Secondary GWs and PBHs in string inflation

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PBH Dark Matter

Many candidates for DM, among which:

PRIMORDIAL BLACK HOLES

The curvature perturbations re-enter the Hubble horizon and undergo a gravitational collapse $\Longrightarrow$ PBH (2109.01398)

Fraction of PBH DM

We define the fraction of the total energy density in PBH at formation time:

$$\beta_f(M_{\rm PBH}) := \left.\frac{\rho_{\rm PBH}}{\rho_{tot}}\right|_f$$

Gaussian distribution (1811.07857, 2103.01056):

$$\beta_f(M_{\rm PBH}) = \frac{1}{\sqrt{2\pi}\sigma_M}\int_{\zeta_c}^{\infty} \mathrm{e}^{-\frac{\zeta^2}{2\sigma_M^2}}\mathrm{d}\zeta \approx \frac{\sigma_M}{\sqrt{2\pi}\zeta_c}\mathrm{e}^{-\frac{\zeta_c^2}{2\sigma_M^2}}.$$

Redshift and rescale up to present time assuming only SM degrees of freedom:

$$\frac{\sigma_M}{\sqrt{2\pi}\zeta_c}\mathrm{e}^{-\frac{\zeta_c^2}{2\sigma_M^2}} \approx 10^{-8}\left(\frac{M_{\rm PBH}}{M_\odot}\right)^{\frac{1}{2}} \mathfrak{f}_{\rm PBH}(M_{\rm PBH})$$

Power spectrum

Two-point function $\sigma_M^2 \sim \langle\zeta\zeta\rangle$ related to the power spectrum:

$$\langle\zeta_{\mathrm{k}}\zeta_{\mathrm{p}}\rangle = |\zeta_{\mathrm{k}}|^2\delta^{(3)}(\mathrm{k}+\mathrm{p}) =: \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)\delta^{(3)}(\mathrm{k}+\mathrm{p})$$

Assumptions: $M_{\mathrm{PBH}} \sim 10^{-12} M_\odot$, $\mathfrak{f}_{\mathrm{PBH}}(10^{-12}M_\odot) = 1$ (1810.12218), $\zeta_c = 0.6$ (1908.11357). Then:

$$\frac{\sigma_M}{\sqrt{2\pi}0.6}\mathrm{e}^{-\frac{0.36}{2\sigma_M^2}} \approx 10^{-14} \Longrightarrow \sigma_M \approx 0.078 \Leftrightarrow \mathcal{P}_\zeta(k_{\mathrm{PBH}}) \sim 10^{-3}$$

Power spectrum at CMB scales: $\mathcal{P}_\zeta(k_{\mathrm{CMB}}) = 2\cdot 10^{-9} \Longrightarrow$

ENHANCEMENT OF 6 ORDERS OF MAGNITUDE

Inflationary dynamics

Dynamics of expanding universe:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\delta V[\phi]}{\delta \phi} = 0,$$

$$\dot{H} = \frac{\dot{\phi}^2}{2}.$$

Hubble slow-roll parameters:

$$\epsilon := -\frac{\dot{H}}{H^2}, \ \eta := \frac{\dot{\epsilon}}{\epsilon H}, \ \kappa := \frac{\dot{\eta}}{\eta H}$$

- $|\eta| \ll 1 \implies$ slow-roll
- $\eta = -\mathcal{O}(1) \implies$ ultra-slow-roll

Ultra-slow-roll

Power spectrum given by the solution of the Mukhanov-Sasaki equation ($z := \sqrt{2\epsilon}a$, $u_k := z\zeta_k$):

$$u_k''(\tau) + \left(k^2 - \frac{z''}{z}\right) u_k(\tau) = 0 \Longrightarrow \mathcal{P}_\zeta(k) = \frac{k^3}{2\pi^2} \left|\zeta_k\right|^2$$

$$\frac{z''}{z} = (aH)^2 \left(2 - \epsilon + \frac{3\eta}{2} - \frac{\epsilon\eta}{4} + \frac{\eta^2}{4} + \frac{\eta\kappa}{2}\right)$$

Ultra-slow-roll: $\eta = -\mathcal{O}(1) = \text{const.}$ Definition of η:

$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = \frac{\epsilon_N}{\epsilon} \Longrightarrow \epsilon(N) = \epsilon_0 e^{\eta N}.$$

Ultra-slow-roll

Mukhanov-Sasaki equation in N:

$$u_{NN}+(1-\epsilon)u_N+\left[\left(\frac{k}{aH}\right)^2-\left(2-\epsilon+\frac{3\eta}{2}-\frac{\epsilon\eta}{4}+\frac{\eta^2}{4}+\frac{\eta\kappa}{2}\right)\right]u=0$$

In USR for super-horizon scales
($\epsilon \ll 1,\ \eta=-\mathcal{O}(1),\ \kappa=0,\ k\ll aH$):

$$u_{NN}+u_N-\left(2+\frac{3\eta}{2}+\frac{\eta^2}{4}\right)u=0.$$

Solution: $u(N)=u_0e^{\lambda_\pm N},\ \lambda_\pm=\frac{-1\pm|\eta+3|}{2}$. Physical variable:

$$\zeta(N)=\zeta_0e^{\omega_+(\eta)N}+\bar{\zeta}_0e^{\omega_-(\eta)N},\ \omega_\pm(\eta):=\frac{\pm|\eta+3|-(\eta+3)}{2}.$$

PBHs and inflation

Number of efoldings $\mathrm{d}N := H\mathrm{d}t = \mathrm{d}\log a$ (1706.06784):

$$\Delta N_{\mathrm{CMB}}^{\mathrm{PBH}} = 18.4 - \frac{1}{12}\log\left(\frac{g_{*f}}{g_{*0}}\right) + \frac{1}{2}\log\gamma - \frac{1}{2}\log\left(\frac{M_{\mathrm{PBH}}}{M_\odot}\right)$$

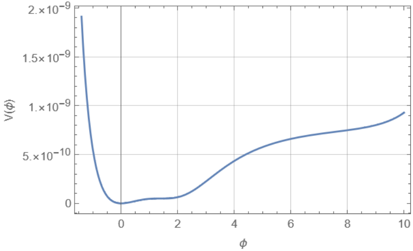
$M_{\mathrm{PBH}} \sim 10^{-12} M_\odot \Longrightarrow \Delta N_{\mathrm{CMB}}^{\mathrm{PBH}} \sim 32$

Fibre inflation

String inflationary model in the framework of type IIB flux compactification (0808.0691), (1803.02837).
Inflaton is one of the Kähler moduli:

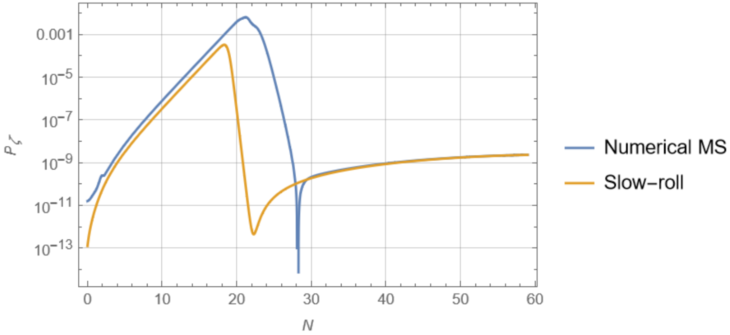
$$\frac{V_{inf}(\phi)}{V_0} = C_1 - e^{-\hat{\phi}/\sqrt{3}}\left(1 - \frac{C_2}{1 - C_3 e^{-\hat{\phi}/\sqrt{3}}}\right) + \ldots$$

$$\ldots + e^{2\hat{\phi}/\sqrt{3}}\left(C_4 - \frac{C_5}{1 + C_6 e^{\hat{\phi}\sqrt{3}}}\right)$$



Power spectrum

Studies of reheating after fibre inflation: $N_{CMB} = 53$ before the end of inflation (1809.01159) $\Longrightarrow$
$N_{PBH} = 21$ before the end of inflation for $M_{PBH} = 10^{-12} M_{\odot}$



GW production

PBH formation $\Longrightarrow$ production of a stochastic background of secondary GWs (1804.07732). Equation of motion:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\mathcal{P}_{ij}^{rs}\mathcal{S}_{rs},$$

source term:

$$\mathcal{S}_{ij} = 4\Psi\partial_i\partial_j\Psi + 2\partial_i\Psi\partial_j\Psi - \partial_i\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right)\partial_j\left(\frac{\Psi'}{\mathcal{H}} + \Psi\right).$$

GW production

Energy density parameter for GWs (9909001):

$$\Omega_{GW}(\tau, \mathrm{k}) := \frac{\rho_{GW}(\tau, \mathrm{k})}{\rho_c} = \frac{1}{24}\left(\frac{k}{a\mathcal{H}}\right)^2 \overline{\mathcal{P}_h(\tau, \mathrm{k})}$$

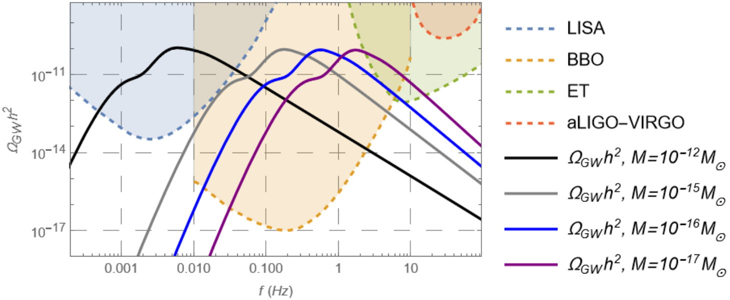
Perform the average over conformal time and redshift and rescale up to present time:

$$\Omega_{GW}(\tau_0, \mathrm{k}) = \frac{c_g \Omega_{r,0}}{36} \int_{1/\sqrt{3}}^{\infty} \mathrm{d}s \int_0^{1/\sqrt{3}} \mathrm{d}d \left[\frac{\left(s^2 - \frac{1}{3}\right)\left(d^2 - \frac{1}{3}\right)}{s^2 - d^2}\right]^2 \cdot \ldots$$

$$\ldots \cdot \mathcal{P}_\zeta\left(\frac{k\sqrt{3}}{2}(s+d)\right) \mathcal{P}_\zeta\left(\frac{k\sqrt{3}}{2}(s-d)\right) \cdot \ldots$$

$$\ldots \cdot \left[\mathcal{I}_c^2(s,d) + \mathcal{I}_s^2(s,d)\right]$$

GW production



$M_{\rm PBH}$ $[M_\odot]$	$\mathcal{P}_\zeta^{\rm peak}$	$f_{\rm PBH}$	$f_{\rm GW}^{\rm peak}$ [Hz]	$\Omega_{\rm GW}^{\rm peak} h^2$
$\mathcal{O}(10^{-12})$	$6.6 \cdot 10^{-3}$	$\mathcal{O}(1)$	$5.96 \cdot 10^{-3}$	$1.08 \cdot 10^{-10}$
$\mathcal{O}(10^{-15})$	$6.4 \cdot 10^{-3}$	$\mathcal{O}(1)$	0.178	$9.32 \cdot 10^{-11}$
$\mathcal{O}(10^{-16})$	$6.2 \cdot 10^{-3}$	$\mathcal{O}(1)$	0.567	$8.99 \cdot 10^{-11}$
$\mathcal{O}(10^{-17})$	$6.1 \cdot 10^{-3}$	$\mathcal{O}(10^{-3})$	1.73	$8.80 \cdot 10^{-11}$

Secondary GWs without PBHs

Exponential dependence on σ_M:

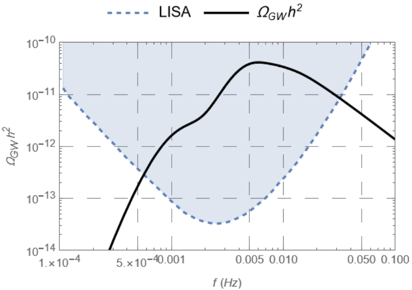
$$\frac{\sigma_M}{\sqrt{2\pi}0.6} \mathrm{e}^{-\frac{0.36}{2\sigma_M^2}} \approx 10^{-8} \left(\frac{M_{\rm PBH}}{M_\odot}\right)^{\frac{1}{2}} \mathfrak{f}_{\rm PBH}(M_{\rm PBH})$$

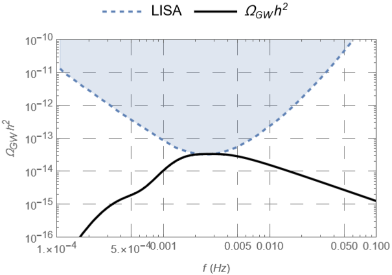
Example:

$$M = 10^{-12} M_\odot, \; \mathfrak{f}_{\rm PBH}(10^{-12} M_\odot) = 10^{-10} \Longrightarrow \sigma_M = 0.059$$

We can still have a detectable GW signal with PBHs contributing to DM with a small (even negligible) fraction

Secondary GWs without PBHs





$M_{\rm PBH}$ $[M_\odot]$	$\mathcal{P}_\zeta^{\rm peak}$	$\mathfrak{f}_{\rm PBH}$	$f_{\rm GW}^{\rm peak}$ [Hz]	$\Omega_{\rm GW}^{\rm peak}h^2$
$\mathcal{O}(10^{-12})$	$4.1\cdot10^{-3}$	$\mathcal{O}(10^{-8})$	$5.97\cdot10^{-3}$	$8.80\cdot10^{-11}$
$\mathcal{O}(10^{-12})$	$1.1\cdot10^{-4}$	0	$2.80\cdot10^{-3}$	$3.35\cdot10^{-14}$

Conclusions

- nature of DM: Primordial Black Holes?
- PBH formation: re-entry of the Hubble horizon
- Fibre Inflation: inflection point in the inflaton potential
- GW production with and without PBH being DM

THANK YOU FOR THE ATTENTION