Combined neutrino oscillation analysis between Super-Kamiokande and T2K

Aoi Eguchi (University of Tokyo)
on behalf of Super-Kamiokande Collaboration and T2K Collaboration

Outline

- **This is the first time we show the data fit results of joint SK + T2K oscillation analysis.**

- Introduction
  - Super-Kamiokande (SK) and T2K experiments
  - Motivation of the joint analysis

- Analysis method
  - Treatment of systematic uncertainties
  - Two fitters used for the Bayesian analysis

- Data fit result (Bayesian analysis)
  - Credible intervals
  - Bayes factors for different hypothesis
  - Study of the robustness of the analysis

- Conclusion

Introduction

- Super-Kamiokande (SK) and T2K experiments
- Motivation of the joint analysis

Neutrino Oscillation

- Neutrino flavor (interaction) eigenstates are the superimposition of the mass eigenstates.
  - Mixing is usually described with the PMNS matrix:

Atmospheric, Reactor, Solar

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}}_{\text{Atmospheric}} \cdot \underbrace{\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{\mathrm{CP}}}\\ 0 & 1 & 0\\ -s_{13}e^{-i\delta_{\mathrm{CP}}} & 0 & c_{13}\end{pmatrix}}_{\text{Reactor}} \cdot \underbrace{\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}}_{\text{Solar}} \cdot \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}$

$s_{ij} \equiv \sin\theta_{ij}$

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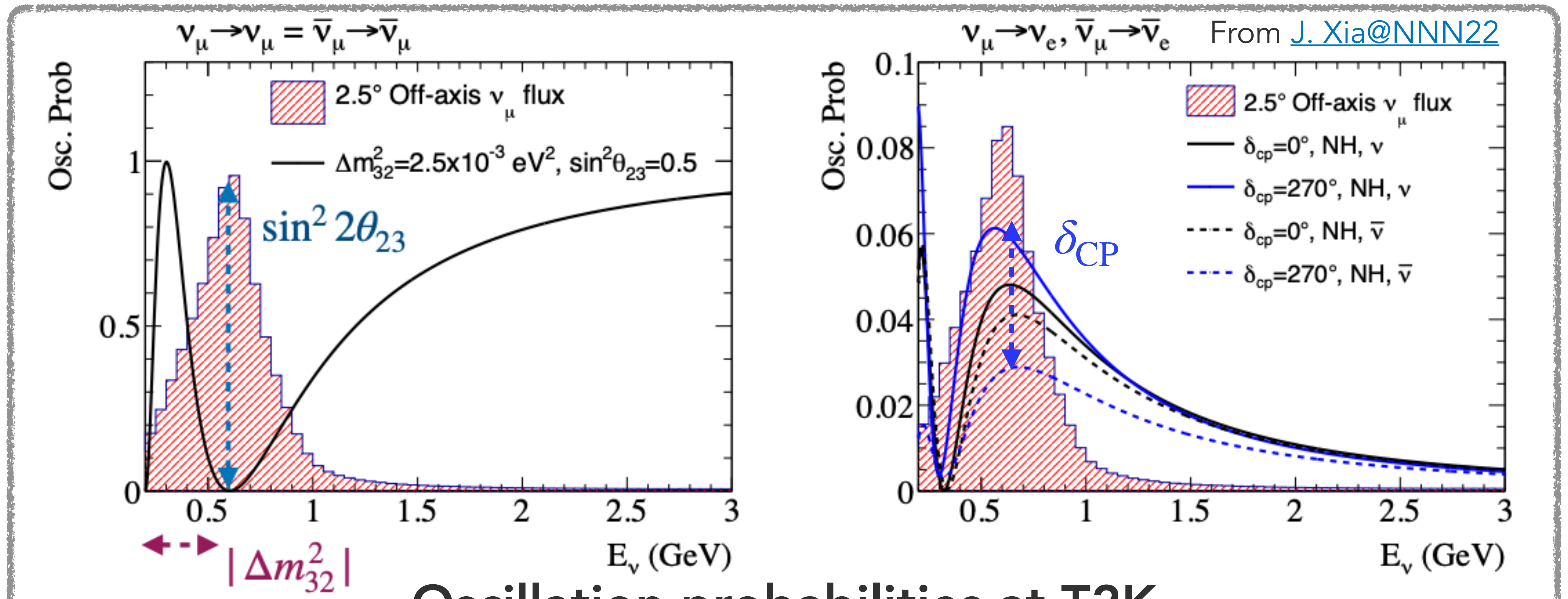
- Oscillation probability depends on
  - Neutrino energy E_ν
  - Mixing angles θ_{ij}
  - CP phase $\delta_{\rm CP}$
  - Square mass difference $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$.
    - We can constrain these parameters through the measurement of oscillation probabilities.



Oscillation probabilities at T2K

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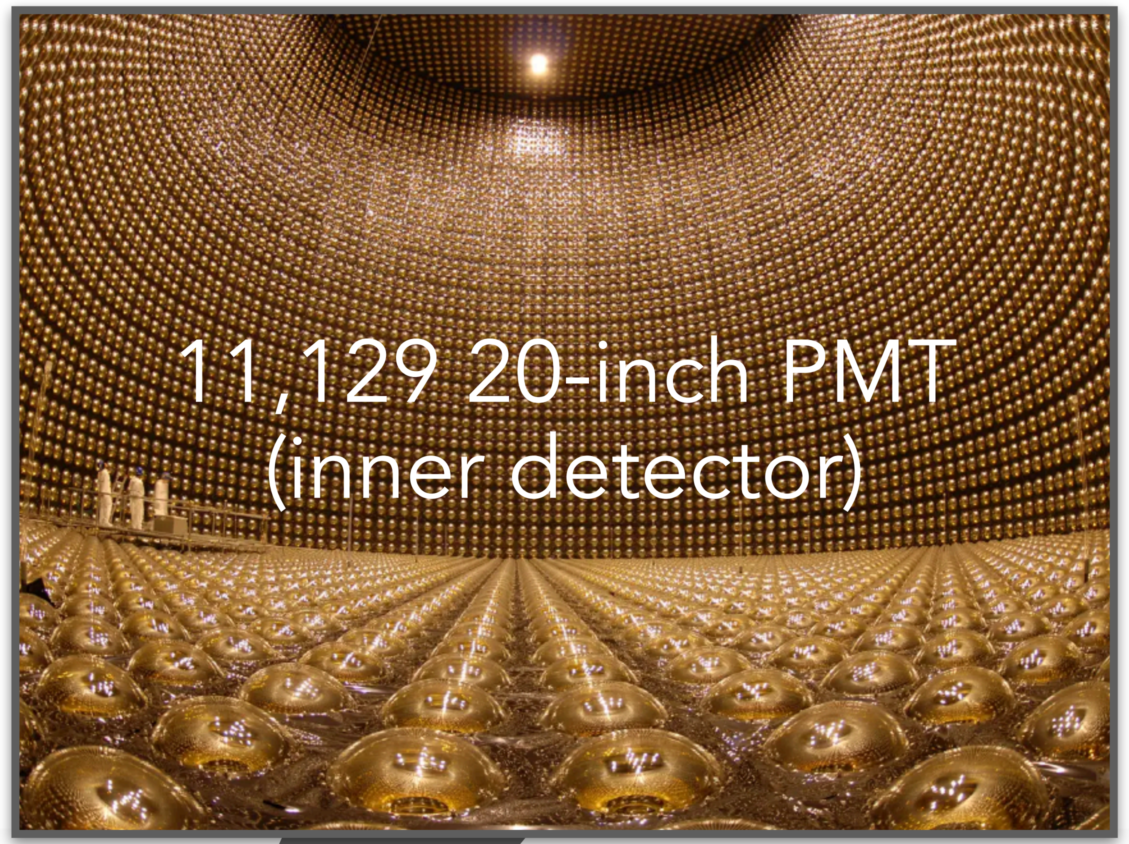
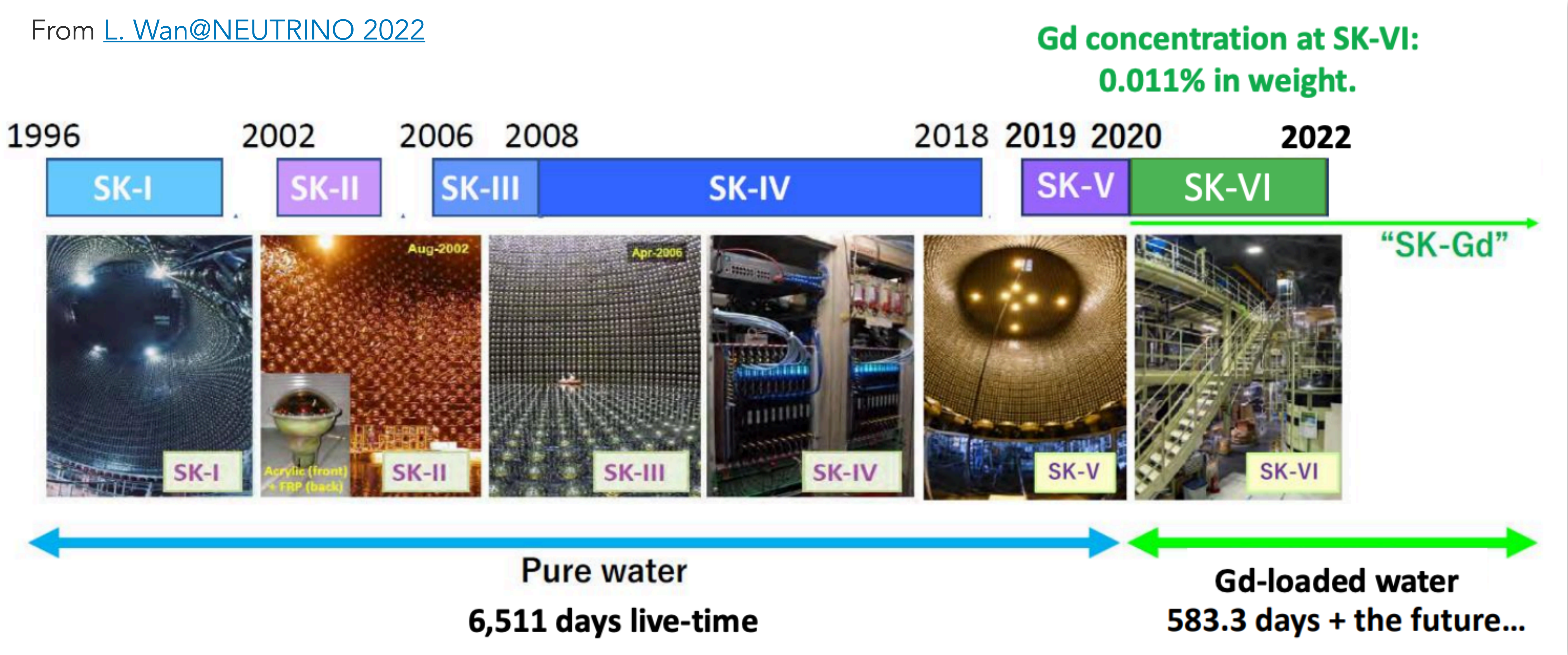
- Neutrino changes its flavors while it propagates long distances → **Neutrino oscillation**

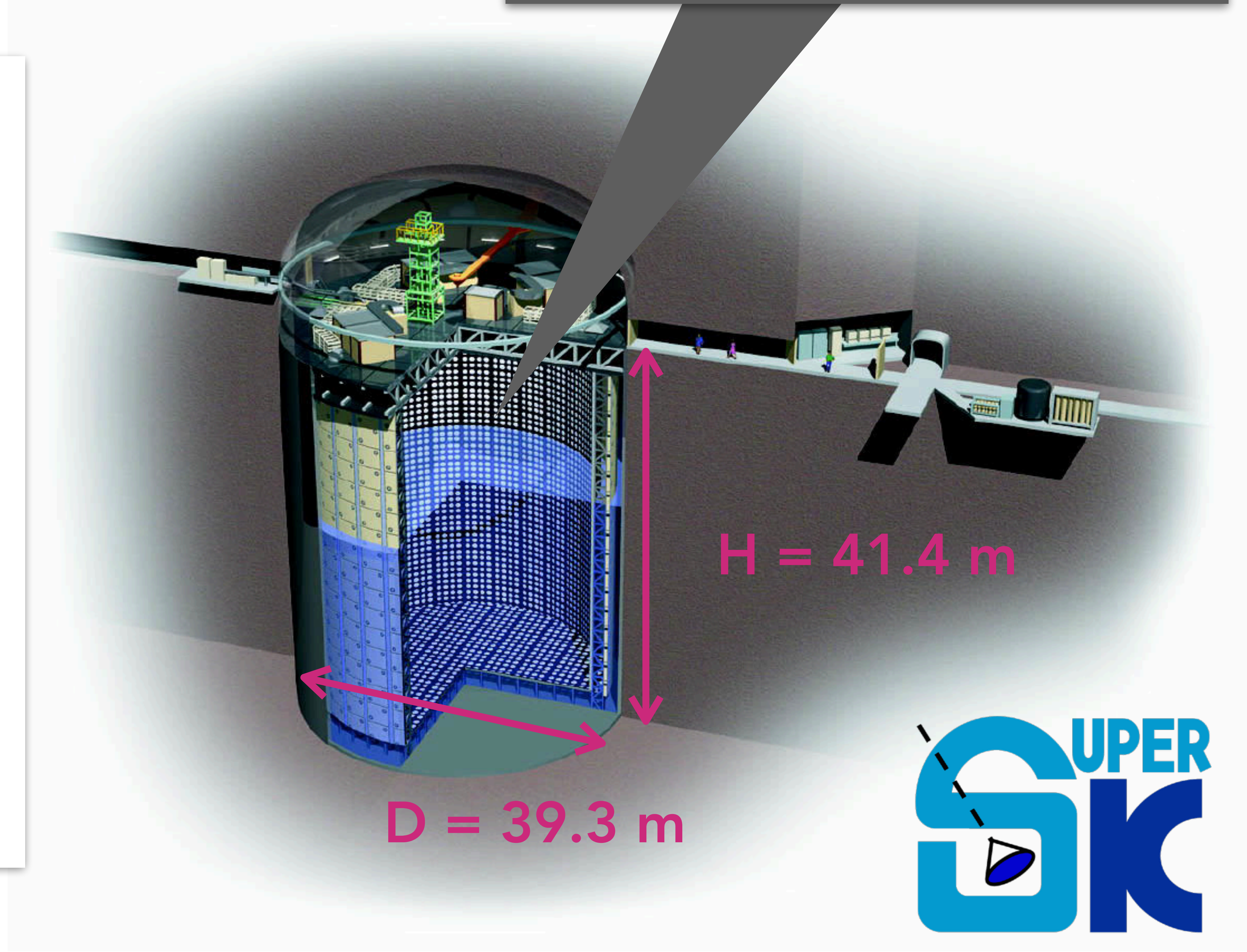
- Open questions in neutrino oscillation physics:
  - **CP symmetry** (value of δ_{CP})
  - **Mass ordering** (sign of Δm^2_{32})
  - **Octant of θ_{23}** ($\theta_{23} < \pi/4$ or $\pi/4 < \theta_{23}$)

Super-Kamiokande Experiment

- Super-Kamiokande (SK) experiment
  - 50 kton water Cherenkov detector located in Gifu, Japan
    - Good e/μ separation (mis-PID < 1% at 1 GeV)
    - Cannot separate $\nu/\bar{\nu}$ event-by-event basis







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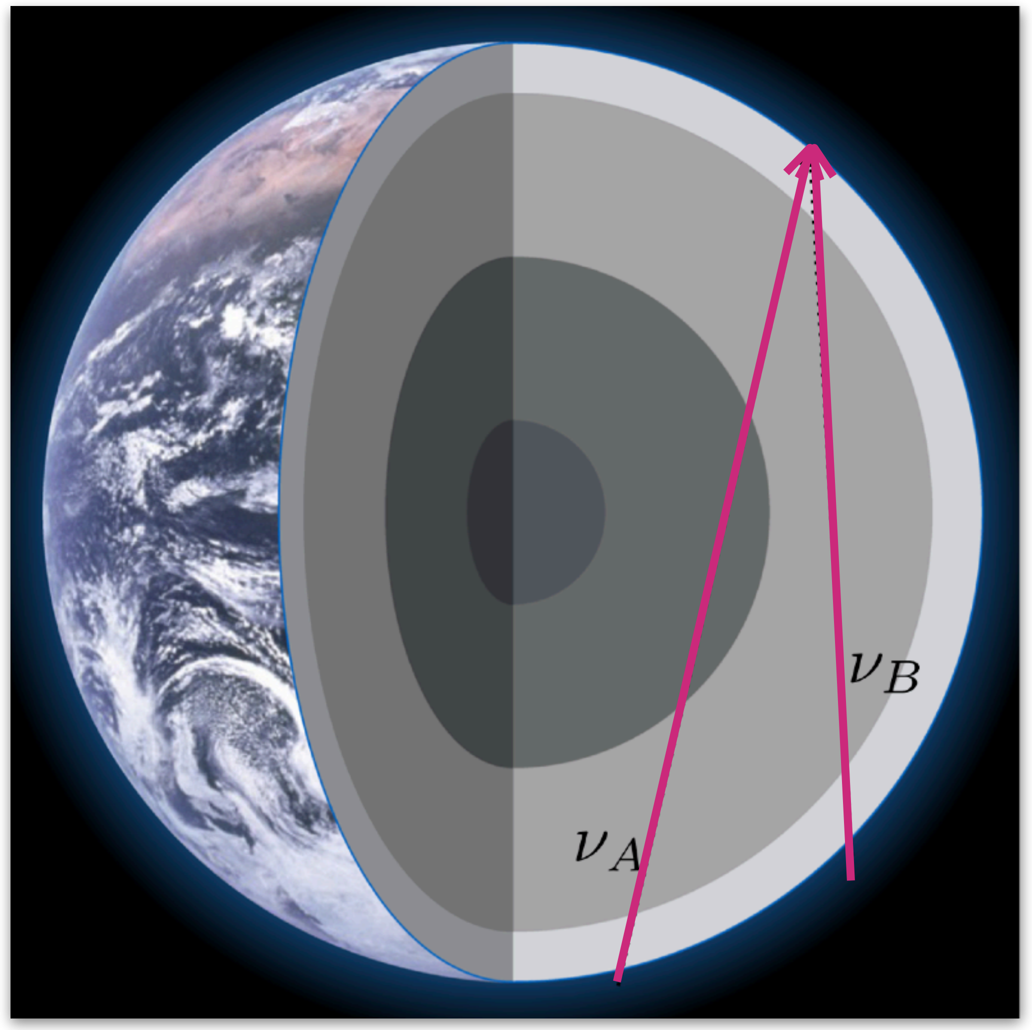
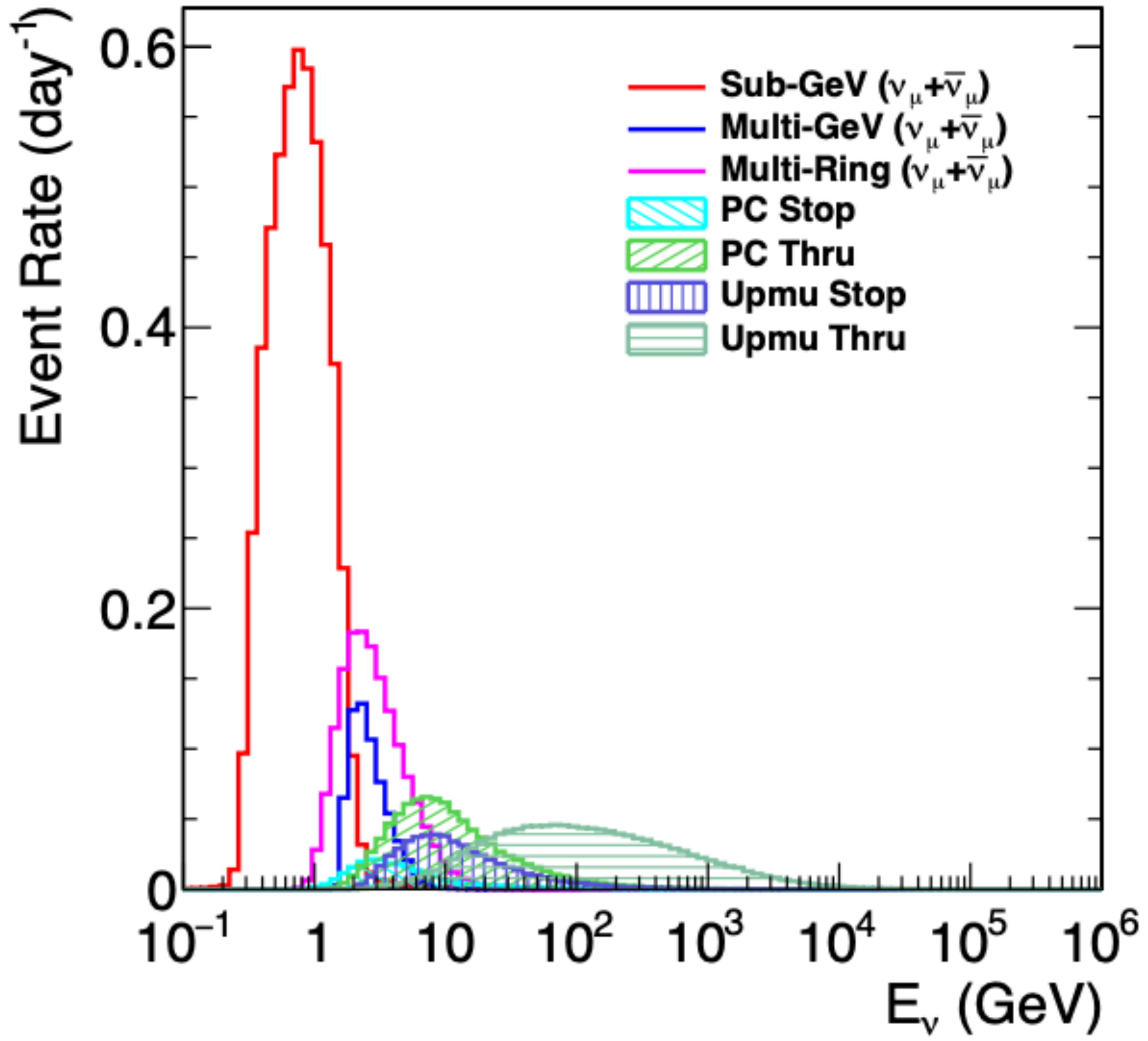
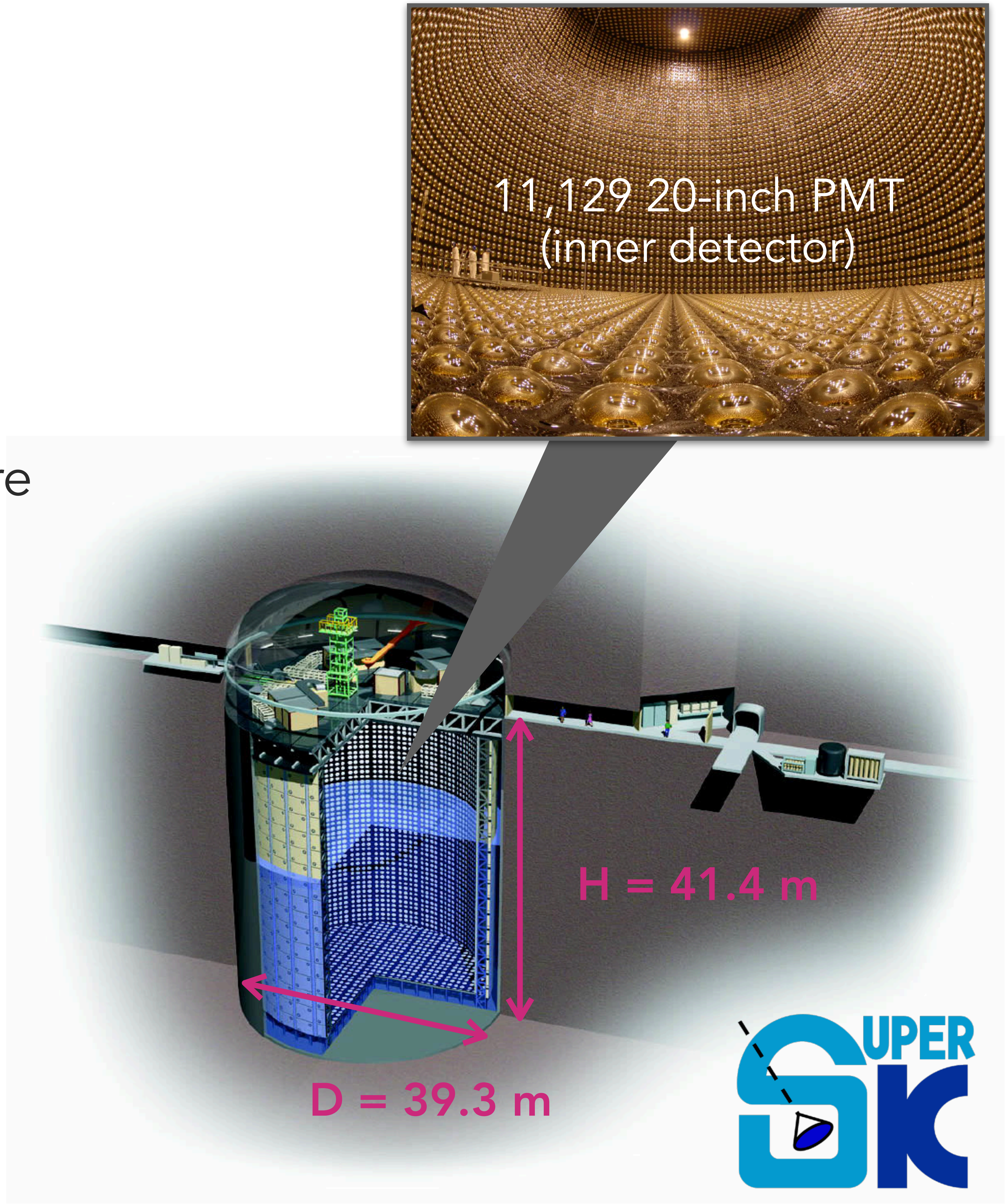
    - Cannot separate $\nu/\bar{\nu}$ event-by-event basis
  - SK atmospheric neutrinos have wide ranges of E_ν and L.
    - Oscillations of neutrinos passing through the Earth are largely **affected by the matter effect**.







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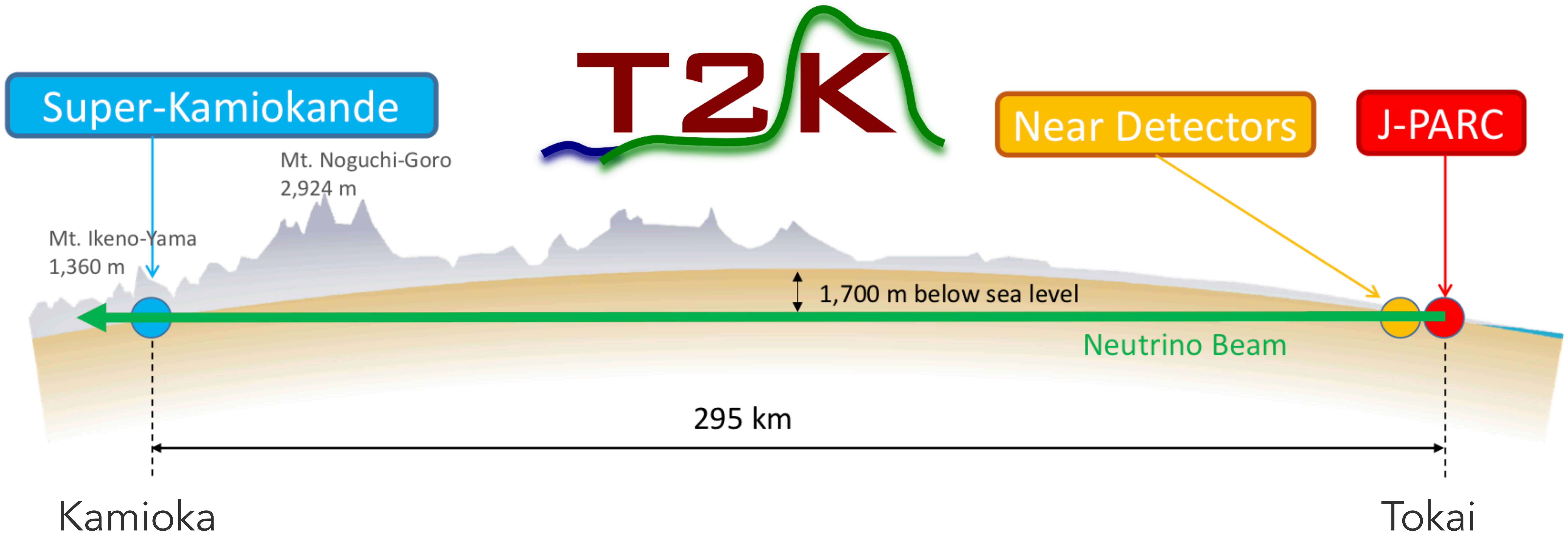
  - Long baseline neutrino oscillation experiment ($L \simeq 295$ km)



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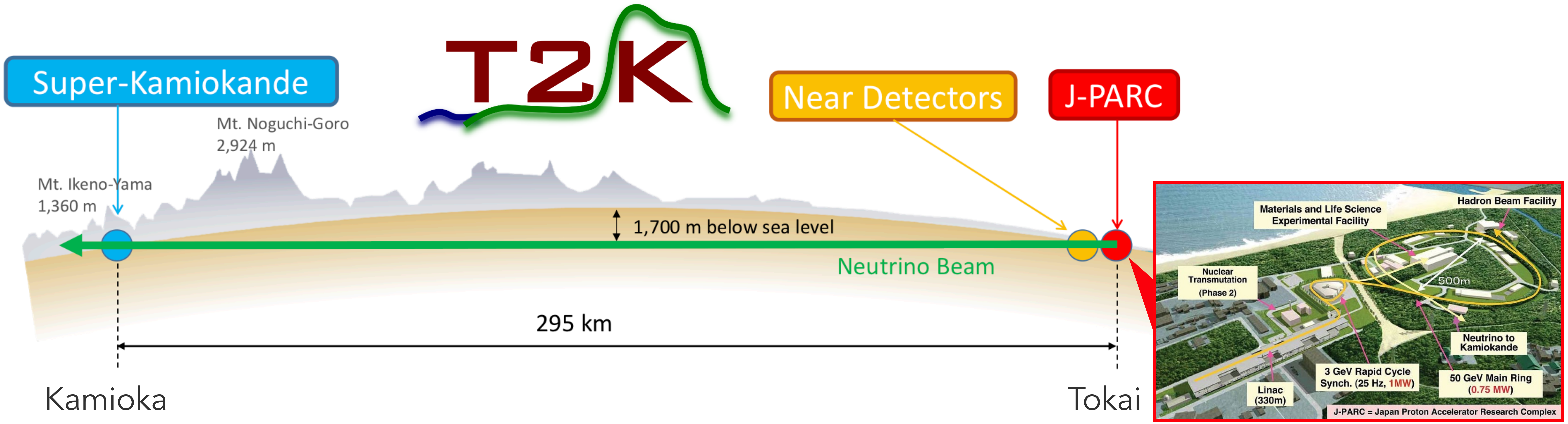

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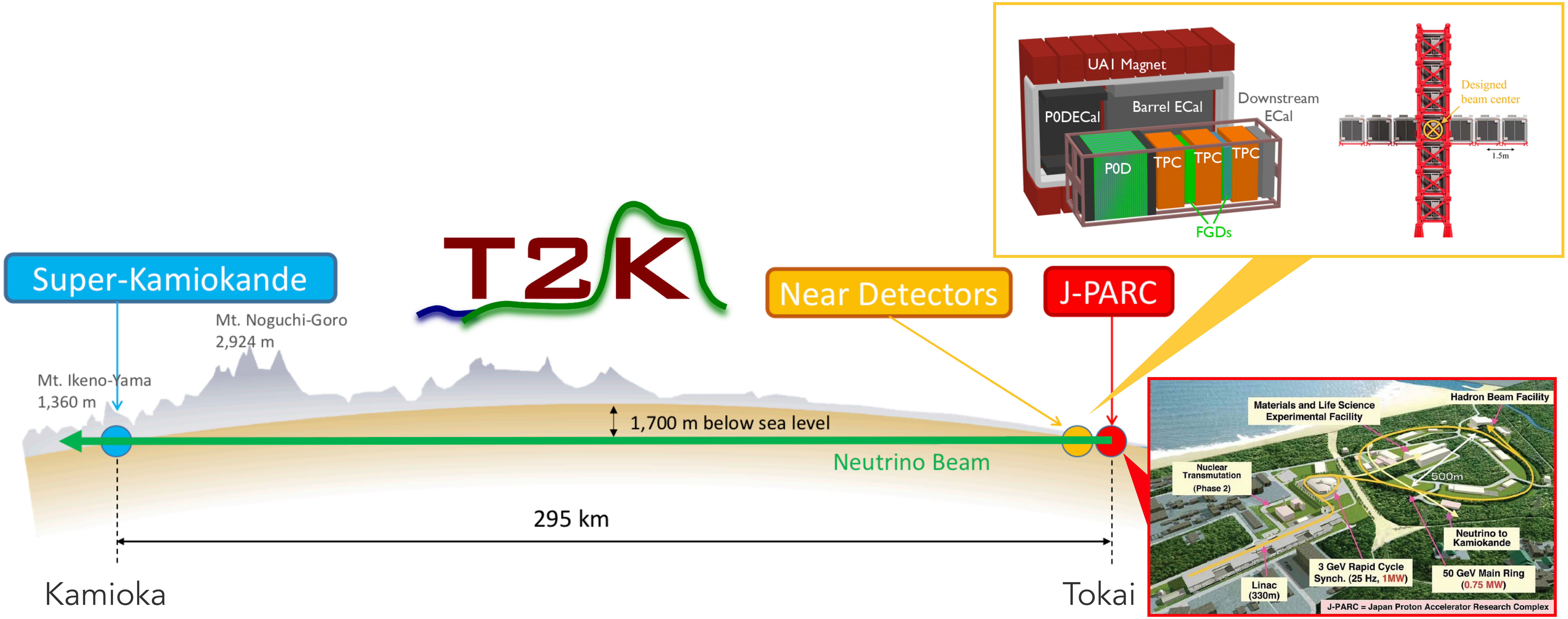

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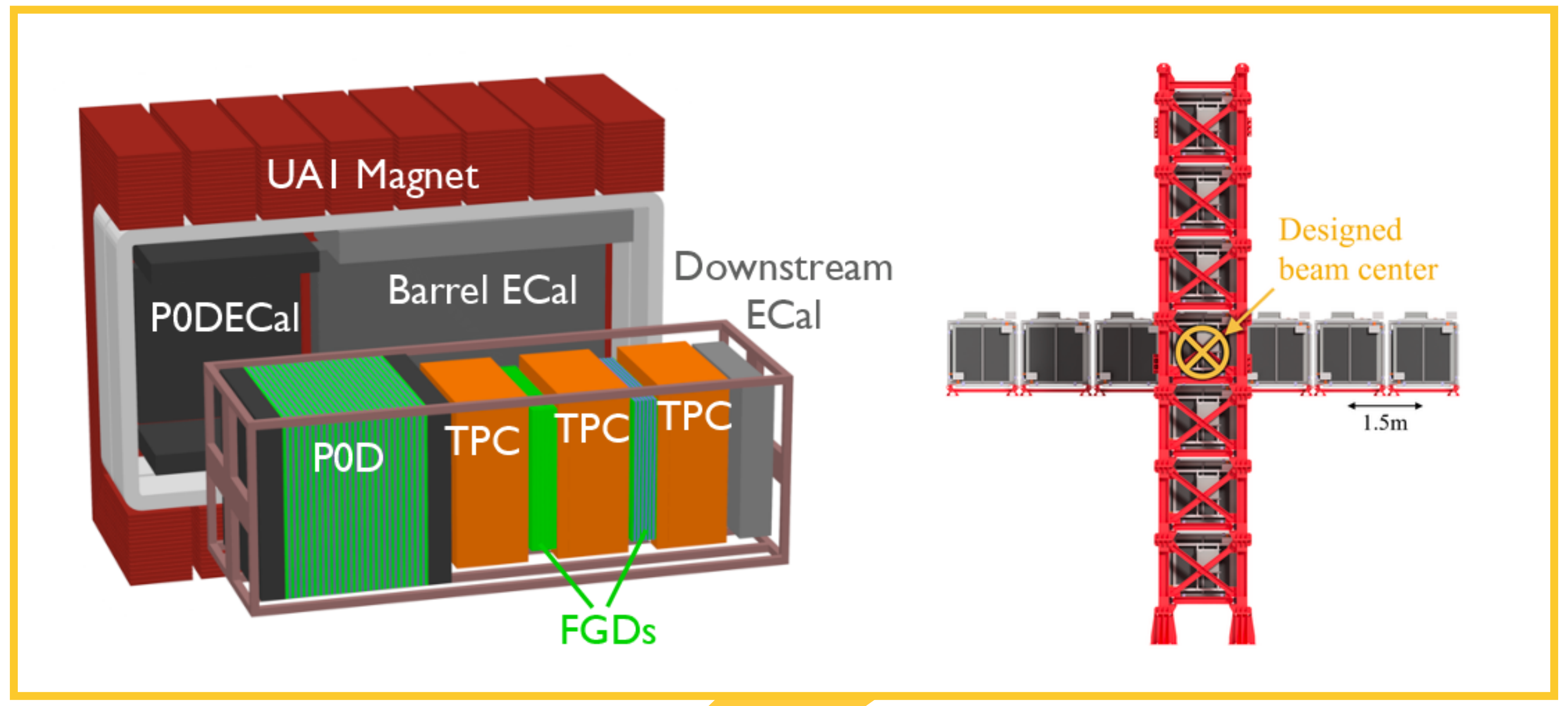

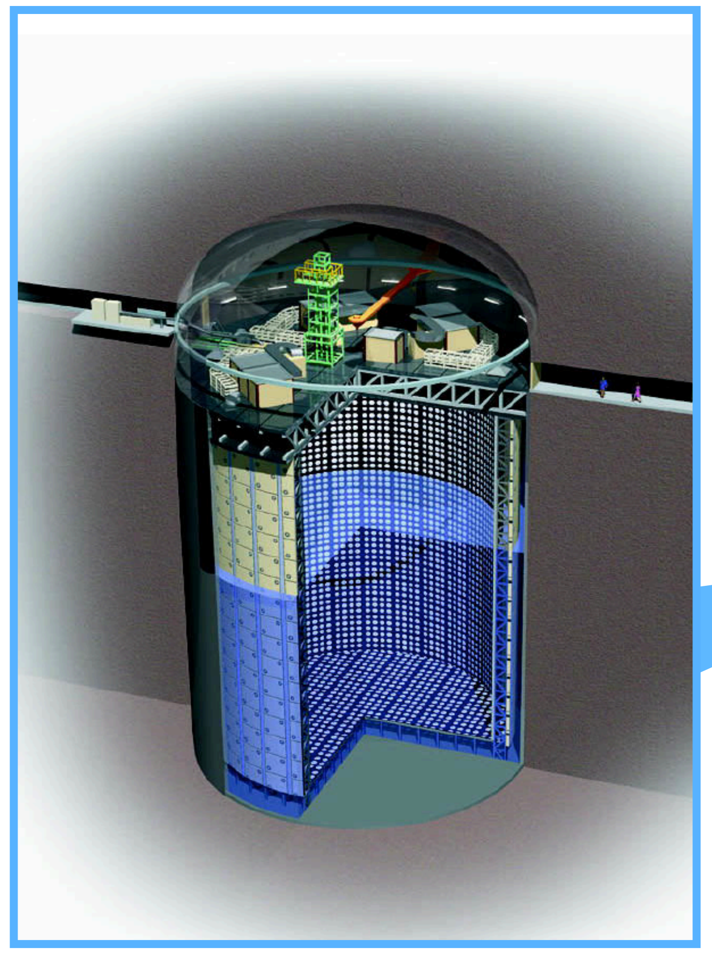

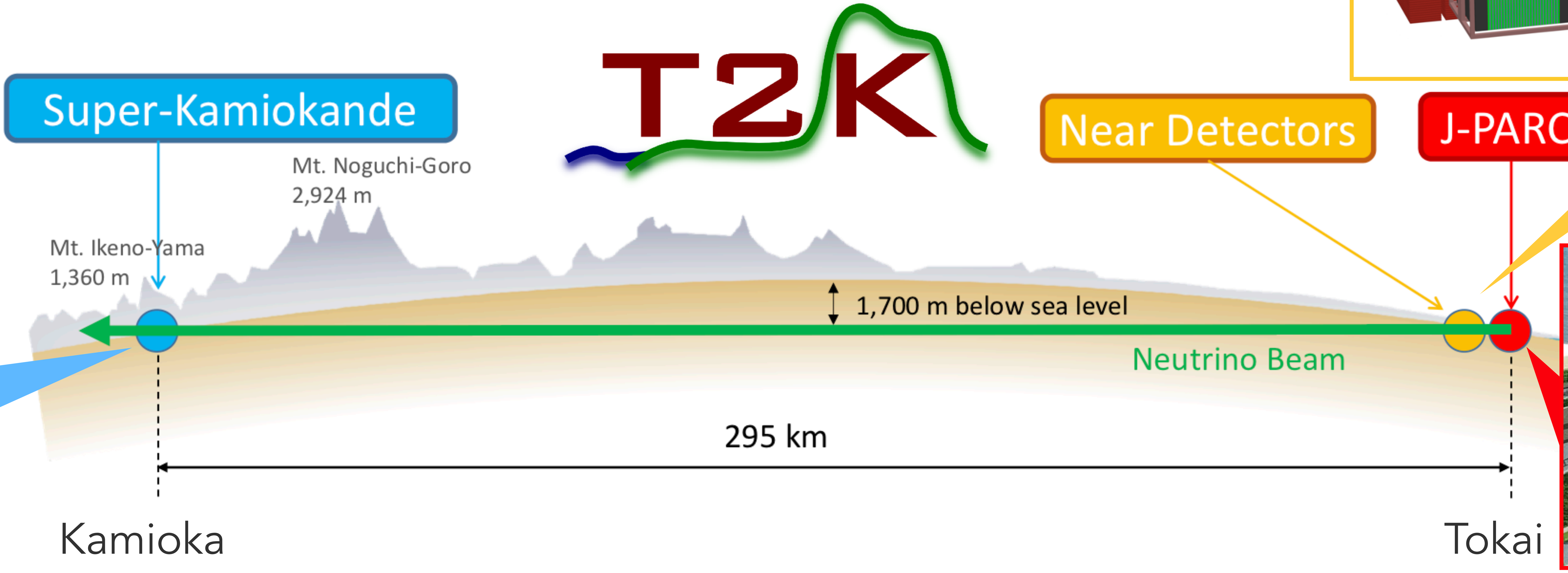

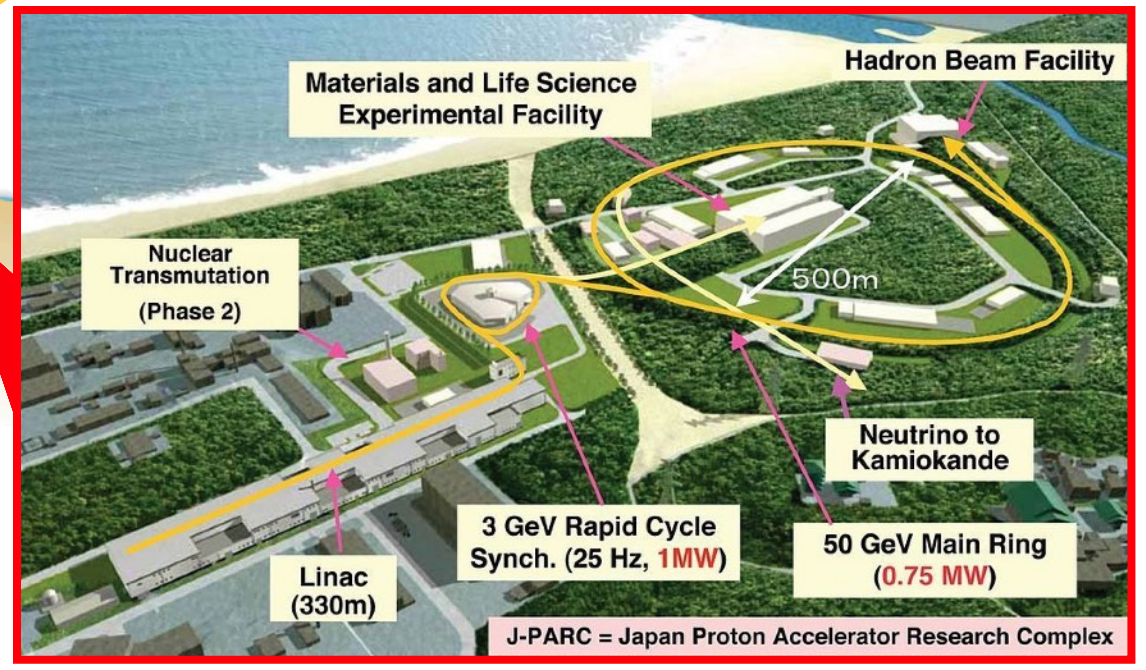

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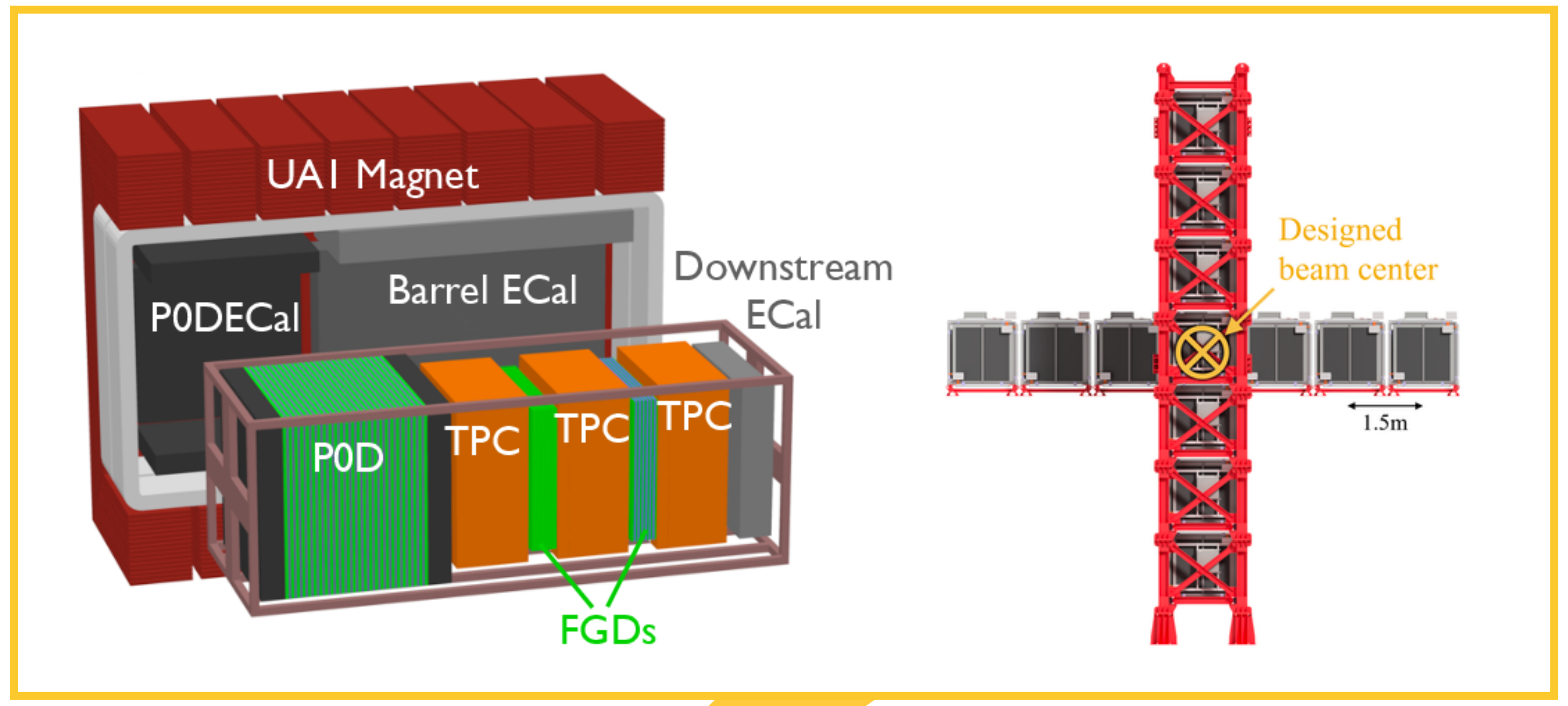
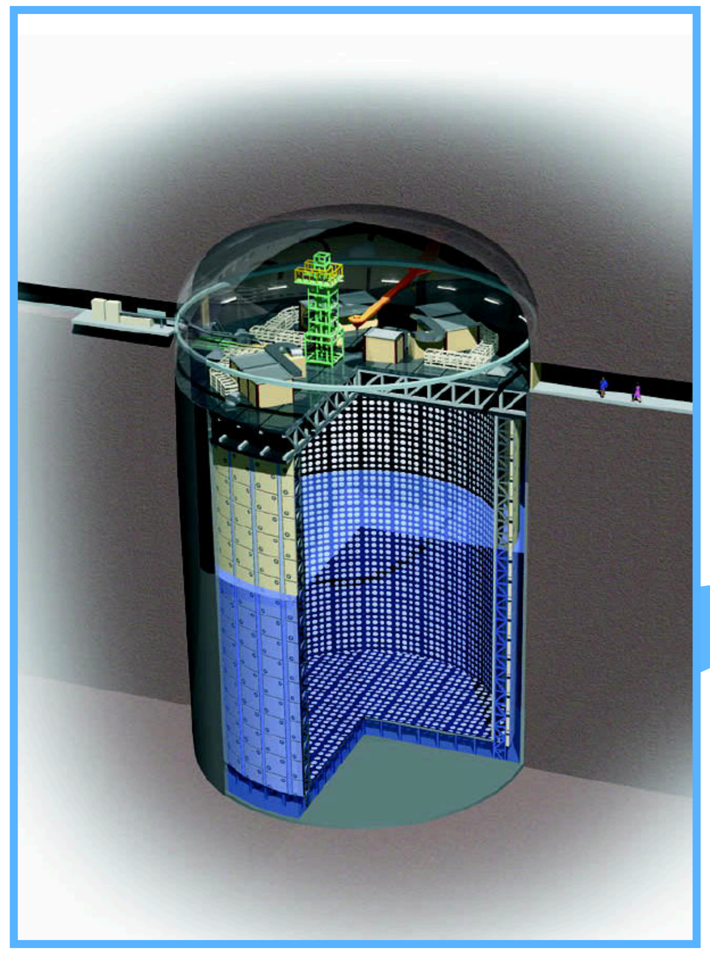
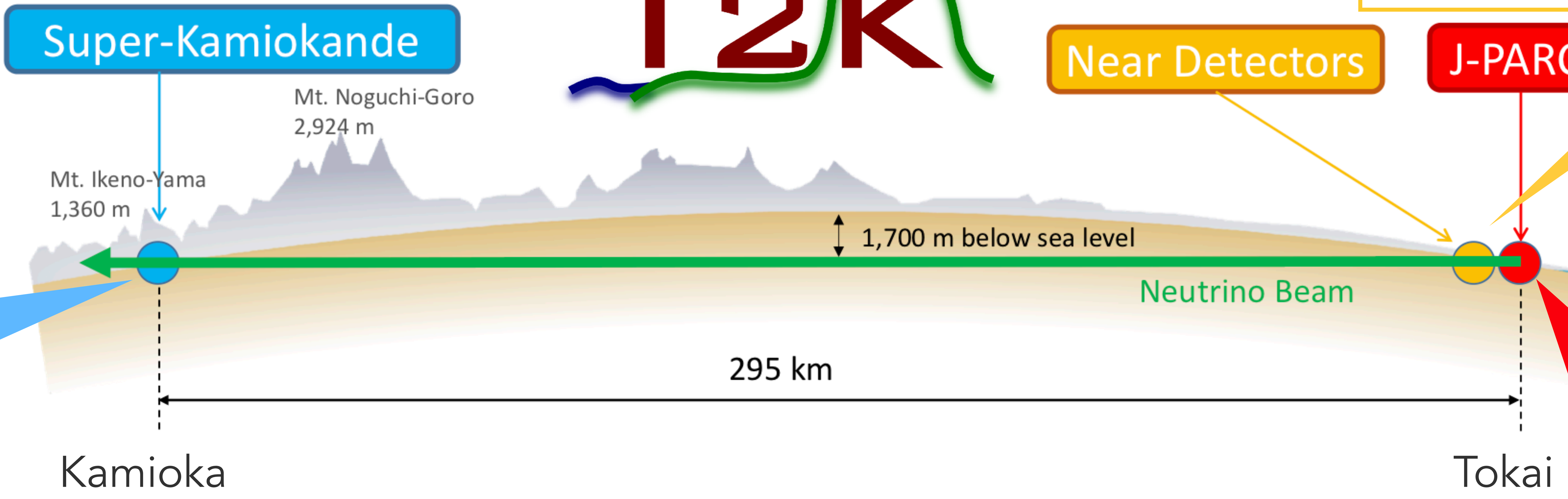
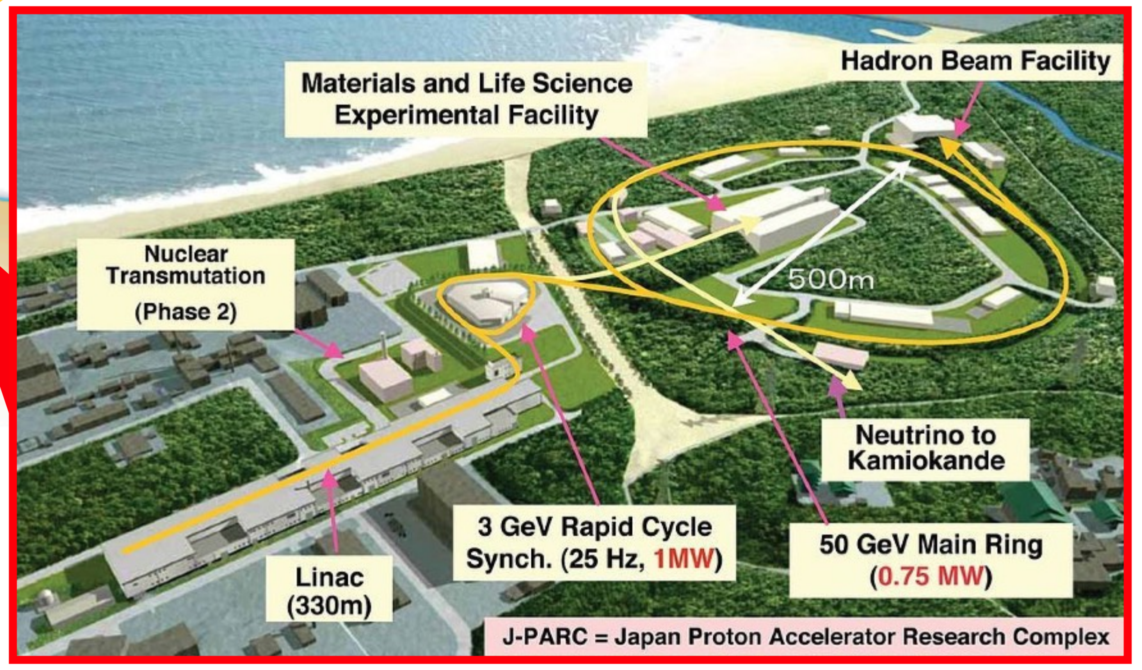


- T2K can measure both the disappearance $\nu_\mu(\bar{\nu}_\mu) \to \nu_\mu(\bar{\nu}_\mu)$ and appearance $\nu_\mu(\bar{\nu}_\mu) \to \nu_e(\bar{\nu}_e)$ probabilities using almost pure $\nu_\mu(\bar{\nu}_\mu)$ flux.





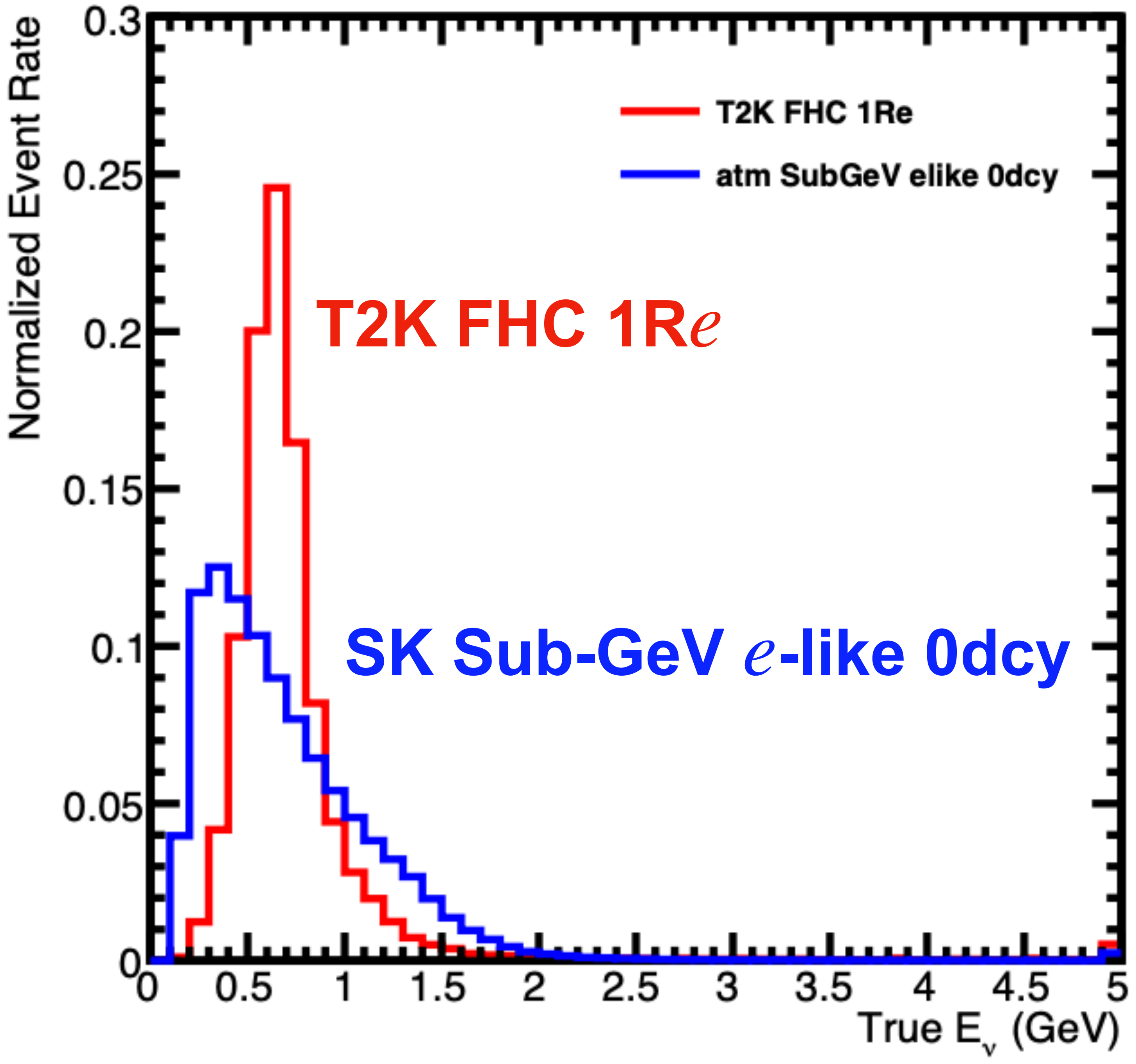


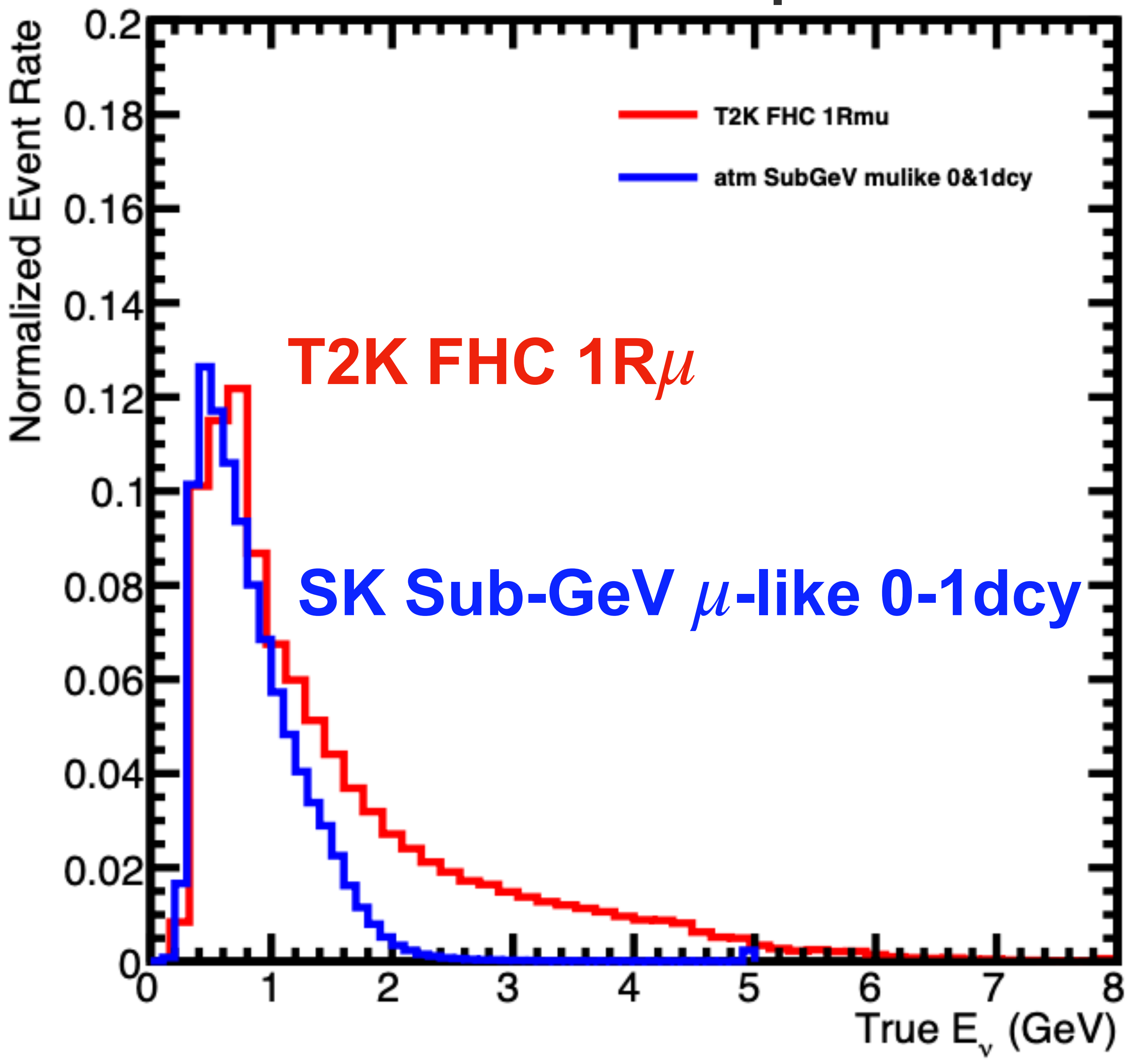
Motivation of the Joint Analysis

- We expect to have **several benefits beyond the increased statistics**.

- T2K and SK use the same detector and have samples with similar energy ranges and similar selections.
  - We can take into account the **correlations of the systematic uncertainties**.
  - **T2K near detector** can be used to constrain the cross-section uncertainties for the low-energy atmospheric samples as well.

Comparison of the normalized flux of the selected samples





Motivation of the Joint Analysis

- The additional benefit of the joint fit
  - T2K has **better sensitivity to** δ_{CP} from $\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_e$ oscillation and to $\theta_{23}, \Delta m^2_{32}$ from $\overset{(-)}{\nu}_\mu \rightarrow \overset{(-)}{\nu}_\mu$

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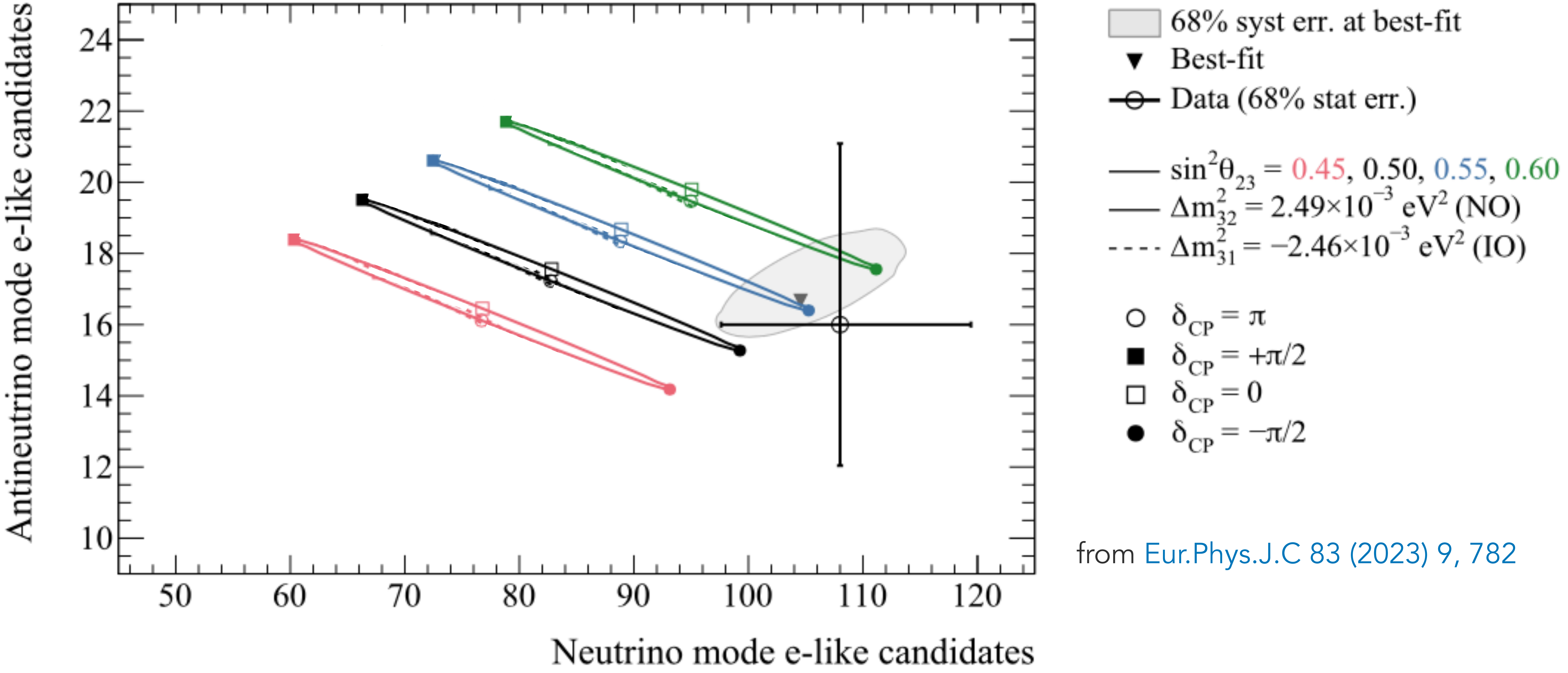
- The additional benefit of the joint fit
  - The event rate of $\nu_e/\bar{\nu}_e$ depends on the value of δ_{CP}

Number of T2K beam $\nu_e/\bar{\nu}_e$ events with different sets of oscillation parameter values



from Eur.Phys.J.C 83 (2023) 9, 782

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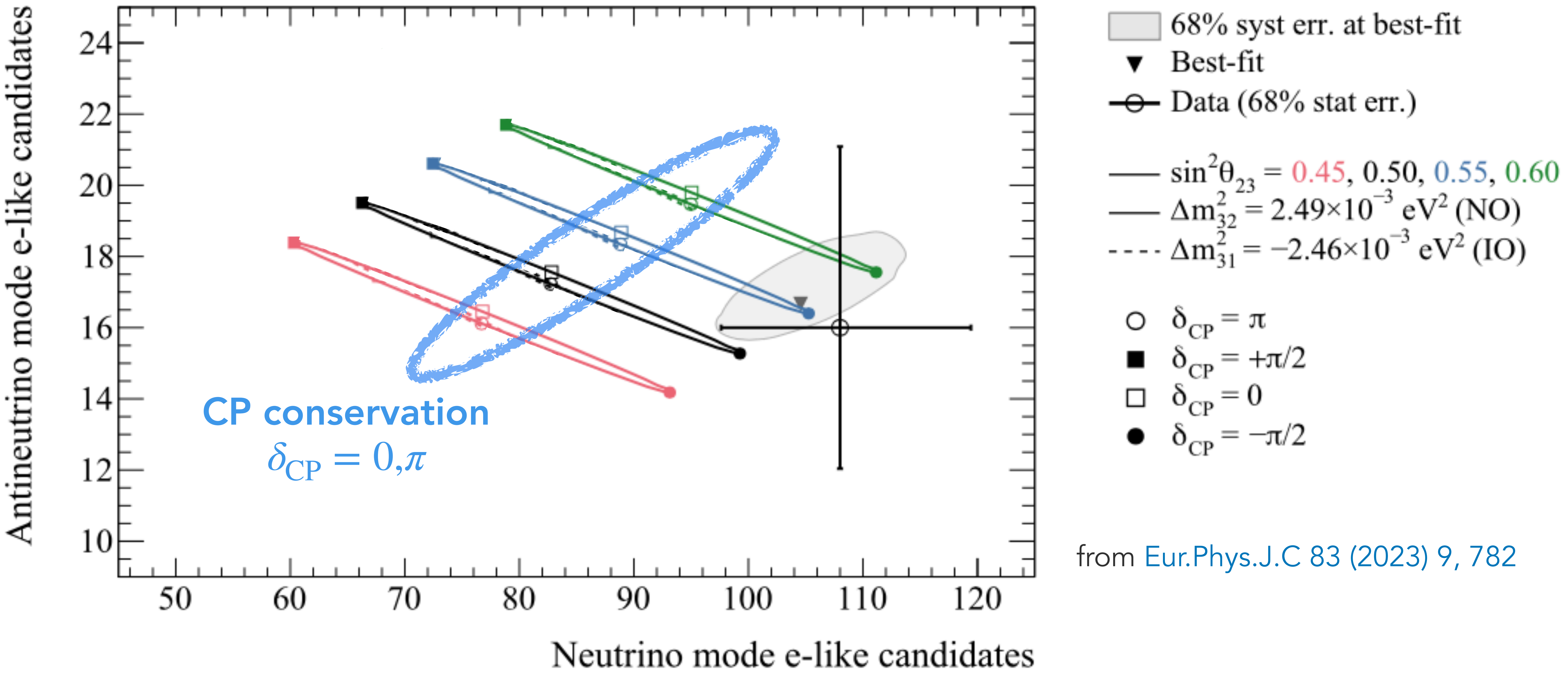

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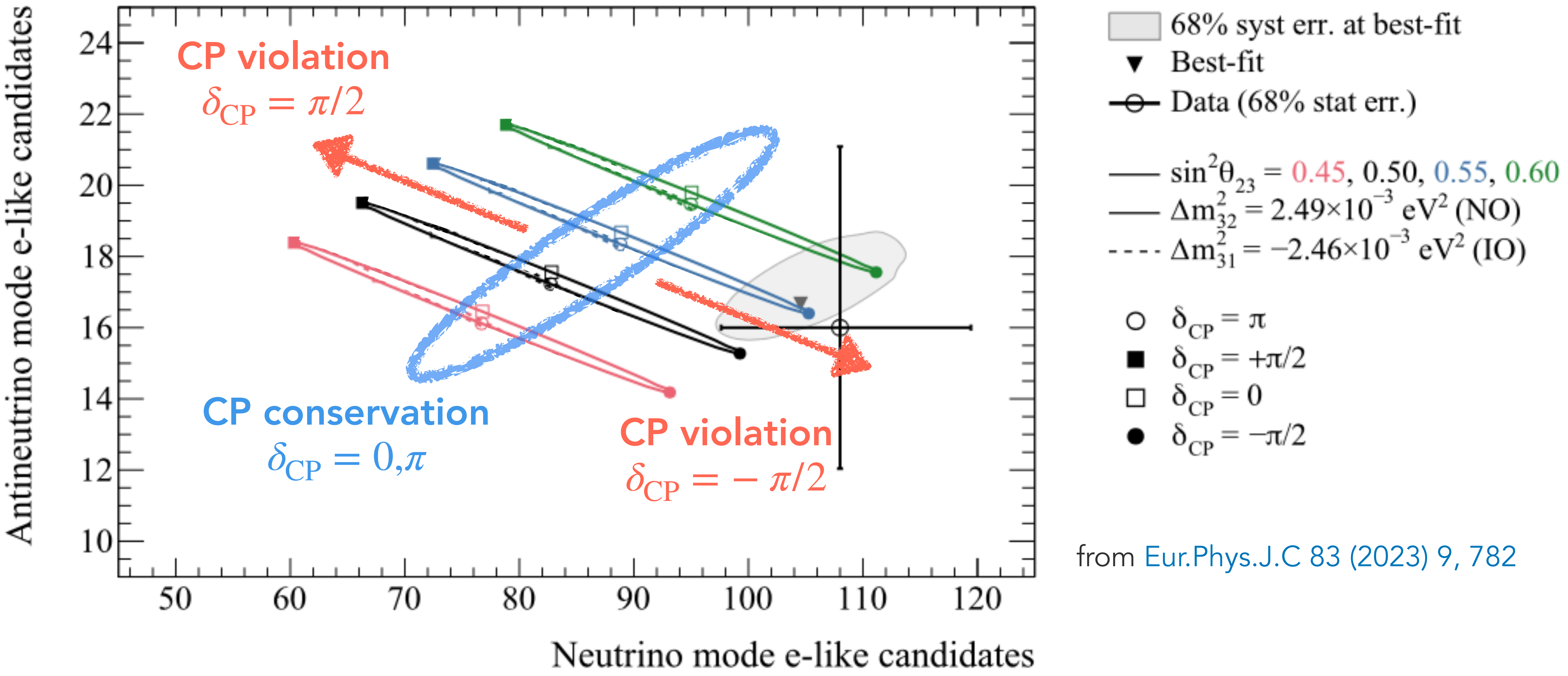

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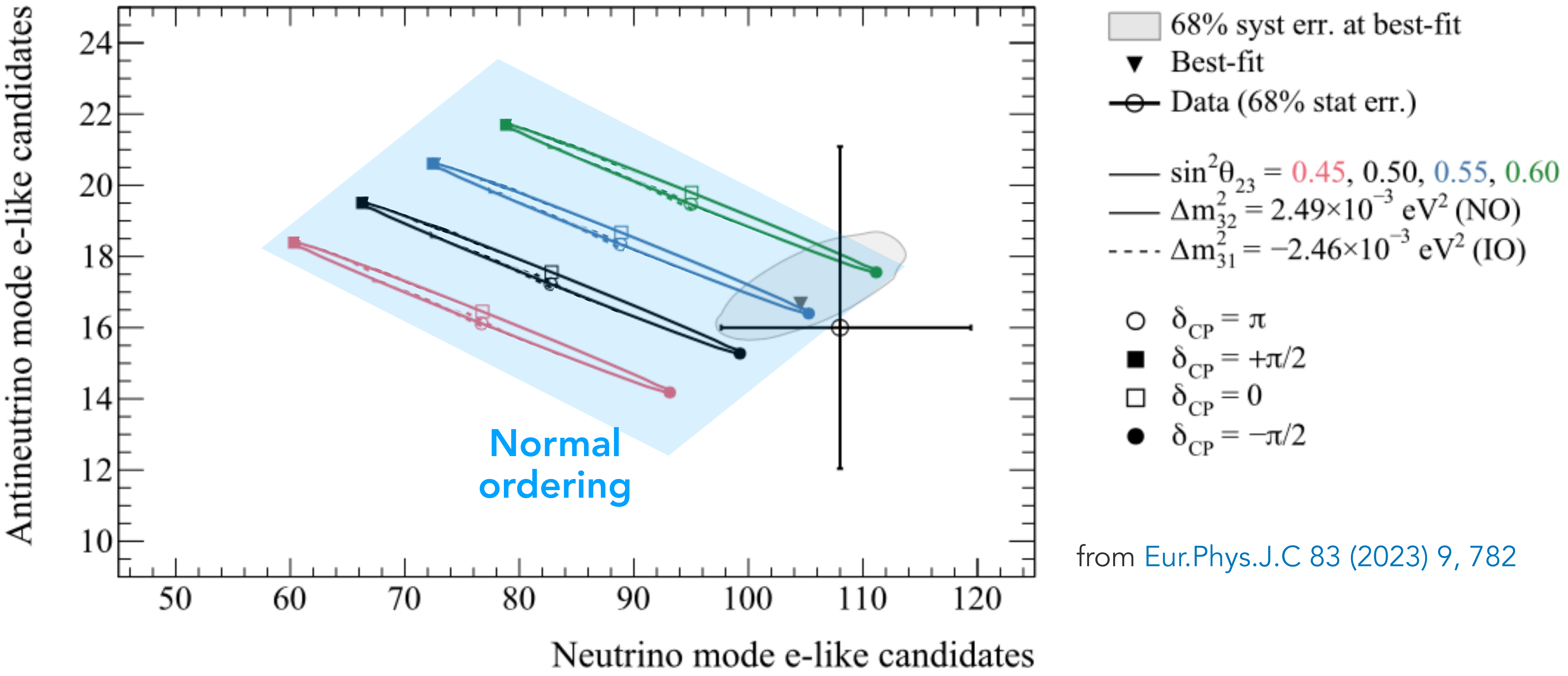

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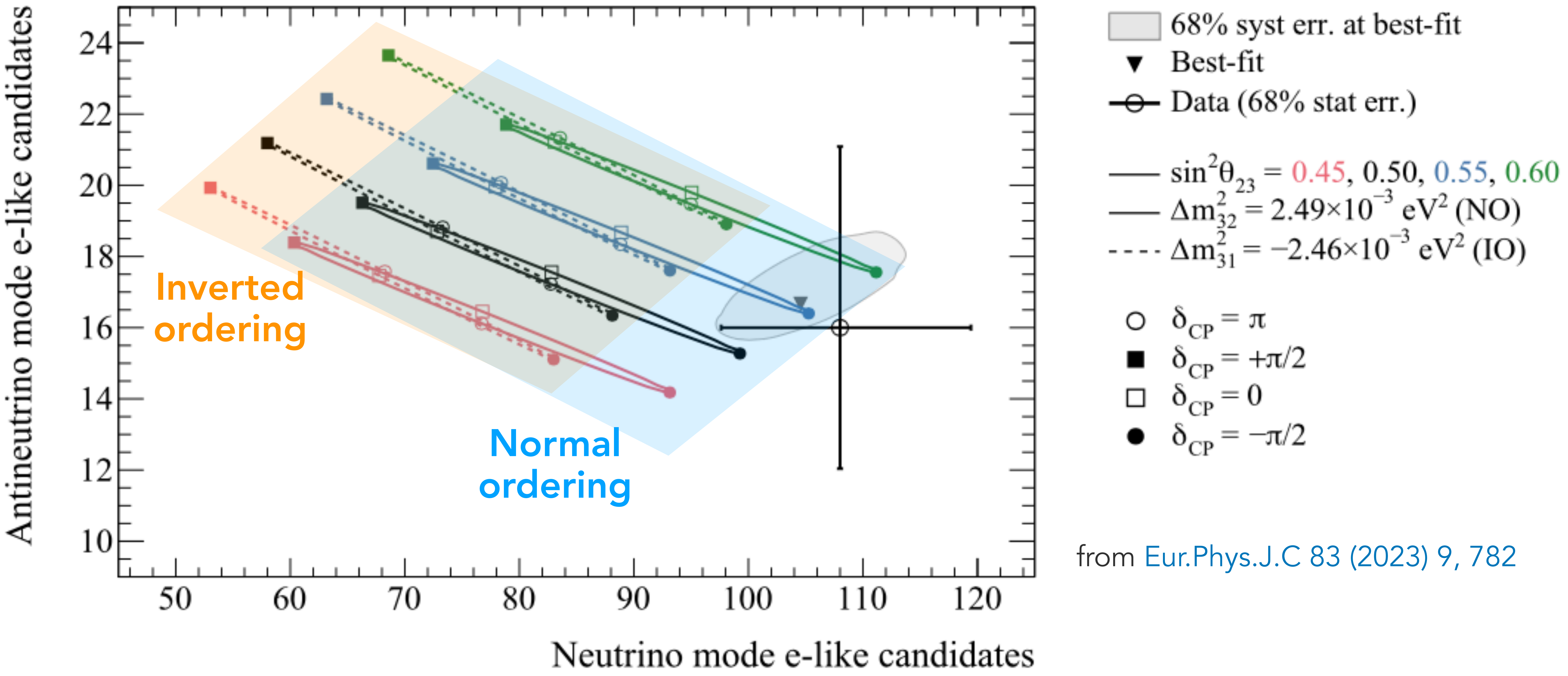
Motivation of the Joint Analysis

- The additional benefit of the joint fit
  - The event rate of $\nu_e/\bar{\nu}_e$ depends on the value of δ_{CP}.
  - However, δ_{CP} and **neutrino mass ordering** have a similar effect to the $\nu_e/\bar{\nu}_e$ event rates we observe in T2K (we call this "**degeneracy**" of the oscillation parameter).

Number of T2K beam $\nu_e/\bar{\nu}_e$ events with different sets of oscillation parameter values



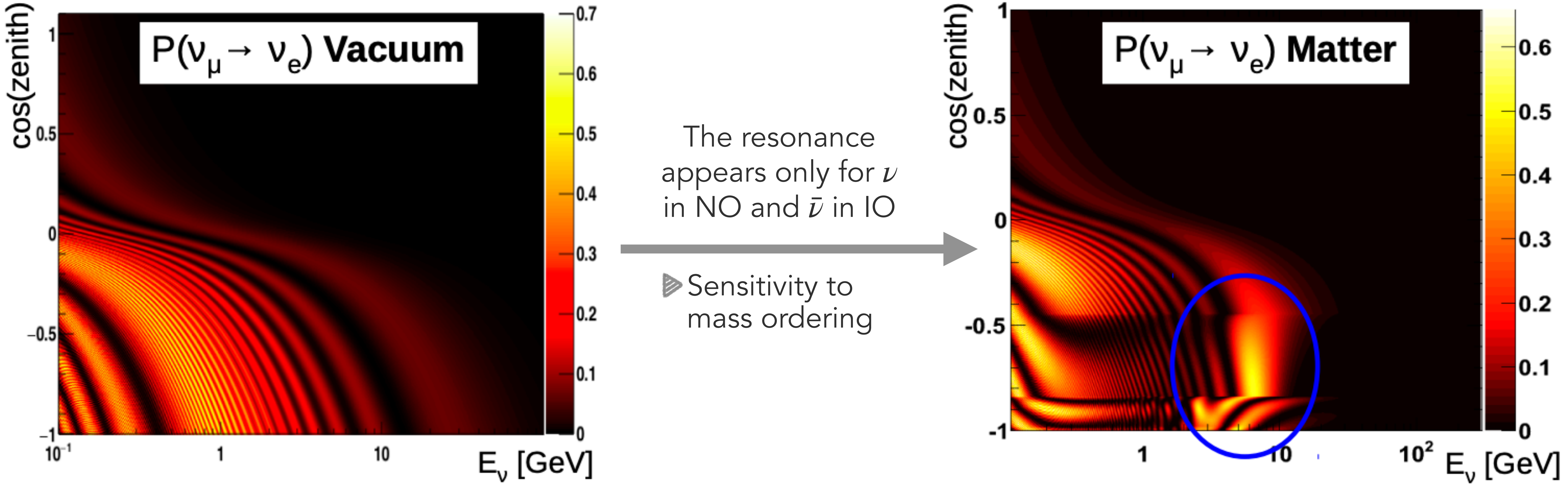
from Eur.Phys.J.C 83 (2023) 9, 782

Motivation of the Joint Analysis

- The additional benefit of the joint fit
  - SK has **stronger discrimination of the mass ordering** thanks to the **matter effect** at the few GeV regions, which is not degenerate with δ_{CP}.

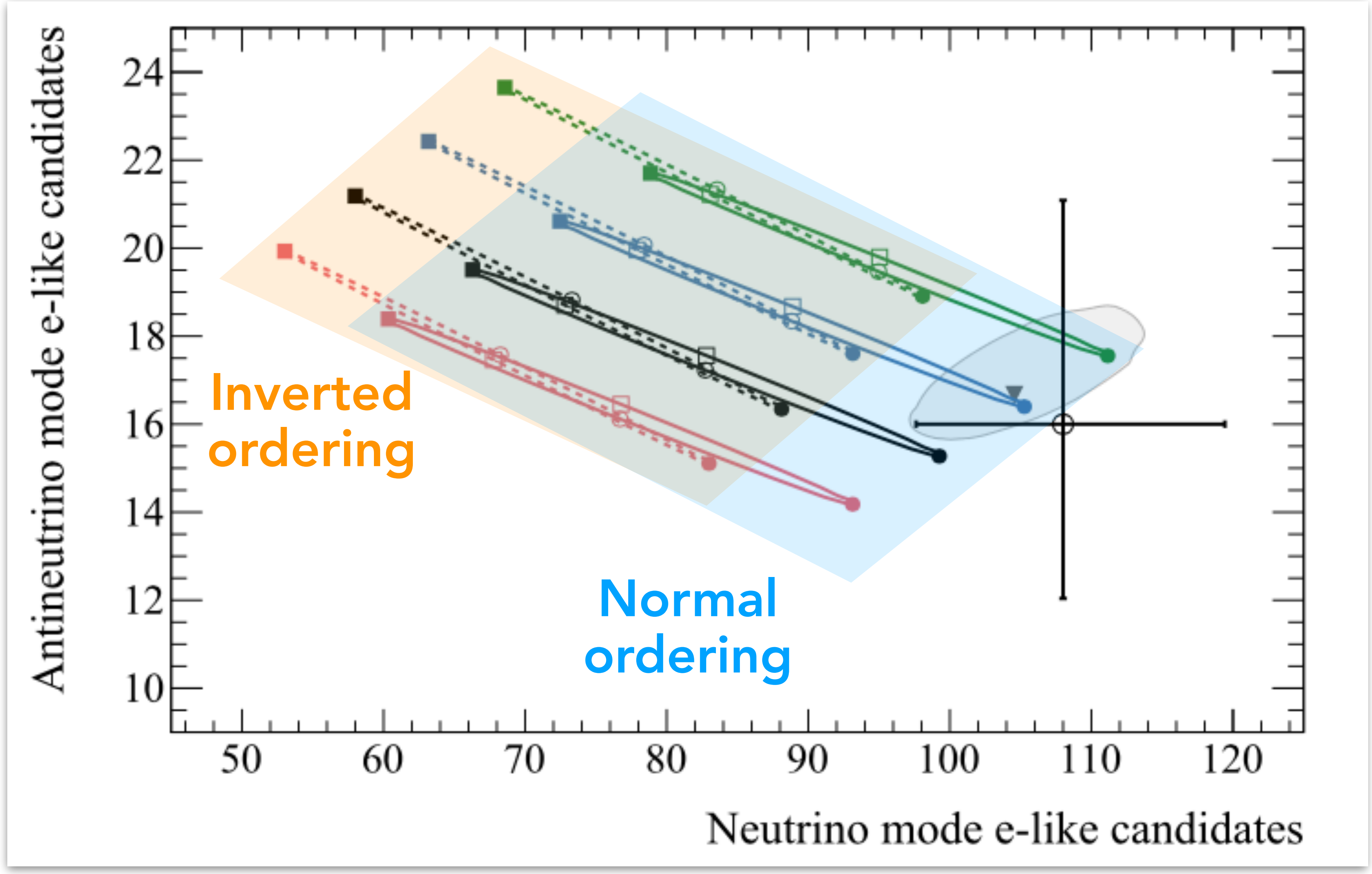
Atmospheric neutrino oscillation probability (normal ordering)

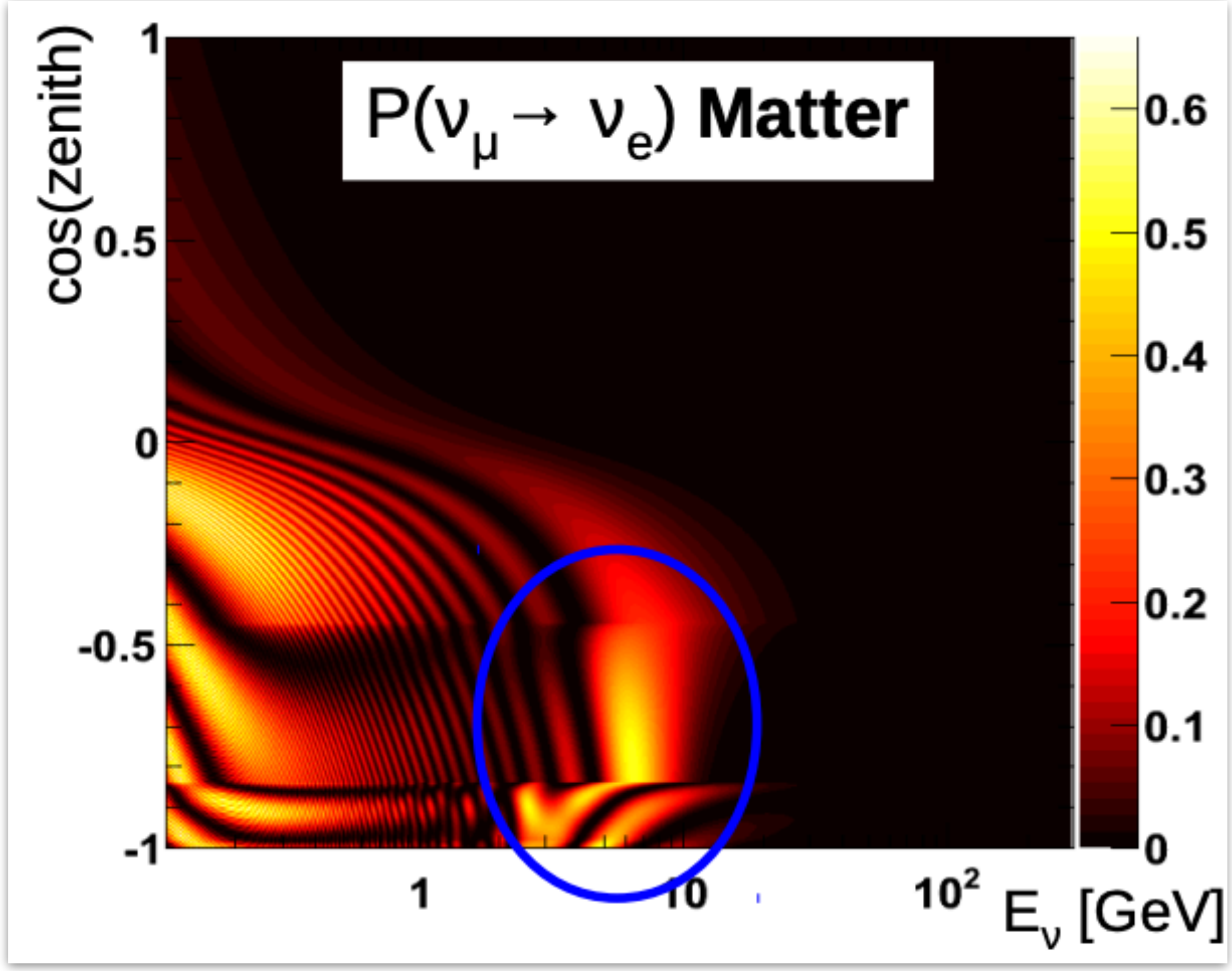
from C. Bronner @ PANE 2018



Motivation of the Joint Analysis

- We expect to **break the degeneracy** by combining these two experiments to get better sensitivity to oscillation parameters and mass ordering.
  - The atmospheric neutrino oscillation can break the **degeneracy between δ_{CP} and mass ordering**.
  - **T2K can constrain $\sin^2\theta_{23}$ better** and it improves the **mass ordering sensitivity** in the atmospheric oscillations as the resonance is $\sin^2\theta_{23}$-dependent.



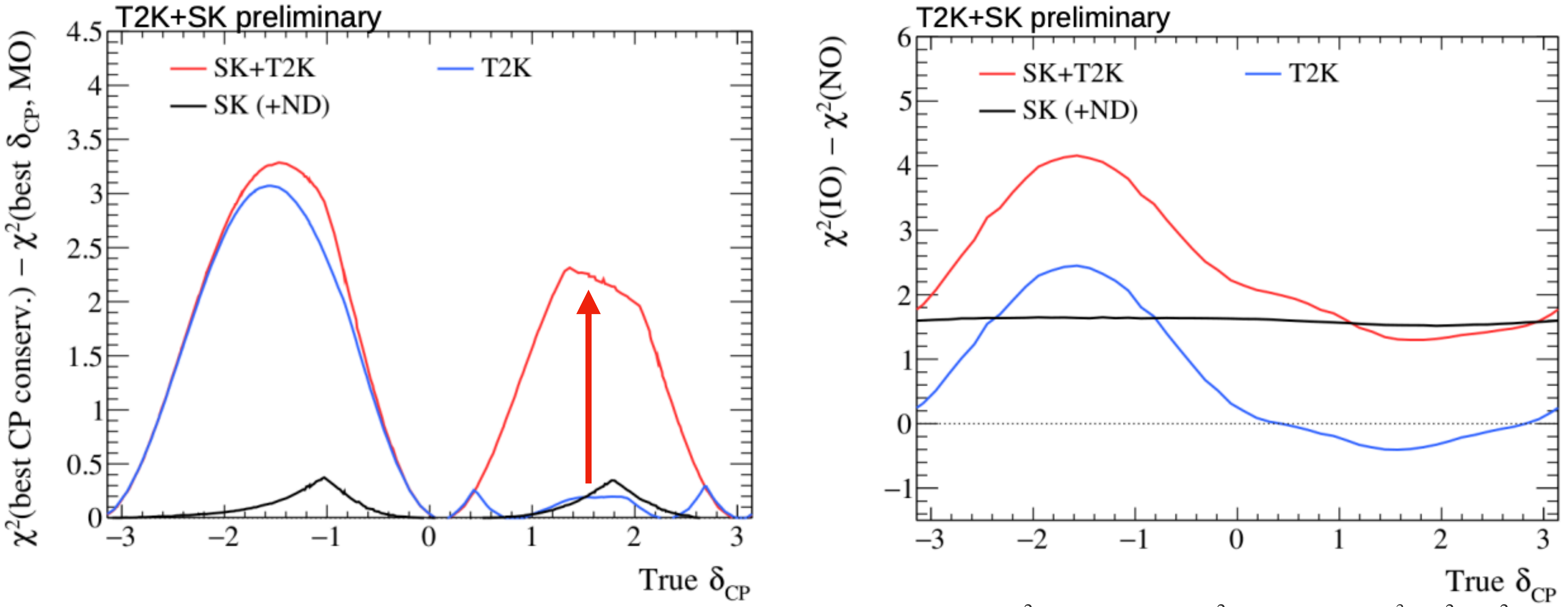


Better constraints on δ_{CP}, $\sin^2\theta_{23}$, Δm^2_{32}, and MO

From the Sensitivity Study

- Preliminary sensitivity study was shown in C. Bronner@Neutrino 2022 and J. Xia@NOW2022
  - Joint fit breaks the degeneracy between δ_{CP} and MO, and **increases the ability to reject CP conservation** beyond the simple sum of two experiments in some regions.

Ability to reject (left) CP conservation and (right) wrong mass ordering as a function of true δ_{CP}



"SK (+ND)": SK-only fit with T2K ND constraint on the low-E cross-section model. True values assumed: $\sin^2\theta_{23} = 0.528$, $\Delta m^2_{32} = 2.509 \times 10^{-3}$ eV^2, $\sin^2\theta_{13} = 0.0218$, NO

Analysis Method

- Samples used in this analysis
- Treatment of systematic uncertainties
- Two fitters used for the Bayesian analysis
- Study of the robustness of the model using the simulated data

Data Set and Samples

- **5 T2K beam samples** and **18 SK atmospheric samples** are fitted simultaneously.
  - **T2K near detector** is used to constrain the beam flux and low-energy cross-section parameters.
  - Data set before Gd loading is used.

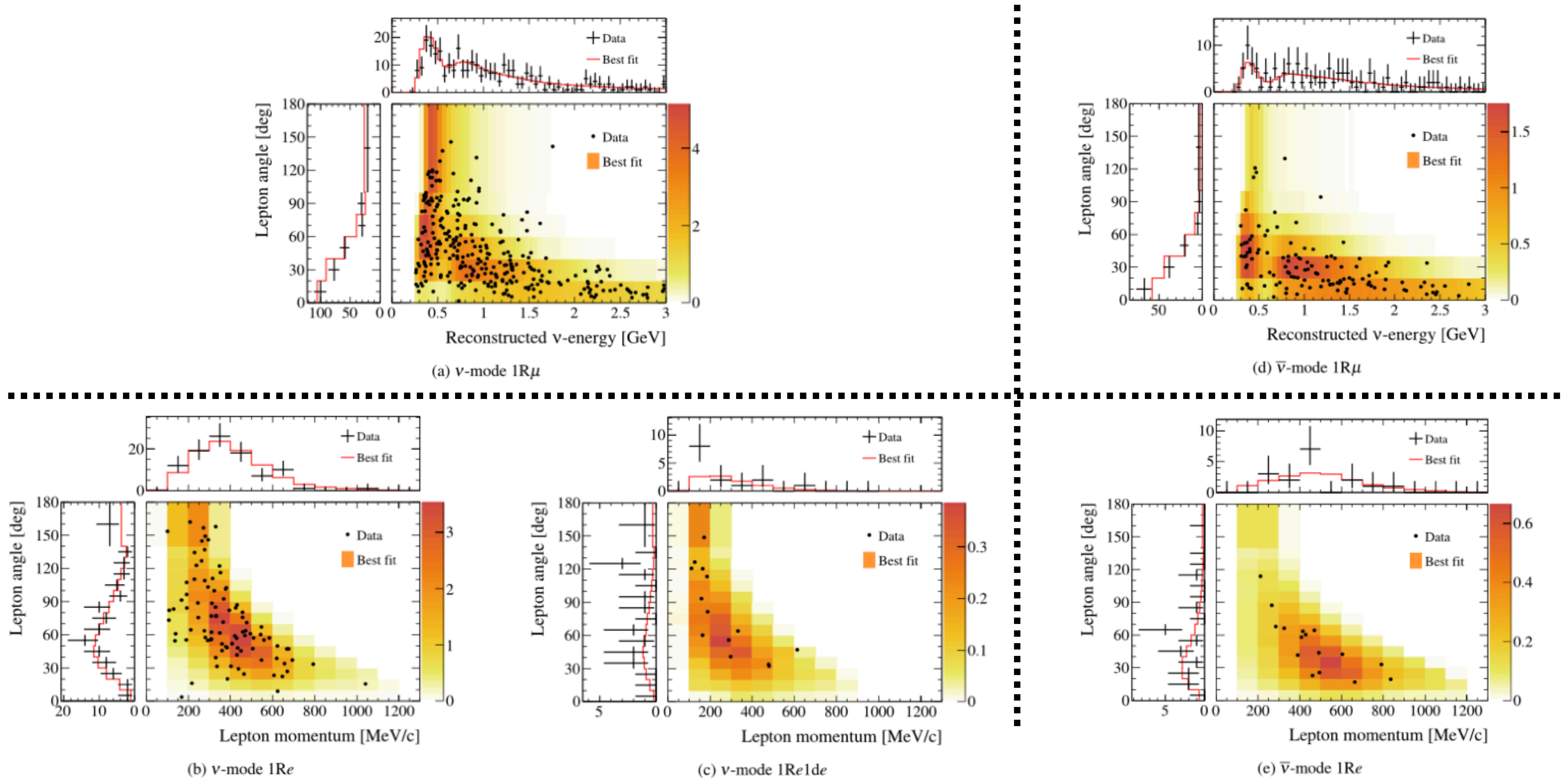
T2K Run 1-10 (not the latest analysis)
[Eur.Phys.J.C 83 (2023) 9, 782]

- Neutrino mode: 19.7×10^{20} POT
- Antineutrino mode: 16.3×10^{20} POT
- Mean neutrino energy ~ 0.6 GeV

ν-mode **$\bar{\nu}$-mode**

μ-like

e-like



CCQE-like **CC1π-like**

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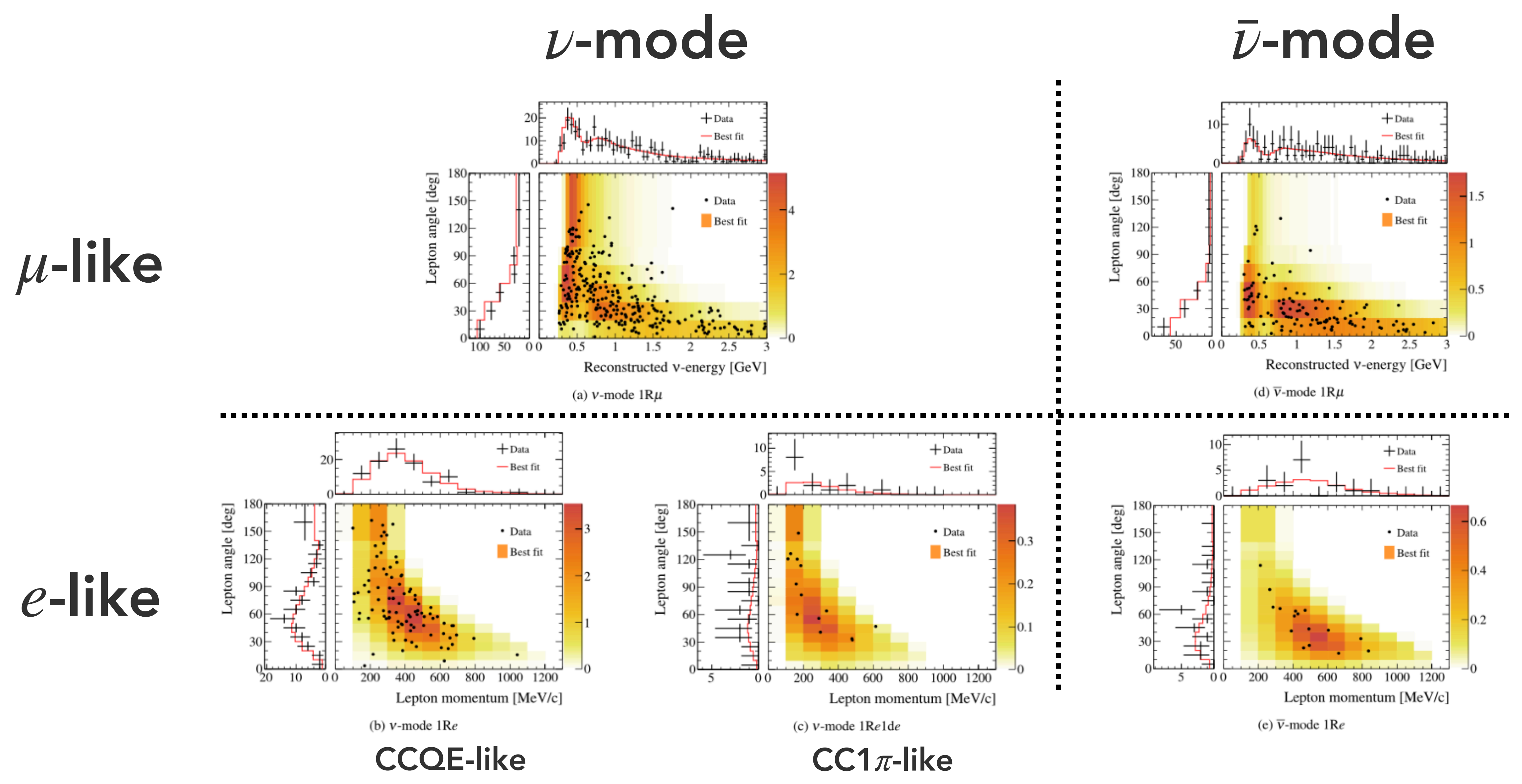
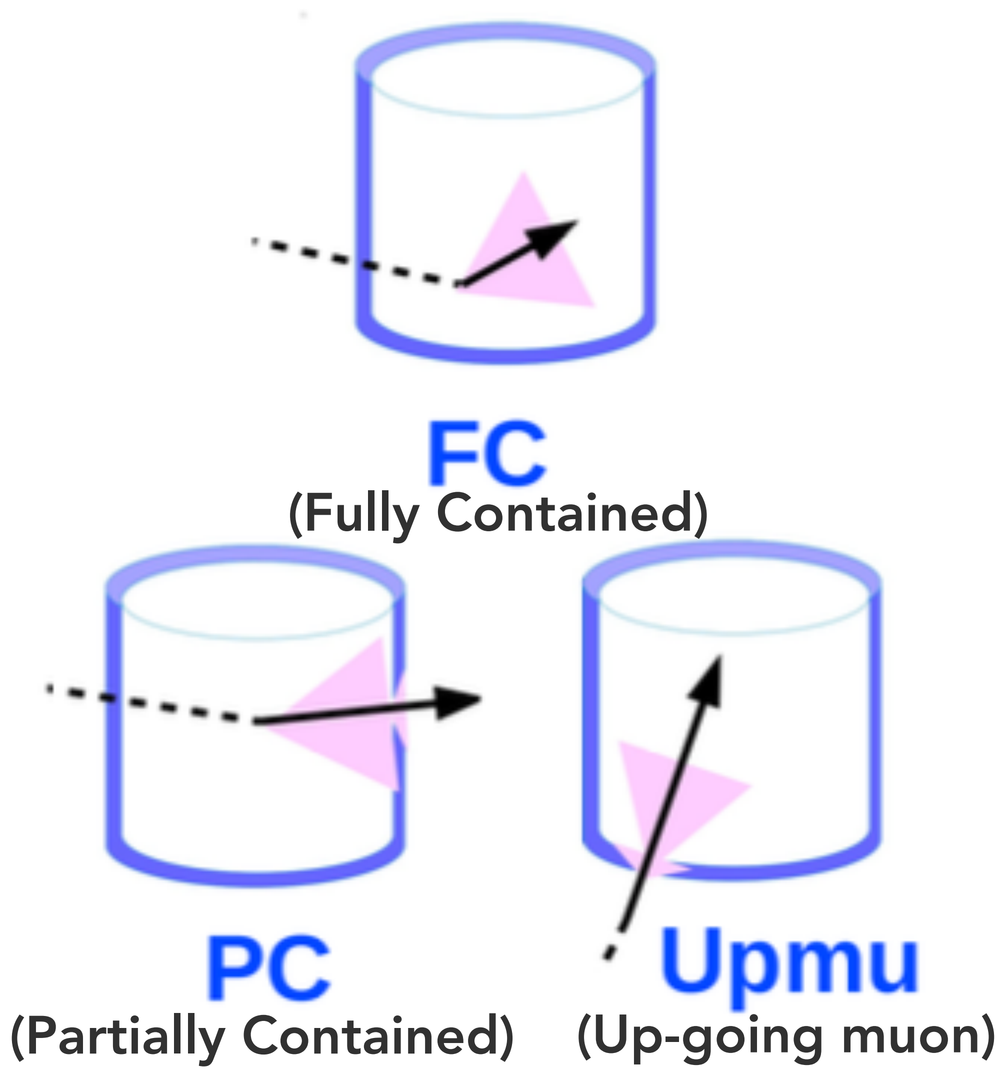
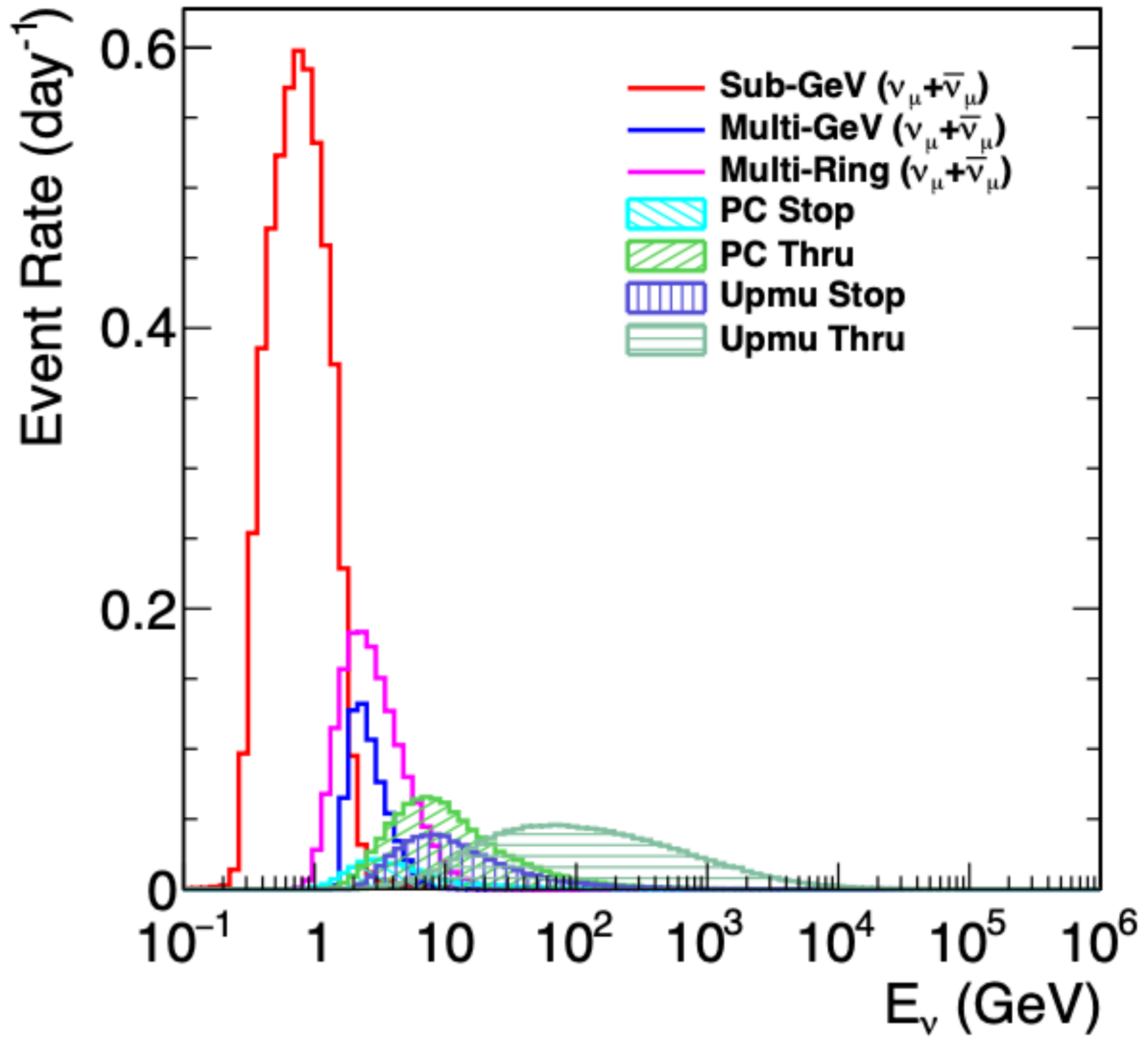


SK IV atmospheric neutrinos
[PTEP 2019 (2019) 5, 053F01]

- 3244.4 days of data taking
- 18 samples depending on the event topologies and neutrino energies
- **Wider energy ranges** than T2K

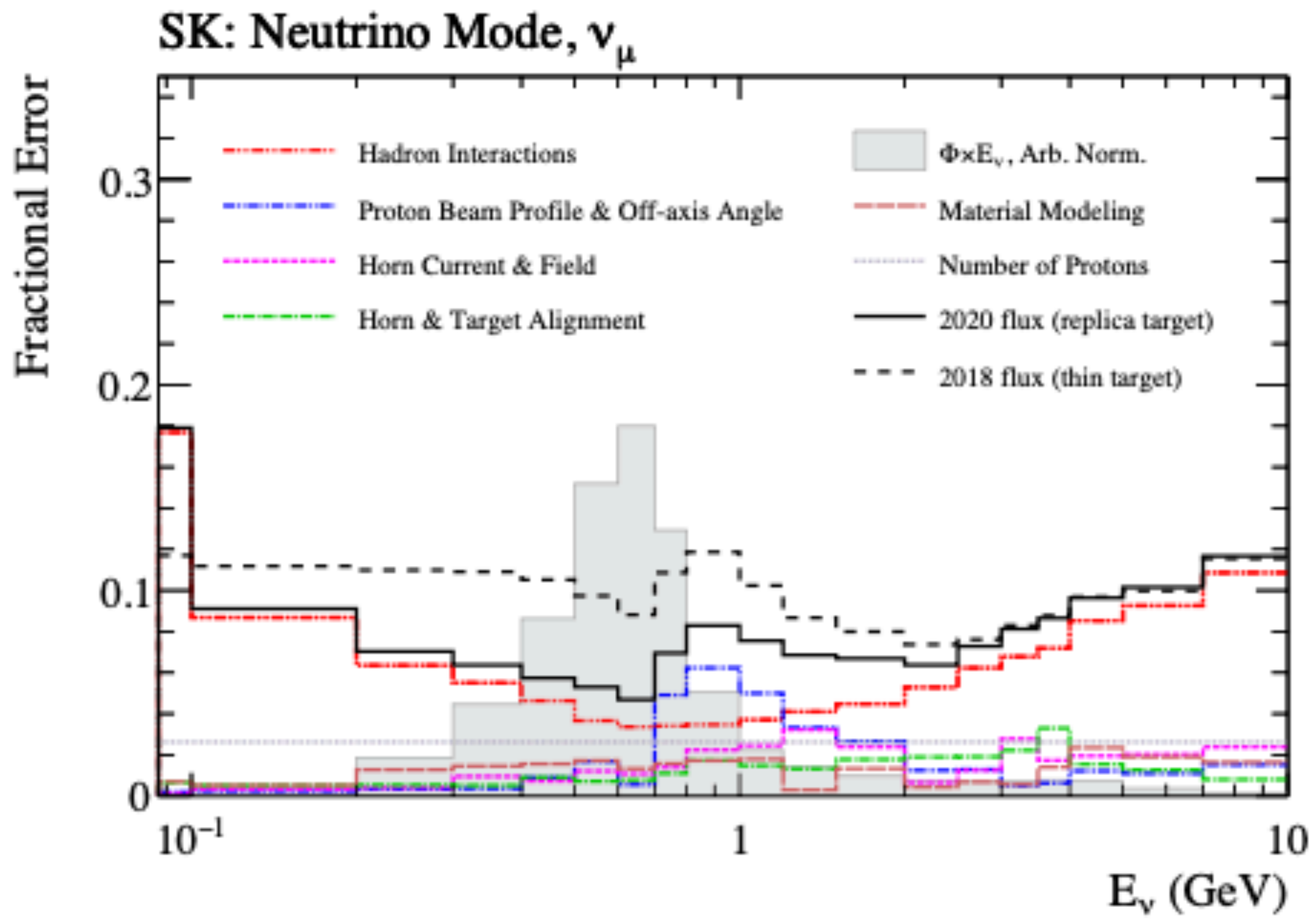




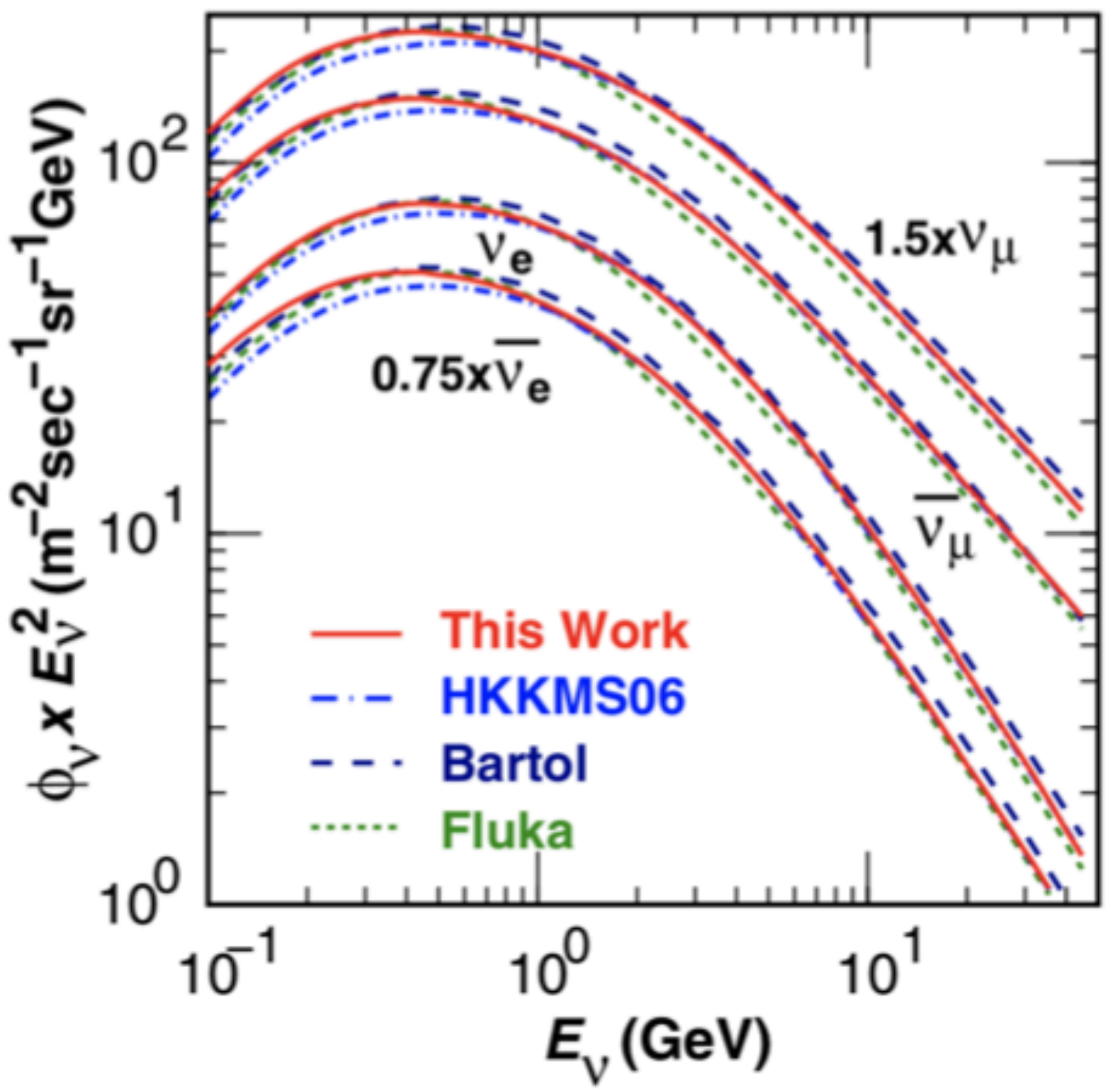
Flux

- Flux systematics are taken from each experiment and treated as **uncorrelated between T2K and SK**.
  - **T2K**: FLUKA2011 and GEANT3 MC simulation tuned to the NA61/SHINE experiment at CERN which uses the T2K replica target [Eur.Phys.J.C 76 (2016) 11, 617].
  - **SK**: Honda flux calculation [Phys.Rev.D 83 (2011) 123001]
    - Correlation of the flux models is one of the possible updates in future analysis (work ongoing at SK to implement tuning to the same or similar external hadron production measurements)

T2K Run 1-10



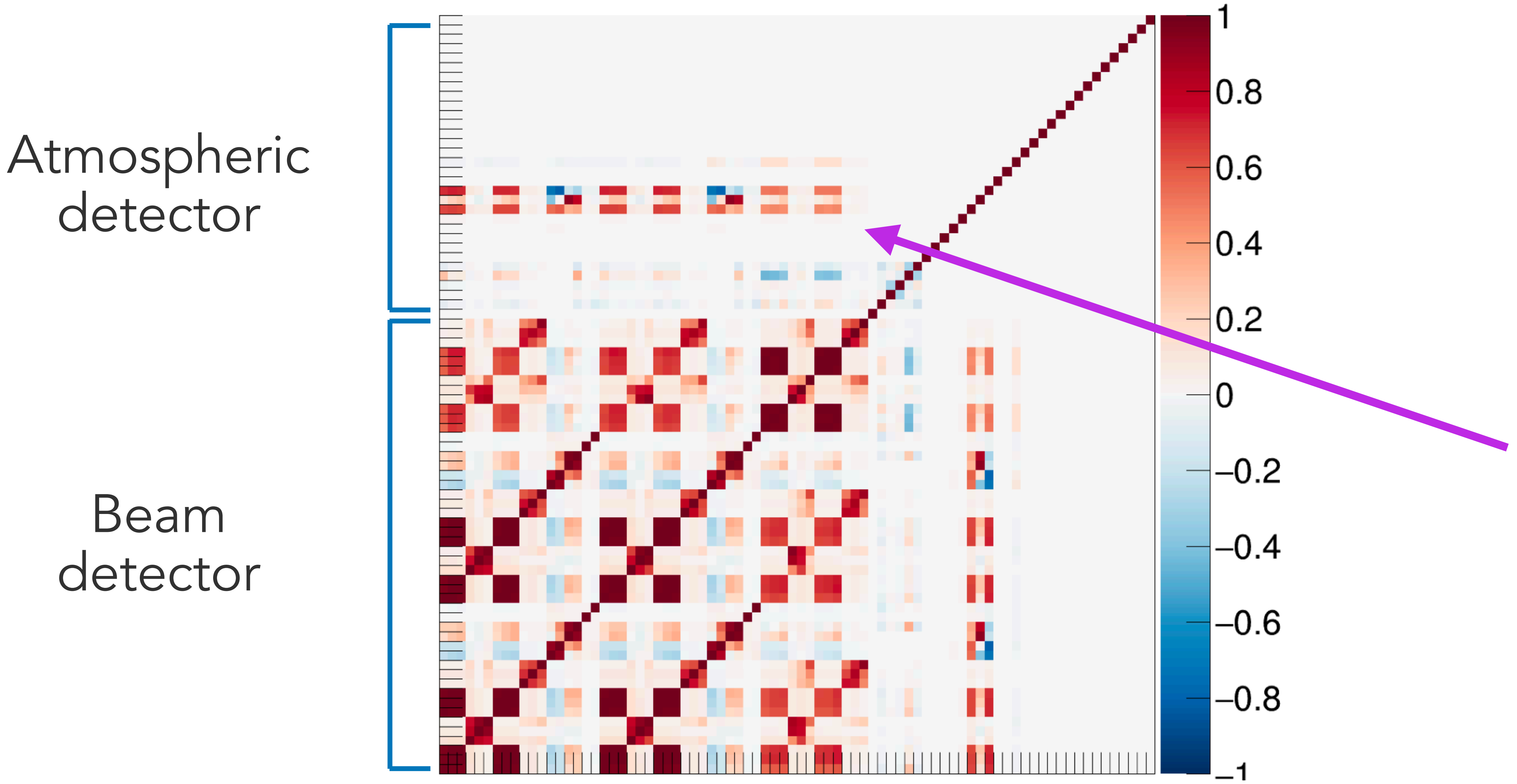
SK IV Atmospheric



Detector

- The same detector/simulation software/reconstruction tool is used for the samples of both experiments.
  - **Correlations between T2K and SK detector systematics are taken into account** by reevaluating the T2K detector systematics using the atmospheric MC (reweighted to the T2K flux and the T2K selection is applied).

Correlated detector systematics



Among the atmospheric parameters,
- Single-ring PID
- Fiducial volume
- Ring separation

have strong correlations with the beam part

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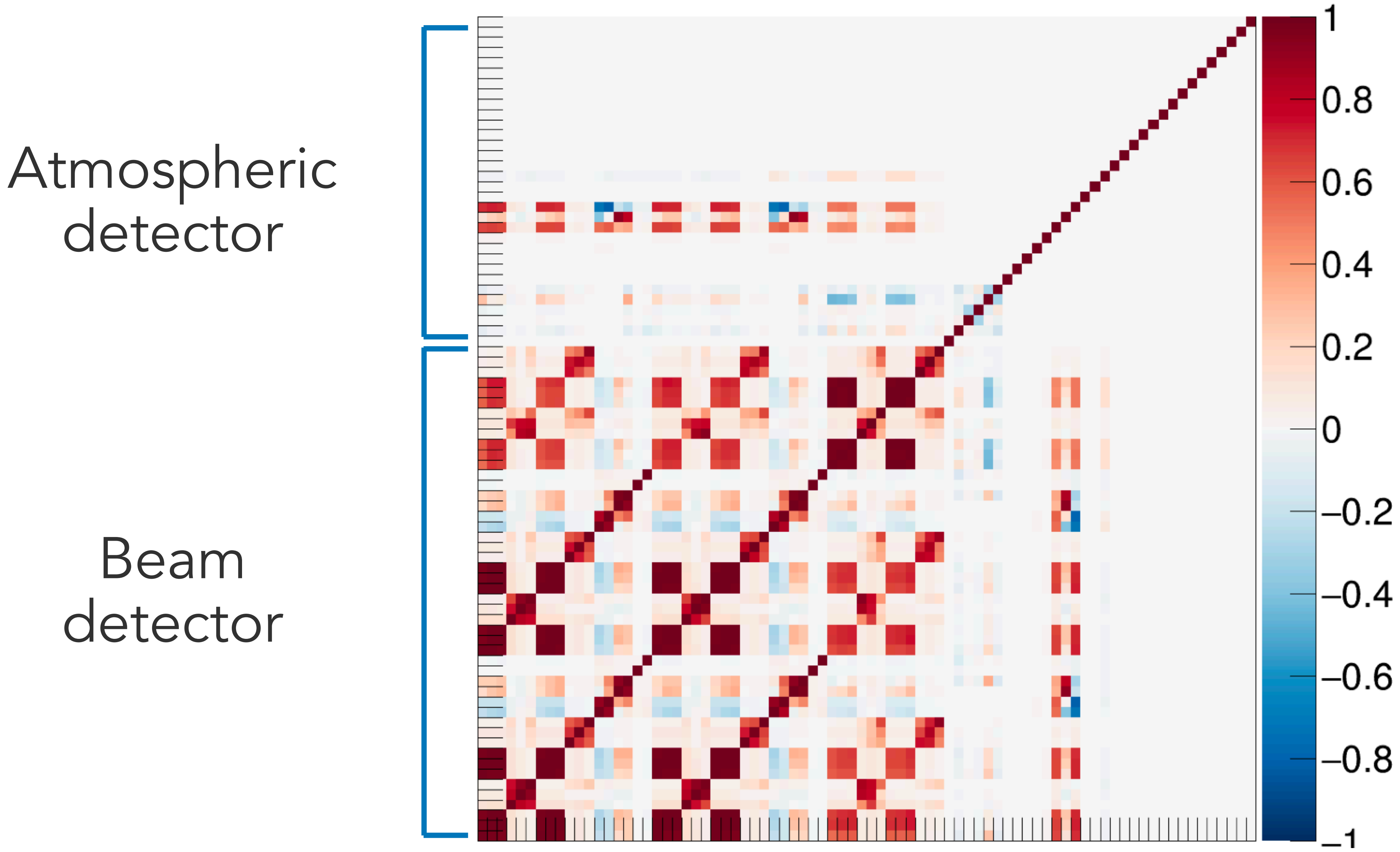
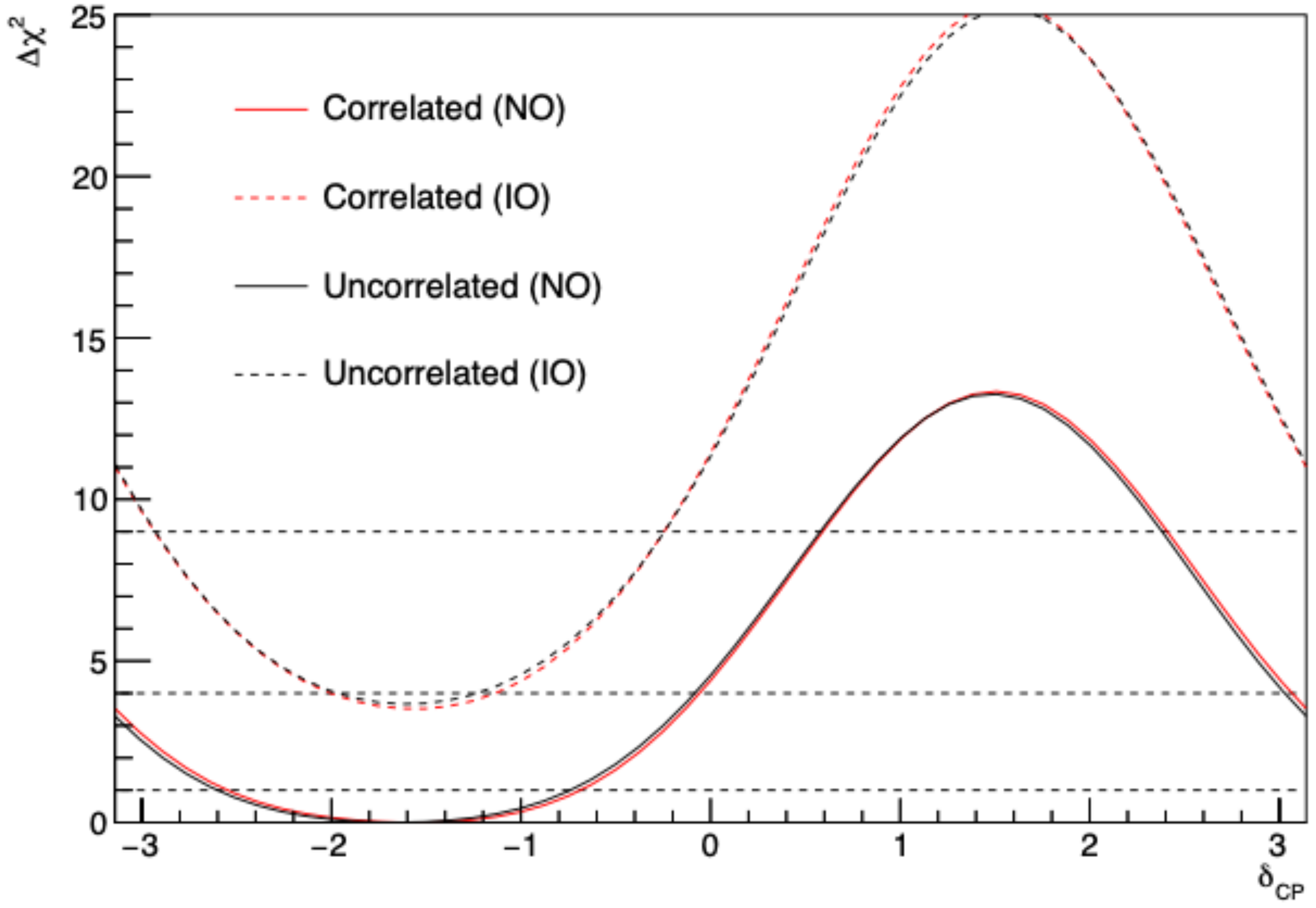
  - **Correlations between T2K and SK detector systematics are taken into account** by reevaluating the T2K detector systematics using the atmospheric MC (reweighted to the T2K flux and the T2K selection is applied).
  - Including the correlations makes the analysis more robust but has a **limited impact at current statistics**.

Correlated detector systematics

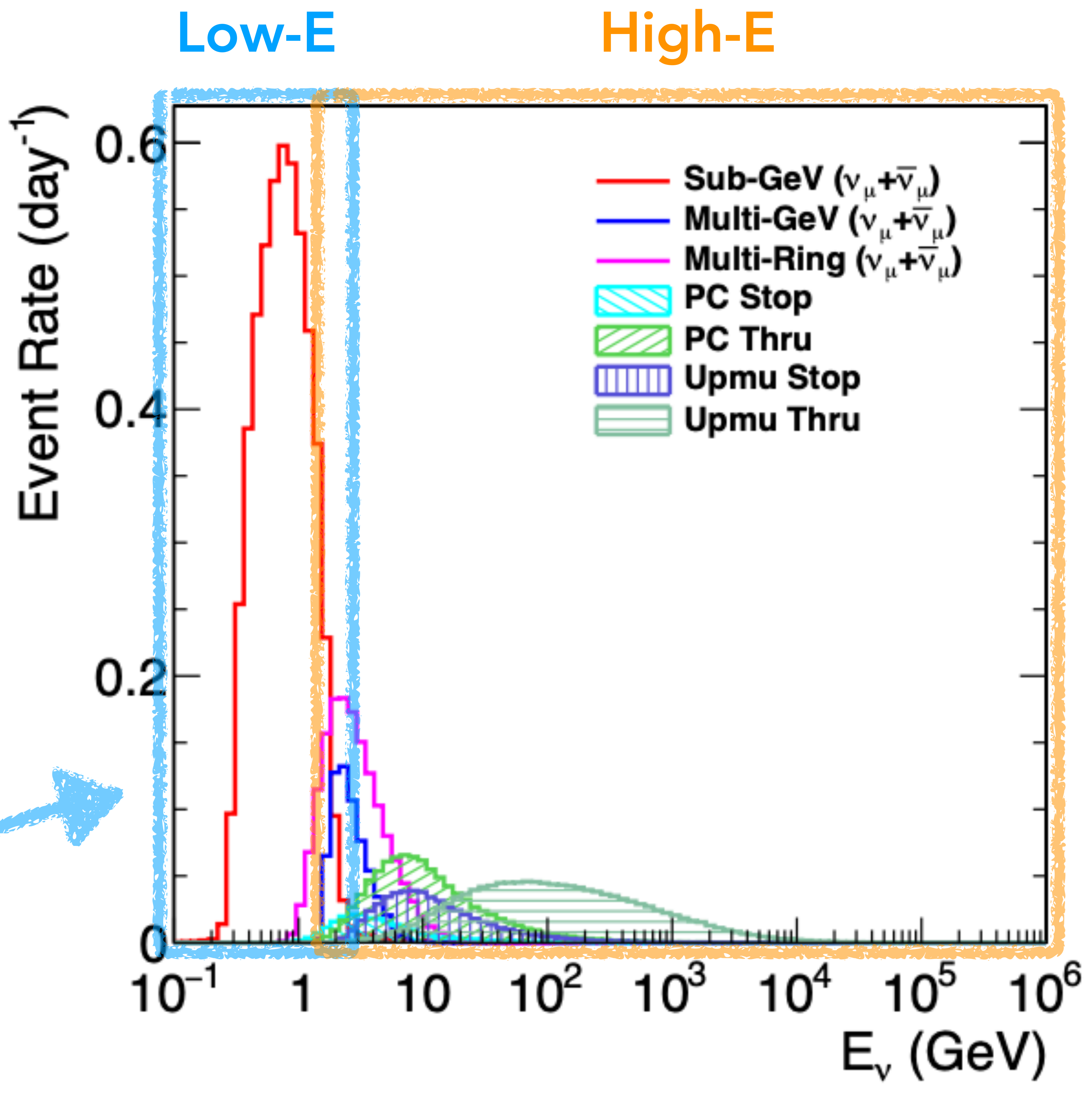


Effect of the correlation on the sensitivity



Cross-Section

- SK atmospheric covers a wider range of energies than T2K.
  - Use different models for low-energy and high-energy samples.



Use a common cross-section model with T2K

	Low-energy sub-GeV atm + beam	**High-energy** multi-GeV atm
CCQE	T2K model with ND280 constraint, correlated in low-E/highE (except for high-Q^2)	
	high-Q^2 params w/ND280	high-Q^2 params w/o ND
	add ν_e/ν_μ ratio unc. (CRPA)	
2p2h	T2K model w/ND280	SK model (100% error) + T2K-style shape
Resonant	T2K model w/ND280 + new pion momentum dial + NC1π0 uncertainties	SK model for 3 dials common with T2K, use more recent larger T2K priors
DIS	T2K model w/ND280	SK model
ν_τ	SK model (25% norm on top of other syst) for other systematics checked that we have no numerically unstable values	
FSI	T2K model w/ND280	T2K model w/o ND280 should be mostly same as SK model
SI	T2K model, correlated in low-E/high-E only applied to FC and PC for atm, PN not applied to atm	

Cross-Section

- SK atmospheric covers a wider range of energies than T2K.
  - Use different models for low-energy and high-energy samples.

- **Low energy** (beam and atmospheric Sub-GeV samples)
  - Use the T2K model [ref] as the base which is **constrained by the T2K near detector**.
  - Some extra parameters are added to cover important uncertainties for the atmospheric analysis.

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CCQE	T2K model with ND280 constraint, correlated in low-E/highE (except for high-Q^2)	
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Cross-Section

- SK atmospheric covers a wider range of energies than T2K.
  - Use different models for low-energy and high-energy samples.

- **Low energy** (beam and atmospheric Sub-GeV samples)
  - Use the T2K model [ref] as the base which is **constrained by the T2K near detector**.
  - Some extra parameters are added to cover important uncertainties for the atmospheric analysis.

- **High energy** (rest of atmospheric samples)
  - Use a modified SK model [ref] including additional systematics uncertainties.

	Low-energy sub-GeV atm + beam	**High-energy** multi-GeV atm
CCQE	T2K model with ND280 constraint, correlated in low-E/highE (except for high-Q^2)	
	high-Q^2 params w/ND280	high-Q^2 params w/o ND
	add ν_e/ν_μ ratio unc. (CRPA)	
2p2h	T2K model w/ND280	SK model (100% error) + T2K-style shape
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Analysis Method

- **Frequentist analysis** (not shown in this talk)

- **Bayesian analysis** (shown in this talk)

Analysis Method

- **Frequentist analysis** (not shown in this talk)
  - For the construction of frequentist CL, the fixed-$\Delta\chi^2$ method ($\Delta\chi^2 = 1,4,\ldots$ for $1\sigma,2\sigma,\ldots$) does not guarantee the correct coverage in the neutrino oscillation analysis.
    - Conditions for Wilks' theorem are not met (e.g. small statistics, parameters with boundary (e.g. $\sin\delta_{\mathrm{CP}}$), degeneracy, etc).
  - Feldman-Cousins (FC) method is frequently used in which we need to fit many toys but this is computationally prohibitive for the joint analysis due to the longer time to fit toys.
  - Ongoing work to perform FC and CLs in a reasonable timescale for the joint analysis.

- **Bayesian analysis** (shown in this talk)

Analysis Method

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  - Ongoing work to perform FC and CLs in a reasonable timescale for the joint analysis.

- **Bayesian analysis** (shown in this talk)
  - Bayesian credible intervals can be constructed from the posterior probability distributions without fitting many toys.
    - The conclusion may depend on the prior choice.

posterior　likelihood　prior

$$p(\theta|x) = \frac{\mathscr{L}(\theta|x)\pi(\theta)}{p(x)}$$

Analysis Method

- We check the validity of the results by performing multiple analyses.
  - Four analysis methods have been prepared for this joint analysis.
  - All of them use **the same analysis model** but use **different implementations and fitting methods**.

Analysis Method

- We check the validity of the results by performing multiple analyses.
  - Four analysis methods have been prepared for this joint analysis.
  - All of them use **the same analysis model** but use **different implementations and fitting methods**.
- In this talk, we will show the results of two analyses that can produce Bayesian results.

	Analysis 1	**Analysis 2**
Oscillation probability Systematic response	Binned	Event by event / Binned
T2K sample binning	$(E_{\mathrm{rec}}, \theta)$ for μ-like samples (p, θ) for e-like samples	(E_{rec}) for μ-like samples $(E_{\mathrm{rec}}, \theta)$ for e-like samples
T2K near detector constraint	Gaussian approximation (Sequential fit)	Full likelihood (Simultaneous fit)
Fast oscillation smearing	Semi-analytic averaging	Down-sampling finer to coarser grid
Earth density	Average density + deviations	Average density

Data Fit Result (Bayesian Analysis)

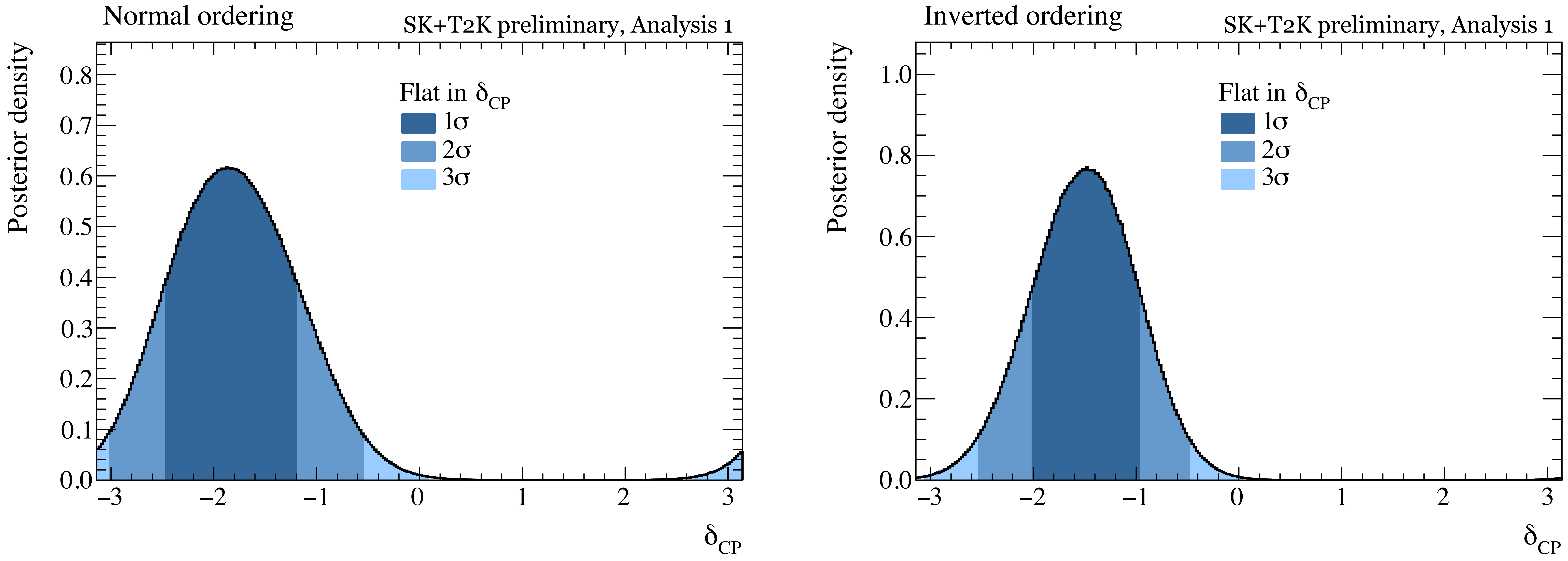
- Credible intervals
- Bayes factors for different hypothesis

In the following results, a Gaussian constraint of $\sin^2 2\theta_{13} = 0.0853 \pm 0.0027$ is applied from the reactor experiments.

Data Fit δ_{CP} Credible Intervals

- δ_{CP} credible intervals
  - CP conserving values ($\delta_{CP} = 0, \pi$) are **excluded at** 2σ when the flat prior in δ_{CP} is applied.

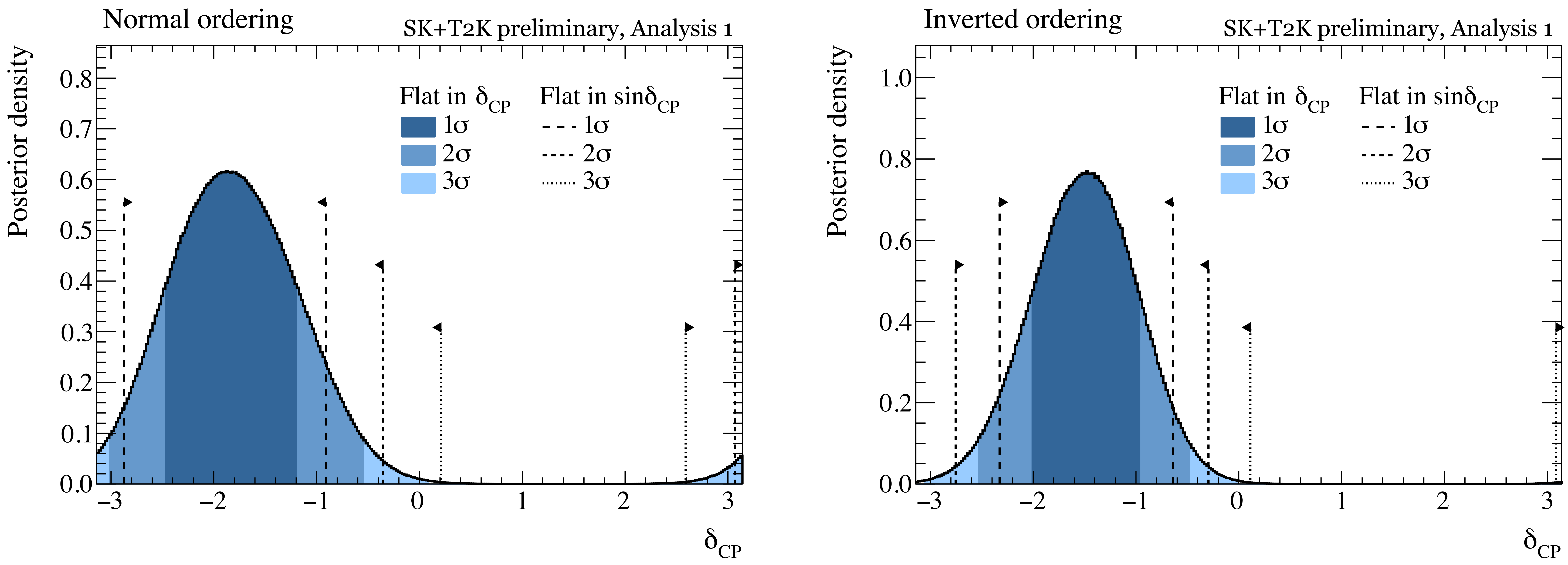
δ_{CP} posterior distributions with reactor constraint



Data Fit δ_{CP} Credible Intervals

- δ_{CP} credible intervals
  - CP conserving values ($\delta_{CP} = 0,\pi$) are **excluded at** 2σ when the flat prior in δ_{CP} is applied.
  - However, $\delta_{CP} = \pi$ **is not excluded in normal ordering** when the flat $\sin\delta_{CP}$ prior is applied. [see backup for these prior choices]

δ_{CP} posterior distributions with reactor constraint



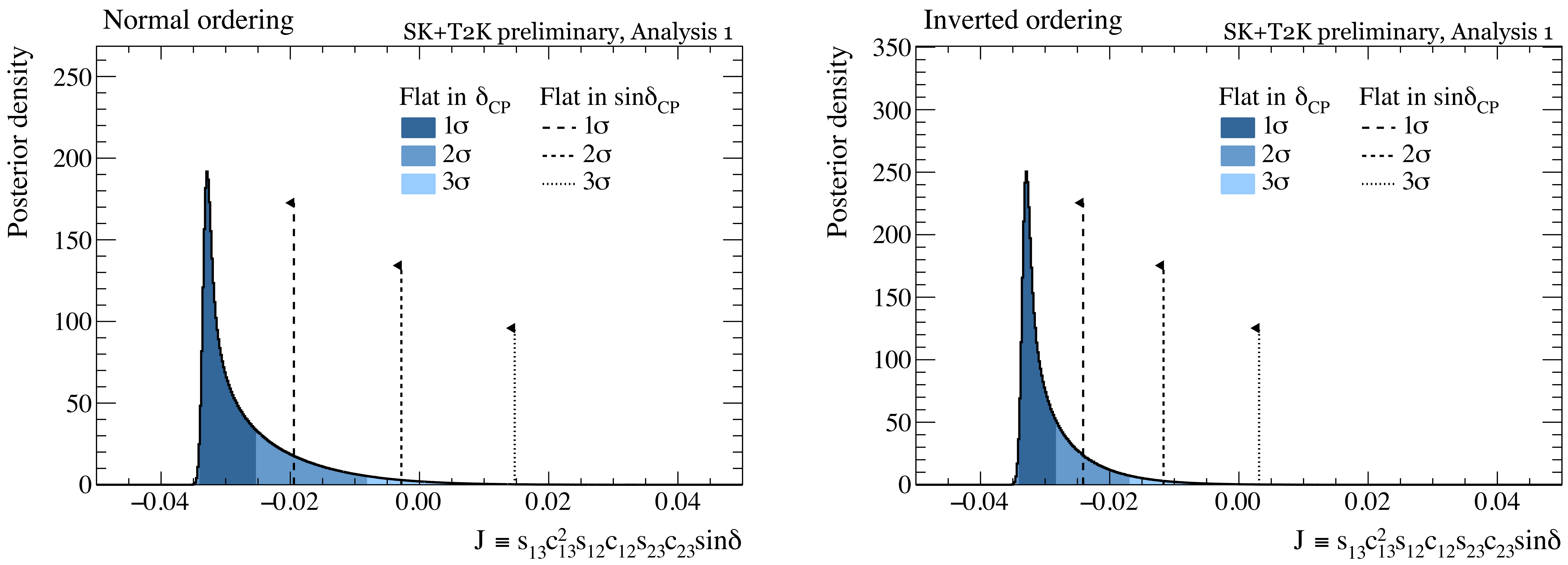
Data Fit Jarlskog Invariant Intervals

- Jarlskog invariant credible intervals

$$J_{CP} = s_{13}c_{13}^2 s_{12}c_{12}s_{23}c_{23}\sin\delta_{CP}$$

  - CP conserving value ($J_{CP} = 0$) is **excluded at 2σ** with both the flat δ_{CP} and flat $\sin\delta_{CP}$ priors.

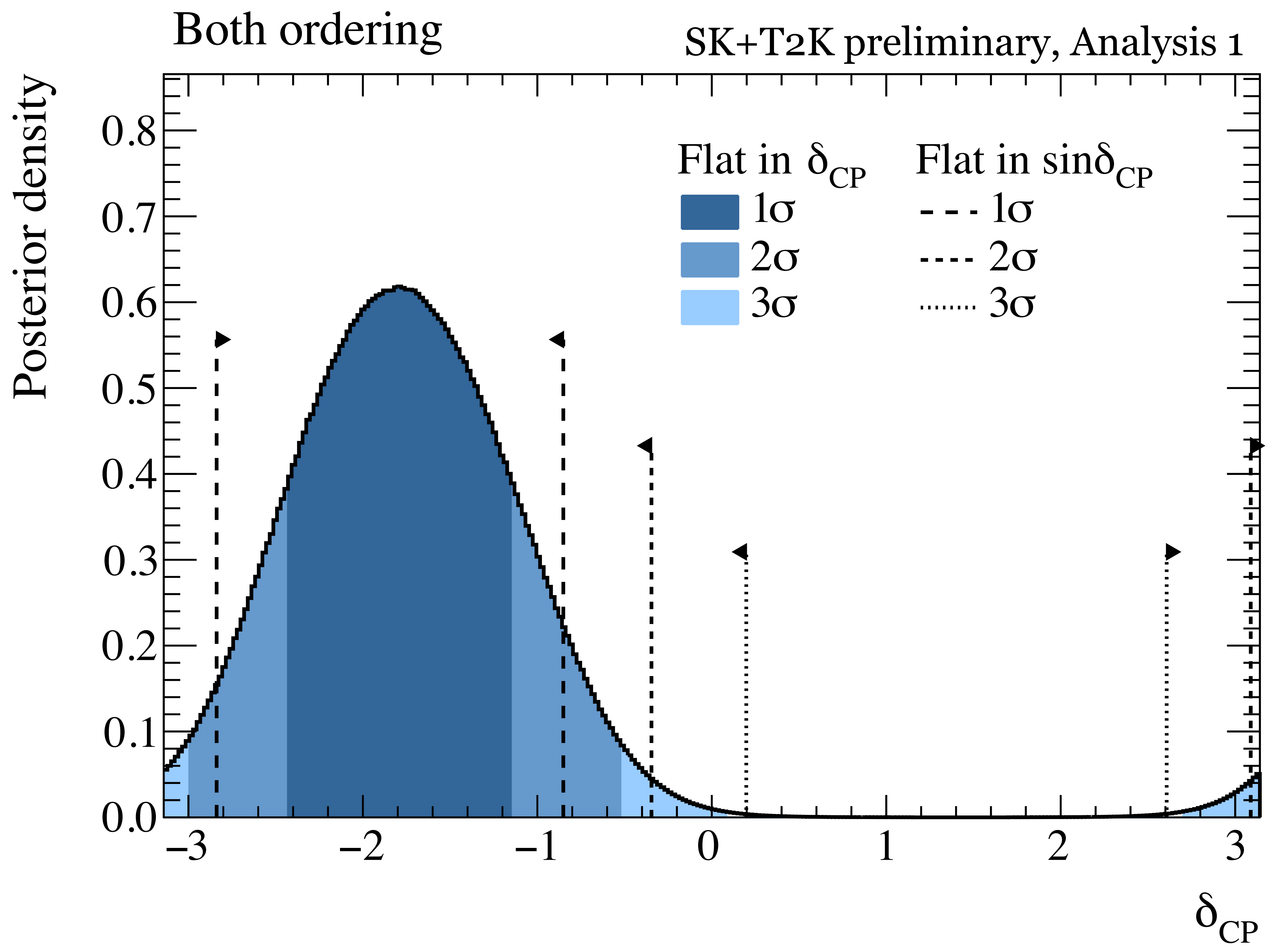
J_{CP} posterior distributions with reactor constraint

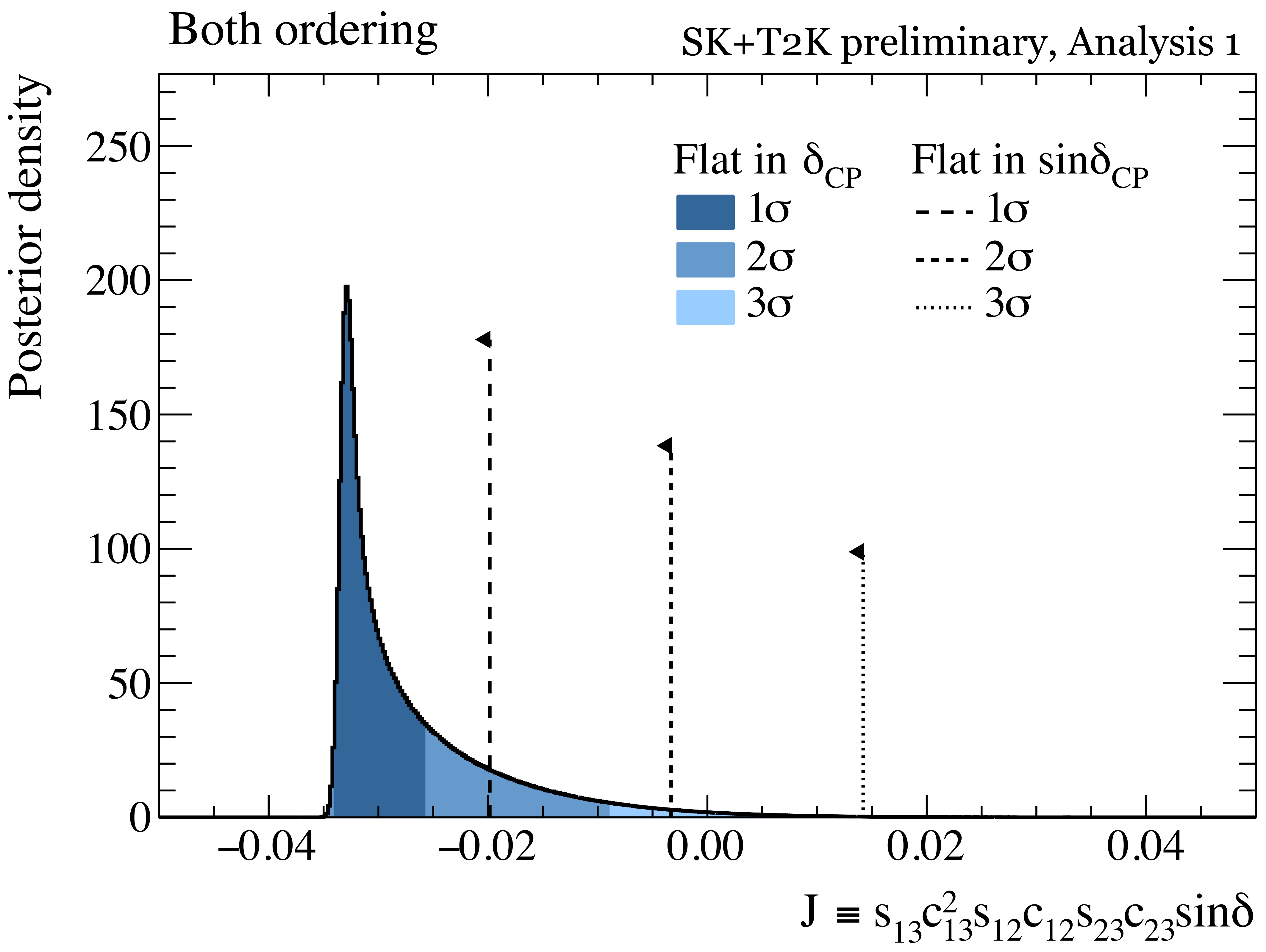


Data Fit δ_{CP} and Jarlskog (Both Ordering)

- δ_{CP} and Jarlskog invariant credible intervals marginalized over both mass ordering.
  - CP conserving values are excluded around 2σ.

δ_{CP} and J_{CP} **posterior distributions with reactor constraint marginalized over both mass ordering**





Data Fit Atmospheric Parameters

- Δm^2_{32} posterior distribution shows a clear preference for the normal ordering ($\Delta m^2_{32} > 0$).

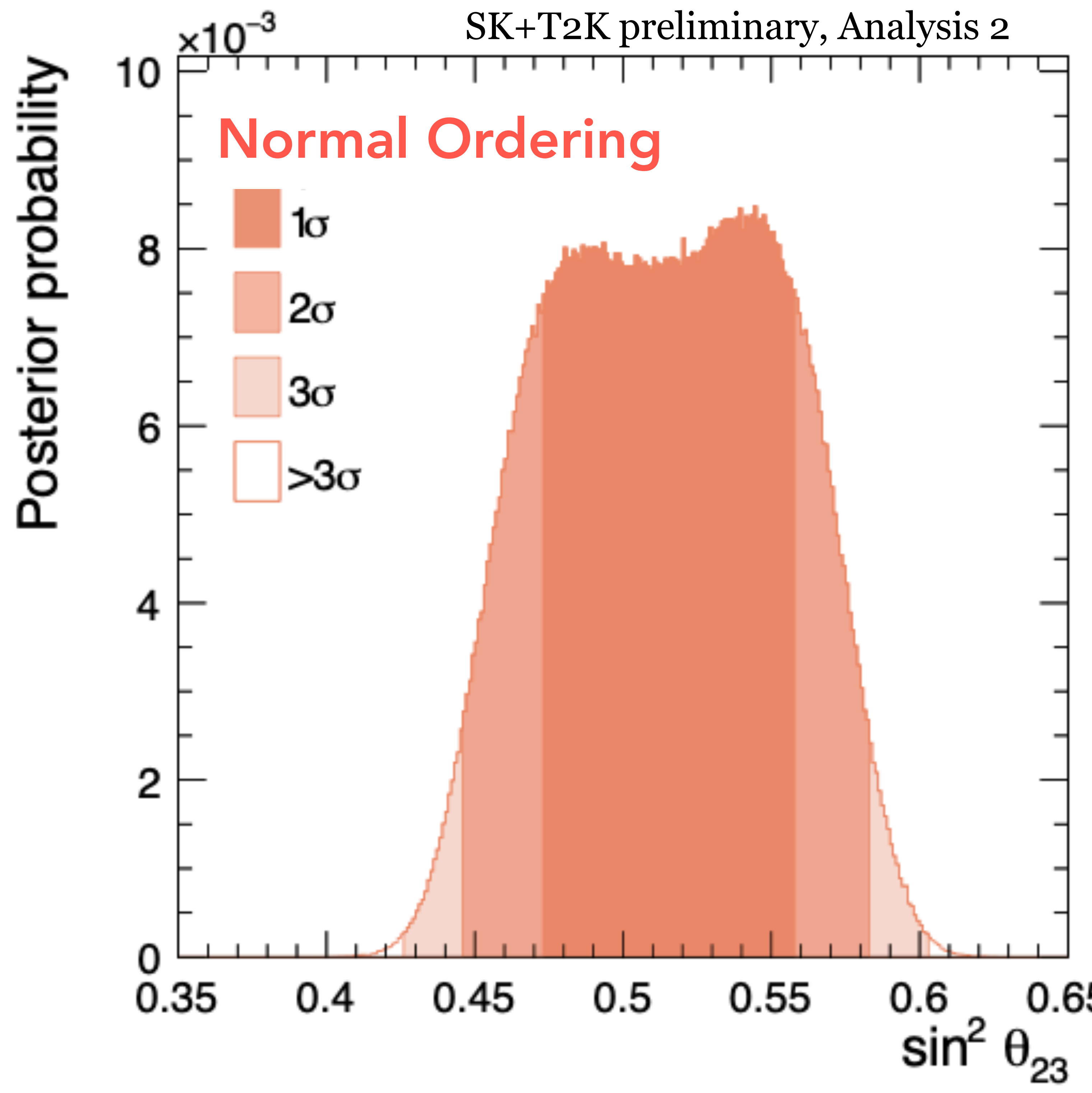
Gaussian smearing of $\sigma = 3.6 \times 10^{-5}\ \mathrm{eV}^2$ is applied to take into account the possible bias from the out-of-model effect [backup].

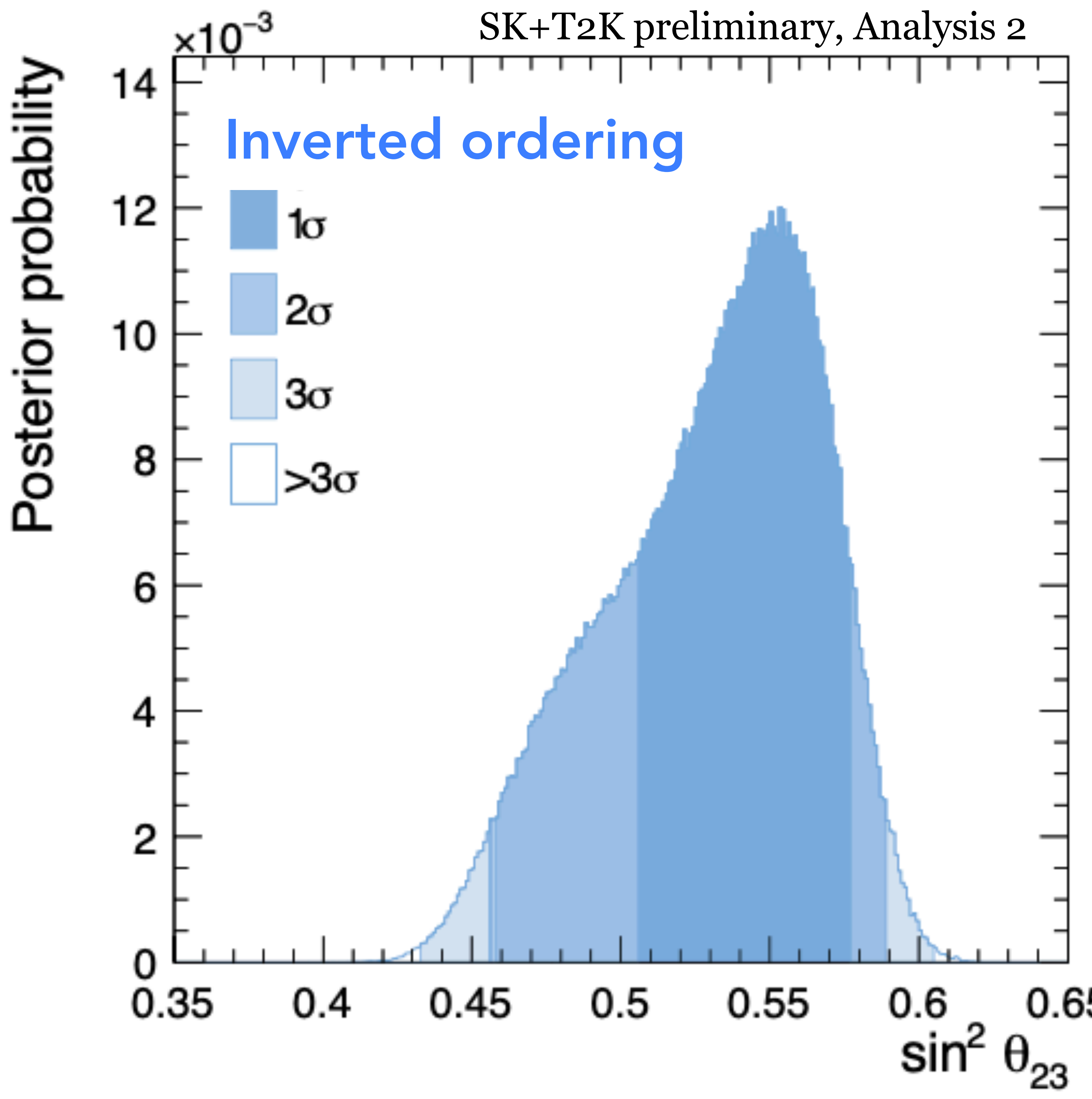
Δm^2_{32} posterior distributions with reactor constraint

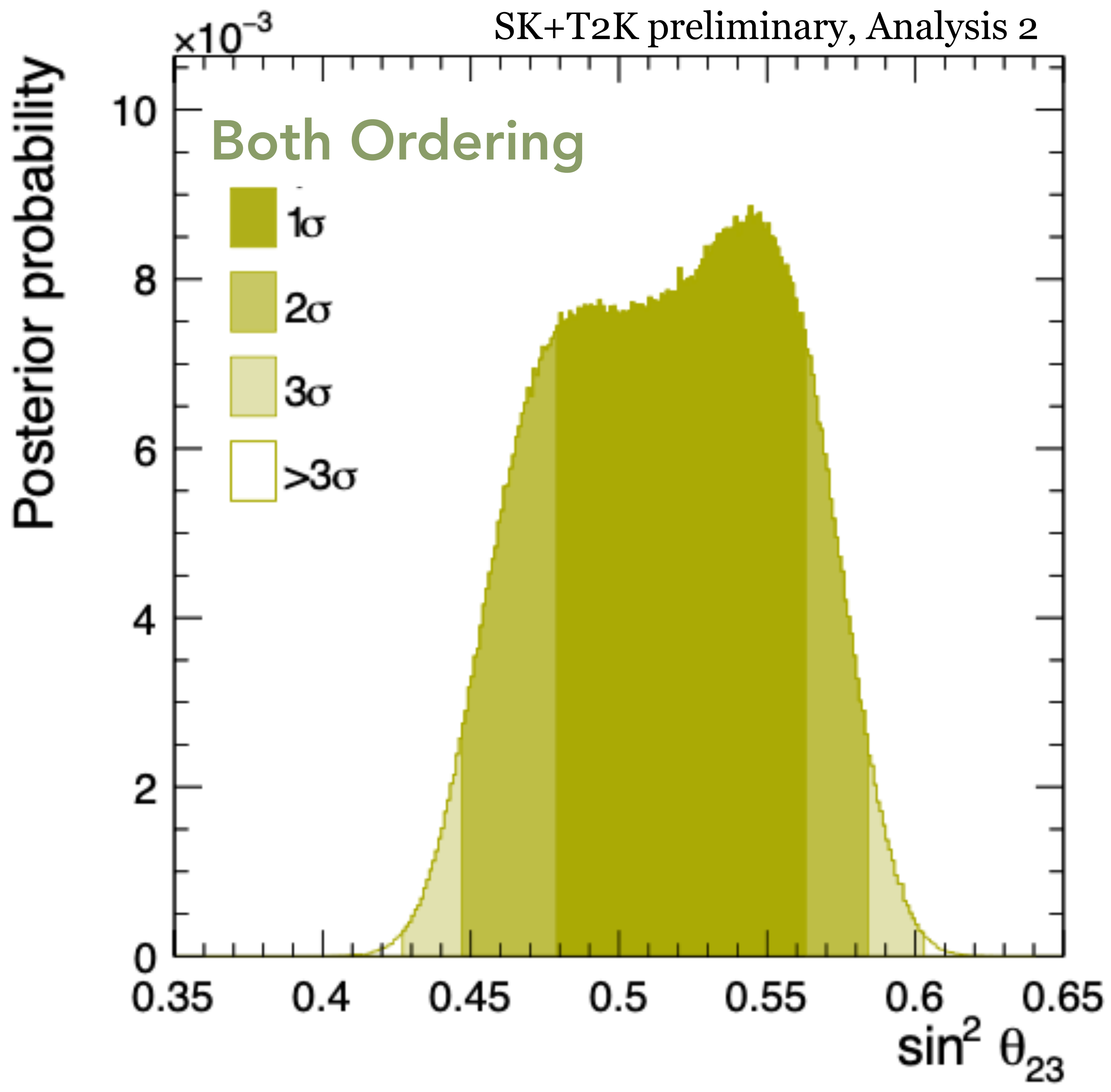
Data Fit Atmospheric Parameters

- $\sin^2\theta_{23}$ shows an **almost equal preference for both the upper and lower octant in normal ordering**, while it prefers the upper octant in the inverted ordering.

$\sin^2\theta_{23}$ **posterior distributions with reactor constraint**



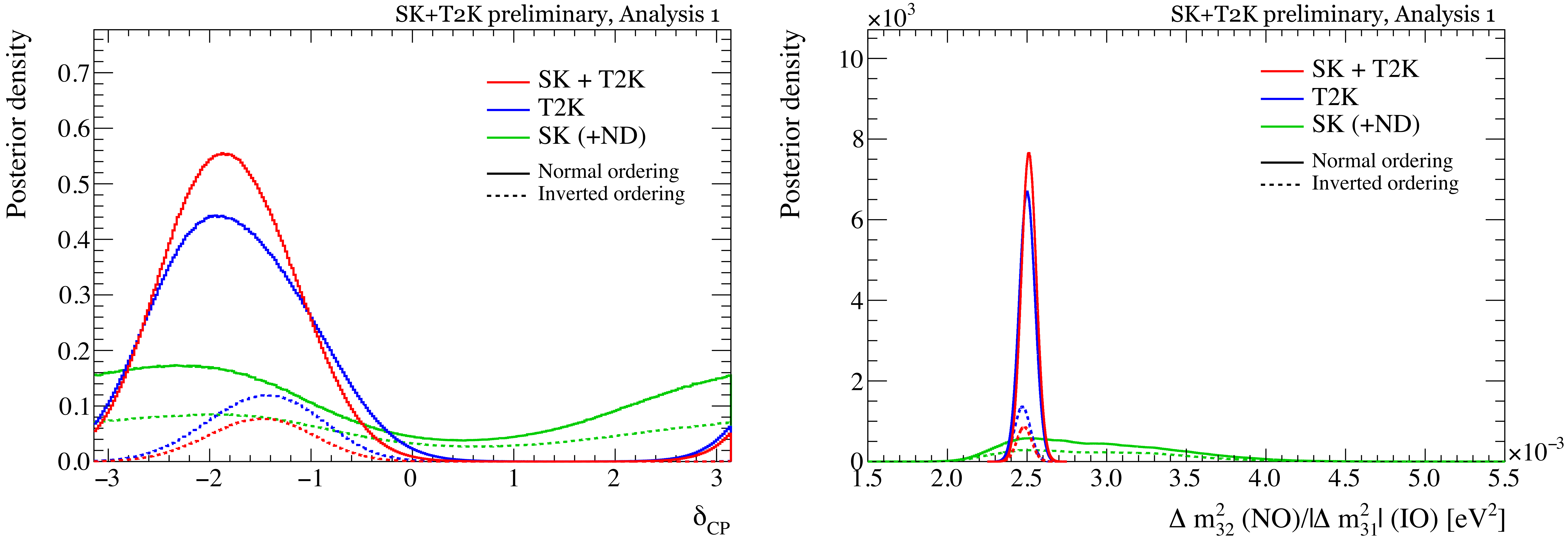




Data Fit SK+T2K/T2K/SK Comparison

- To understand the contributions of each sample, T2K-only and SK-only (with T2K near detector constraint) fits are also performed.
  - The δ_{CP} and Δm^2_{32} constraints are dominated by T2K, but SK also contributed.

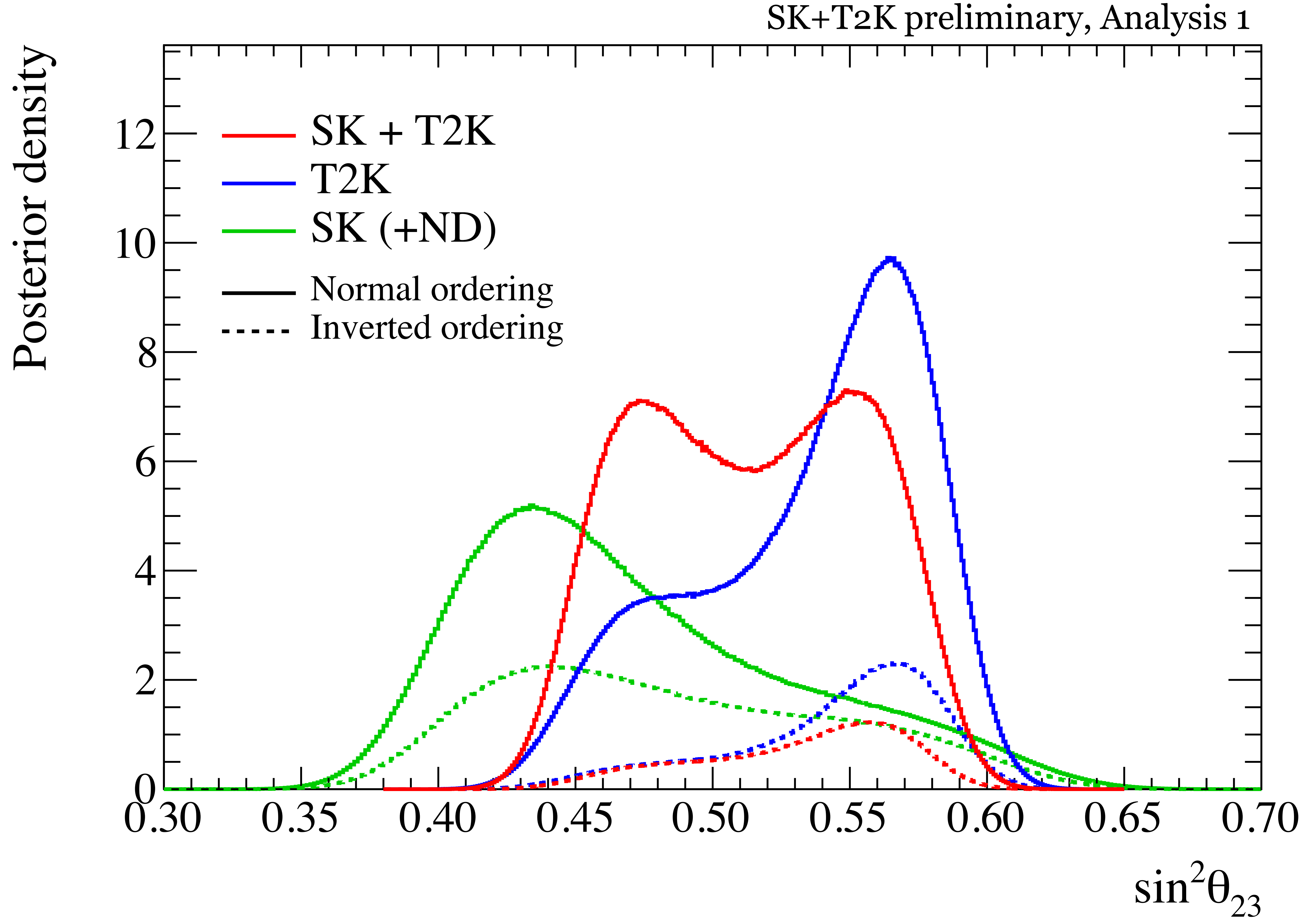
Comparison of the δ_{CP} and Δm^2_{32} posterior distribution for the fit with different sets of samples



Data Fit SK+T2K/T2K/SK Comparison

- There is some tension between SK and T2K for $\sin^2\theta_{23}$, and the joint fit therefore has a very similar likelihood in both lower and upper octant.

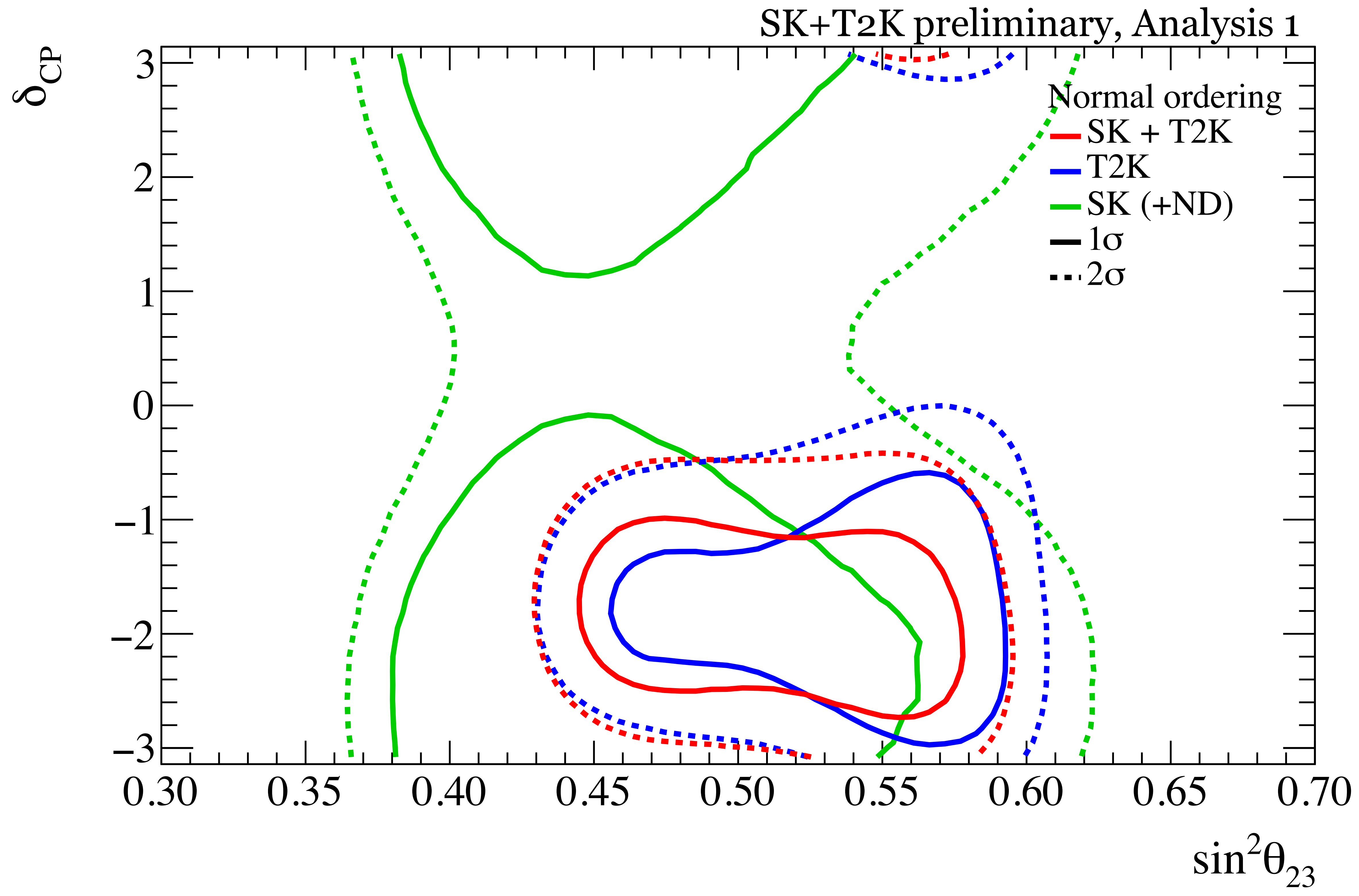
Comparison of the $\sin^2\theta_{23}$ posterior distribution for the fit with different sets of samples

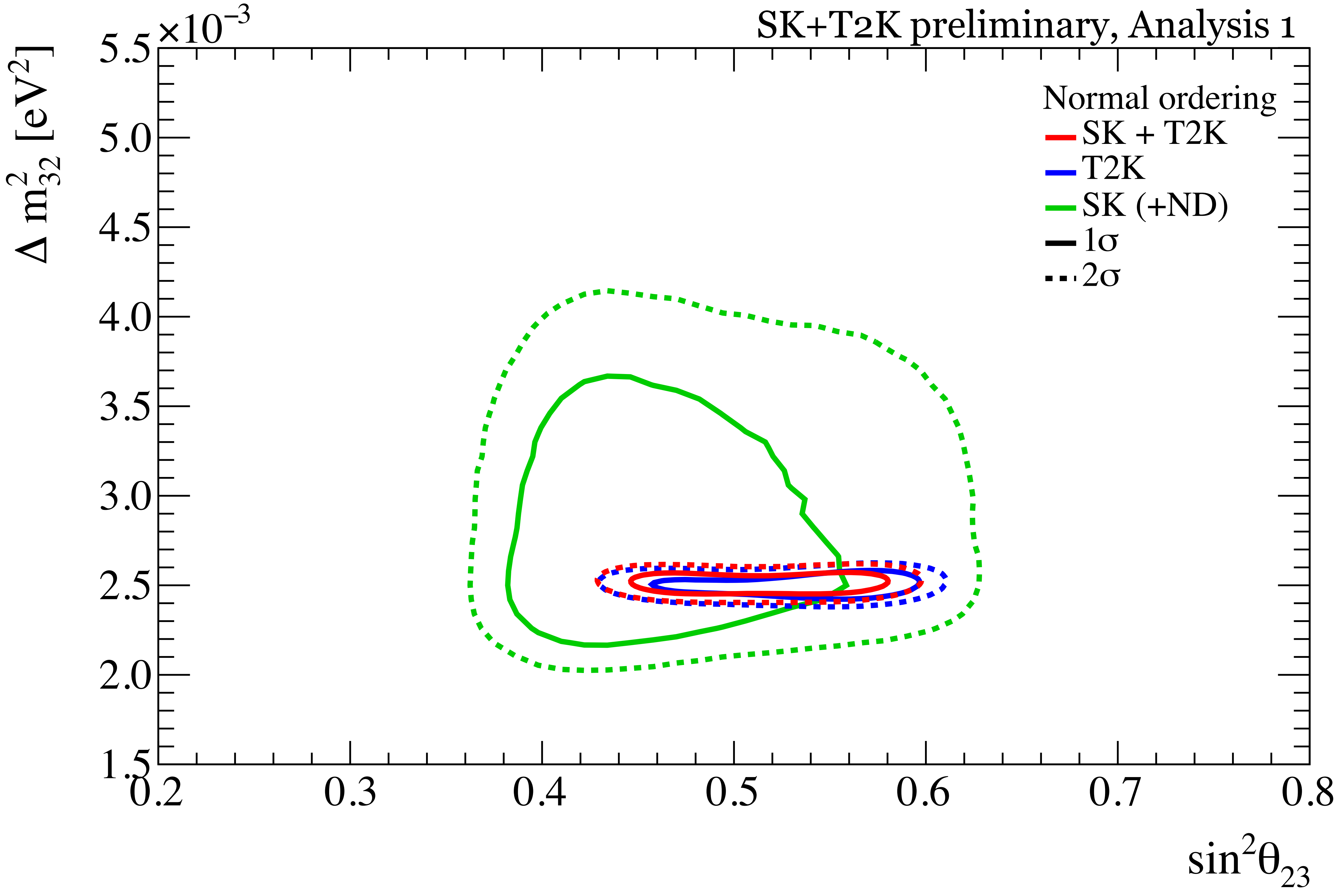


Data Fit SK+T2K/T2K/SK Comparison

- 2D posterior distributions for T2K-only and SK-only (with T2K near detector constraint) fits compared to the joint SK+T2K fit.
  - The constraints are largely dominated by T2K but SK also has a significant contribution on the octant.

Comparison of the 2D posterior distribution for the fit with different sets of samples





Data Fit Mass Ordering and Octant

- The mass ordering Bayes factor is ~9.0, suggesting a **weak preference for normal ordering**.

  - This corresponds to the probability of obtaining a $\sim 1.64\sigma$ deviation in a Gaussian assuming equal prior probabilities and is not **enough to claim the rejection of inverted ordering**.

Posterior probabilities of different combinations of the mass ordering and octant

	T2K+SK	
	Lower octant	Upper octant
Normal ordering	0.367	0.533
Inverted ordering	0.022	0.078
MO Bayes factor B(NO/IO)	$\mathbf{8.98 \pm 0.06}$	
Octant Bayes factor B(UO/LO)	**1.57**	

Data Fit Mass Ordering and Octant

- The mass ordering Bayes factor is ~9.0, suggesting a **weak preference for normal ordering**.
  - The **mass ordering Bayes factor is larger than the T2K-only or SK-only fits**.
  - While the octant Bayes factor becomes smaller.

Posterior probabilities of different combinations of the mass ordering and octant

SK+T2K preliminary, Analysis 1

	T2K+SK		T2K		SK (+ND)	
	Lower octant	Upper octant	Lower octant	Upper octant	Lower octant	Upper octant
Normal ordering	0.367	0.533	0.190	0.642	0.468	0.186
Inverted ordering	0.022	0.078	0.025	0.142	0.214	0.132
MO Bayes factor $B(\mathrm{NO/IO})$	$\mathbf{8.98 \pm 0.06}$		$\mathbf{4.96 \pm 0.02}$		$\mathbf{1.886 \pm 0.008}$	
Octant Bayes factor $B(\mathrm{UO/LO})$	**1.57**		**3.65**		**0.47**	

Validation of the Model Robustness

- We evaluate the **possible bias** in the oscillation parameter measurement due to the possible mis-modeling [see backup].
  - We tested 14 alternative models before the data fit and some of them showed non-negligible biases in Δm^2_{32}.
  - Therefore we decided to **smear the Δm^2_{32} contour in the data fit** by convolving the posterior distribution with a Gaussian ($\sigma = 3.6 \times 10^{-5}$ eV^2).

- We also tested whether it would change our conclusion on the significance of CP violation.
  - The size of the shift in the credible interval edges of δ_{CP} and Jarlskog invariant was checked.
  - None of them caused a shift of 2σ interval edges over the value of interest ($\delta_{CP} = 0, \pi$, $J_{CP} = 0$) for Analysis 1. The 2σ interval edge of the Jarlskog invariant in Analysis 2 could be affected by the possible bias.
  - **It does not change our conclusion on CP violation around 2σ.**

Main Conclusion on CP Symmetry

- The table below summarizes the conclusion on CP symmetry from different analyses.
  - Based on this, we concluded that **CP conservation ($\delta_{CP} = 0, \pi$, $J_{CP} = 0$) is excluded around** 2σ when the reactor constraint is applied.

SK+T2K preliminary

Analysis	Prior	CP conserving value	1σ	90%	2σ	3σ
Analysis 1	Flat in δ_{CP}	$\delta_{CP} = 0, \pi$	✓	✓	✓	×
		$J_{CP} = 0$	✓	✓	✓	×
	Flat in $\sin\delta_{CP}$	$\delta_{CP} = 0, \pi$	✓	✓	×	×
		$J_{CP} = 0$	✓	✓	✓	×
Analysis 2	Flat in δ_{CP}	$\delta_{CP} = 0, \pi$	✓	✓	✓	×
		$J_{CP} = 0$	✓	✓	✓	×
	Flat in $\sin\delta_{CP}$	$\delta_{CP} = 0, \pi$	✓	✓	×	×
		$J_{CP} = 0$	✓	✓	✓(×)	×

✓: excluded ×: not excluded ✓(×): excluded but may not be robust against the possible bias from an out-of-model effect

Summary

- The joint oscillation analysis of SK atmospheric + T2K accelerator neutrino has been performed.
  - Increased sensitivity was expected thanks to not only the increased statistics but also the **correlated systematics** and **broken degeneracies**.

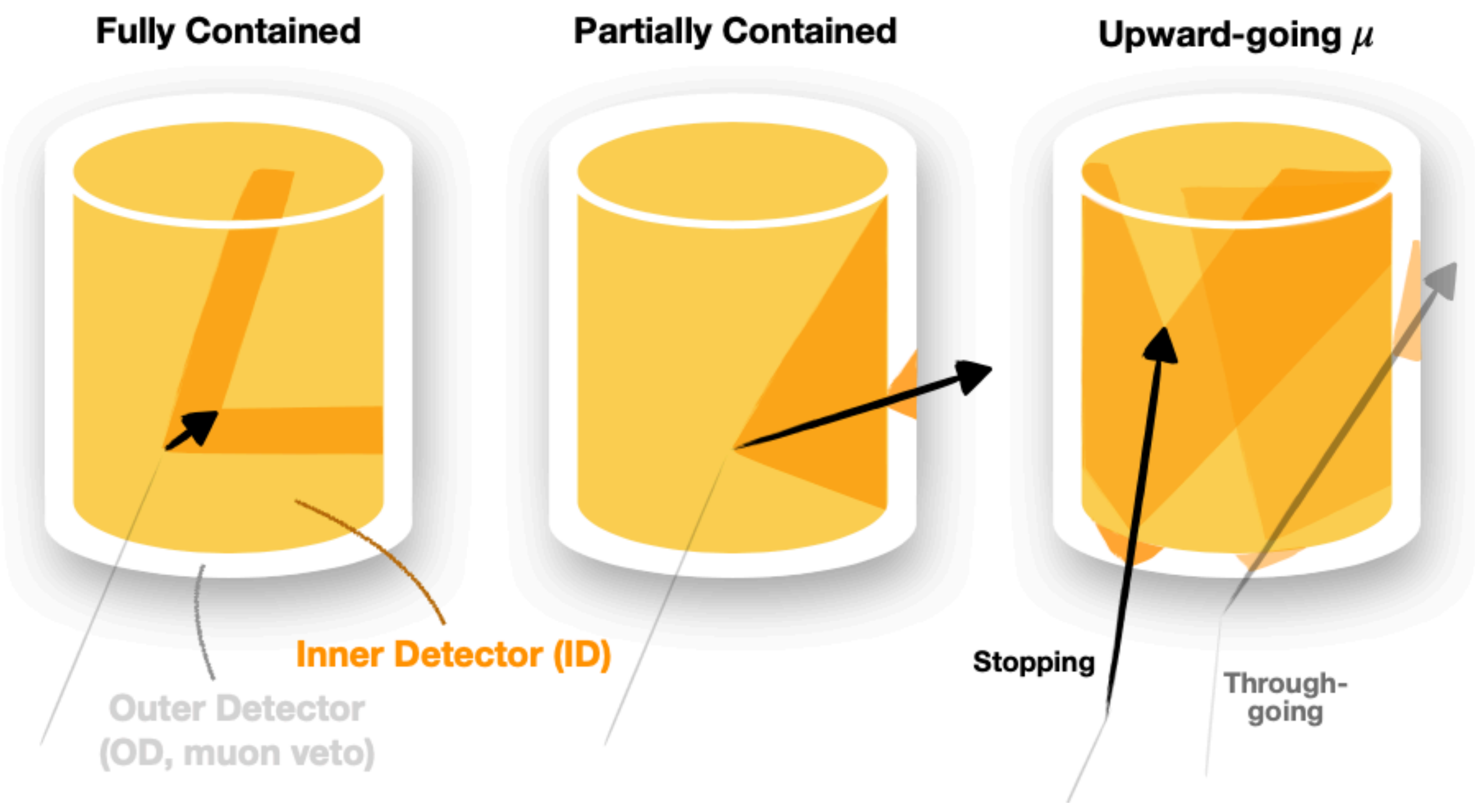
- So far we have performed Bayesian analysis using two analysis methods and obtained consistent results from both analyses.
  - **CP conservation ($\delta_{\mathrm{CP}} = 0, \pi$, $J_{\mathrm{CP}} = 0$) is excluded around** 2σ when the reactor constraint is applied.
  - **Slightly prefers the normal mass ordering** with the Bayes factor of ~9.0.
  - Almost equal preference to the upper and lower octant.
  - Posterior predictive p-values of 0.236 (shape) and 0.189 (rate) indicate a **good fit in joint analysis** which is comparable to the individual analysis by each experiment [see backup].

- The frequentist analysis will be reported in the near future. Stay tuned!

Backup

List of SK Atmospheric Samples

- 18 samples are used in this analysis [more details: PTEP 2019 (2019) 5, 053F01]

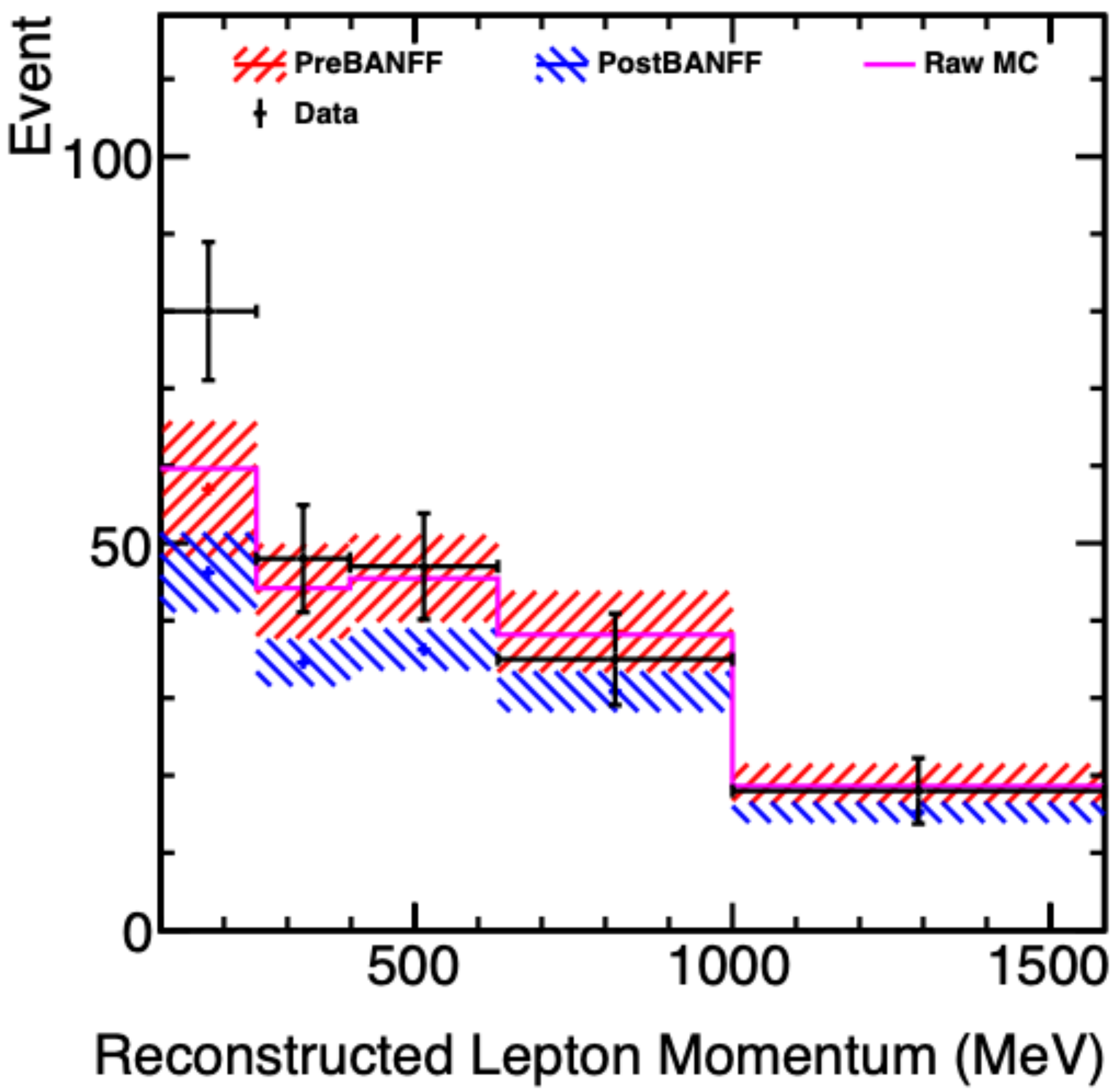


Sample Name	Category	Selection			
SubGeV elike 0dcy	Fully Contained (FC)	Sub-GeV	Single-ring	e-like	0 decay-e
SubGeV elike 1dcy					1 decay-e
SubGeV mulike 0dcy				μ-like	0 decay-e
SubGeV mulike 1dcy					1 decay-e
SubGeV mulike 2dcy					$\leq$ 2 decay-e
SubGeV pi0like			Multi-ring	Two e-like rings	
MultiGeV elike nue		Multi-GeV	Single-ring	e-like	$\leq$ 1 decay-e
MultiGeV elike nuebar					0 decay-e
MultiGeV mulike				μ-like	
MultiRing elike nue			Multi-ring	e-like	ν_e-like
MultiRing elike nuebar					$\bar{\nu}_e$-like
MultiRing mulike					other
MultiRingOther 1				μ-like	
PCStop	Partially Contained (PC)	No charge deposition in OD			
PCThru		Charge deposition in OD			
UpStop mu	Up-going Muon (UpMu)	Stopping			
UpThruNonShower mu		Through-going Non-showering			
UpThruShower mu		Through-going Showering			

Model Updates

- After reporting the sensitivity study in 2022 (C. Bronner@Neutrino 2022 and J. Xia@NOW2022), we have made various updates to perform the data fit.
  - In particular, there was a data/MC excess in the down-going CC1π atmospheric events (used for the validation as it is less affected by oscillation) and several countermeasures were taken.

Atmospheric sub-GeV e-like 1de sample down-going events with prediction



- The updates include the following:
  - Add a new Michel-electron and neutron separation
  - Update the definition of the Adler angle systematics
  - Add new systematics to take into account the possible worse PID at low momentum ($p < 0.25$ GeV)
  - Re-evaluate the detector systematics and include the correlation between atmospheric and beam

Posterior Predictive P-Values

- The posterior predictive p-values are computed for each set of SK+T2K, T2K-only, and SK-only fits.
  - The result shows a good agreement between the data and the model.
  - It was also consistent with those values produced by the official analysis of T2K and SK (the total p-value is 0.049 [ref]).

from Eur.Phys.J.C 83 (2023) 9, 782

Table 12 Breakdown of posterior predictive *p*-values by sample, quoted separately using a shape or rate based calculation, demonstrating good compatibility between the model and the data

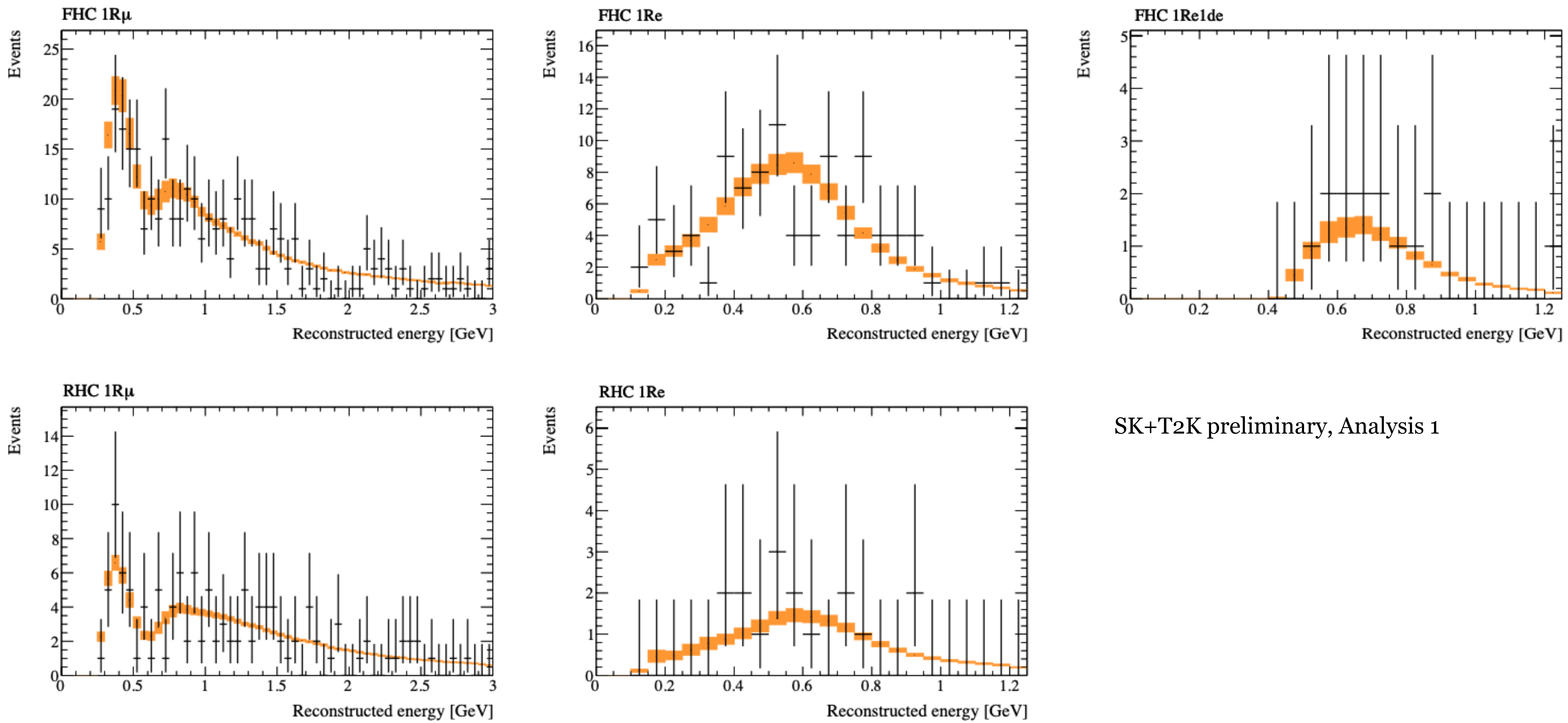
Selection		*p*-value	
		Shape	Rate
1Rμ	ν-mode	0.48	0.18
	$\bar{\nu}$-mode	0.85	0.74
1R*e*	ν-mode	0.19	0.49
	$\bar{\nu}$-mode	0.61	0.39
1R*e*1de	ν-mode	0.86	0.22
All		0.73	0.30

SK+T2K preliminary, Analysis 2

Sample	Shape-based	Norm-based
T2K FHC 1Rmu	0.240	0.175
T2K RHC 1Rmu	0.643	0.735
T2K FHC 1Re	0.209	0.519
T2K RHC 1Re	0.625	0.483
T2K FHC 1Re1de	0.912	0.456
SK SubGeV-elike-0dcy	0.056	0.491
SK SubGeV-elike-1dcy	0.519	0.519
SK SubGeV-mulike-0dcy	0.433	0.549
SK SubGeV-mulike-1dcy	0.435	0.517
SK SubGeV-mulike-2dcy	0.565	0.565
SK SubGeV-pi0like	0.091	0.091
SK MultiGeV-elike-nue	0.107	0.230
SK MultiGeV-elike-nuebar	0.868	0.459
SK MultiGeV-mulike	0.848	0.515
SK MultiRing-elike-nue	0.635	0.379
SK MultiRing-elike-nuebar	0.632	0.445
SK MultiRing-mulike	0.578	0.283
SK MultiRingOther-1	0.741	0.349
SK PCStop	0.356	0.550
SK PCThru	0.338	0.546
SK UpStop-mu	0.346	0.537
SK UpThruNonShower-mu	0.030	0.499
SK UpThruShower-mu	0.436	0.502
Total	0.236	0.189

Posterior Predictive Distributions

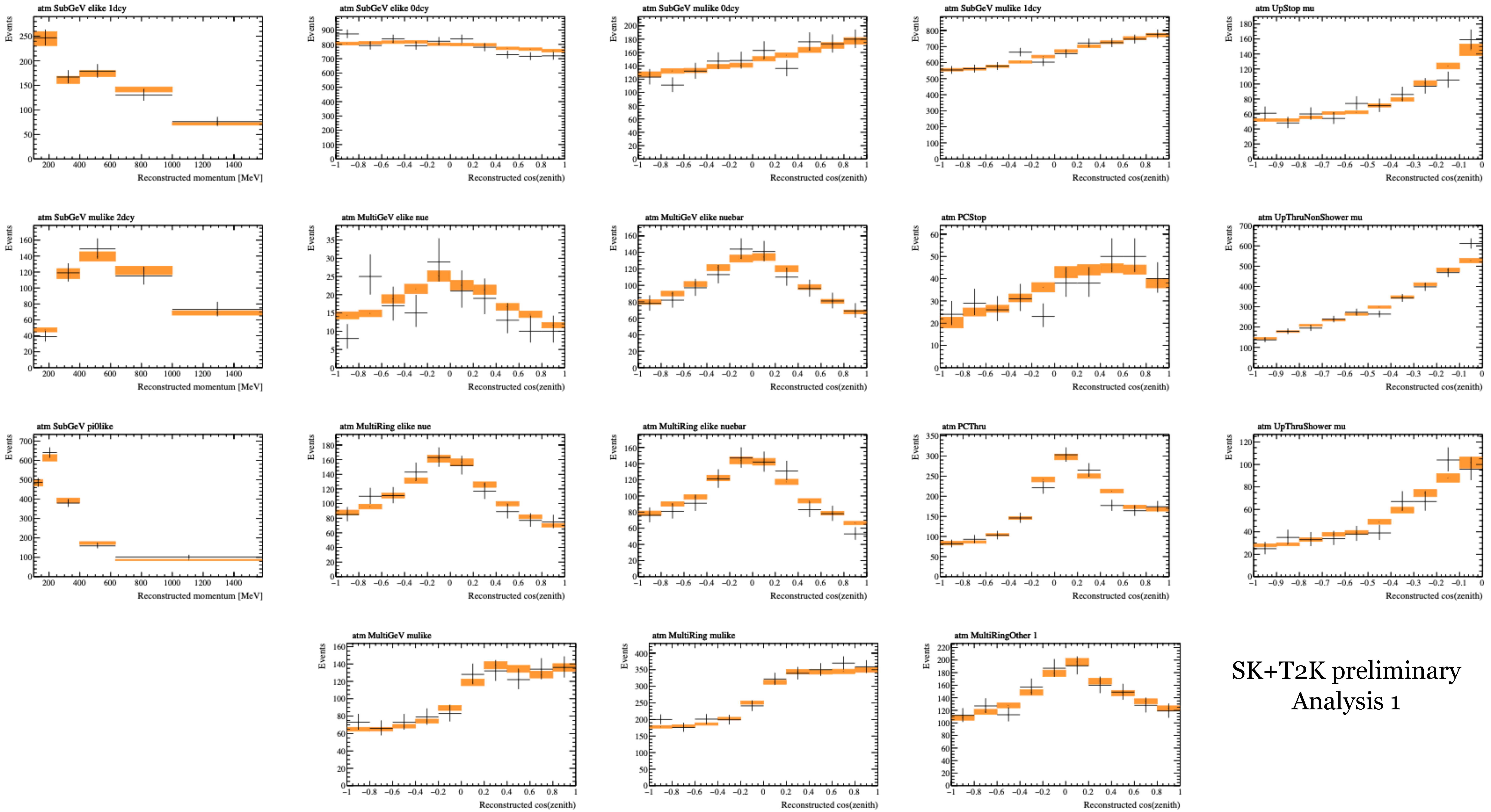
- The posterior predictive distributions for the five T2K beam samples.



SK+T2K preliminary, Analysis 1

Posterior Predictive Distributions

- The posterior predictive distributions for the 18 SK atmospheric samples.

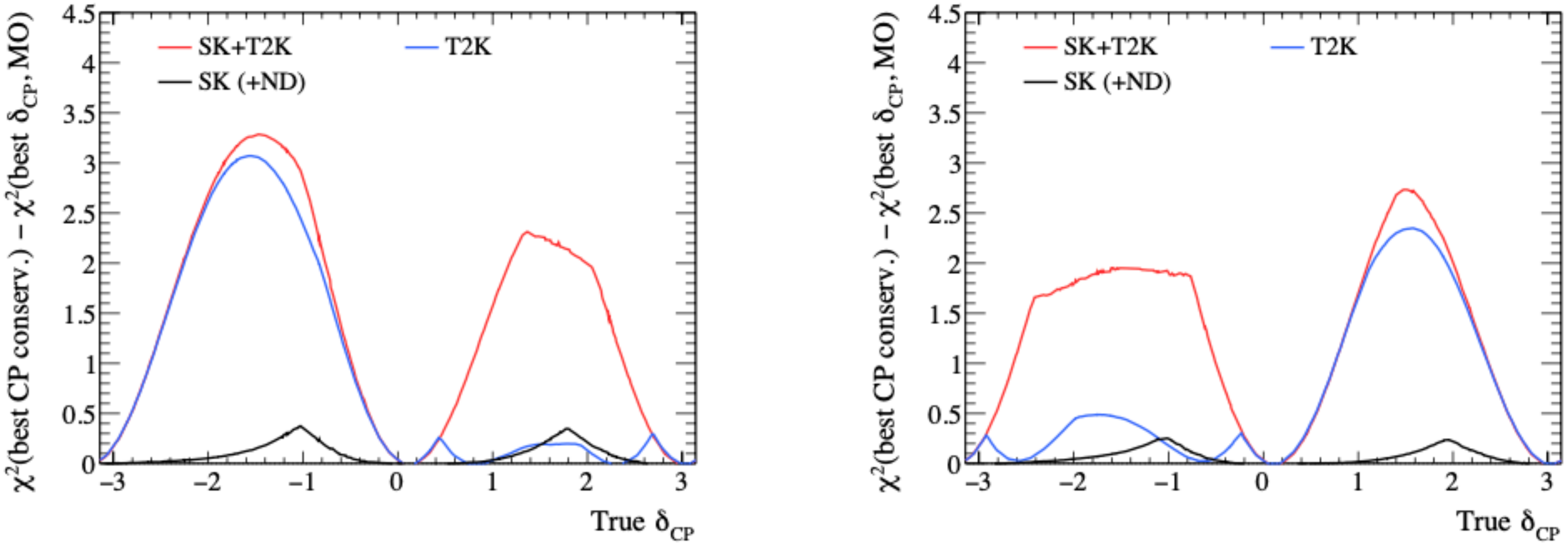


SK+T2K preliminary
Analysis 1

Sensitivity Study (Old Model)

- CP conservation rejection power at various values of true δ_{CP}.
  - CP violation sensitivity is dominated by T2K.
  - The sensitivity is low in $\delta_{CP} > 0$ ($\delta_{CP} < 0$) for NO (IO) due to the degenerated effect of mass ordering and δ_{CP}.
  - This degeneracy can be broken by the combined analysis.

Asimov sensitivity for rejecting CP conservation hypothesis for various values of true δ_{CP}

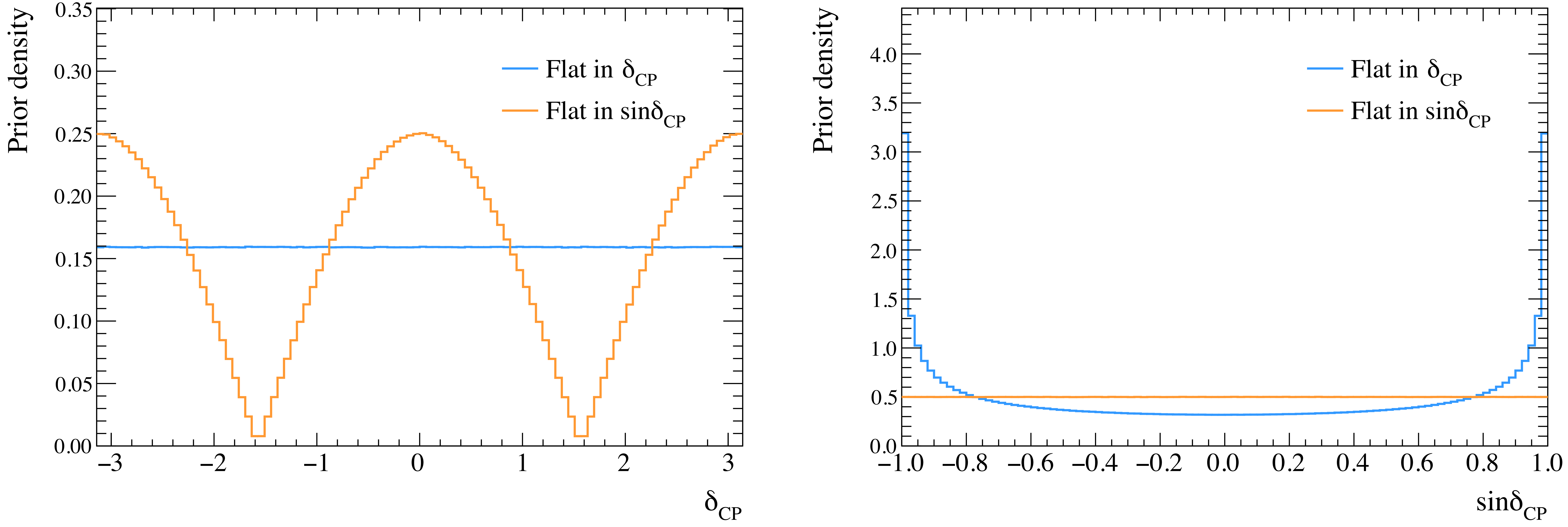


(a) True NO (b) True IO

Bayesian Prior Choice

- Two widely accepted non-informative priors were tested in our analysis of CP violation.
  - Uniform δ_{CP}: closer to Jensen's prior for $U(3)$ Haar measure
  - Uniform $\sin\delta_{\mathrm{CP}}$: closer to Jeffreys' prior ($\propto \sqrt{\det I_{\mathrm{Fisher}}}$) for this analysis

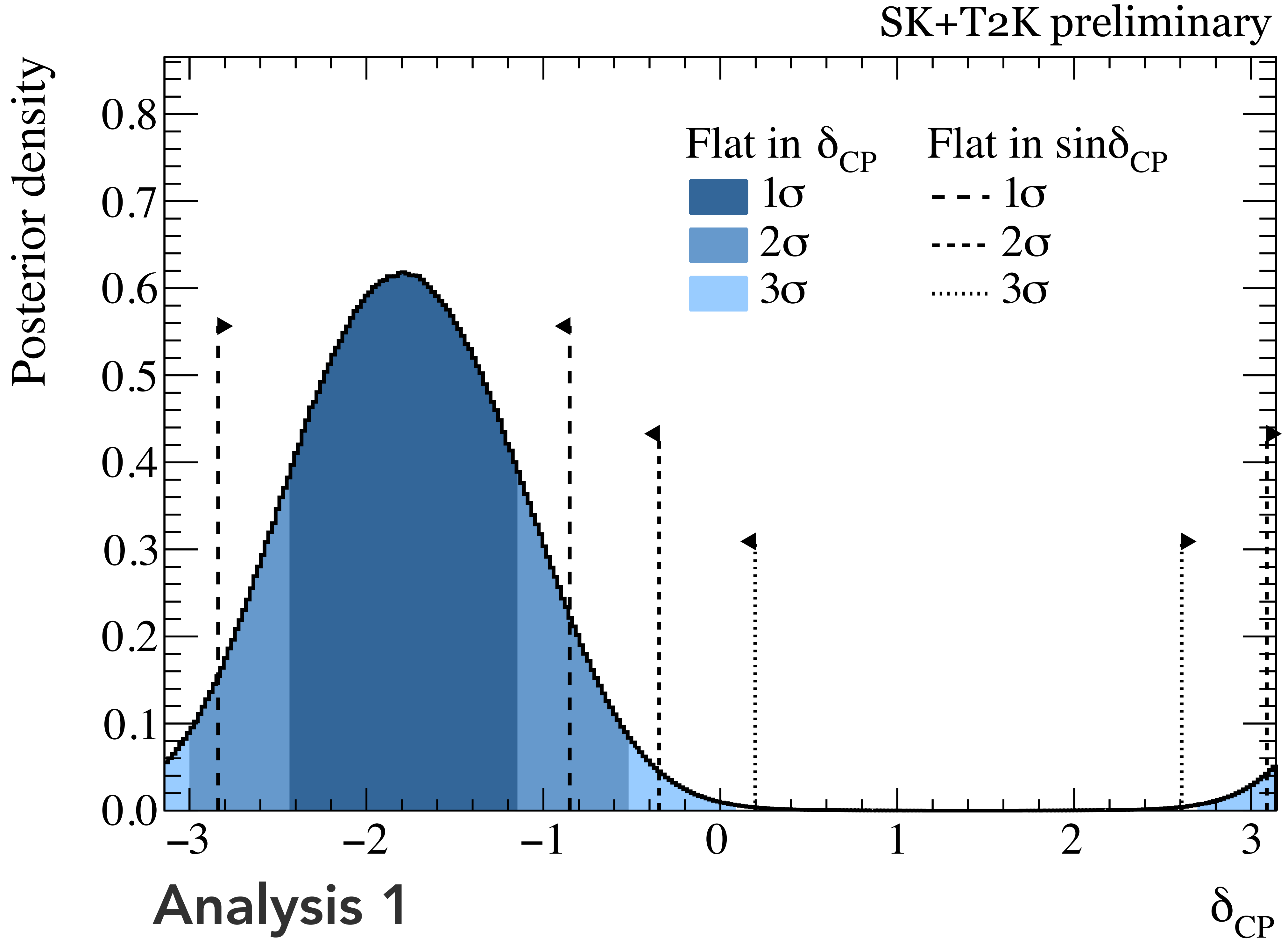
Comparison of the two prior distributions in different parameter spaces



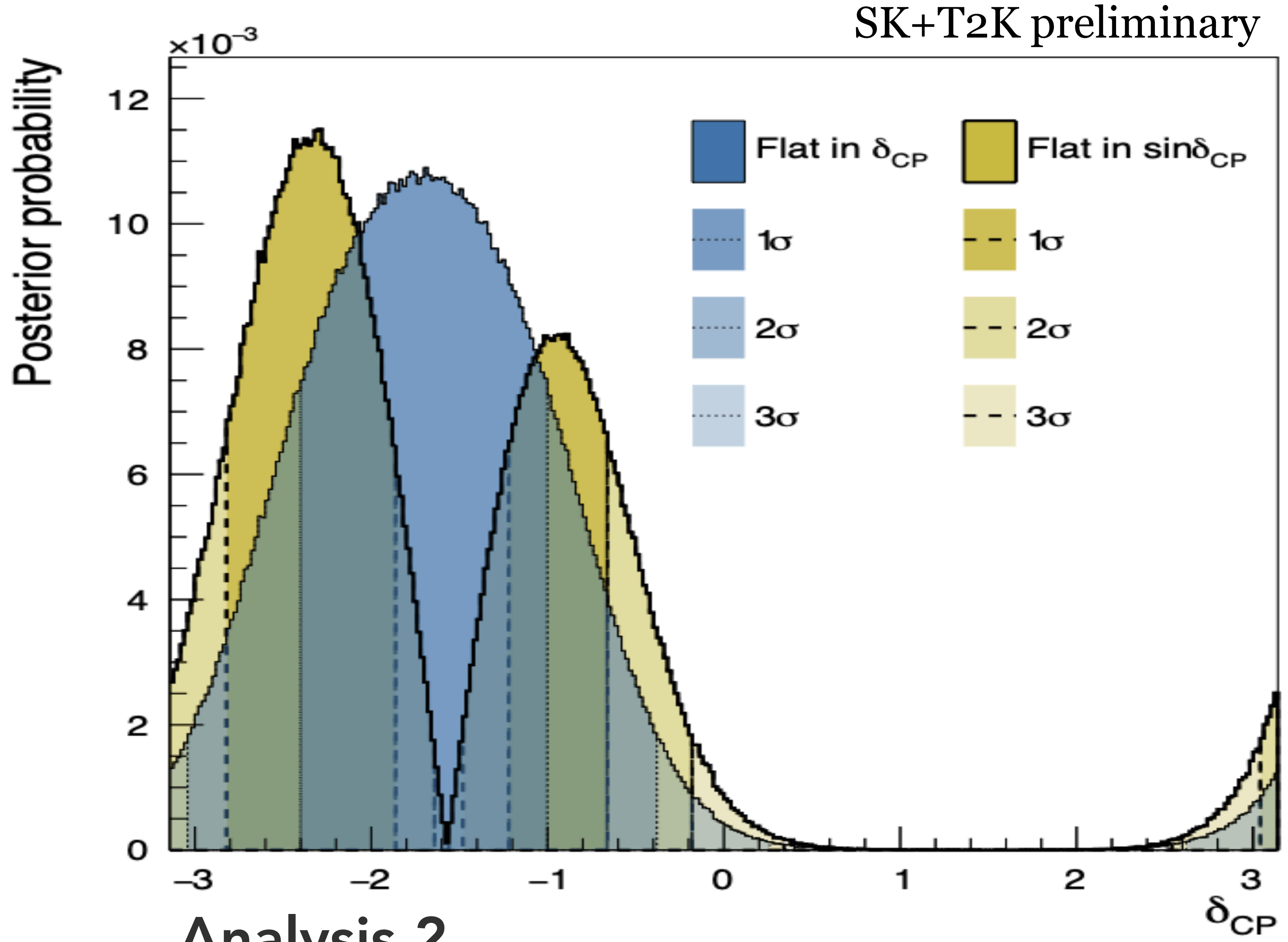
Comparison of Two Methods

- δ_{CP} credible intervals

δ_{CP} posterior distributions with reactor constraint and marginalized over both mass ordering



Analysis 1



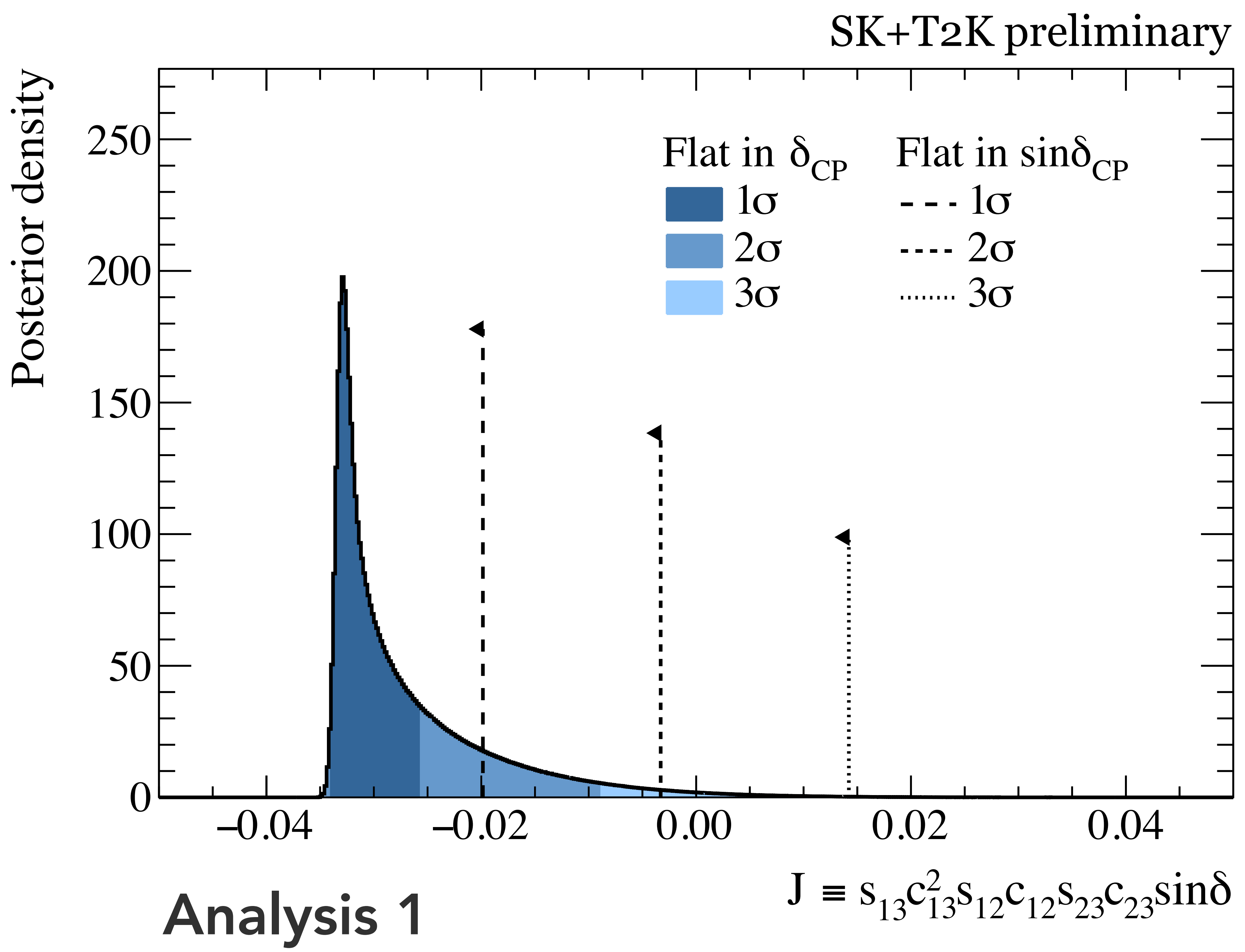
Analysis 2

Comparison of Two Methods

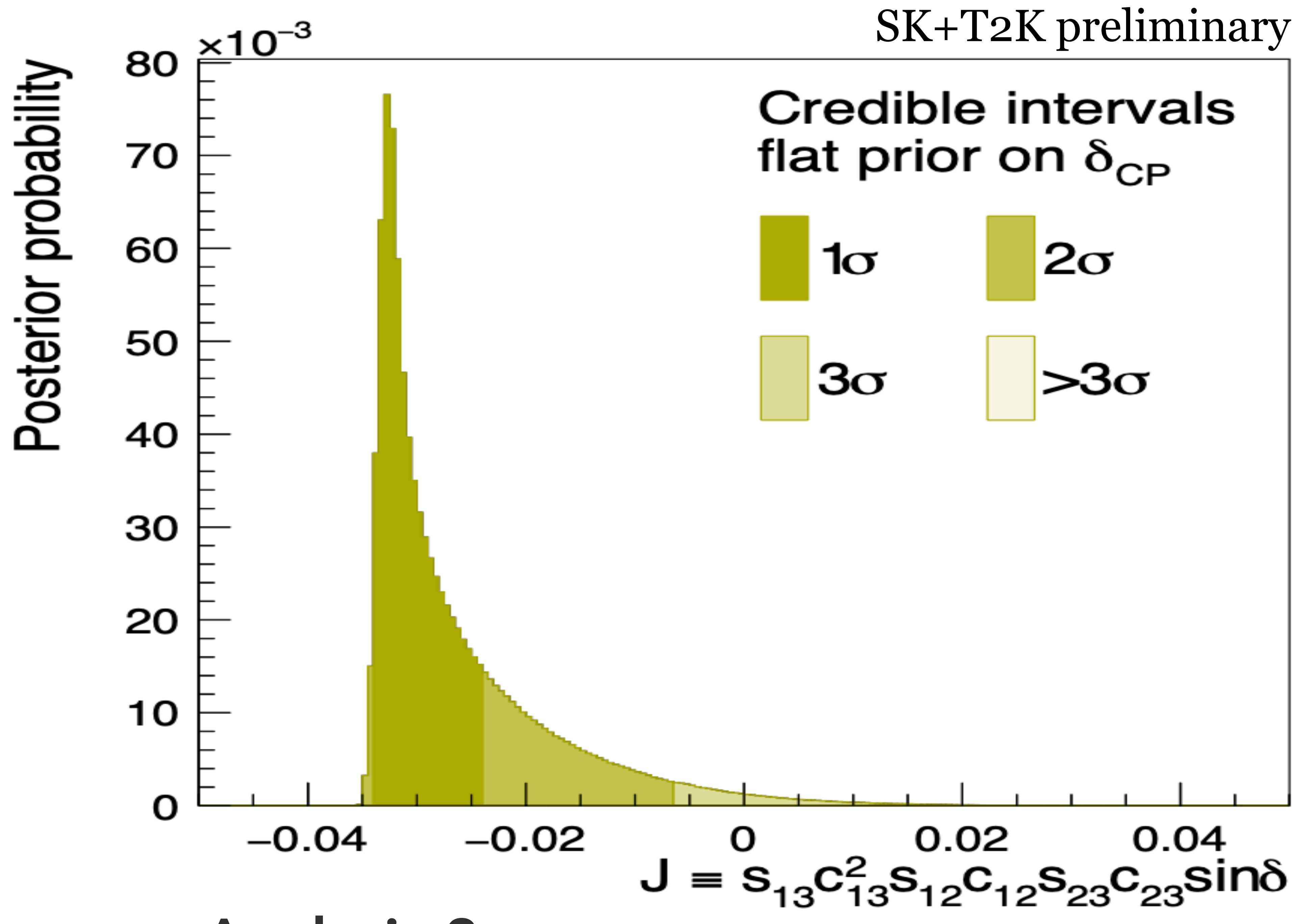
- Jarlskog invariant credible intervals

$$J_{\mathrm{CP}} = s_{13}c_{13}^2 s_{12}c_{12}s_{23}c_{23} \sin\delta_{\mathrm{CP}}$$

J_{CP} posterior distributions with reactor constraint and marginalized over both mass ordering



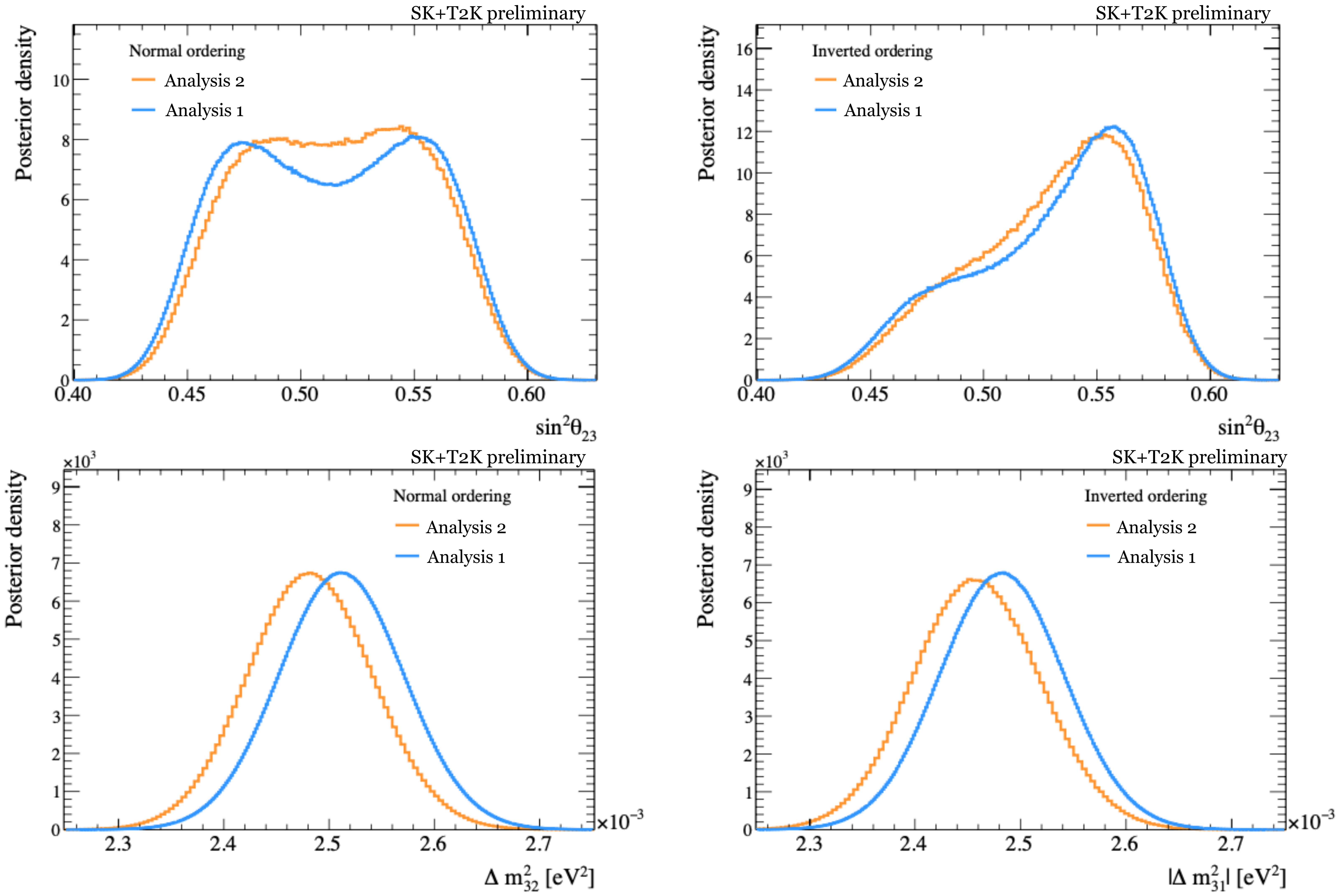
Analysis 1



Analysis 2

Comparison of Two Methods

- Atmospheric parameters
  - These differences are understood to be originating from the implementation differences.
  - It does not affect our overall conclusions.



Summary of the Oscillation Parameter

- The table below summarizes the 1σ credible intervals for oscillation parameters.

SK+T2K preliminary, Analysis 1

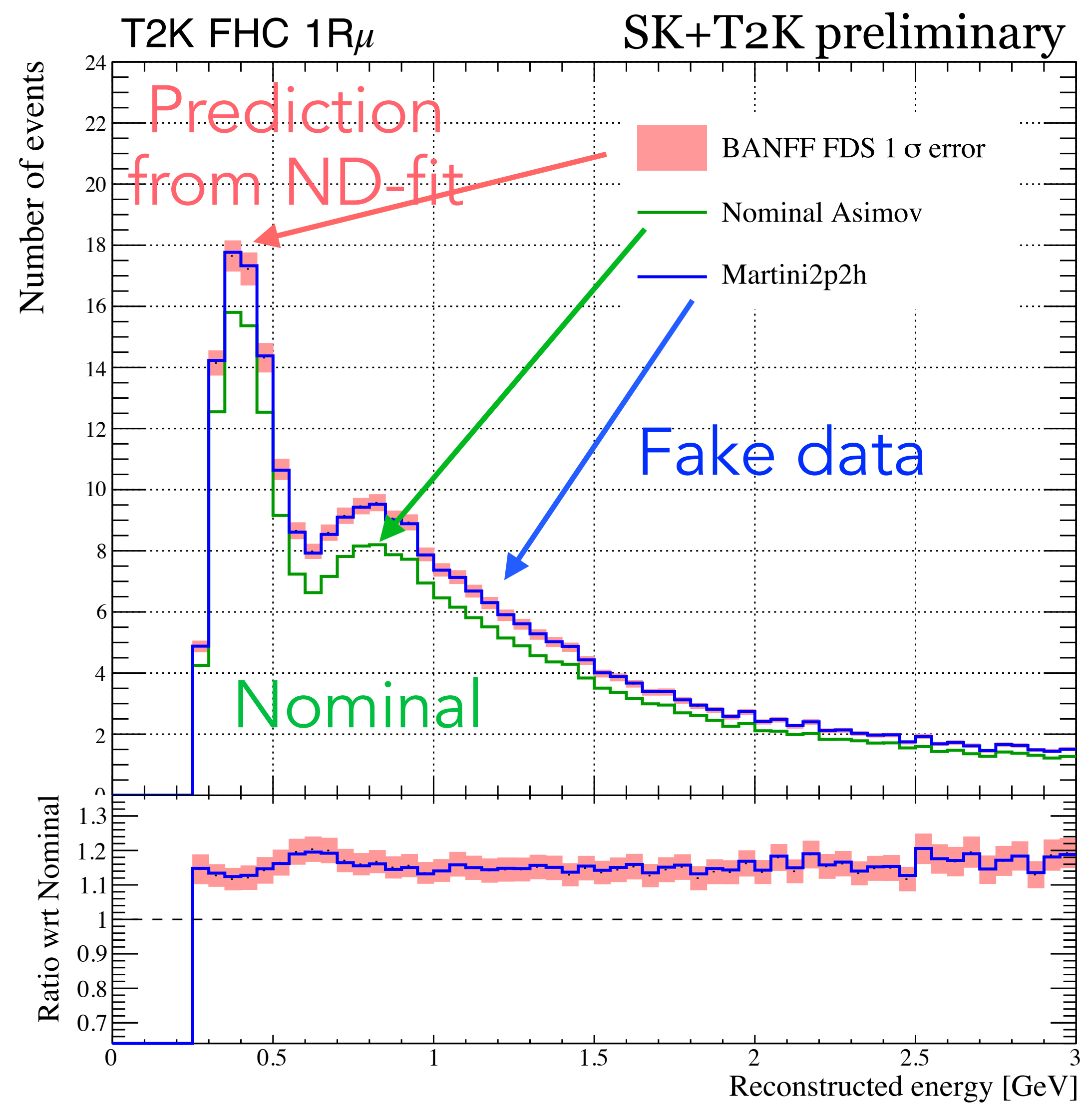
Normal ordering	$\sin^2\theta_{13}$	δ_{CP}	Δm^2_{32} [10^{-3} eV^2]	$\sin^2\theta_{23}$	J_{CP}
Most probable value	0.0219	-1.872	2.511	0.549	-0.033
1σ	[0.0212, 0.0226]	[-2.464, -1.205]	[2.452, 2.571]	[0.459, 0.505] and [0.521, 0.568]	[-0.034, -0.026]

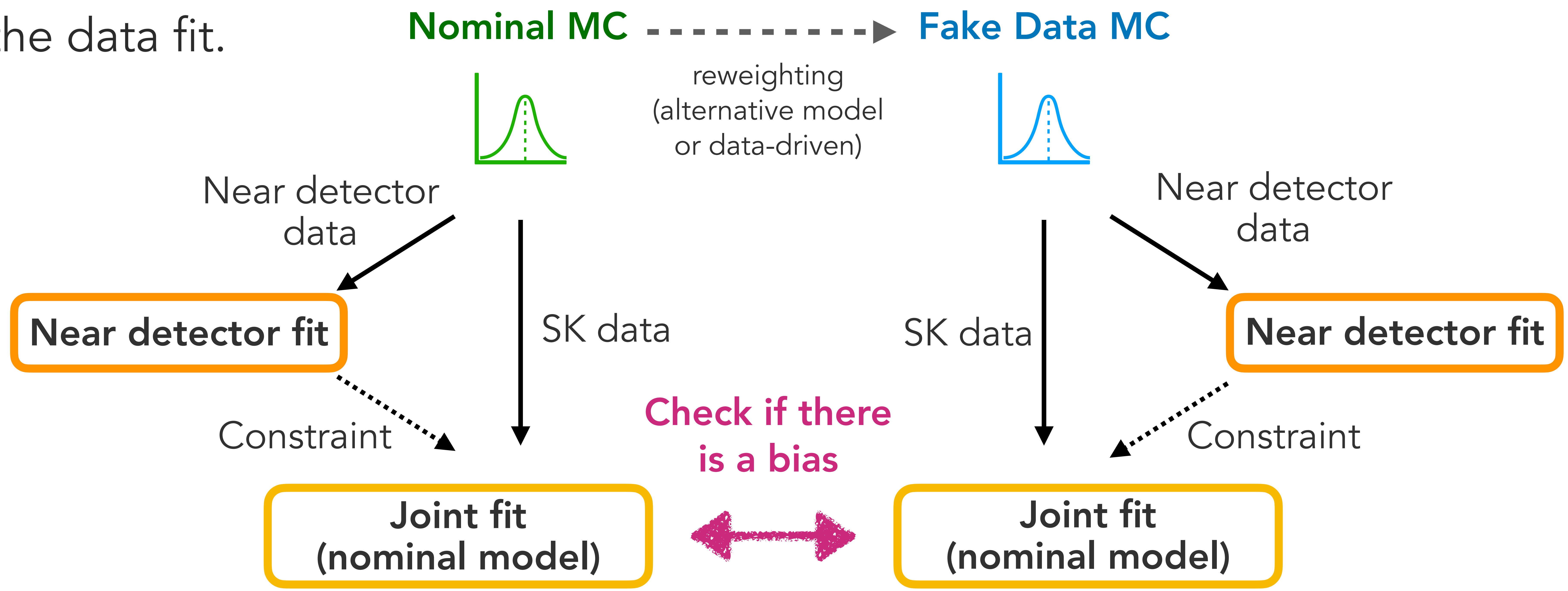
Inverted ordering	$\sin^2\theta_{13}$	δ_{CP}	Δm^2_{32} [10^{-3} eV^2]	$\sin^2\theta_{23}$	J_{CP}
Most probable value	0.0220	-1.476	2.484	0.558	-0.033
1σ	[0.0213, 0.0227]	[-2.003, -0.976]	[2.424, 2.541]	[0.508, 0.581]	[-0.034, -0.029]

Both ordering	$\sin^2\theta_{13}$	δ_{CP}	Δm^2_{32} [10^{-3} eV^2]	$\sin^2\theta_{23}$	J_{CP}
Most probable value	0.0219	-1.797	2.510	0.549	-0.033
1σ	[0.0212, 0.0226]	[-2.417, -1.159]	[2.449, 2.568]	[0.461, 0.503] and [0.520, 0.570]	[-0.034, -0.026]

Validation of the Model Robustness

- We evaluate the possible bias in the oscillation parameter measurement due to the possible mis-modeling.
  - Generate a simulated data set using an alternative model and fit it with our nominal model.
  - If there is a significant bias, we update our model with additional systematics or apply smearing on the oscillation parameter.
  - This first step is done based on MC, before performing the data fit.



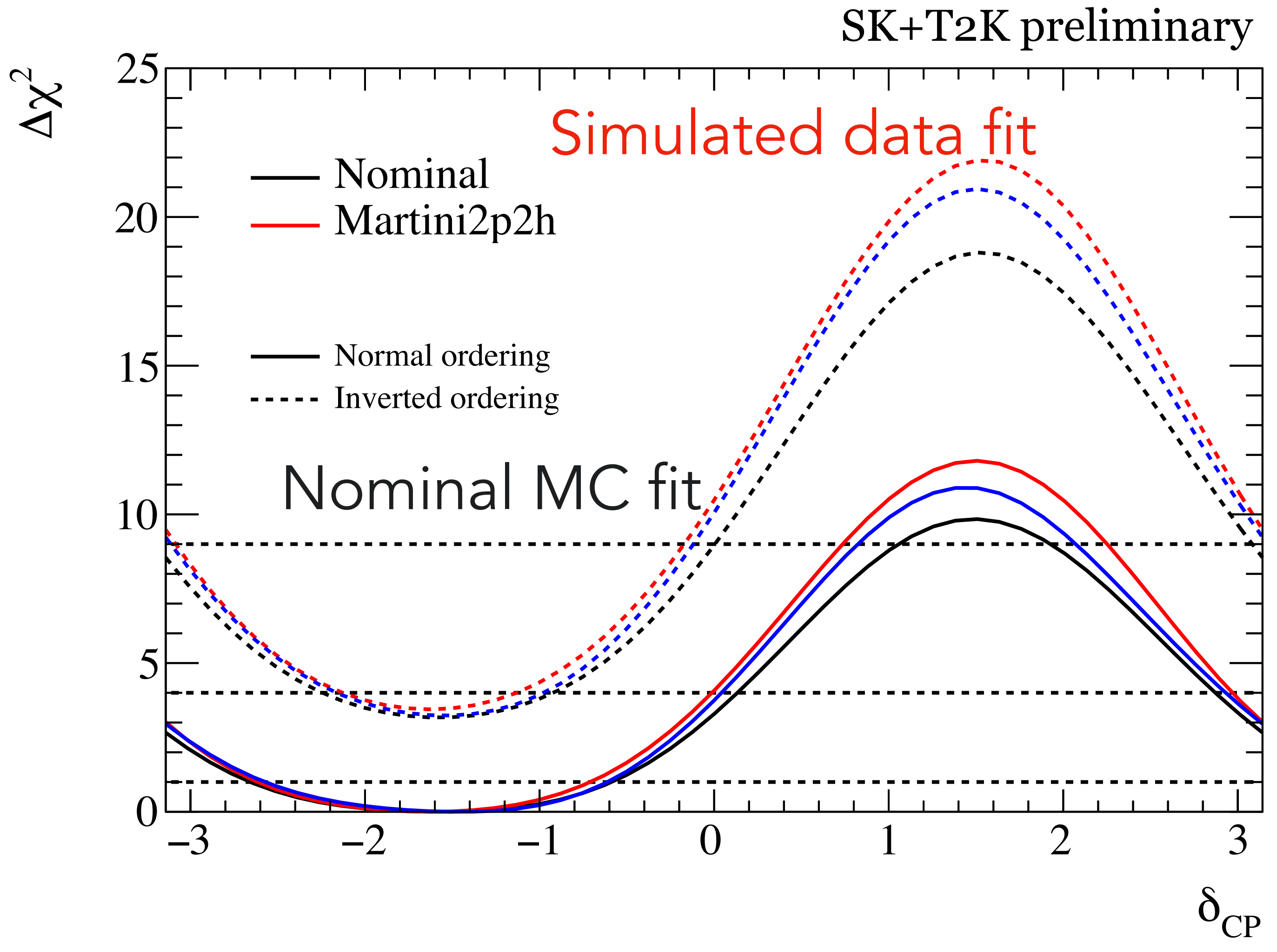


Validation of the Model Robustness

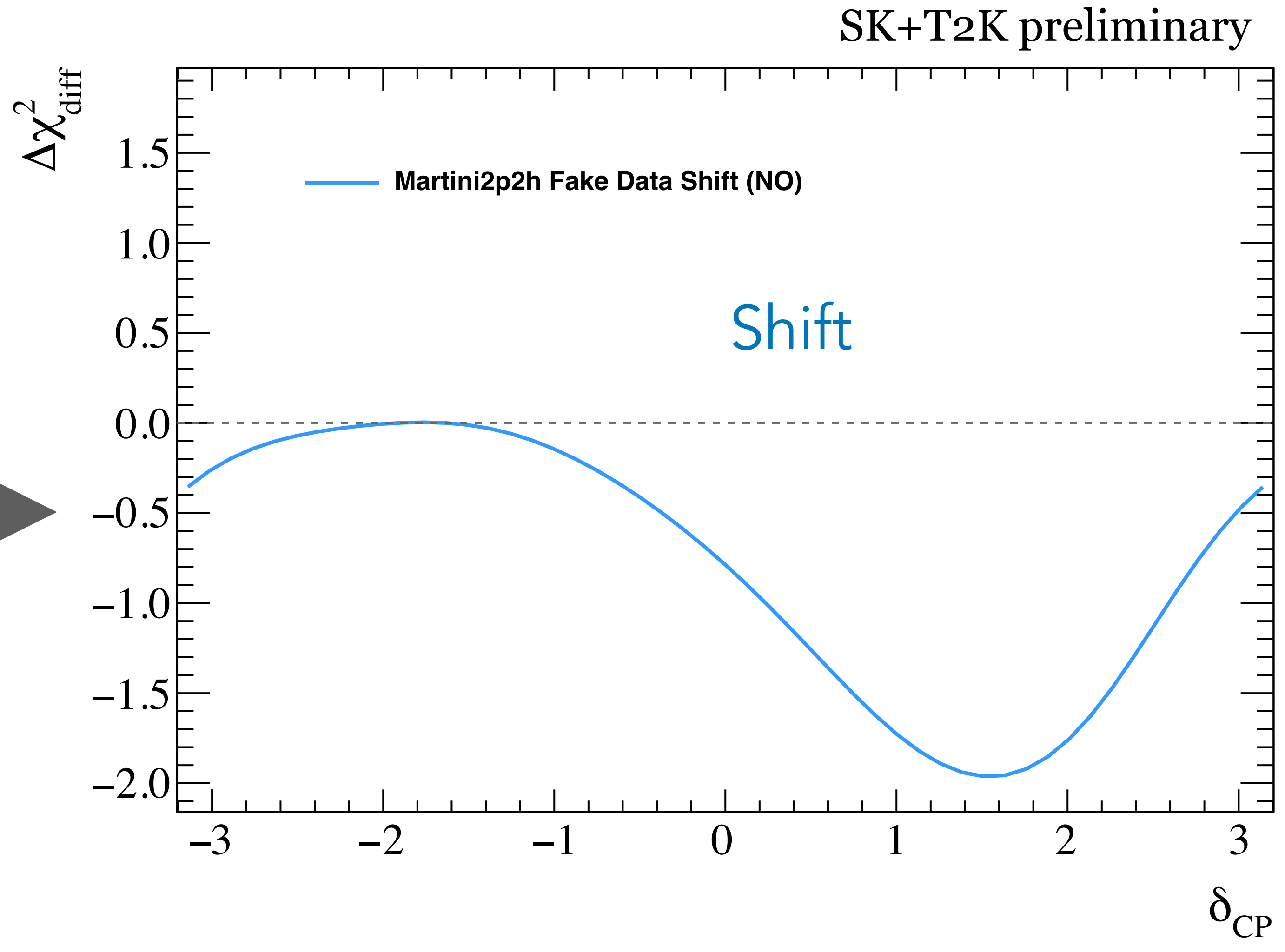
- The second step of the robustness test is done after the data fit.
  - We take the difference between nominal fit and simulated data fit results.
  - Impose this shift to the data fit to see if the bias in the interval edges can change our conclusion.
  - This effect is tested on δ_{CP} and Jarlskog invariant (relevant to our CP statement).

MC

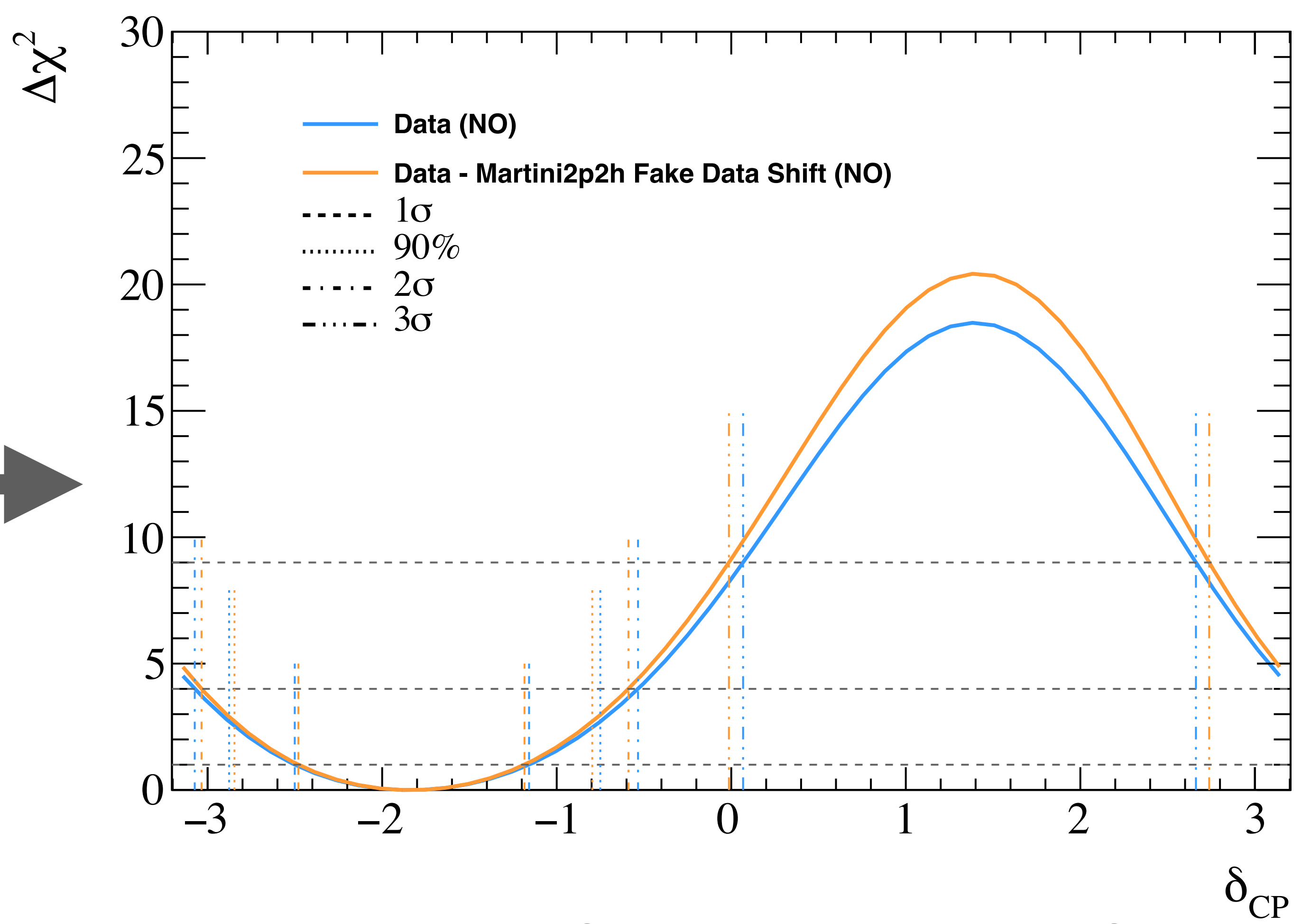
Difference between simulated data and nominal MC fit



Shift is extracted



Data



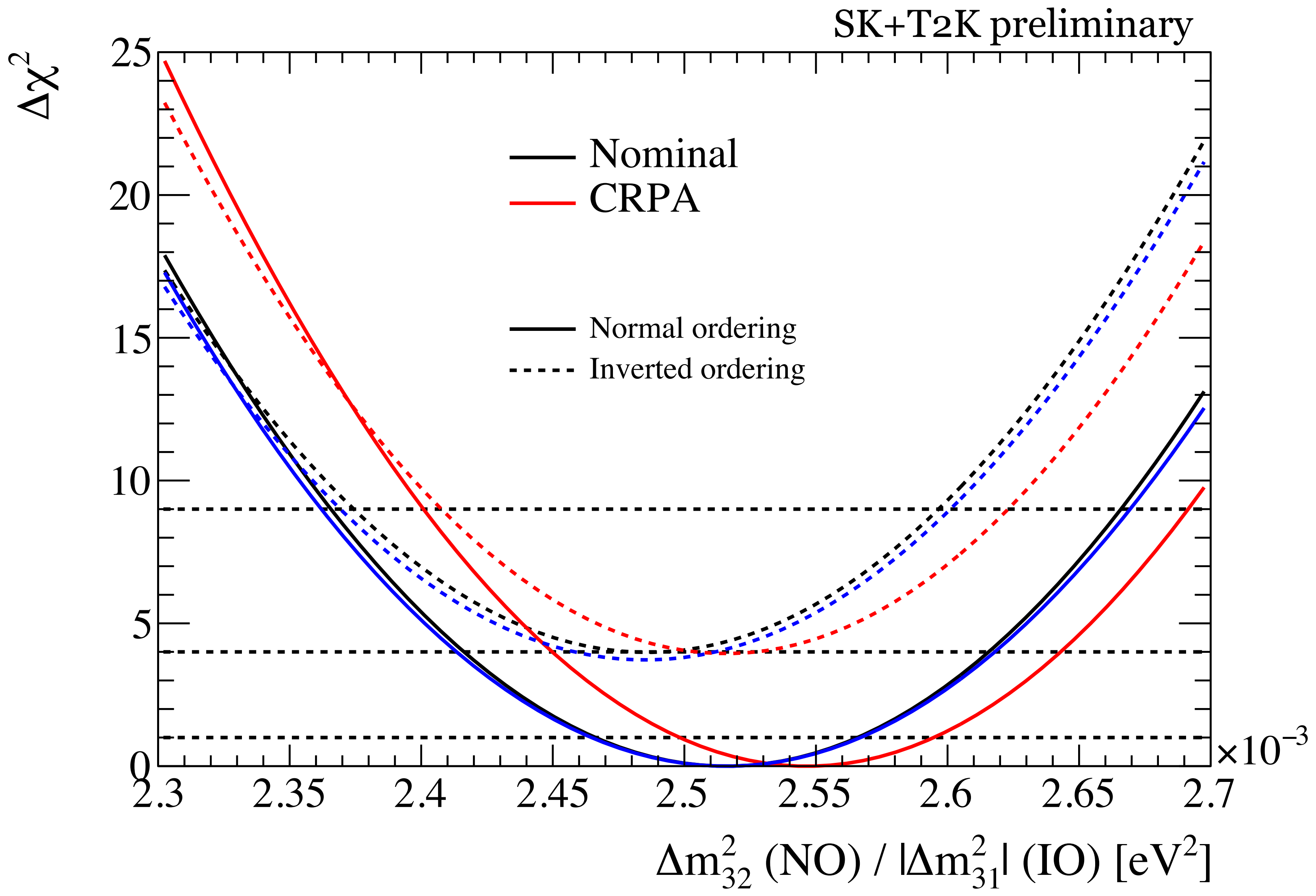
Impose the shift on the data fit result

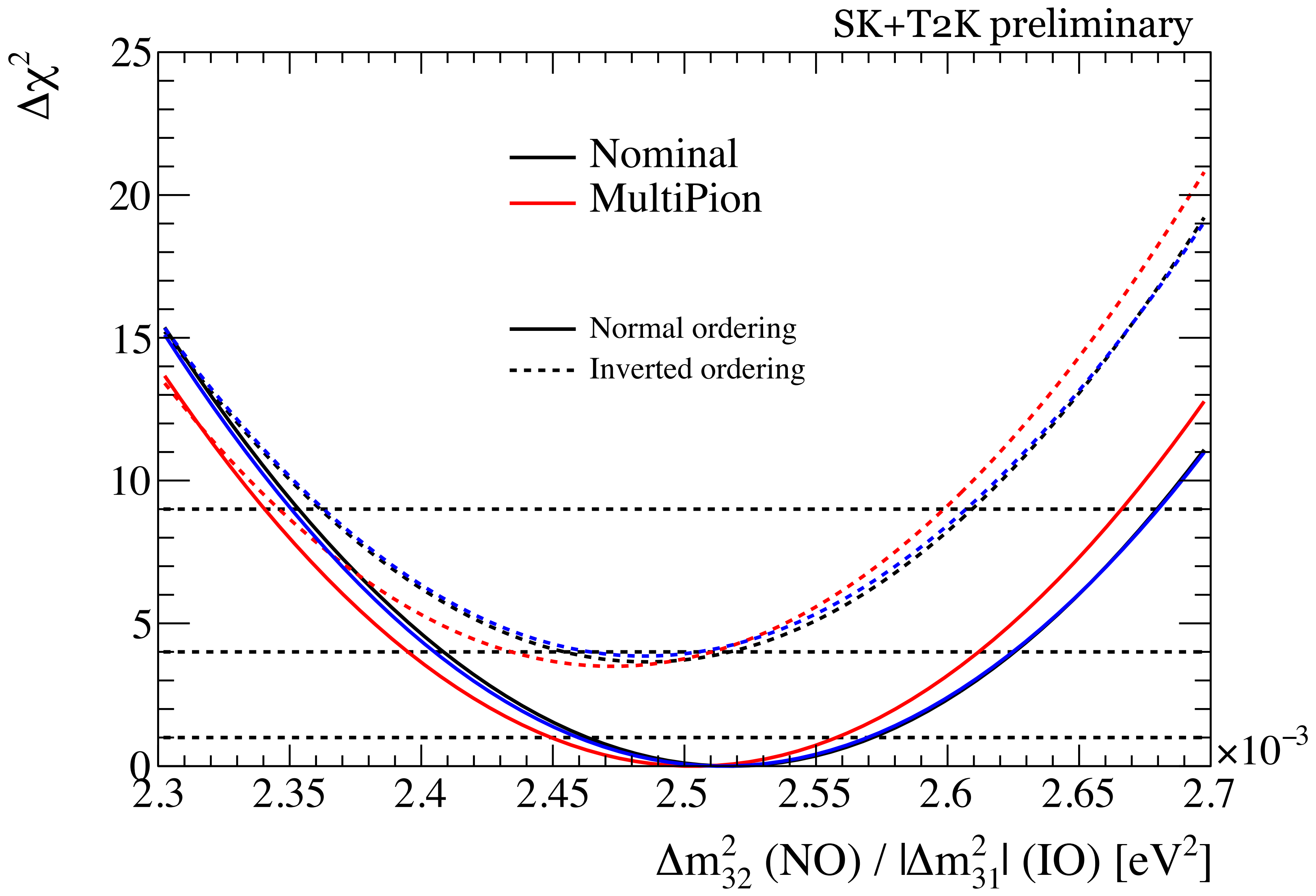
*Here $\Delta\chi^2 = 1,4,9$ lines are shown but it does not guarantee the correct coverage

Result of the Robustness Study

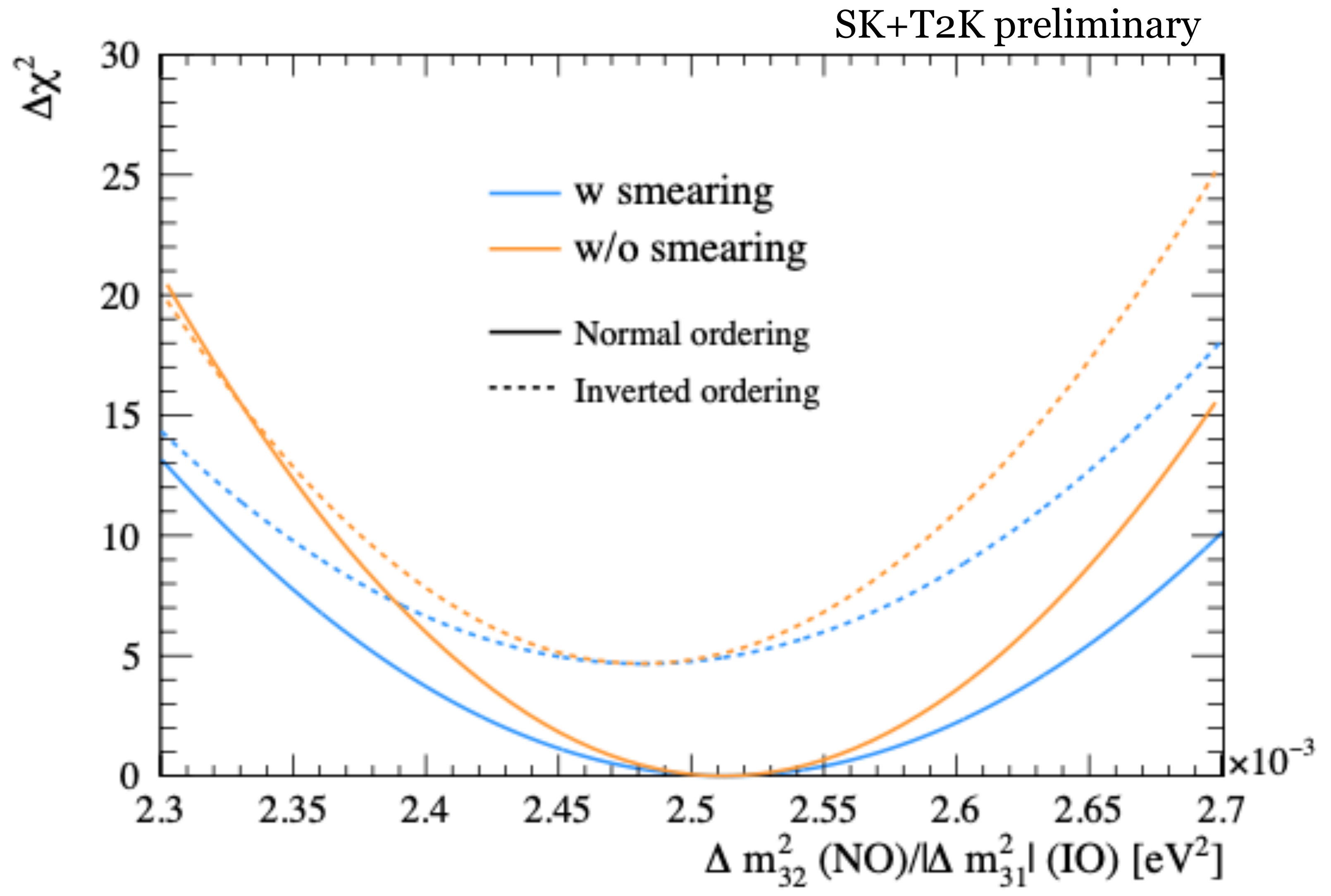
- We have tested 14 alternative models before the data fit.
  - Some fake data studies showed non-negligible biases in Δm^2_{32}.
  - Therefore we decided to smear the Δm^2_{32} contour in the data fit by convolving the posterior distribution with a Gaussian ($\sigma = 3.6 \times 10^{-5}\ \mathrm{eV}^2$).

Example of the fake data fit results that showed large biases





Gaussian smearing applied on data

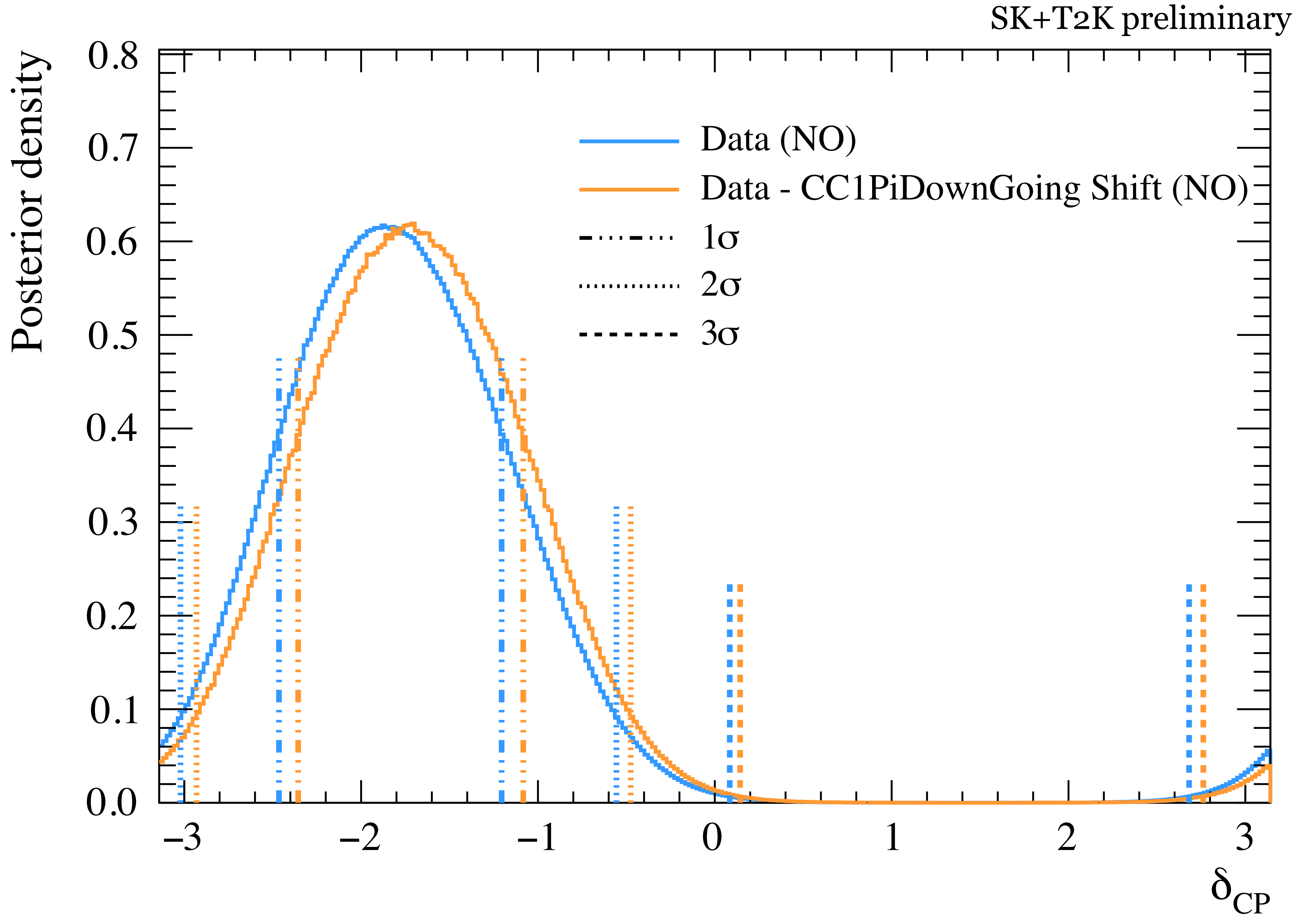


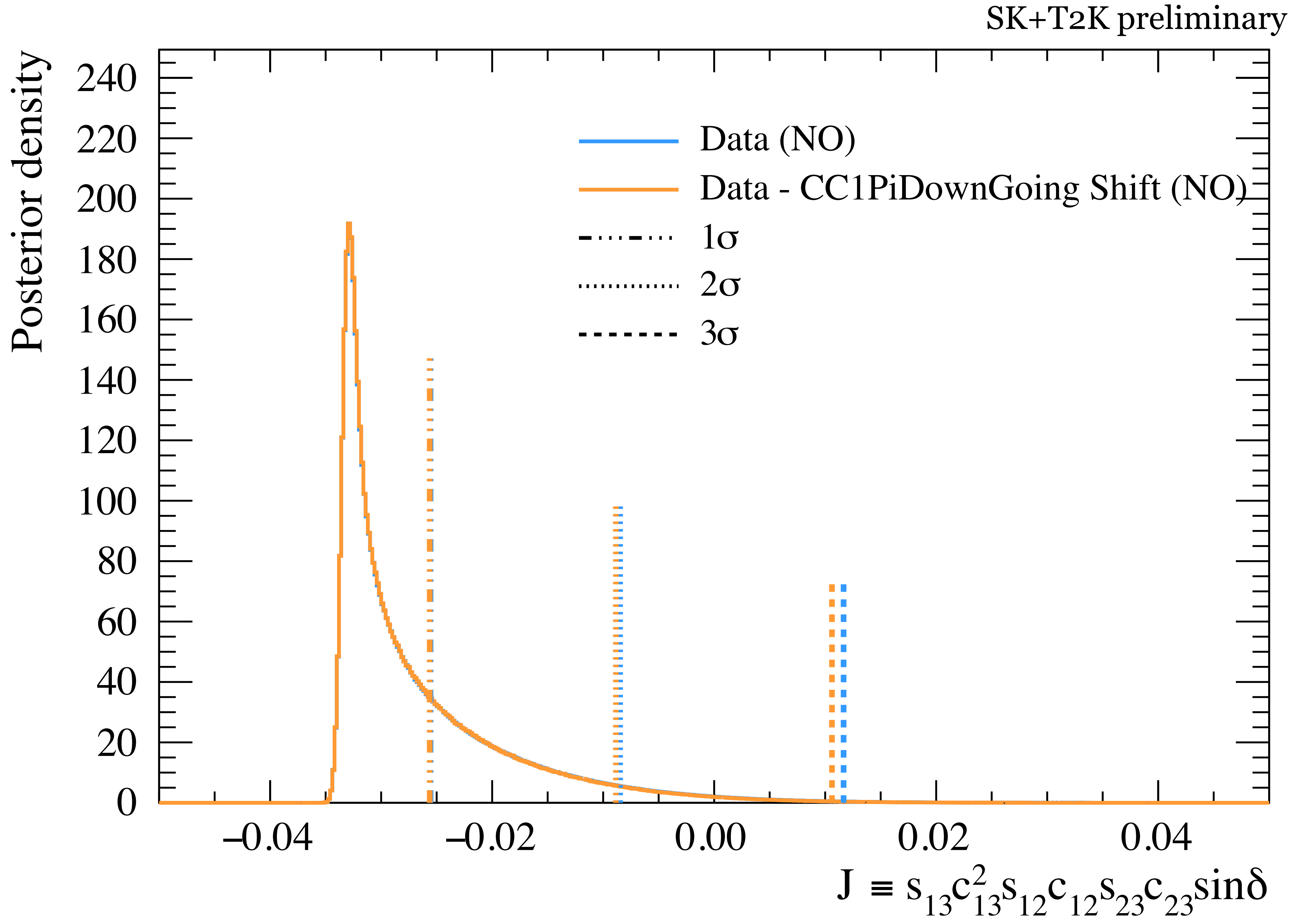
*Here $\Delta\chi^2 = 1,4,9$ lines are shown but it does not guarantee the correct coverage

Result of the Robustness Study

- We also tested whether it can change our conclusion on the significance of CP violation.
  - The size of the shift in the credible interval edges of δ_{CP} and Jarlskog invariant was checked.
  - None of them caused a shift of 2σ interval edges over the value of interest ($\delta_{CP} = 0, \pi$, $J_{CP} = 0$)

- Therefore it does not change our conclusion on CP violation around 2σ.





List of Robustness Test

- 14 simulated data studies have been performed to test a possible bias in the analysis.

  - The first six studies are taken from Appendix B of Eur.Phys.J.C 83 (2023) 9, 782.

  - Two alternative nuclear models are tested (our baseline model is SF)
    - LFG+RPA [ref]
    - HF+CRPA [ref]

  SF: Spectral Function
  LFG: Local Fermi Gas
  RPA: Random Phase Approximation
  HF: Hartree-Fock
  CRPA: Continuum Random Phase Approximation

  - The last six studies were included to test possible problems that would come with the joint fit.

	Model component
Martini 2p2h	2p2h
ND280 data-driven pion kinematics	CC1π
CC0π non-QE alteration	CC0π
Removal energy	Nuclear Model
Axial form factors	CCQE
Pion SI bug fix	CC1π, CC$n\pi$
LFG	Nuclear model
CRPA	Nuclear model
Pion multiplicity	CC$n\pi$
Energy-dependent $\sigma_{\nu_e}/\sigma_{\nu_\mu}$	$\sigma_{\nu_e}/\sigma_{\nu_\mu}$
Xsec-only fit	Fit
Atmospheric down-going CC1π	CC1π
Atmospheric full-zenith CC1π	CC1π
No-migration energy scale fit	Fit