The revised TMD shape function in SIDIS

Talk @ SarWorS 2023

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In collaboration with:
D. Boer, J. Bor, C. Pisano & F. Yuan

Outline

- **TMD factorization** involving a **shape function** for the quarkonium
- **Extraction** of the TMD shape function via a **matching procedure**
  - Relevance of the hard amplitude **pole structure**
- **Process dependence** of the TMD shape function
- Conclusions and outcome

Quarkonia & gluon TMDs

Processes involving Quarkonia are **sensitive to gluons**

hadron collisions

- $p + p \to \eta_Q + X$
- $p + p \to \chi_Q + X$
- $p + p \to J/\psi + J/\psi + X$
- $p + p \to J/\psi + X$ **?**

ep **collisions**

- $e + p \to e' + J/\psi + X$
- $e + p \to e' + J/\psi + jet + X$

and more…

Quarkonia & gluon TMDs

Processes involving Quarkonia are **sensitive to gluons**

hadron collisions

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- $p + p \to J/\psi + J/\psi + X$
- $p + p \to J/\psi + X$ **?**

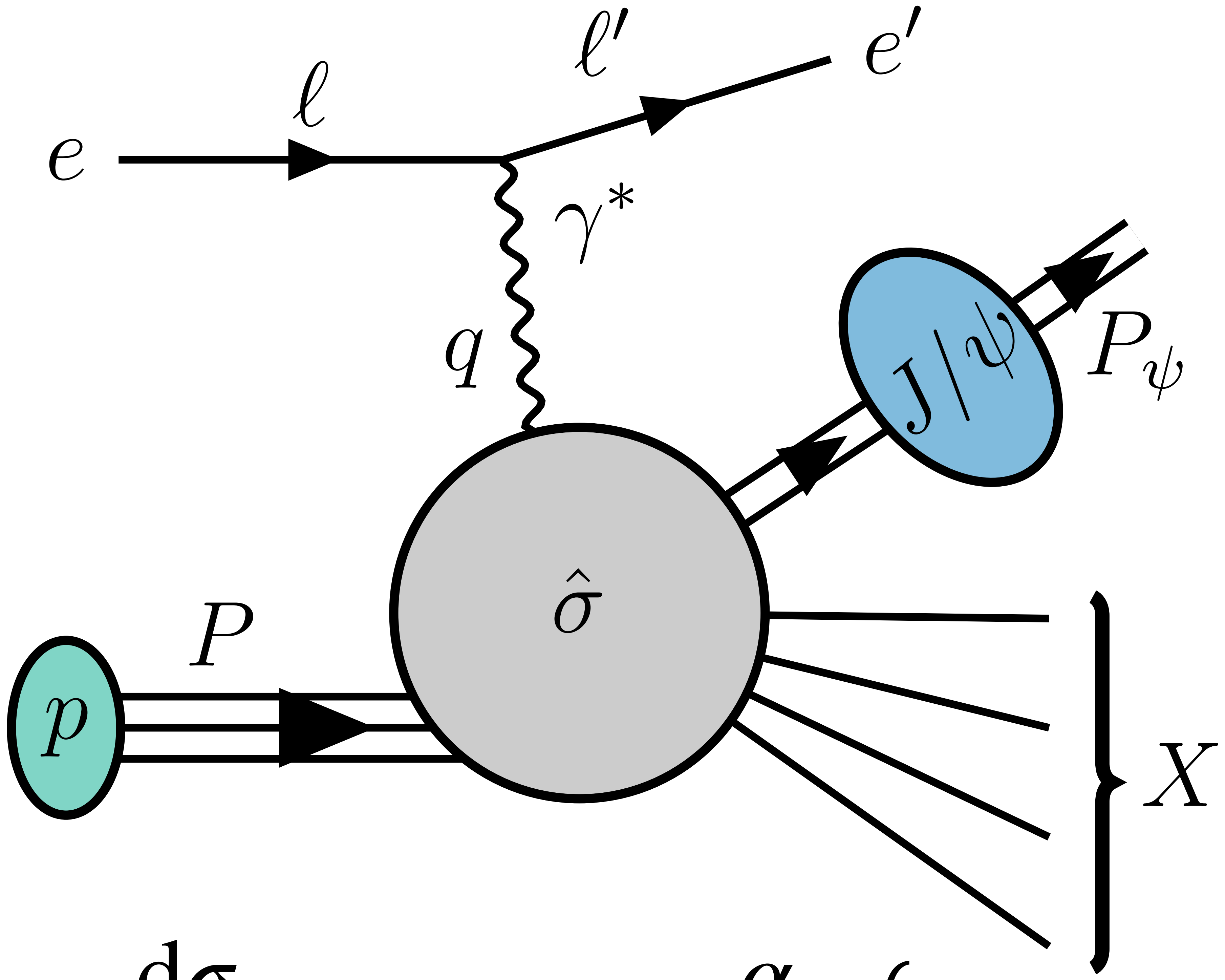
ep **collisions**

- $\boxed{e + p \to e' + J/\psi + X}$
- $e + p \to e' + J/\psi + jet + X$

and more…

Theoretical framework

$$e\,(\ell) + p\,(P) \to e'\,(\ell') + \gamma^*\,(q) + p\,(P) \to e'\,(\ell') + J/\psi\,(P_\psi) + X$$



SIDIS variables

$$q^2 = -Q^2,\ S \approx 2P\cdot\ell$$

$$x_B = \frac{Q^2}{2\cdot q},\ y = \frac{P\cdot q}{P\cdot \ell},\ z = \frac{P\cdot P_\psi}{P\cdot q}$$

Phase spaces

$$\frac{\mathrm{d}^3\ell'}{2E'} = 2\pi\, yS\,\mathrm{d}x_B \mathrm{d}y$$

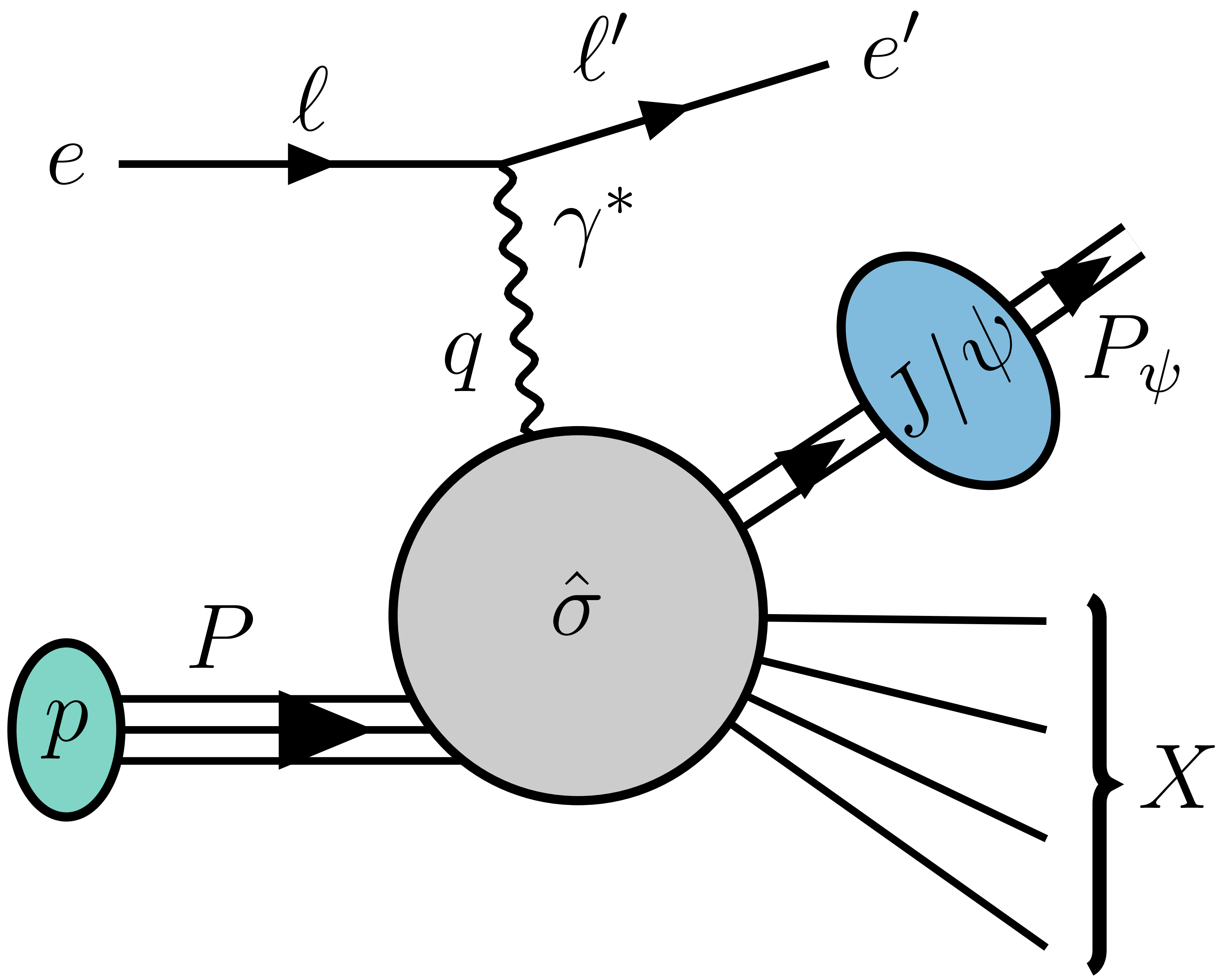
$$\frac{\mathrm{d}^3P_\psi}{2E_\psi} = \frac{\mathrm{d}z}{z}\mathrm{d}^2P_{\psi\perp}\mathrm{d}\phi_\psi$$

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, *JHEP* ***02*** (2007)

Sun, Zhang, *EJPC* ***77*** (2017)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\mathbf{P}^2_{\psi\perp}\,\mathrm{d}\phi_\psi} = \frac{\alpha}{yQ^2}\Big\{\left[1+(1-y)^2\right] F_{UUT} + 4(1-y)\,F_{UUL}$$
$$+(2-y)\sqrt{1-y}\cos\phi_\psi\,F_{UU}^{\cos\phi_\psi} + 4(1-y)\,\cos 2\phi_\psi\,F_{UU}^{\cos 2\phi_\psi}\Big\}$$

The TMD shape function



"light-hadron" SIDIS

$$\sigma^{ep\to e'hX} = \boxed{\hat{\sigma}^{[a]}(\mu_H)} \otimes \boxed{f_p(\hat{x};\mu_H)} \otimes \boxed{D_{a\to h}(\hat{z};\mu_H)}$$

Bodwin, Braaten, Lepage, *PRD* ***51*** (1997)

"Quarkonium" SIDIS (adopting NRQCD)

$$\sigma^{ep\to e'J/\psi X} = \boxed{\hat{\sigma}^{[n]}(\mu_H)} \otimes \boxed{f_p(\hat{x};\mu_H)} \otimes \boxed{\langle \mathcal{O}_\psi[n]\rangle\, \delta(\hat{z}-z)}$$

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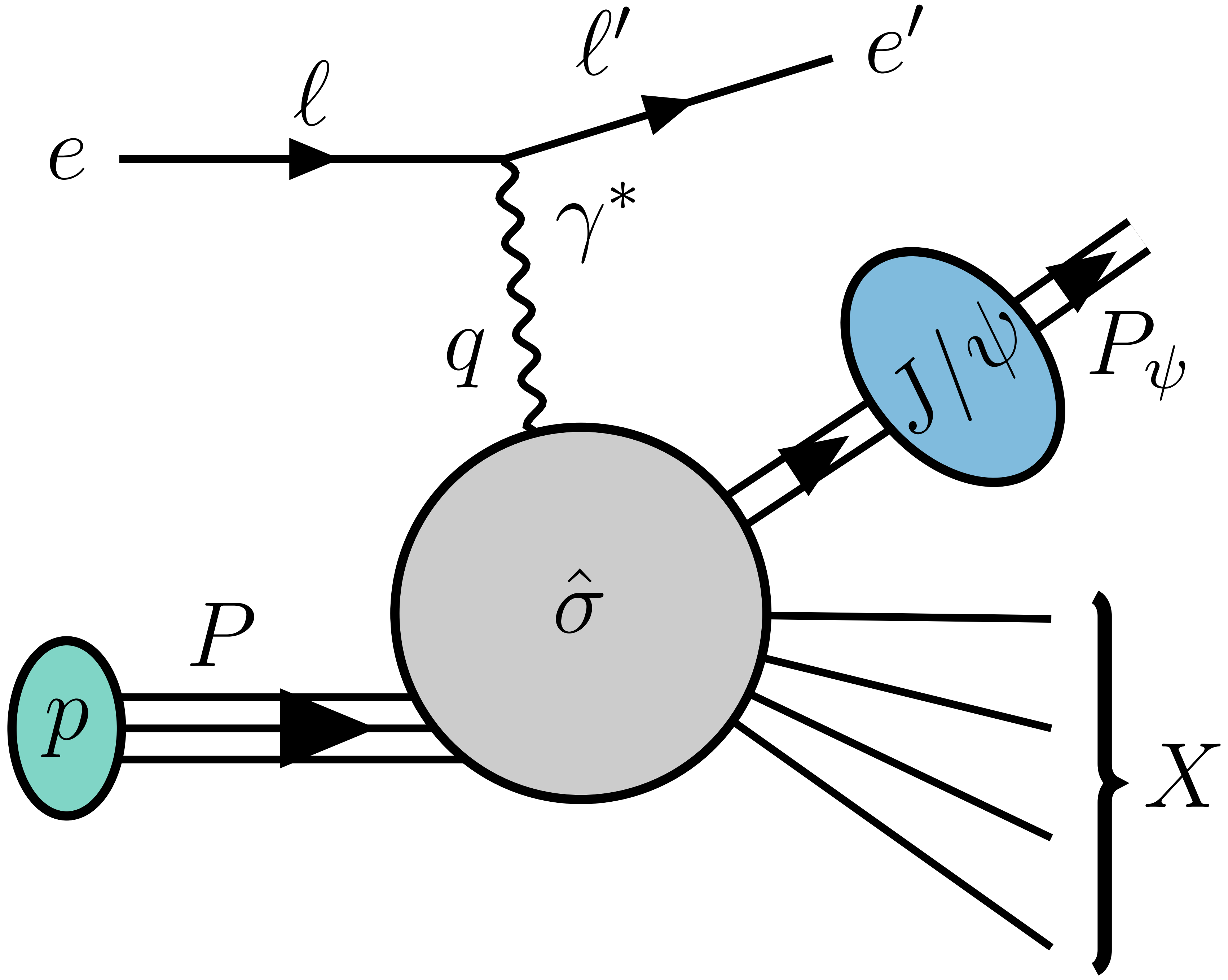

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As for $D_{a \to h}(\hat{z}) \to D_{a \to h}(\hat{z}, k_T)$, we have $\langle \mathcal{O}_\psi[n] \rangle \delta(\hat{z} - z) \to \Delta^{[n]}(\hat{z}, k_T)$

Echevarría, *JHEP* ***144*** (2019)

Fleming, Markis, Mehen, *JHEP* ***112*** (2020)

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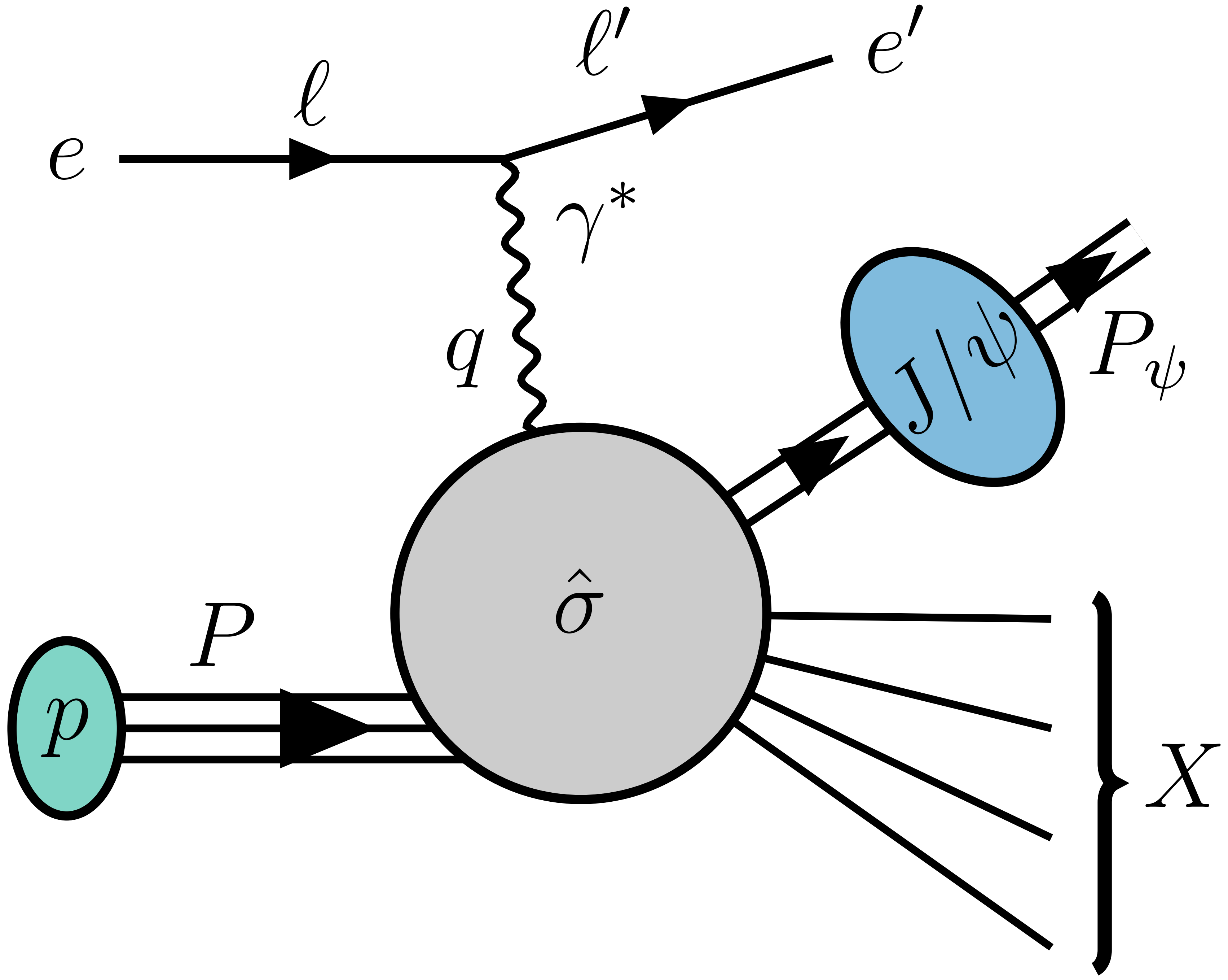

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$\Delta^{[n]}$ encodes hadronization
plus
exchange of soft gluons

Echevarría, *JHEP* ***144*** (2019)

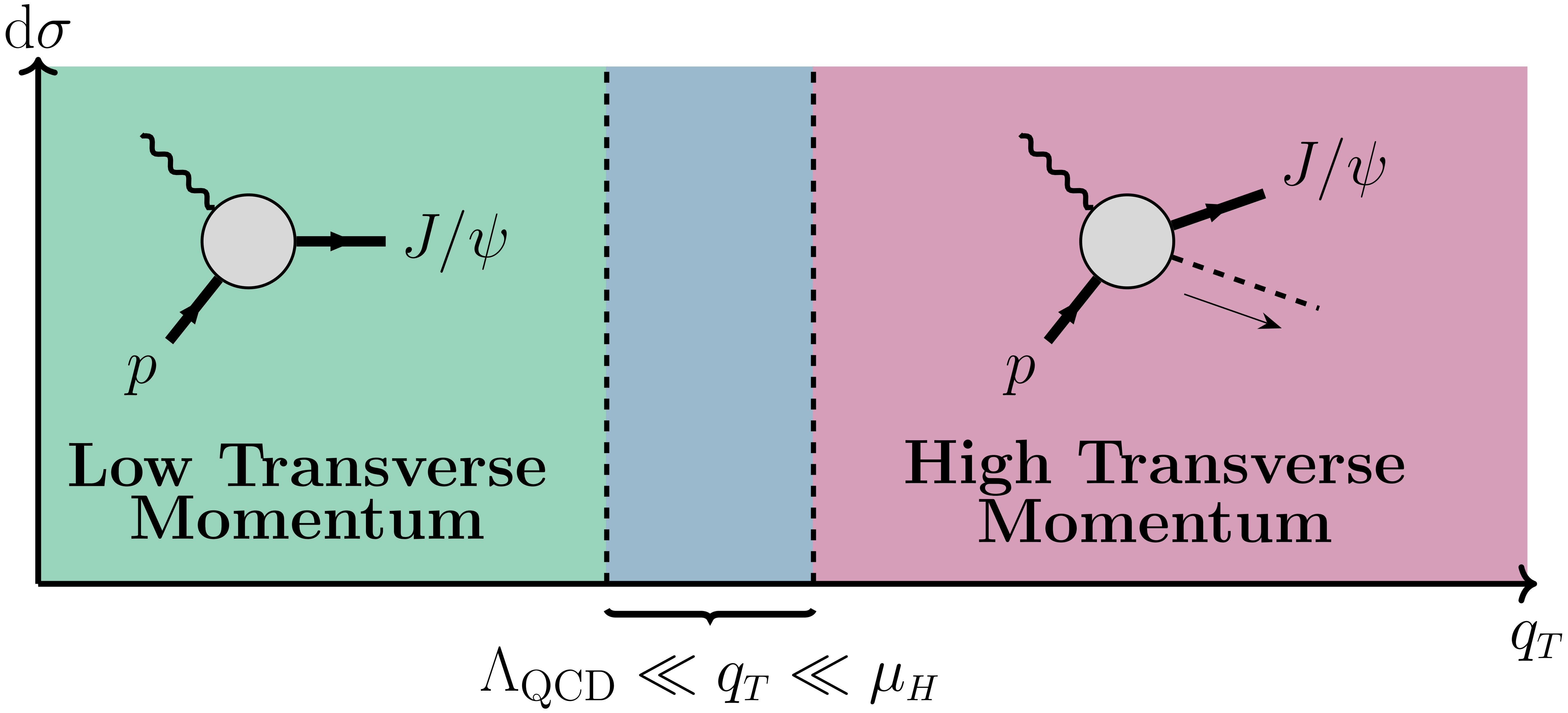
Fleming, Markis, Mehen, *JHEP* ***112*** (2020)

Matching procedure

q_T → photon transverse momentum

$P_{\psi\perp}$ → J/ψ transverse momentum

$$\Rightarrow \quad q_T = \frac{P_{\psi\perp}}{\hat{z}}$$



TMD factorization $\quad q_T \ll \mu_H$

↓

Overlapping at $\quad \Lambda_{\rm QCD} \ll q_T \ll \mu_H$

↑

Collinear factorization $\quad q_T \gg \Lambda_{\rm QCD}$

Matching procedure

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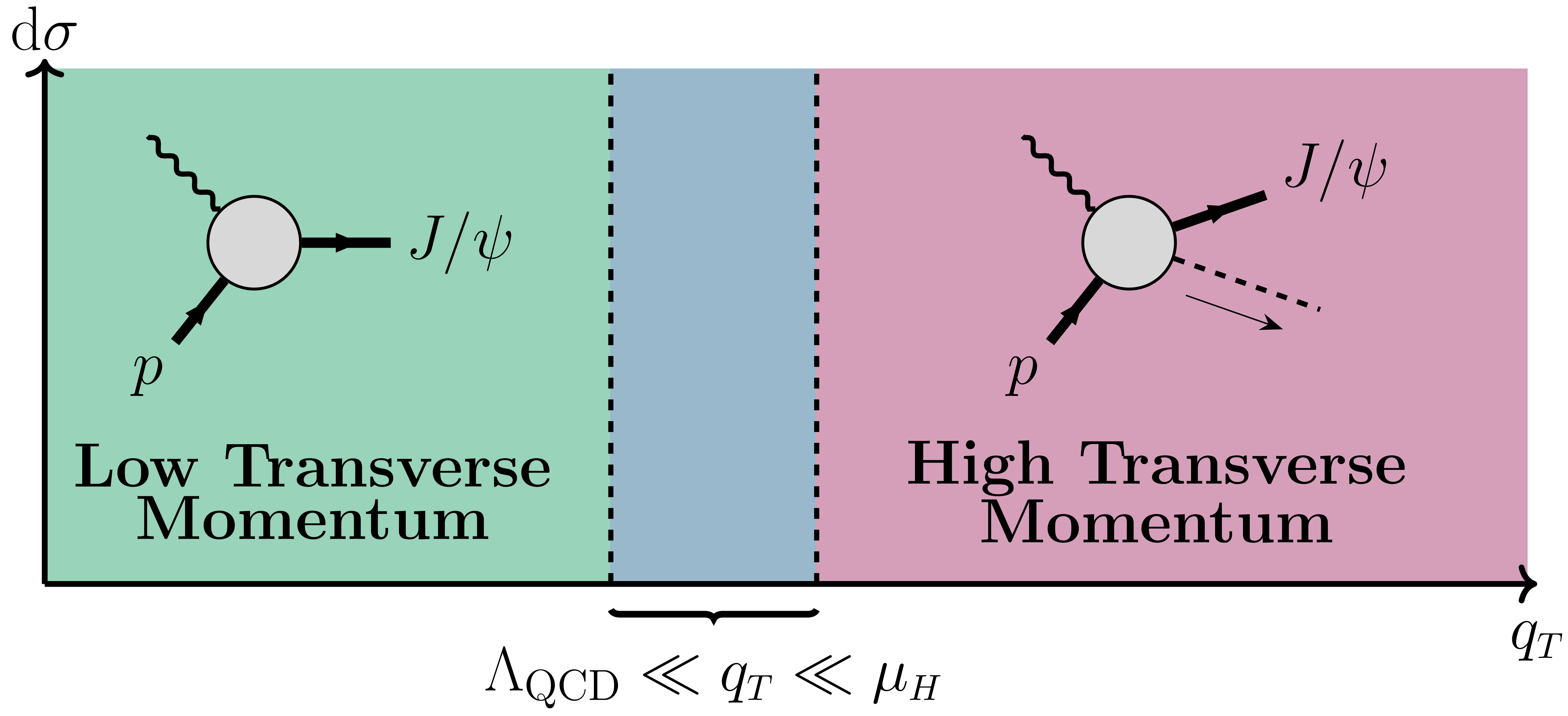


Description of the same dynamics?

Thus they should match!

TMD factorization $q_T \ll \mu_H$

↓

Overlapping at $\Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

↑

Collinear factorization $q_T \gg \Lambda_{\mathrm{QCD}}$

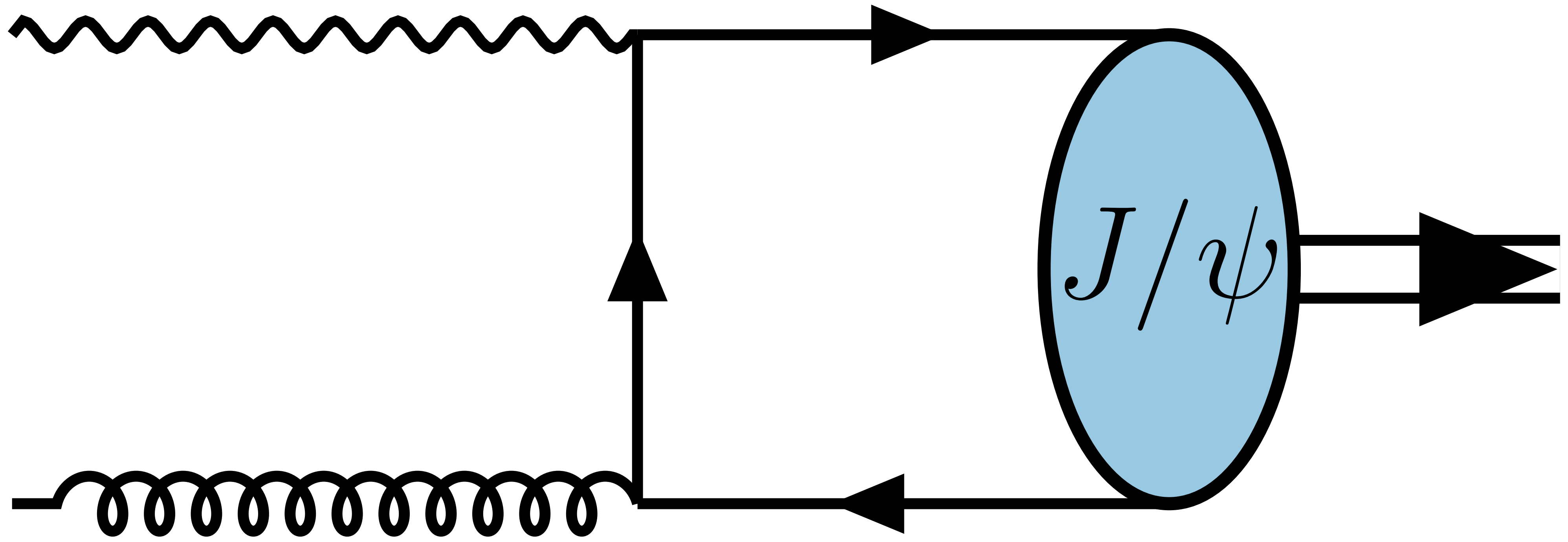
Bacchetta, Boer, Diehl, Mulders, *JHEP* ***08*** (2008)

Boer, D'Alesio, Murgia, Pisano, Taels, *JHEP* ***09*** (2020)

Structure function at small-q_T (TMD region)

J/ψ production at the lowest α_s-order: $\gamma^* + g \to c\bar{c}[n]$

Bacchetta, Boer, Pisano, Taels, *EPJC* ***80*** (2020)



Kinematics fixes most of the variables:

- $\hat{x} = x$ $\left(\text{where } x = x_B \dfrac{M_\psi^2 + Q^2}{Q^2}\right)$
- $\hat{z} = 1$
- $p_{aT} = q_T$

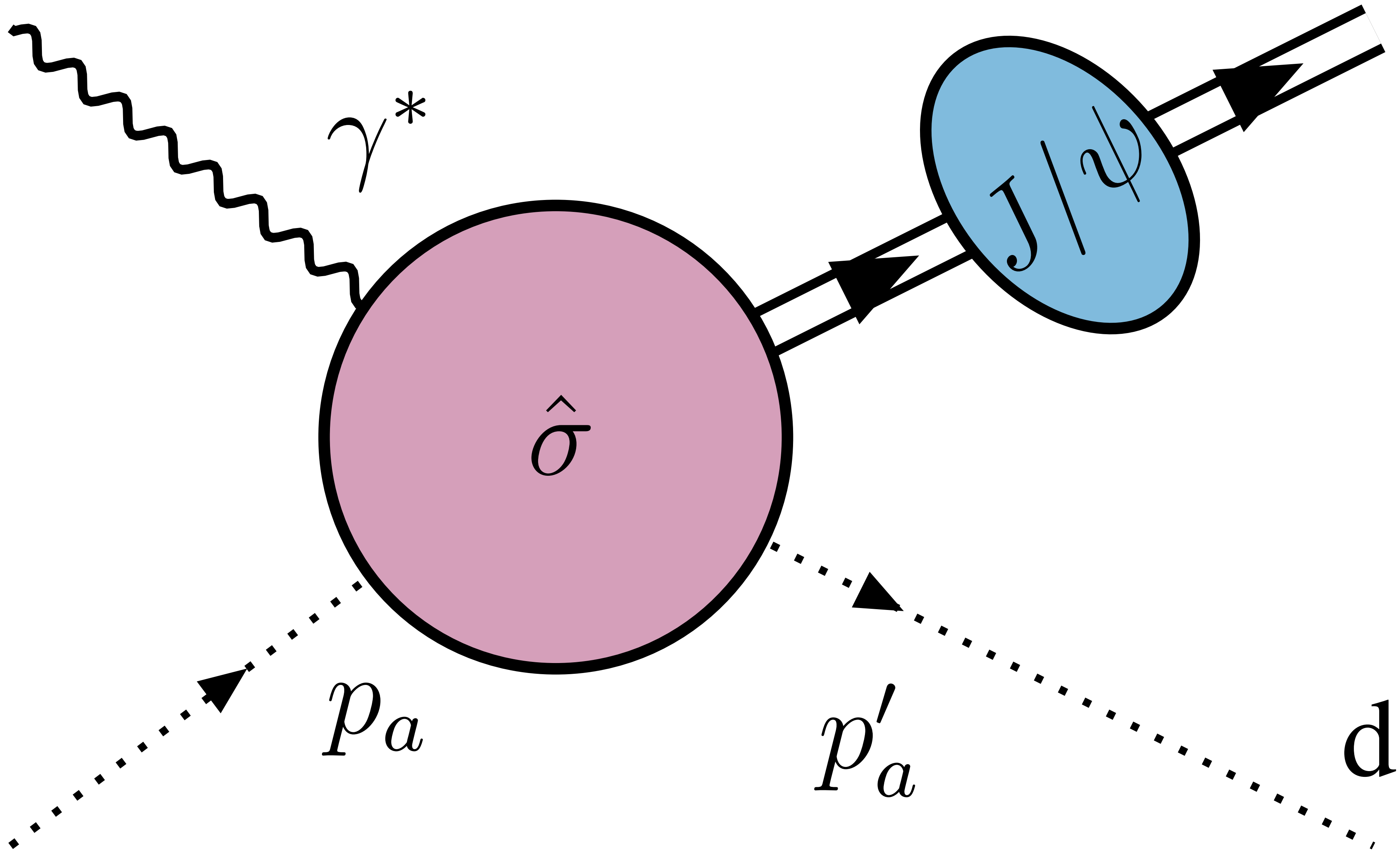
$$\mathrm{d}\sigma|_{\mathrm{TMD}} = \frac{\alpha}{yQ^2}\left\{\left[1+(1-y)^2\right]\mathcal{F}_{UUT} + 4(1-y)\left(\mathcal{F}_{UUL} + \cos 2\phi\, \mathcal{F}_{UU}^{\cos 2\phi}\right)\right\}$$

Involves the convolutions: $\begin{cases} \mathcal{C}\left[f_1^g \Delta^{[n]}\right](x, q_T) \\ \mathcal{C}\left[w\, h_1^{\perp g} \Delta_h^{[n]}\right](x, q_T) \end{cases}$

Structure function at high-q_T (collinear region)

J/ψ production at the lowest α_s-order: $\gamma^* + a \to c\bar{c}[n] + a$

$(a = q,\ \bar{q},\ g)$



$$\mathrm{d}\sigma^{ep\to e'J/\psi X} = \mathrm{d}\hat{\sigma}^{a\,[n]}(\mu_H) \otimes f_p^a(\hat{x};\mu_H) \otimes \langle \mathcal{O}_\psi[n]\rangle\, \delta(\hat{z}-z)$$

Lepton tensor from

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, *JHEP* ***02*** (2007)

$$\mathrm{d}\hat{\sigma}^{a\,[n]} \propto \int \frac{\mathrm{d}\hat{x}}{\hat{x}} \frac{\mathrm{d}\hat{z}}{\hat{z}} \frac{L^{\mu\nu}}{Q^4} H_\mu^{a\,[n]} H_\nu^{*\,a\,[n]}\ \delta(\hat{x}', \hat{z})$$

$$\boxed{\hat{x}' = \frac{x_B}{\hat{x}} \frac{M_\psi^2 + Q^2}{Q^2}}$$

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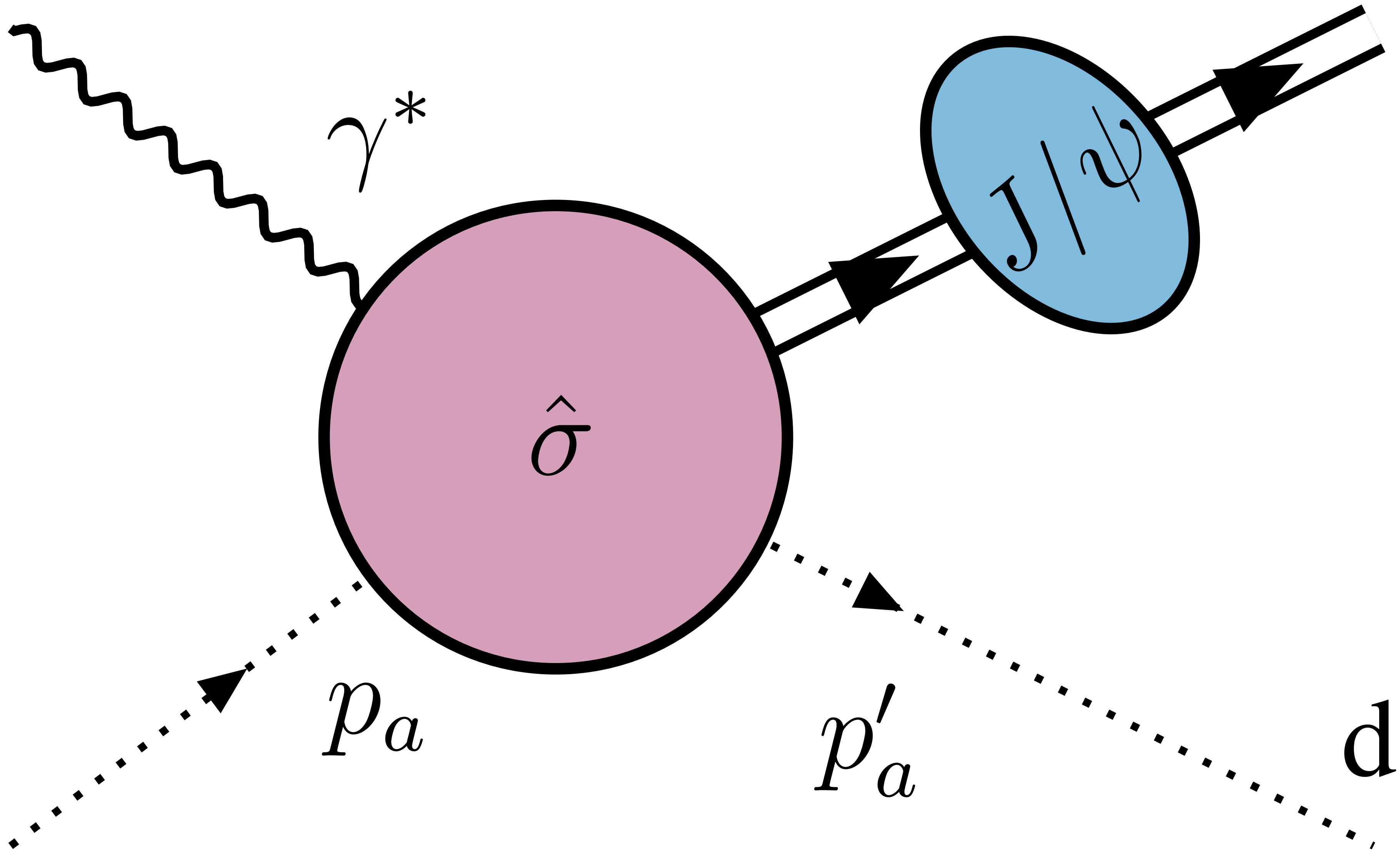

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$$\boxed{\hat{x}' = \frac{x_B}{\hat{x}}\frac{M_\psi^2+Q^2}{Q^2}}$$

$$\delta(\hat{x}',\hat{z}) = \delta\left(\frac{(1-\hat{x}')(1-\hat{z})}{\hat{x}'\hat{z}} + \frac{1-\hat{z}}{\hat{z}}\frac{\hat{z}-\hat{x}'}{\hat{x}'\hat{z}}\frac{M_\psi^2}{Q^2} + \frac{q_T^2}{Q^2}\right)$$

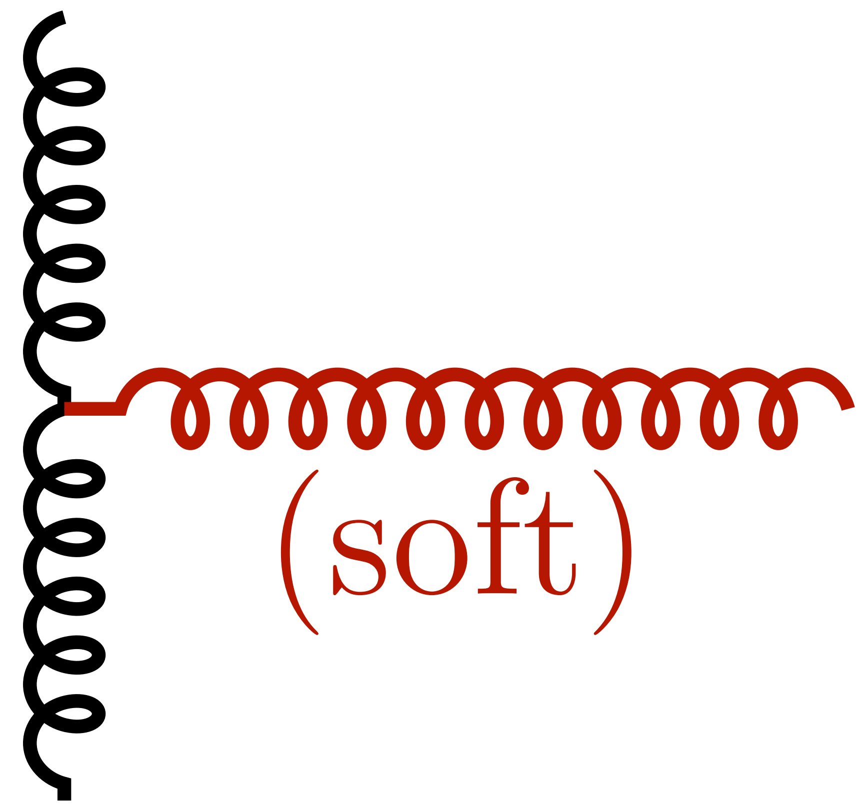
for $M_\psi \ll Q$ in agreement with

Meng, Olness, Soper *JHEP* ***11*** (2019)

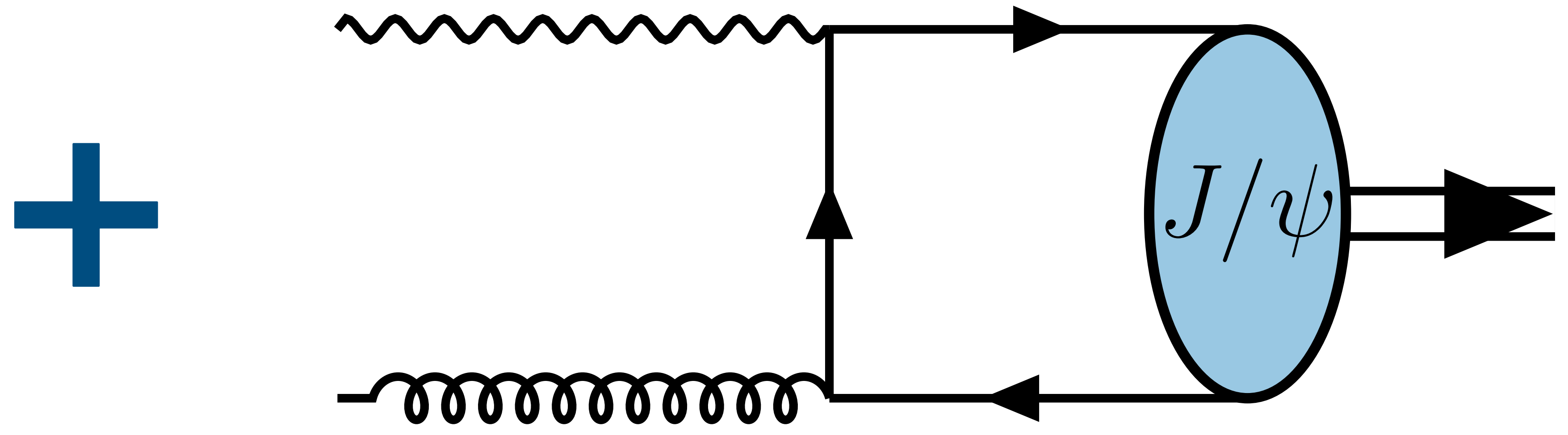
Schematic small-q_T limit valid at $\Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

$\Lambda_{\mathrm{QCD}} \ll q_T \ll \mu_H$

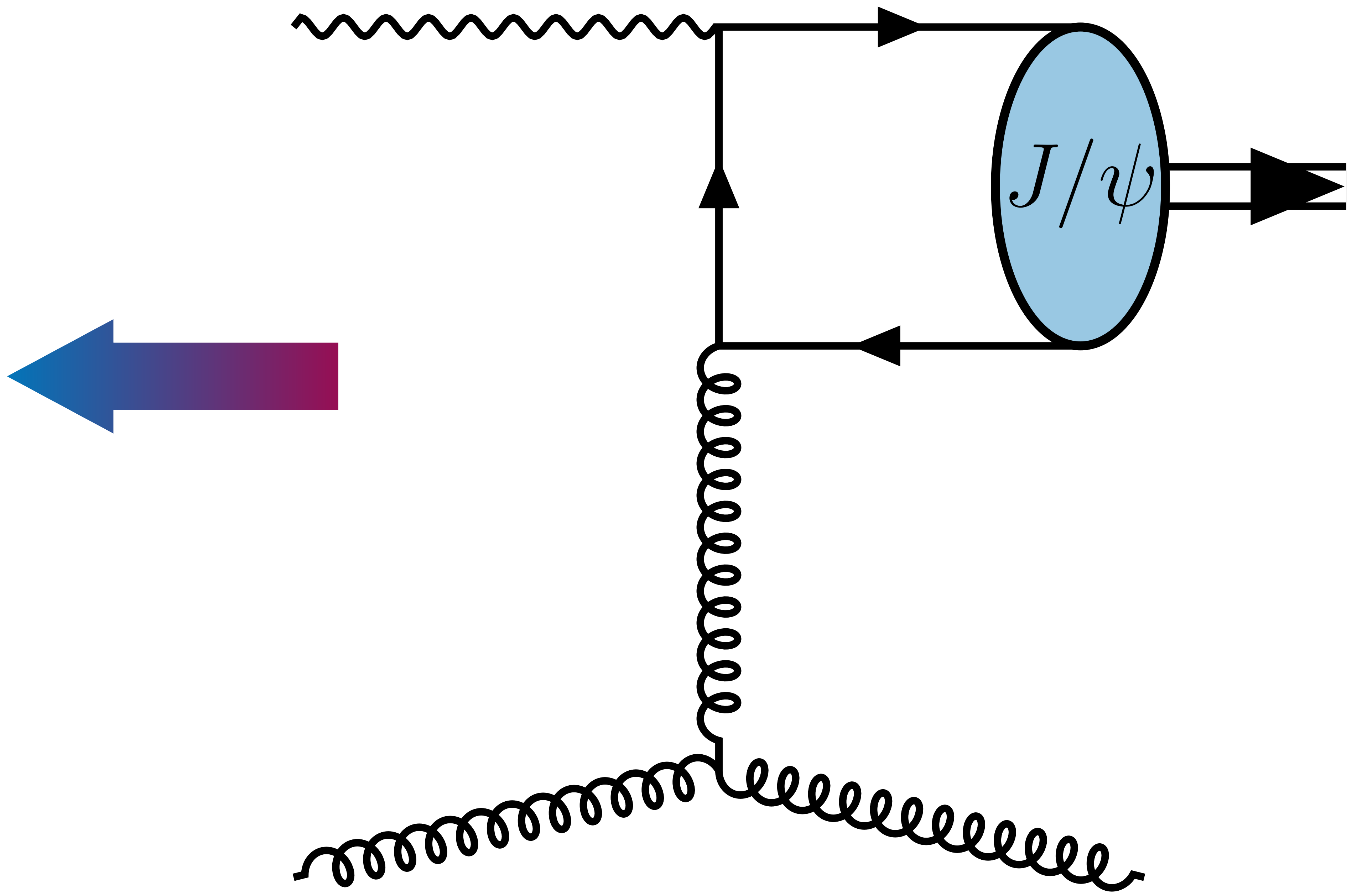
$q_T \gg \Lambda_{\mathrm{QCD}}$



q_T divergent behavior
(resummed)



same partonic interaction
appearing in the TMD region



Limit is obtained by expanding $\delta(\hat{x}', \hat{z})$ at small-q_T

Structure functions’ pole structure

Boer, D’Alesio, Murgia, Pisano, Taels, *JHEP* ***09*** (2020)

$$\delta(\hat{x}', \hat{z}) \sim \frac{\hat{x}'}{(1-\hat{x})_+}\,\delta(1-\hat{z}) + \log\frac{M_\psi^2 + Q^2}{q_T^2}\,\delta(1-\hat{x}')\,\delta(1-\hat{z})$$

Limit based on
a continuous test function

Boer, Bor, LM, Pisano, Yuan, *2304.09473* (2023)

$$F_{UU}\,(\hat{x}', \hat{z}) = F_{UU}^{(0)}\,(\hat{x}', \hat{z}) + \sum_{k=1}\left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^k F_{UU}^{(k)}\,(\hat{x}', \hat{z})$$

(general notation)

Continuous functions of $\hat{x}'$ and $\hat{z}$

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Boer, D’Alesio, Murgia, Pisano, Taels, *JHEP* ***09*** (2020)

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Limit based on
a **continuous** test function

Boer, Bor, LM, Pisano, Yuan, *2304.09473* (2023)

Has an impact on the double delta

$$F_{UU}\left(\hat{x}',\hat{z}\right) = \overset{\checkmark}{F^{(0)}_{UU}}\left(\hat{x}',\hat{z}\right) + \sum_{k=1}\left(\frac{1-\hat{z}}{1-\hat{x}'}\right)^k \overset{\times}{F^{(k)}_{UU}}\left(\hat{x}',\hat{z}\right)$$

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Relevant for $\gamma^* g$

in $F_{UUT}^{(k)}$ and $F_{UUL}^{(k)}$

with $k = 1,\, 2$

$$\log\frac{M_\psi^2 + Q^2}{q_T^2} \to \frac{1}{2}\left(\log\frac{M_\psi^2 + Q^2}{q_T^2} - 1 - \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right)$$

Eikonal method

Same term is found by considering the soft gluon emission

$$x_g = \frac{p_a \cdot p_g}{p_a \cdot q}$$

$$\mathrm{d}\sigma_1 \propto \int_{\frac{-p_{g\perp}^2}{M_\psi^2+Q^2}}^{1} \frac{\mathrm{d}x_g}{x_g}\left[2\,S_g(p_a,P_\psi)+S_g(P_\psi,P_\psi)\right] \propto \underline{\frac{1}{2}\left(\log\frac{M_\psi^2+Q^2}{q_T^2}-1-\log\frac{M_\psi^2}{M_\psi^2+Q^2}\right)}$$

from momentum conservation and $2p_g^+p_g^- = -p_{g\perp}^2 = -P_{\psi\perp}^2$

$$S_g(v_1,v_2) = \frac{v_1 \cdot v_2}{(v_1 \cdot p_g)(v_2 \cdot p_g)}$$

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Relation to quark-pair Fragmentation Function?

Kang, Ma, Qiu, Sterman, *PRD* ***90*** (2014) & *PRD* ***91*** (2015)

Ma, Qiu, Sterman, Zhang, *PRL* ***113*** (2014)

TMD shape function perturbative tail

Comparison at $\Lambda_{\text{QCD}} \ll q_T \ll \mu_H$ obtained evolving TMDs according to

Echevarria, Kasemets, Mulders, Pisano, *JHEP* ***07*** (2015)

Sun, Xiao, Yuan, *PRD* ***84*** (2011)

$$\mathcal{F}_{UU}^{\cos 2\phi}\big|_{\text{TMD}} = F_{UU}^{\cos 2\phi}\big|_{\text{coll}} \quad \Longrightarrow \quad \Delta_{h,\psi}^{[n]} = \delta^{(2)}(k_T^2)\,\langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

Boer, Bor, LM, Pisano, Yuan, *2304.09473* (2023)

$$\left.\begin{array}{l} \mathcal{F}_{UUT}\big|_{\text{TMD}} \neq F_{UUT}\big|_{\text{coll}} \\ \mathcal{F}_{UUL}\big|_{\text{TMD}} \neq F_{UUL}\big|_{\text{coll}} \end{array}\right\} \quad \Longrightarrow \quad \Delta_{\psi}^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log\frac{M_\psi^2}{M_\psi^2 + Q^2}\right)\langle \mathcal{O}_\psi[n]\rangle\,\delta(1-z)$$

Up to the precision considered, bulk of the expression given by **CO waves**

$$\underbrace{{}^1S_0^{(8)} \qquad {}^3P_J^{(8)}}_{\textbf{CO}}$$

TMD shape function in other processes?

Previous results are obtained for $\mu_H \equiv \sqrt{M_\psi^2 + Q^2}$

Boer, Bor, LM, Pisano, Yuan, *2304.09473* (2023)

In general we get

(in b_T-space)

$$\Delta_{ep}^{[n]}(\mu_H) = \frac{1}{2\pi}\left[1 + \frac{\alpha_s}{2\pi}C_A\left(1 + \log\frac{M_\psi^2\,\mu_H^2}{\left(M_\psi^2 + Q^2\right)^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle\mathcal{O}[n]\rangle\,\delta(1-z)$$

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It is process related!

Note that: a Q^2 dependent soft-factor is present in the open-quark production too

Zhu, Sun, Yuan, *Phys. Lett. B* ***727*** (2013)

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Boer, Bor, LM, Pisano, Yuan, *2304.09473* (2023)

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split up: $\boldsymbol{\Delta_{ep}^{[n]}(\mu_H) = \Delta_\psi^{[n]}(\mu_H)\times S_{ep}(\mu_H)}$

$$\Delta_\psi^{[n]}(\mu_H) = \frac{1}{2\pi}\left[1+\frac{\alpha_s}{2\pi}C_A\left(1+\log\frac{M_\psi^2}{\mu_H^2}\right)\log\frac{\mu_H^2}{\mu_b^2}\right]\langle\mathcal{O}[n]\rangle\,\delta(1-z)$$

➡ **Universal**

$$S_{ep}(\mu_H) = 1+\frac{\alpha_s}{2\pi}C_A\left(2\log\frac{\mu_H^2}{M_\psi^2+Q^2}\right)\log\frac{\mu_H^2}{\mu_b^2}$$

➡ **Process dependent**

e.g. $S_{pp}(\mu_H)$

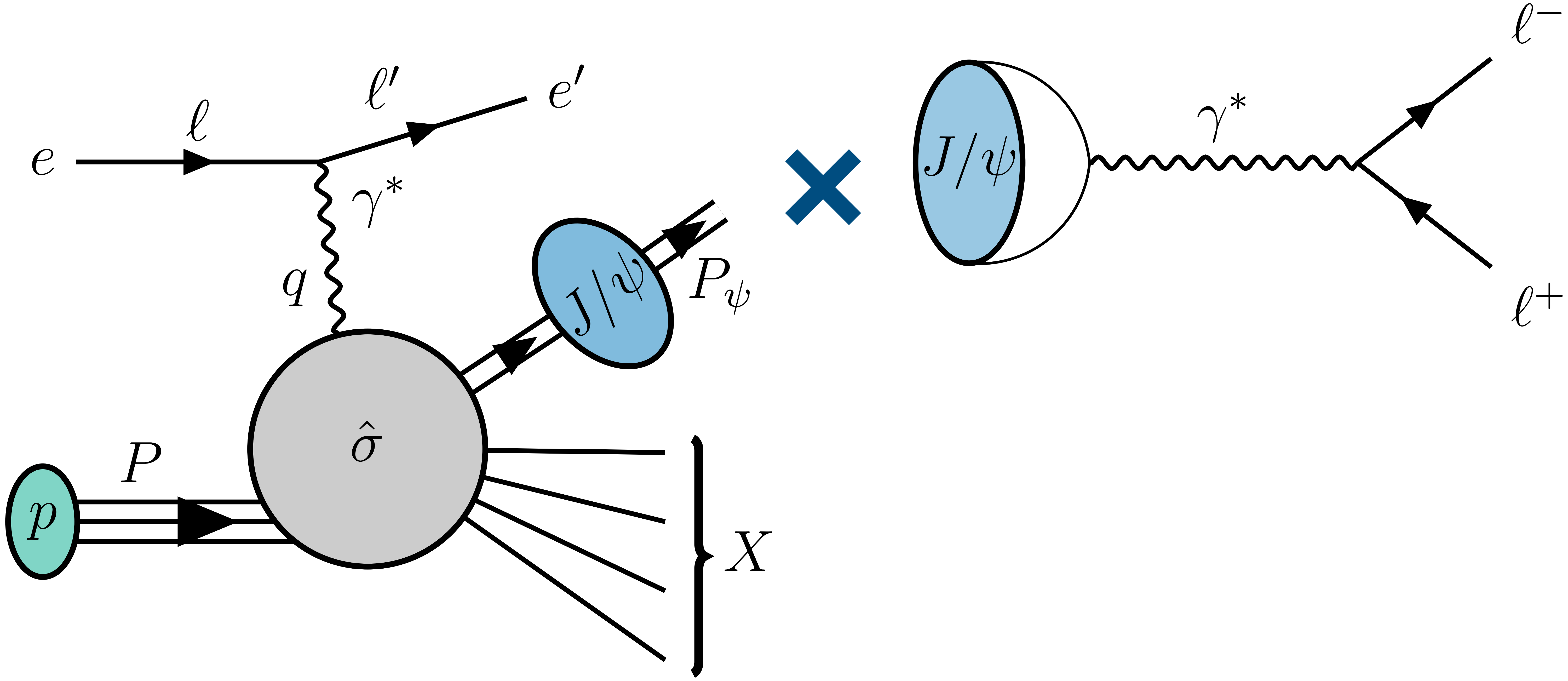
$$1+\frac{\alpha_s}{2\pi}C_A\left(3\log\frac{\mu_H^2}{M_\psi^2}\right)\log\frac{\mu_H^2}{\mu_b^2}$$

What about polarization?

Coming sections in collaboration with:
U. D'Alesio, F. Murgia, C. Pisano & R. Sangem

We can study the J/ψ polarization by considering its decay into a lepton pair

$(\mathscr{B}_{e^+e^-} \sim \mathscr{B}_{\mu^+\mu^-} \sim 6\,\%)$



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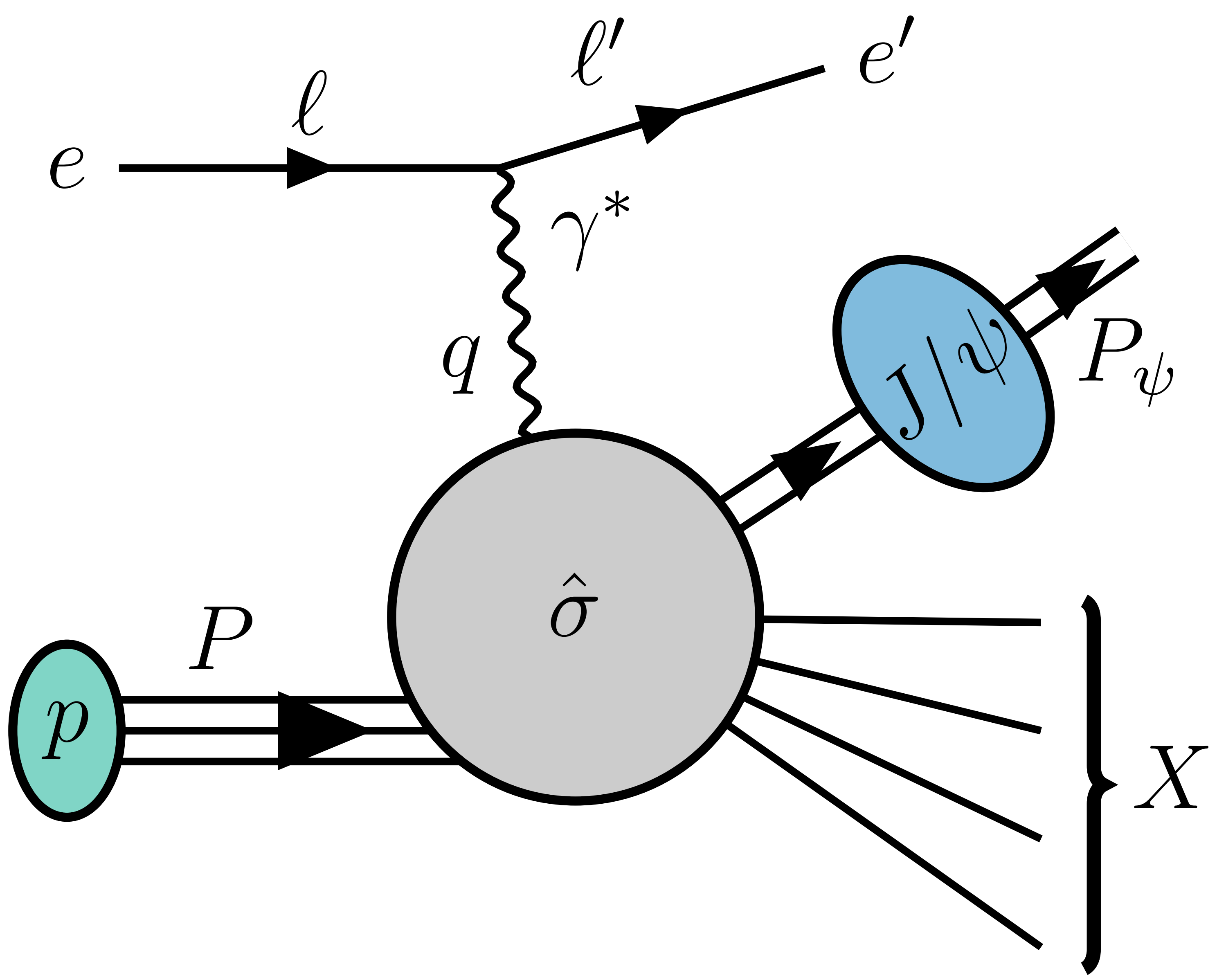

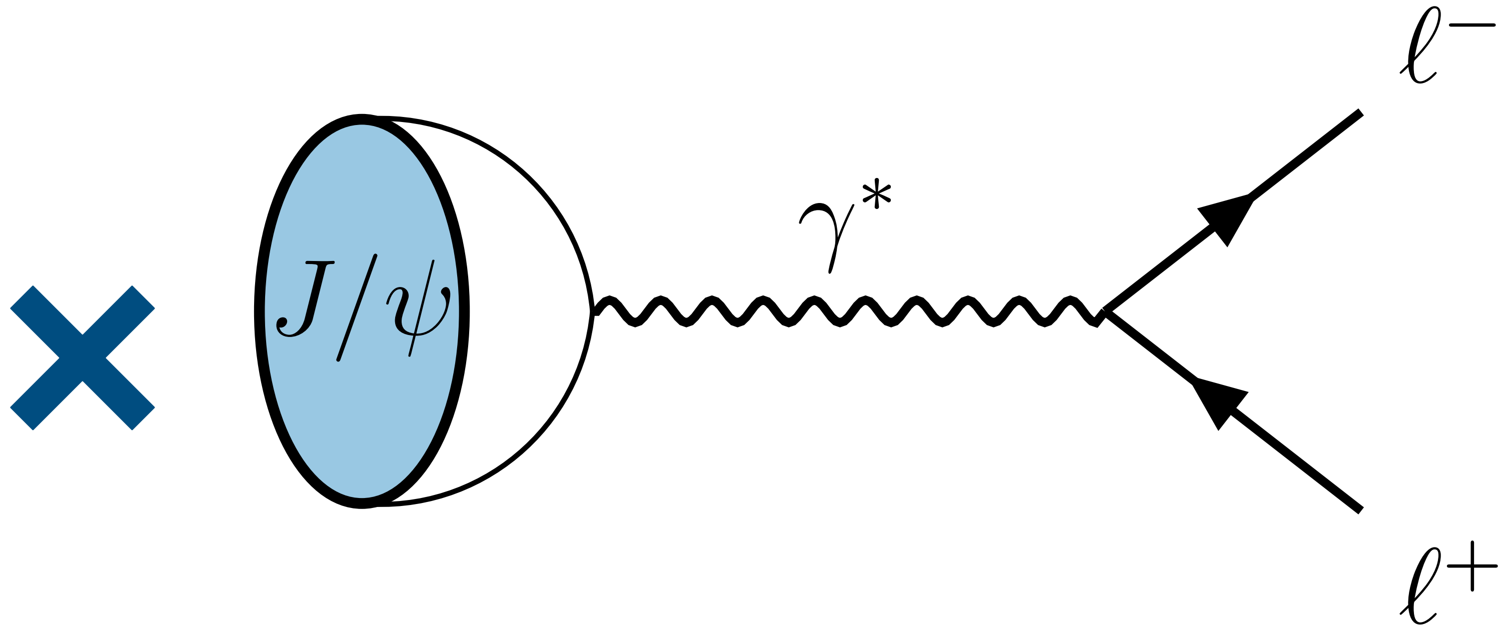

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}^4P_\psi\,\mathrm{d}\Omega} \propto 1+\lambda\cos^2\theta+\mu\cos 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi$$

J/ψ polarisation in SIDIS at low-q_T

Angular parameters within **TMD** factorization

D'Alesio, LM, Murgia, Pisano, Sangem, *JHEP* ***03*** (2022)

$$\lambda = \frac{\mathscr{W}_T - \mathscr{W}_L}{\mathscr{W}_T + \mathscr{W}_L} \longrightarrow \begin{cases} \mathscr{C}\left[f_1^g \Delta_T^{[n]}\right] \\ \\ \mathscr{C}\left[f_1^g \Delta_L^{[n]}\right] \end{cases}$$

$$\xcancel{\mu = \frac{\mathscr{W}_\Delta}{\mathscr{W}_T + \mathscr{W}_L}}$$

$$\nu = \frac{2\mathscr{W}_{\Delta\Delta}}{\mathscr{W}_T + \mathscr{W}_L} \longrightarrow \mathscr{C}\left[w\, h_1^{\perp g} \Delta_{\Delta\Delta}^{[n]}\right]$$

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Angular parameters within **TMD** factorization

D'Alesio, LM, Murgia, Pisano, Sangem, *JHEP* ***03*** (2022)

$$\lambda = \frac{\mathscr{W}_{\textcircled{\scriptsize T}} - \mathscr{W}_{\textcircled{\scriptsize L}}}{\mathscr{W}_{\textcircled{\scriptsize T}} + \mathscr{W}_{\textcircled{\scriptsize L}}} \longrightarrow \begin{cases} \mathscr{C}\left[f_1^g \Delta^{[n]}_{\textcircled{\scriptsize T}}\right] \\ \\ \mathscr{C}\left[f_1^g \Delta^{[n]}_{\textcircled{\scriptsize L}}\right] \end{cases}$$

$$\xcancel{\mu = \frac{\mathscr{W}_{\Delta}}{\mathscr{W}_T + \mathscr{W}_L}}$$

$$\nu = \frac{2\mathscr{W}_{\textcircled{\scriptsize $\Delta\Delta$}}}{\mathscr{W}_{\textcircled{\scriptsize T}} + \mathscr{W}_{\textcircled{\scriptsize L}}} \longrightarrow \mathscr{C}\left[w\, h_1^{\perp g} \Delta^{[n]}_{\textcircled{\scriptsize $\Delta\Delta$}}\right]$$

J/ψ polarisation in SIDIS at low-q_T

Angular parameters within **TMD** factorization

D'Alesio, LM, Murgia, Pisano, Sangem, *JHEP* ***03*** (2022)

Boer, Bor, LM, Pisano, Yuan, *2304.09473* (2023)

Matching procedure works
in the same way!

$$\lambda = \frac{\mathscr{W}_T - \mathscr{W}_L}{\mathscr{W}_T + \mathscr{W}_L} \longrightarrow \begin{cases} \mathscr{C}\left[f_1^g \Delta_T^{[n]}\right] \\ \mathscr{C}\left[f_1^g \Delta_L^{[n]}\right] \end{cases}$$

$$(\Lambda_\psi = T,\, L)$$

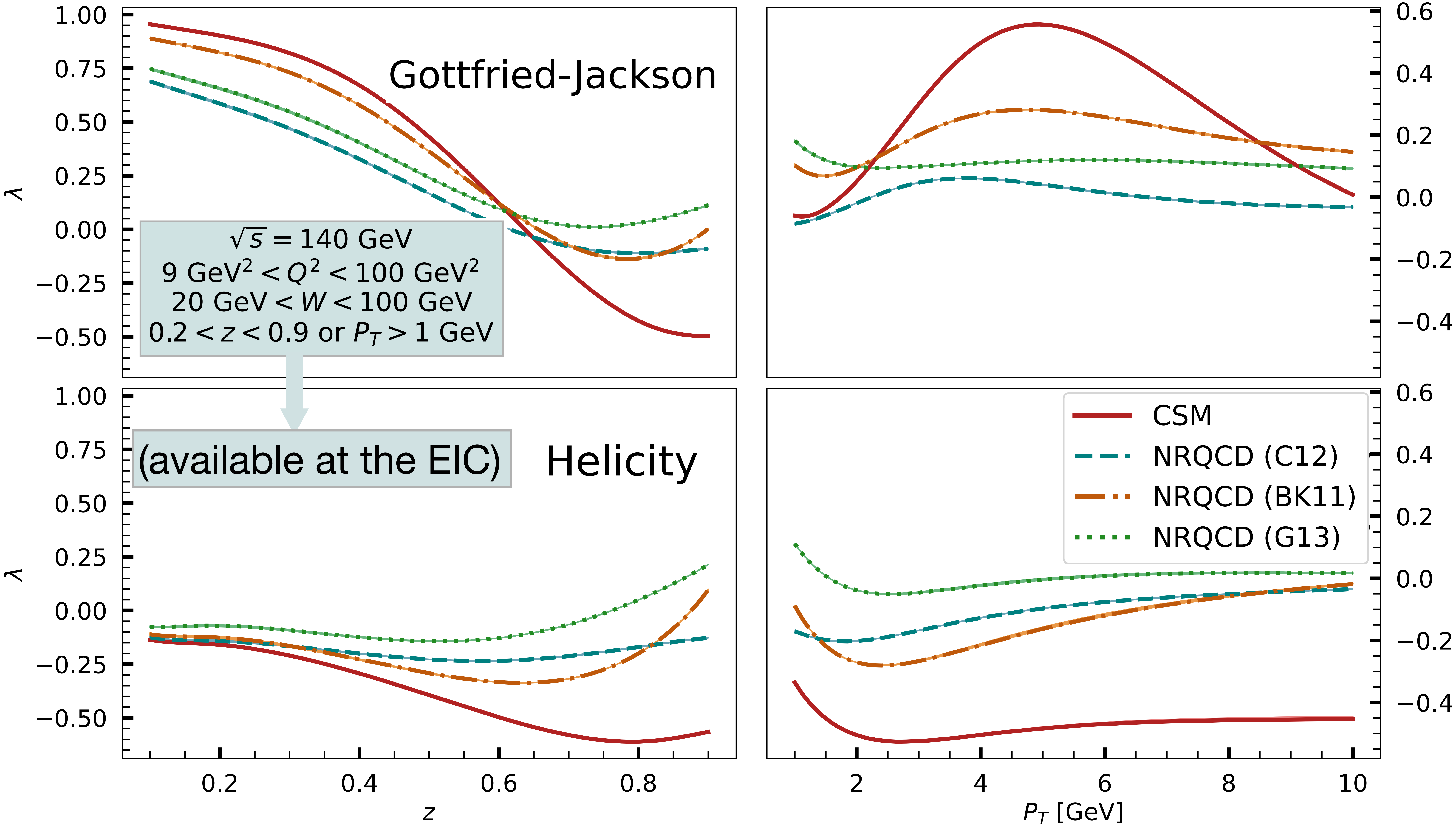
$$\Delta_{\Lambda_\psi}^{[n]} = -\frac{\alpha_s}{2\pi^2 k_T^2} C_A \left(1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathscr{O}_\psi[n] \rangle\, \delta(1-z)$$

$$\cancel{\mu = \frac{\mathscr{W}_\Delta}{\mathscr{W}_T + \mathscr{W}_L}}$$

$$\nu = \frac{2\mathscr{W}_{\Delta\Delta}}{\mathscr{W}_T + \mathscr{W}_L} \longrightarrow \mathscr{C}\left[w\, h_1^{\perp g} \Delta_{\Delta\Delta}^{[n]}\right] \longrightarrow \Delta_{\Delta\Delta}^{[n]} = ??$$

J/ψ polarization at high-q_T

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD* ***107*** (2023)



$\sqrt{s} = 140$ GeV
$9\ \text{GeV}^2 < Q^2 < 100\ \text{GeV}^2$
$20\ \text{GeV} < W < 100$ GeV
$0.2 < z < 0.9$ or $P_T > 1$ GeV

(available at the EIC)

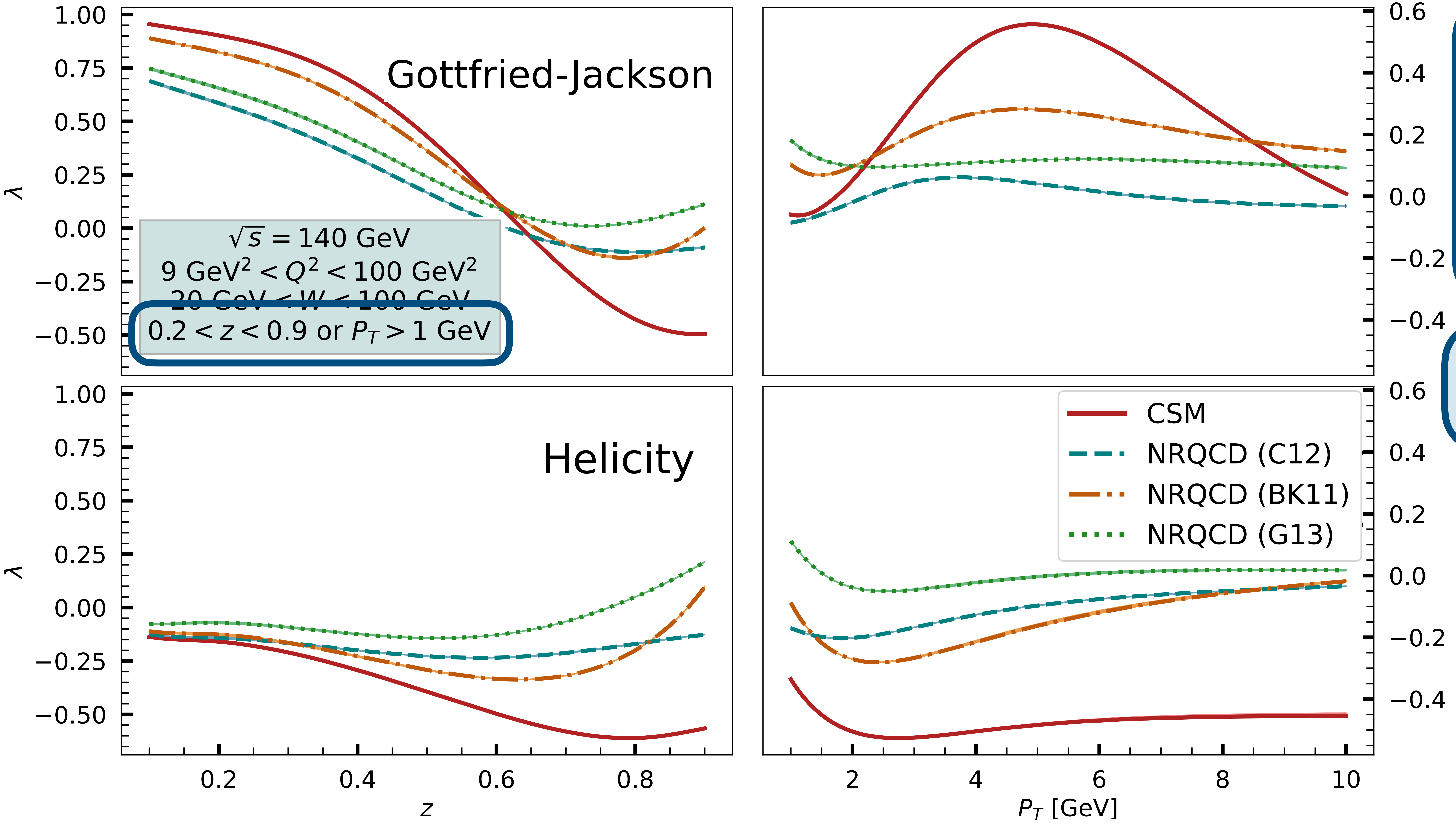
Frames:

GJ: $\hat{Z} = \dfrac{\boldsymbol{q}}{|\boldsymbol{q}|}$

HX: $\hat{Z} = -\dfrac{\boldsymbol{P}+\boldsymbol{q}}{|\boldsymbol{P}+\boldsymbol{q}|}$

J/ψ polarization at high-q_T

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD* ***107*** (2023)



Frames:

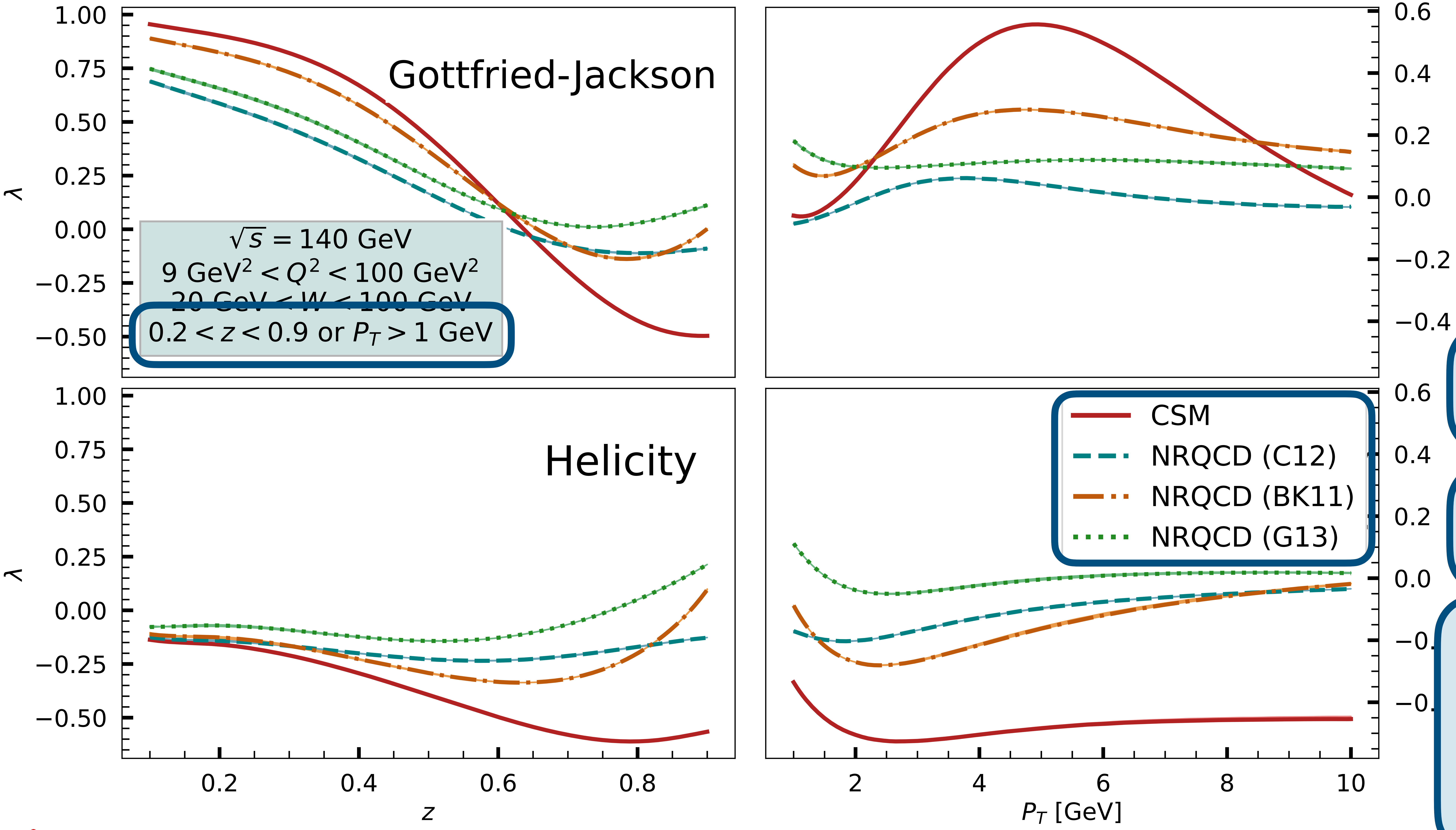
GJ: $\hat{Z} = \dfrac{\boldsymbol{q}}{|\boldsymbol{q}|}$

HX: $\hat{Z} = -\dfrac{\boldsymbol{P}+\boldsymbol{q}}{|\boldsymbol{P}+\boldsymbol{q}|}$

$\mathbf{z < 0.9}$ to avoid **TMD** region

J/ψ polarization at high-q_T

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD* ***107*** (2023)



Frames:

GJ: $\hat{Z} = \frac{\boldsymbol{q}}{|\boldsymbol{q}|}$

HX: $\hat{Z} = -\frac{\boldsymbol{P}+\boldsymbol{q}}{|\boldsymbol{P}+\boldsymbol{q}|}$

$z < 0.9$ to avoid **TMD** region

Comparison between **CSM** and **NRQCD**

LDMEs:

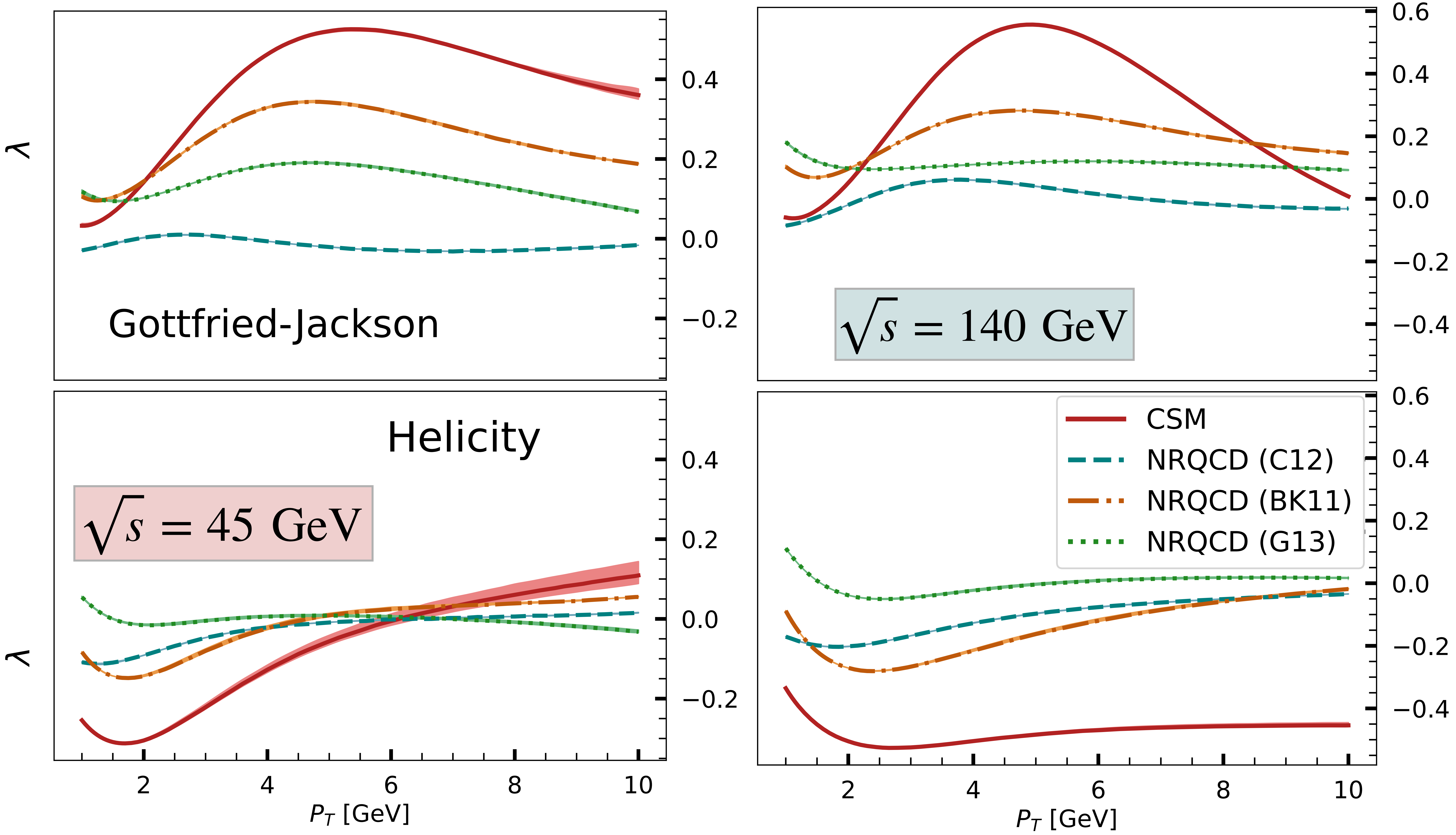
C12: *PRL* ***108*** (2012)

BK11: *PRD* ***84*** (2011)

G13: *PRL* ***110*** (2013)

J/ψ polarization at high-q_T

D’Alesio, LM, Murgia, Pisano, Sangem, *PRD* ***107*** (2023)

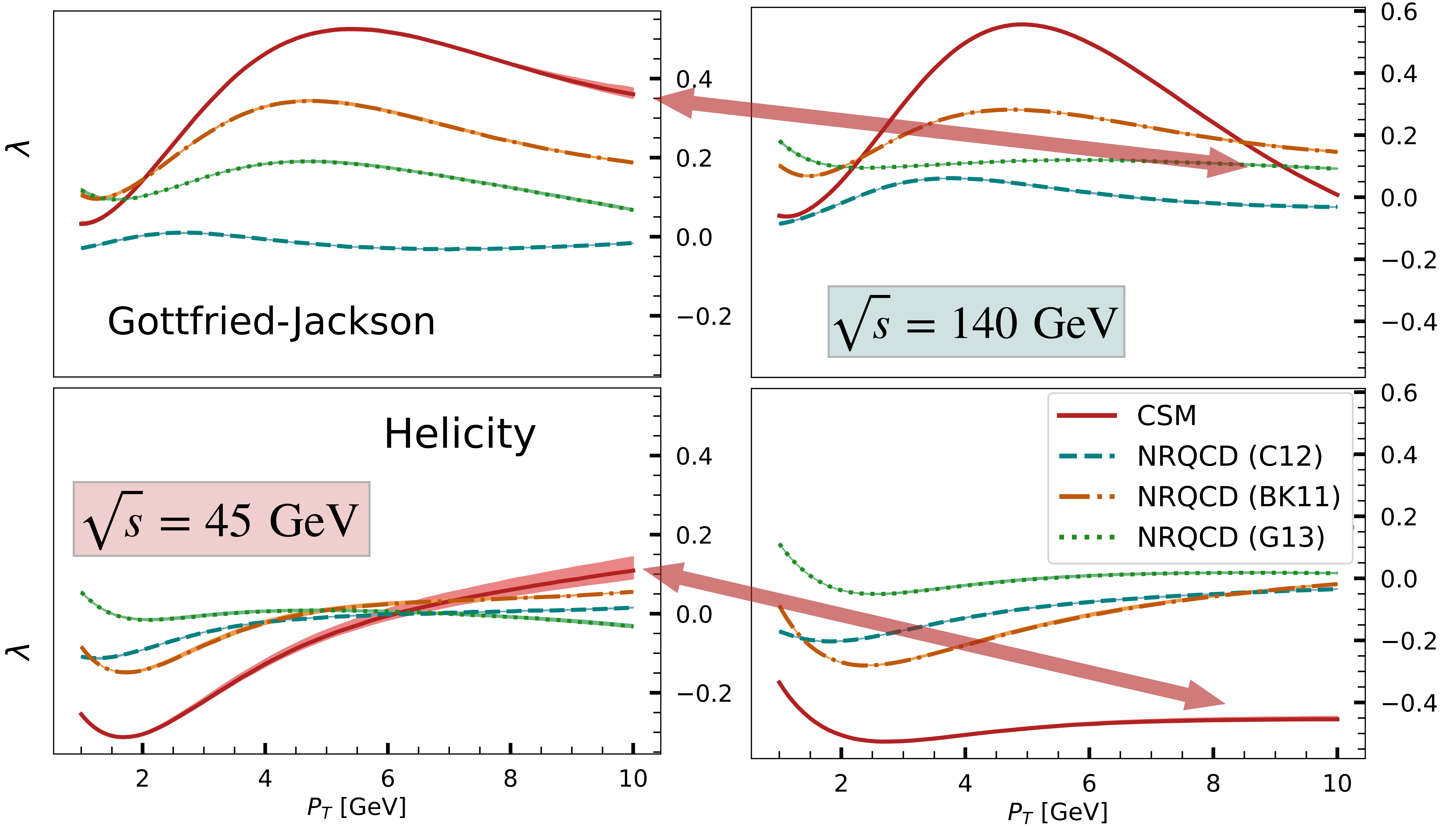


$\sqrt{s} = 140$ GeV
9 GeV$^2 < Q^2 <$ 100 GeV^2
20 GeV $< W <$ 100 GeV
$0.2 < z < 0.9$ or $P_T > 1$ GeV

$\sqrt{s} = 45$ GeV
2.5 GeV$^2 < Q^2 <$ 100 GeV^2
10 GeV $< W <$ 40 GeV
$0.2 < z < 0.9$ or $P_T > 1$ GeV

J/ψ polarization at high-q_T

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD* ***107*** (2023)

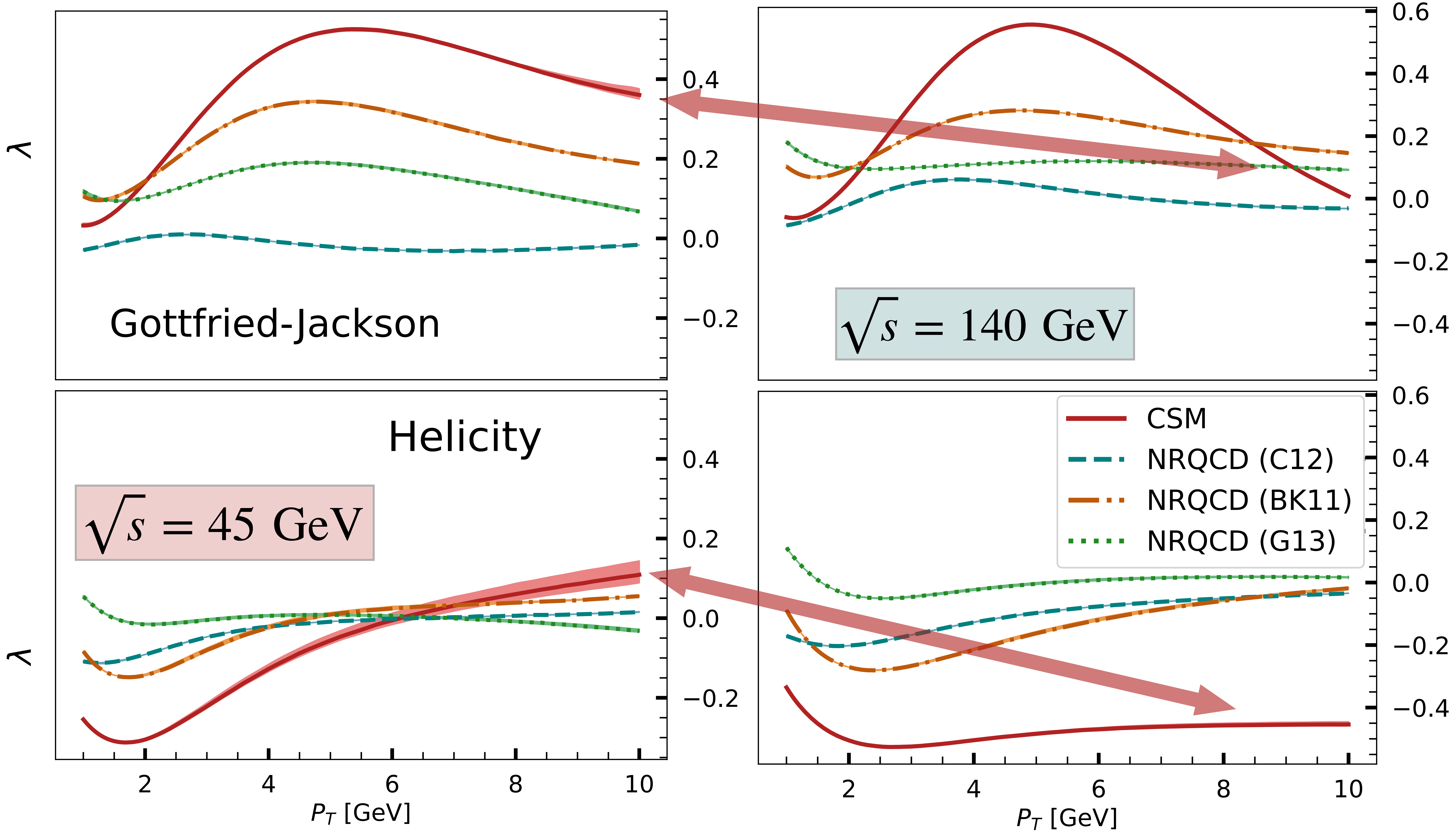


$\sqrt{s} = 140$ GeV
$9\ \text{GeV}^2 < Q^2 < 100\ \text{GeV}^2$
$20\ \text{GeV} < W < 100\ \text{GeV}$
$0.2 < z < 0.9$ or $P_T > 1$ GeV

$\sqrt{s} = 45$ GeV
$2.5\ \text{GeV}^2 < Q^2 < 100\ \text{GeV}^2$
$10\ \text{GeV} < W < 40\ \text{GeV}$
$0.2 < z < 0.9$ or $P_T > 1$ GeV

J/ψ polarization at high-q_T

D'Alesio, LM, Murgia, Pisano, Sangem, *PRD* ***107*** (2023)



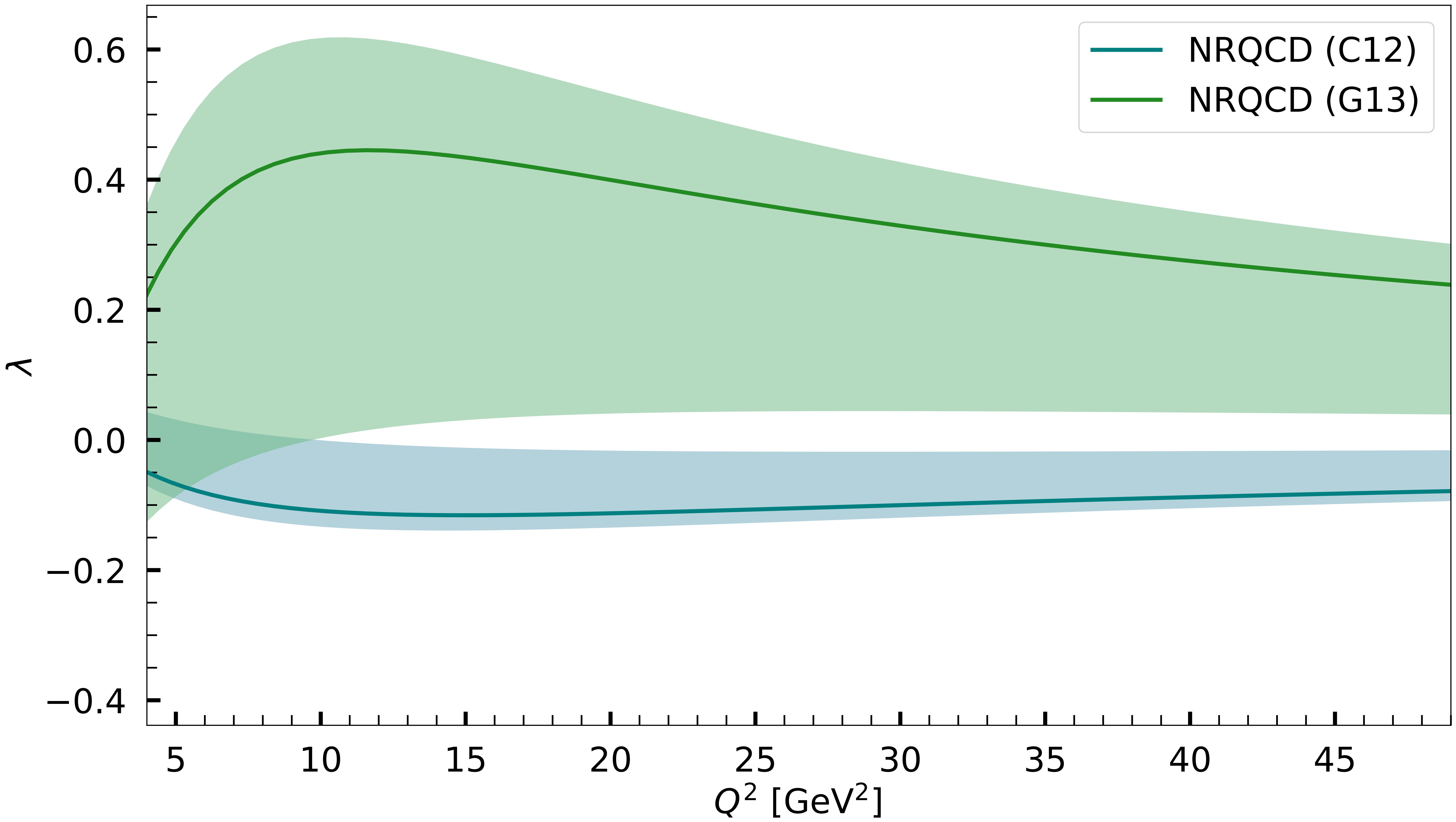
$\sqrt{s} = 140$ GeV
$9\ \text{GeV}^2 < Q^2 < 100\ \text{GeV}^2$
$20\ \text{GeV} < W < 100\ \text{GeV}$
$0.2 < z < 0.9$ or $P_T > 1$ GeV

$\sqrt{s} = 45$ GeV
$2.5\ \text{GeV}^2 < Q^2 < 100\ \text{GeV}^2$
$10\ \text{GeV} < W < 40\ \text{GeV}$
$0.2 < z < 0.9$ or $P_T > 1$ GeV

Different behavior between **CSM** and **NRQCD** due to **virtuality explored**

J/ψ polarization w.r.t. photon virtuality

Analogous to Fleming, Mehen, *PRD* ***57*** (1998)



λ polarisation is related to the ratio

$$R = \frac{\langle \mathcal{O}_8[^3P_0] \rangle}{m_c^2 \langle \mathcal{O}_8[^1S_0] \rangle}$$

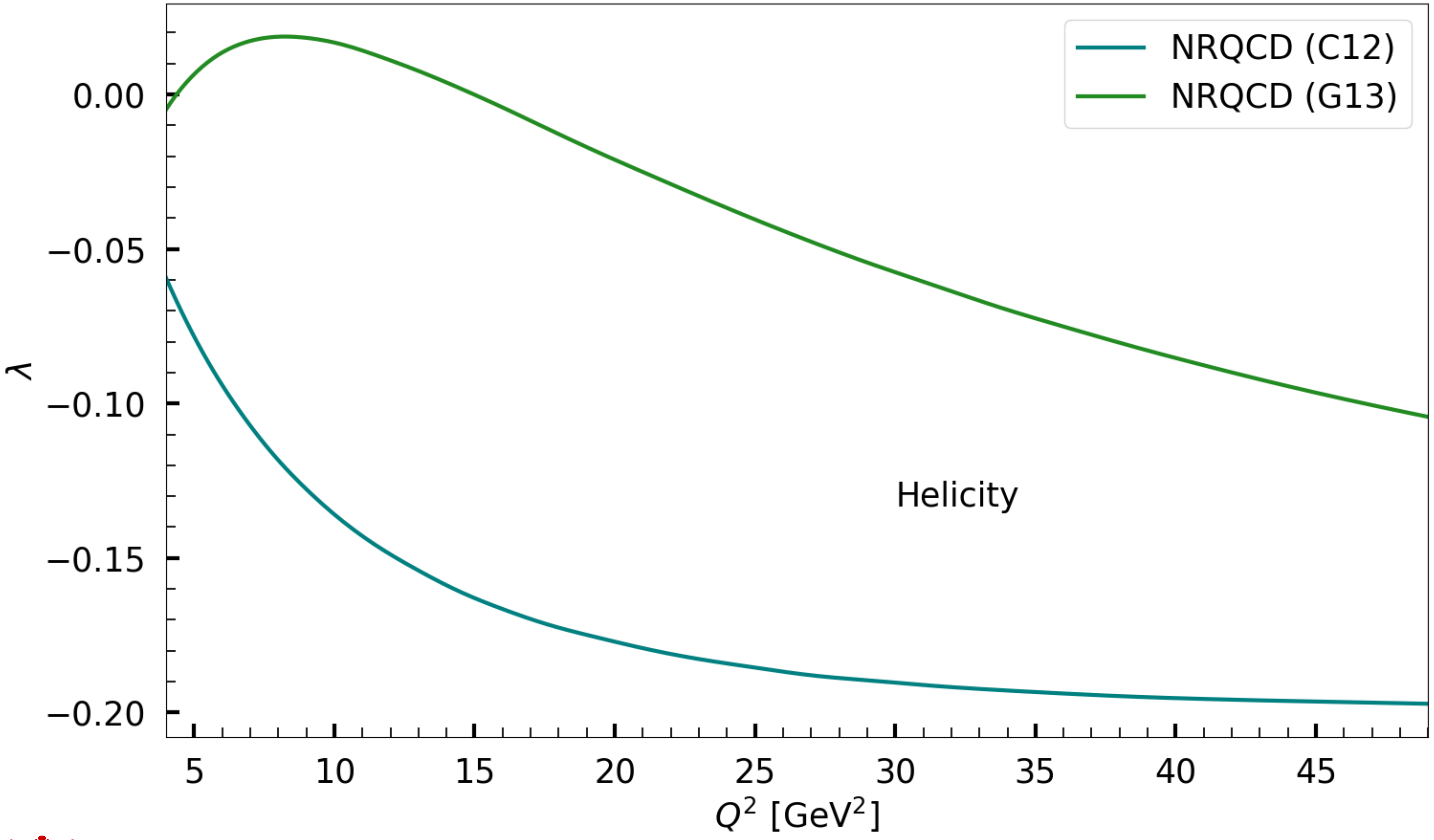
Band given by y variation

$$\left(0 \leq y = \frac{P \cdot q}{P \cdot \ell} \leq 1 \right)$$

Valid only for $z = 1$!

(all variables are fixed)

J/ψ polarization w.r.t. photon virtuality

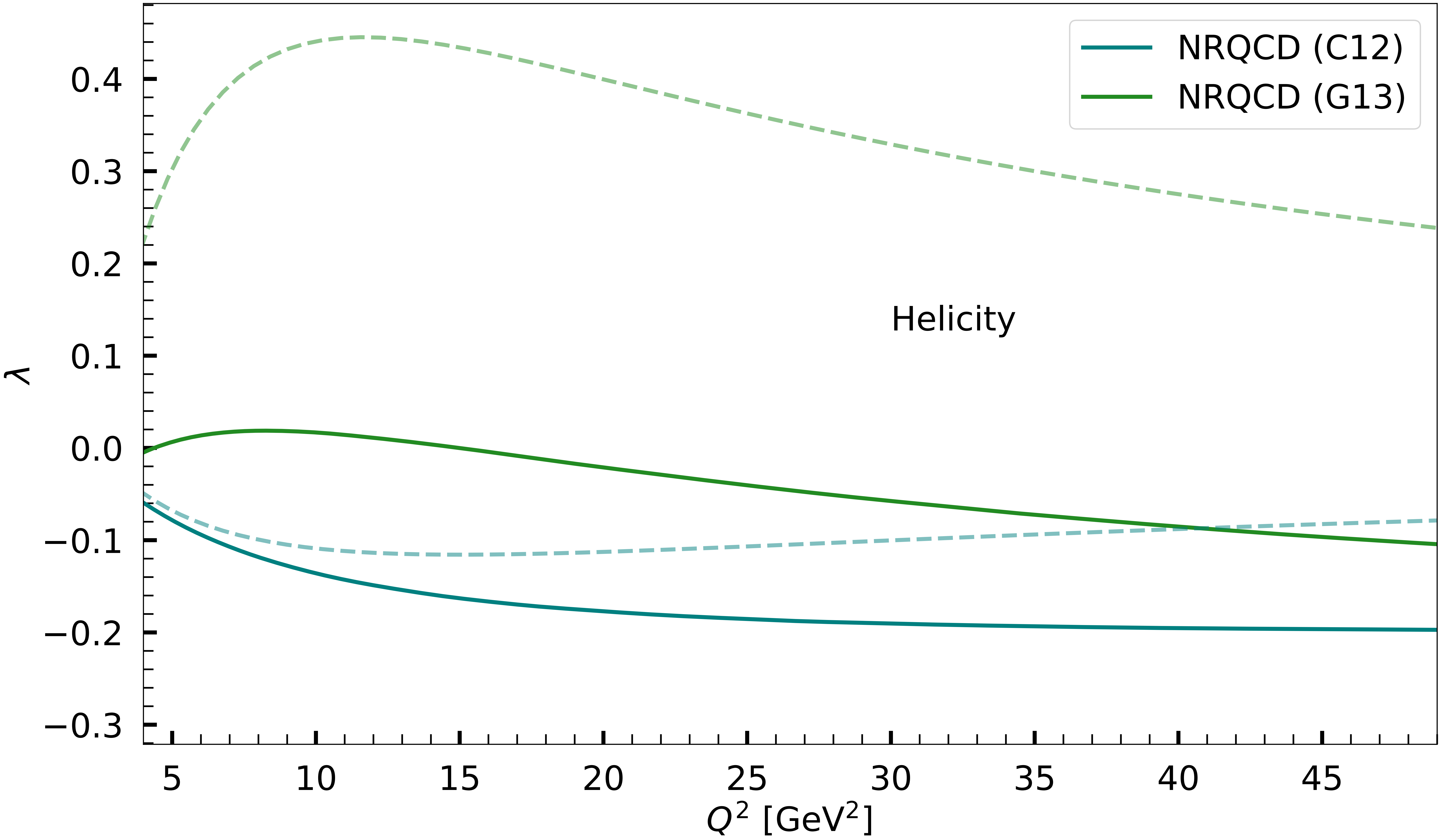


At $P_T \neq 0$ **CSM**
is **relevant**

z **integrated** in a
phase space
accessible at the **EIC**

$\sqrt{s} = 140$ GeV
$P_T > 1$ GeV
$0.2 < z < 0.9$
$y = 0.5$

J/ψ polarization w.r.t. photon virtuality



At $P_T \neq 0$ **CSM** is **relevant**

z **integrated** in a phase space accessible at the **EIC**

$\sqrt{s} = 140$ GeV
$P_T > 1$ GeV
$0.2 < z < 0.9$
$y = 0.5$

Smooth **connection** between **TMD** and **collinear** regions **is neeed!**

Summary and outlook

- Factorization involves the presence of TMD shape functions
- We present a matching procedure to extract the TMDShF perturbative tail
- TMD shape functions present universal and process-dependent components

- Perturbative tail at higher order → Relevant for $\Delta_h^{[n]}$
- Non-perturbative dependence
- Extraction of the TMDShF universal component
- The advent of the EIC may shed a light on the role of the TMDShF and its properties