Theoretical Overview of Neutrino Oscillations

22nd International Workshop on Next Generation Nucleon Decay and Neutrino Detectors (NNN23)

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INFN

Theory and Phenomenology of Fundamental Interactions
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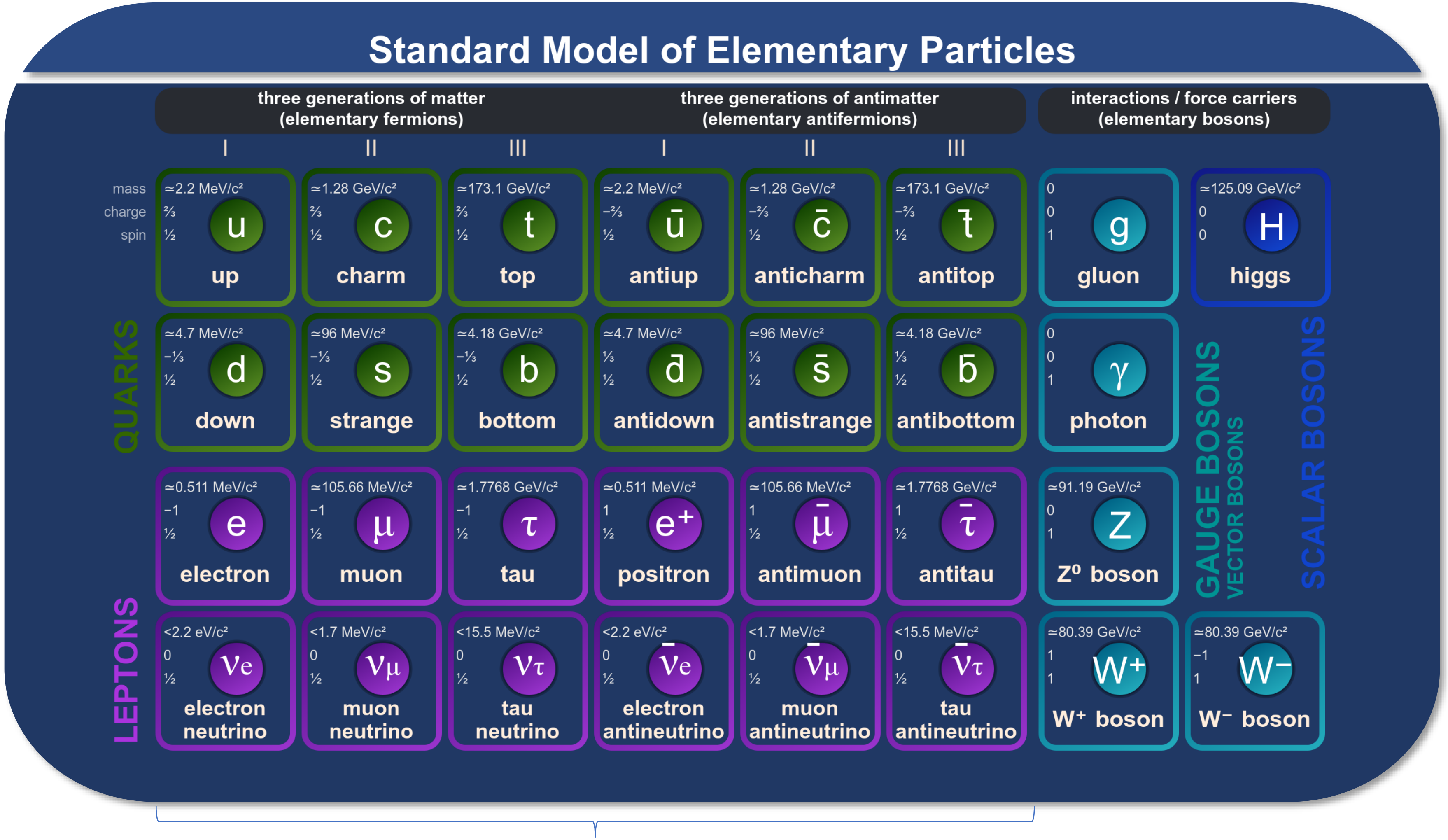
Particle content of the Standard Model



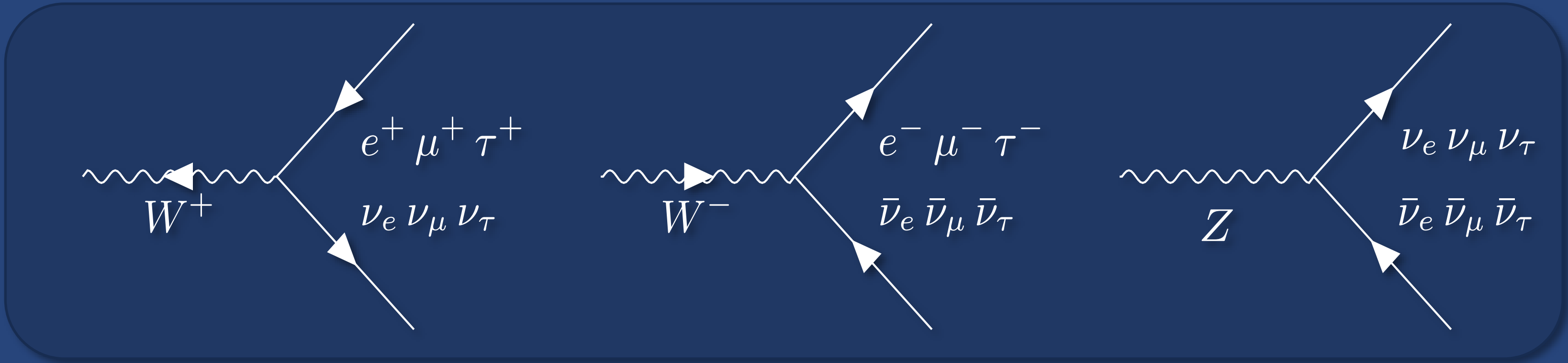
Standard Model of Elementary Particles

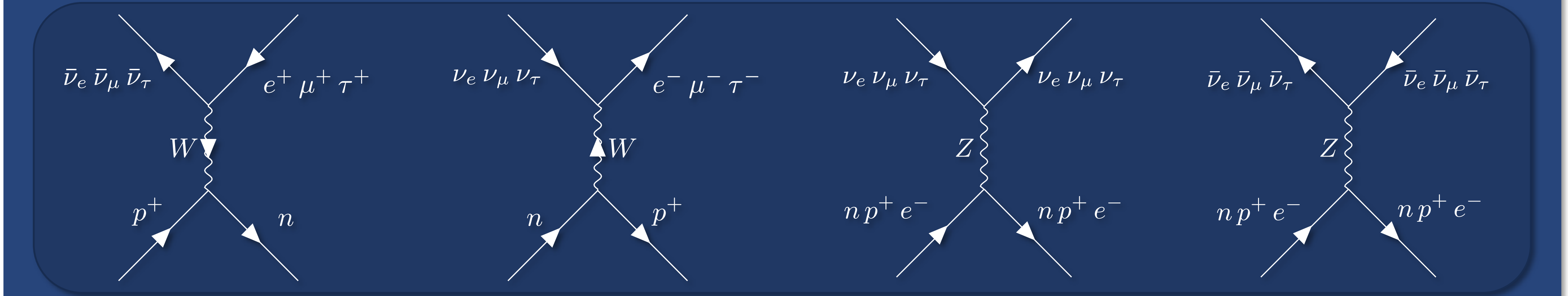
	three generations of matter (elementary fermions)			three generations of antimatter (elementary antifermions)			interactions / force carriers (elementary bosons)	
	I	II	III	I	II	III		
mass charge spin	≃2.2 MeV/c² ⅔ ½ u up	≃1.28 GeV/c² ⅔ ½ c charm	≃173.1 GeV/c² ⅔ ½ t top	≃2.2 MeV/c² −⅔ ½ $\bar{u}$ antiup	≃1.28 GeV/c² −⅔ ½ $\bar{c}$ anticharm	≃173.1 GeV/c² −⅔ ½ $\bar{t}$ antitop	0 0 1 g gluon	≃125.09 GeV/c² 0 0 H higgs
QUARKS	≃4.7 MeV/c² −⅓ ½ d down	≃96 MeV/c² −⅓ ½ s strange	≃4.18 GeV/c² −⅓ ½ b bottom	≃4.7 MeV/c² ⅓ ½ $\bar{d}$ antidown	≃96 MeV/c² ⅓ ½ $\bar{s}$ antistrange	≃4.18 GeV/c² ⅓ ½ $\bar{b}$ antibottom	0 0 1 γ photon	GAUGE BOSONS VECTOR BOSONS SCALAR BOSONS
	≃0.511 MeV/c² −1 ½ e electron	≃105.66 MeV/c² −1 ½ μ muon	≃1.7768 GeV/c² −1 ½ τ tau	≃0.511 MeV/c² 1 ½ e^+ positron	≃105.66 MeV/c² 1 ½ $\bar{\mu}$ antimuon	≃1.7768 GeV/c² 1 ½ $\bar{\tau}$ antitau	≃91.19 GeV/c² 0 1 Z Z^0 boson	
LEPTONS	<2.2 eV/c² 0 ½ ν_e electron neutrino	<1.7 MeV/c² 0 ½ ν_μ muon neutrino	<15.5 MeV/c² 0 ½ ν_τ tau neutrino	<2.2 eV/c² 0 ½ $\bar{\nu}_e$ electron antineutrino	<1.7 MeV/c² 0 ½ $\bar{\nu}_\mu$ muon antineutrino	<15.5 MeV/c² 0 ½ $\bar{\nu}_\tau$ tau antineutrino	≃80.39 GeV/c² 1 1 W^+ W^+ boson	≃80.39 GeV/c² −1 1 W^- W^- boson

Left-handed (right-handed), massless (anti)neutrinos, part of SU(2) doublets

Flavour neutrinos in the SM

Charged Current and Neutral Current electroweak interactions





Sources of Neutrinos

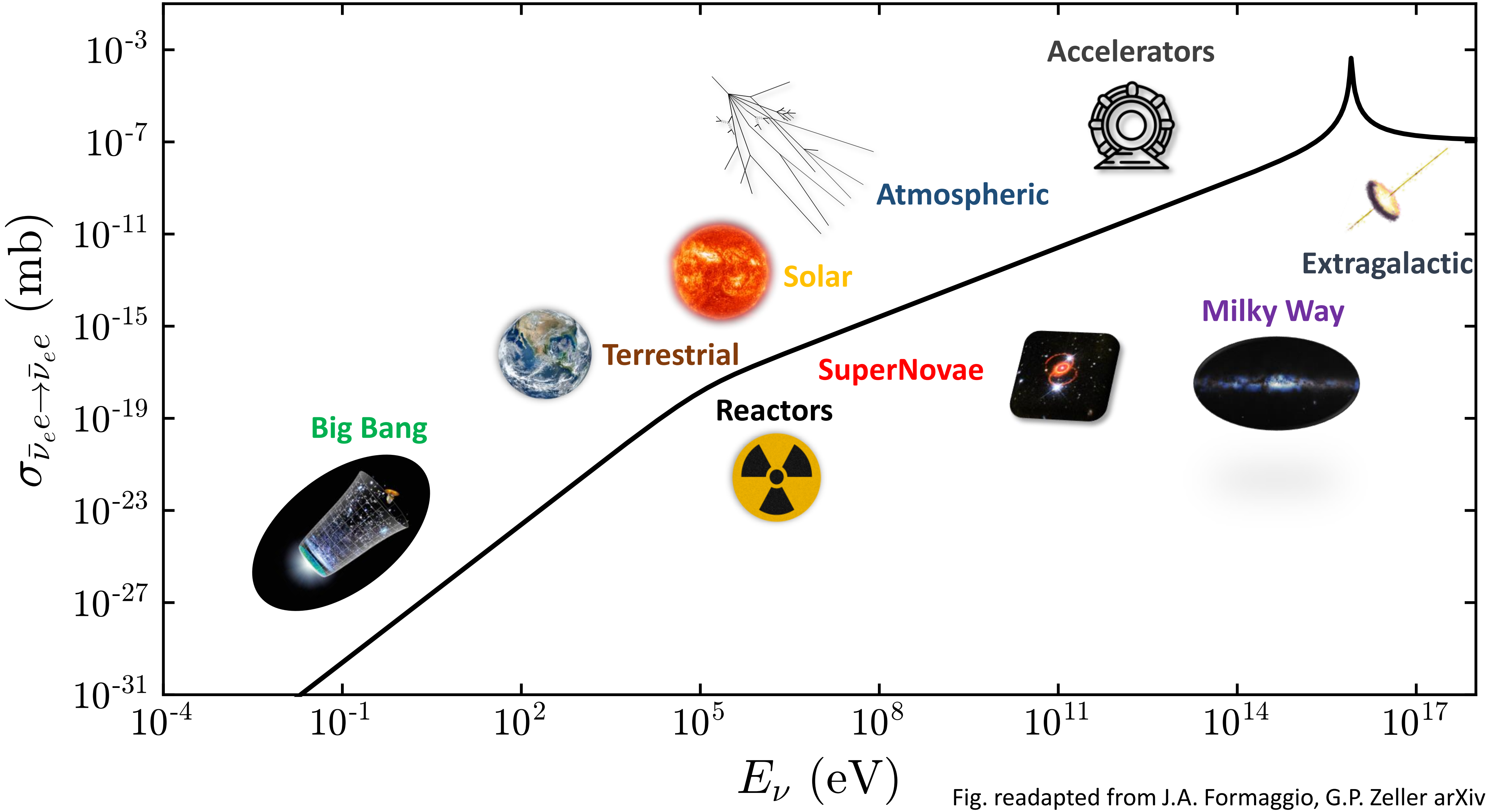
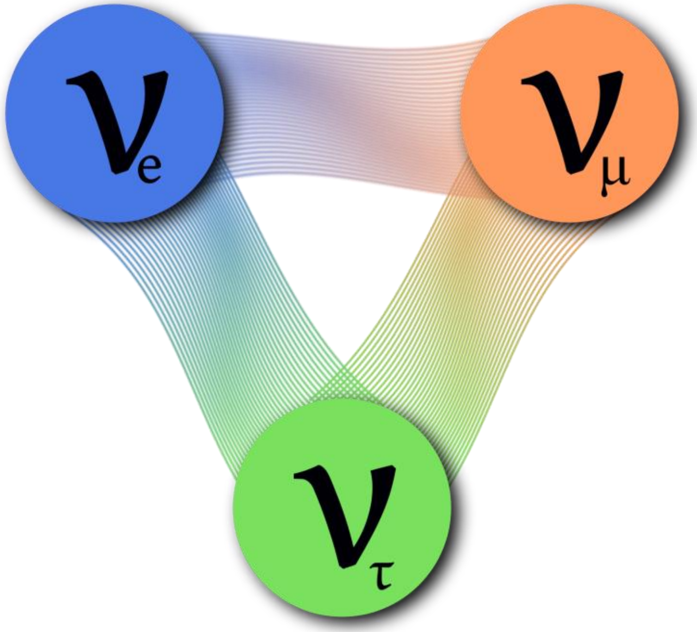


Fig. readapted from J.A. Formaggio, G.P. Zeller arXiv:1305.7513.

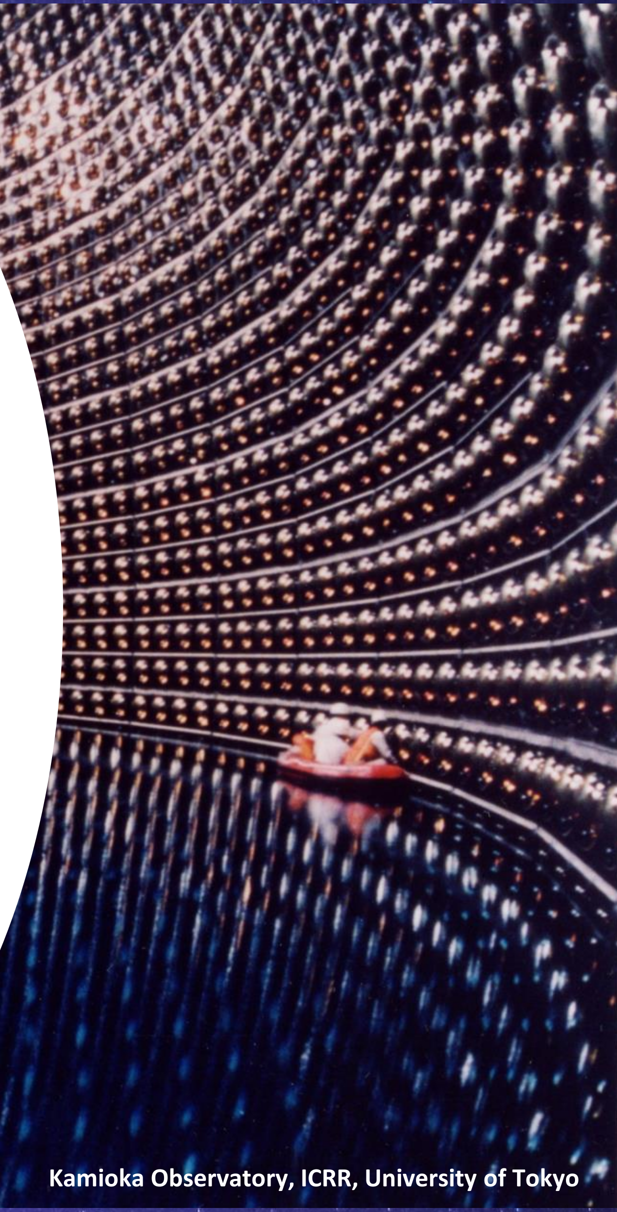
Neutrinos are massive and mix

SNO Detector, Ontario, Canada



Neutrinos **oscillate** and, thus, are **massive!**

Their mass is small ($<$ eV) compared to the other known fundamental particles.



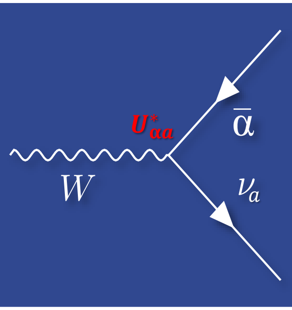
Kamioka Observatory, ICRR, University of Tokyo

The three-neutrino mixing scheme

The flavour neutrinos are a superposition of mass eigenstates

$$\nu_{\alpha L}(x) = \sum_{a=1}^{3} \boldsymbol{U_{\alpha a}} \nu_{aL}(x),$$

where $\nu_a(x)$ have masses $m_{1,2,3}$ and U is the unitary Pontecorvo-Maki-Nakagawa-Sakata (**PMNS**) lepton mixing **matrix**



The PMNS matrix U enters the electroweak interaction

$$\mathcal{L}_{C.C.} = -\frac{g_W}{\sqrt{2}} \boldsymbol{U^*_{\alpha a}} \overline{\nu_{aL}} \gamma^\mu W_\mu \psi_{\alpha L} + \mathrm{h.c.}$$

and regulates in the probability that a neutrino with flavour α and energy E at t = 0, after a distance L is detected with flavour β

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_a \boldsymbol{U_{\alpha a} U^*_{\beta a}}\, e^{-i\frac{\Delta m^2_{a1}}{2E}L} \right|^2$$

Current knowledge of neutrino oscillation parameters

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix

$$\boldsymbol{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\theta_{23},\, |\Delta m^2_{32}|} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}}_{\theta_{13},\, \delta} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\theta_{12},\, \Delta m^2_{21}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}}_{\alpha_{21},\, \alpha_{31}}$$

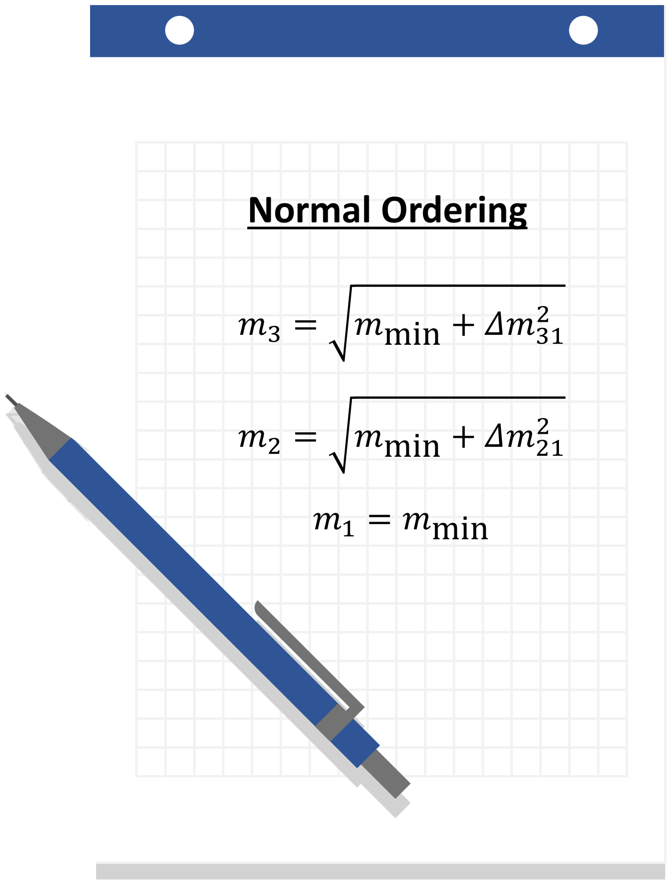
$\theta_{23}, \|\Delta m^2_{32}\|$	θ_{13}, δ	$\theta_{12}, \Delta m^2_{21}$	α_{21}, α_{31}
Accelerator Atmospheric	Reactor Accelerator	Solar Reactor	Double-beta decay

Parameters from global fits

Ordering	θ_{12} (°)	θ_{13} (°)	θ_{23} (°)	δ (°)	Δm^2_{21} $(10^{-5}\,\mathrm{eV}^2)$	$\Delta m^2_{31(32)}$ $(10^{-3}\,\mathrm{eV}^2)$
NO	$33.41^{+0.75}_{-0.72}$	$8.58^{+0.11}_{-0.11}$	$42.1^{+1.1}_{-0.9}$	232^{+36}_{-26}	$7.41^{+0.21}_{-0.20}$	$2.507^{+0.026}_{-0.027}$
IO	$33.41^{+0.75}_{-0.72}$	$8.57^{+0.11}_{-0.11}$	$49.0^{+1.0}_{-1.2}$	276^{+22}_{-29}	$7.41^{+0.21}_{-0.20}$	$-2.486^{+0.025}_{-0.028}$

I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou (2020), NuFIT 5.2 (2022), www.nu-fit.org

Neutrino mass spectrum

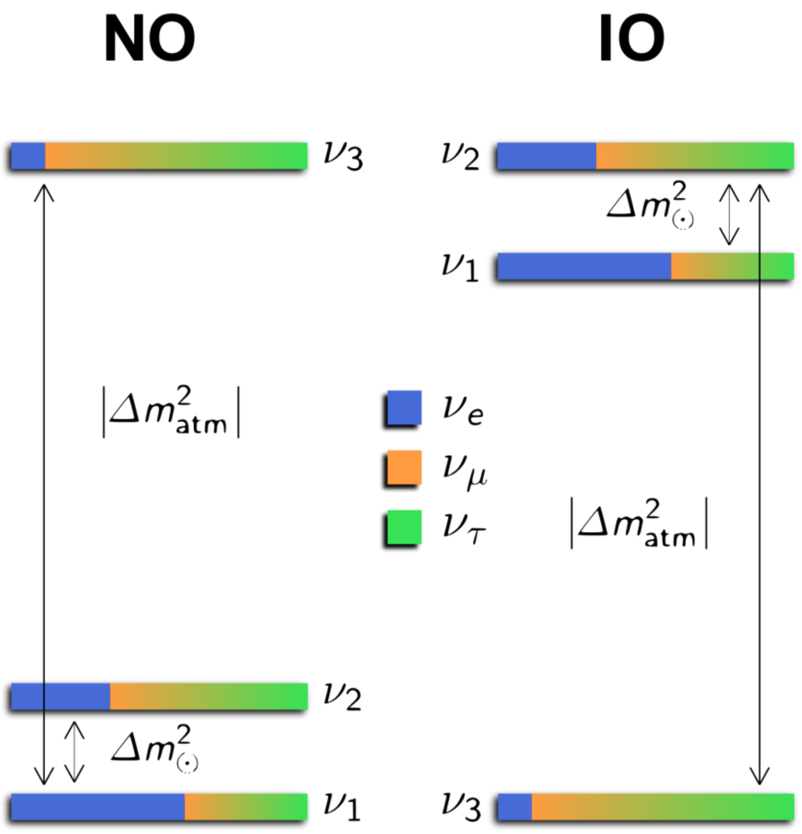


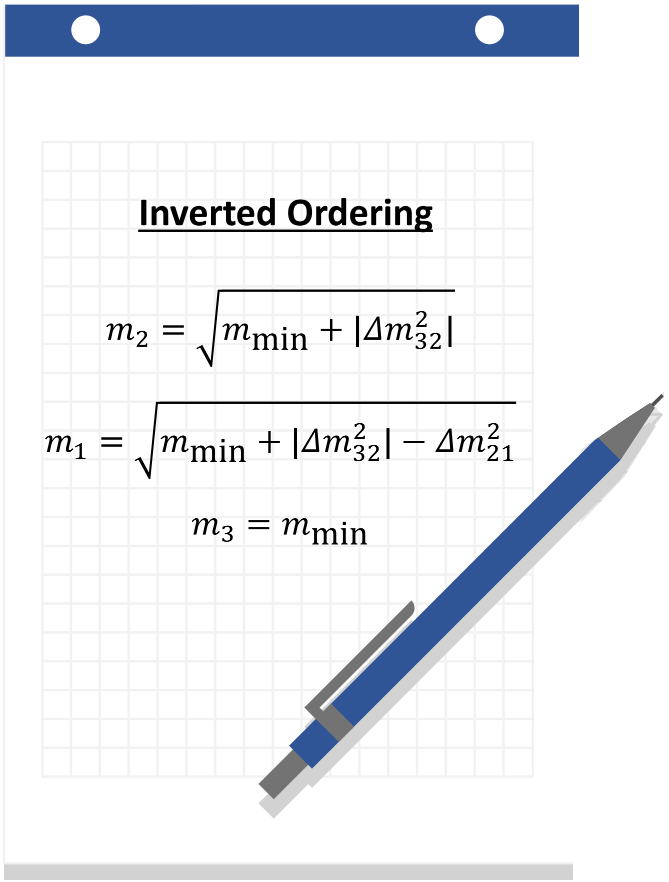
Normal Ordering

$$m_3 = \sqrt{m_{\text{min}} + \Delta m^2_{31}}$$

$$m_2 = \sqrt{m_{\text{min}} + \Delta m^2_{21}}$$

$$m_1 = m_{\text{min}}$$





Inverted Ordering

$$m_2 = \sqrt{m_{\text{min}} + |\Delta m^2_{32}|}$$

$$m_1 = \sqrt{m_{\text{min}} + |\Delta m^2_{32}| - \Delta m^2_{21}}$$

$$m_3 = m_{\text{min}}$$

Measuring the masses requires:

- The absolute mass scale: m_{min} (can be zero)
- The mass ordering: preference of NO

Phenomenological questions for the future

What is the nature of neutrinos, **Dirac** or **Majorana?**

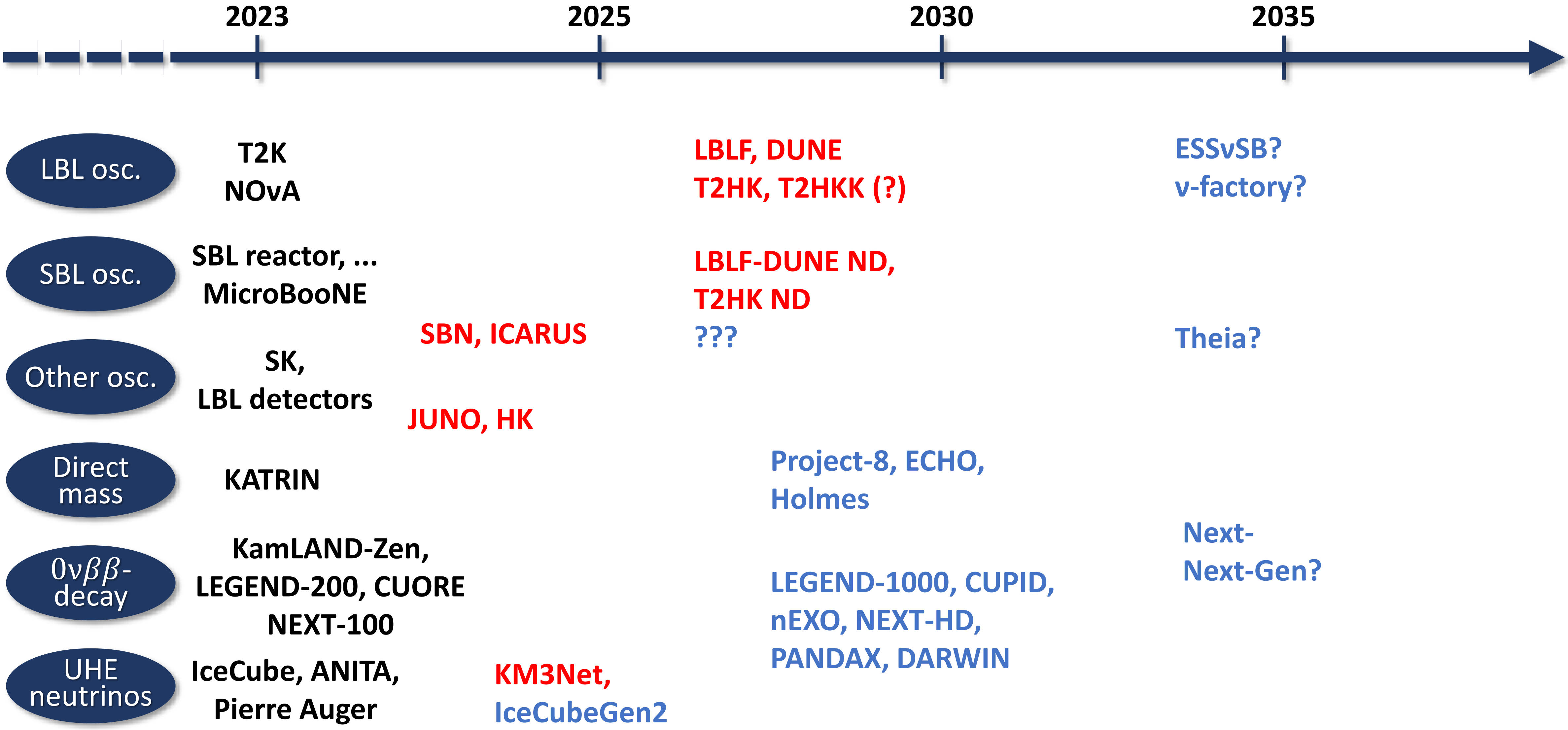
What are the **absolute mass scale** and the **mass ordering**?

Is there **CP-violation** in the PMNS lepton mixing matrix?

What are the **precise** values of the **mixing angles**?

Is the **standard picture correct**? Hints for BSM physics?

Future prospects of neutrino physics experiments



	2023	2025	2030	2035
LBL osc.	T2K NOvA		LBLF, DUNE T2HK, T2HKK (?)	ESSνSB? ν-factory?
SBL osc.	SBL reactor, ... MicroBooNE	SBN, ICARUS	LBLF-DUNE ND, T2HK ND ???	Theia?
Other osc.	SK, LBL detectors	JUNO, HK		
Direct mass	KATRIN		Project-8, ECHO, Holmes	
$0\nu\beta\beta$-decay	KamLAND-Zen, LEGEND-200, CUORE NEXT-100		LEGEND-1000, CUPID, nEXO, NEXT-HD, PANDAX, DARWIN	Next-Next-Gen?
UHE neutrinos	IceCube, ANITA, Pierre Auger	KM3Net, IceCubeGen2		

The nature of massive neutrinos

What is the nature of neutrinos, **Dirac** or **Majorana**?

If neutrinos are **Majorana** particles:

$$\nu = C(\overline{\nu})^T$$

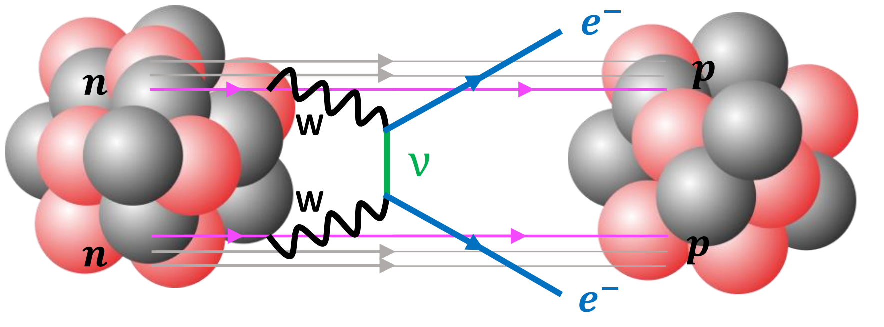
Lepton number is not conserved

LNV processes:

0νββ-decay, μ and τ decays, colliders.

Also crucial for **Leptogenesis**

Neutrinoless double-beta decay



$$m_{\beta\beta} = m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}$$

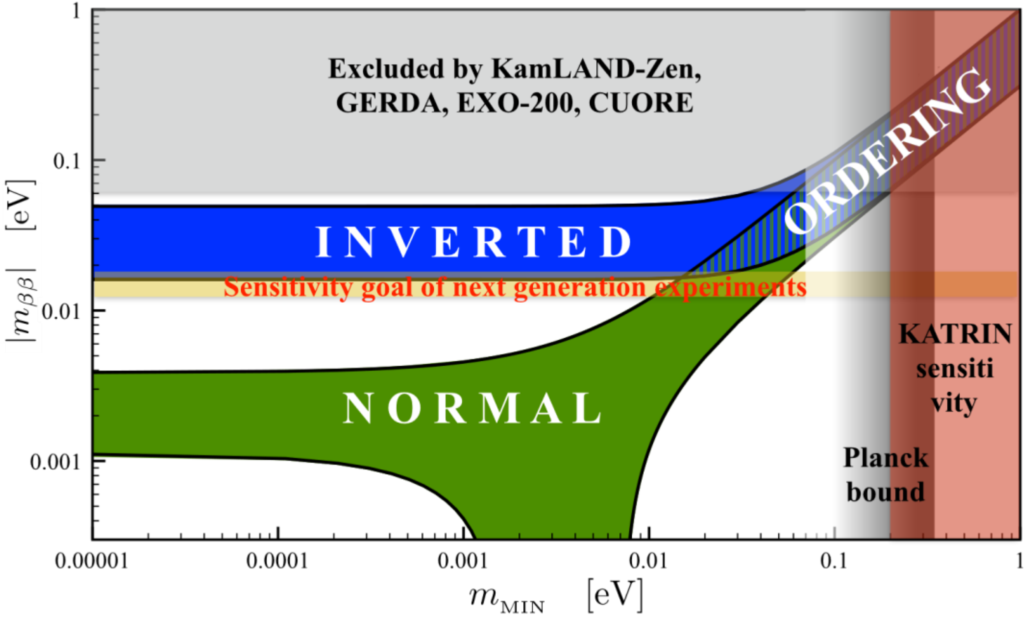


Fig. from APPEC DBD committee, arXiv:1910.04688.

Absolute mass scale and mass ordering

What are the **absolute mass scale** and the **mass ordering**?

Absolute mass scale

- Beta-decay
- 0νββ-decay (interplay with CP-violating phases)
- Cosmology (CMB, SNe, LSS, matter power spectrum)

Mass ordering

- Neutrino oscillations in vacuum (reactor neutrinos) thanks to relatively large θ_{13}
- Neutrino oscillations in matter (atmospheric neutrinos, long baseline neutrino oscillations)
- 0νββ-decay

CP-violation in the lepton mixing matrix

The Dirac phase

- The Dirac phase δ generates CP-violating effects in oscillations

$$P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

- Long baseline neutrino experiments are sensitive to δ.
- Some hints of CP-violation from T2K and NOvA, but still undetermined

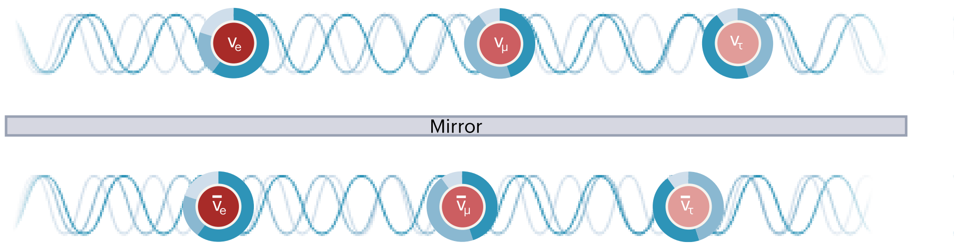


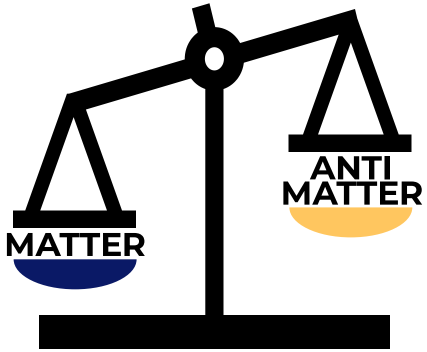
Fig. from Nature, S. Pascoli and J. Turner, 15 April 3020.

Is there **CP-violation** in the PMNS lepton mixing matrix?

The Majorana phases

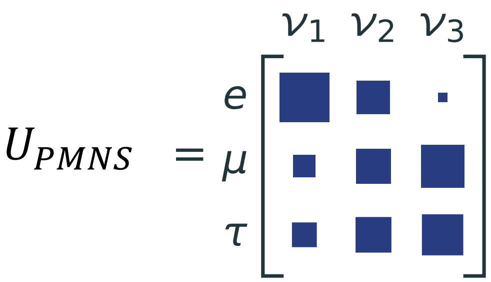
- Neutrino oscillation experiments are insensitive to the Majorana phases.
- Relevant in $0\nu\beta\beta$-decay, cLFV processes ($\mu \to e\gamma, \tau \to \mu\gamma$, etc.) in some models (e.g., with seesaw mechanism).

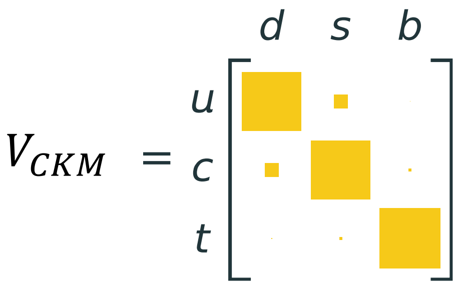
Matter-antimatter asymmetry of the Universe?

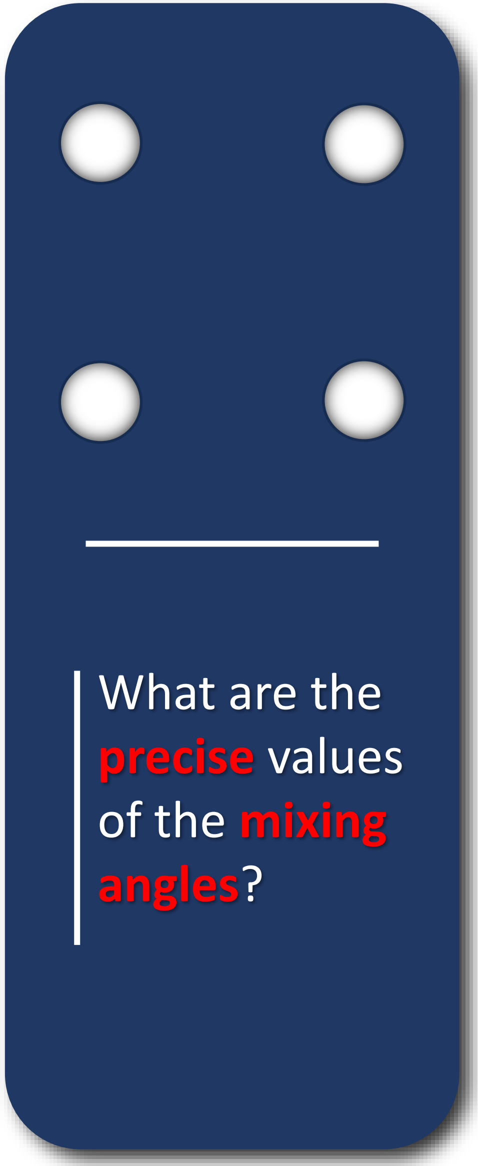


The flavour mixing pattern

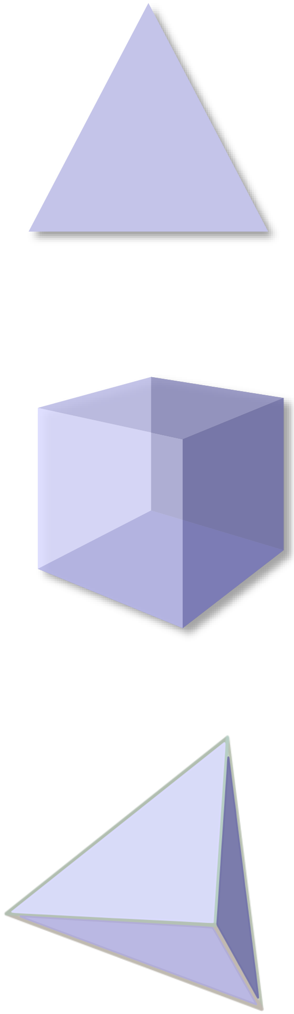
- The angle θ_{23} is currently not known precisely. How close is it to the maximal value, i.e. $\theta_{23} \sim \pi/4$?
- After θ_{12}, θ_{13} , θ_{23} and δ are measured, we should seek for precision. Next generation experiments?
- Crucial for understanding the flavour problem. Is there an underlying (broken) symmetry? E.g. Non-abelian discrete groups and modular invariance (G. Altarelli and F. Feruglio 1002.0211 , F. Feruglio, 1706.08749 and vast literature).





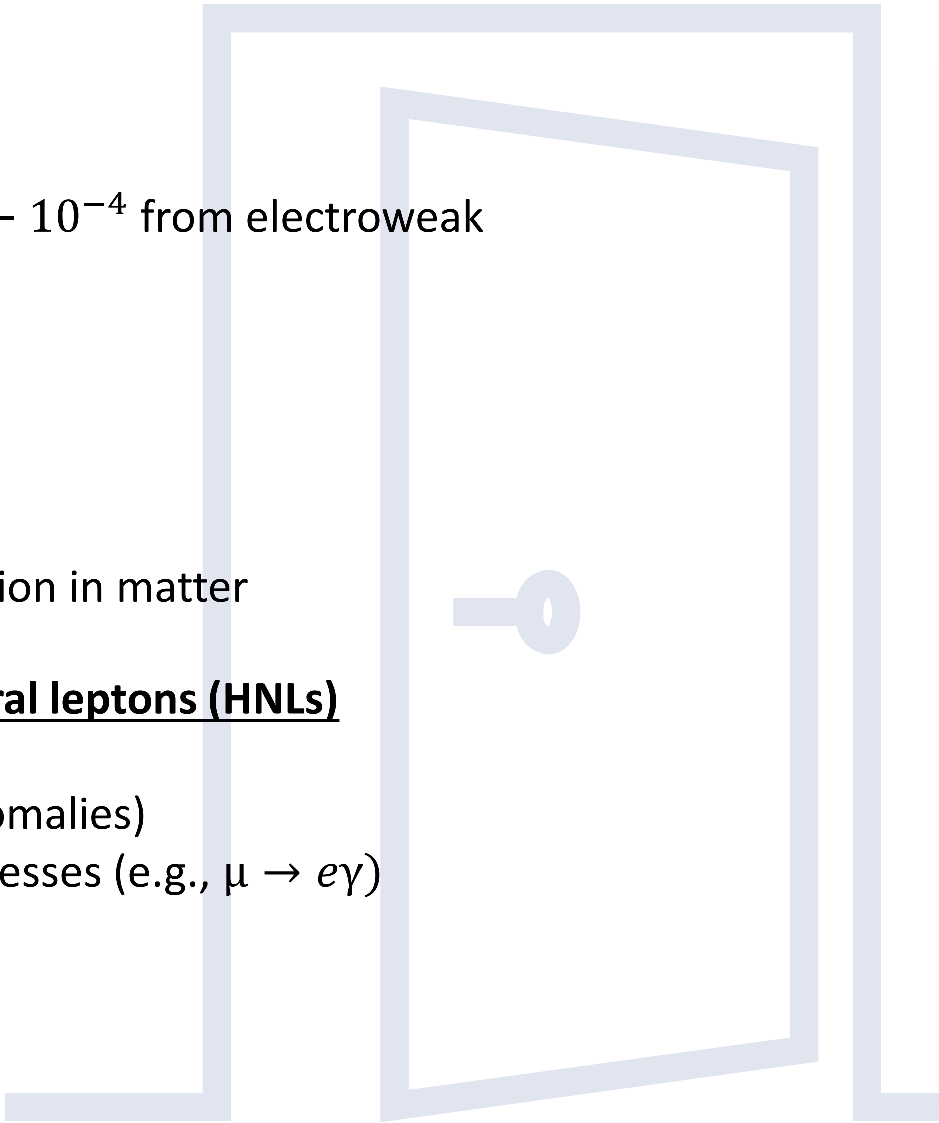


What are the **precise** values of the **mixing angles**?



Beyond the standard picture

Being the least known fundamental particles,
neutrinos provide a door to Physics beyond the SM.



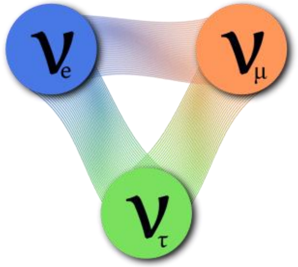
- **Non-unitarity of the PMNS matrix**
  $U = (1 + \eta)U_{PMNS}$, bounds on $|\eta_{\alpha\beta}| \lesssim 10^{-5} - 10^{-4}$ from electroweak precision data and data on flavour observables
- **Dark sector connections**
  Modify the oscillation in a «dark» environment
- **Non-standard interactions**
  Modify the production, detection and propagation in matter
- **Existence of sterile neutrinos, also heavy neutral leptons (HNLs)**
  - Modify the oscillation (some unresolved anomalies)
  - Induce charged lepton flavour violating processes (e.g., $\mu \to e\gamma$)
  - Effects on the $0\nu\beta\beta$-decay rate
  - Leptogenesis
  - Dark matter
  - ...

Is the **standard picture correct?**
Hints for BSM physics?

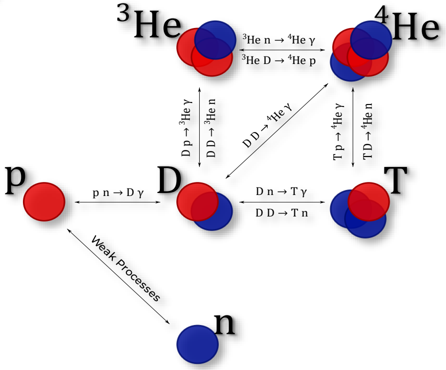
Evidence of physics beyond the SM

There is evidence of physics **beyond the Standard Model**: **neutrinos play a key role!**

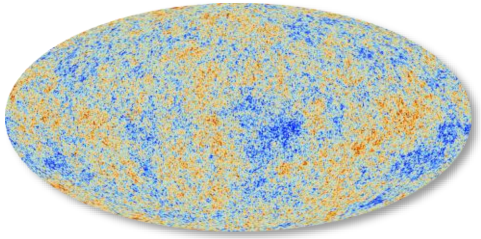
Neutrino masses and mixings



The baryon asymmetry of the Universe



$$\eta_B = \frac{(n_B - n_{\bar{B}})}{n_\gamma} \simeq 6.1 \times 10^{-10}$$



Dark Matter

Key questions

Where do neutrino masses come from?
What is the origin of the lepton mixing pattern?

Origin of neutrino masses

Dirac mass term

$$\mathcal{L}_{\mathrm{Dirac}}(x) = -\left(Y_{\alpha j}\overline{\psi_{\alpha L}}(x)\, i\sigma_2\, \Phi^*(x)\, N_{jR}(x) + \mathrm{h.c.}\right) \xrightarrow{\text{EWSSB}} m_\nu = 0.1\ \mathrm{eV}\left(\frac{v}{100\ \mathrm{GeV}}\right)\left(\frac{Y}{10^{-12}}\right)$$

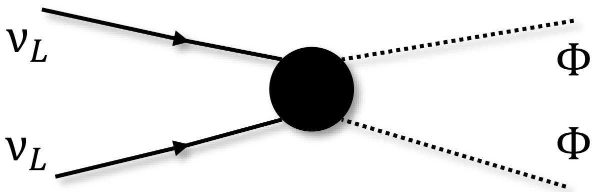
"Unnaturally" small Yukawas!

Total lepton number is conserved!

Majorana mass term

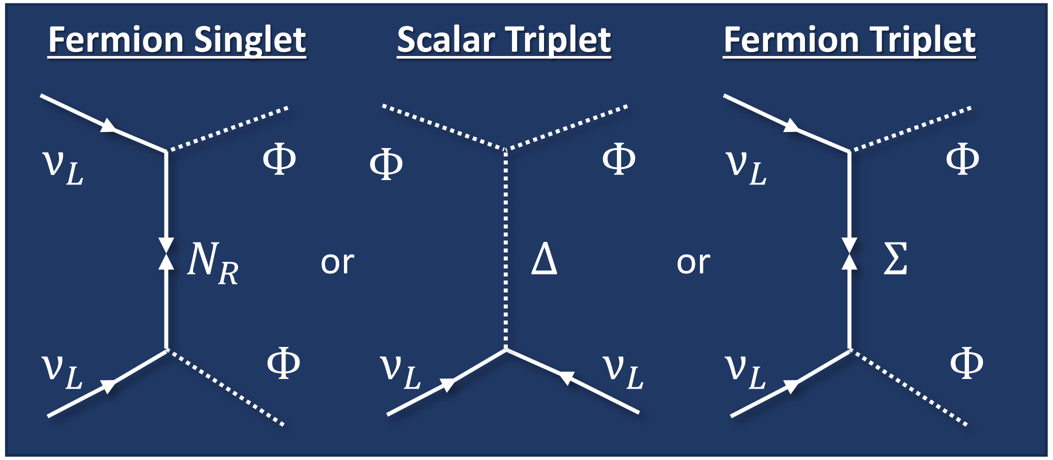
5-dim. effective operator (Weinberg, PRL 1943):

$$\mathcal{L}_{\mathrm{Weinberg}}(x) = -\left(\frac{\lambda}{2}\frac{\Phi\cdot\overline{\psi_L^c}\,\psi_L\cdot\Phi}{M} + \mathrm{h.c.}\right) \to -\frac{1}{2}\frac{v^2}{M}\overline{\nu_L^c}\nu_L + \mathrm{h.c.}$$



Total lepton number is violated!

High-energy renormalisable interactions

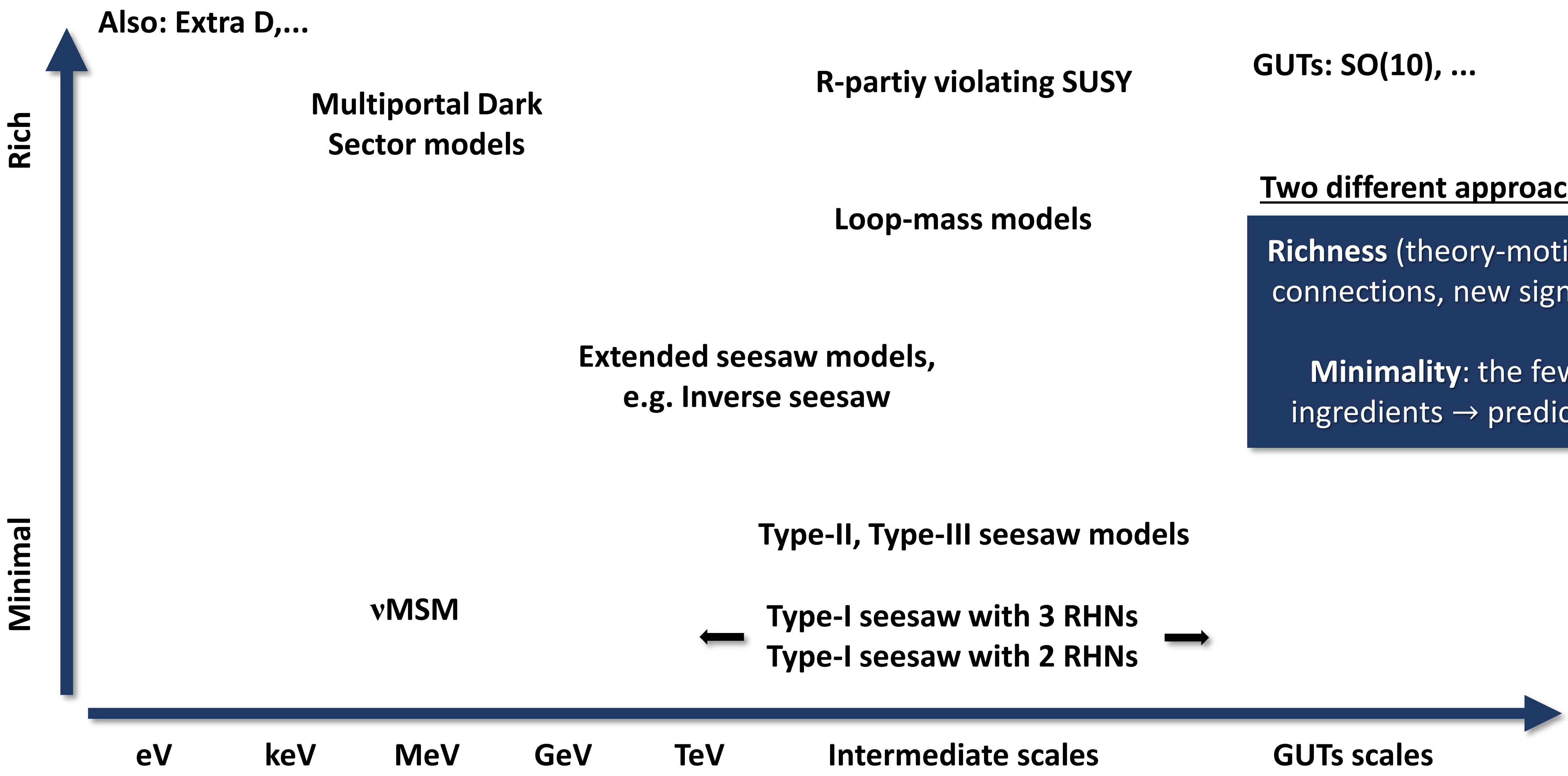


Type-I Seesaw

Type-II Seesaw

Type-III Seesaw

New physics scale, from minimality to richness



Also: Extra D,...

Rich

Multiportal Dark Sector models

R-partiy violating SUSY

GUTs: SO(10), ...

Loop-mass models

Extended seesaw models, e.g. Inverse seesaw

Type-II, Type-III seesaw models

Minimal

νMSM

← Type-I seesaw with 3 RHNs →
Type-I seesaw with 2 RHNs

eV keV MeV GeV TeV Intermediate scales GUTs scales

<u>Two different approaches</u>

Richness (theory-motivated): connections, new signatures

Minimality: the fewest ingredients → predictivity

Type-I seesaw mechanism

Seesaw lagrangian

Yukawa and mass terms

$$\mathcal{L}_{\mathrm{Y,M}}(x) = -\left(Y_{\alpha j}\,\overline{\psi_{\alpha L}}(x)\, i\sigma_2\, \Phi^*(x)\, N_{jR}(x) + h.c.\right) - \frac{1}{2}\, M_j\, \overline{N_j}(x)\, N_j(x)$$

Right-handed neutrinos (RHNs)/ sterile neutrinos/ heavy Majorana neutrinos

Type-I seesaw mechanism

Seesaw lagrangian

Yukawa and mass terms

$$\mathcal{L}_{\mathrm{Y,M}}(x) = -\left(Y_{\alpha j}\overline{\psi_{\alpha L}}(x)\, i\sigma_2\, \Phi^*(x)\, N_{jR}(x) + h.c.\right) - \frac{1}{2} M_j\, \overline{N_j}(x)\, N_j(x)$$

Right-handed neutrinos (RHNs)/ sterile neutrinos/ heavy Majorana neutrinos

Electroweak Symmetry Breaking

Neutrino mass generation

Neutrino mass matrix

$$m_\nu \simeq -(v^2/2) Y \widehat{M}^{-1} Y^T$$

Neutrino mixing

$$\nu_{\alpha L} \simeq U_{\alpha a}\nu_{aL} + \Theta_{\alpha j} N_{jR}^c$$

$$\Theta_{\alpha j} \simeq \left(v/\sqrt{2}\right) Y_{\alpha j}/M_j$$

Mixing angle/Coupling

Type-I seesaw mechanism

Seesaw lagrangian

Yukawa and mass terms

$$\mathcal{L}_{\mathrm{Y,M}}(x) = -\left(Y_{\alpha j}\overline{\psi_{\alpha L}}(x)\, i\sigma_2\, \Phi^*(x)\, N_{jR}(x) + h.c.\right) - \frac{1}{2} M_j\, \overline{N_j}(x)\, N_j(x)$$

Right-handed neutrinos (RHNs)/ sterile neutrinos/ heavy Majorana neutrinos

Electroweak Symmetry Breaking

Neutrino mass generation

Neutrino mass matrix

$$m_\nu \simeq -(v^2/2)\, Y \widehat{M}^{-1} Y^T$$

Neutrino mixing

$$\nu_{\alpha L} \simeq U_{\alpha a}\nu_{aL} + \Theta_{\alpha j} N_{jR}^c$$

$$\Theta_{\alpha j} \simeq (v/\sqrt{2})\, Y_{\alpha j}/M_j$$

Mixing angle/Coupling

Phenomenology

RHNs – SM interaction

$$\mathcal{L}_{\mathrm{C.C.}}(x) = -\frac{g_{\mathrm{W}}}{\sqrt{2}}\overline{\alpha_{\mathrm{L}}}(x)\, \mathrm{W}^{\mu}(x)\gamma_{\mu}\, \Theta_{\alpha j}\, N_{jR}^c(x) + \mathrm{h.c.}$$

$$\mathcal{L}_{\mathrm{N.C.}}(x) = -\frac{g_{\mathrm{W}}}{2c_W}\overline{\nu_{\alpha \mathrm{L}}}(x)\, \mathrm{Z}^{\mu}(x)\gamma_{\mu}\, \Theta_{\alpha j}\, N_{jR}^c(x) + \mathrm{h.c.}$$

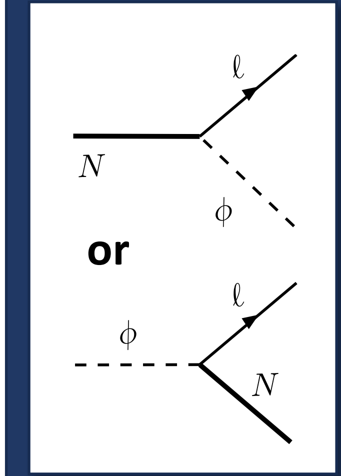
- Signatures of RHNs at low-energy
- BAU generation through Leptogenesis
- keV sterile neutrinos could be DM

See, e.g., the νMSM model (T. Asaka, S. Blanchet & M. Shaposhnikov, PLB 1995)

Leptogenesis within the type-I seesaw mechanism

Lepton Number violating processes via Yukawa coupling

Decays



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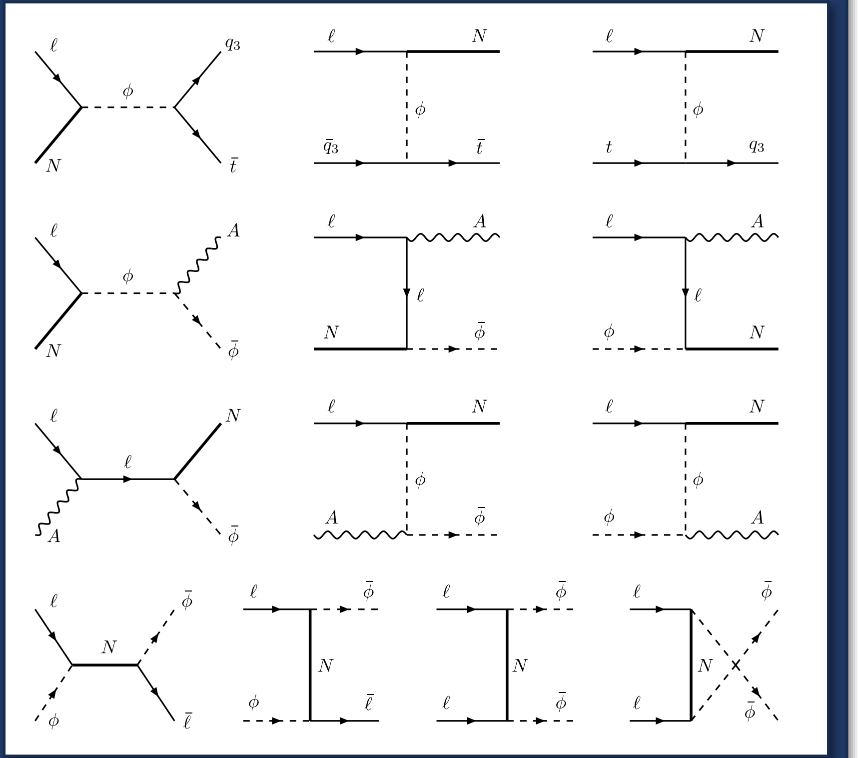
Scatterings



G. F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia hep-ph/0310123
S. Davidson, E. Nardi, Y. Nir arXiv:0802.2962

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CP-violation

$$\epsilon^{\mathrm{CP}} = \frac{\Gamma(N \to \mathrm{l} \ldots) - \Gamma(N \to \bar{\mathrm{l}} \ldots)}{\Gamma(N \to \text{anything})}$$

L. Covi, E. Roulet, F. Vissani
hep-ph/9605319,
W. Buchmuller, M. Plumacher
hep-ph/9710460,
A. Pilaftsis hep-ph/9702393,
...

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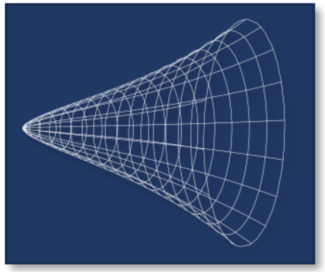
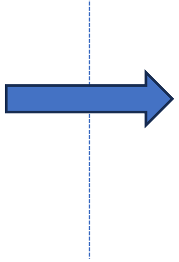
Expansion of the Universe



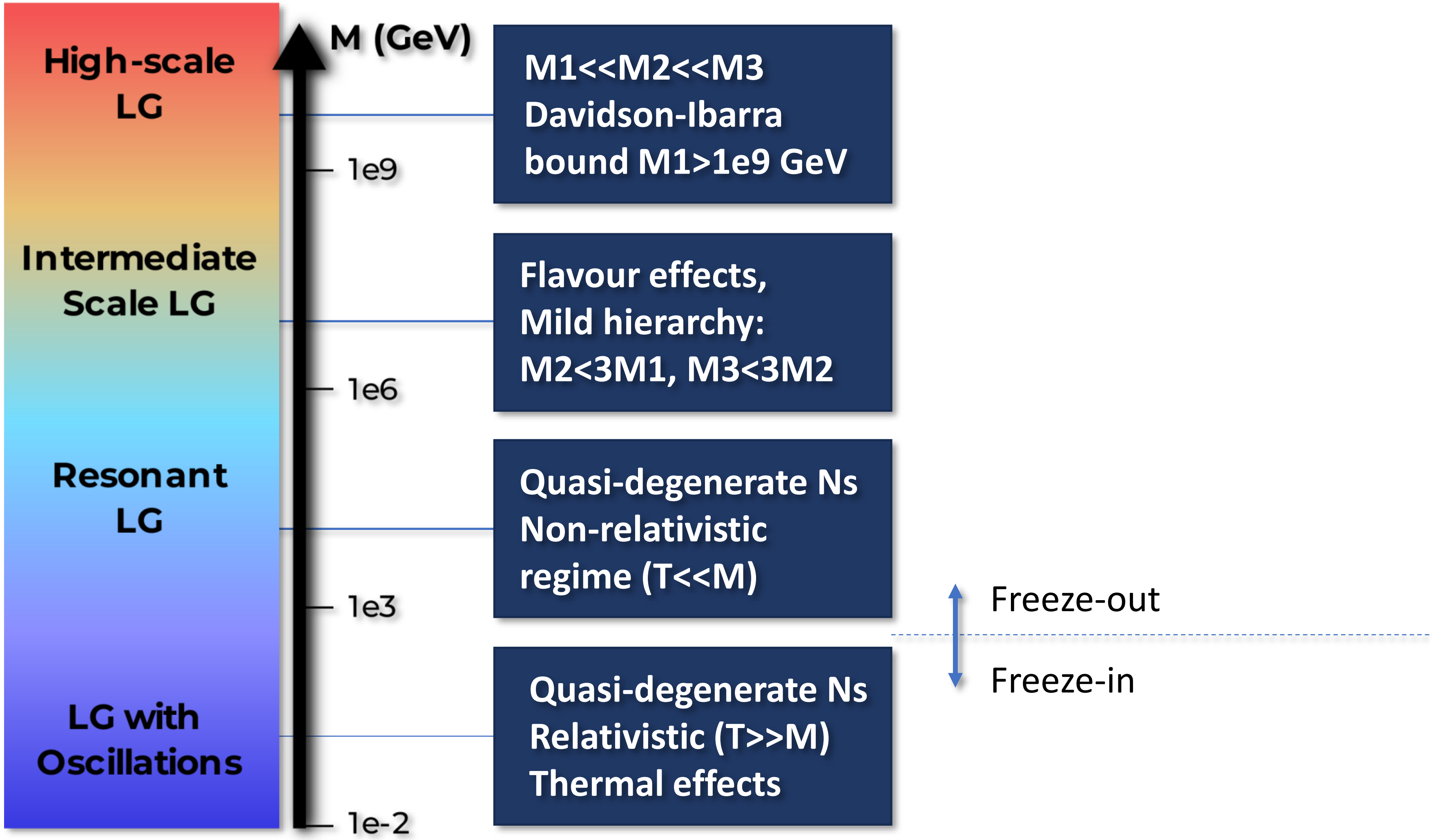


T ~ 130 GeV

BAU

Sphaleron freeze-out

Leptogenesis scales



Fukugita & Yanagida (1986)

High-scale LG

M1<<M2<<M3
Davidson-Ibarra bound M1>1e9 GeV

Racker, Rius & Pena (2012)

Intermediate Scale LG

Flavour effects,
Mild hierarchy:
M2<3M1, M3<3M2

Pilaftsis & Underwood (2003)

Resonant LG

Quasi-degenerate Ns
Non-relativistic regime (T<<M)

Ahmedov, Rubakov & Smirnov (1998)

LG with Oscillations

Quasi-degenerate Ns
Relativistic (T>>M)
Thermal effects

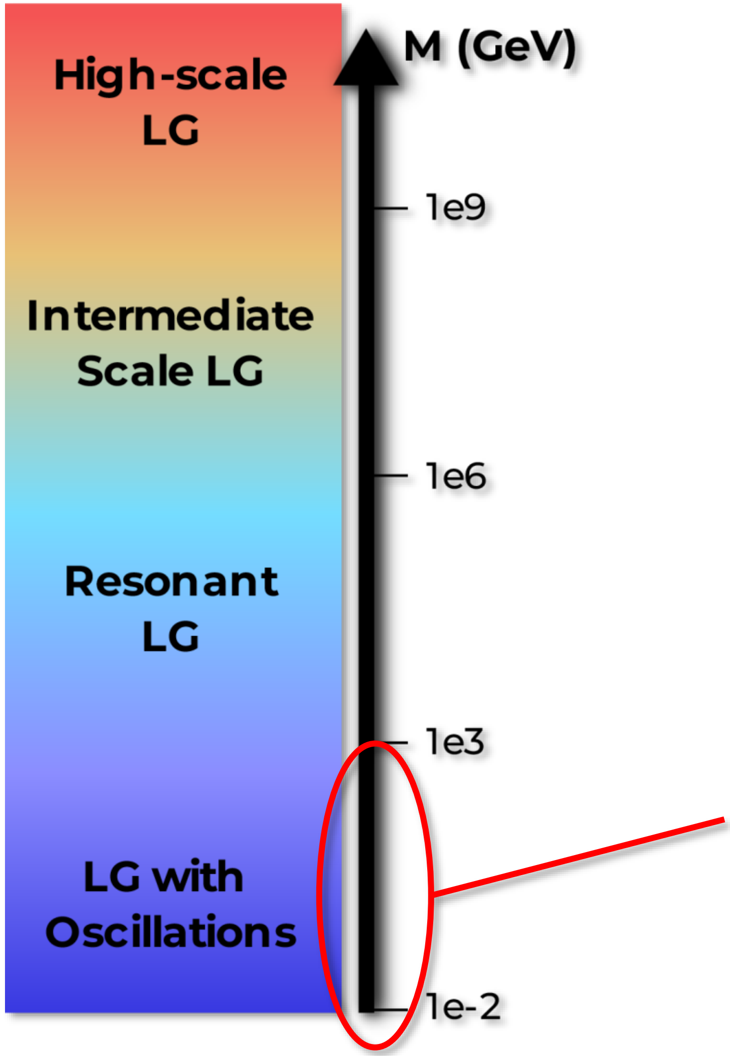
Leptogenesis scales

Fukugita &
Yanagida
(1986)

Racker, Rius &
Pena (2012)

Pilaftsis &
Underwood
(2003)

Ahmedov,
Rubakov &
Smirnov (1998)



High-scale LG

Intermediate Scale LG

Resonant LG

LG with Oscillations

Accessible energies!

Parameter space of viable LG

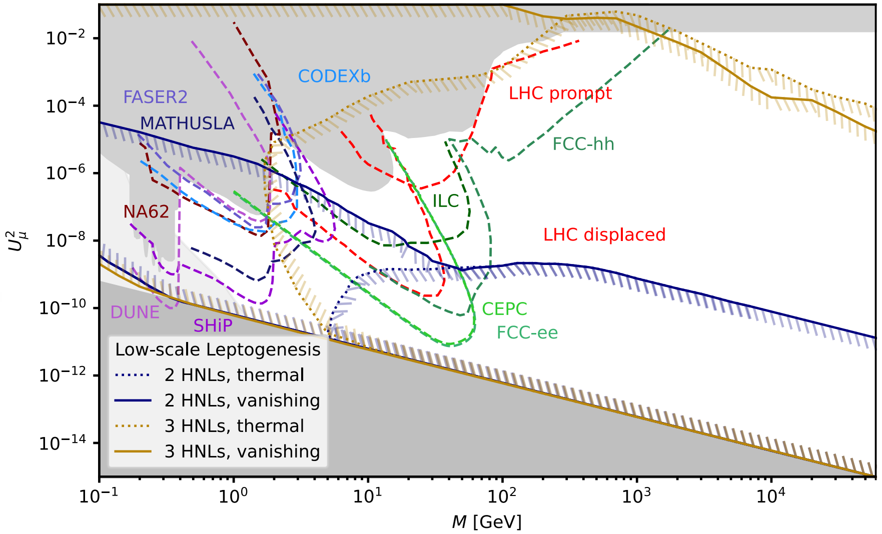


Fig. from A. M. Abdullahi et al., arXiv:2203.08039

Low-energy CP-violation in the type-I seesaw

Casas-Ibarra Parameterisation

$$Y = \pm i(\sqrt{2}/v)U\sqrt{\hat{m}}O^T\sqrt{\hat{M}}$$

Dirac phase δ
Majorana phases α_{21}, α_{31}

Low-energy CP-violation

Direct connection with low-energy experiments on neutrino oscillations and 0νββ-decay

Casas-Ibarra CP-violating phases

CP-symmetry at high-energy

Casas-Ibarra matrix must have entries that are either real or purely imaginary.

Dirac CP-violation:
the **Dirac phase** may be
the unique CP-violating phase in the neutrino sector,
and be responsible for **leptogenesis.**

Leptogenesis with low-energy leptonic CP-violation

The **connections between low-energy CPV and CPV in LG** has been studied in the high-scale scenario

- S. Pascoli, S. T. Petcov, A. Riotto (2007) – *Topic*: CPV properties and generalities
- S. Blanchet. P. Di Bari (2007) – CPV from the Dirac phase
- G. C. Branco, R. Gonzalez Felipe, and F. R. Joaquim (2007); S. Uhlig (2007); A. Anisimov, S. Blanchet, and P. Di Bari (2008); E. Molinaro and S. T. Petcov (2009); G. Bambhaniya, P. S. Bhupal Dev, S. Goswami, S. Khan, and W. Rodej (2017); M. J. Dolan, T. P. Dutka, and R. R. Volkas (2018)
- K. Moffat, S. Pascoli, S. T Petcov, J. Turner (2018), A.G., K. Moffat, S. T. Petcov (2022) – detailed numerical studies.

There are examples of **theoretically-motivated models** where low-energy CPV arise:

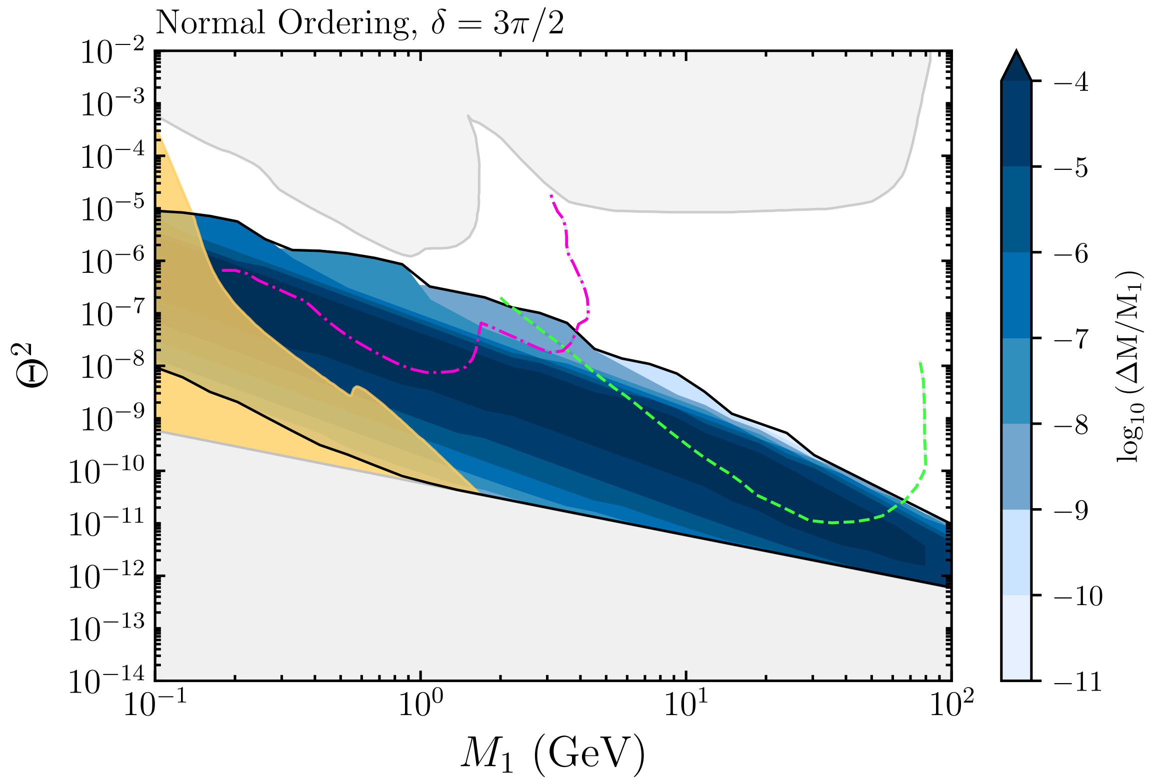
- S. F. King (2007) – based on sequential dominance
- P. Chen, G.-J. Ding, S. F. King (2016), C. Hagedorn, E. Molinaro (2016) – based non-abelian discrete groups and residual CP-symmetries

Low-scale LG with oscillations works with low-energy CPV solely from the Dirac phase:

- A. Granelli, S. Pascoli, S. T. Petcov (2023)

important connections to precise measurements of the **Dirac phase** in neutrino oscillations,
and the **mixing angle** Θ^2 and the **flavour ratios** $\Theta_\tau^2 : \Theta_\mu^2 : \Theta_e^2$ in the searches for heavy neutral leptons.

Low-scale LG with Dirac CP-violation


Normal Ordering, $\delta = 3\pi/2$

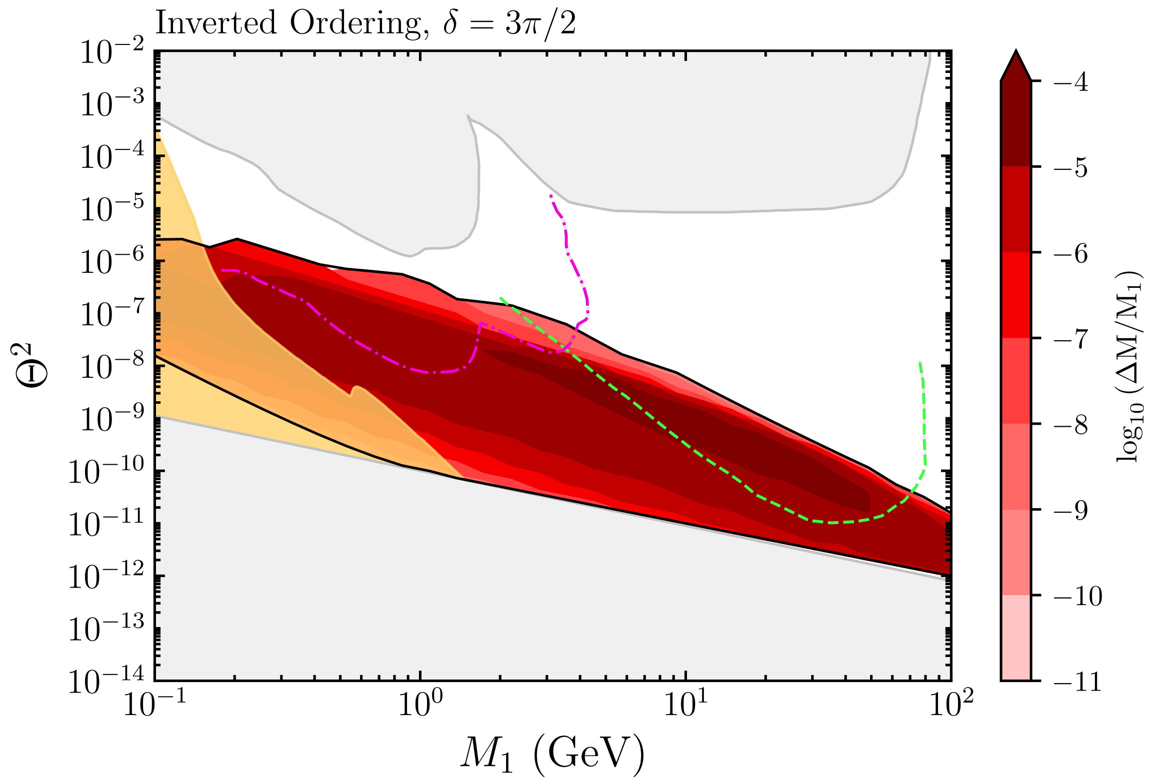
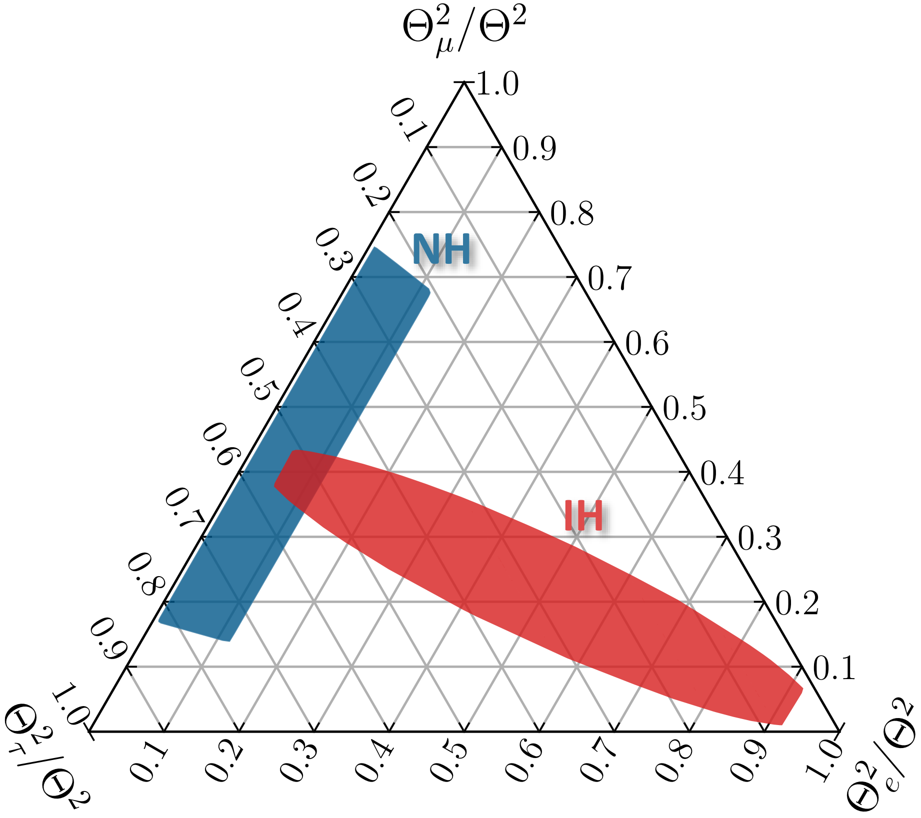

Inverted Ordering, $\delta = 3\pi/2$

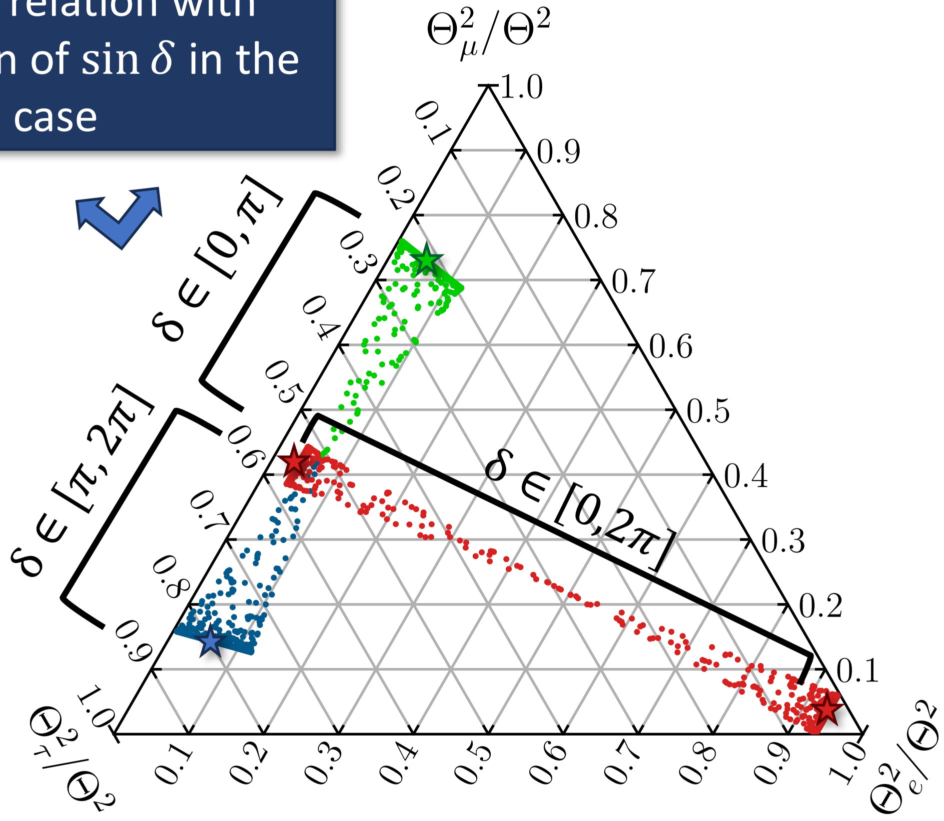
Fig. from A. G., S. Pascoli, S. T. Petcov arXiv:2307.07476.

Flavour ratios compatible with viable LG



LG with low- or high-energy CP-violation

Correlation with sign of sin δ in the NH case



Low-energy Dirac CP-violation

★ Large mixings $\xi > 1$, Θ^2 in the experimental region

Fig. from A. G., S. Pascoli, S. T. Petcov arXiv:2307.07476.

Conclusions

1. **Neutrino masses and mixing** are the first particle physics **evidence** of physics beyond the Standard Model. A vast exciting and promising experimental programme will provide us with **precise measurements** of the **lepton mixing angles, absolute masses** and **ordering, CP-violation** within the **next decades**.

2. Explaining the generation of neutrino masses and the lepton mixing beyond the standard model can have possible connections to the **matter-antimatter asymmetry of the Universe** and **Dark Matter**. Among the many proposed models, the **type-I seesaw mechanism** can explain the **neutrino mass generation**, the matter-antimatter asymmetry via **leptogenesis** and **dark matter**. A vast experimental programme can **test** part of the **viable parameter space of leptogenesis** within the type-I seesaw model in the **next decades**.

3. **Exciting** field of research with many discoveries and precision measurements expecetd in the next decades. Recent observations have opened a **new window of multimessenger astrophysics** with ultra high-energy cosmic neutrinos.

Thanks for your attention!