Quantum aspects of Metric-Affine Gravity

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Analogy with strong interactions. Chiral vs. gravitational dynamics

Chiral field $U \in (SU(2)_L \times SU(2)_R)/SU(2)_V$,

$$U = exp(\pi/f_\pi), \qquad \pi = pion, \quad f_\pi = pion\ decay\ constant.$$

Metric in $\mathbb{R}^4$ $\quad g \in GL(4)/O(1,3)$,

$$g = exp(h/m_P), \qquad h = graviton, \quad m_P = \sqrt{8\pi G} = Planck\ mass.$$

Chiral action

$$S = \int dx \left[\frac{f_\pi^2}{4} tr(U^{-1}\partial U)^2 + \ell_1 tr((U^{-1}\partial U)^2)^2 + \ell_2 tr((U^{-1}\partial U)^2)^2 + O(\partial^6)\right]$$

Gravitational action

$$S = \int dx \sqrt{g}\left[2m_P^2\Lambda + m_P^2 R + \ell_1 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \ell_2 R_{\mu\nu}R^{\mu\nu} + \ell_3 R^2 + O(\partial^6)\right]$$

$R \sim \Gamma\Gamma \sim (g^{-1}\partial g)^2$

Analogy with electroweak interactions. The Higgs phenomenon

Central question in the physics is status of symmetries.

Basic properties of many systems determined by Higgs mechanism.

Gravity is one such system.

$$S = \int \frac{1}{2} (D_\mu \phi^a)^2 - \frac{\lambda}{4} \left(\phi^2 - \upsilon^2\right)^2, \qquad D_\mu \phi^a = \partial_\mu \phi^a + A_\mu{}^a{}_b \phi^b.$$

$$< \phi^2 >= \upsilon^2 \neq 0, \qquad D_\mu \phi^a D^\mu \phi_a \rightarrow \upsilon^2 A_{\mu ab} A^{\mu ab}.$$

The kinetic term of Goldstone bosons is the mass term of the gauge field.

Non-linear sigma model ("Higgsless Higgs mechanism"):

$$S = \int \frac{1}{2} D_\mu \varphi^a D^\mu \varphi^b h_{ab}(\varphi), \qquad \varphi^a \varphi^b h_{ab}(\varphi) = 1.$$

unitary gauge $< \varphi^a >= 0$:

$$\upsilon^2 h_{ab}(\varphi) D_\mu \varphi^a D^\mu \varphi^b \rightarrow \upsilon^2 A_{\mu ab} A^{\mu ab}.$$

Gravity. GL(4) invariance

Consider arbitrary bases $\{e_a\}$ in the tangent spaces and $\{e^a\}$ in the cotangent spaces. Given a coordinate system x^μ, they are related to the coordinate bases by

$$e_a = \theta_a{}^\mu \partial_\mu \ , \qquad e^a = \theta^a{}_\mu dx^\mu \ .$$

The transformation matrices $\theta_a{}^\mu$ and $\theta^a{}_\mu$ are called the frame field and coframe field (a.k.a. soldering form). They can also be given a global geometrical meaning as isomorphisms between two bundles.

Then $\theta^a{}_\mu$ and the metric g_{ab} are nonlinear objects

- metric $g_{ab} \in GL(4)/O(1,3)$,
- frame field $\theta^a{}_\mu \in GL(4)$, $(\det\theta \neq 0)$,

They carry nonlinear realizations of $GL(4)$. Think of them as Goldstone bosons.

$$g_{\mu\nu} = \theta^a{}_\mu \, \theta^b{}_\nu \, g_{ab} \ ,$$
$$A_\lambda{}^\mu{}_\nu = \theta_a{}^\mu A_\lambda{}^a{}_b \theta^b{}_\nu + \theta_a{}^\mu \partial_\lambda \theta^a{}_\nu \ .$$

Two known gauge choices for $GL(4)$:

- $\theta^a_\mu = \delta^a_\mu$: coordinate frames - breaks $GL(4)$ completely
  $\rightarrow$ metric formulation
- $g_{ab} = \eta_{ab}$: orthonormal frames - breaks $GL(4)$ to $O(1,3)$ (Local Lorentz group)
  $\rightarrow$ vierbein formulation

The torsion tensor and the nonmetricity tensor Q

$$Q_{\lambda ab} = -D_\lambda g_{ab} = -\partial_\lambda g_{ab} + A_\lambda{}^c{}_a\, g_{cb} + A_\lambda{}^c{}_b\, g_{ac},$$
$$T_\mu{}^a{}_\nu = \partial_\mu \theta^a{}_\nu - \partial_\nu \theta^a{}_\mu + A_\mu{}^a{}_b\, \theta^b{}_\nu - A_\nu{}^a{}_b\, \theta^b{}_\mu$$

are the covariant derivatives of the Goldstone bosons.

$$F_{\mu\nu}{}^a{}_b = \partial_\mu A_\nu{}^a{}_b - \partial_\nu A_\mu{}^a{}_b + A_\mu{}^a{}_c A_\nu{}^c{}_b - A_\nu{}^a{}_c A_\mu{}^c{}_b,$$

	coefficients	covariant derivative	curvature
Independent connection	$A_\mu{}^a{}_b$	D_μ	$F_{\mu\nu}{}^a{}_b$
LC connection	$\Gamma_\mu{}^a{}_b$	∇_μ	$R_{\mu\nu}{}^a{}_b$

Gravitational Higgs mechanism

$$S_G(\theta, \gamma, A) = m^2 \int d^4x \sqrt{|g|} \left[T_{\dots} T^{\cdots} + Q_{\dots} Q^{\cdots} + T_{\dots} Q^{\cdots} \right]$$

expanding around flat background: $A = 0$, $\theta = \mathbf{1}$, $g = \eta$

$$\begin{aligned} T_\mu{}^a{}_\nu &= A_\mu{}^a{}_\nu - A_\nu{}^a{}_\mu \\ Q_{\mu ab} &= A_{\mu ab} + A_{\mu ba} \end{aligned}$$

kinetic term of Goldstone bosons becomes

$$S_G = m^2 \int d^4x \sqrt{|g|}\, A_{\dots} A^{\cdots}$$

In general a non-degenerate quadratic form.

EFT at low energies in presence of Higgs phenomenon

- Superconductivity: for $p \ll m_\rho$: $\quad \rho = \rho_0, \quad D_\mu \varphi = 0, \quad B = 0$.
- Electroweak: for $p \ll \upsilon$: $\quad D\sigma = 0, \quad F^a_{\mu\nu}|_{a \in (SU(2)_L \times U(1)_Y)/U(1)_Q} = 0.$
- MAG: or $p \ll M_{T,Q}$: $\quad Q = -Dg = 0, \quad T = d_\Delta \theta = 0, \quad F_{\mu\nu}{}^a{}_b = R_{\mu\nu}{}^a{}_b.$

Is Ostrogradsky ghost problem a problem? [Solomon, Trodden '18]

MAGs and Poincaré gauge theory

$\mathbb{R}^{1,3} \rtimes O(1,3)$

$e_a{}^{\mu}(x) - \delta_a{}^{\mu} =$ translation gauge field $\qquad T_{\mu}{}^{a}{}_{\nu} = \partial_\mu e^a{}_\nu + A_\mu{}^a{}_b\, e^b{}_\nu - (\mu \leftrightarrow \nu),$

$A_\mu{}^a{}_b(x) =$ Lorentz gauge field $\qquad F_{\mu\nu}{}^a{}_b = \partial_\mu A_\nu{}^a{}_b + A_\mu{}^a{}_c A_\nu{}^c{}_b - (\mu \leftrightarrow \nu).$

Same theory can be represented in various forms, for example via the distorsion tensor

$$A_\mu{}^a{}_b = \Gamma_\mu{}^a{}_b + \phi_\mu{}^a{}_b.$$

Actions can be written in different ways:

$$\mathcal{L} = \mathcal{L}(F_{\mu\nu\rho\lambda}, T, Q, D_\mu(A)),$$

$$\mathcal{L} = \mathcal{L}(R_{\mu\nu\rho\lambda}, \phi, T, Q, \nabla_\mu(\Gamma)).$$

Counting terms

Antisymmetric MAG (R&T):

R^2	$(\nabla T)^2$	$R\,\nabla T$	$R\,T^2$	$T^2\nabla T$	T^4	Total
3	9	2	14	31	33	92

[Christensen '80]

Symmetric MAG (R&Q):

R^2	$(\nabla Q)^2$	$R\,\nabla Q$	$R\,Q^2$	$Q^2\nabla Q$	Q^4	Total
3	16	4	22	59	69	173

General MAG (R&T&Q):

R^2	$(\nabla \phi)^2$	$R\,\nabla \phi$	$R\,\phi^2$	$\phi^2\nabla \phi$	ϕ^4	Total
3	38	6	56	315	504	922

In the general case we have 59 contributions to the flat-space 2-point function.

Spin projectors

	s	a
TT	2^+_4, 0^+_5	1^+_4
TL	1^-_7	1^-_8
LL	0^+_6	-

	ts	hs	ha	ta
TTT	3^-, 1^-_1	2^-_1, 1^-_2	2^-_2, 1^-_3	0^-
$TTL + TLT + LTT$	2^+_1, 0^+_1	-	-	1^+_3
$\frac{3}{2}LTT$	-	2^+_2, 0^+_2	1^+_2,	-
$TTL + TLT - \frac{1}{2}LTT$	-	1^+_1	2^+_3, 0^+_3	-
$TLL + LTL + LLT$	1^-_4	1^-_5	1^-_6	-
LLL	0^+_4	-	-	-

$SO(3)$ spin content of projection operators for a rank-2 and rank-3 tensors in $d = 4$ (a/s=(anti)symmetric,ts/ta=totally (anti)symmetric; hs/ha=hook (anti)symmetric).

In addition to the metric degrees of freedom, MAG may propagate:
3^-, $2^+ \times 3$, $2^- \times 2$, $1^+ \times 3$, $1^- \times 6$, $0^+ \times 4$, 0^-.

RG flow of Poincaré gauge theory

Consider the following action (in metric gauge):

$$\mathcal{S} = \frac{1}{2}\int d^4x\sqrt{g}\left(-a_0 F + c_1 F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda} + a_1 T_{\mu\nu\rho}T^{\mu\nu\rho}\right).$$

It leads to a kinetic operator with nonminimal terms:

$$F = -c_1\delta A_{\nu\rho\lambda}\Box\delta A^{\nu\rho\lambda} - c_1\delta A_{\mu\rho\lambda}\nabla_\mu\nabla^\nu\delta A^{\nu\rho\lambda} - \frac{a_0}{2}h_{\mu\nu}\Box h^{\mu\nu} - a_0 h^{\mu\nu}\nabla_\mu\nabla_\rho h^\rho{}_\nu$$
$$-\frac{1}{\alpha}\left(h_{\mu\nu}\nabla^\nu + \beta h\nabla_\mu - \gamma\,\delta T_\mu\right)\left(\nabla_\rho h^{\rho\mu} + \beta\nabla^\mu h + \gamma\,\delta T^\mu\right) + \dots$$

In general, there are three ways to deal with nonminimal operators:

1. gauge choice
2. off-diagonal heat kernel
3. field decomposition (transverse–longitudinal, York)

Fixing the Local Lorentz symmetry

$$\mathcal{S} = \frac{1}{2}\int d^4x\sqrt{g}\ det\ e\ \left(-a_0 F + c_1 F_{\mu\nu}{}^a{}_b F^{\mu\nu a}{}_b + a_1 T_\mu{}^a{}_\rho T^\mu{}_a{}^\rho\right).$$

Introduce new field variables

$$X^\mu{}_\nu := \bar{e}_a{}^\mu \delta e^a{}_\nu,$$
$$Z_\mu{}^\lambda{}_\nu := \bar{e}_a{}^\lambda \delta A_\mu{}^a{}_b \bar{e}^b{}_\nu, \qquad Z_{\mu\rho\nu} = Z_{\mu\nu\rho}.$$

$$\delta g_{\mu\nu} = X_{\mu\nu} + X_{\nu\mu} + X_{a\nu} X^a{}_\mu + \ldots,$$
$$\delta A_\mu{}^\alpha{}_\beta = -X^\alpha{}_\sigma Z_\mu{}^\sigma{}_\beta + X^\sigma{}_\beta Z_\mu{}^\alpha{}_\sigma + Z_\mu{}^\alpha{}_\beta - X^\alpha{}_\sigma \nabla_\mu X^\sigma{}_\beta + \nabla_\mu X^\alpha{}_\beta + \ldots.$$

This formulation possesses a symmetry:

$$X^\mu{}_\nu \to X^\mu{}_\nu - \epsilon^\mu{}_\nu + \epsilon^a{}_\nu \epsilon^\mu{}_a + \ldots$$
$$Z_\mu{}^\lambda{}_\nu \to Z_\mu{}^\lambda{}_\nu + \nabla_\mu \epsilon^\lambda{}_\nu - \epsilon^\lambda{}_\sigma \nabla_\mu \epsilon^\sigma{}_\nu + \ldots,$$

Fixing the Local Lorentz symmetry

For the former we take the gauge fixing condition to be

$$F_\mu = 2\sqrt{a_1}\nabla_\alpha X_\mu{}^\alpha = 0,$$

and we fix the gauge by adding the following term into the effective action:

$$S_{g.f.diffeos} = \frac{1}{2}\int d^4x\sqrt{-g}F_\mu g^{\mu\nu}F_\nu = a_1\int d^4x\sqrt{-g}\nabla_\alpha X_\mu{}^\alpha\nabla_\beta X^{\mu\beta}.$$

For the $O(1,3)$ symmetry we impose

$$\chi^\lambda{}_\rho = 2\sqrt{c_1}\nabla^\alpha Z_\alpha{}^\lambda{}_\rho = 0,$$

and the corresponding action term is

$$S_{g.f.GL(4)} = \frac{1}{2}\int d^4x\sqrt{-g}\chi^\mu{}_\nu g_{\mu\lambda}g^{\nu\rho}\chi^\lambda{}_\rho = \int d^4x\sqrt{-g}\; c_1\;\nabla^\alpha Z_{\alpha\lambda}{}^\rho\nabla^\beta Z_\beta{}^\lambda{}_\rho.$$

This way does not work for more general Lagrangians.

Unimodular Gauge Fixing

First, we partially break the symmetry by fixing the determinant of the metric to be one:

$$\sqrt{g} = \omega^2(x).$$

This leaves the theory invariant under the special (volume-preserving) diffeomorphisms *SDiff*.

$$g_{\mu\nu} = \bar{g}_{\mu\rho}\left(e^h\right)^\rho{}_\nu, \qquad h^\mu{}_\mu = 0.$$

To break it further we apply a second condition which the trace-free version of the de Donder condition:

$$\nabla^\nu h_{\mu\nu} - \nabla_\mu h + \delta T_\mu = \chi_\mu,$$

According to the Faddeev-Popov procedure, we add to the EA

$$S_{g.f.} = \lim_{\alpha\to\infty}\frac{1}{\alpha}\int d^4x\ \left(g-\omega^2\right) + \int d^4x\ \sqrt{g}\left(\chi_\mu\Box\chi^\mu\right),$$

UV divergences in the *Diff*-invariant version of a theory and the Unimodular version of it are the same. [de Brito, OM, Percacci, Pereira '21]

$$S = \frac{1}{2}\int d^4x\sqrt{g}\left[-m_0^2 R - b_1 T_{\nu\rho\lambda}\Box T^{\nu\rho\lambda} - b_2 T_{\nu\rho\lambda}\Box T^{\nu\lambda\rho} - b_3 T_\nu\Box T^\nu - \dots\right]$$

Schwinger-DeWitt (Heat Kernel) technique

[Schwinger '51, DeWitt '65]

A formal way to treat functional traces and determinants of local pseudo-differential operators (including but not necessarily Laplace-type). We define

$$H(s) = e^{-s\Delta}\,.$$

for $\Delta = -\Box + E$. It gives, for example, definitions of the propagator and 1-loop effective action as

$$\frac{1}{\Delta} = \int_0^\infty ds\; e^{-s\Delta} \qquad\qquad \Gamma_{1-\mathrm{loop}} = \frac{1}{2}\mathrm{Tr}\log\Delta = \frac{1}{2}\int_0^\infty \frac{ds}{s}\mathrm{Tr}e^{-s\Delta}$$

$$\mathrm{Tr}\;H(s) = \frac{1}{(4\pi s)^{d/2}}\sum_{n\geq 0}\int d^dx\sqrt{g}s^n\,\mathrm{tr}\,\overline{a_n}\,,$$

$\overline{a_n}(x)$ are local functions of the curvature invariants and their covariant derivatives.

What happens in the nonminimal case?

$$\Delta_{\mu\nu} = -g_{\mu\nu}\Box + \nabla_\mu\nabla_\nu + V_\mu\nabla_\nu + E_{\mu\nu}$$

1-loop EA in the nonminimal case.

$$F(\lambda) = F_{min} + \lambda N$$

[Barvinsky, Vilkovisky '85]

$$\Gamma_{1-loop} = \frac{1}{2} Tr\ log\ F$$

$$\Gamma_{1-loop}(\lambda) = \Gamma(\lambda = 0) + \frac{1}{2}\int_0^\lambda d\lambda\ Tr\left[\ F^{-1}(\lambda)\cdot\frac{dF}{d\lambda}\right],$$

$$\Gamma_{1-loop}(\lambda = 1) = \frac{1}{2} Tr\ log\ F_{min} + \frac{1}{2}\int_0^1 d\lambda\ Tr\left[G(\lambda)\cdot N\right].$$

Generalised Schwinger-DeWitt (Off-diagonal Heat Kernel) technique

$$F\ \ G = 1.$$

$$F_0\ \ G_0 = 1, \qquad \nabla_\mu \to n_\mu, \quad R \to 0, \quad T \to 0.$$

$$F\ \ G_0|_{n_\mu \to \nabla_\mu} = 1 + M(\nabla, R, R^2, T, T^2, R\nabla T, etc.),$$

$$G = G_0 \frac{1}{1+M} = G_0 \left[1 - M + M^2 - \dots\right]$$

sort derivatives: commute all contracted derivatives to the right (to form $\Box$'s)

$$[X, f(\Box)] = \sum_{n=1}^{\infty} \frac{1}{n!}(-1)^{n-1} \left[X\, ,\, \Box\right]_n f^{(n)}(\Box)$$

Universal functional traces

$$\mathrm{Tr}[\nabla_{\mu_1}\dots\nabla_{\mu_n} f(\Delta)] = \int d^d x \sqrt{g} \int ds\, \tilde{f}(s) H_{\mu_1\dots\mu_N}(x,s) =$$

$$= \frac{1}{(4\pi)^{d/2}} \sum_{n\geq 0} Q_{-n+\frac{d}{2}+\lfloor N/2 \rfloor}[f] \cdot tr \int d^d x\, \sqrt{g} K^{(n)}_{\mu_1\dots\mu_N}(x)$$

$$H(x,s) = (4\pi s)^{-d/2} \sum_{n\geq 0} s^n \overline{a_n}, \qquad \Delta = -\Box + E$$

$$H_\mu(x,s) = (4\pi s)^{-d/2} \sum_{n\geq 0} s^n \overline{\nabla_\mu a_n}, \qquad \Omega_{\mu\nu}\varphi = [\nabla_\mu, \nabla_\nu]\varphi$$

$$\overline{a_0} = 1 \qquad \overline{a_1} = -E + \frac{1}{6}R,$$

$$\overline{a_2} = -\frac{1}{6}\Box E + \frac{1}{2}E^2 - \frac{1}{6}RE + \frac{1}{12}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{72}R^2 - \frac{1}{180}R_{\mu\nu}^{\ 2} + \frac{1}{180}R_{\mu\nu\alpha\beta}^{\ 2},$$

$$\overline{\nabla_{(\nu}\nabla_{\mu)} a_0} = \frac{1}{6}R_{\nu\mu},$$

$$\overline{\nabla_{(\alpha}\nabla_\nu\nabla_{\mu)} a_0} = \frac{1}{4}R_{(\nu\mu;\alpha)}.$$

[*Groh*, *Saueressig*, *Zanusso*'11]

Gravity with propagating torsion

$$
\begin{aligned}
T_\mu &= T_\mu{}^\alpha{}_\alpha, \\
\check{T}_\mu &= \epsilon_{\mu\nu\rho\lambda} T^{\nu\rho\lambda}, \\
\hat{t}_{\alpha\beta\gamma} &= T_{\alpha\beta\gamma} - T_{[\alpha\beta\gamma]} - \frac{1}{6} g_{[\alpha\beta} T_{\gamma]}.
\end{aligned}
$$

$$
\begin{aligned}
S = \frac{1}{2}\int d^4x\sqrt{g}\Big[& -m_0^2 R + m_1 T_\mu T^\mu + m_2 \check{T}_\mu \check{T}^\mu + m_3 t_{\mu\nu\rho} t^{\mu\nu\rho} + \zeta R^2 + \eta C_{\mu\nu\rho\lambda} C^{\mu\nu\rho\lambda} \\
& + r_1 R\ \nabla_\mu T^\mu + r_2 C_{\mu\nu\rho\lambda} \nabla_\mu \hat{t}_{\nu\rho\lambda} - d_1 T_\mu \Box T^\mu + d_2 T_\mu \nabla_\mu \nabla^\nu T^\nu - d_3 \check{T}_\mu \Box \check{T}^\mu \\
& + d_4 \check{T}_\mu \nabla_\mu \nabla^\nu \check{T}^\nu - d_5 \hat{t}_{\mu\nu\rho} \Box \hat{t}^{\mu\nu\rho} + d_6 \hat{t}_{\mu\rho\lambda} \nabla^\mu \nabla_\nu \hat{t}^{\nu\rho\lambda} + d_7 \hat{t}_{\mu\rho\lambda} \nabla^\mu \nabla_\nu \hat{t}^{\nu\lambda\rho} \\
& + d_8 T_\mu \nabla_\nu \nabla_\rho \hat{t}^{\rho\nu\mu} + d_9 \epsilon_{\mu\nu\rho\lambda} T^\mu \nabla^\lambda \nabla_\sigma \hat{t}^{\sigma\nu\rho} + \ldots \Big].
\end{aligned}
$$

Even the first curvature squared term contributes to all other terms allowed by the symmetries.

Can field redefinitions help?

$$
T_{\alpha\beta\gamma} \to \alpha_1\ T_{\alpha\beta\gamma} + \alpha_2\ T_{\alpha\gamma\beta} + \alpha_3\ g_{\alpha\beta} T_\gamma + ?
$$

On-shell reduction of the effective action

Consider the following infinitesimal redefinitions of the fields and the corresponding change of the effective action:

$$\varphi \to \varphi + \Psi[\varphi], \qquad \varphi \gg \Psi[\varphi], \qquad \Gamma[\varphi] \to \Gamma[\varphi] + \frac{\delta \Gamma}{\delta \varphi}\Psi[\varphi].$$

Let us assume that the usual perturbative expansion of the effective action is valid:

$$\Gamma = S + \sum_{k=1}^{\infty} \hbar^k \Gamma^{(k)}, \qquad S \gg \hbar \Gamma^{(1)}, \qquad \Gamma^{(k)} \gg \hbar \Gamma^{(k+1)} \quad \forall k > 0.$$

This means that higher order (in $\hbar$) terms proportional to the equations of motion obtained from the lower order (in $\hbar$) terms can be eliminated from the effective action by appropriate field redefinitions. The obtained action we will refer to as the on-shell effective action:

$$\Gamma^{(1)} \approx \Gamma^{(1)}_{on-shell} + \frac{\delta S}{\delta \varphi}\Psi[\varphi],$$

$$\Gamma^{(k)} \approx \Gamma^{(k)}_{on-shell} + \frac{\delta}{\delta \varphi}\left(S + \sum_{l=1}^{k-1} \Gamma^{(l)}\right)\Psi[\varphi].$$

On-shell reduction of the effective action

Assuming that we are in a regime of validity of EFT:

$$|p| \ll \Lambda_{cutoff} \sim m_{Pl},$$

contributions of lower mass dimension are dominant.

$$\frac{\delta S}{\delta g_{\mu\nu}} g_{\mu\nu} \simeq \sum \text{all dim } \leq 2 \text{ terms}$$
$$2\frac{\delta S}{\delta g_{\mu\nu}} R_{\mu\nu} \simeq m_0^2 R_{\mu\nu} R^{\mu\nu} - \frac{1}{2} m_0^2 R^2$$
$$\frac{\delta S}{\delta g_{\mu\nu}} g_{\mu\nu} R \simeq -\frac{1}{2} m_0^2 R^2$$

$$\frac{\delta S}{\delta g_{\mu\nu}} \nabla_\gamma T_{\mu\nu}{}^{\gamma} \simeq \frac{1}{2} m_0^2 H_{RT}^5$$
$$2\frac{\delta S}{\delta g_{\mu\nu}} g_{\mu\nu} \nabla_\alpha T^\alpha \simeq -m_0^2 H_{RT}^3 + \frac{1}{2} m_0^2 H_{RT}^5$$

etc.

On-shell reduction of the effective action

$$
\begin{aligned}
g_{\mu\nu} &\to \gamma g_{\mu\nu} + \eta_1 R g_{\mu\nu} + \eta_2 R_{\mu\nu} + \eta_3 \nabla_{(\mu} T_{n)} + \eta_4 \nabla^{\rho} T_{\rho(\mu\nu)} + \dots \\
T_{\alpha\beta\gamma} &\to \alpha_1 \; T_{\alpha\beta\gamma} + \alpha_2 \; T_{\alpha\gamma\beta} + \alpha_3 \; g_{\alpha\beta} T_{\gamma} \\
&\quad + \epsilon_i^1 \nabla_{.} R_{..} + \epsilon_i^2 \nabla_{.} \nabla_{.} T_{...} + \epsilon_i^3 R_{....} T_{...} + \dots
\end{aligned}
$$

As a result, all the operators that give contributions to the propagator are "inessential".

$$
\Gamma^{(1)} \approx \frac{\delta S}{\delta \varphi} \Psi[\varphi] + \text{``}\textit{interaction terms}\text{''}.
$$

This means that on-shell the form of the propagator can be maintained.

Takeaways:

1. Gravity can be viewed in a way that is similar to particle physics (Chiral theory of pions, Electroweak Symmetry Breaking, Higgs mechanism).
2. When treated as an EFT, MAG explains why we see the Levi-Civita connection at low energies.
3. Even though it lacks predictivity at very high energies, it is consistent and predictive below the Plank mass. Quantum effects will come as loop corrections to G.
4. Field redefinitions are essential to understand whether a given Lagrangian is closed under renormalization. In the on-shell scheme, quantum corrections do not alter the form of the propagator.

Thank you!