Minimum “length scale”: gravity as a regulator redux

Roberto Casadio

D.I.F.A. “A. Righi”

Bologna University

INFN

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Gravity as a regulator

Planck scale:

$$m = m_{\rm p} \equiv \sqrt{\frac{c\,\hbar}{G_{\rm N}}}$$

$$\lambda_{\rm C} = \ell_{\rm p} \equiv \sqrt{\frac{\hbar\,G_{\rm N}}{c^3}}$$

$$\frac{\hbar}{m\,c} \equiv \lambda_{\rm C} \sim R_{\rm H} \equiv \frac{2\,G_{\rm N}\,m}{c^2}$$

Gravity as a regulator of (mathematical) divergences [1]

[1] W. Pauli, Letter of Heisenberg to Peierls (1930), in Scientific Correspondence, editor K. von Meyenn (Springer-Verlag, 1985) p. 15, Vol II

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Gravity as a regulator of (mathematical) divergences [1]:

1. Planck length (mass) as a geometric minimum length (maximum energy) ~ spacetime crystal / lattice

2. Planck scale as an emergent / dynamical regulator (from non-linearity and quantum) [2,3]

[2] R.C., I. Kuntz, *Revisiting the minimum length in the Schwinger-Keldysh formalism,* EPJ C 80 (2020) 958 [arXiv:2006.08450]

[3] R.C., W. Feng, I. Kuntz, F. Scardigli, *Minimum length (scale) in QFT, GUP and the non-rinormalisability of gravity* [arXiv:2210.12801]

EFT metric corrections

EFT of gravity [4]:

$$S = \frac{1}{8\,\pi\,G_\mathrm{N}} \int d^4x\,\sqrt{-g}\,\left(R + c_1\,R^2 + c_2\,R_{\mu\nu}\,R^{\mu\nu} + \ldots\right)$$

Field redefinition $\mapsto$ effective massive(less) GW

$$\mapsto \quad \int d^4x\,\sqrt{-g}\,\left[\frac{R}{8\,\pi\,G_\mathrm{N}} - \frac{1}{2}\,(\nabla\chi_i)^2 - m_i^2\,\chi_i^2 + \ldots\right]$$

Background field approx:

$$g_{\mu\nu}(x) = \bar{g}_{\mu\rho}(x)\left(e^{\sqrt{\frac{32\,\pi\,\ell_\mathrm{p}}{m_\mathrm{p}}}\,h(x)}\right)^{\rho}_{\ \nu} \simeq \eta_{\mu\nu} + \sqrt{\frac{32\,\pi\,\ell_\mathrm{p}}{m_\mathrm{p}}}\,h_{\mu\nu}(x) + \frac{16\,\pi\,\ell_\mathrm{p}}{m_\mathrm{p}}\lim_{x\to y} h_{\mu\rho}(x)\,h^{\rho}_{\ \nu}(y) + \ldots$$

[4] L.E. Parker, D. Toms, *QFT in curved spacetime: quantised fields and gravity* (Cambridge University Press, 2009)

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Geometric length [2,5]: $$\ell^2_{\rm in-in} = \langle 0_{\rm in}\,|\,\hat g_{\mu\nu}\,|\,0_{\rm in}\rangle\,dx^\mu\,dx^\nu = \ell^2 + \frac{16\,\pi\,\ell_{\rm p}}{m_{\rm p}}\,\lim_{x\to y}\,\langle 0_{\rm in}\,|\,\hat h_{\mu\rho}(x)\,\hat h^{\rho}{}_{\nu}(y)\,|\,0_{\rm in}\rangle\,dx^\mu\,dx^\nu$$
$$\equiv \ell^2 + \frac{16\,\pi\,\ell_{\rm p}}{m_{\rm p}}\,\lim_{x\to y}\,G^{\rm R}{}_{\mu\rho}{}^{\rho}{}_{\nu}(x,y)\,dx^\mu\,dx^\nu$$

Length scale [6]: $$\ell^2_{\rm in-out} = \langle 0_{\rm out}\,|\,\hat g_{\mu\nu}\,|\,0_{\rm in}\rangle\,dx^\mu\,dx^\nu = \ell^2 + \frac{16\,\pi\,\ell_{\rm p}}{m_{\rm p}}\,\lim_{x\to y}\,\langle 0_{\rm out}\,|\,\hat h_{\mu\rho}(x)\,\hat h^{\rho}{}_{\nu}(y)\,|\,0_{\rm in}\rangle\,dx^\mu\,dx^\nu$$
$$\equiv \ell^2 + \frac{16\,\pi\,\ell_{\rm p}}{m_{\rm p}}\,\lim_{x\to y}\,G^{\rm F}{}_{\mu\rho}{}^{\rho}{}_{\nu}(x,y)\,dx^\mu\,dx^\nu$$

[5] L.V. Keldysh, Zh. Eksp. Teor. Fiz. 47 (1964) 1515
[6] T. Padmanabhan, *Planck length as the lower bound to all physical length scales*, GRG 17 (1985) 215

EFT metric corrections

Geometric length ($\ell_{\text{in-in}} \in \mathbb{R}$): metric ~ geodesics ~ (free) retarded (GW) propagator

$$G^{\text{R}}_{\mu\nu\rho\sigma}(x,y) = \sum_i \left[-\frac{\theta(x^0 - y^0)}{2\,\pi}\,\delta(\ell^2) + \theta(x^0 - y^0)\,\theta(\ell^2)\,\frac{m_i\,J_1(m_i\,\ell)}{4\,\pi\,\ell} \right] \hbar\,P^i_{\mu\nu\rho\sigma} \sim \ell^0 + \ell^{-1}$$

Length scale: metric ~ graviton scatterings ~ Feynman propagator

$$G^{\text{F}}_{\mu\nu\rho\sigma}(x,y) = \sum_i \frac{\hbar\,P^i_{\mu\nu\rho\sigma}}{4\,\pi^2\,(x-y)^2} + \mathcal{O}(|x-y|) \sim \ell^{-2}$$

$i = 1,\ldots$ effective massive(less) GW: $P^i_{\mu\nu\rho\sigma} = \alpha_i\,\eta_{\mu\rho}\,\eta_{\nu\sigma} + \beta_i\,\eta_{\mu\sigma}\,\eta_{\nu\rho} + \gamma_i\,\eta_{\mu\nu}\,\eta_{\rho\sigma}$

Background length: $\ell^2 = \eta_{\mu\nu}\,dx^m u\,dx^\nu$

EFT minimum lengths

Coincidence limit $dx^\mu \sim \ell \to 0$

Minimum geometric length: $\ell^2_{\text{in-in}} \sim \left(\ell^0 + \ell^{-1}\right) \ell^2 \to 0$

Free propagation not affected by ℓ_p [2]

Minimum length scale:

$$\ell^2_{\text{in-out}} \sim \begin{cases} e^{-\frac{\ell_\text{p}^2}{\ell^2}|\Sigma|} \ell^2 \to 0 & \Sigma \equiv \sum_i (\alpha_i + 4\beta_i + \gamma_i) \le 0 \\ \ell_\text{p}^2 \frac{\Sigma}{\ell^2} \ell^2 \to \ell_\text{p}^2 |\Sigma| & \Sigma \equiv \sum_i (\alpha_i + 4\beta_i + \gamma_i) > 0 \end{cases}$$

Scattering processes (measurements) cannot probe below ℓ_p [2]

[2] R.C., I. Kuntz, EPJ C 80 (2020) 958 [arXiv:2006.08450]

EFT and GUP

Minimum length scale:

$$\ell^2_{\text{in-out}} \sim \begin{cases} e^{-\frac{\ell_p^2}{\ell^2}|\Sigma|}\,\ell^2 \to 0 & \Sigma \equiv \sum_i(\alpha_i + 4\beta_i + \gamma_i) \le 0 \\ \ell_p^2 \frac{\Sigma}{\ell^2}\,\ell^2 \to \ell_p^2 |\Sigma| & \Sigma \equiv \sum_i(\alpha_i + 4\beta_i + \gamma_i) > 0 \end{cases}$$

Scattering processes (measurements) cannot probe below ℓ_p

Heisenberg microscope [7] / uncertainty relation modified to GUP [8]:

$$\Delta x\, \Delta p \ge \frac{\hbar}{2}$$

$$\Delta x\, \Delta p \ge \frac{\hbar}{2}\left(1 + \delta_0 \frac{\Delta p^2}{m_p^2}\right) \longrightarrow \ell = \ell_p \sqrt{\delta_0} = \ell_p \sqrt{\Sigma}$$

$$\delta_0 = \frac{2}{\pi} \sum_i (\alpha_i + 4\,\beta_i + \gamma_i)$$

N.B. (much) larger minimum length scales still possible in many-body systems [9]

[7] W. Heisenberg, *The physical principles of the quantum theory* (University of Chicago Press, 1930)

[8] M.P. Bronstein, Phys. Zeithchr. Der Sowjetunion 9 (1936) 140

[9] R.C. *A quantum bound on the compactness*, EPJ C 82 (2022) 10 [arXiv:21-3.14582] ~ talk at FLaG in TN 2022

Gravity as a regulator

Planck scale as an emergent / dynamical regulator in non-renormalisable theories [3]:

$$\delta_0^{\mathrm{GR}} = \frac{8}{\pi} \quad \longleftrightarrow \quad \delta_0^{\mathrm{Stelle}} = 0 \quad [10]$$

$$\delta_0^{\text{Spacetime crystal}} \leq 0 \quad [11]$$

Minimum scale makes theory finite (not just effective!)

Possibly fundamental (“complete”) theories with no minimum scale

[3] R.C., W. Feng, I. Kuntz, F. Scardigli, [arXiv:2210.12801]
[10] K.S. Stelle, PRD 16 (1977) 953
[11] P. Jizba, H. Kleinert, F. Scardigli, PRD 81 (2010) 084030

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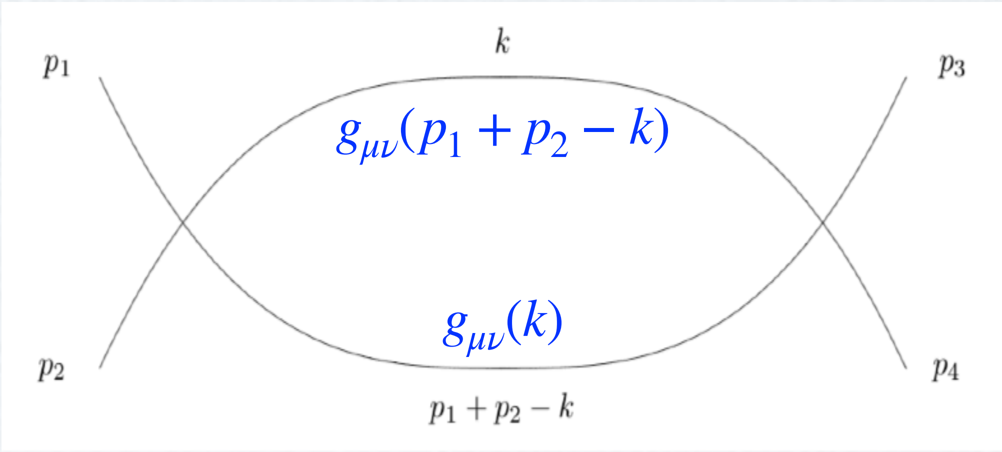
Non-linearity ~ beyond background field approx

Momentum-dependent propagators in loops [12]:

$$G^{\rm F}(k) \quad \Longrightarrow \quad G^{\rm F}_{p_1+p_2-k}(k) \sim \Omega(p_1+p_2-k)\, G^{\rm F}(k)$$

$$G^{\rm F}(p_1+p_2-k) \quad \Longrightarrow \quad G^{\rm F}_{k}(p_1+p_2-k) \sim \Omega(k)\, G^{\rm F}(p_1+p_2-k)$$



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Thank you!

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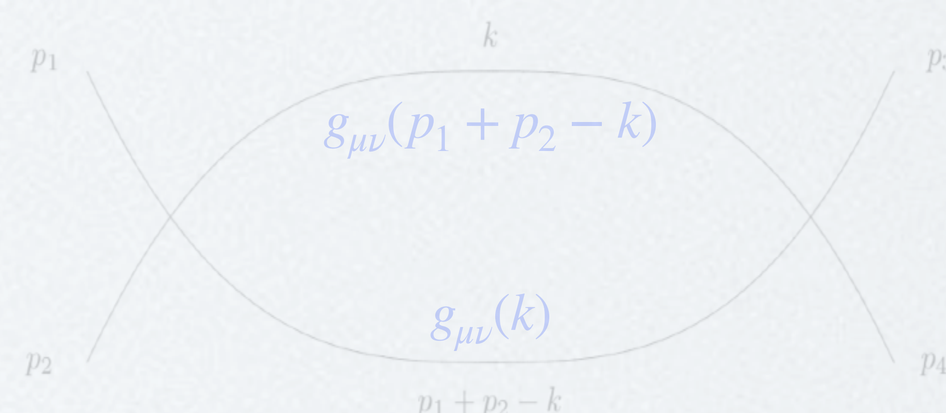

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