Self-tuning inflation

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Introduction

Despite the success of general relativity, there are still unsolved problems, such as:

- GR describes only the classical aspects of gravitational interaction;
- The accelerated stages of the expansion of the Universe can be explained only beyond the Standard Model;
- There is no mechanism ensuring the observed smallness of the CC: $\Lambda \sim 10^{-122} m_{\rm Pl}^2$.

In an attempt to resolve these, a huge number of modified gravity models have been developed, and one of the most promising ways to extend the GR is $f(R)$ gravity.

Inflationary scenarios have increased interest in $f(R)$ theories, beginning with the seminal Starobinsky model (Starobinsky, Phys. Lett. B, 1980):

$$S_{\rm Starob} = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{|g_4|} \left(a_{\rm Starob} R^2 + R \right),$$

which has remained very successful up to now. The inflationary predictions originally calculated to the lowest order (Mukhanov, Chibisov, ZhETF Pisma, 1981):

$$n_s \simeq 1 - \frac{2}{N_{\rm e}} \simeq 0.9649 \pm 0.0042, \quad r \simeq \frac{12}{N_{\rm e}^2} < 0.032$$

are in good agreement with the Planck 2018 data with a combination of BICEP/ Keck Array 2018 and BAO (Tristram et al., Phys. Rev. D, 2022) for $50 < N_{\rm e} < 60$.

The R^2 multiplier obtained from the COBE normalization (Planck results, A. & A., 2020) is

$$a_{\rm Starob} \simeq 1.12 \cdot 10^9 \left(\frac{N_{\rm e}}{60} \right)^2 m_{\rm Pl}^{-2}.$$

$$S_{\mathrm{Starob}} = \frac{m_{\mathrm{Pl}}^2}{2} \int d^4x \sqrt{|g_4|} \Big(\underbrace{a_{\mathrm{Starob}}}_{\sim 10^9} R^2 + R + \underbrace{\ldots}_{\sim 10^{-122}} \Big)$$

Nevertheless, **there is no natural expectation for the coefficient** in front of the R^2-term in the action **to be large** and in the IR limit the Starobinsky model implies that the **cosmological constant must be fine-tuned**

These features of the Starobinsky model can be resolved (Asaka et al., PTEP, 2016) by considering it as a low-energy effective theory of a multidimensional theory due to the compactification of extra dimensions

$$S_{\mathrm{D}} = \tilde{\Lambda}^{\mathrm{D}} \int d^{\mathrm{D}}x \sqrt{|g_{\mathrm{D}}|} \sum_{\mathrm{k}=0} A_{\mathrm{k}} \left(\frac{R_{\mathrm{D}}}{\tilde{\Lambda}^2} \right)^{\mathrm{k}} \to S_4 = \tilde{\Lambda}^{\mathrm{D}} \mathcal{V}_{\mathrm{D}-4} \int d^4x \sqrt{|g_4|} \sum_{\mathrm{k}=0} A_{\mathrm{k}} \left(\frac{R}{\tilde{\Lambda}^2} \right)^{\mathrm{k}},$$

$$\text{where} \quad \tilde{\Lambda}^{\mathrm{D}-2} \mathcal{V}_{\mathrm{D}-4} \frac{A_1}{A_2} = \frac{m_{\mathrm{Pl}}^2}{2} \quad \text{and} \quad \tilde{\Lambda}^{\mathrm{D}-4} \mathcal{V}_{\mathrm{D}-4} = \frac{m_{\mathrm{Pl}}^2}{12 m^2} \equiv \frac{m_{\mathrm{Pl}}^2}{2} a_{\mathrm{Starob}} \sim 10^9 \quad \Rightarrow$$

$$S_4 = \frac{m_{\mathrm{Pl}}^2}{2} \int d^4x \sqrt{|g_4|} \left(R + a_{\mathrm{Starob}} R^2 + \sum_{\mathrm{k}=3}^{\infty} \frac{A_{\mathrm{k}}}{A_2^{\mathrm{k}-1}} \frac{R^{\mathrm{k}}}{a_{\mathrm{Starob}}^{\mathrm{k}-1}} \right).$$

However, the fine tuning of the CC and the extra space stabilization problem remain.

The model

We develop an inflationary model without small parameters on the basis of quadratic $f(R)$ gravity with a minimally coupled scalar field φ in $\mathrm{D} = 4 + \mathrm{n}$ dimensions

$$S = \frac{m_{\mathrm{D}}^{\mathrm{D}-2}}{2} \int d^{\mathrm{D}}x\sqrt{|g_{\mathrm{D}}|}\left(aR^2 + R + c + \partial^{\mathrm{M}}\varphi\,\partial_{\mathrm{M}}\varphi - 2V(\varphi)\right),$$

for the chosen metric of $M_1 \times M_3 \times M_{\mathrm{n}}$ manifold as

$$ds^2 = dt^2 - e^{2\alpha(t)}\delta_{ij}dx^i dx^j - e^{2\beta(t)}m_{\mathbf{D}}^{-2}\left((dx^4)^2 + r^2(x^4)(dx^5)^2 + \ldots + r^2(x^4)\prod_{\mathbf{k}=5}^{\mathbf{D}-2}\left(\sin^2 x^{\mathbf{k}}\right)(dx^{\mathbf{k}+1})^2\right)$$

The model is described by two stages:

1. The first one begins at energy scales about $m_{\mathrm{D}} \sim 10^{14}$ GeV and ends with the *de Sitter* metric of our space and *maximally symmetric* extra space.
2. At the second stage, the quantum scalar field fluctuations produce a wide set of *inhomogeneous* extra metrics in causally disconnected regions quickly generated in the de Sitter space.

A specific extra space metric that leads to the effective Starobinsky model that fits the observational data was found.

The first stage: high-energy space expansion

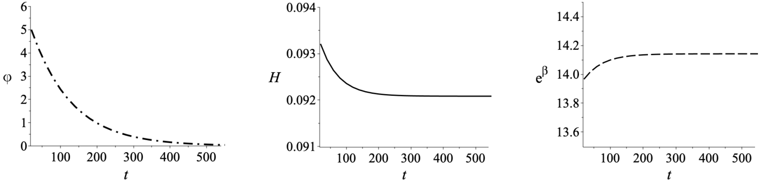
Let us consider metric, where M_{n} is the n-dimensional sphere, i.e., $r(x^4) = \sin x^4$,

$$ds^2 = dt^2 - \mathrm{e}^{2\alpha(t)}\delta_{ij}dx^i dx^j - \mathrm{e}^{2\beta(t)} m_{\mathrm{D}}^{-2} d\Omega_{\mathrm{n}}^2, \quad i,j = \overline{1,3}$$

and all dynamical variables depend only on time. Then, the system of equations for (tt), $(x^1x^1) = (x^2x^2) = (x^3x^3)$, $(x^4x^4) = \dots = (x^{\mathrm{D}-1}x^{\mathrm{D}-1})$–components and field equation lead to the asymptotic behavior of the metric and the scalar field: $\varphi_{as} = 0$

$$H_{as}^2 = \frac{-(\mathrm{n}+2) \pm \sqrt{(\mathrm{n}+2)^2 - 4a\mathrm{n}(\mathrm{n}+4)c}}{6a\mathrm{n}(\mathrm{n}+4)};$$

$$\mathrm{e}^{-2\beta_{as}} = \frac{-(\mathrm{n}+2) \pm \sqrt{(\mathrm{n}+2)^2 - 4a\mathrm{n}(\mathrm{n}+4)c}}{2a\mathrm{n}(\mathrm{n}+4)(\mathrm{n}-1)\, m_{\mathrm{D}}^2} \equiv \mathrm{e}^{-2\beta_{\mathbf{c}}}.$$



$$m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{as}} = \frac{3H_{as}^2}{(\mathrm{n}-1)}$$

The first stage: high-energy space expansion

Action takes the following form after integration over the extra coordinates

$$S^{I}_{eff} = \frac{m_{\rm Pl}^2}{2}\int d^4x\sqrt{|g_4|}\Big(a_{eff}R_4^2 + R_4 + c_{eff}\Big)$$

with effective parameters and the relationship between the 4D and D-dim Planck masses

$$a_{eff}\,m_{\rm Pl}^2 = \frac{1}{2}\,\mathcal{V}_{\rm n}m_{\rm D}^2 e^{{\rm n}\beta_{\rm c}} f_{RR}\big(R_{\rm n}\big), \quad m_{\rm Pl}^2 = \mathcal{V}_{\rm n}m_{\rm D}^2 e^{{\rm n}\beta_{\rm c}} f_R\big(R_{\rm n}\big), \quad c_{eff}\,m_{\rm Pl}^2 = \mathcal{V}_{\rm n}m_{\rm D}^2 e^{{\rm n}\beta_{\rm c}} f\big(R_{\rm n}\big)$$

here the n-dim Ricci scalar $R_{\rm n} = {\rm n}\big({\rm n}-1\big)\,m_{\rm D}^2 e^{-2\beta_{\rm c}}$ and volume $\mathcal{V}_{\rm n} = 2\pi^{\frac{\rm n+1}{2}}/\,\Gamma\left(\frac{\rm n+1}{2}\right)$.

In the case of

$$a = 20m_{\rm D}^{-2}, \quad c = -0.95m_{\rm D}^2, \quad {\rm n} = 6,$$

the effective parameter values become

$$a_{eff}\,m_{\rm Pl}^2 \simeq 2.65\cdot 10^9, \quad m_{\rm Pl}^2 \simeq 9.26\cdot 10^8\,m_{\rm D}^2, \quad c_{eff}\,m_{\rm Pl}^2 \simeq -4.63\cdot 10^7\,m_{\rm D}^4,$$

hence, it is the non-zero value of c_{eff} that causes the de Sitter metric of our 4D space.

The D-dim Planck mass to 4D relation gives expression in physical units

$$m_{\rm D} \sim 10^{14}\,{\rm GeV}, \quad H \sim 10^{13}\,{\rm GeV}\,, \quad \text{and} \quad e^{\beta_{\rm c}} \sim 10^{-27}\ {\rm cm}.$$

for $m_{\rm Pl} = M_{\rm Pl}/\sqrt{8\pi} = 2.4\cdot 10^{18}$ GeV.

The second stage: observable inflation, inhomogeneous extra dimensions

The high-energy stage is finished with 4D dS metric and maximally symmetrical extra space. As usual, the space expands exponentially, producing more and more causally disconnected volume, each of which is characterized by a specific extra metric and a scalar field distribution. The scalar field fluctuations at the dS stage can break the symmetry of the extra space metric, resulting in inhomogeneous metric formation.

Here we consider an inhomogeneous n-dimensional extra metric

$$ds^2 = dt^2 - e^{2Ht}\Big(\big(dx^1\big)^2 + \big(dx^2\big)^2 + \big(dx^3\big)^2\Big) - e^{2\beta_c} m_D^{-2}\Big(du^2 + r^2(u)\, d\Omega^2_{n-1}\Big).$$

To obtain a numerical solution, we use the definition of the Ricci scalar R as the additional unknown function to avoid 3rd and 4th order derivatives in the equations[1], resolving the corresponding system of equations with respect to unknown functions $r(u)$, $\varphi(u)$, and $R(u)$ with boundary conditions

$$r(u_{\mathbf{min}}) = 0, \quad r'(u_{\mathbf{min}}) = 1, \quad R(u_{\mathbf{min}}) = R_{\mathbf{0}}, \quad \varphi(u_{\mathbf{min}}) = \varphi_{\mathbf{0}},$$
$$R'(u_{\mathbf{min}}) = 0, \quad \varphi'(u_{\mathbf{min}}) = 0.$$

We suppose that $u_{\mathbf{min}} = 0$ is the regular center and $R_{\mathbf{0}}$ and $\varphi_{\mathbf{0}}$ values are linked by the constraint equation on the coupled second-order differential equations:

$$\Big(3(n+3)H^{\mathbf{2}} - R\Big) f_R + \frac{f(R)}{2} = -\frac{(\varphi')^{\mathbf{2}}}{2} m_D^{\mathbf{2}} e^{-\mathbf{2}\beta_c} + V(\varphi),$$

coming from the combination of equations $\Big((tt) - (x^{\mathbf{5}}x^{\mathbf{5}})\Big) \cdot (n-1) - (uu) - R \cdot f_R$.

[1] Note that equations are not independent:
$$(x^{\mathbf{5}}x^{\mathbf{5}}) = (uu) + \left(\frac{d}{du}(uu) + \frac{f_R}{2} \cdot \frac{d}{du}(R) + \varphi' \cdot (\varphi\varphi)\right) \cdot \frac{r}{r'\,(n-1)}$$

Observable inflation: numerical results

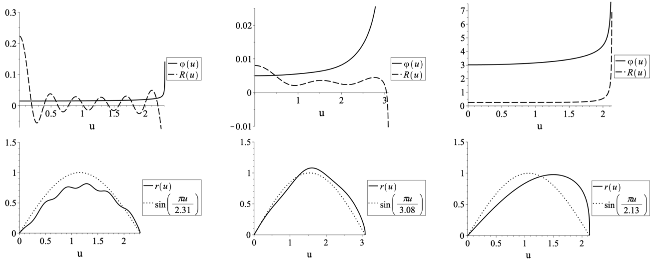
In the case of homogeneous scalar field distribution there is a solution $R_{\mathbf{c}}$ of constant curvature the constraint equation on the coupled second-order differential equations:

$$\Big(3(\mathrm{n}+3)H^{2}-R\Big)f_R+\frac{f(R)}{2}=-\frac{(\varphi')^{2}}{2}m_{\mathbf{D}}^{2}\mathrm{e}^{-2\beta_{\mathbf{c}}}+V(\varphi),$$

for any form of $f(R)$ function. For constant curvature the difference of $(tt)-(uu)$-comp. allows

$$r(u)=m_{\mathbf{D}}\mathrm{e}^{-\beta_{\mathbf{c}}}\frac{\sqrt{(\mathrm{n}-1)}}{\sqrt{3}H}\sin\left(\frac{\sqrt{3}H}{\sqrt{(\mathrm{n}-1)}}\mathrm{e}^{\beta_{\mathbf{c}}}m_{\mathbf{D}}^{-1}u\right)$$

to find analytically the function $r(u)$, $\forall f_R(R_{\mathbf{c}})\neq 0$ and then $R_{\mathbf{c}}=12H^{2}+3\mathrm{n}H^{2}$.



(a) : $a=-1.5$, $c=-0.005$, $\mathrm{n}=3$, $m=0.01$, $\beta_{\mathrm{c}}=3.80$, $H=0.018$, $\varphi_0=0.015$ and $R_0\simeq 0.223$.

(b) : $a=-45$, $c=-0.0021$, $\mathrm{n}=4$, $m=0.01$, $\beta_{\mathrm{c}}=4.48$, $H=0.011$, $\varphi_0=0.005$ and $R_0\simeq 0.008$.

(c) : $a=20$, $c=-0.95$, $\mathrm{n}=6$, $m=0.05$, $\beta_{\mathrm{c}}=2.65$, $H=0.092$, $\varphi_0=3.02$ and $R_0\simeq 0.254$.

As can be seen from some examples above for $\mathrm{n}=3,4,6$, and potential $V(\varphi)=\frac{1}{2}m^{2}\varphi^{2}$, the properties of inhomogeneous extra space vary significantly depending on the parameter values.

The deformed configuration leads to an alternation of the Lagrangian parameters a_{eff} and c_{eff} and launches the second, low-energy step of inflation.

At this stage, after integration over extra coordinates, action turns to the effective thy

$$S_{eff}^{II} = \frac{m_{\mathrm{Pl}}^2}{2} \int d^4x \sqrt{|g_4|} \left(a_{eff} R_4^2 + R_4 + c_{eff} \right)$$

with effective values of the parameters

$$a_{eff} = \mathcal{V}_{\mathrm{n}-1} \mathrm{e}^{\mathrm{n}\beta_{\mathbf{c}}} \frac{m_{\mathrm{D}}^2}{2m_{\mathrm{Pl}}^2} \int\limits_{u_{\mathbf{min}}}^{u_{\mathbf{max}}} f_{RR}\left(R_{\mathrm{n}}(u)\right) r^{\mathrm{n}-1}(u)\, du,$$

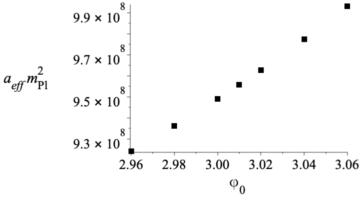
$$\frac{m_{\mathrm{Pl}}^2}{m_{\mathrm{D}}^2} = \mathcal{V}_{\mathrm{n}-1} \mathrm{e}^{\mathrm{n}\beta_{\mathbf{c}}} \int\limits_{u_{\mathbf{min}}}^{u_{\mathbf{max}}} f_R\left(R_{\mathrm{n}}(u)\right) r^{\mathrm{n}-1}(u)\, du,$$

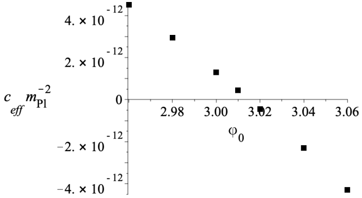
$$c_{eff} = \mathcal{V}_{\mathrm{n}-1} \mathrm{e}^{\mathrm{n}\beta_{\mathbf{c}}} \frac{m_{\mathrm{D}}^2}{m_{\mathrm{Pl}}^2} \int\limits_{u_{\mathbf{min}}}^{u_{\mathbf{max}}} \Big(f\left(R_{\mathrm{n}}(u)\right) - \left(\varphi'(u)\right)^2 m_{\mathrm{D}}^2 \mathrm{e}^{-2\beta_{\mathbf{c}}} - 2V\left(\varphi(u)\right) \Big)\, r^{\mathrm{n}-1}(u)\, du.$$

As follows from the more general form for c_{eff} above, it depends on the extra space metric and the scalar field distribution.

Observable inflation: fitting to observational data

The parameter values $a = 20 m_D^{-2}$, $c = -0.95 m_D^2$, n $= 6$, $m = 0.05 m_D$ suit our aims.





The parameter c_{eff} changes its sign somewhere near $\varphi_0 \simeq 3.015$. This means that we can find an extra metric for which $c_{eff} \simeq 0$ with arbitrary good accuracy. It allows one to calculate numerically other effective parameters, i.e., a_{eff} and the ratio m_{Pl}^2 / m_D^2.

As a result, we have $m_D \sim 10^{14}$ GeV, $H \sim 10^{13}$ GeV, $a_{eff} \simeq a_{Starob} \sim 10^9 m_{Pl}^{-2}$ and c_{eff} is negligibly small at the boundary value of the scalar field $\varphi_0 \simeq 3.015$.

This means that **the Starobinsky model is restored**, and **the values of the initial parameters have a reasonable deviation from unity.**

We can also reach suitable effective values for other sets of parameters, including another dimension of the extra space: the set $a = 50 m_D^{-2}$, $c = -0.25 m_D^2$, n $= 5$, $m = 0.02\, m_D$ also reproduces the Starobinsky model with appropriate parameters.

$H \to 0$ limit

Let us simplify the form of the cosmological constant. To this end, we integrate (tt) - comp. over the extra dimensions $\int d^{\mathrm{n}}x\sqrt{|g_{\mathrm{n}}|}(tt)$ and the comparison with c_{eff} gives

$$c_{eff} = \mathcal{V}_{\mathrm{n}-1} e^{\mathrm{n}\beta_c} \frac{m_{\mathrm{D}}^2}{m_{\mathrm{Pl}}^2} \int\limits_{u_{\mathrm{min}}}^{u_{\mathrm{max}}} \Big(6H^2 f_R\left(R_{\mathrm{D}}(u)\right) - f\left(R_{\mathrm{D}}(u)\right) + f\left(R_{\mathrm{n}}(u)\right)\Big) r^{\mathrm{n}-1}(u)\, du.$$

Here, $R_{\mathrm{D}} = 12H^2 + R_{\mathrm{n}}$ so that $c_{eff} \xrightarrow{H\to 0} 0$, that reasonable for our effective action. By expanding the integrand in expression above, one obtains

$$c_{eff} = -6H^2\, \mathcal{V}_{\mathrm{n}-1} e^{\mathrm{n}\beta_c} \frac{m_{\mathrm{D}}^2}{m_{\mathrm{Pl}}^2} \int\limits_{u_{\mathrm{min}}}^{u_{\mathrm{max}}} f_R\left(R_{\mathrm{n}}(u)\right) r^{\mathrm{n}-1}(u)\, du,$$

which, being combined with

$$\frac{m_{\mathrm{Pl}}^2}{m_{\mathrm{D}}^2} = \mathcal{V}_{\mathrm{n}-1} e^{\mathrm{n}\beta_c} \int\limits_{u_{\mathrm{min}}}^{u_{\mathrm{max}}} f_R\left(R_{\mathrm{n}}(u)\right) r^{\mathrm{n}-1}(u)\, du,$$

yields the well-known relation $H^2 = \Lambda/3$ for the standard notations $c_{eff} = -2\Lambda$, where Λ is the cosmological constant.

Note that we can find a numerical solution for a compact inhomogeneous subspace at $H \simeq 0$, in contrast to the case of a maximally symmetric one:

$$m_{\mathrm{D}}^2 e^{-2\beta_c} = \frac{3H^2}{(\mathrm{n}-1)}.$$

Conclusion and outlook

- Many inflationary models explain observational data at the cost of using a small parameter to account for the smallness of the Hubble parameter $H \sim 10^{-6} m_{\mathrm{Pl}}$. Also, it is implicitly assumed that one of the model parameters related to the cosmological constant is extremely small (fine-tuned).

- In this work, we elaborate the inflationary model without unacceptably small or large parameters of the Lagrangian: $a = 20 m_{\mathrm{D}}^{-2}$, $c = -0.95 m_{\mathrm{D}}^2$, $m = 0.05 m_{\mathrm{D}}$, and n $= 6$. The effective parameters $a_{eff}\, m_{\mathrm{Pl}}^2 \sim 10^9$, $c_{eff}\, m_{\mathrm{Pl}}^{-2} \simeq 0$ suitable for the experimental data are formed by the inhomogeneous extra metric.

  We also show the way to a significant decrease in the cosmological constant.

- It is of interest to investigate the stability of the obtained inhomogeneous extra space and explore their effects on inflationary predictions and observable low energy physics, e.g. on the electroweak scale.

Thank for your attention!

Backup

$$-\left(3\dot{\alpha}+\mathrm{n}\dot{\beta}\right)\dot{R}f_{RR}+\left(3\ddot{\alpha}+\mathrm{n}\ddot{\beta}+3\dot{\alpha}^2+\mathrm{n}\dot{\beta}^2\right)f_R-\frac{f(R)}{2}=-\frac{\dot{\varphi}^2}{2}-V(\varphi),$$

$$\dot{R}^2 f_{RRR}+\left(\ddot{R}+(2\dot{\alpha}+\mathrm{n}\,\dot{\beta})\dot{R}\right)f_{RR}-\left(\ddot{\alpha}+3\dot{\alpha}^2+\mathrm{n}\dot{\alpha}\dot{\beta}\right)f_R+\frac{f(R)}{2}=-\frac{\dot{\varphi}^2}{2}+V(\varphi),$$

$$\dot{R}^2 f_{RRR}+\left(\ddot{R}+(3\dot{\alpha}+(\mathrm{n}-1)\dot{\beta})\dot{R}\right)f_{RR}-\left(\ddot{\beta}+3\dot{\alpha}\dot{\beta}+\mathrm{n}\dot{\beta}^2+(\mathrm{n}-1)m_{\mathbf{D}}^2e^{-2\beta(t)}\right)f_R+$$

$$+\frac{f(R)}{2}=-\frac{\dot{\varphi}^2}{2}+V(\varphi),$$

$$\ddot{\varphi}+\left(3\dot{\alpha}+\mathrm{n}\dot{\beta}\right)\dot{\varphi}+V'_{\varphi}=0,$$

In the slow-roll approximation, $\dot{\varphi}^2 \ll V(\varphi)$, $|\ddot{\varphi}| \ll |V'_{\varphi}|$, $\dot{\alpha} \simeq \text{const} \equiv H$ and $\ddot{\beta} \simeq \dot{\beta} \simeq 0$

$$3H^2 f_R-\frac{f(R)}{2}\simeq -V(\varphi),$$

$$-(\mathrm{n}-1)\,m_{\mathbf{D}}^2e^{-2\beta_{\mathbf{c}}}f_R+\frac{f(R)}{2}\simeq V(\varphi),$$

$$3H\dot{\varphi}+V'_{\varphi}\simeq 0.$$

In the absence of extra dimensions and for the standard linear gravity, $f(R)=R$, equations yield the well-known relations

$$3H^2 \simeq V(\varphi)\big|_{m_{\mathbf{Pl}}=1}, \qquad 3H\dot{\varphi}\simeq -V'_{\varphi}.$$

Equations also lead to the relationship, valid for any form of $f(R)$ and $V(\varphi)$:

$$m_{\mathbf{D}}^2e^{-2\beta_{\mathbf{c}}}=\frac{3H^2}{(\mathrm{n}-1)}.$$

$$\left((R')^2 f_{RRR} + \left(R'' + (n-1)\frac{r'}{r}R'\right) f_{RR}\right) m_{\mathbf{D}}^2 e^{-2\beta_c} + 3H^2 f_R - \frac{f(R)}{2} = -\frac{(\varphi')^2}{2} m_{\mathbf{D}}^2 e^{-2\beta_c} - V(\varphi),$$

$$(n-1)\left(\frac{r'}{r}R' f_{RR} - \frac{r''}{r} f_R\right) m_{\mathbf{D}}^2 e^{-2\beta_c} - \frac{f(R)}{2} = \frac{(\varphi')^2}{2} m_{\mathbf{D}}^2 e^{-2\beta_c} - V(\varphi),$$

$$\left((R')^2 f_{RRR} + \left(R'' + (n-2)\frac{r'}{r}R'\right) f_{RR}\right) m_{\mathbf{D}}^2 e^{-2\beta_c} -$$

$$- \left(\frac{r''}{r} + (n-2)\left(\frac{r'}{r}\right)^2 - \frac{(n-2)}{r^2}\right) m_{\mathbf{D}}^2 e^{-2\beta_c} f_R - \frac{f(R)}{2} = -\frac{(\varphi')^2}{2} m_{\mathbf{D}}^2 e^{-2\beta_c} - V(\varphi),$$

$$\left(\varphi'' + (n-1)\frac{r'}{r}\varphi'\right) m_{\mathbf{D}}^2 e^{-2\beta_c} - V'_\varphi = 0,$$

$$R(u) = 12H^2 - (n-1)\left(\frac{2r''}{r} + (n-2)\left(\frac{r'}{r}\right)^2 - \frac{(n-2)}{r^2}\right) m_{\mathbf{D}}^2 e^{-2\beta_c}$$

Note that the solution to

$$\left(3(n+3)H^2 - R\right) f_R + \frac{f(R)}{2} = V(\varphi)$$

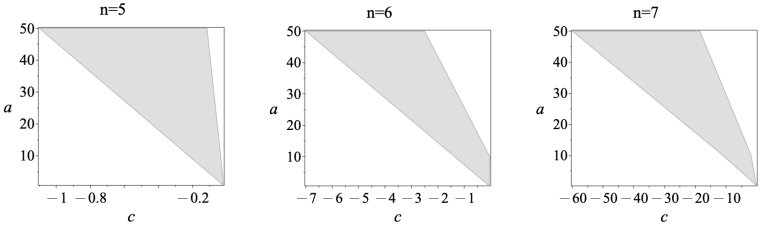
is $f(R) = C_0\sqrt{R - 3(n+3)H^2}$ if $V = 0$. This solution with $C_0 = 2\sqrt{3H^2}\left(6a(n+4)H^2 + 1\right)$ is equivalent to relation

$$H_{as}^2 = \frac{-(n+2) \pm \sqrt{(n+2)^2 - 4an(n+4)c}}{6an(n+4)}$$

for chosen function $f(R) = aR^2 + R + c$ and constant curvature $R = R_c$.

Backup

Figures below show some acceptable ranges that ensure $a_{eff} \sim 10^9 m_{\rm Pl}^{-2}$, $m_{\rm D} \sim 10^{14}$ GeV, and $H \sim 10^{13}$ GeV for a given dimension of subspace n



As one can see, the number of extra dimensions influences mostly the range of the parameter c. The choice of the remaining parameters, such as m and boundary conditions φ_0 or R_0, is owing to achieving $c_{eff} \simeq 0$.