Regular Evaporating BH with stable cores

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Overview

- Gravitational collapse
- regular BH
- CH instability and mass-inflation
- Stability of BH core
- Conclusions

Regular BH

- The occurrence of singularities is often taken as an indicator that general relativity is incomplete and should be generalized to a quantum theory expelling this feature.
- Regular BH geometries are *ad hoc* modifications which respect the limiting curvature hypothesis
- The black hole singularity is replaced by a regular piece of de Sitter space.
- Typically come with two spacetime horizons: an (outer) event horizon and an (inner) Cauchy horizon.
- Global hyperbolicity is hampered by the presence of a “quantum" generated Cauchy horizon: is the CH stable?

Limiting curvature hypothesis: de Sitter core

Early works by Sakharov (1966), Frolov, Markov, Mukhanov (1989), Dimnykova (1992)

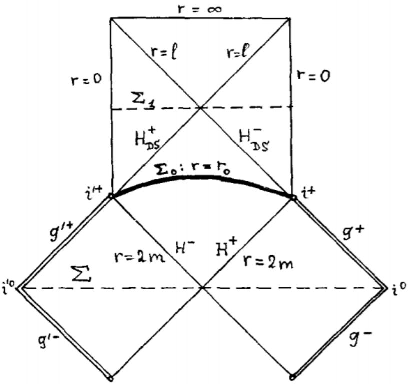
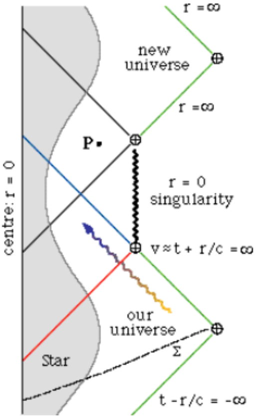


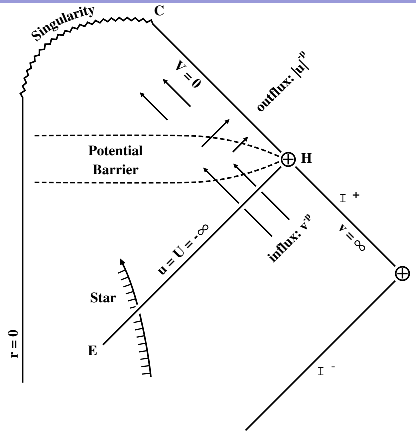
Fig. 1. Conformal Penrose diagram for the spacetime of a spherically symmetric eternal black hole with a de Sitter space in its interior. Σ_0 is a junction surface which represents the thin tran-

RN


r = ∞
new universe
r = ∞
P
centre: r = 0
r = 0 singularity
v ≈ t + r/c = ∞
our universe
Σ
Star
t - r/c = -∞

CH singularity

- Poisson and Israel, 1989, 1991
- Ori's model
- AB, Droz, Morsink, Israel



Mass-inflation

The behavior of the curvature invariants near the Cauchy horizon is rather insensitive to the details of the local fields trapped inside the event horizon. The rate of divergence of the Coulomb component of the Weyl curvature $\Psi_2 \equiv C_{\mu\nu\rho\sigma} l^\mu m^\nu \bar{m}^\rho n^\sigma$, can then be characterized by the *anomalous dimension* of the instability

$$\nu = \frac{d \ln |\Psi_2|}{d \ln v} \,. \tag{1}$$

In the limit $v \to \infty$, $\Psi_2 \sim v^{-p} \, \mathrm{e}^{\kappa_- \, v}$, entailing that the anomalous dimension behaves as $\nu \sim \kappa_- \, v - p$. Since $\Psi_2 \propto M(r, v)$ this effect has has been dubbed *mass-inflation*.

An exact MI solution

A generic spherically symmetric line element can always be written as $ds^2 = g_{ab}dx^a dx^b + r^2 d\Omega^2$, $a, b = 0, 1$, where $r = r(x^a)$ is the radius of the 2-spheres with x^a being constant. It is convenient to introduce a generalized Schwarzschild mass function $M(x^a)$ by means of the gradient of $r(x^a)$,

$$g^{ab}\partial_a r \partial_b r = f(x^a) = 1 - \frac{2M(x^a)}{r} \,. \qquad (2)$$

The taxonomy of different static black hole geometries is encoded in the radial behavior of $M(r)$. For a Schwarzschild black hole $M = m$ where m is the mass of the object at large distances $r \gg m$. In the physical picture proposed by Sakharov, a phase transition to a false vacuum occurs at Planckian distances from the center so that a de Sitter core eventually develops and $M(r) \sim r^3$ for small r. An explicit model realizing such a phase transition is the Hayward model where

$$M(r) = \frac{mr^3}{r^3 + 2ml^2} \,. \tag{3}$$

Here l is a free parameter whose value should be fixed by the underlying quantum gravity model.

RBH from Asymptotic Safety

AS model of regular BH (Bonanno & Reuter, PRD (2000) At very high energies the RG evolution of Newton's constant reads

$$G(k) = \frac{G(k_0)}{1 + \omega\, G(k_0)\, [k^2 - k_0^2]} \qquad (4)$$

which, upon assuming $k \sim 1/d(P)$, $d(P) = \int_{\mathcal{C}} \sqrt{|ds^2|}$ we obtain

$$G(r) = \frac{G_0\, r^3}{r^3 + \tilde{\omega}\, G_0\, [r + \gamma G_0 M]} \qquad (5)$$

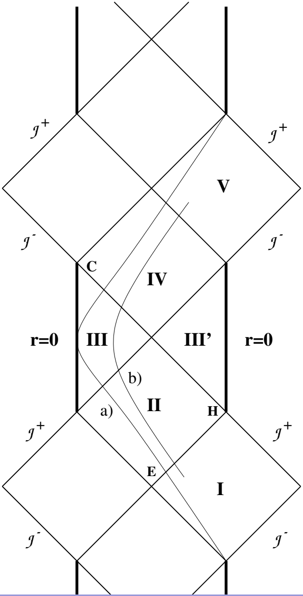
At large distances, the leading correction to Newton's constant is given by

$$G(r) = G_0 - \tilde{\omega}\ \frac{G_0^2}{r^2} + O(\frac{1}{r^3}). \tag{6}$$

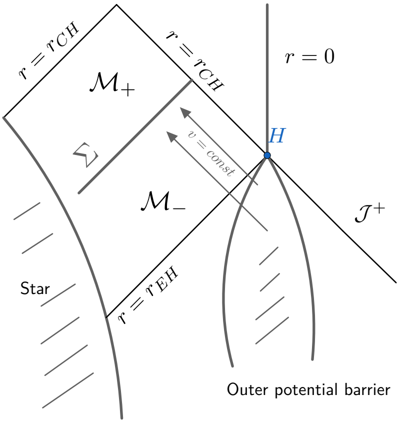
For small distances $r \to 0$, it vanishes very rapidly:

$$G(r) = \frac{r^3}{\gamma \tilde{\omega} G_0 M} + O(r^4) \tag{7}$$

Causal structure of AS BH



MI in regular BH



Ori's model (PRL 1991)

In the Ori model the outgoing energy flux is modelled by a thin pressureless null shell Σ which divides the spacetime in two regions $\mathcal{M}_\pm$, inside $(+)$ and outside $(-)$ the shell. This shell acts as a catalyst to trigger the divergence of the mass function. Assuming spherical symmetry, the metric in each sector of spacetime can be written as

$$ds^2 = -f_\pm(r, v_\pm)dv_\pm^2 + 2drdv_\pm + r^2 d\Omega^2\,, \tag{8}$$

where $f_\pm = 1 - 2M(r, v_\pm)/r$.

Israel-Barrabes formalism

The equality of the induced metric on Σ forces r to be the same on $\mathcal{M}_{\pm}$. For this reason it is convenient to choose r as a parameter (not necessarily affine) along the null generators $s^{\mu}_{\pm} = dx^{\mu}_{\pm}/dr = (2/f_{\pm}, 1, 0, 0)$ of Σ. Einstein's equations on each sector of the spacetime can then be expressed in terms of the mass function $M(r, v)$ as

$$\frac{\partial M}{\partial r} = -4\pi r^2 T_v{}^v \,, \qquad \frac{\partial M}{\partial v} = 4\pi r^2 T_v{}^r \,. \tag{9}$$

Continuity of the flux across Σ requires

$$[T_{\mu\nu} s^\mu s^\nu] = 0 \tag{10}$$

where the square brackets indicate the "jump" of a scalar quantity across the shell. This condition is consistent with the assumption of Σ being pressureless. In terms of the lapse and mass-function, we have

$$\frac{1}{f_+^2} \frac{\partial M_+}{\partial v_+} = \frac{1}{f_-^2} \frac{\partial M_-}{\partial v_-} \,, \tag{11}$$

The functional dependence of the two is fixed by noting that the position of the null hypersurface Σ in the two coordinate systems is

$$f_+ dv_+ = f_- dv_- \quad \text{along } \Sigma\,. \tag{12}$$

In the following we shall use (12) to express all physical quantities in terms of $v \equiv v_-$.

We now specify this general framework to the mass function $M(r,v)$ given in (3), setting

$$f_\pm = 1 - \frac{2m_\pm(v_\pm)r^2}{r^3 + 2m_\pm(v_\pm)l^2}\,. \tag{13}$$

Since the shell moves light-like, we have $f_- dv_- = 2dr$, implying

$$\dot{R}(v) = \frac{1}{2} - \frac{m_-\,R^2}{R^3 + 2\,l^2\,m_-}\,. \tag{14}$$

Here the dot stands for derivative with respect to v and it is understood that r and $m_\pm$ are v-dependent functions specifying the position of Σ and the mass density.

Combining eqs. (11) and (12) furthermore gives a differential equation for the mass function

$$\begin{aligned}
&\frac{m_+'(v)}{\left(2l^2 m_+(v) + R(v)^3\right)\left(2m_+(v)\left(l^2 - R(v)^2\right) + R(v)^3\right)} \\
&= \frac{m'(v)}{\left(2l^2 m(v) + R(v)^3\right)\left(2m(v)\left(l^2 - R(v)^2\right) + R(v)^3\right)} \\
&= \frac{1}{2\left(2l^2 m(v) + R(v)^3\right)^2}\, m'(v) \left(\frac{dR(v)}{dv}\right)^{-1} \qquad (15)
\end{aligned}$$

The system (14) and (15) is a coupled system of non-linear differential equations encoding the evolution of $r(v)$ and $m_+(v)$.

The boundary condition at the event horizon is determined by the Price's tail behavior so that in the past sector of the shell the mass function $m_{-}(v)$ can be written (in the optical geometric limit) as

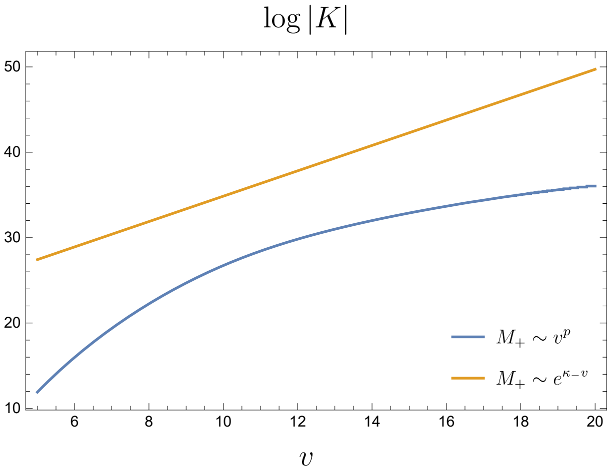
$$m_{-}(v) = m_0 - \frac{\beta}{(v/v_0)^p} \tag{16}$$

where $p = 2(4\ell + 1)$ for a multipole of order ℓ. It is possible to show that (always in the limit $v \to \infty$)

$$m_{+} = -\frac{3\beta p r_0 v^{-p-1}}{32{\kappa_0}^2 l^2 {m_0}^3} - \frac{3\beta {r_0}^3 v^{-p}}{8\kappa_0 l^2 {m_0}^2} - \frac{{r_0}^3}{2l^2} \tag{17}$$

where $\kappa_0 > 0$ is the surface gravity at the CH.

$K = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ is singular!



A power law behavior replaces the exponential one

Strength of the singularity: original PI model

If we indicate with v_+ the advanced coordinate in the future sector of the shell, at large values of v_+ the metric near the CH reads

$$ds^2 \approx 2\frac{dv_+}{r}(rdr + m_+(v_+)dv_+) + r^2 d\Omega^2. \tag{18}$$

The coordinate u defined by

$$du = (rdr + m_+(v_+)dv_+) \tag{19}$$

is regular at the CH and the metric becomes

$$ds^2 \approx 2\frac{dv_+ du}{r} + r^2 d\Omega^2 \tag{20}$$

which is manifestly regular at the CH

Strength of the singularity

- Therefore the singularity of the CH already in the classical case appears to be rather weak because it is possible to find a coordinate system where the metric is regular.
- This fact has profound consequences: that the mass-inflation singularity does not satisfies the necessary conditions to be strong in the Tipler sense.
- A null singularity is said to be strong according to Tipler if there exists at least one component of the Riemann tensor that in a parallelly propagated frame the twice integration with respect to the affine parameter τ does not converge
- The physical meaning of this requirement is that the tidal distorsion is not finite as an observer crosses a strong scalar singularity. A measure of the latter quantity can be obtained by twice integrating the tidal acceleration, as provided by the square of the Weyl curvature.

Strength of the singularity

In the case of the standard MI scenario one finds

$$C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} \propto \frac{1}{V^2(\log(-V))^{2p}} \tag{21}$$

where $V \propto -e^{-k_0 v}$ is a Kruskal coordinate adapted to the inner horizon and $V \propto \tau$ in this case. The tidal distorsion is obtained by twice integrating (21) which is therefore finite. It has further been argued by Ori that this behavior could be sufficient to determine a C^1 extension of the spacetime along the CH.
However, according to Krolak (21) is still a strong singularity, as the expansion of the congruence is divergent (Krolak 1987). In fact in this case if we we integrate the Riemann-Christoffel tensor only once, the integral does not converge on the singularity

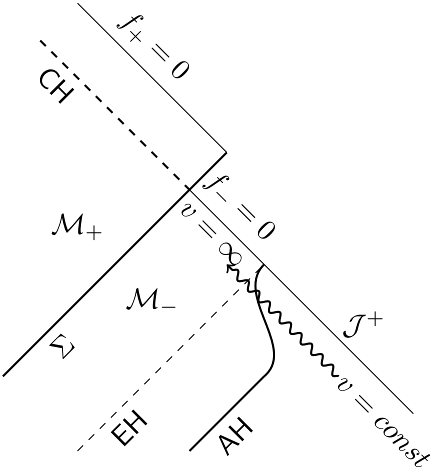
Strength of the singularity

In the case of a regular BH the divergence of the Weyl curvature is further weakened. In particular we finds

$$C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} \propto (\log(-V))^{3p} \qquad (22)$$

which is directly integrable. At variance with the original Poisson-Israel model, the singularity produced at the CH in regular black holes is Krolak weak: not only the volume of the congruence (Tipler weak) but also the expansion does not diverge (negatively) at the CH. Therefore this results further support the possibility that a C^1 extension of the spacetime beyond the singularity is possible for regular black holes.

Including the Hawking radiation



Promoting the temperature and horizon area to functions of v and using the adiabatic approximation the mass-loss of the geometry can be computed from

$$\frac{\partial m(v)}{\partial v} = -\frac{\pi^2}{30}\,T(v)^4\,A(v)\,,$$

where the area of the event horizon is approximated dynamically from the location of the apparent horizon in. The previous equation turns into a closed equation determining the v-dependence of m. The late time behavior can be determined analytically,

$$m(v) \simeq m_{cr} + \frac{10935\,l^4\,\pi}{8v} + \mathcal{O}(v^{-3/2})\,.$$

the strength of the Singularity is universal!

$$C^2\Big|_{\Sigma} = \frac{19683\, v^6}{4096\, m_{cr}^{10}}\,, \qquad K|_{\Sigma} = \frac{59049\, v^6}{4096\, m_{cr}^{10}}\,.$$

Geodesic Equation

In order to determine the geodesic structure of the spacetime, we consider the motion of radially infalling observers and compute the relation between the coordinate v and the observer's proper time τ. The v-component of the geodesic equation is

$$\ddot{v} = -\frac{1}{2}\frac{\partial f}{\partial r}\dot{v}^2 \,,$$

where the dot represents a derivative with respect to proper time. The normalization for the four-velocity of the radial observer furthermore supplies the relation

$$L = \frac{1}{2}(f\dot{v}^2 - 2\dot{r}\dot{v}) = \frac{\epsilon}{2} \,. \tag{23}$$

with $\epsilon = 0, 1$ for lightlike or timelike geodesics, respectively.

Geodesic equation

For a static geometry where f is independent of v,

$$\frac{\partial L}{\partial \dot{v}} = const$$

is conserved along geodesic motion. To understand the implications of the curvature singularities induced by the late-time attractor we solve these equations for timelike geodesics close to the Cauchy horizon. We first note that near the inner horizon

$$-\frac{1}{2}\frac{\partial f}{\partial r} \simeq \kappa_- > 0\,.$$

and therefore

$$\tau = \frac{1}{\kappa_-} e^{-\kappa_- v} + const\,.$$

The important insight from this result is that a massive observer can reach the singularity at $v = \infty$ *in finite proper time*.

Geodesic completeness

In the presence of Hawking radiation this analysis is radically altered though. In this case the late-time structure of spacetime is given by an extremal black hole where $\kappa_- = 0$. One finds the linear relation

$$\tau = cv + const\,.$$

with c an integration constant. The crucial difference is that the timelike geodesic requires *an infinite amount of proper time to reach* $v = \infty$.

Is the spacetime geodesically complete?

Conclusions

- Regular BH in spherical symmetry have Krolak weak singularities
- The spacetime can be extended beyond the CH
- If the backreaction of the Hawking radiation is included, the spacetime could be geodesically complete
- Future work: extension to the rotating case