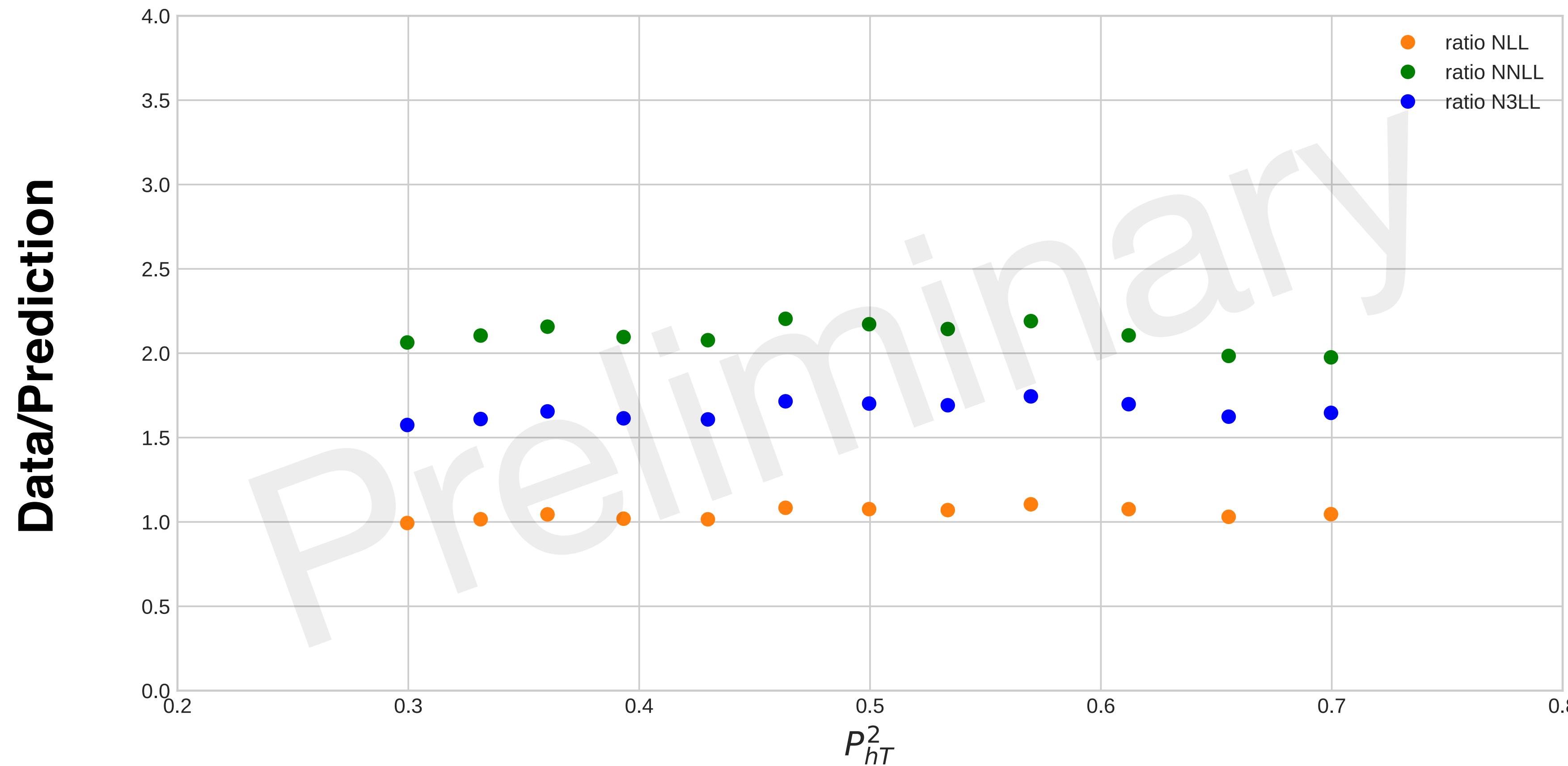
MAPTMD22: Normalization of SIDIS

COMPASS multiplicities (one of many bins)



MAPTMD22: Normalization of SIDIS

COMPASS multiplicities (one of many bins)



For different orders the discrepancy amounts to a nearly constant factor

MAPTMD22: Normalization of SIDIS

MAPTMD22: Normalization of SIDIS

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dxdQ \cancel{dzdP_{hT}}} \Bigg/ \frac{d\sigma}{dxdQ}$$

MAPTMD22: Normalization of SIDIS

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ \cancel{dz} dP_{hT}} \Bigg/ \frac{d\sigma}{dx dQ}$$

Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ \cancel{dz}}$$

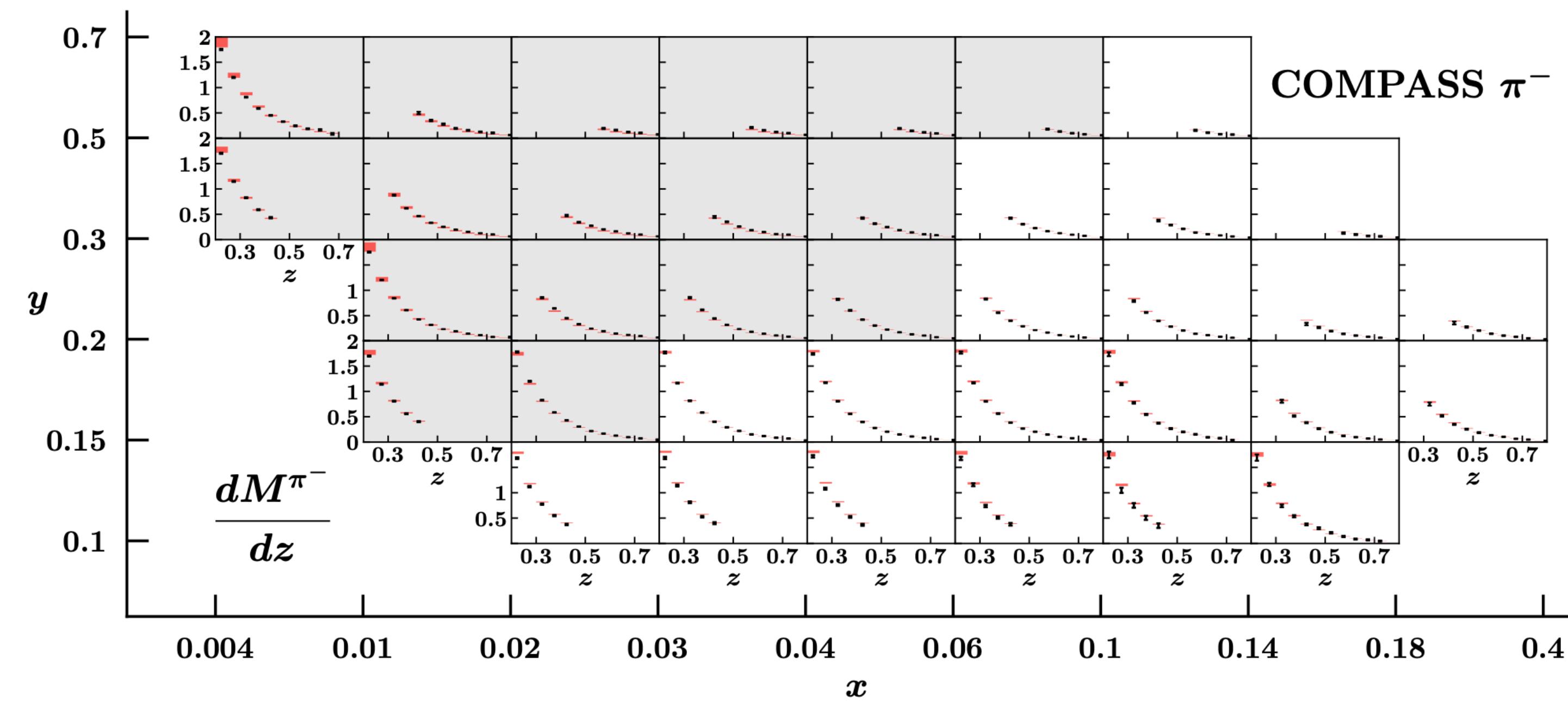
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Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ dz}$$



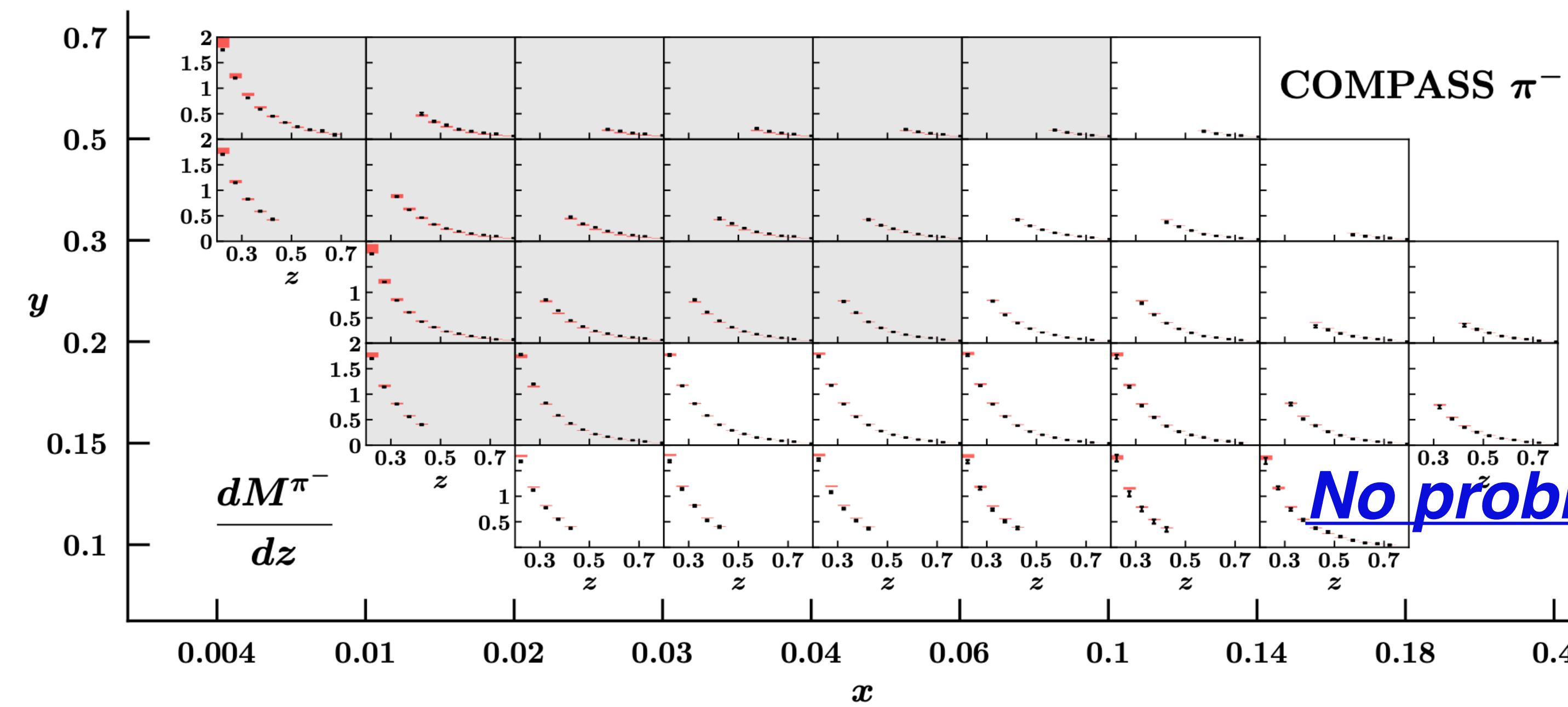
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Collinear SIDIS cross section

$$\frac{d\sigma}{dxdQ \cancel{dz}}$$

$$\int dP_{hT} \frac{d\sigma}{dxdQ \cancel{dz} dP_{hT}} = \frac{d\sigma}{dxdQ \cancel{dz}}$$

MAPTMD22: Normalization of SIDIS

SIDIS multiplicity

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Collinear SIDIS cross section

$$\frac{d\sigma}{dx dQ dz}$$

$$\int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}} = \frac{d\sigma}{dx dQ dz}$$

$$w(x, z, Q) = \frac{d\sigma}{dx dQ dz} \Bigg/ \int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}}$$

MAPTMD22: Normalization of SIDIS

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Bigg/ \frac{d\sigma}{dx dQ}$$

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$$\frac{d\sigma}{dx dQ dz}$$

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SIDIS multiplicity

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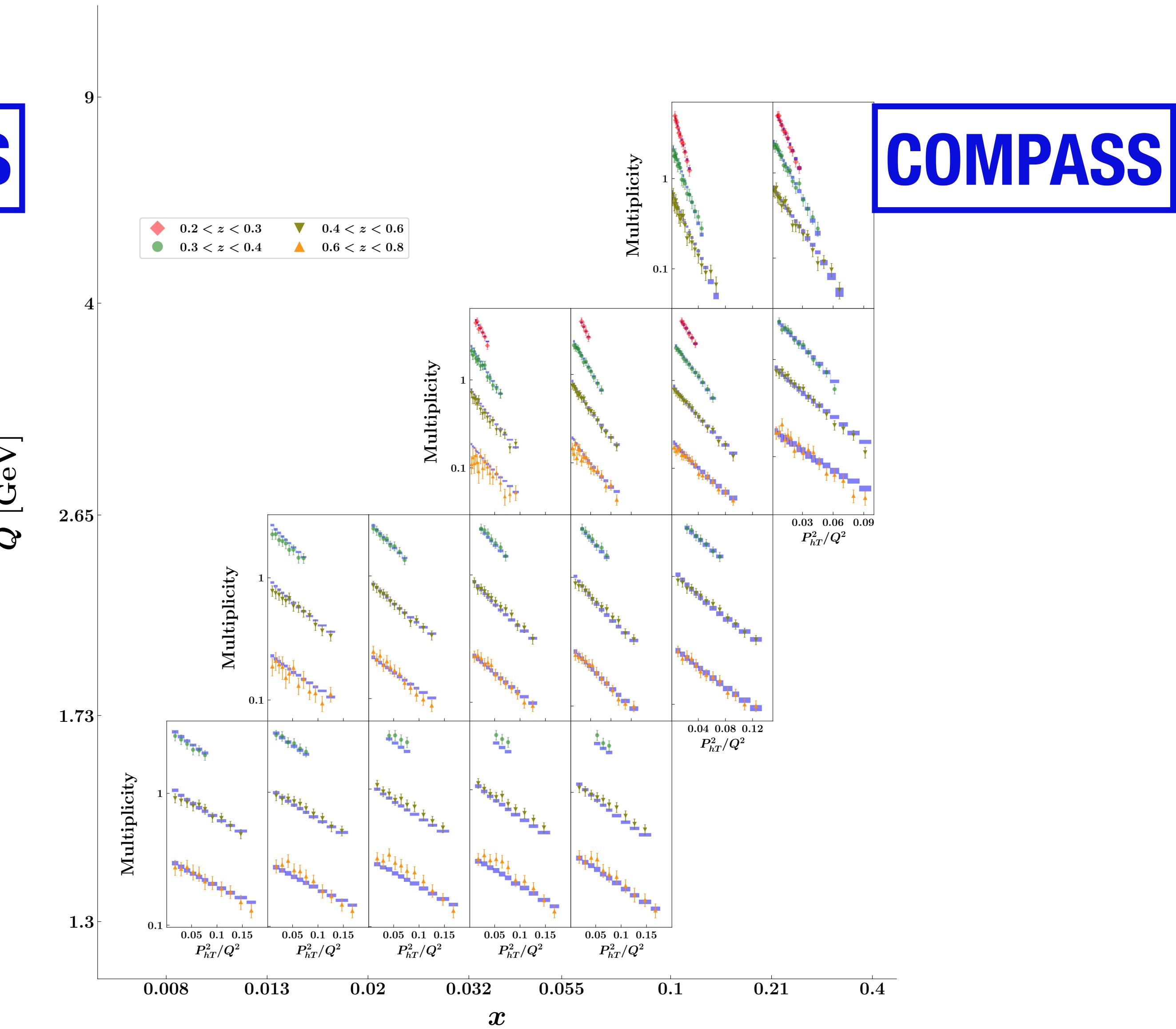
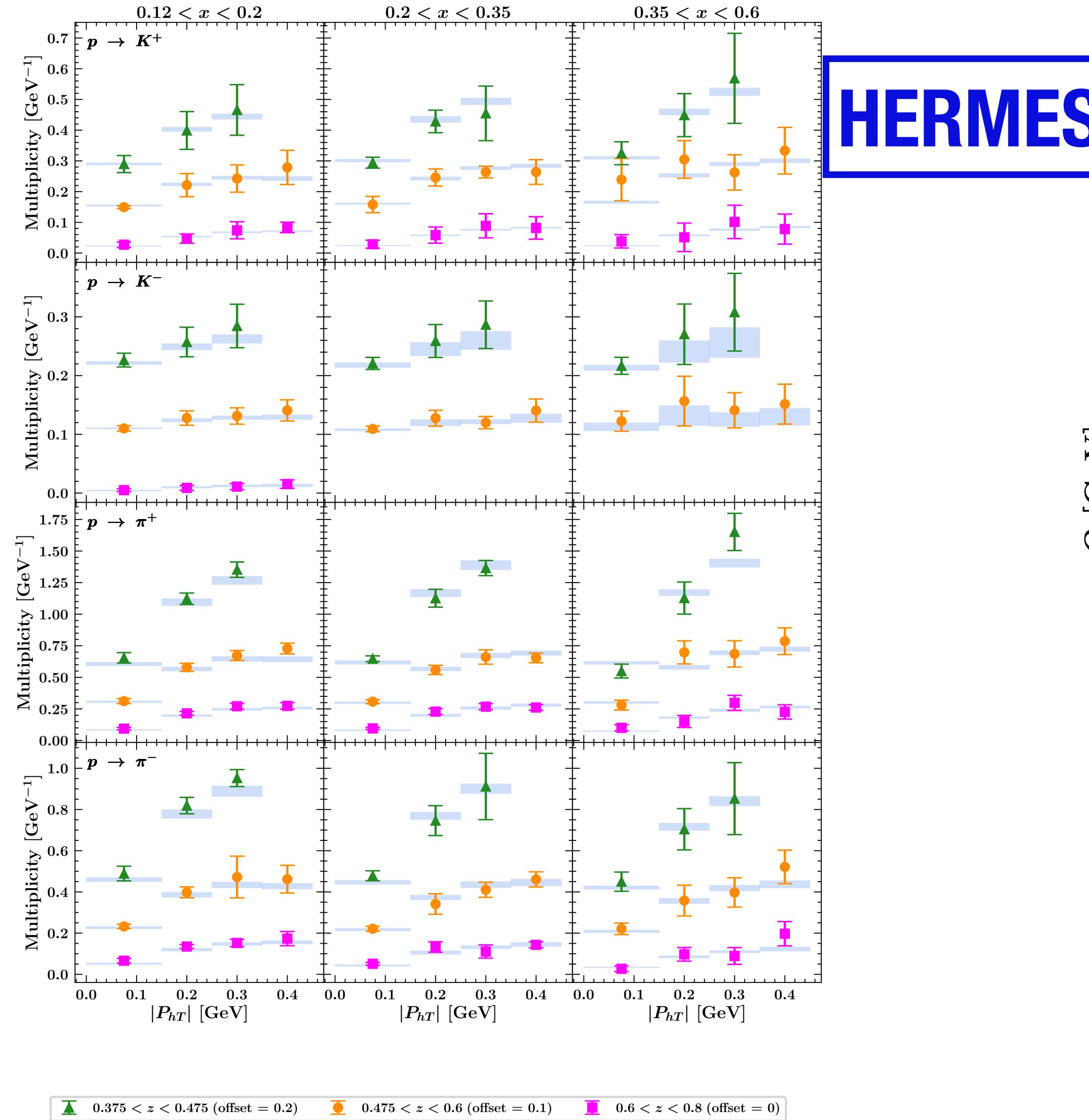
$$\int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}} = \frac{d\sigma}{dx dQ dz}$$

***Fitting parameters
independent***

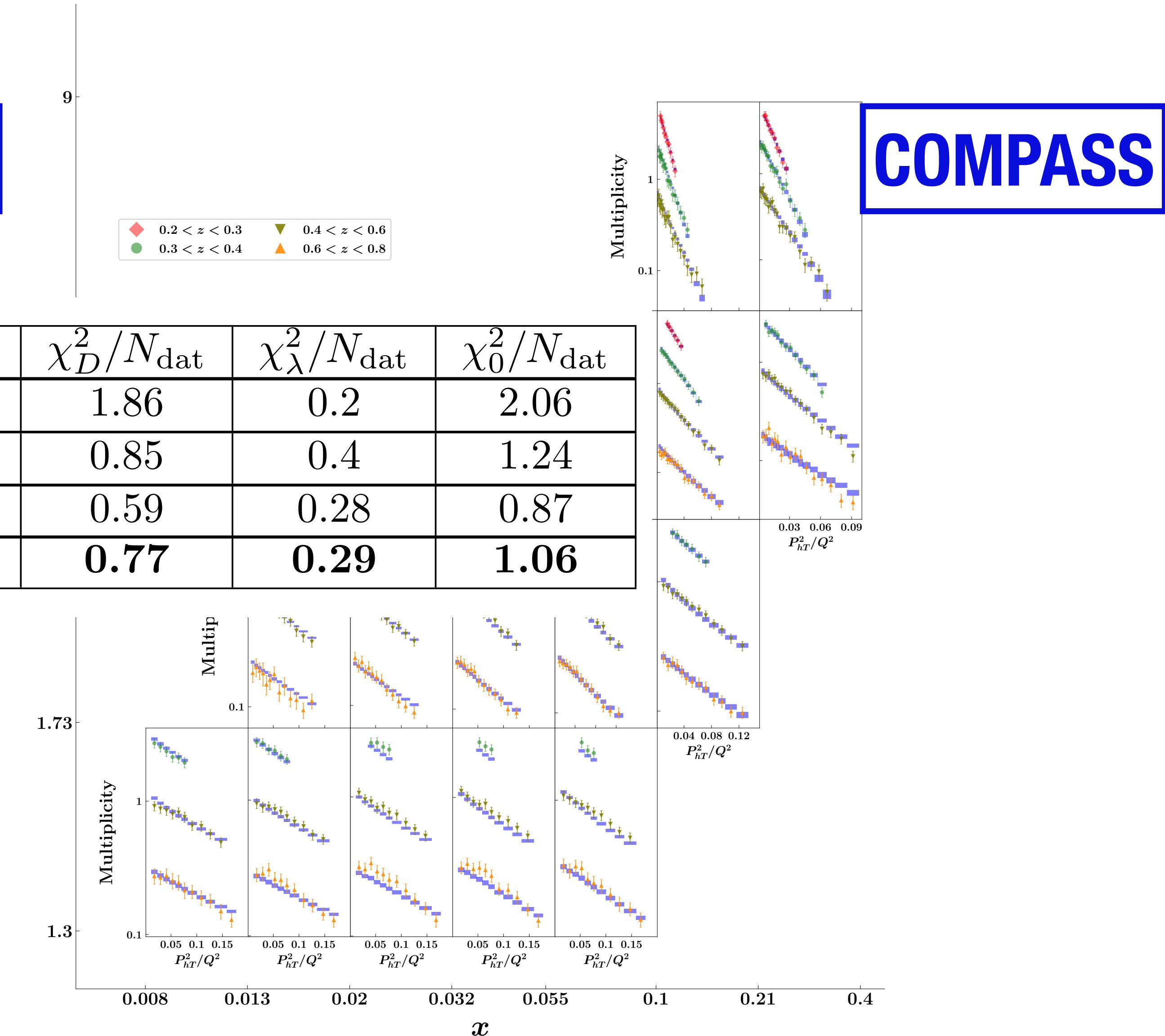
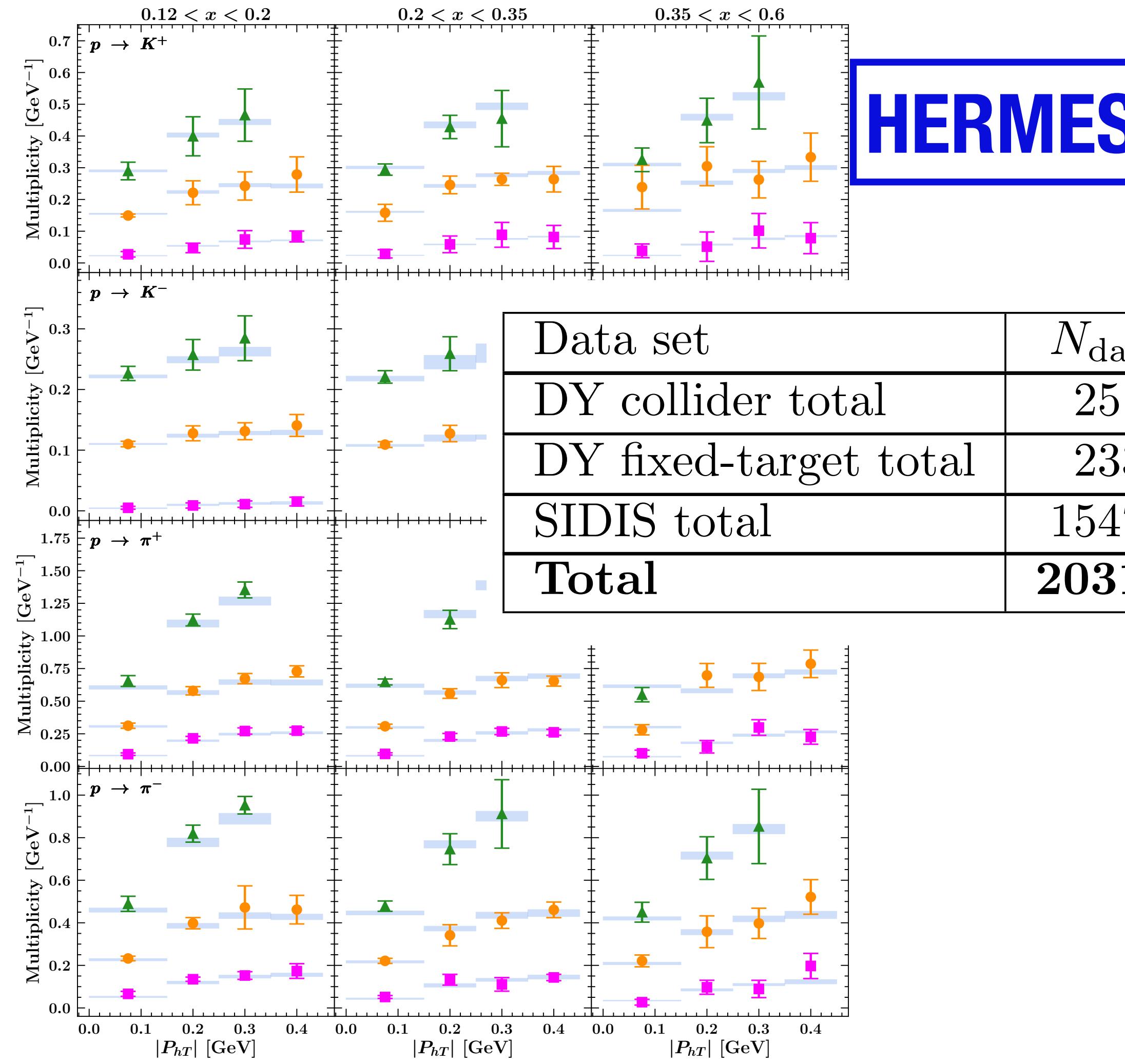
$$w(x, z, Q) = \frac{d\sigma}{dx dQ dz} \Bigg/ \int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}}$$

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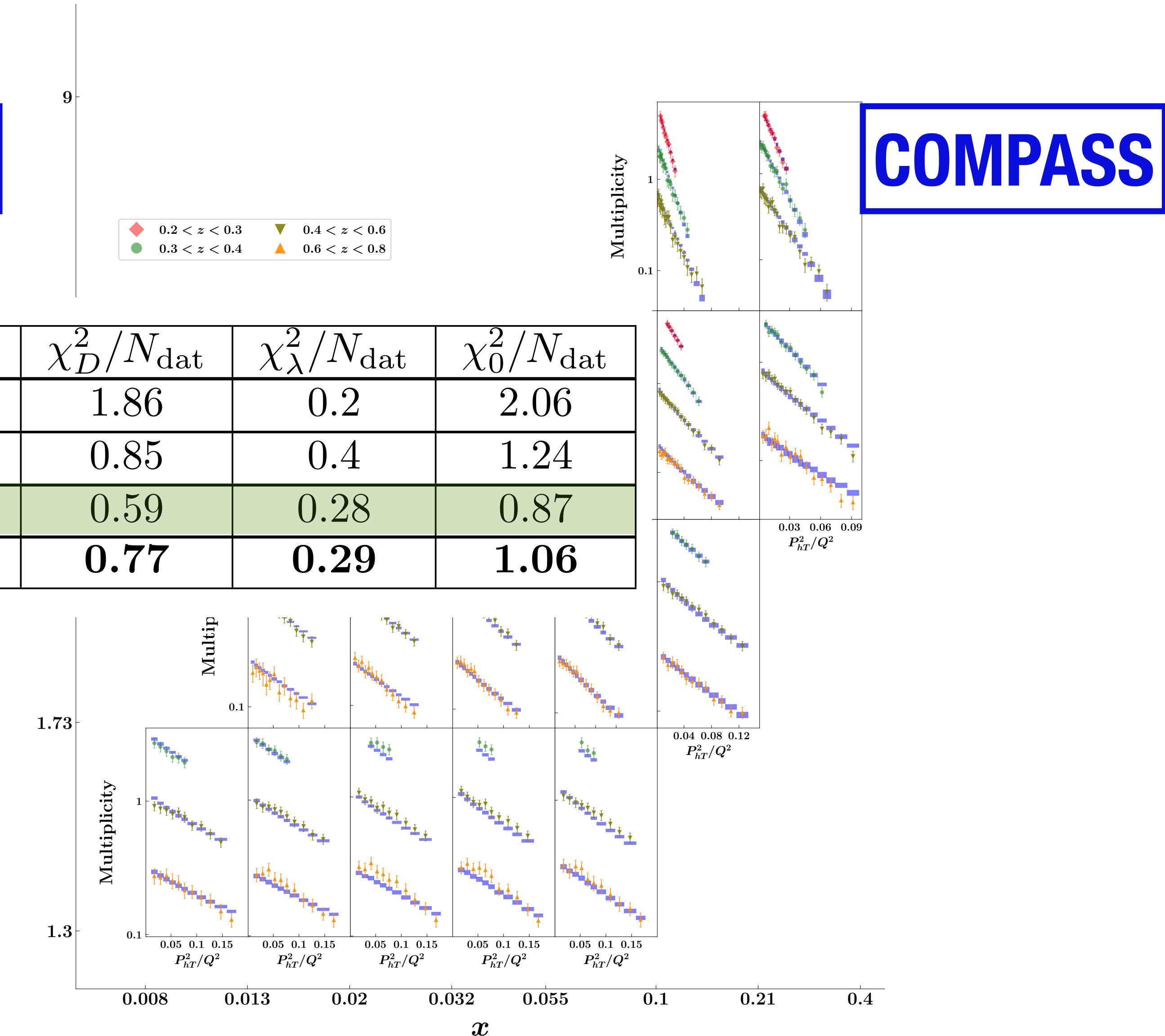
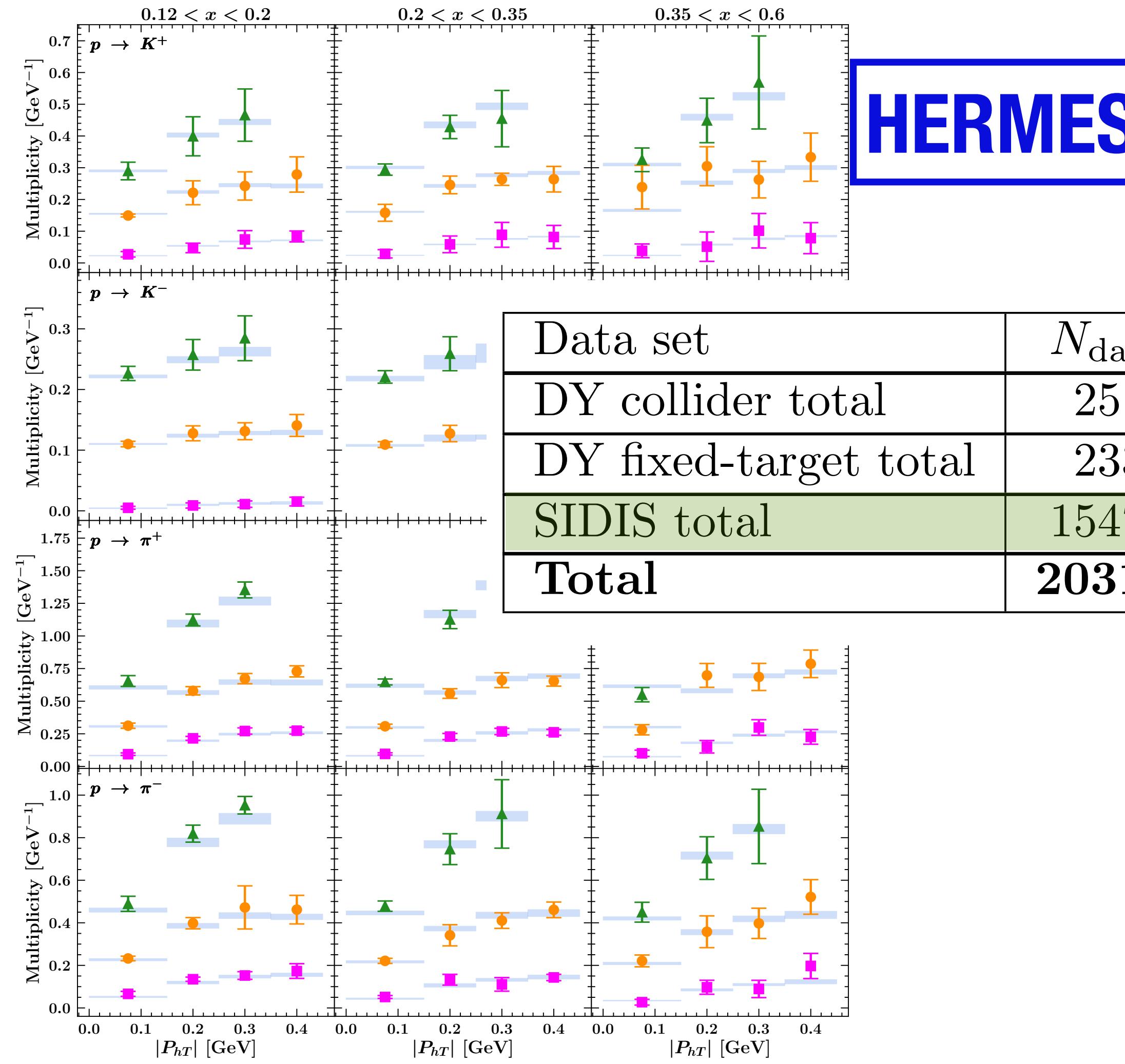
MAPTMD22 – Results of the fit $\chi^2/N_{data} = 1.06$



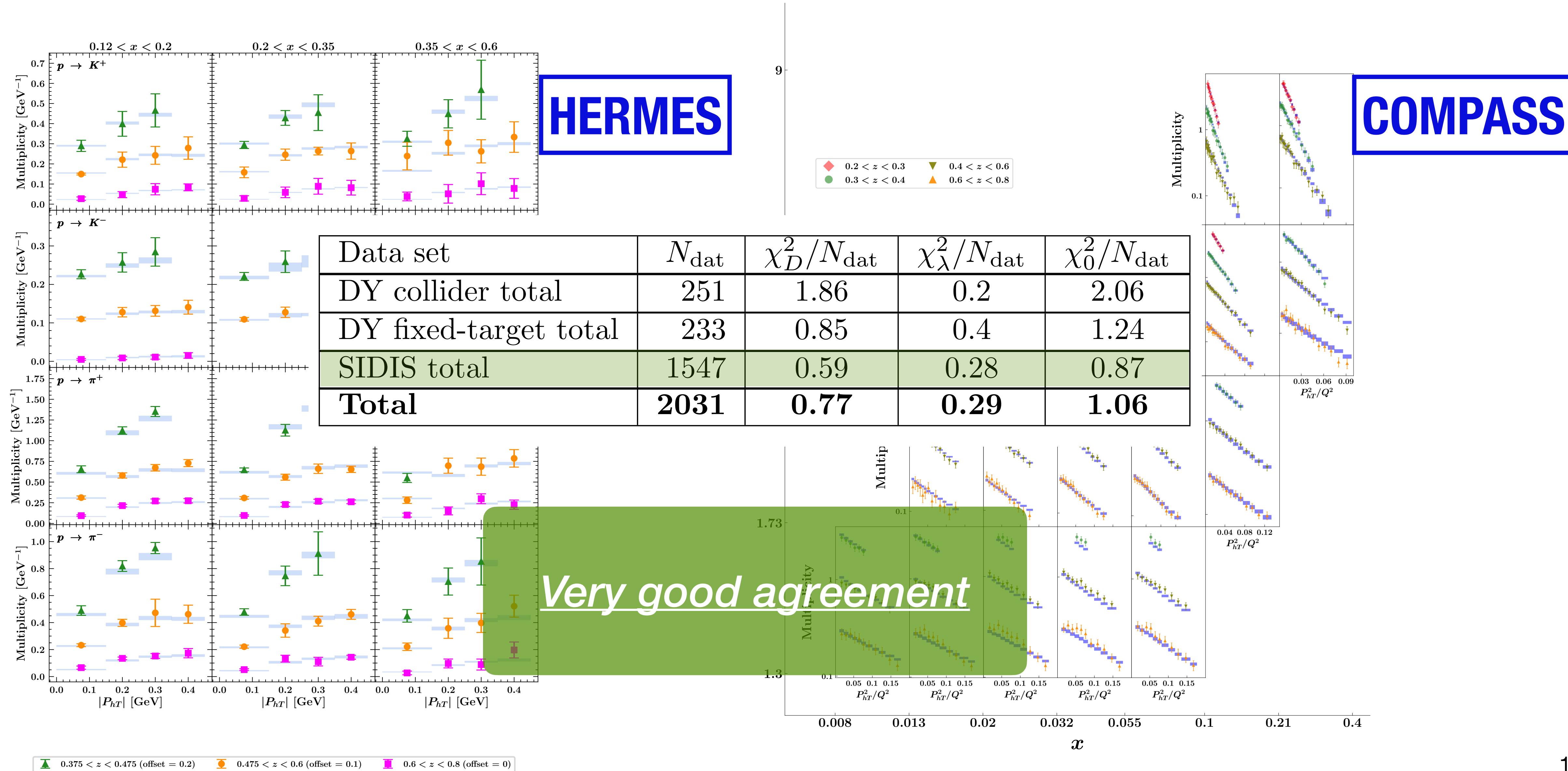
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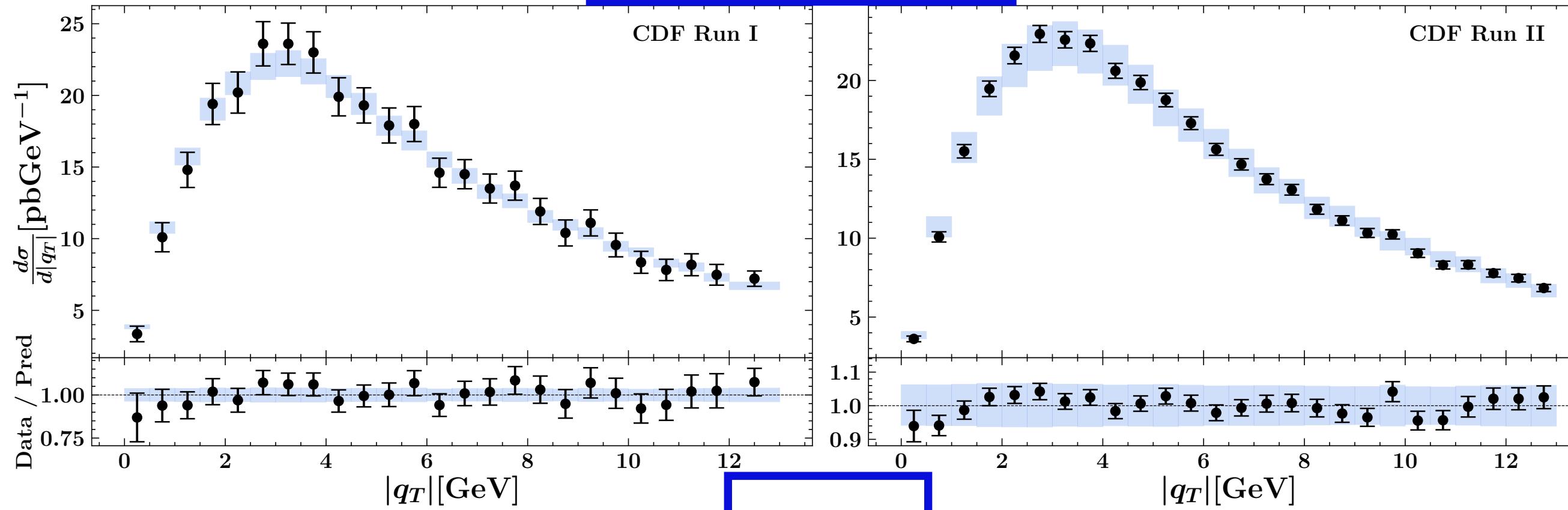


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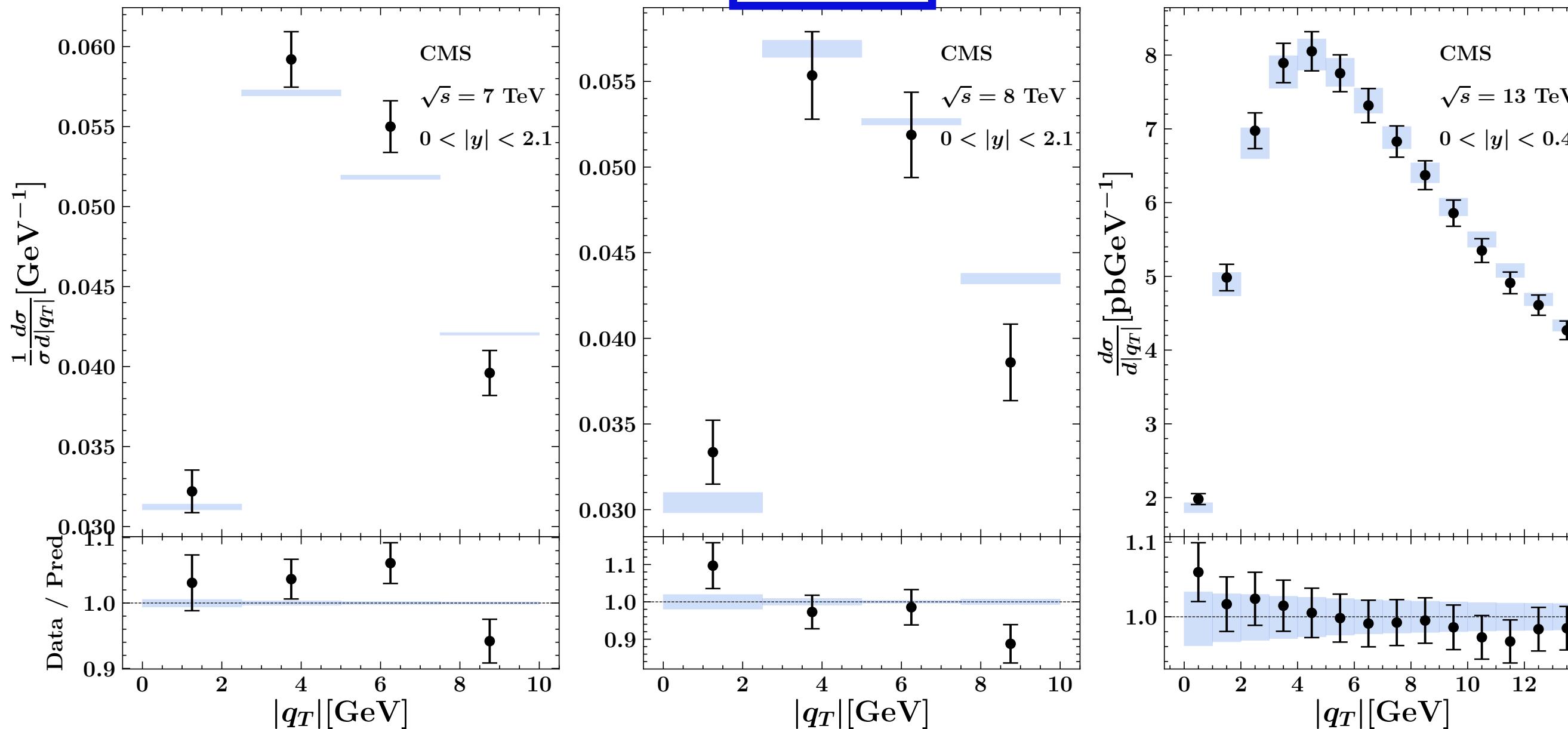


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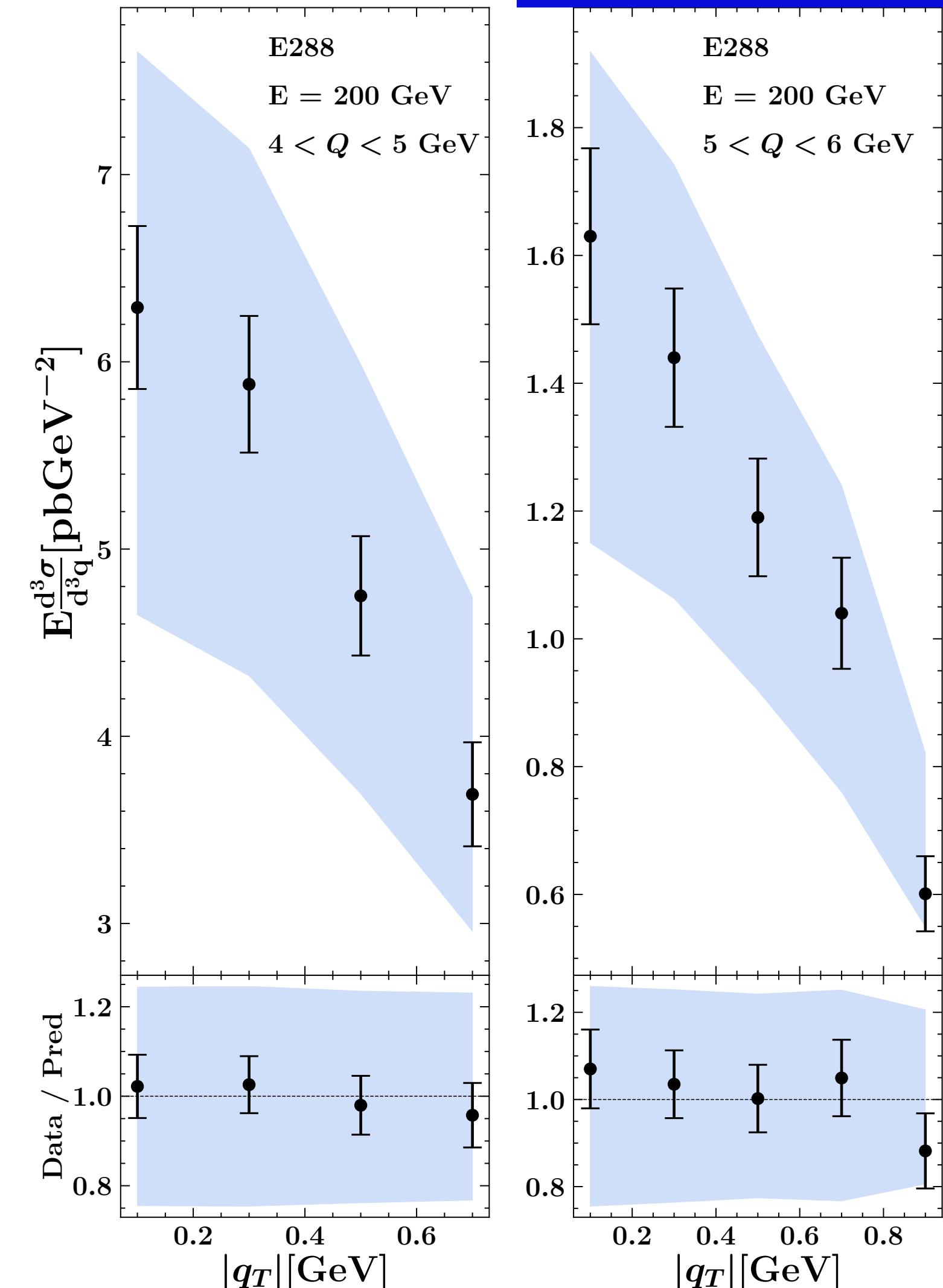
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CMS

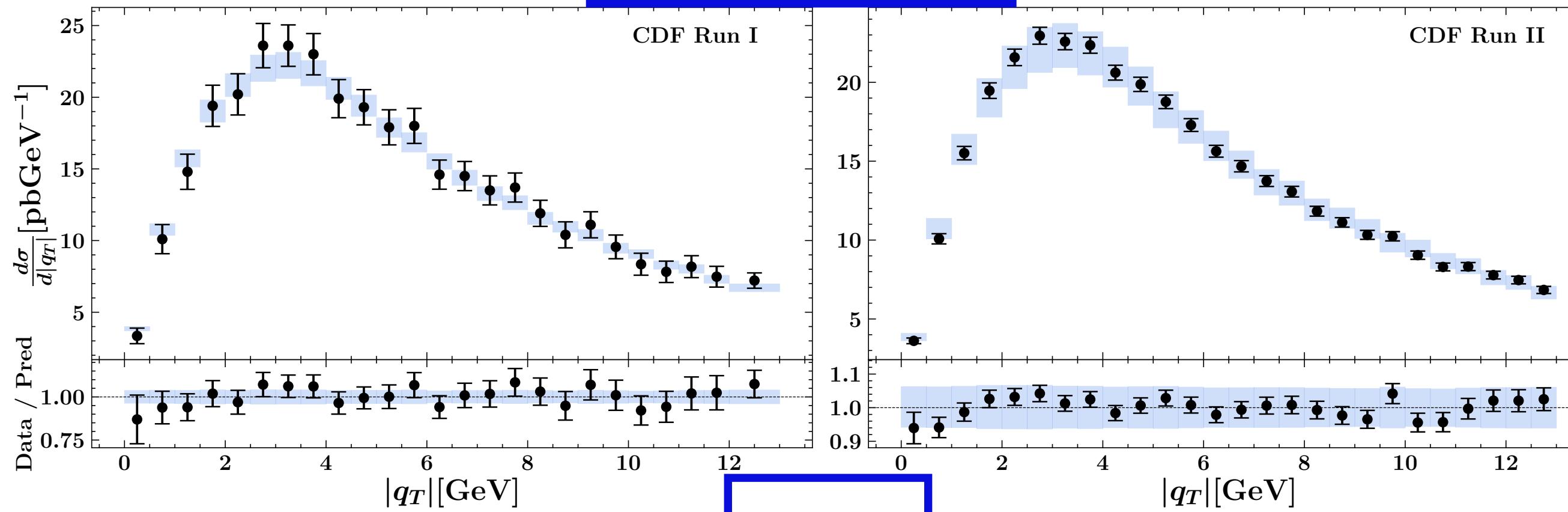


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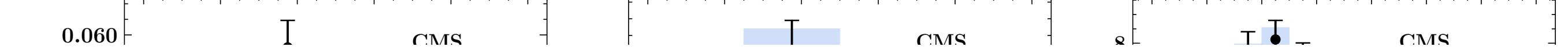


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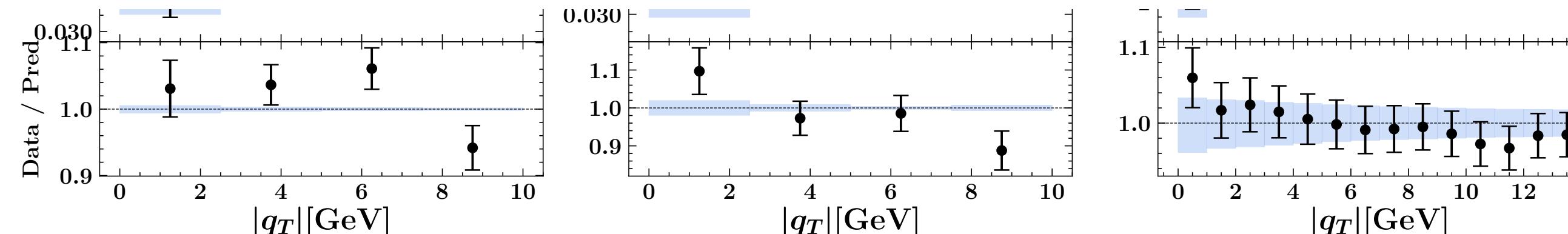
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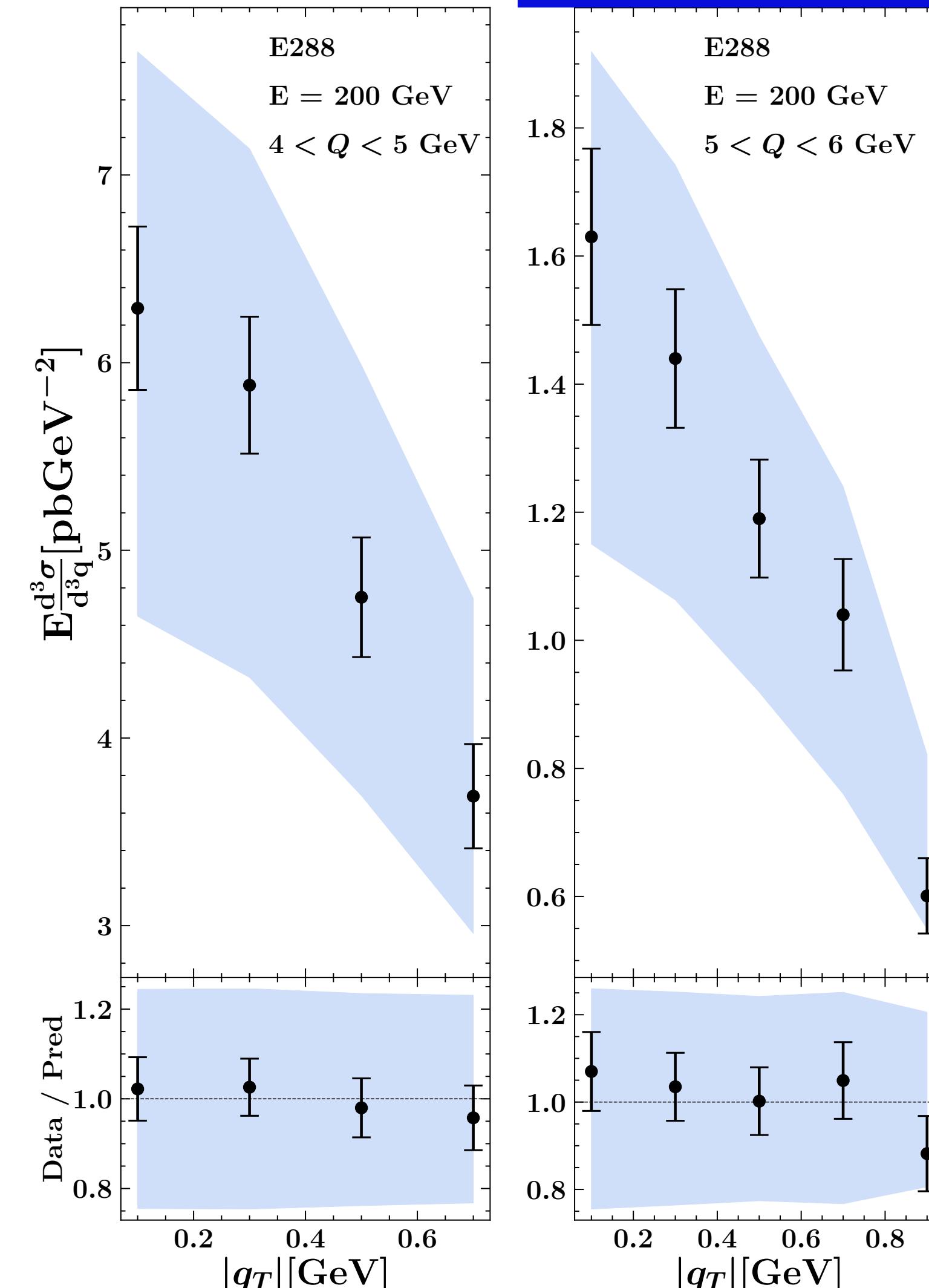
CMS



Data set	N_{dat}	χ^2_D/N_{dat}	χ^2_λ/N_{dat}	χ^2_0/N_{dat}
DY collider total	251	1.86	0.2	2.06
DY fixed-target total	233	0.85	0.4	1.24
SIDIS total	1547	0.59	0.28	0.87
Total	2031	0.77	0.29	1.06

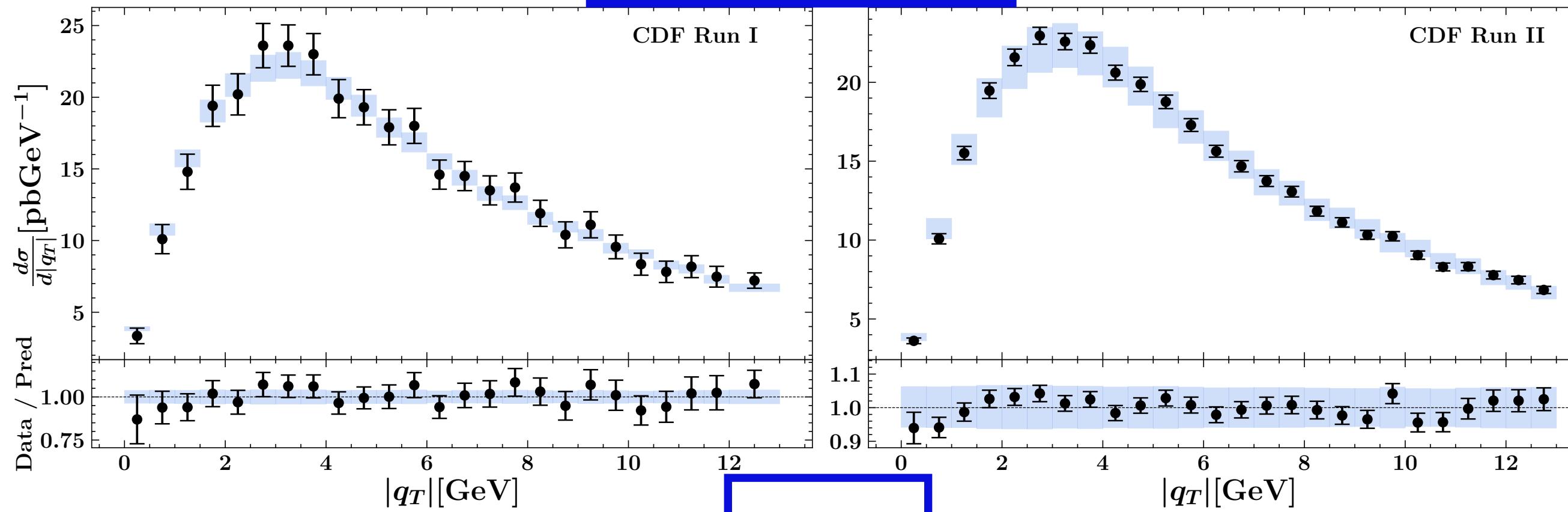


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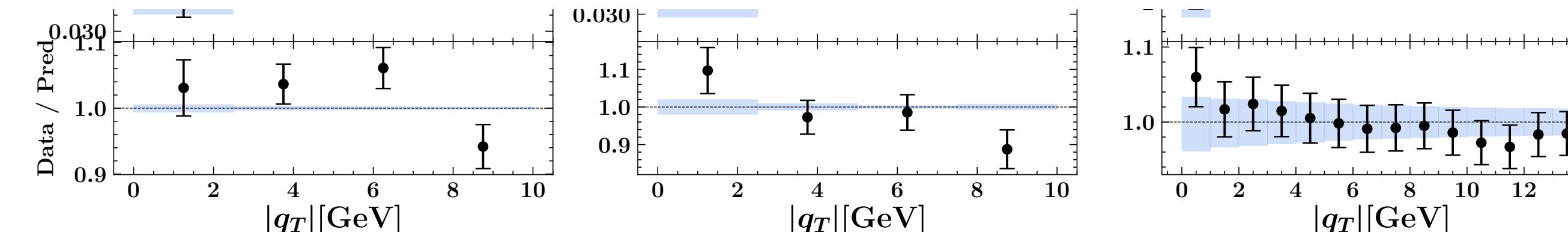
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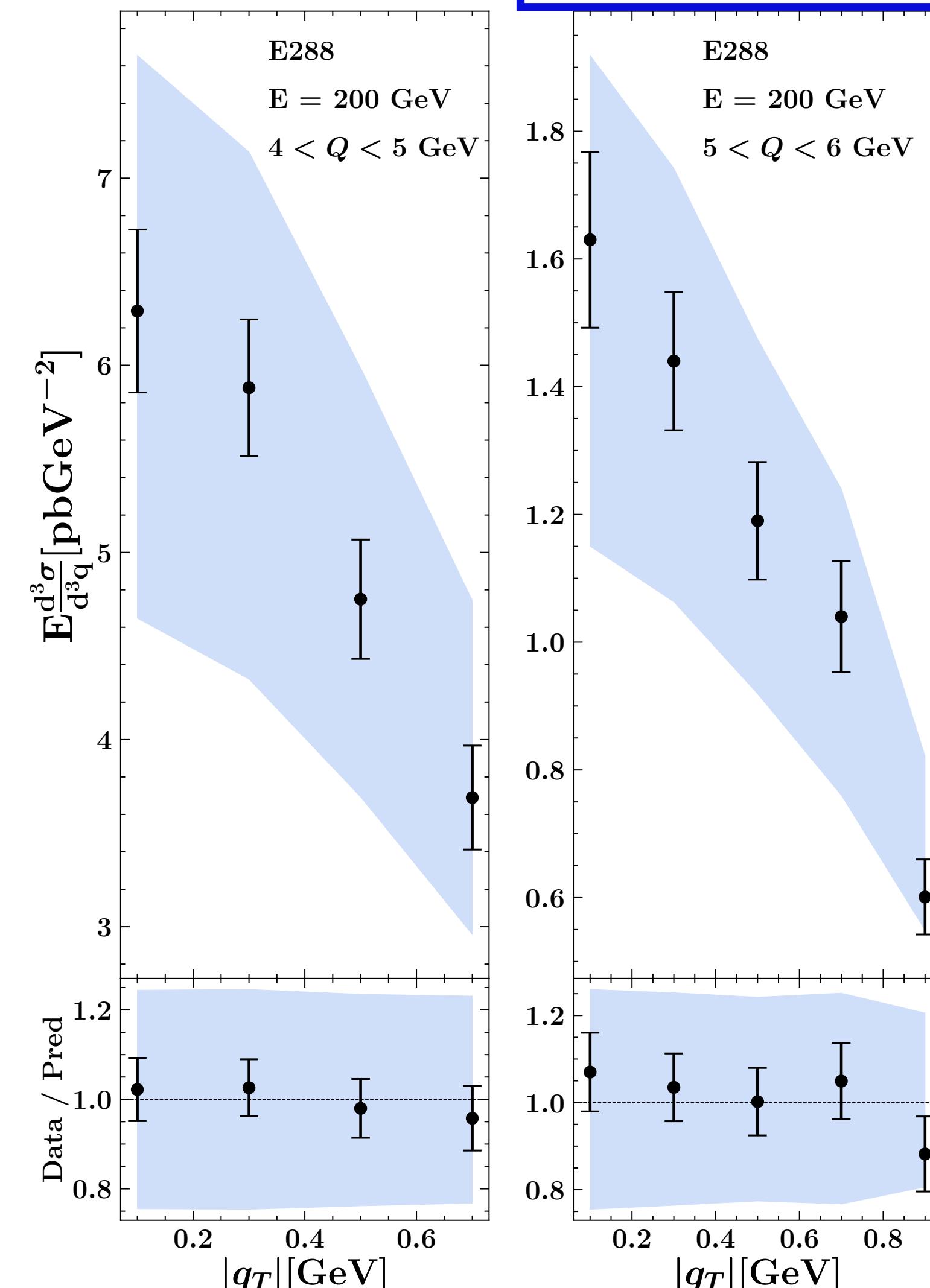
CMS



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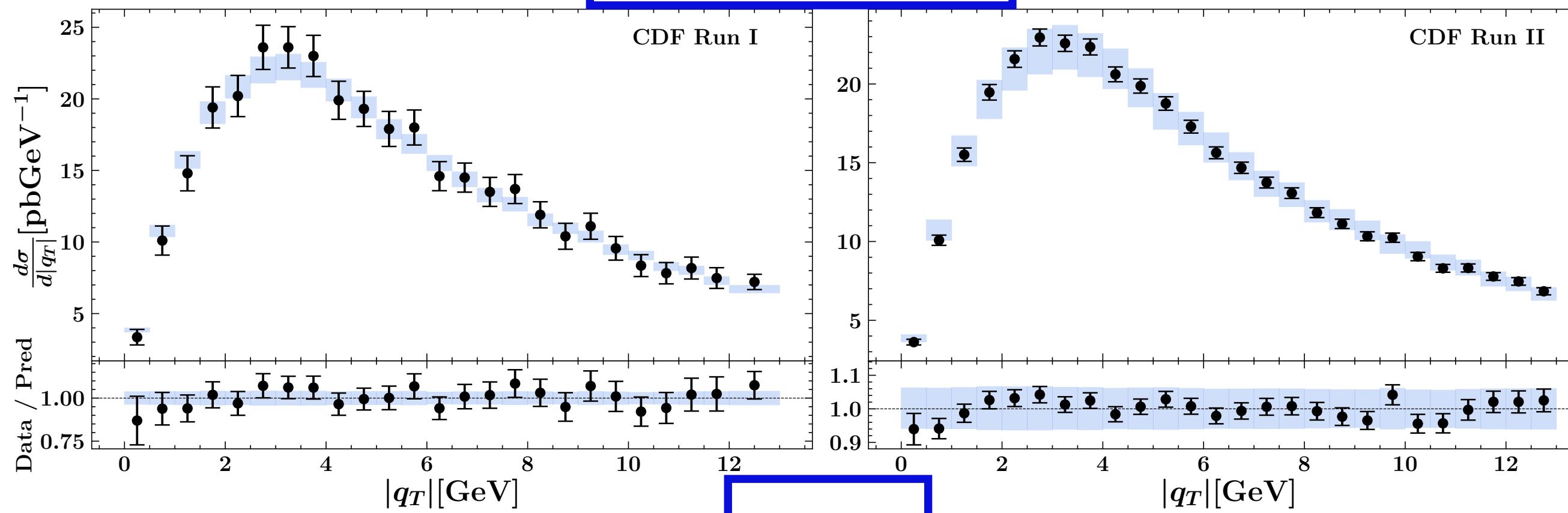


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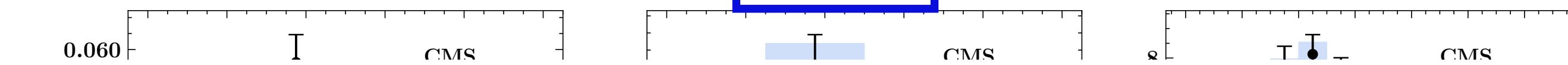


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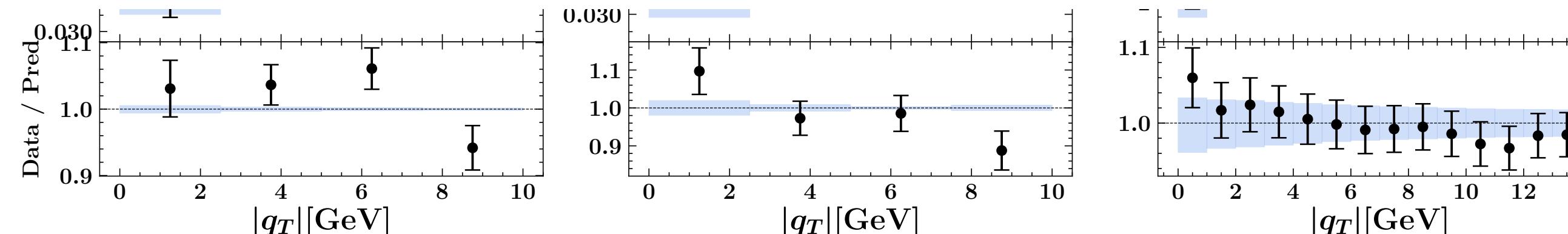
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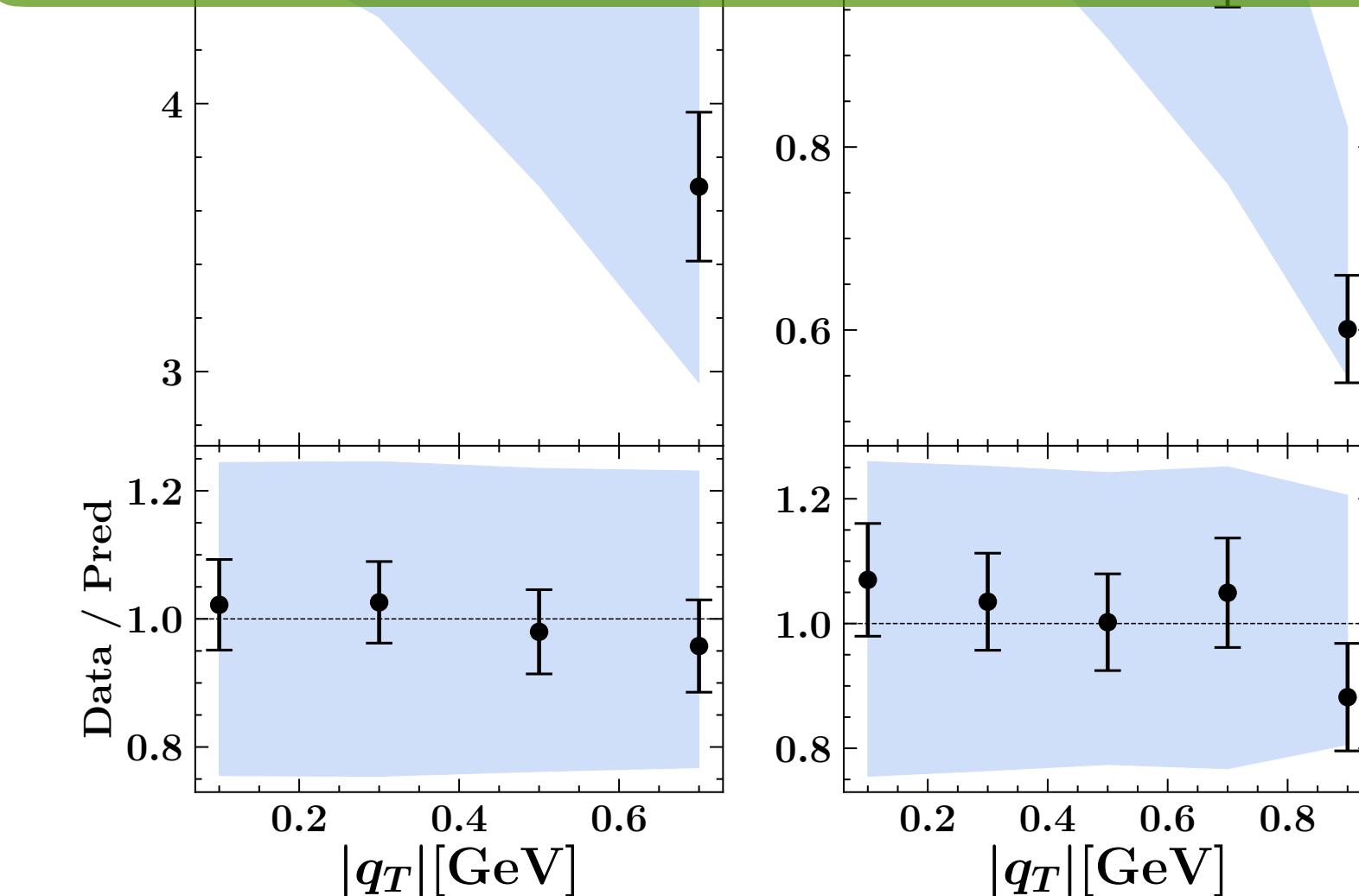
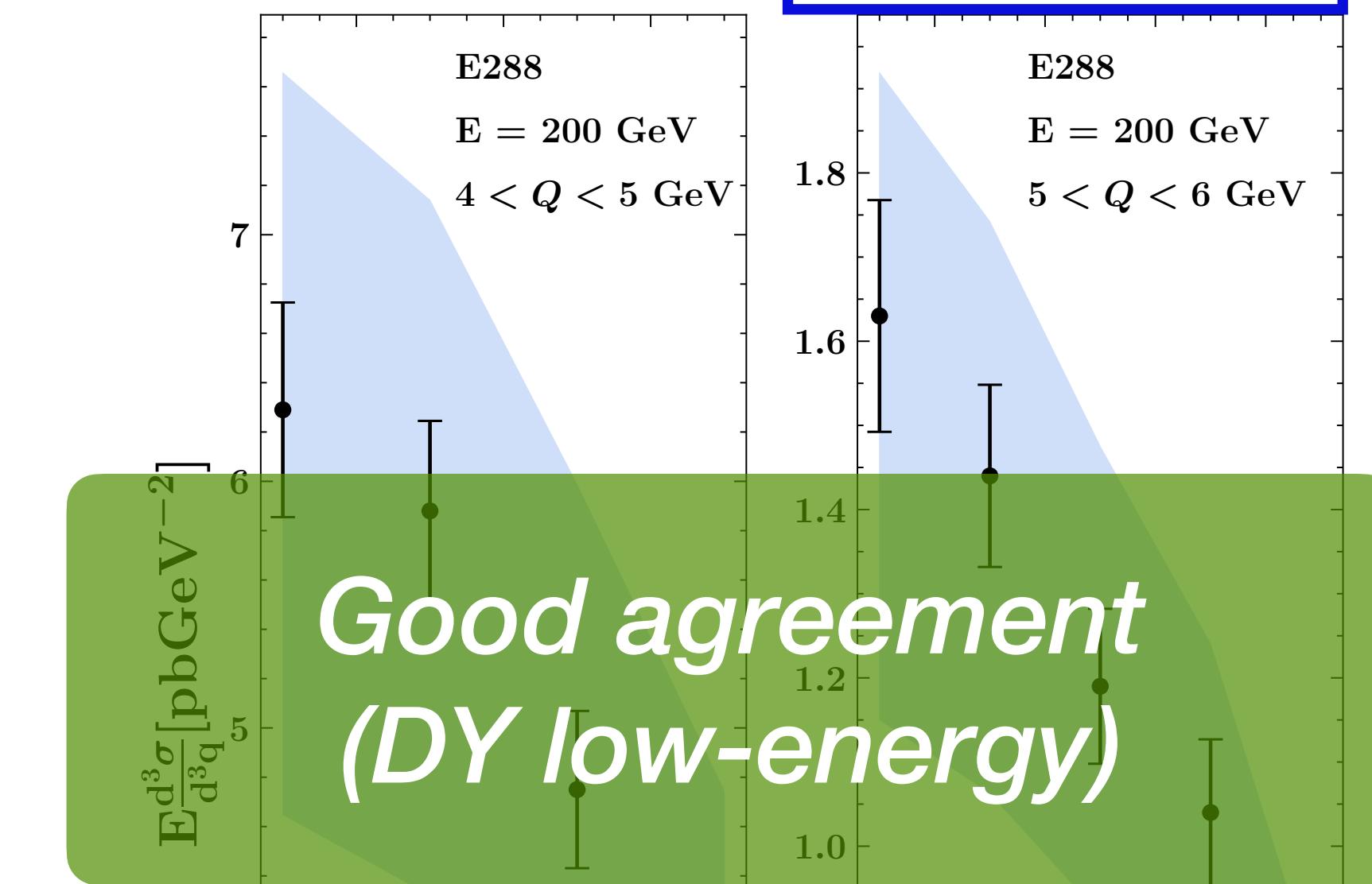
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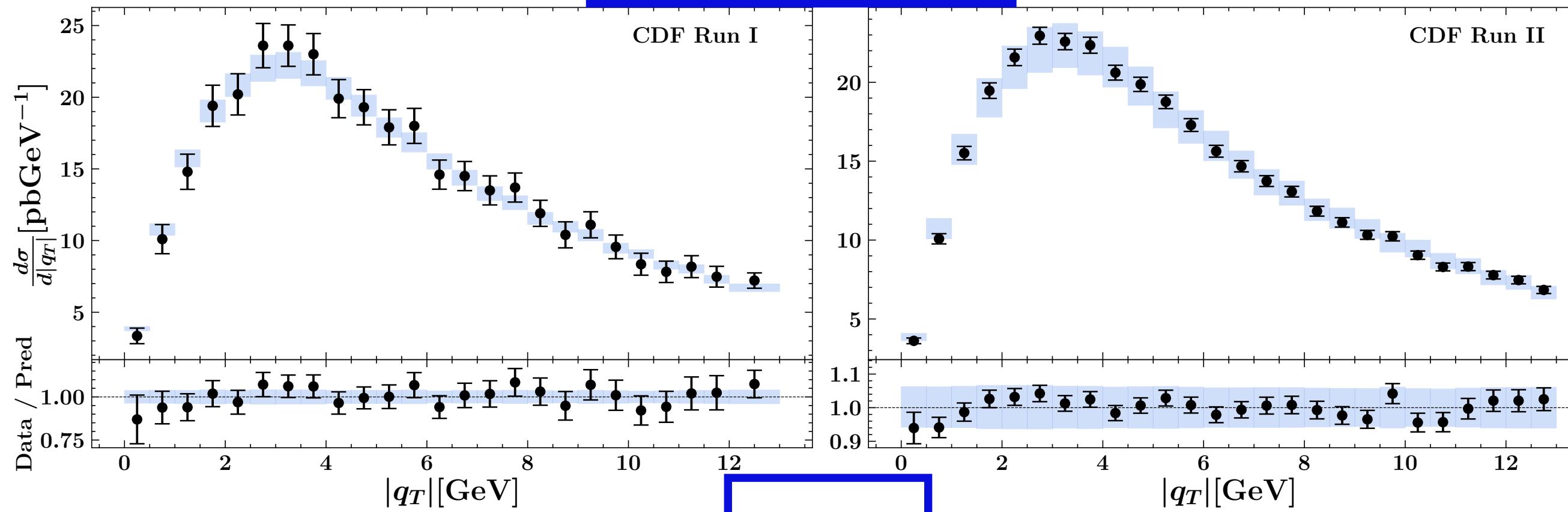


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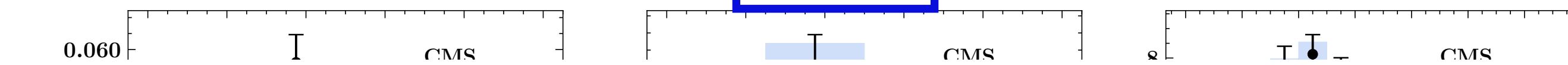


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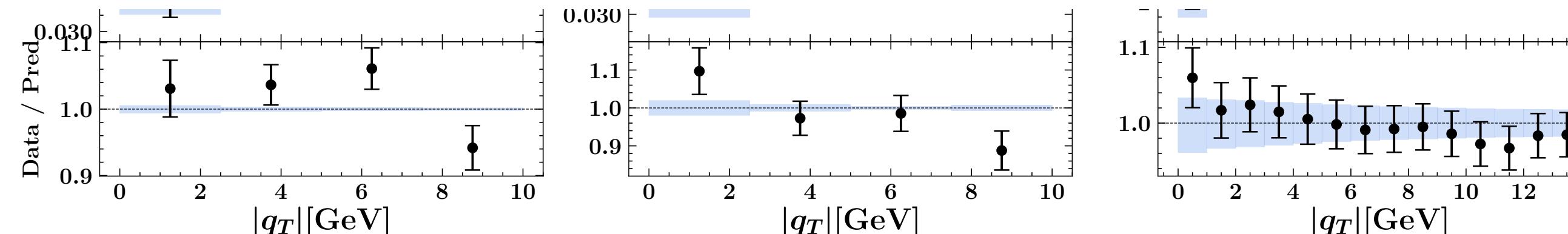
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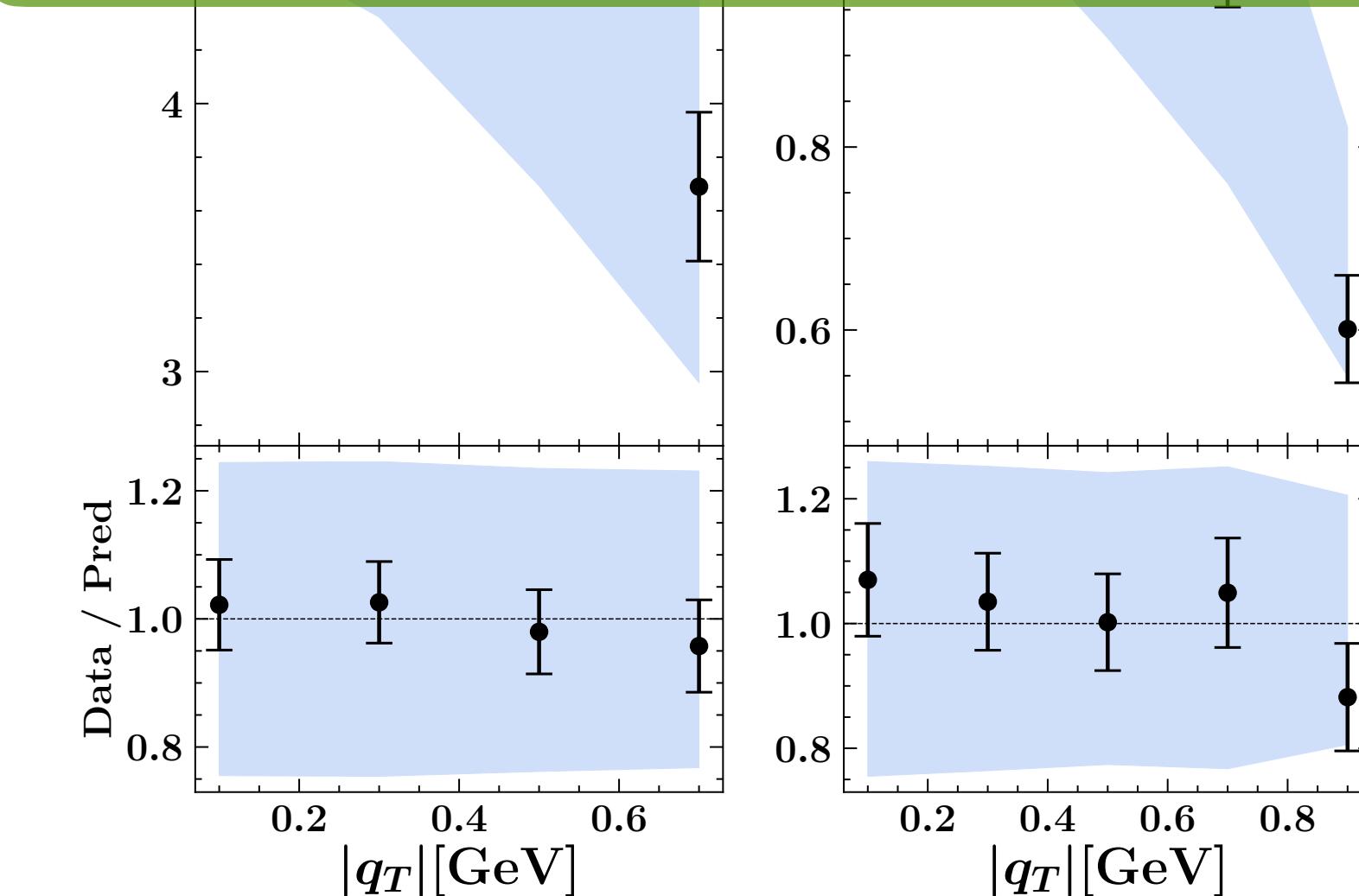
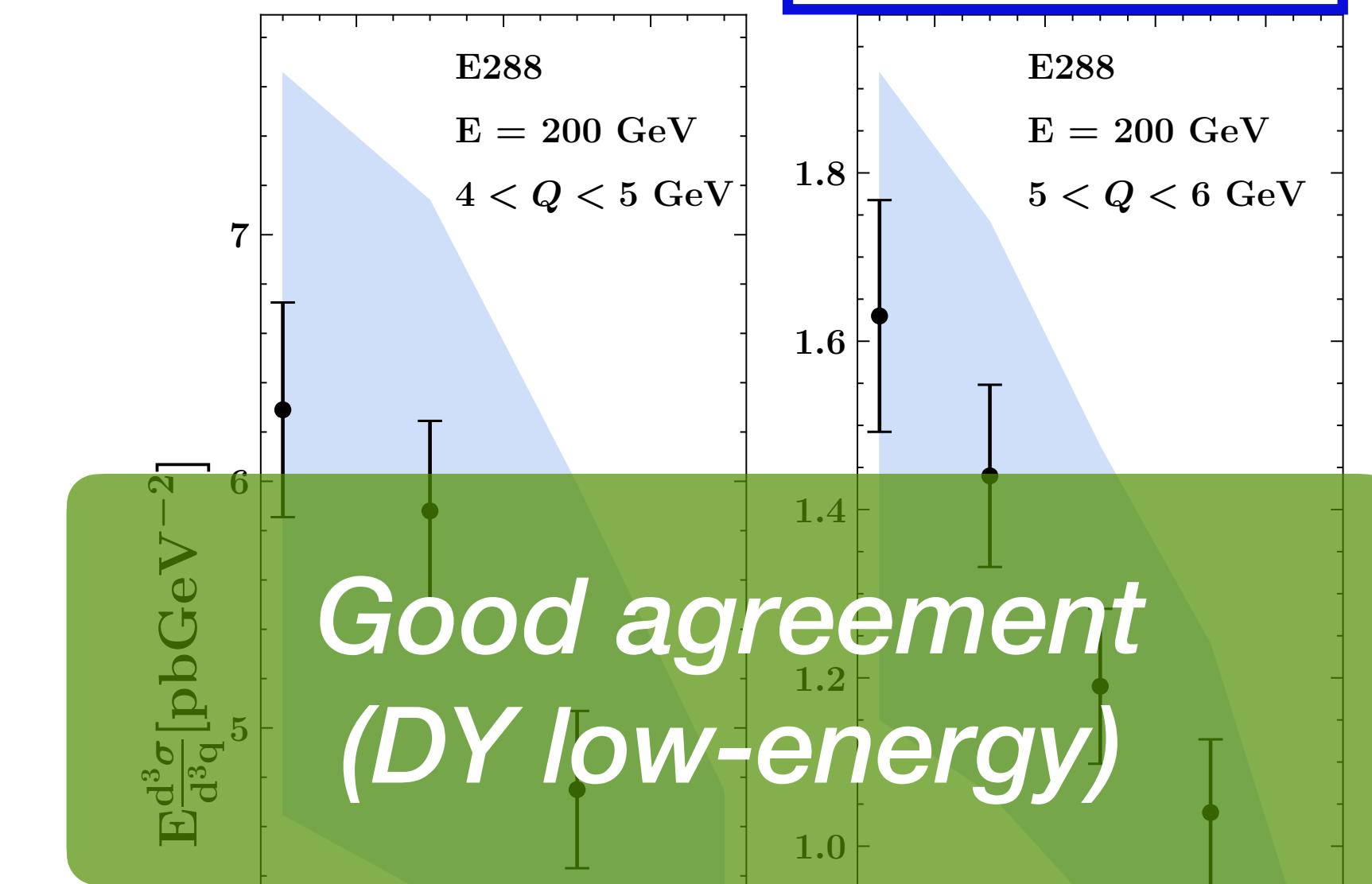
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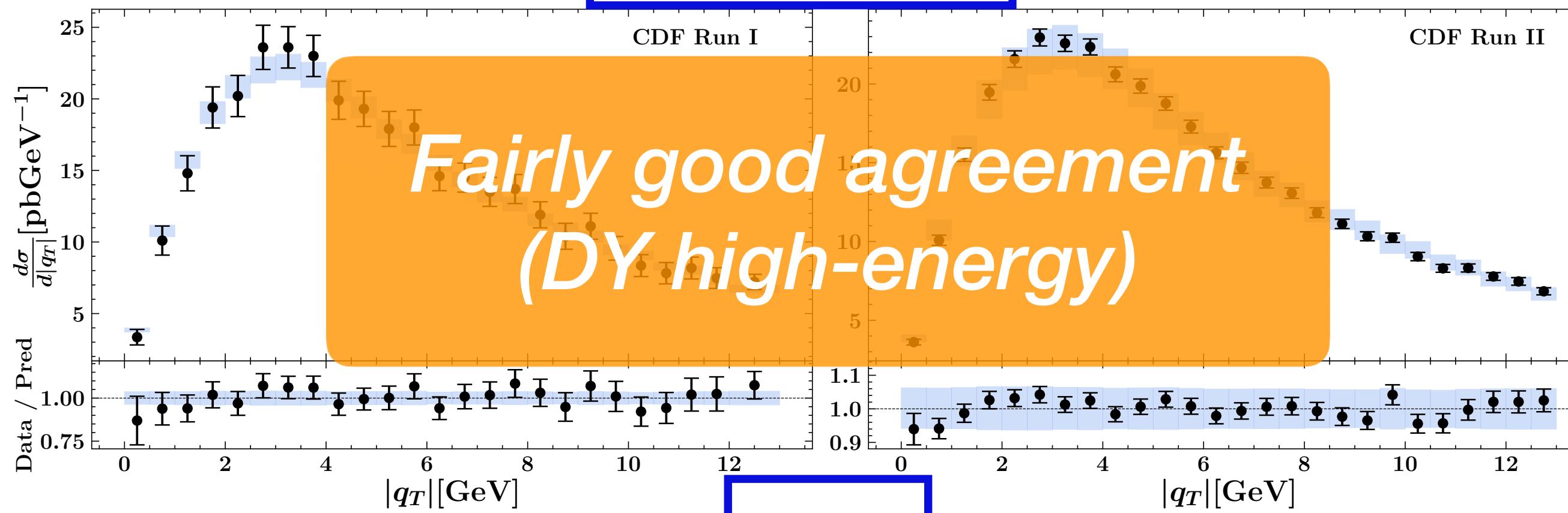


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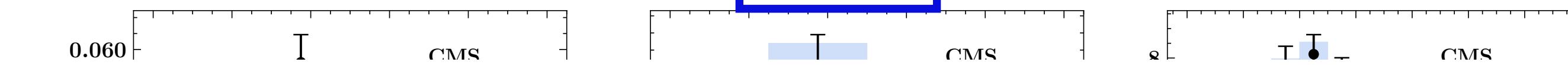


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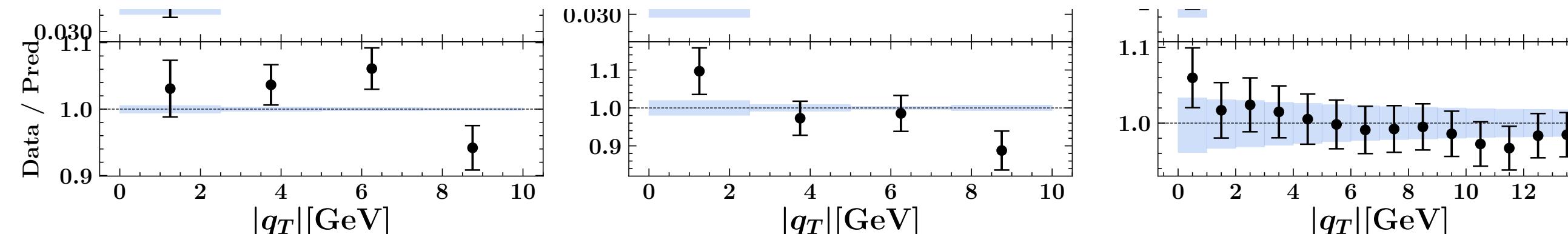
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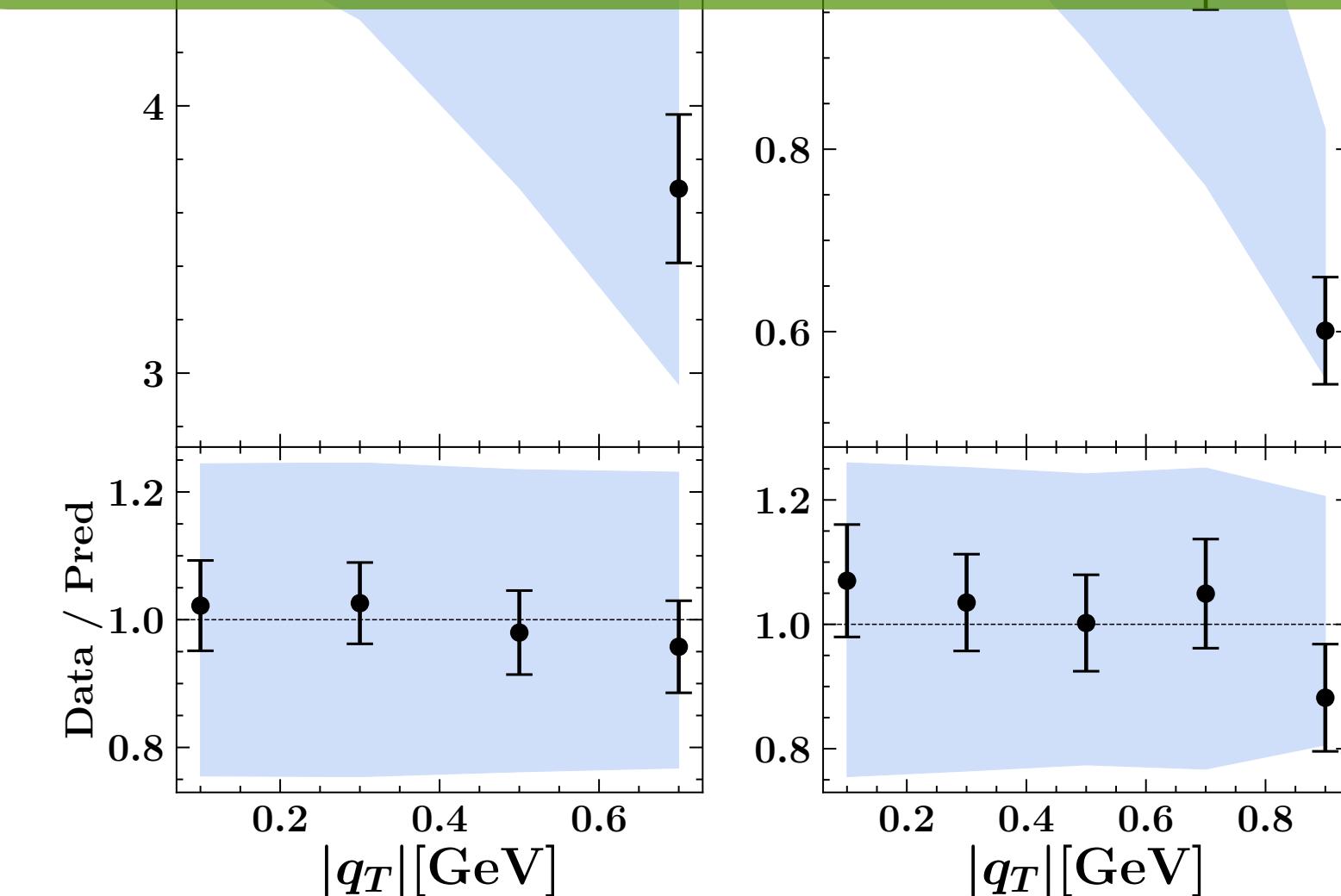
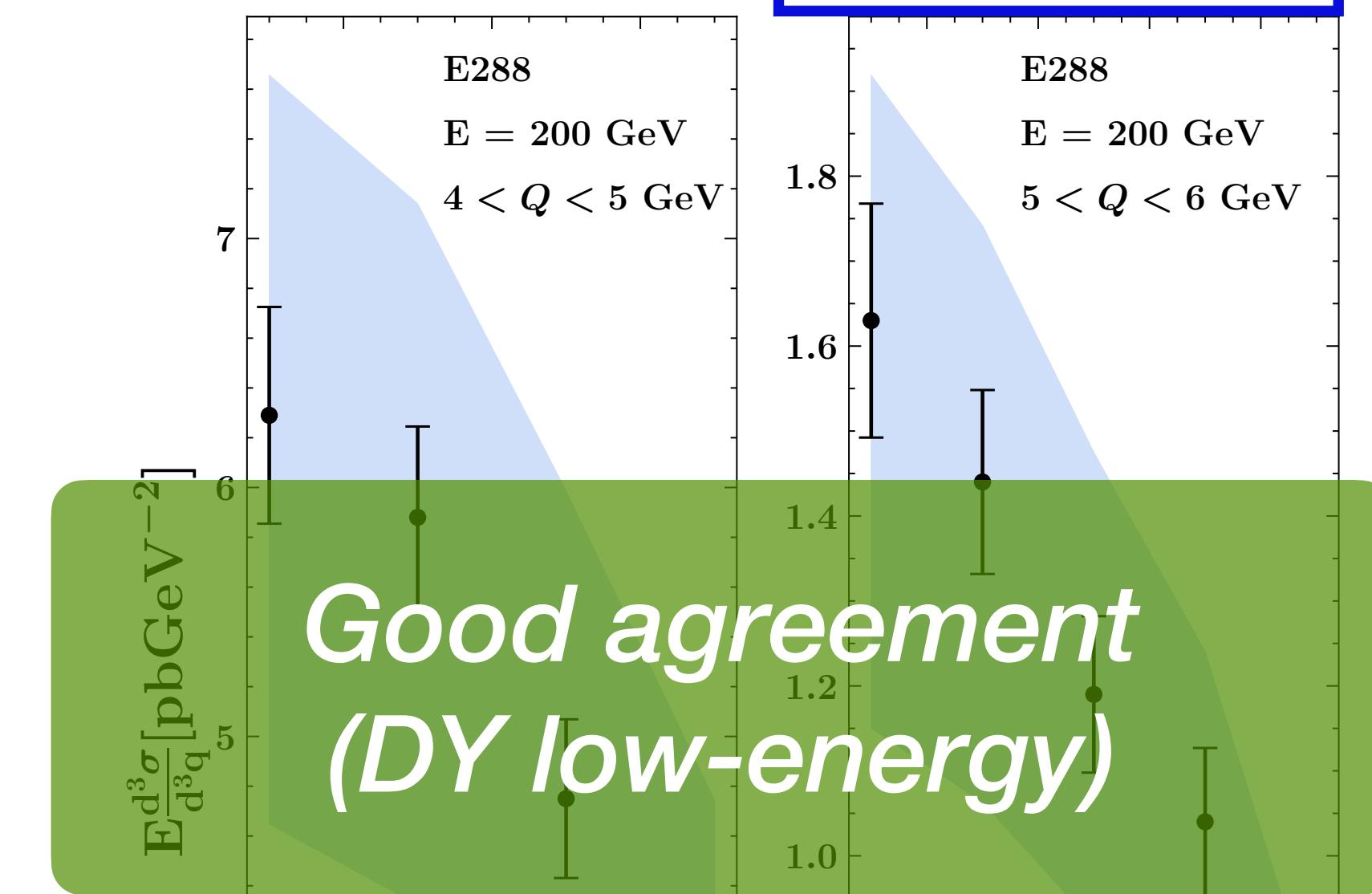
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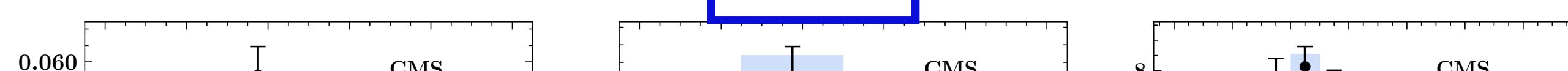
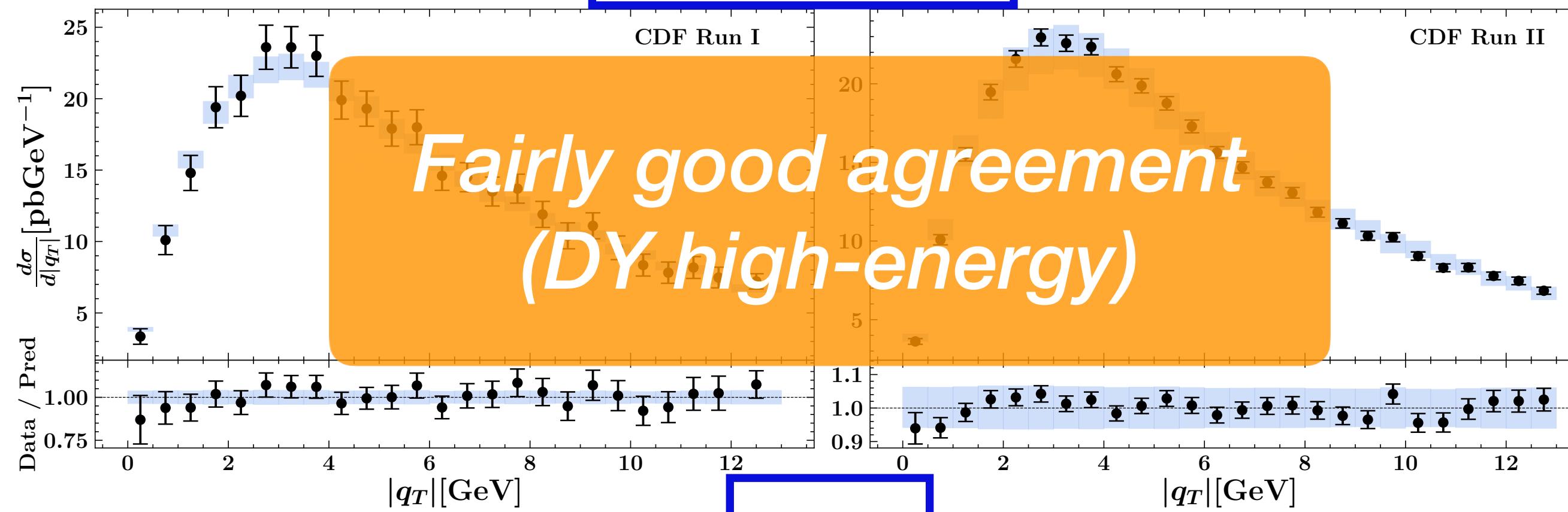


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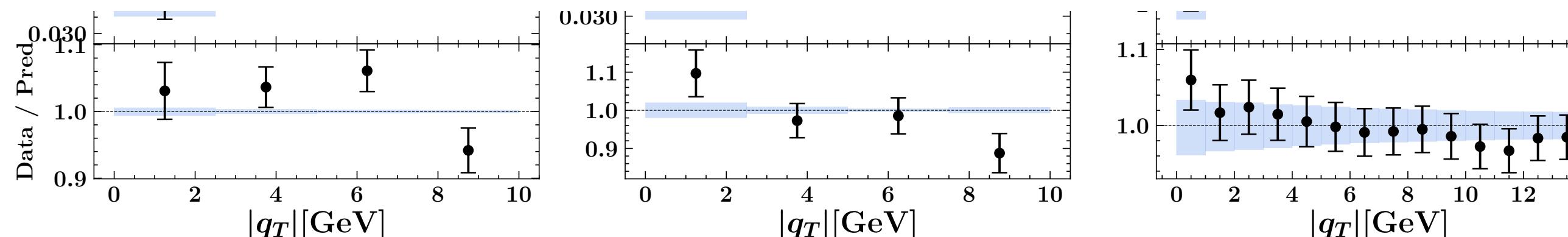


MAPTMD22 – Results of the fit $\chi^2/N_{data} = 1.06$

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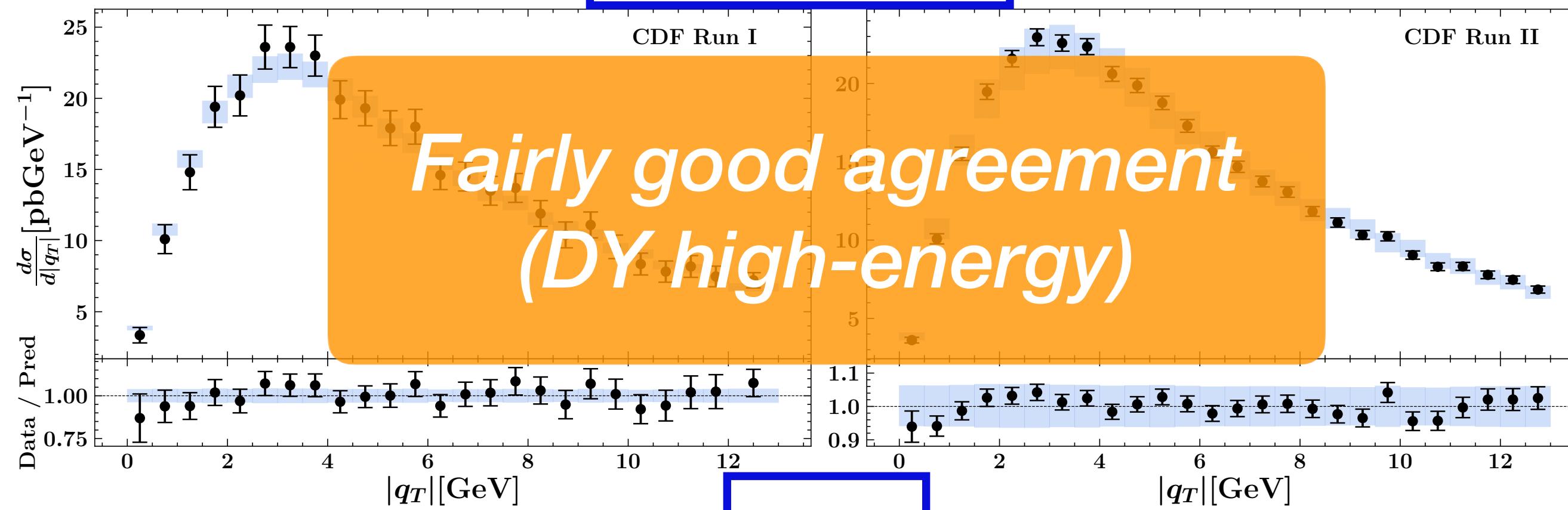


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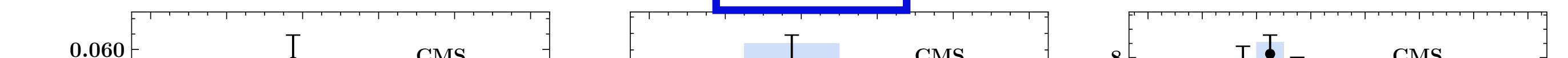


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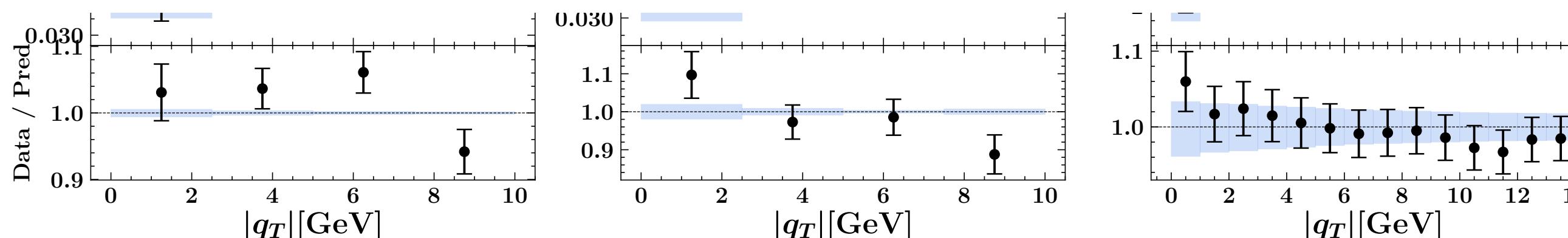
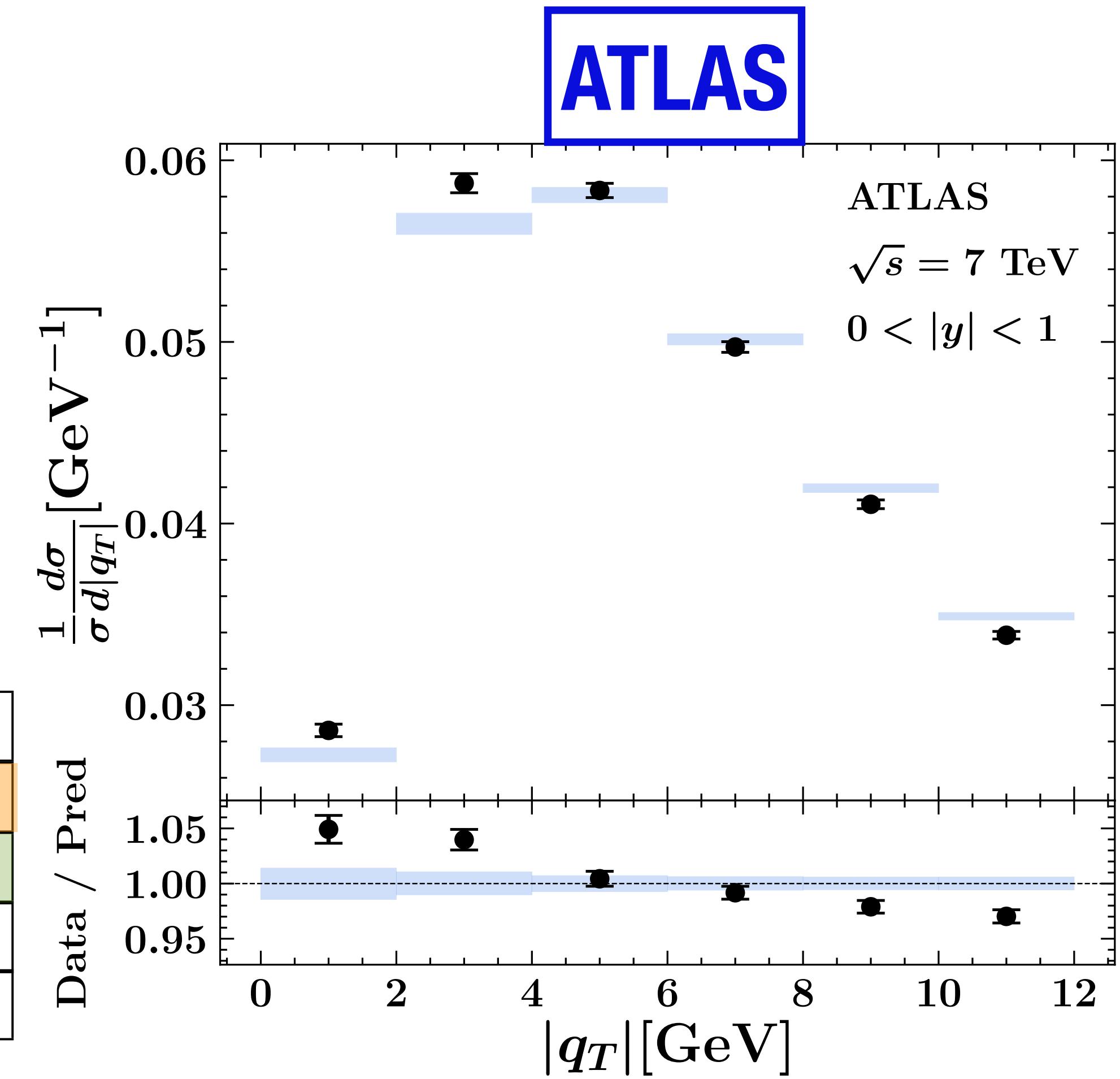
TEVATRON



CMS

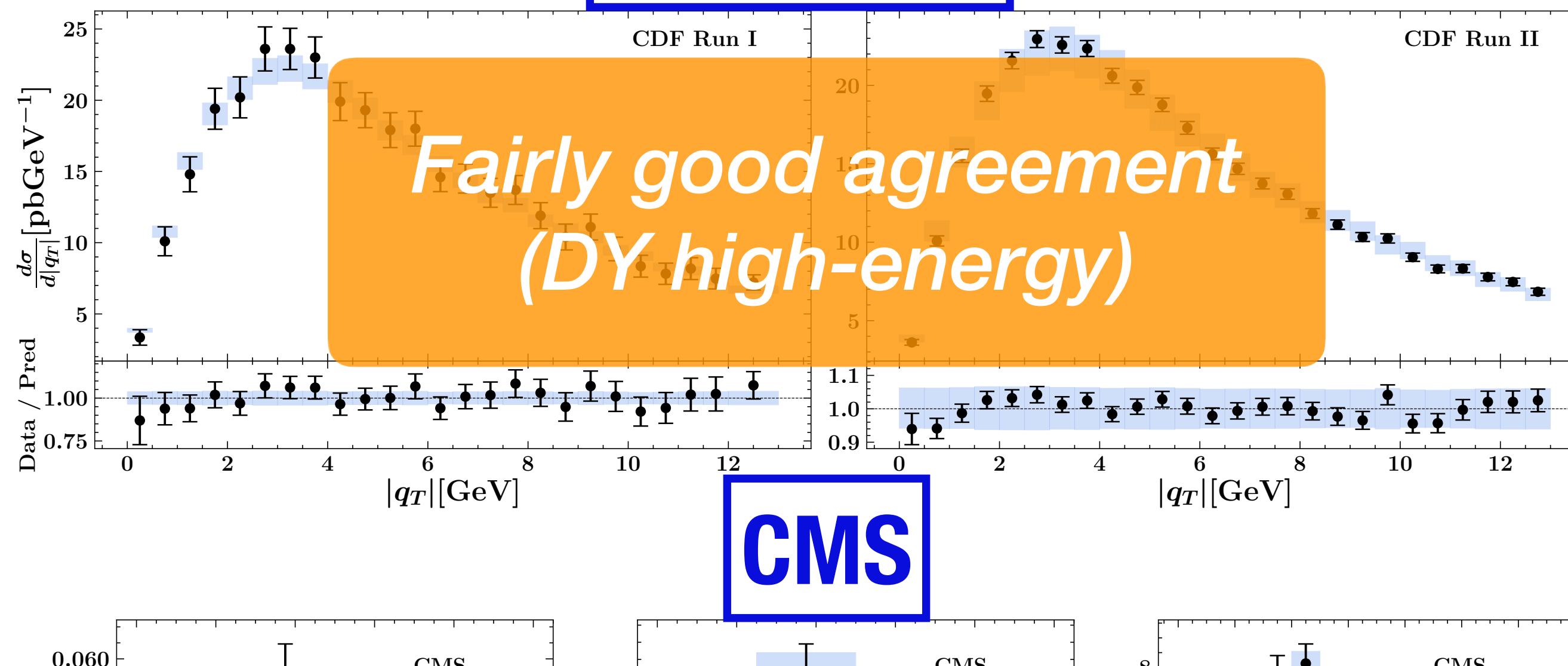


Data set	N_{dat}	χ^2_D/N_{dat}	χ^2_λ/N_{dat}	χ^2_0/N_{dat}
DY collider total	251	1.86	0.2	2.06
DY fixed-target total	233	0.85	0.4	1.24
SIDIS total	1547	0.59	0.28	0.87
Total	2031	0.77	0.29	1.06



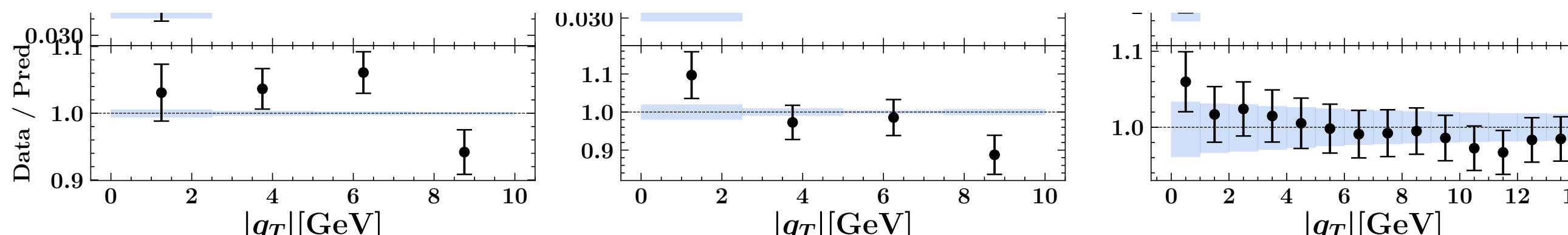
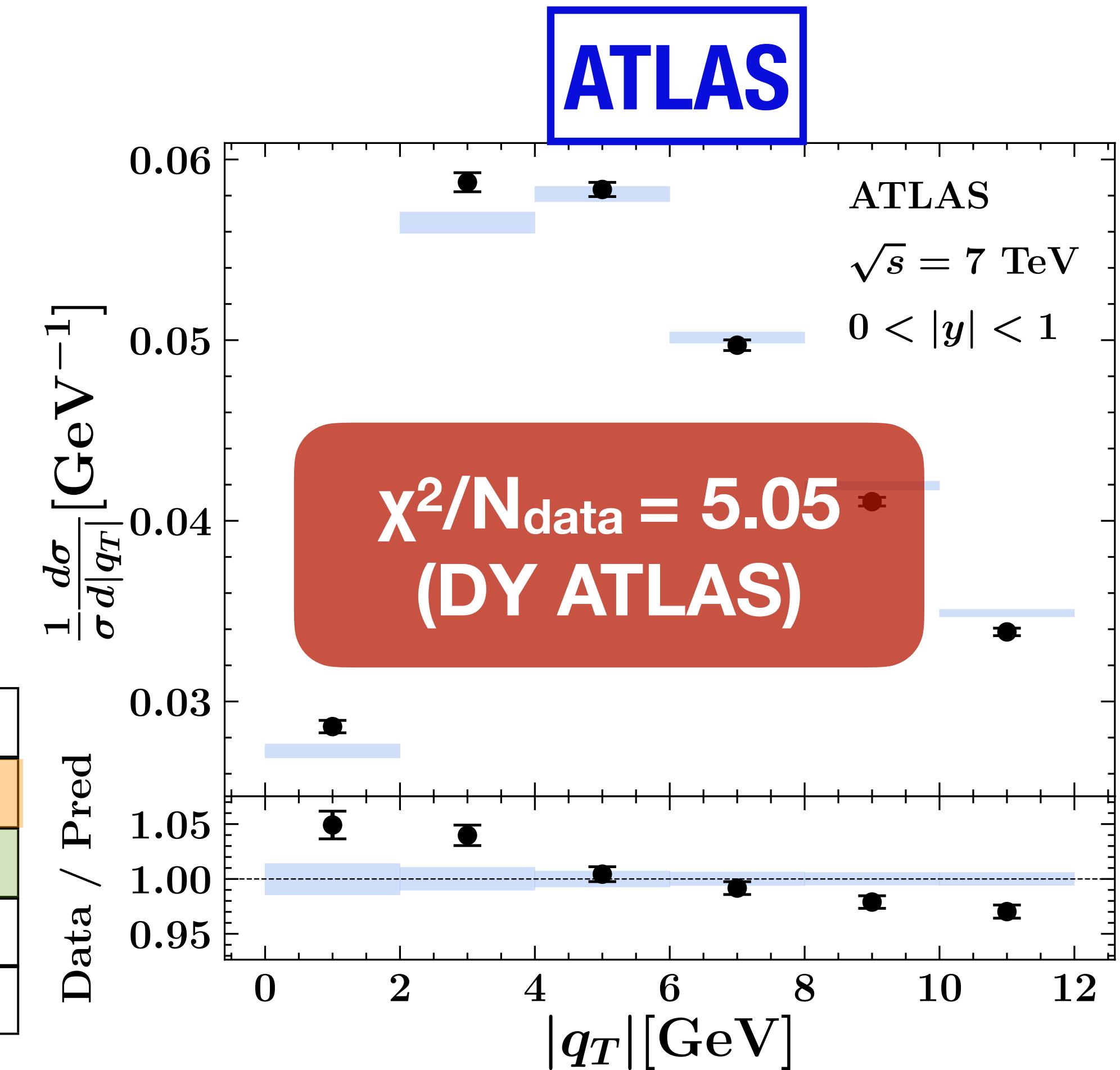
MAPTMD22 – Results of the fit $\chi^2/N_{data} = 1.06$

TEVATRON

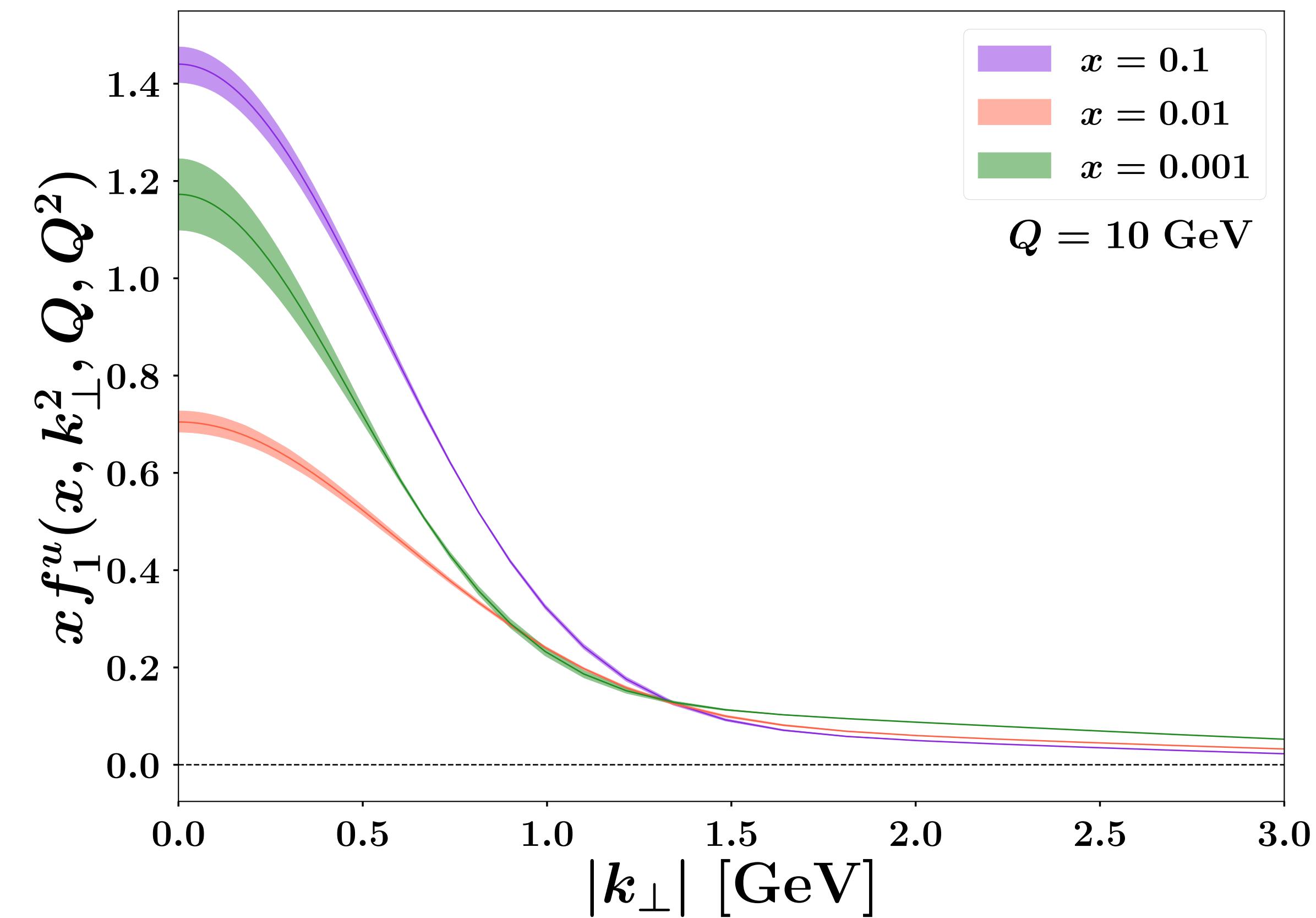
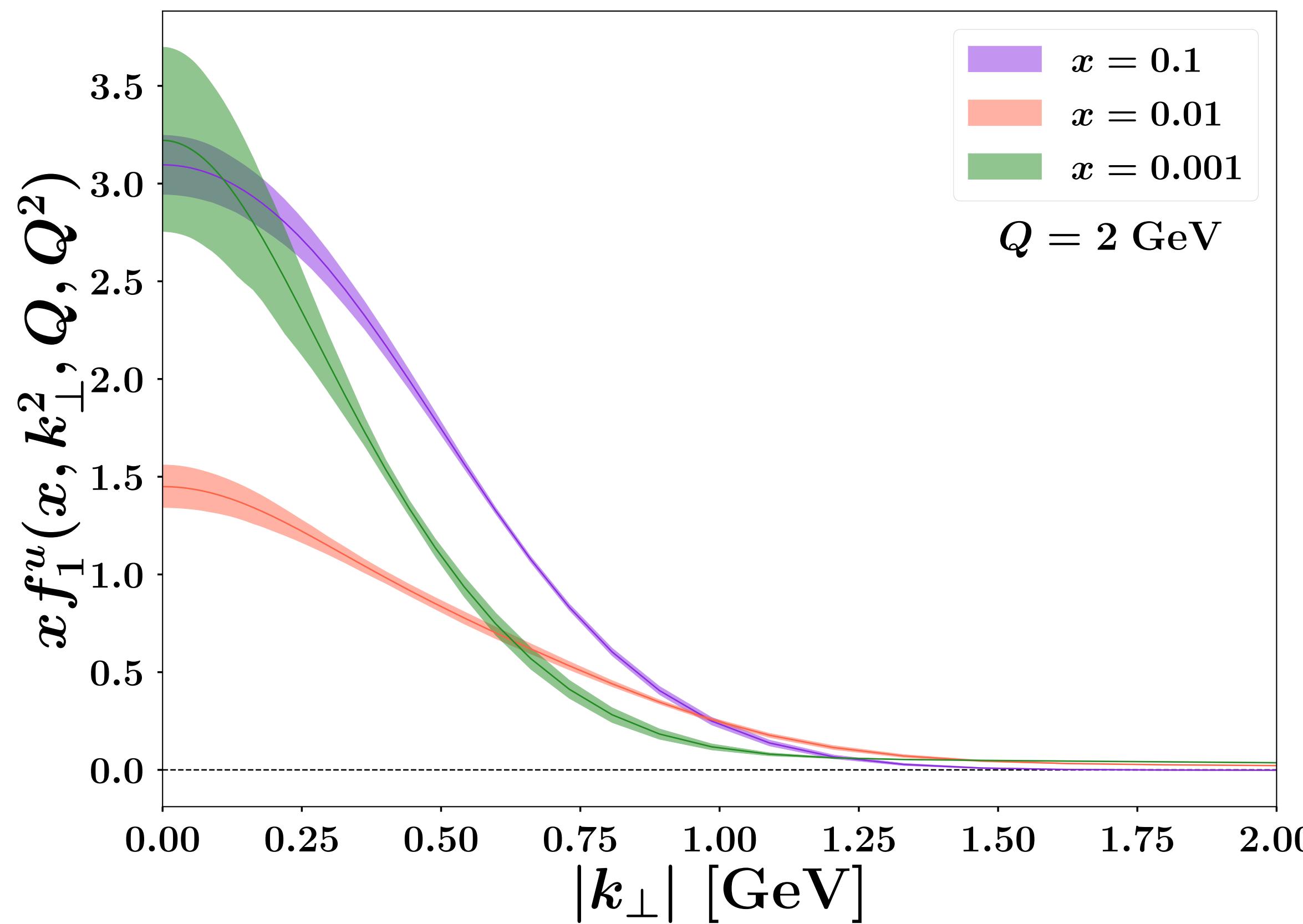


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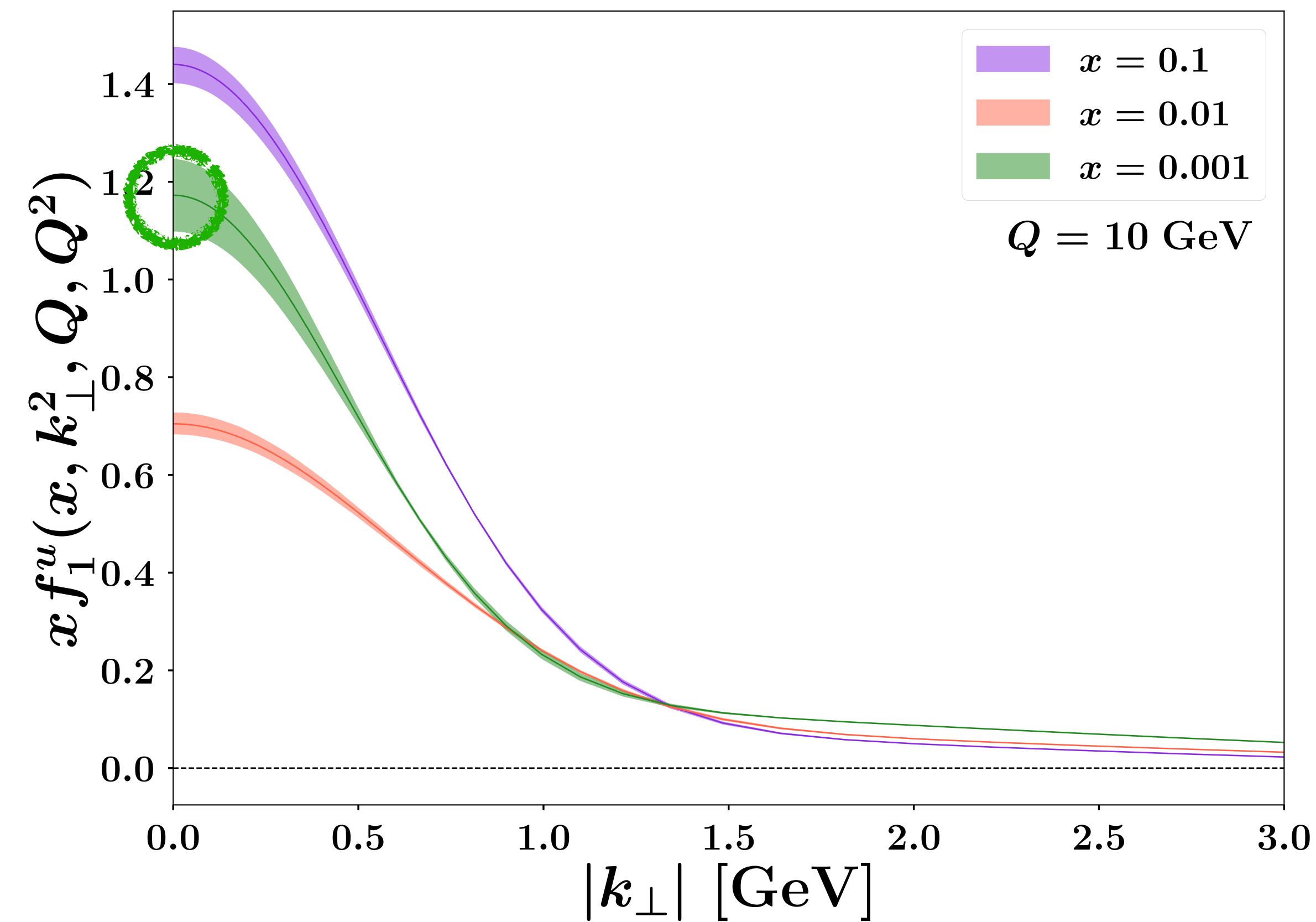
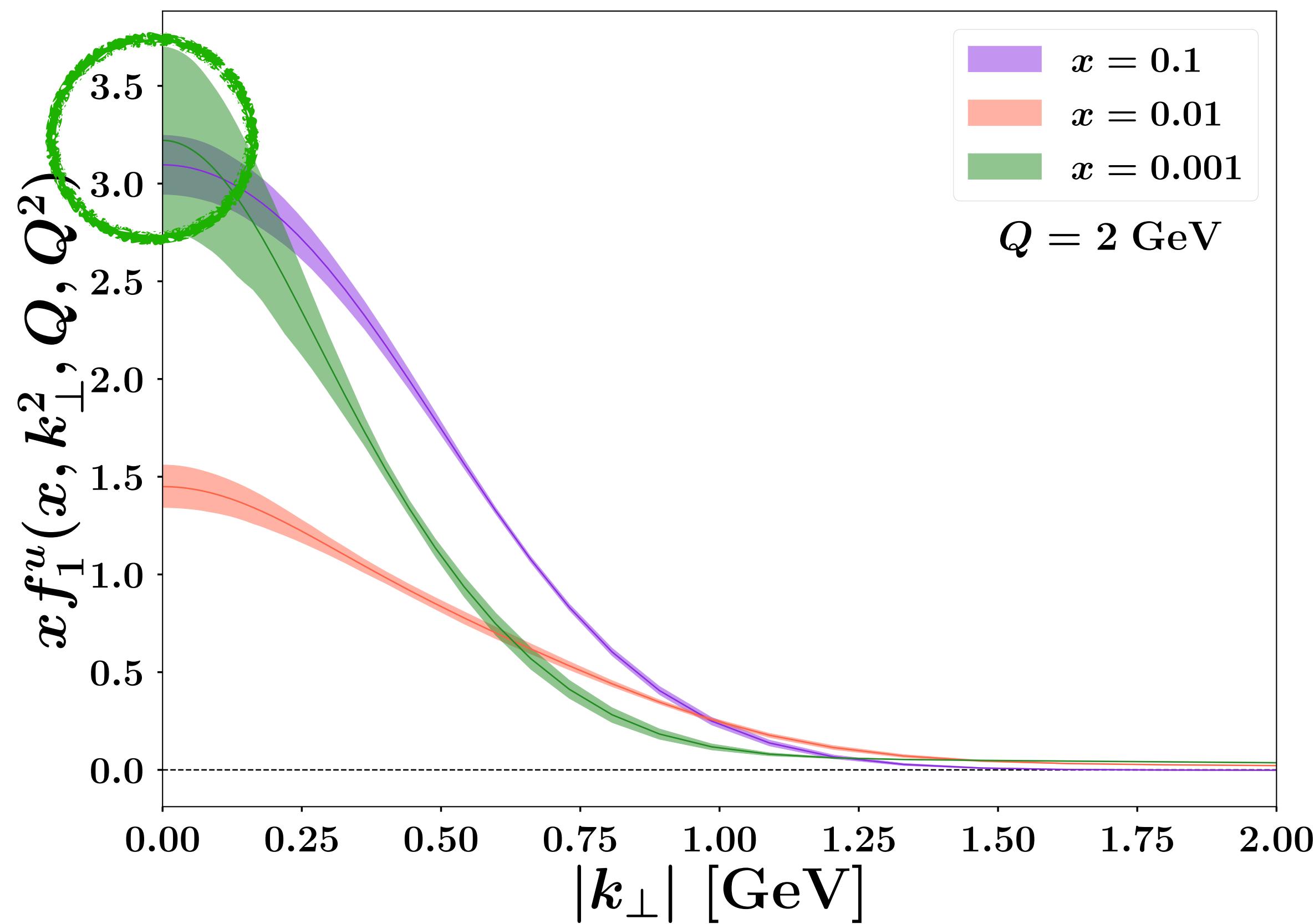
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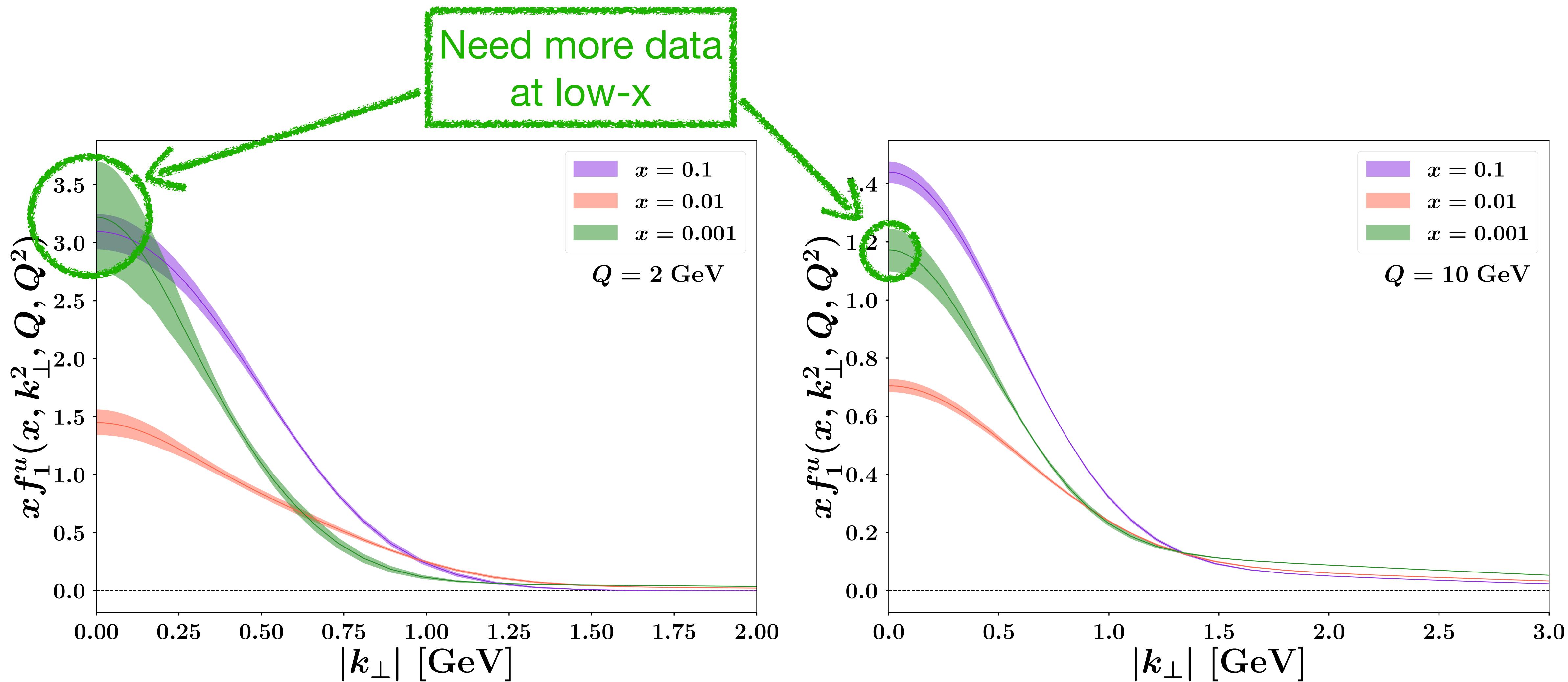
Visualization of TMD PDFs



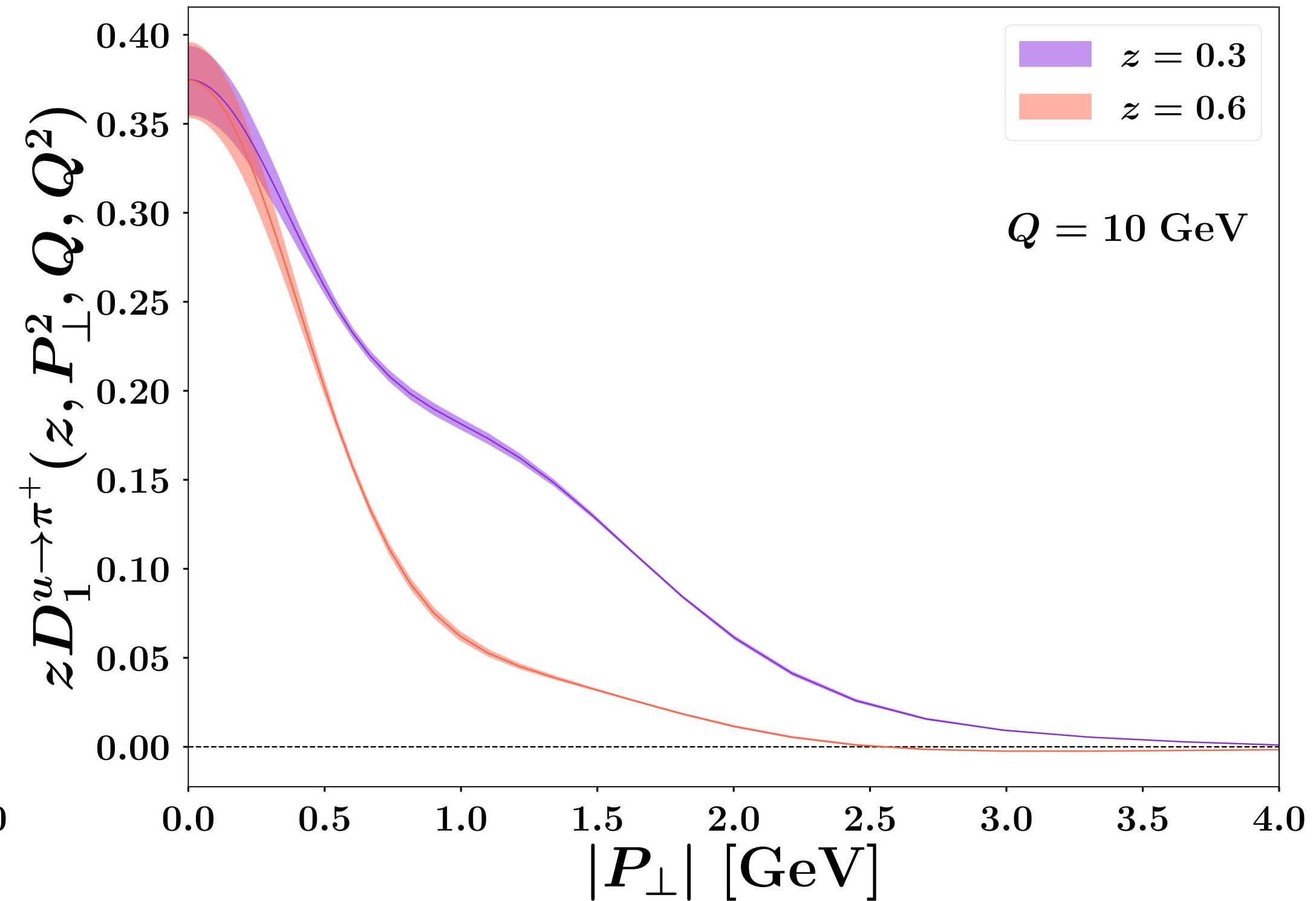
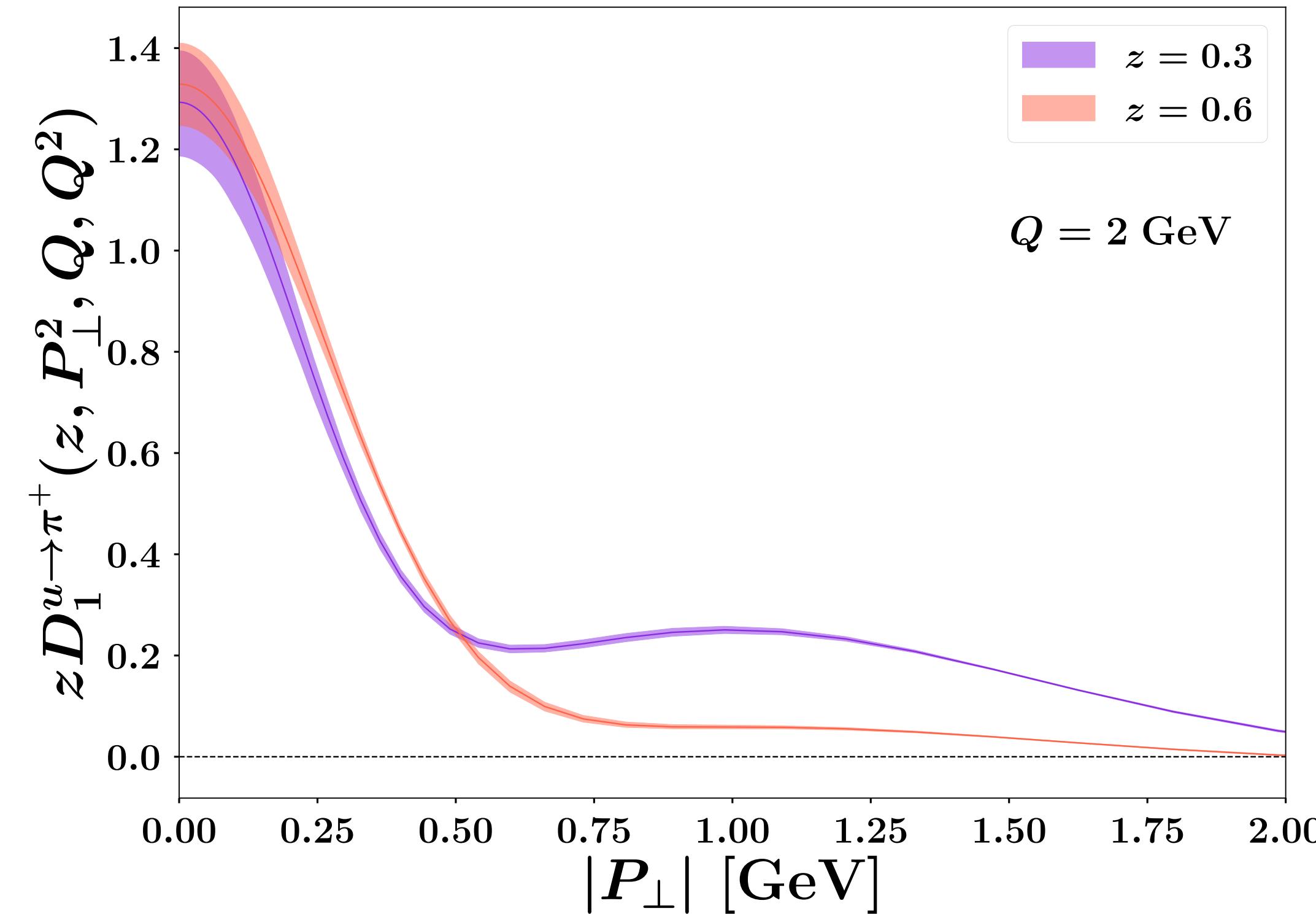
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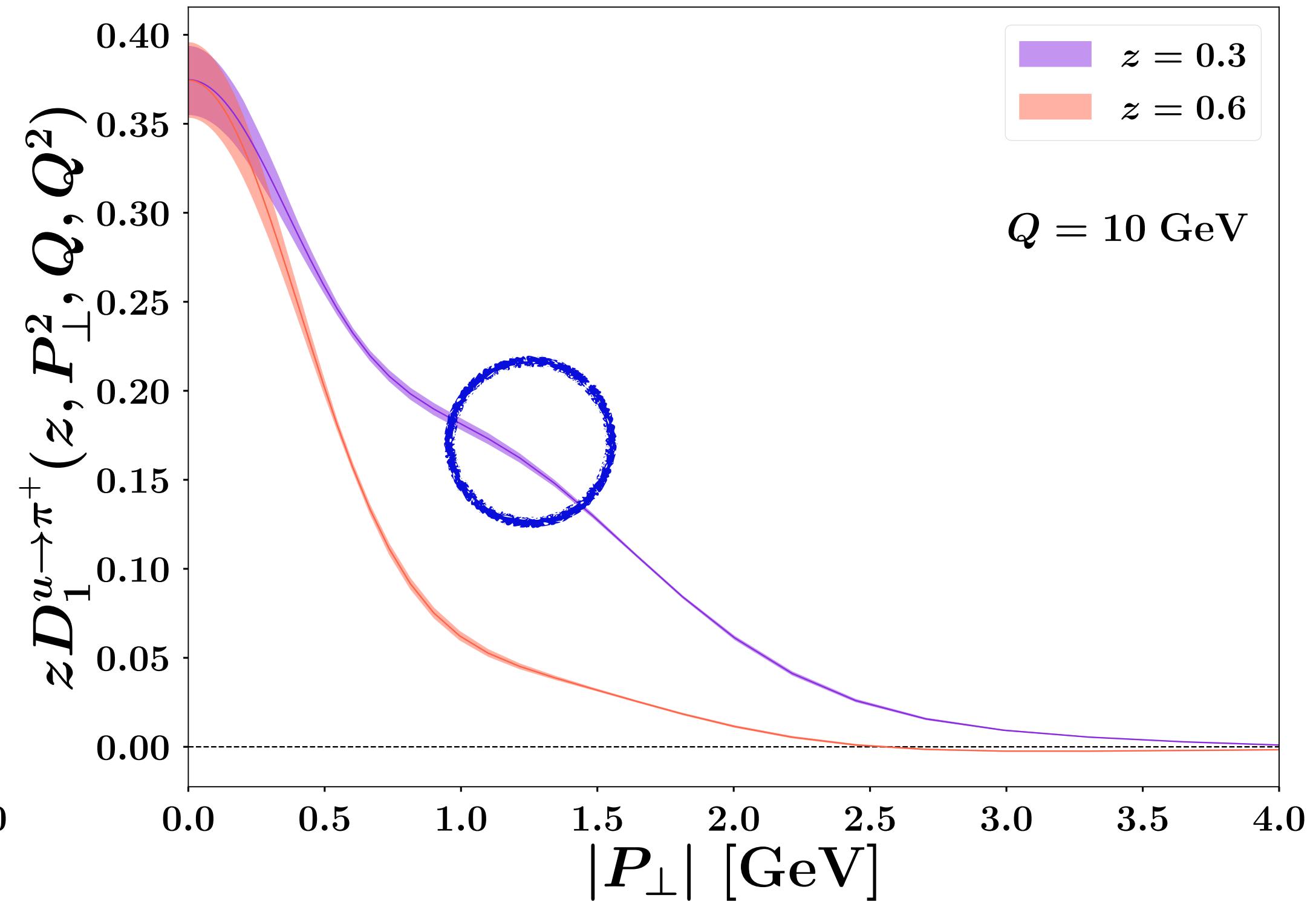
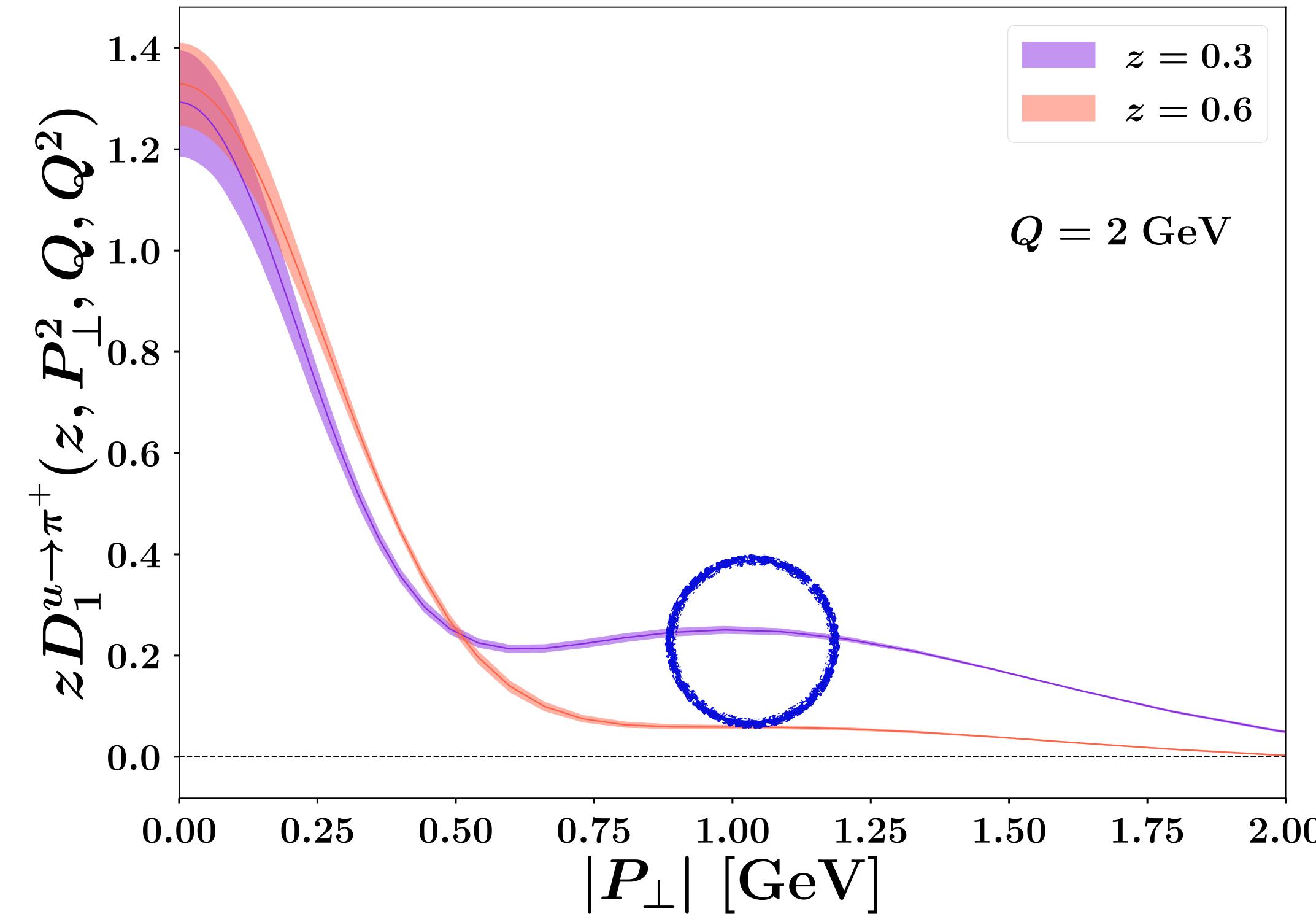
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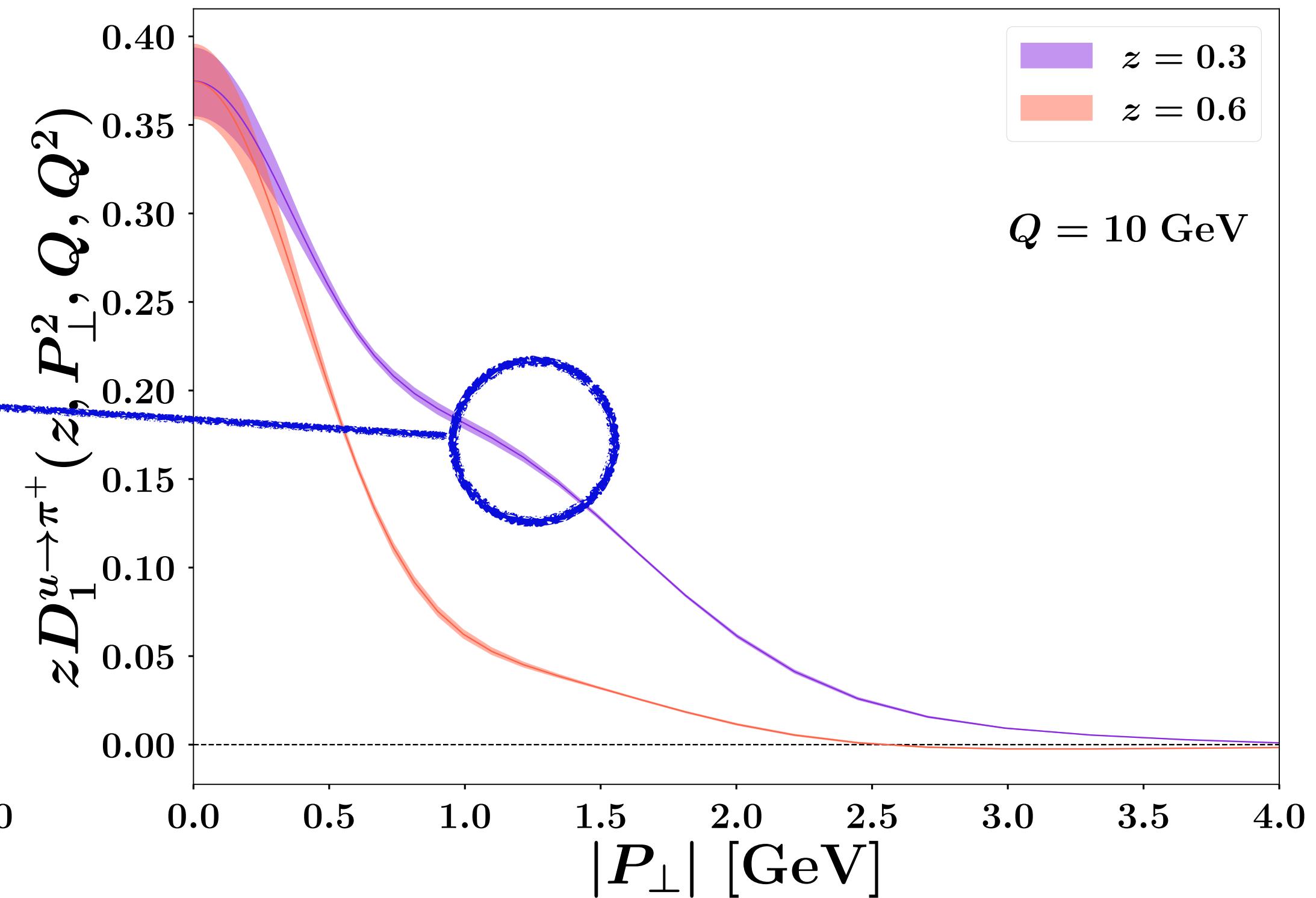
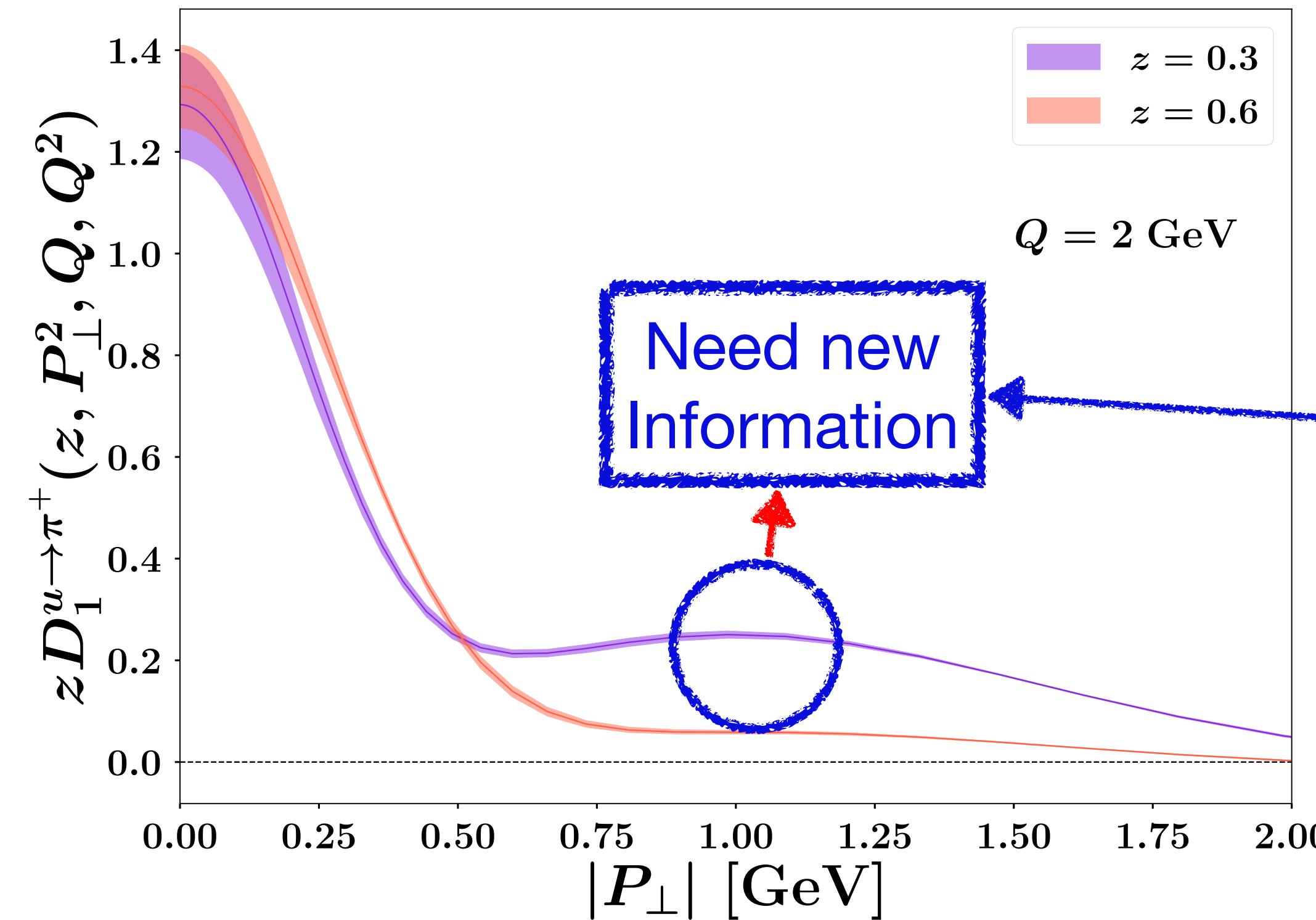
Visualization of TMD FFs



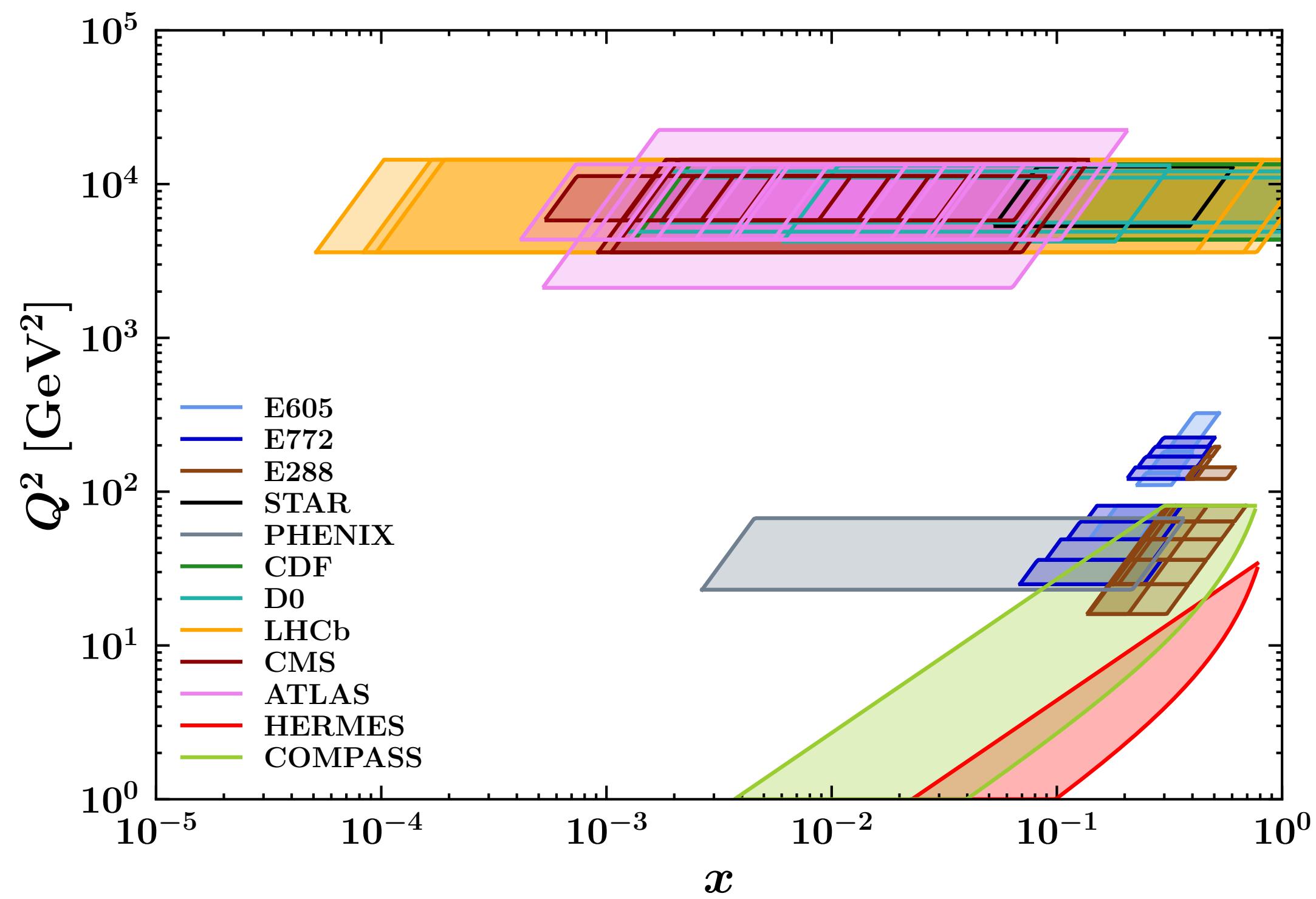
Visualization of TMD FFs



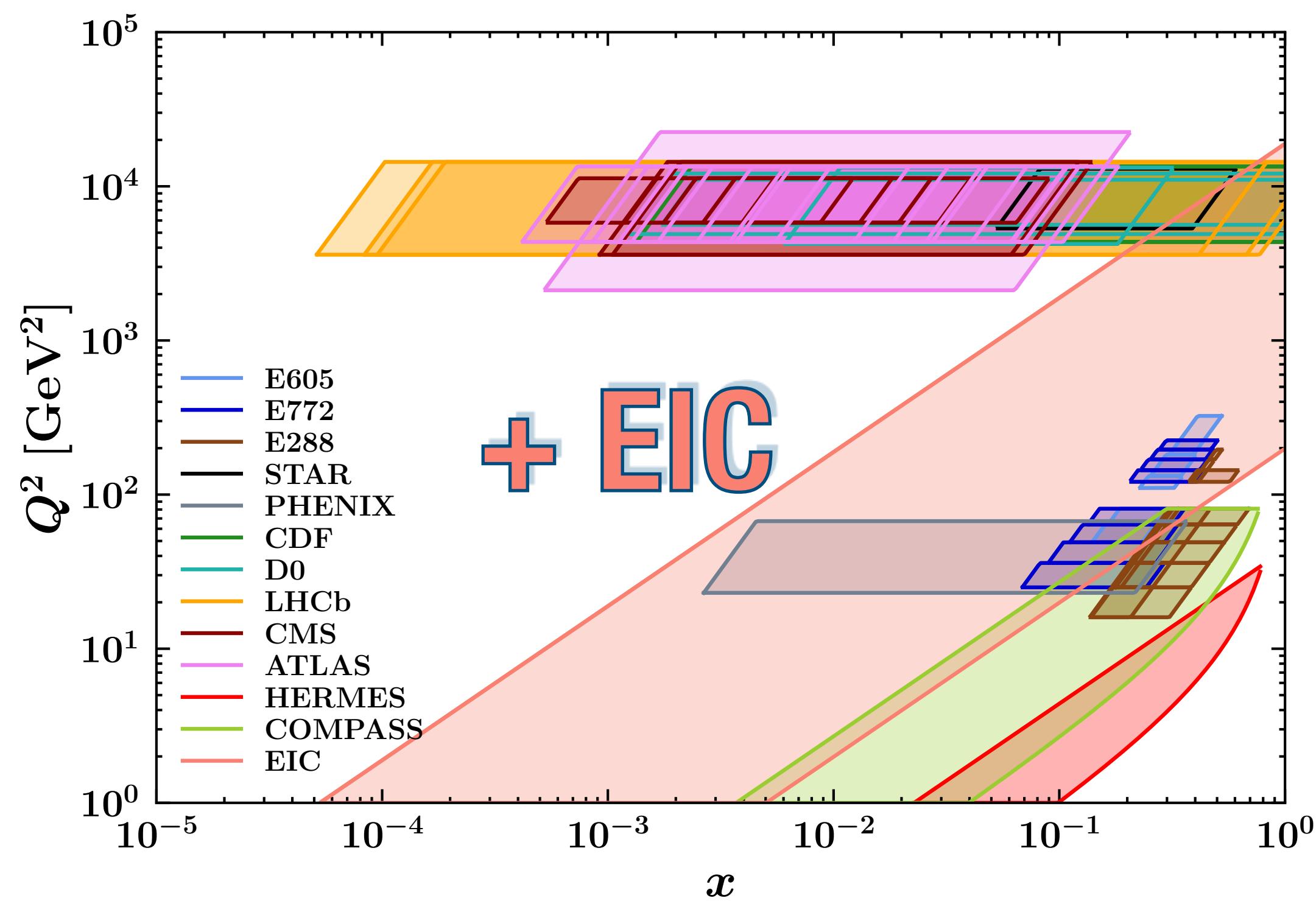
Visualization of TMD FFs



Impact studies

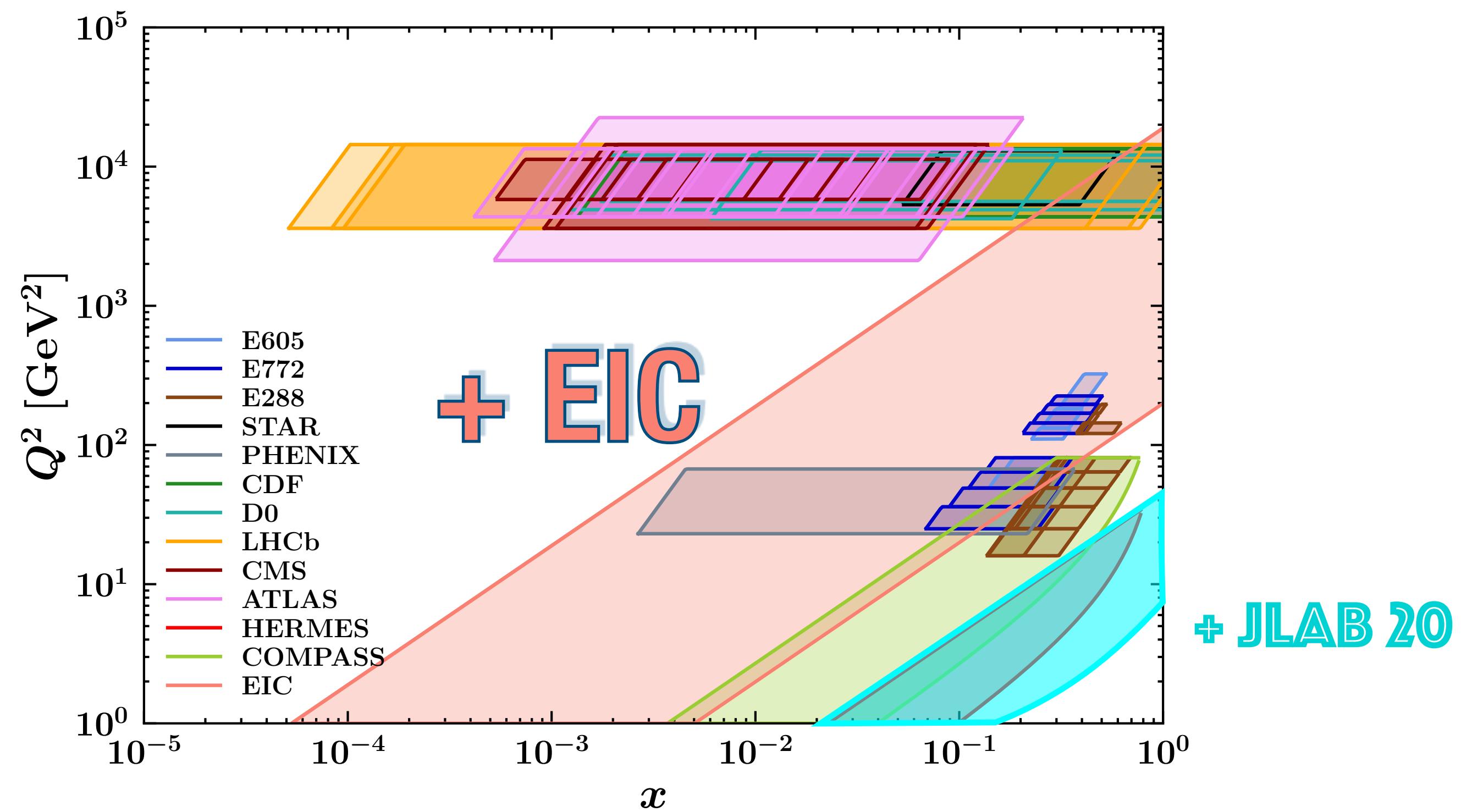


Impact studies



ELECTRON ION COLLIDER

Impact studies

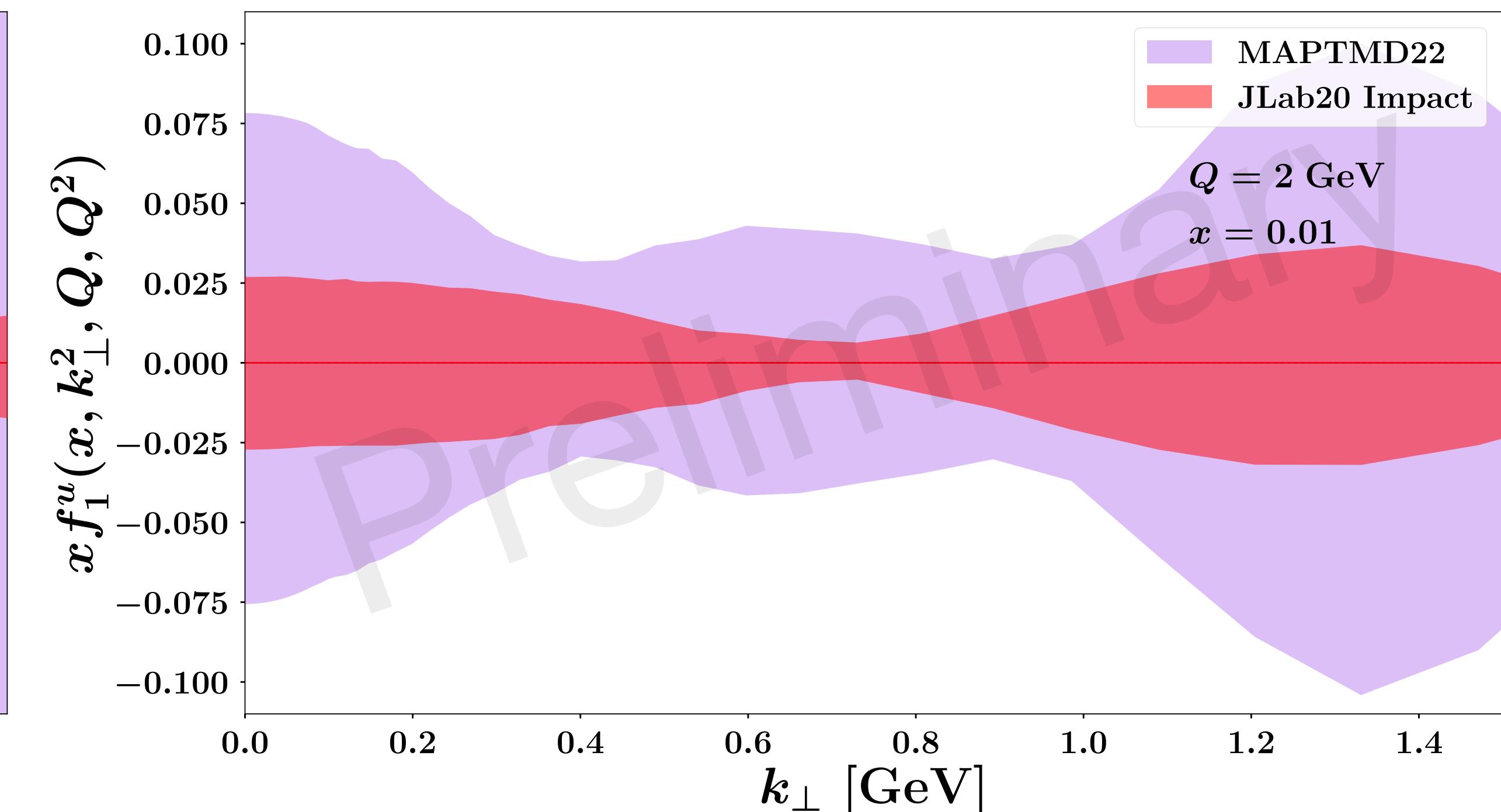
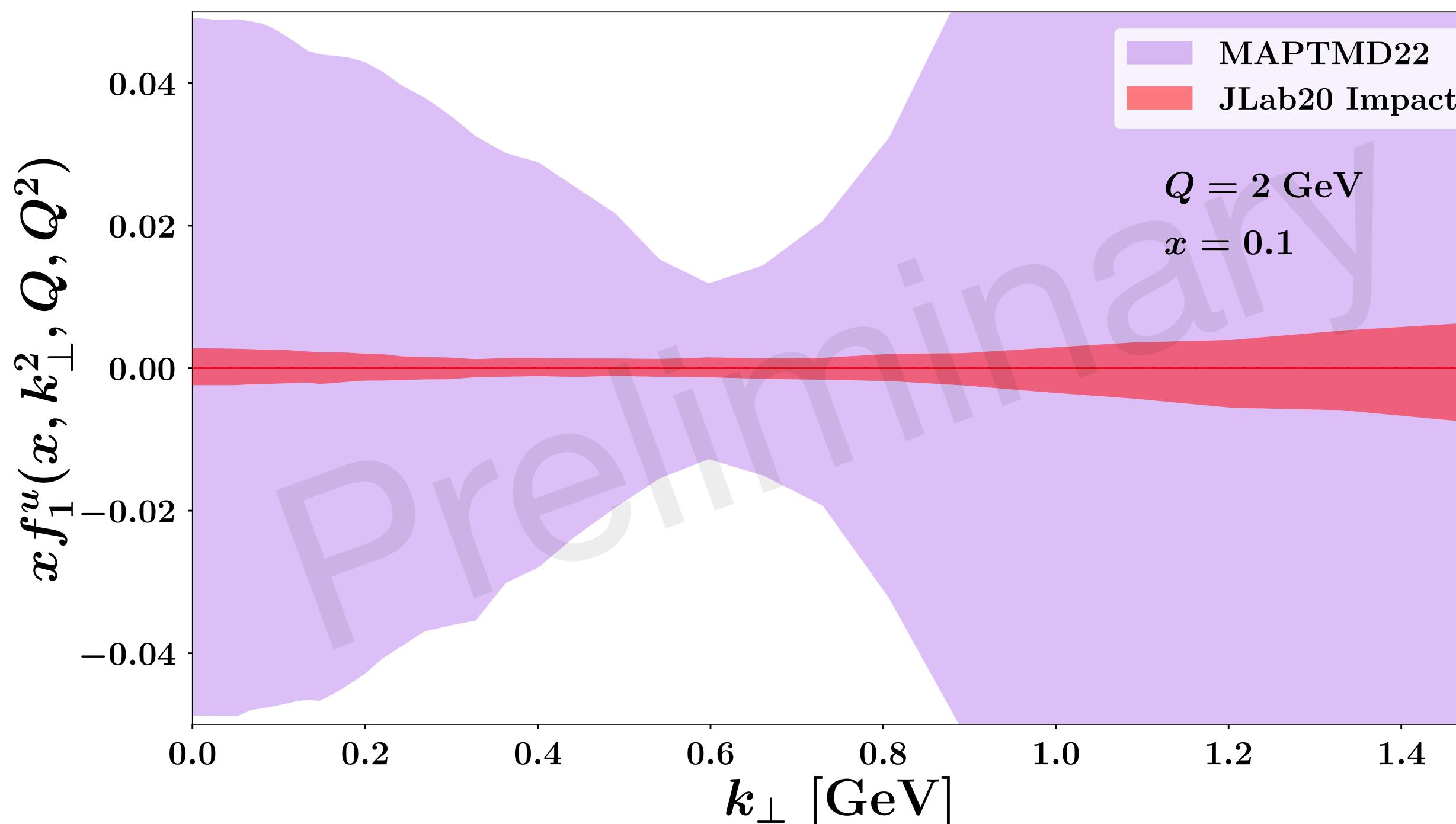


ELECTRON ION COLLIDER

JEFFERSON LAB 20+

+ JLAB 20

Impact studies - JLab 20+



Better constrain at high x and low Q

A new global fit: MAPTMD22

	Accuracy	SIDIS	DY	Z production	N of points	χ^2/N_{data}
Pavia 2017 arXiv:1703.10157	NLL —	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL	✓	✓	✓	1039	1.06
MAPTMD22	N^3LL-	✓	✓	✓	2031	1.06

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And the
other hadrons?

Available fits of Pion TMDs

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	Accuracy	DY	N of points	χ^2/N_{data}
Wang et al, 2017 arXiv:1707.05207	NLL	✓	96	1.61
VPion 2019 arXiv:1907.10356	$N^2 LL'$	✓	80	1.44
MAPTMDPion22	$N^3 LL^-$	✓	138	1.54
Jam 2023 arXiv:2302.01192	$N^2 LL$	✓	93	1.37

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Best theoretical accuracy

High number of included data

Fairly good description

MAPTMDPion22: Included datasets

MAPTMDPion22: Included datasets

Pion-induced Drell-Yan process

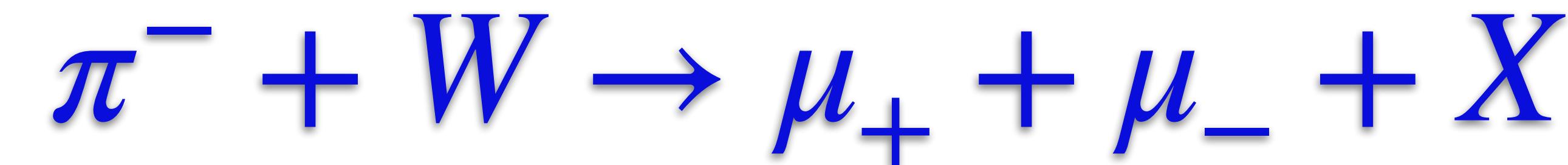
MAPTMDPion22: Included datasets

Pion-induced Drell-Yan process

$$\pi^- + W \rightarrow \mu_+ + \mu_- + X$$

MAPTMDPion22: Included datasets

Pion-induced Drell-Yan process



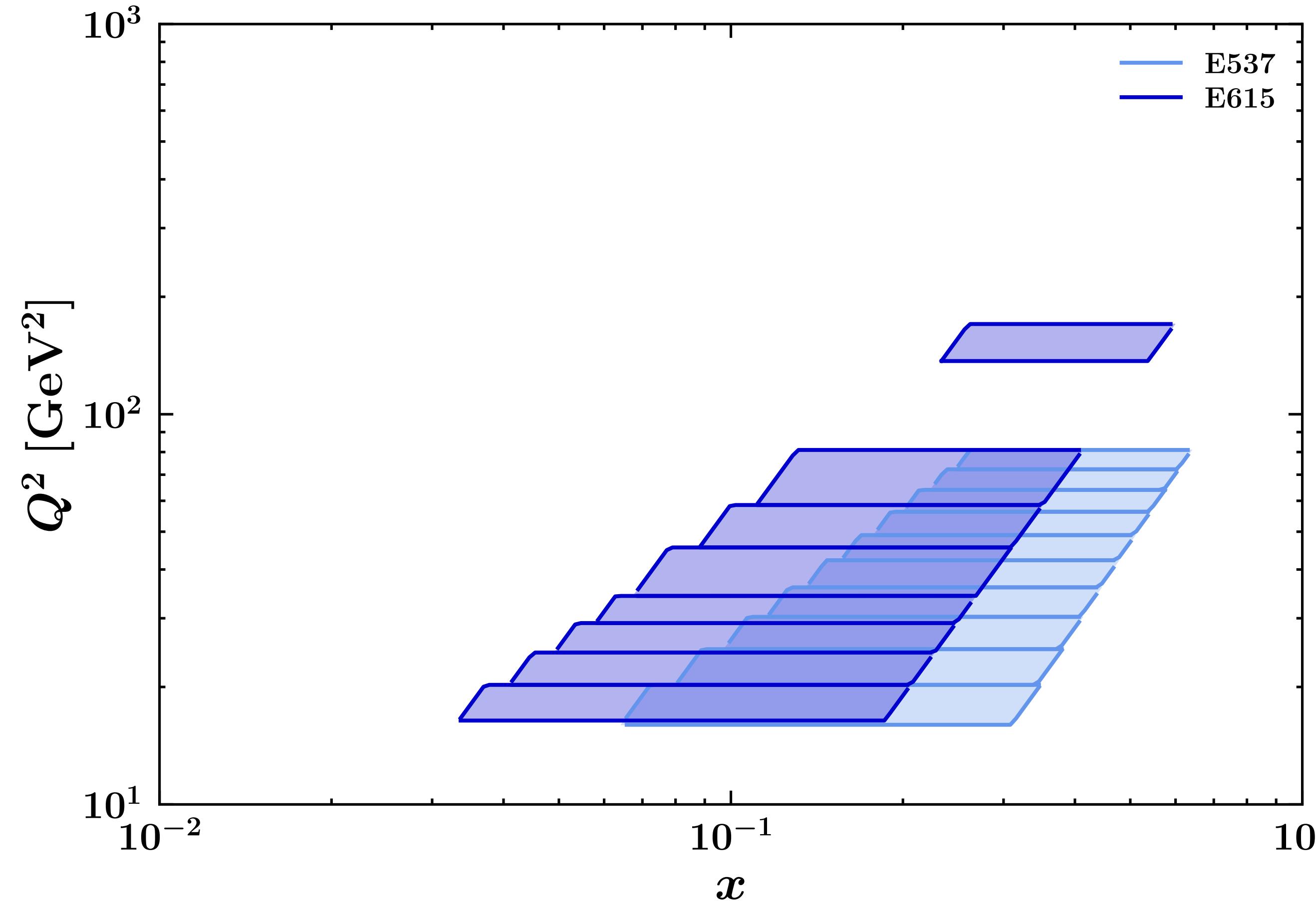
Experiment	\sqrt{s} [GeV]	Q [GeV]	N_{bins}	x_F
E615 (Q-diff)	21.8	$4.05 < Q < 13.05$	10 (8)	$0 < x_F < 1$
E537 (Q-diff)	15.3	$4.0 < Q < 9.0$	10	$-0.1 < x_F < 1$

W. J. Stirling et al. 1993

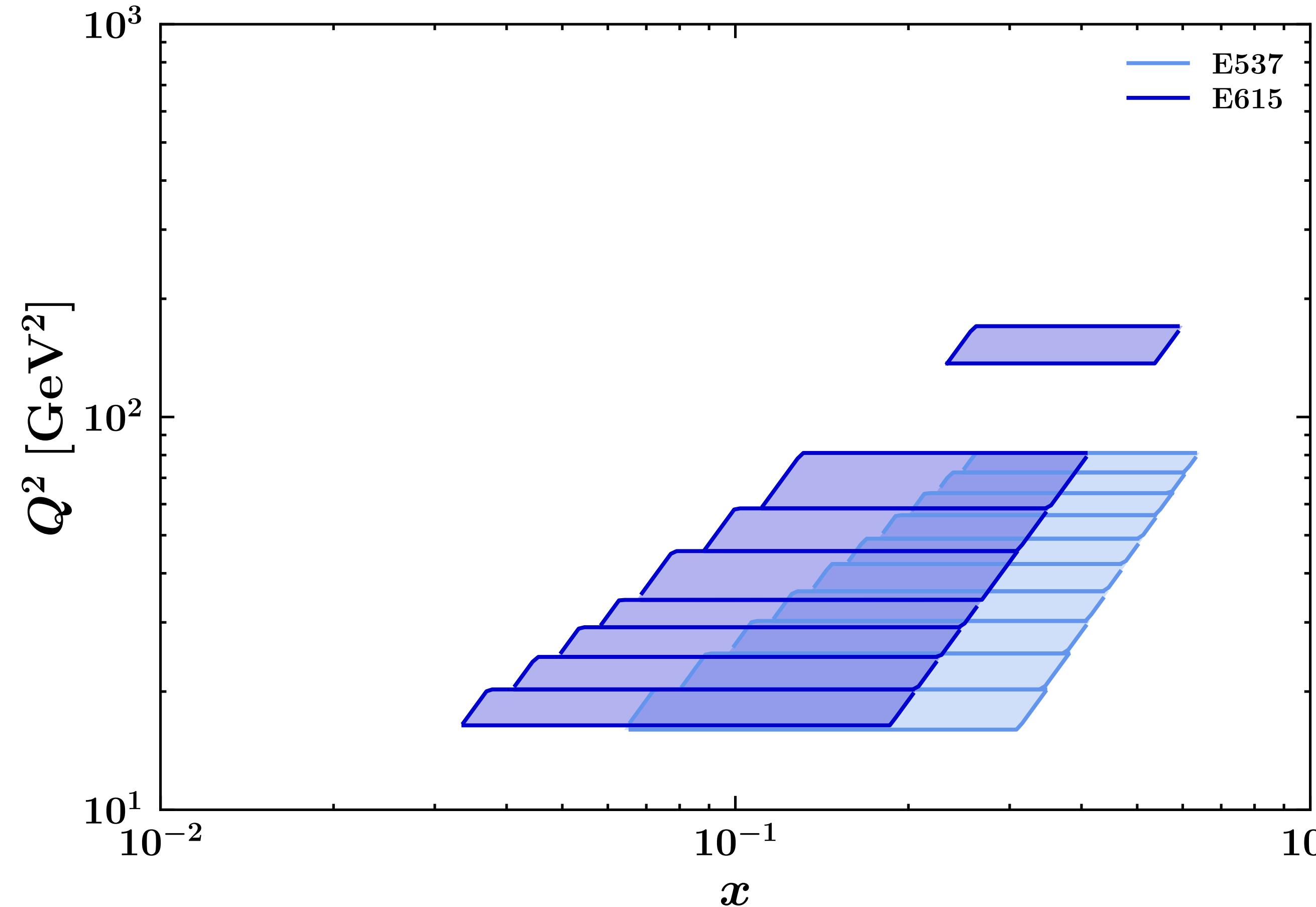
E. Anassontzis et al. 1988

MAPTMDPion22: Included datasets

MAPTMDPion22: Included datasets



MAPTMDPion22: Included datasets



**Small region covered by
the available datasets**

MAPTMDPion22: Included datasets

Experiment	Number of points	Statistical errors	Systematic errors	Theoretical errors
E615 (Q-diff)	74/155	5%	16%	5-8%
E537 (Q-diff)	64/150	15-20%	8%	5-8%
Total	138/305	Large Uncertainties	Large Normalization Errors	Extra uncertainties

MAPTMDPion22: Included datasets

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Presence of many and different kind of errors

MAPTMDPion22: Models

MAPTMDPion22: Models

Proton

MAPTMDPion22: Models

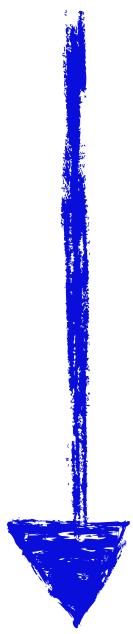
Proton



MAPTMD22

MAPTMDPion22: Models

Proton



Pion

MAPTMD22

MAPTMDPion22: Models

Proton



MAPTMD22

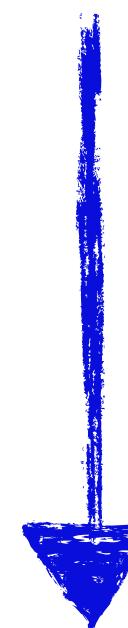
Pion



xFitter20

MAPTMDPion22: Models

Proton



MAPTMD22

$$f_{1\pi}^{NP}(x, \zeta, \mathbf{b}_T) = e^{-g_{1\pi}(x) \frac{\mathbf{b}_T^2}{4}}$$

Pion



xFitter20

MAPTMDPion22: Models

Proton



MAPTMD22

Pion



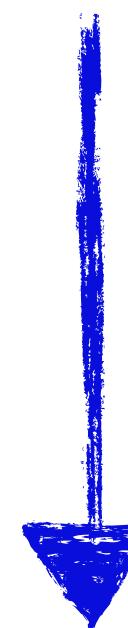
xFitter20

$$f_{1\pi}^{NP}(x, \zeta, \mathbf{b}_T) = e^{-g_{1\pi}(x) \frac{\mathbf{b}_T^2}{4}}$$

$$g_{1\pi}(x) = N_{1\pi} \frac{x^{\sigma_\pi} (1-x)^{\alpha_\pi^2}}{\hat{x}^{\sigma_\pi} (1-\hat{x})^{\alpha_\pi^2}}$$

MAPTMDPion22: Models

Proton



MAPTMD22

Pion



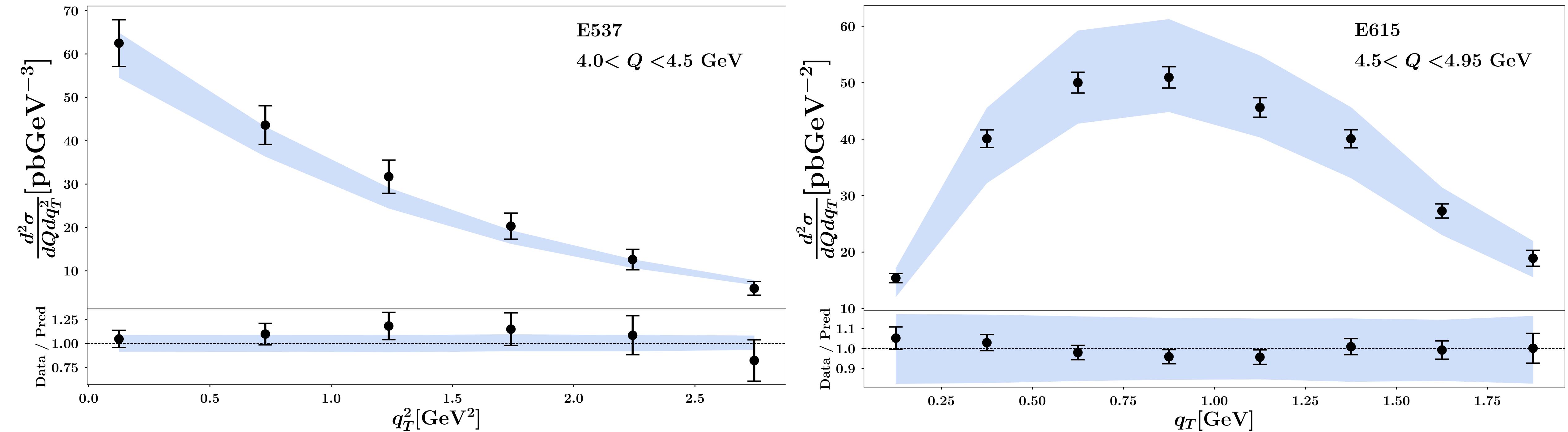
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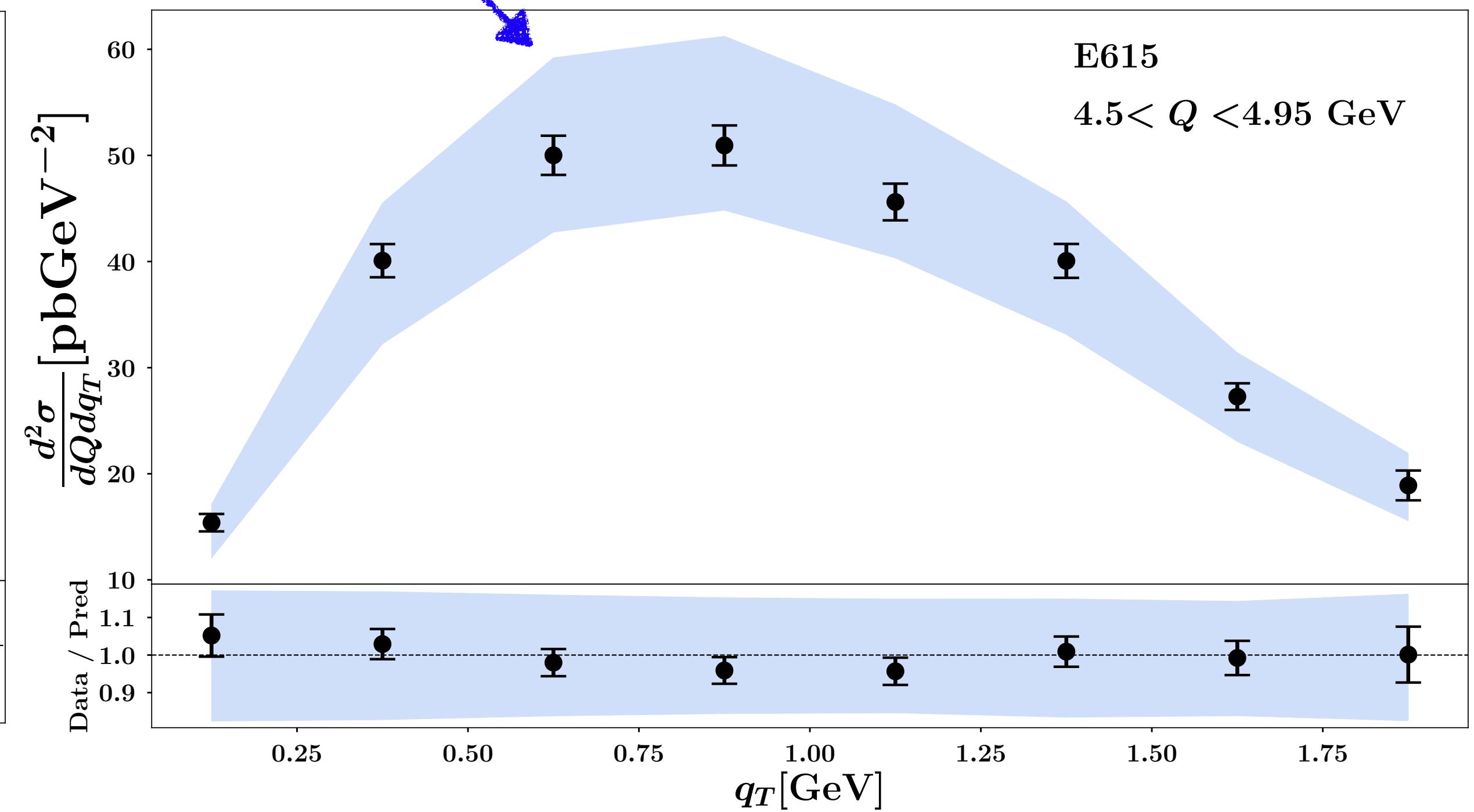
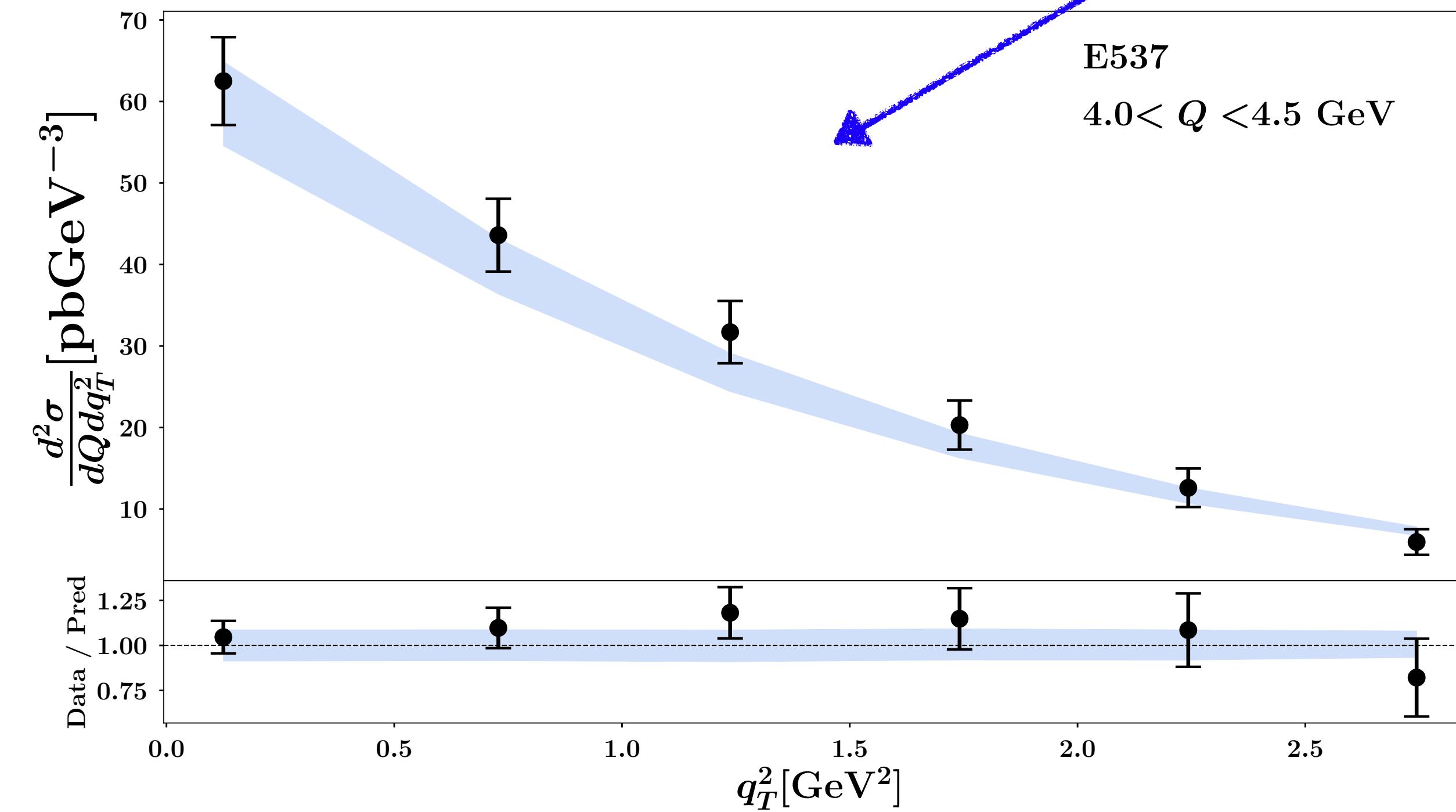
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3 fitting parameters

MAPTMDPion22: Fit Results

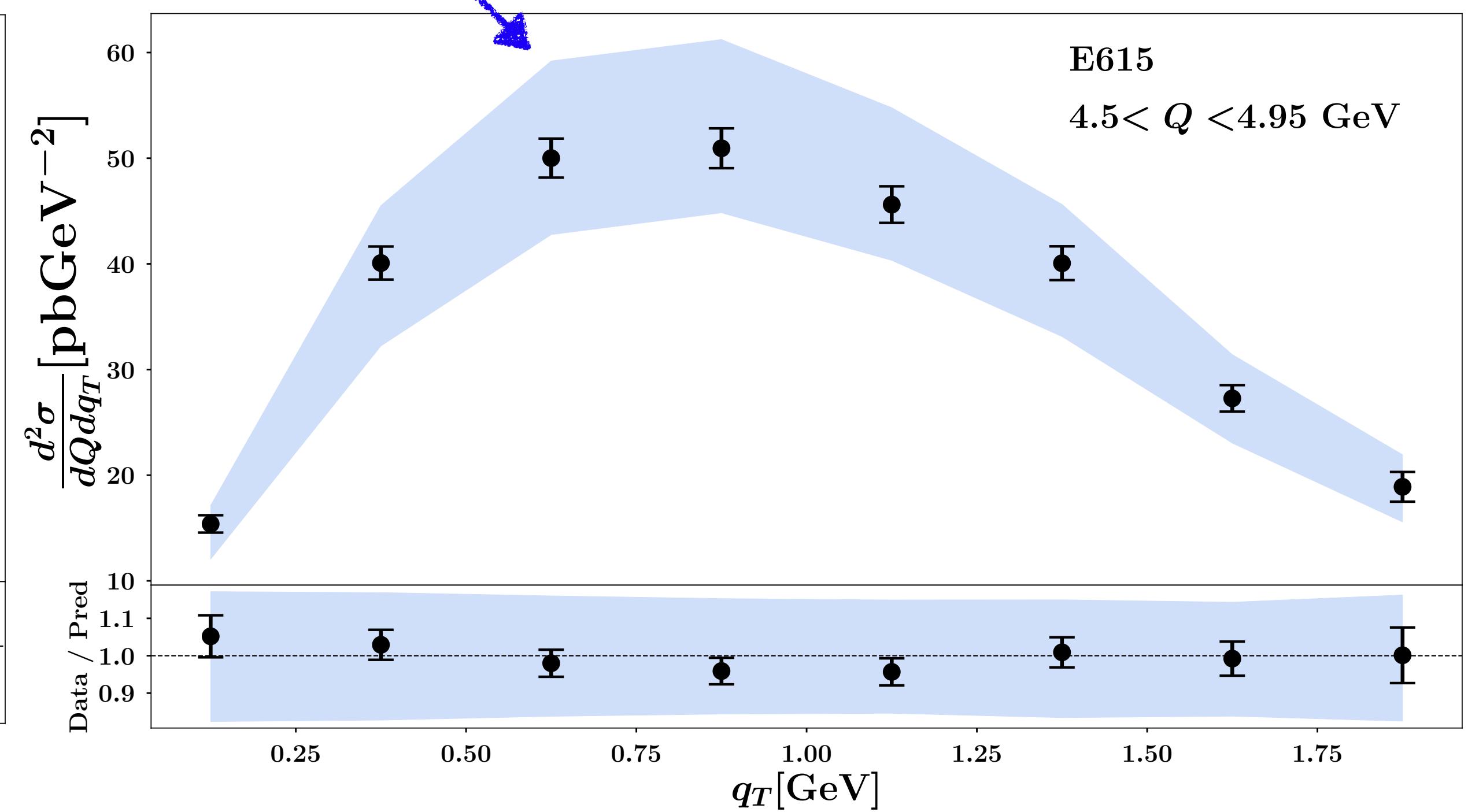
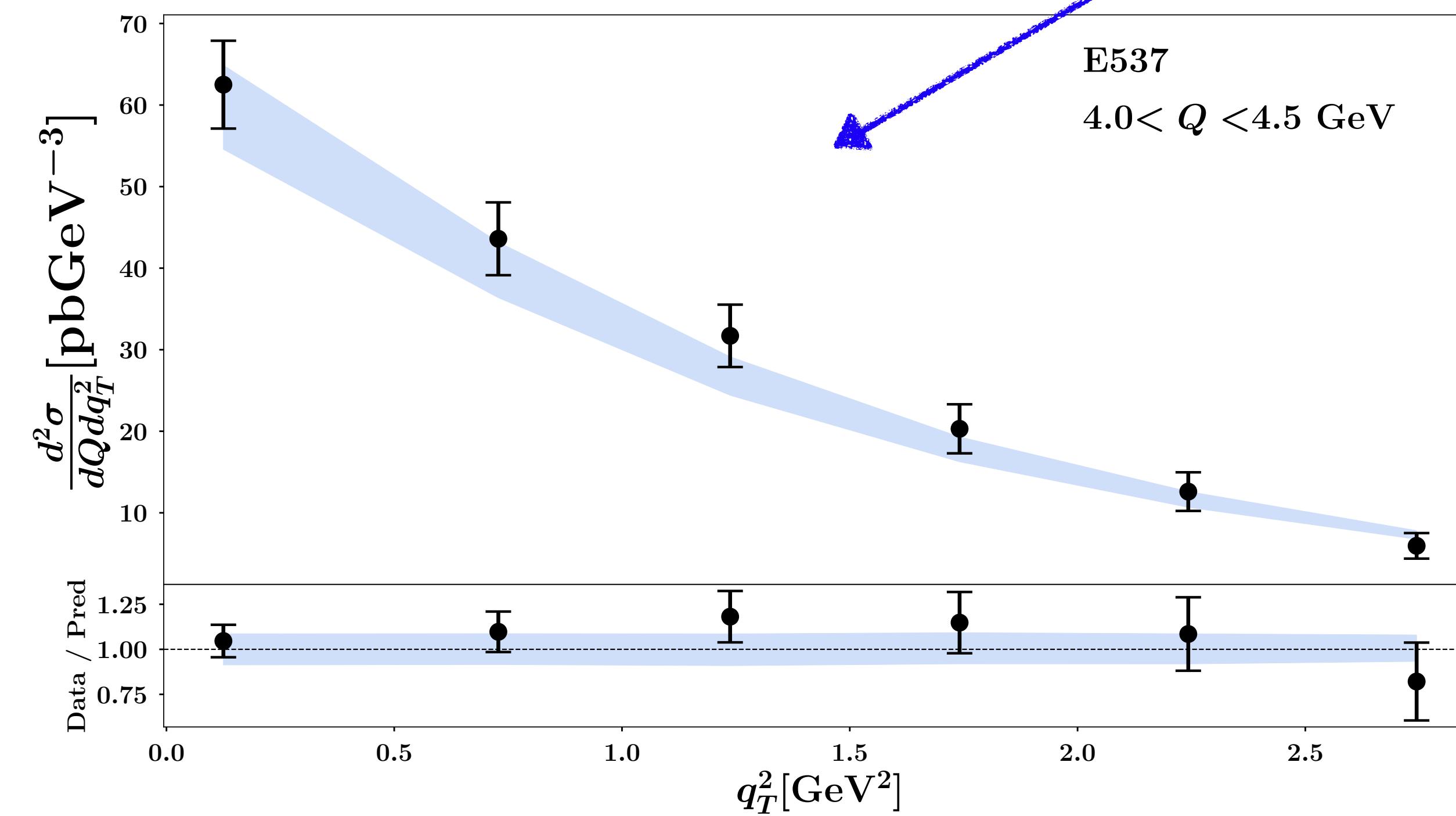


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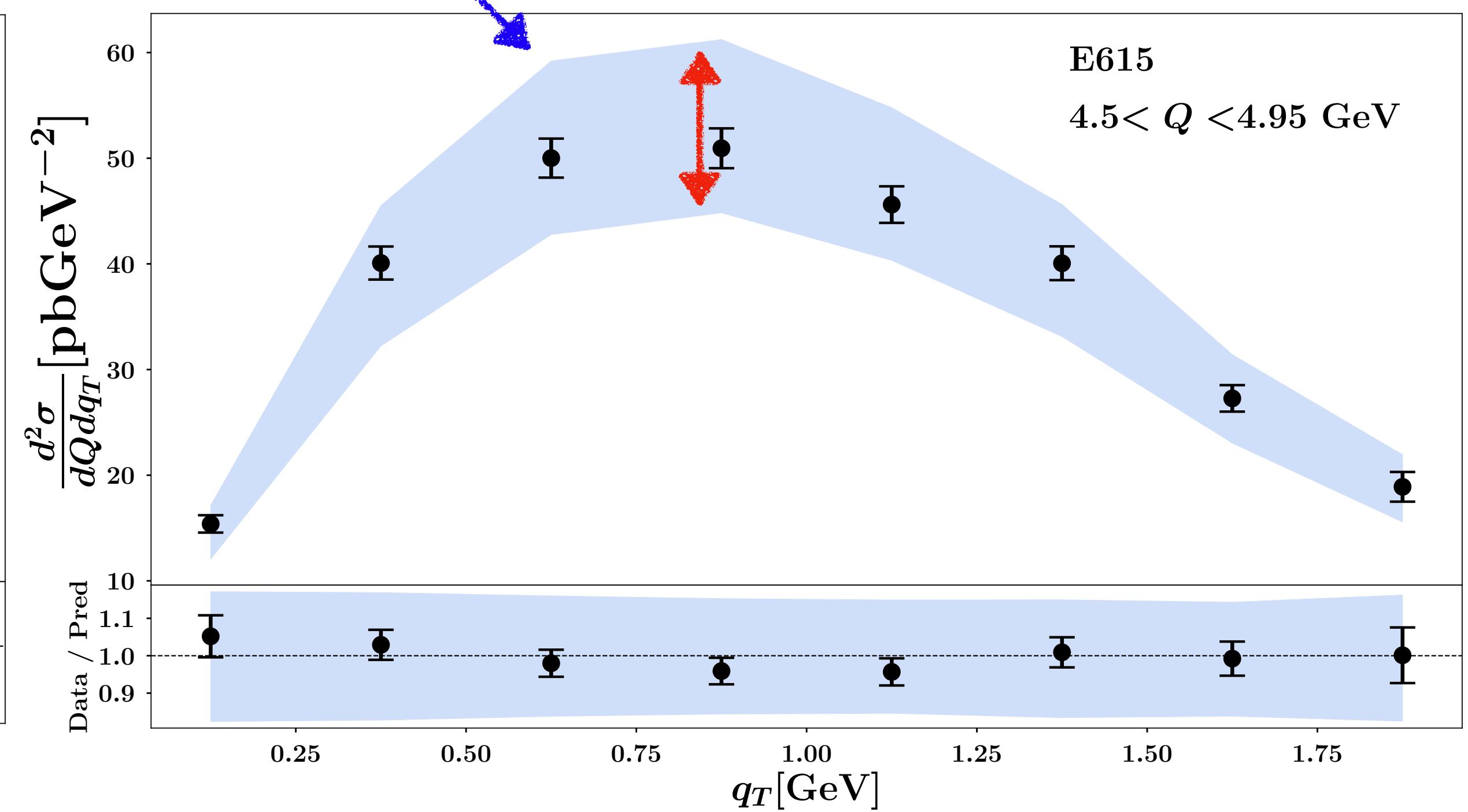
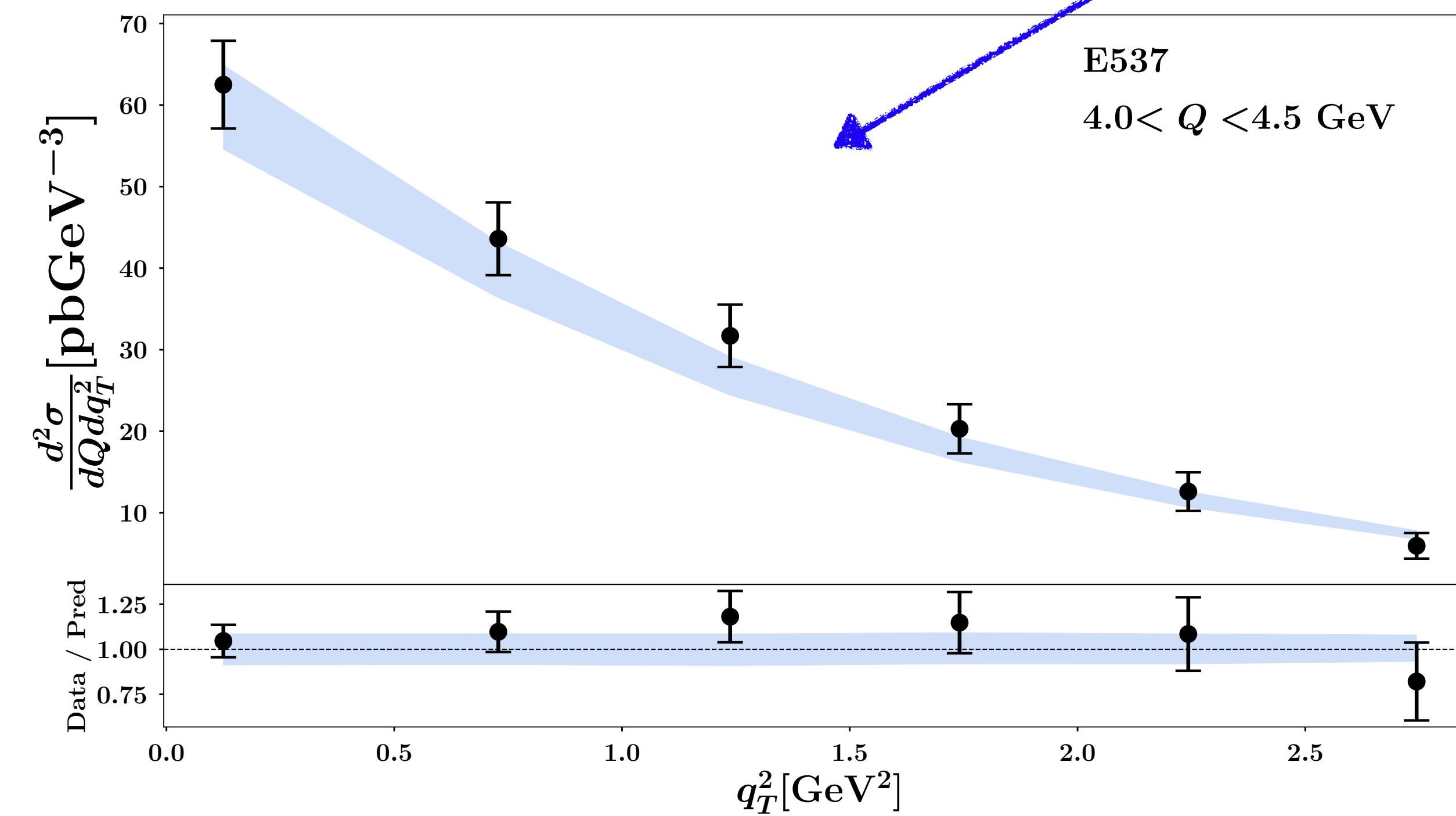
MAPTMDPion22: Fit Results

Good agreement
in the shape

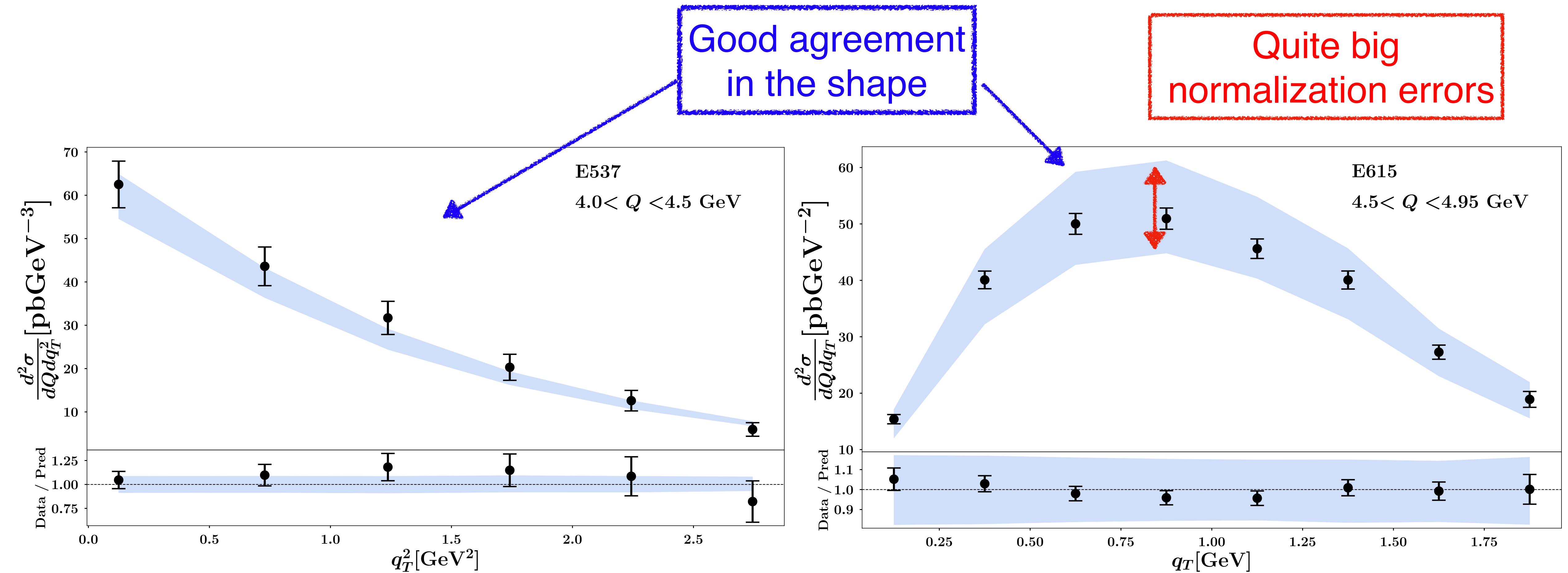


MAPTMDPion22: Fit Results

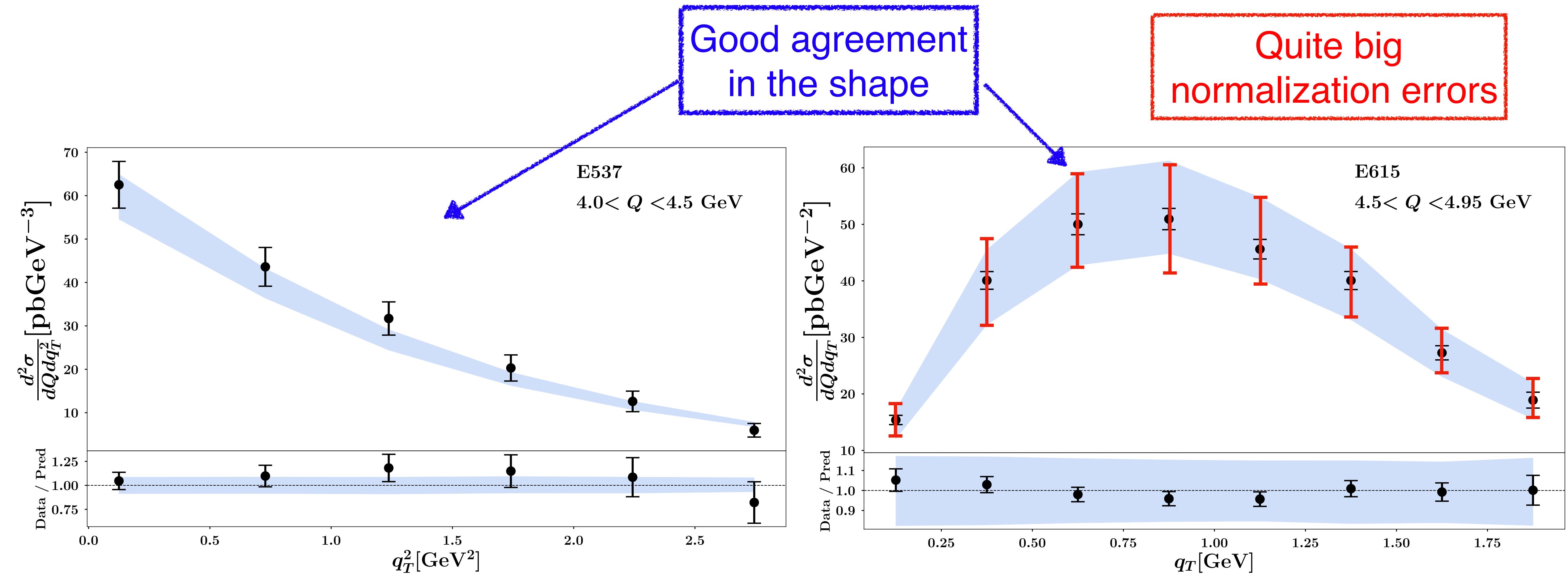
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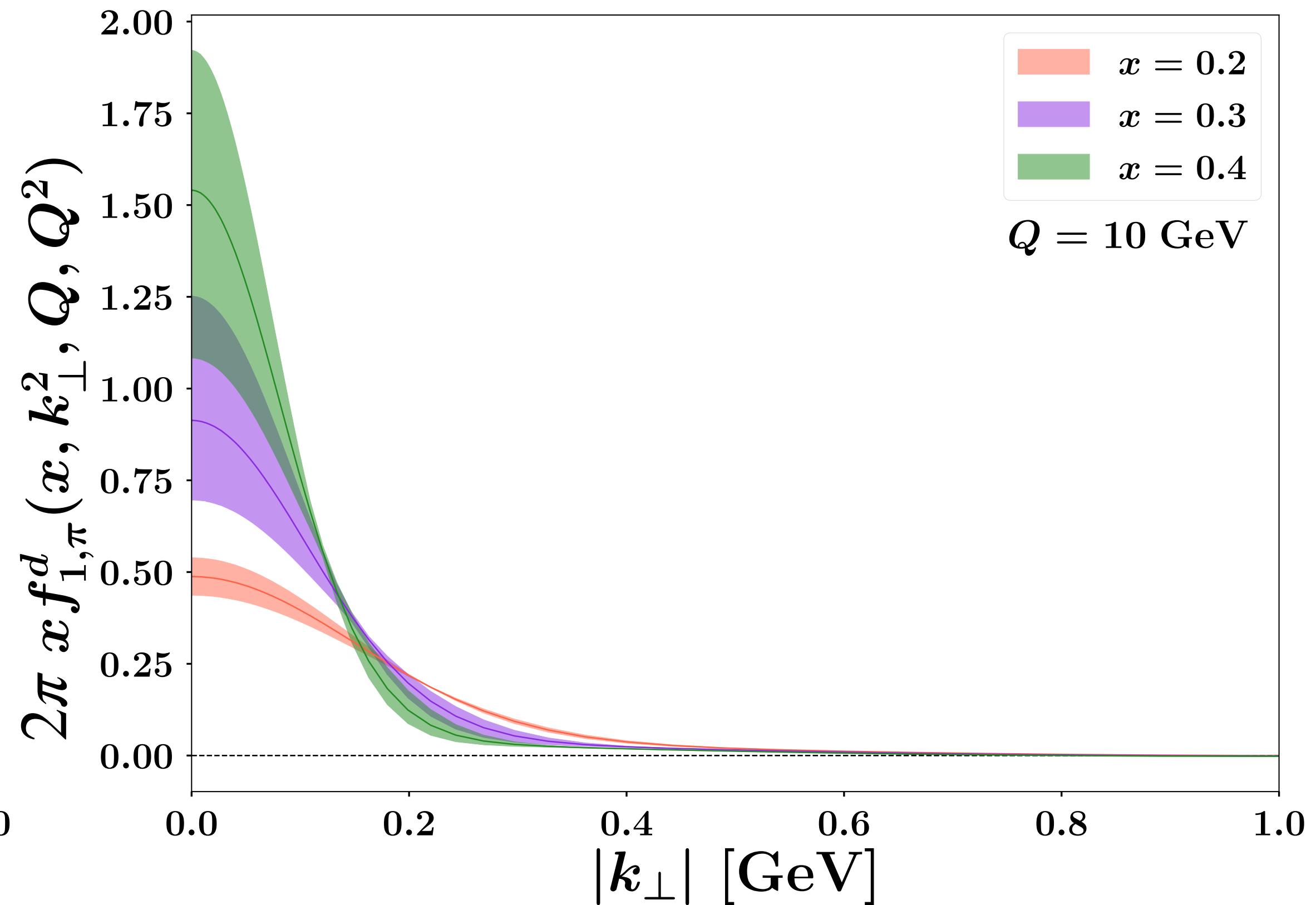
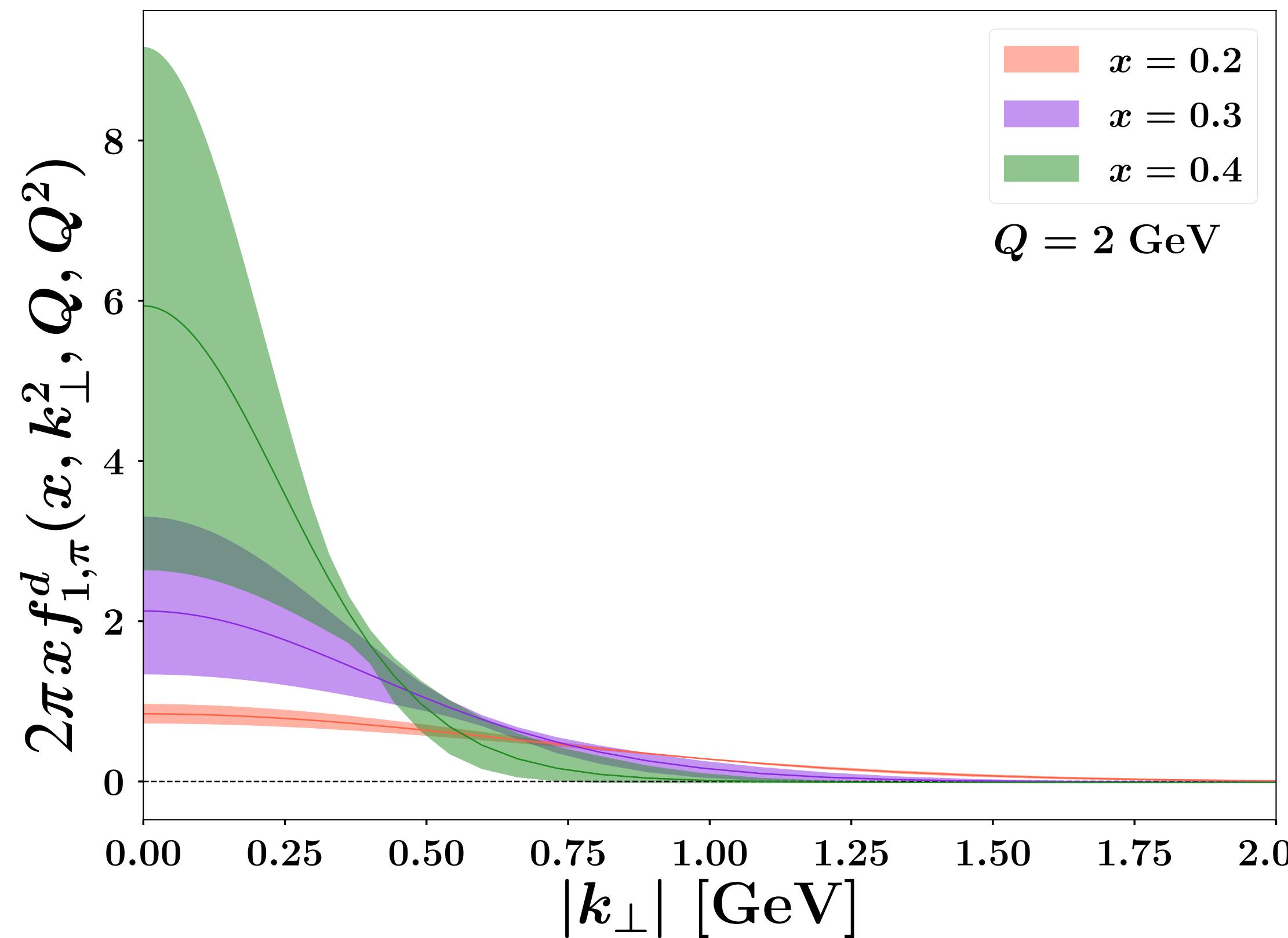


Visualization of Pion TMD PDFs

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$$\sigma_\pi = 4.50 \pm 2.25$$

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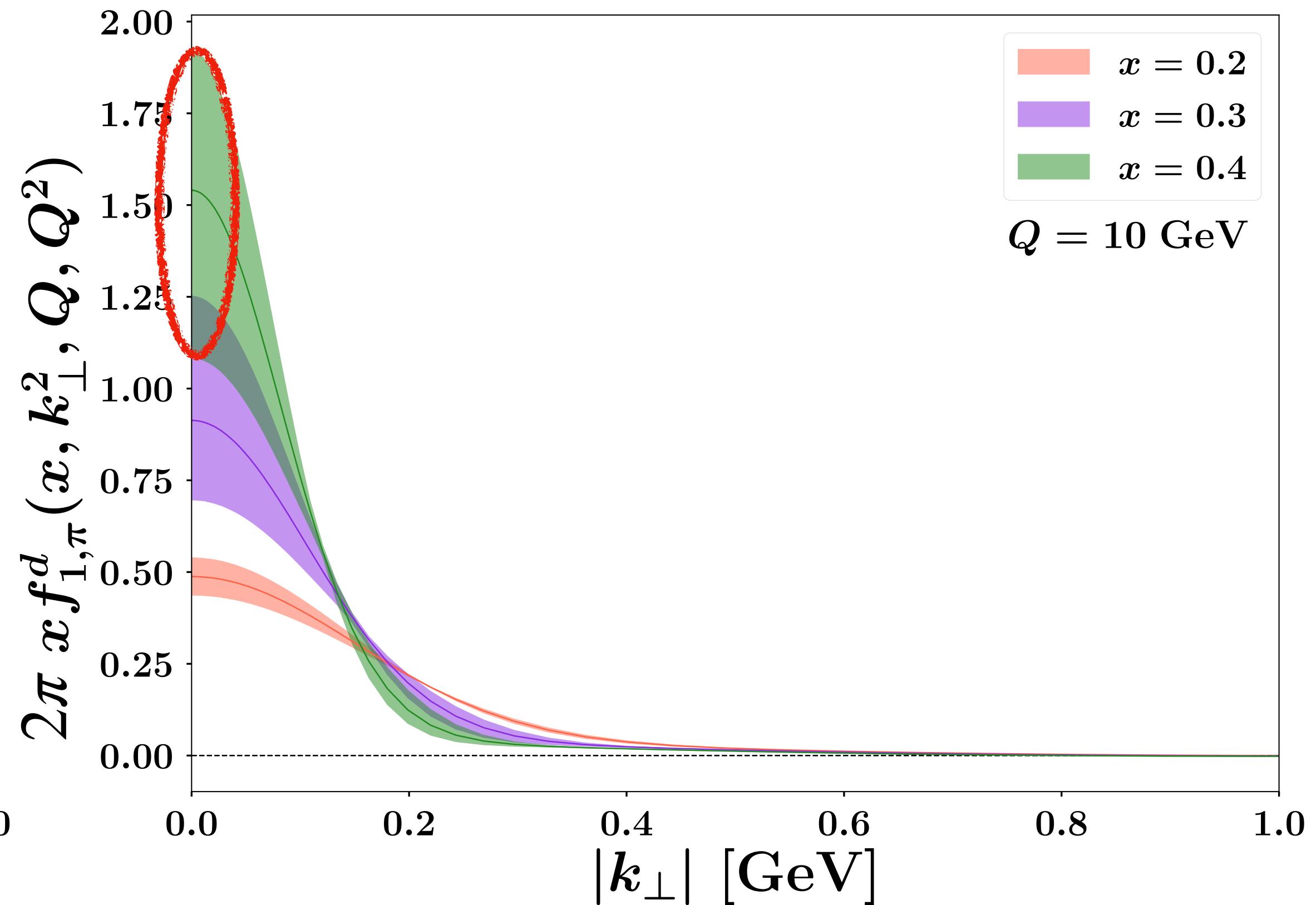
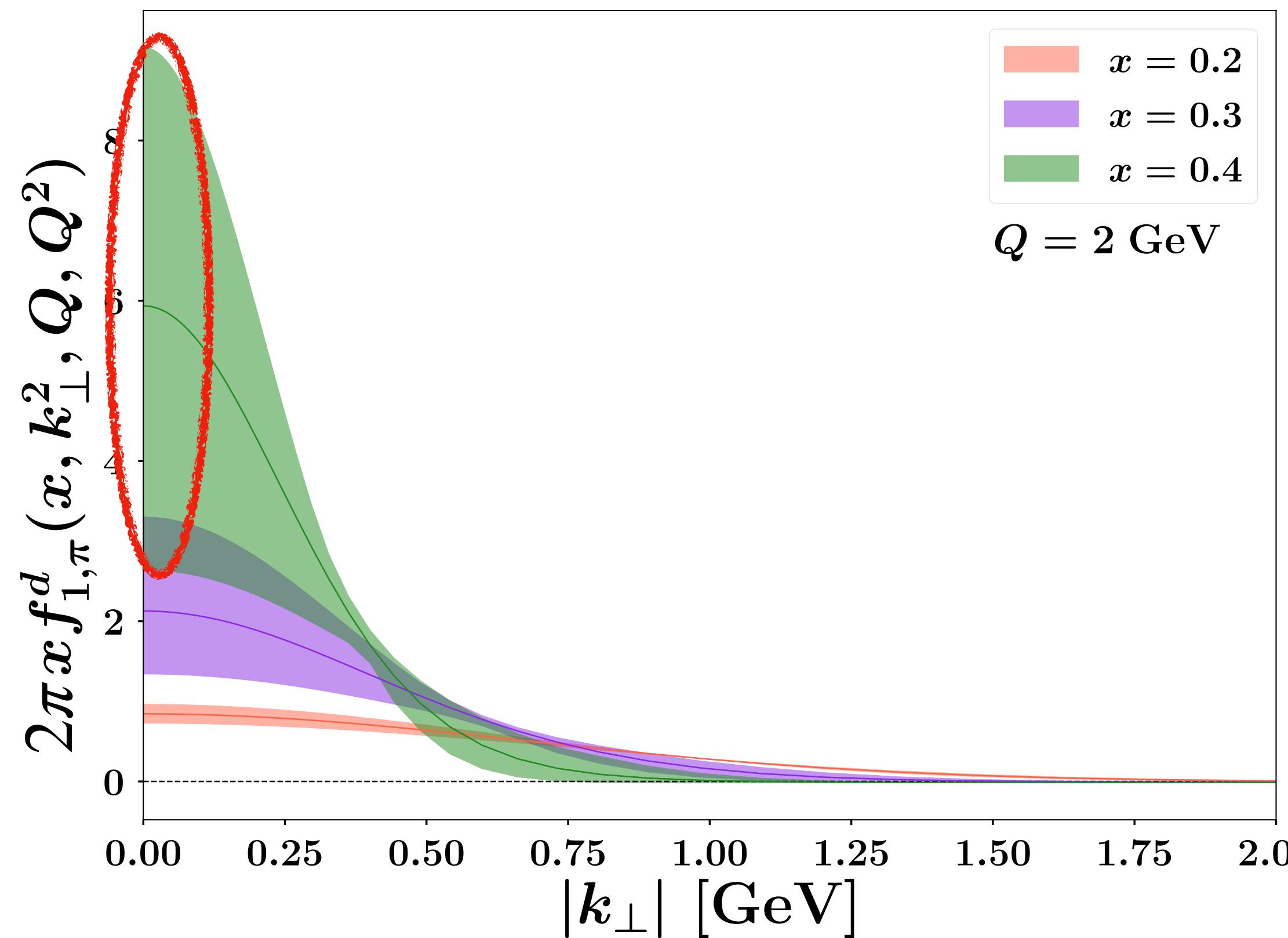


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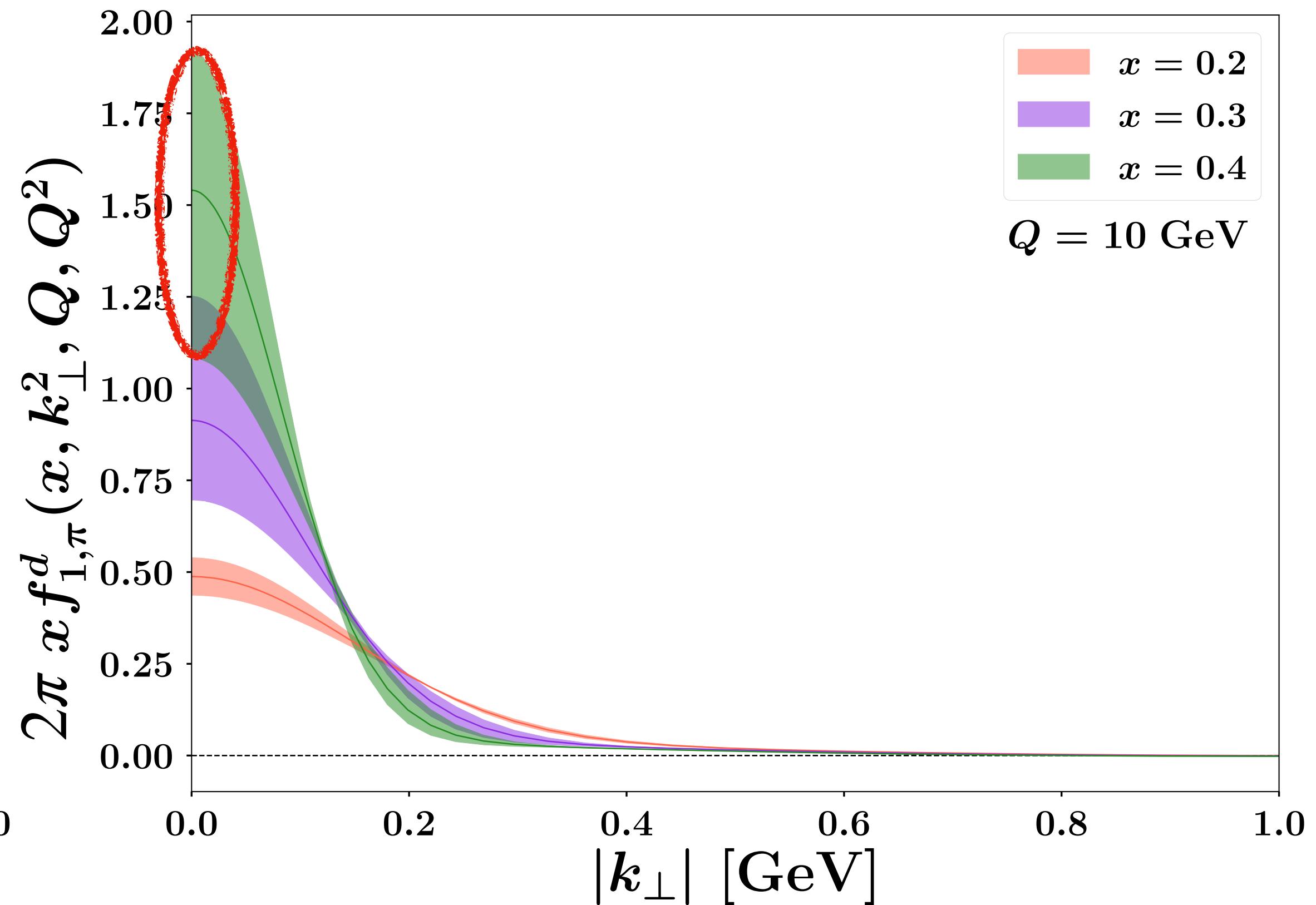
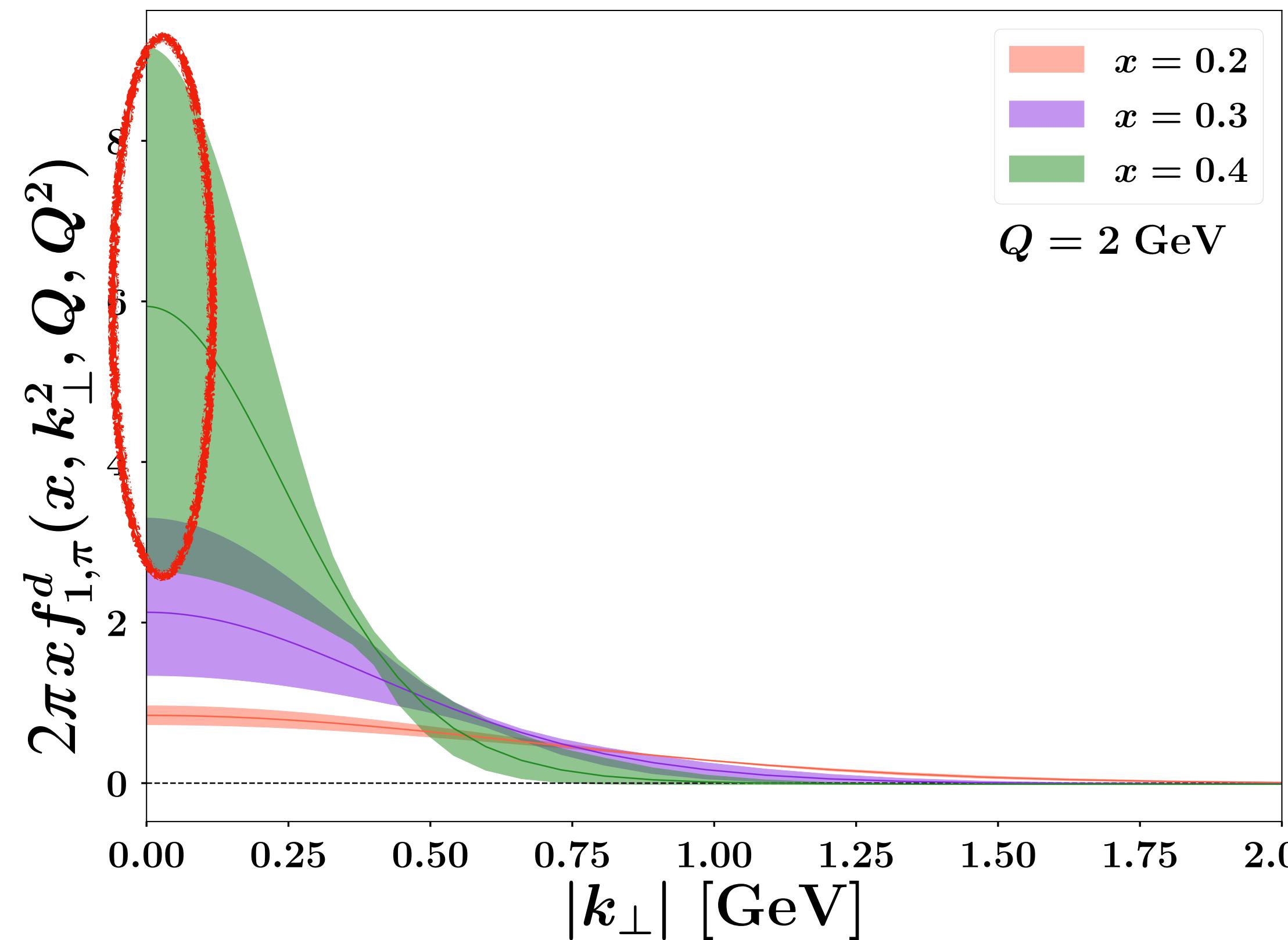


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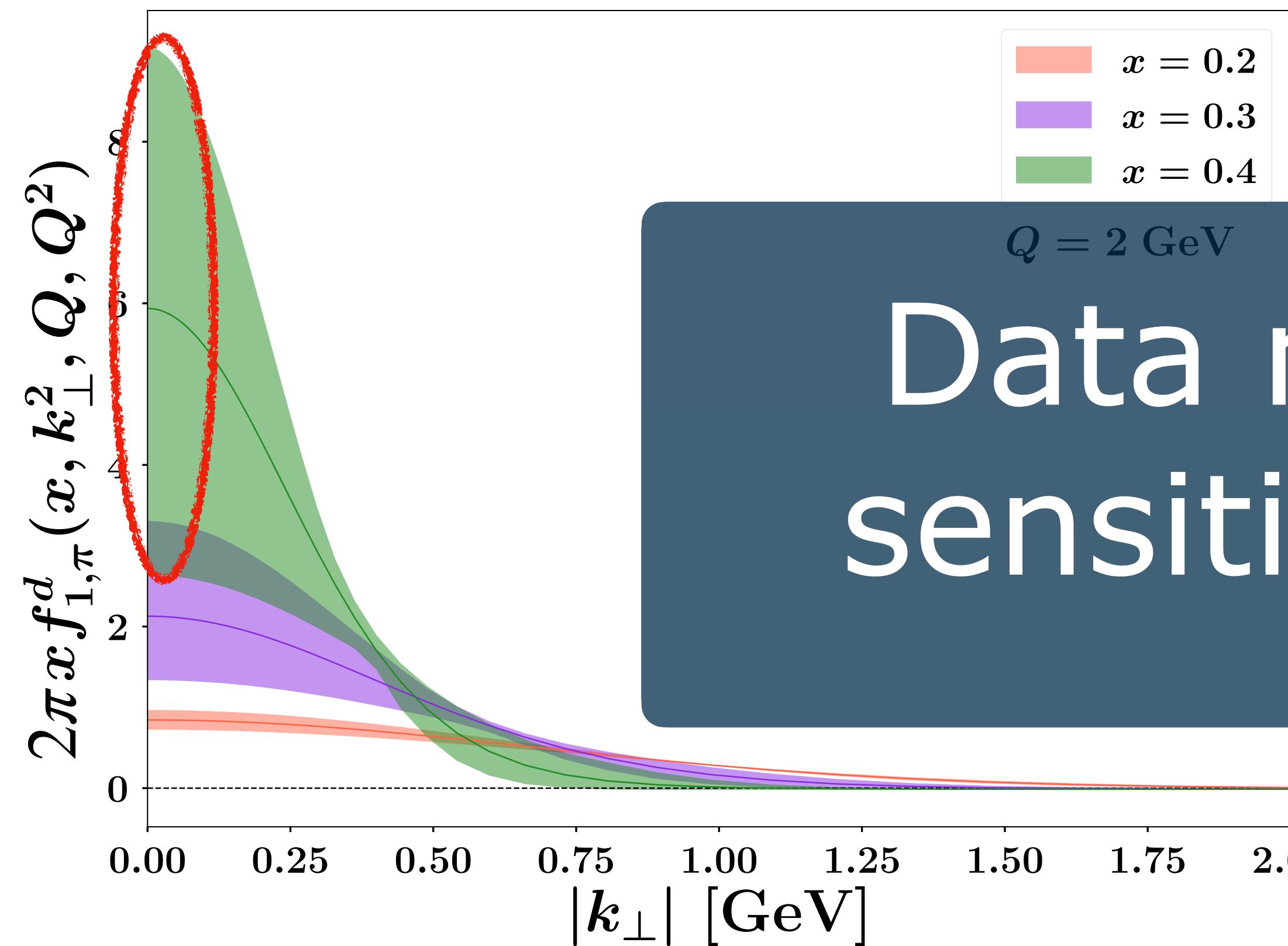


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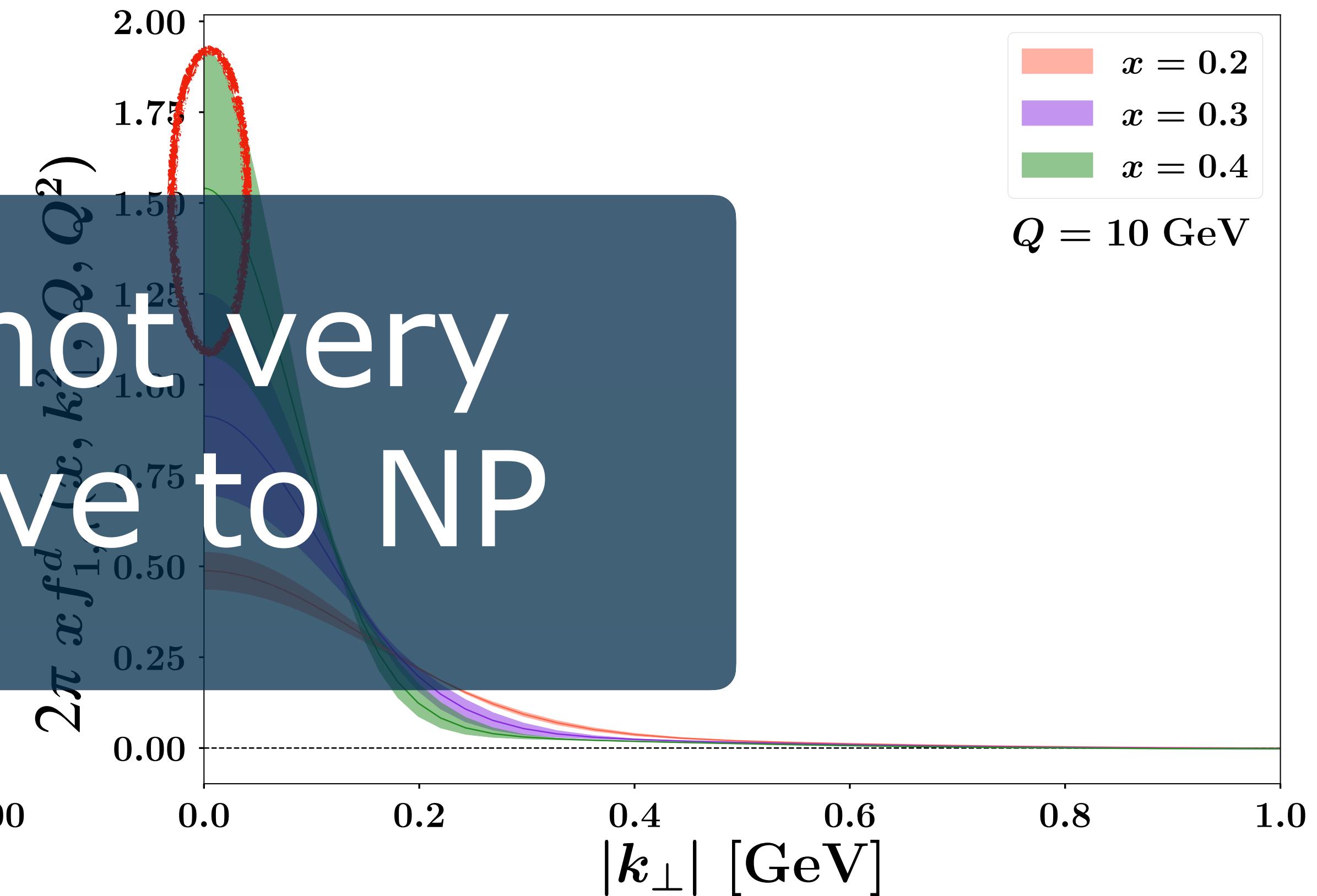
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Data not very
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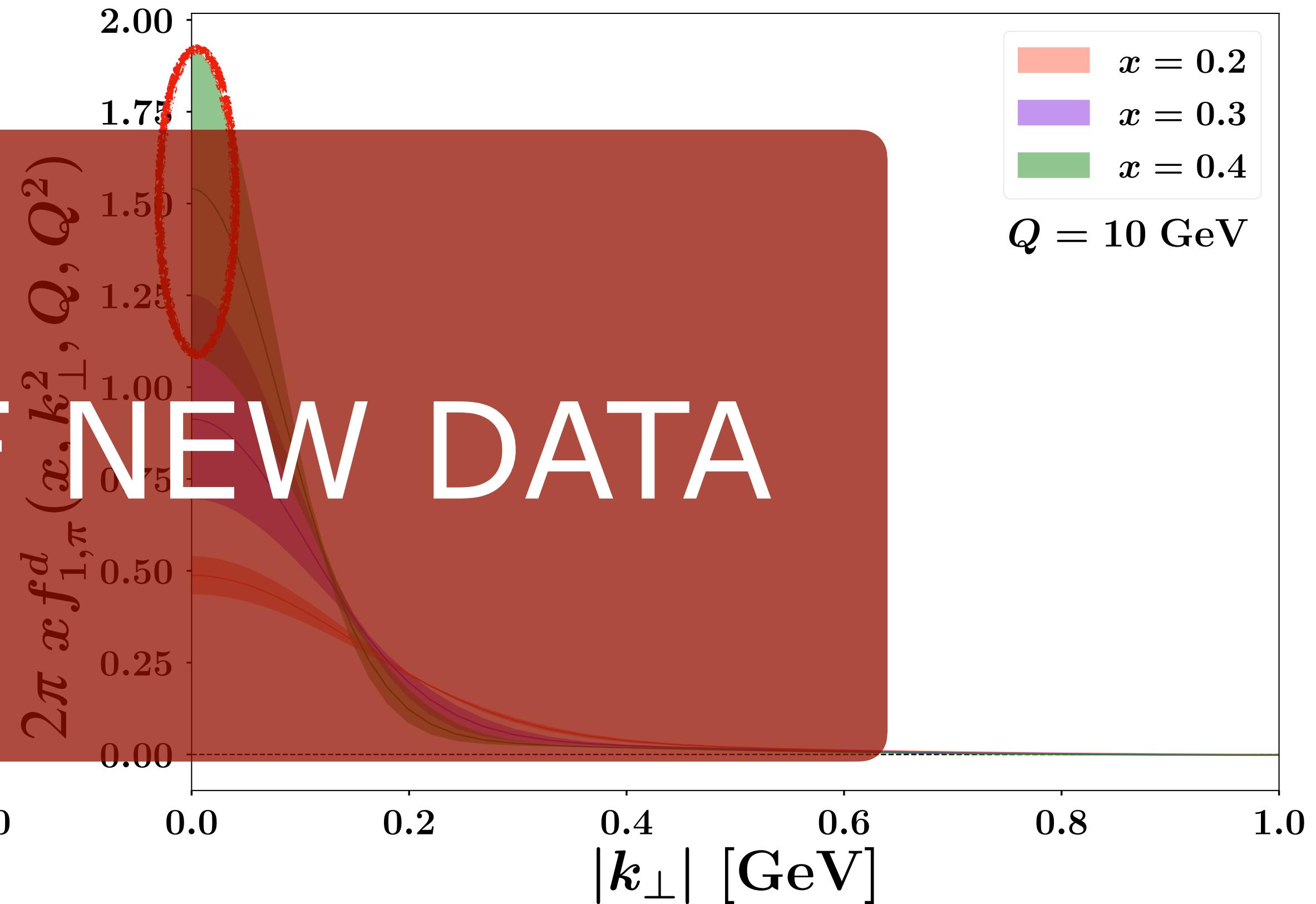
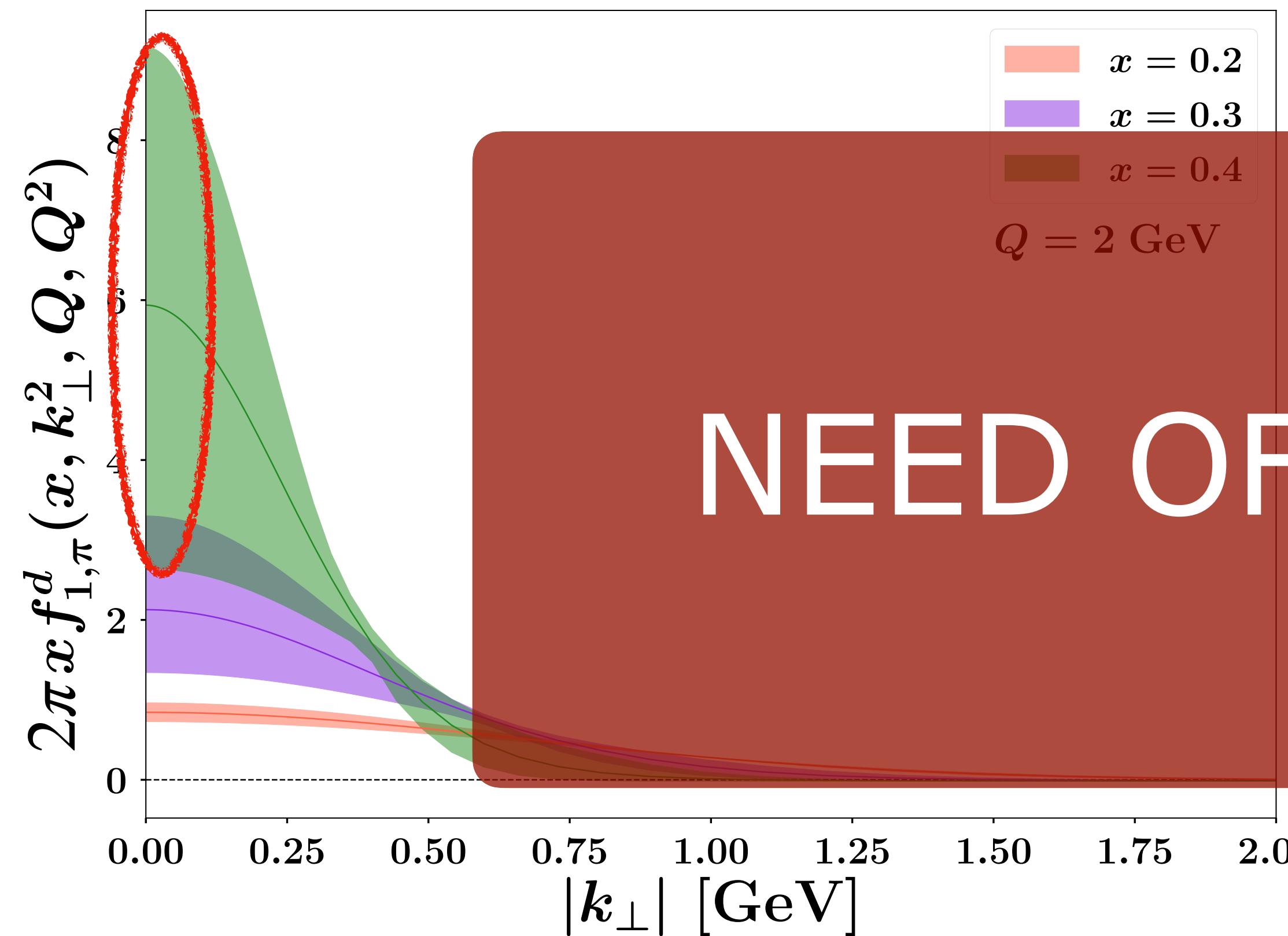


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NEED OF NEW DATA

Conclusions and outlooks

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- Flavour Dependence
- Refinement of Pion TMDs (COMPASS data, Power corrections, PRV model)