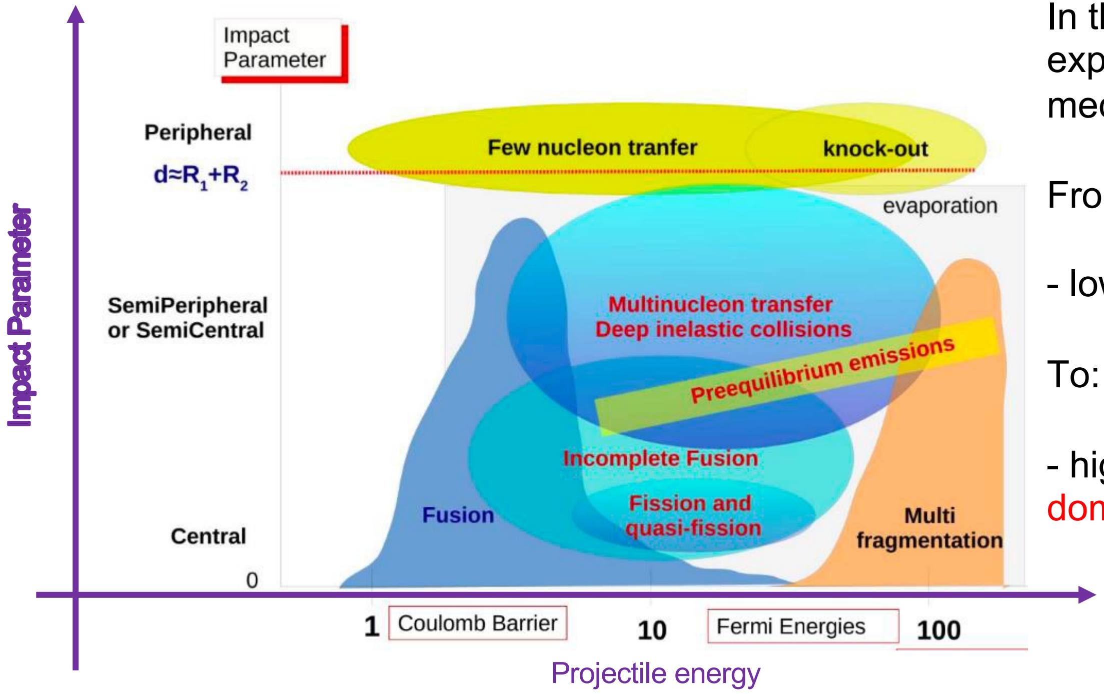
Nucleus-Nucleus Collisions at Intermediate Energies: modelling the Total Reaction Cross Section

Nuclear reactions



In the region of E/A =20—200 MeV it is expected to have a gradual transition in the mechanism of reactions.

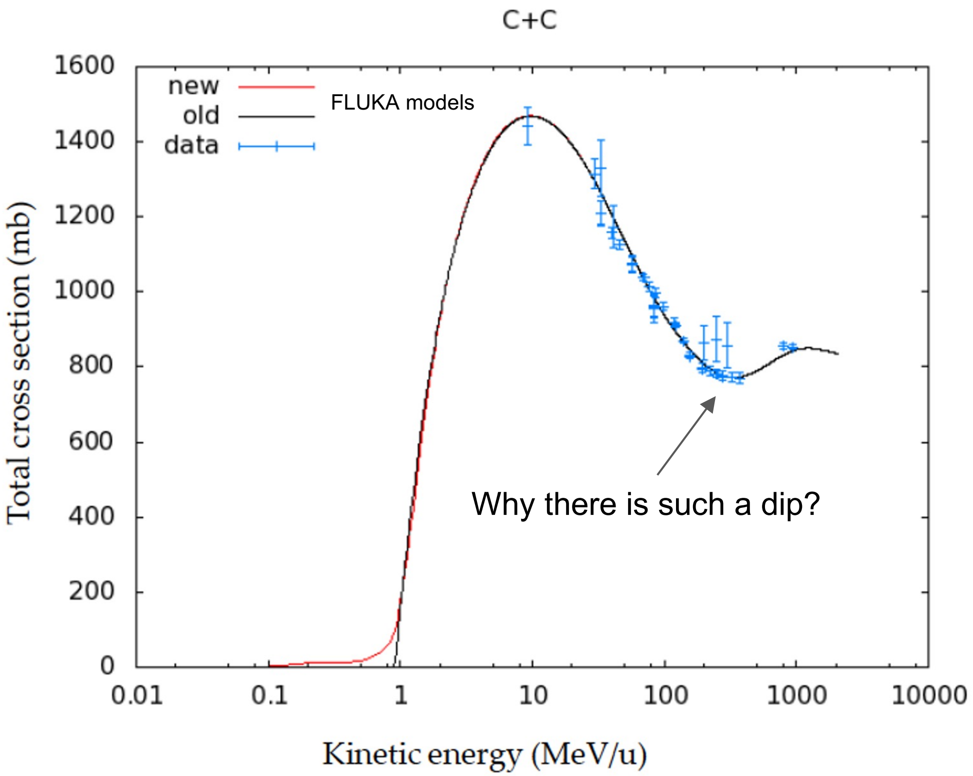
From

- low-energy: mean-field-dominated reactions

To:

- high-energy: nucleon-nucleon collision-dominated reactions

The description in terms of nucleon-nucleon collisions can allow us to understand many features of nuclear cross sections



- Why do total reaction Nucleus-Nucleus cross sections at intermediate energies *(see data points in the plot)* have this shape as a function of energy?
- Can this behaviour be explained at least qualitatively?
- Can these cross sections be approximately calculated?

Definitions

Total Reaction Cross Section:

Experimentally, direct measurement: in this technique one simply counts the number of beam particles incident on the target (N_{inc}), and the corresponding number of outgoing particles which *have not undergone any reaction* in the target (N_{el}). This latter number includes the particles elastically scattered and the residual beam particles (which are not distinguished in this experimental approach).

The difference between these two numbers (N_{inc} and N_{el}) represents the number of reactions (N_{reac}) which occurred in the target for N_{inc} incident particles. If X is the target thickness, A its atomic mass, and N is Avogadro's number, then the value of σ_R can be expressed as

$$\sigma_R = \frac{(N_{inc} - N_{el})}{N_{inc}} \frac{A}{X\ N}$$

→ $\sigma_R = \sigma_{Total} - \sigma_{el}$

We shall see how, in this energy range (and beyond):

1) complex nuclear reactions in the intermediate energy range <u>can be described in terms of individual nucleon-nucleon (n-n) collisions</u>

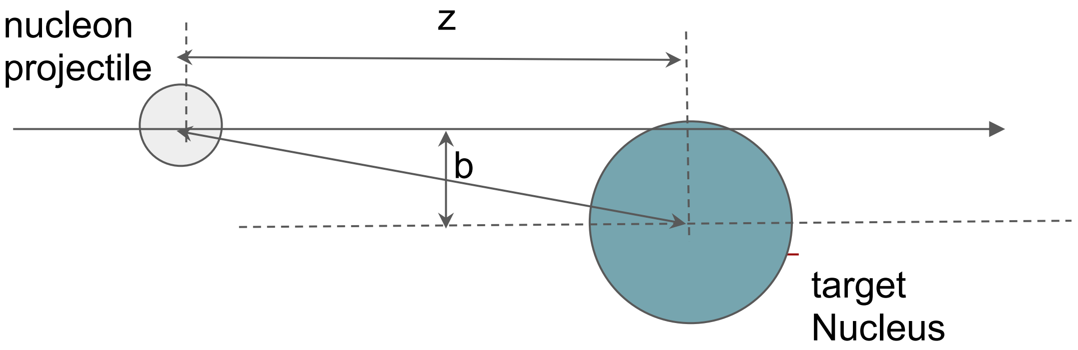
2) the nucleon-nucleon total cross section behaviour as a function of incident energy, strongly determines the shape of the Nucleus-Nucleus cross section

3) Simplified microscopic models, incorporating the measured total nucleon-nucleon cross sections, allow to understand and reproduce, using a limited set of free parameters, the behaviour exhibited by the existing experimental data.

Simplified microscopic models: nucleon-Nucleus interaction

Simplified microscopic models are very useful to understand the basic behaviour.
Basic approach: Glauber (or modified-Glauber) model

Let us start with nucleon-Nucleus collisions. The geometry of the problem is the following:



Simplified microscopic models: nucleon-Nucleus interaction

The physical ingredients to implement the models are simply:

(i) the matter density distribution of the target, and

(ii) and n-n total scattering cross sections: σ_T^{nn}

These quantities are used to construct a local mean free path in the target:

$\Lambda(r)=1/[\rho(r)\, \sigma_T^{\overline{nn}}]$

$\rho(r)$ is the target nuclear matter density and $\sigma_T^{\overline{nn}}$ is the nucleon-nucleon **total** scattering cross section (averaged over isospin, i.e. over p-n and p-p interactions).

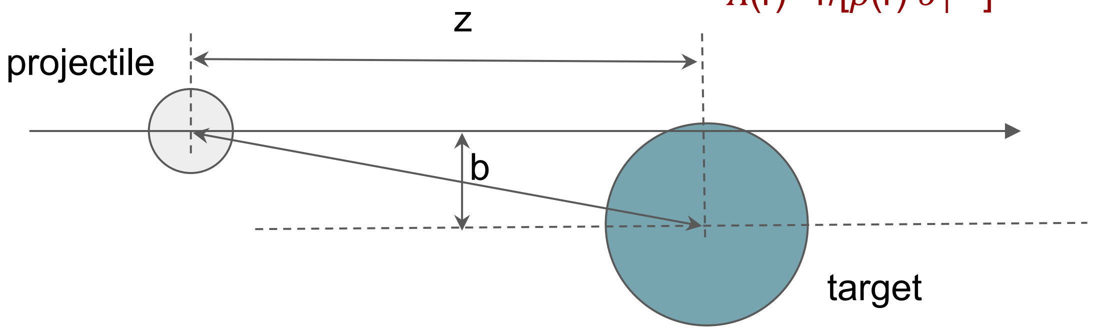
After integrating the local mean free path over the whole trajectory of the projectile, one obtains the *probability for **no** n-n interaction $T_0(b)$* at a given impact parameter *b* . This quantity, for each *b* value, can be expressed as:

"transmission" or

"transparency" function

$$T_0(b) = \exp\left[\int_{-\infty}^{+\infty} -\frac{dz}{\Lambda(r)}\right]$$

$\Lambda(r) = 1/[\rho(r)\ \sigma_T^{nn}]$

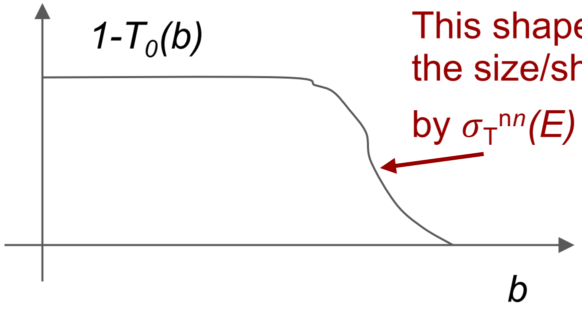


This is the reason why we need the total n-n cross section (elastic+inelastic), and not just the inelatic cross section

Within this microscopic model, the reaction cross section $\boldsymbol{\sigma_R}$ is then derived by assuming that any n-n scattering process leads to a nuclear reaction event between the complex colliding nuclei.

Under this assumption, σ_R is related to $T_0(b)$ (or better to its complement to 1) by the following relation:

$$\sigma_R = \int_0^\infty 2\pi b[1 - T_0(b)]db$$



This shape is determined by the size/shape of nuclei and by $\sigma_T^{nn}(E)$ behaviour

The Glauber Model was first developed for the relativistic energy range, straight-line trajectories of projectile. Also, total cross sections for free n-n scattering were generally employed.

This kind of model can give reasonable results also at intermediate energies provided that other corrections and effect are taken into account. For example: non-straight trajectories due to the Coulomb and nuclear potential, Fermi motion, Pauli blocking etc.

From nucleon-Nucleus to Nucleus-Nucleus collisions

In Nucleus-Nucleus collisions the mean free path of the projectile is obtained by averaging over the mean free path of the different incident nucleons in the target. This is achieved by constructing the volume overlap of the interacting nuclei.

At a given distance ***r*** between the centers of the colliding nuclei, the local mean free path of the projectile in the target is given by:

$$\Lambda(r) = \left[\sigma_T^{\overline{NN}} \int d\tilde{r} \int_V \rho_p(\mathbf{s}) \rho_t(\mathbf{r}-\mathbf{s}) d\mathbf{s} \right]^{-1}$$

$\rho_p(\boldsymbol{s})$ and $\rho_t(\boldsymbol{s})$ being, respectively, the nuclear density distributions of the projectile and target;

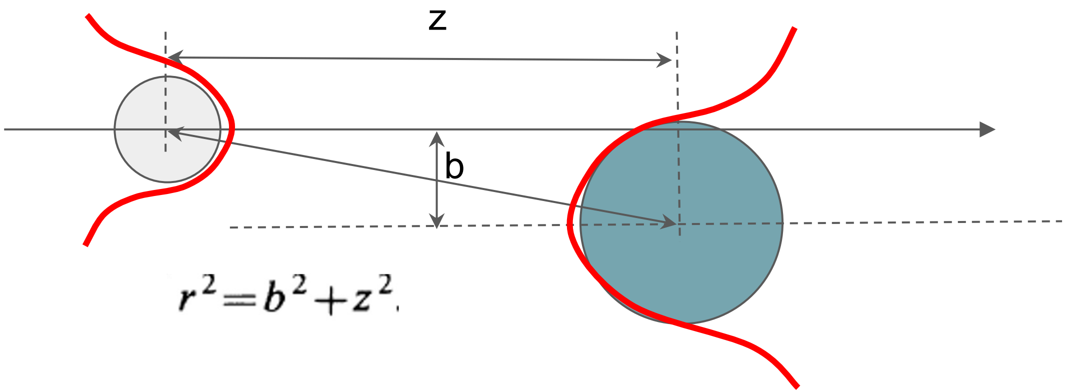
r is the distance between the centers of the nuclei

V is the nuclear volume overlap at the distance ***r***,

s is the position variable for the integration over the overlap volume, and

$d\tilde{r}$ is the angular part of variable ***r***

$\sigma_T^{\overline{nn}}$ is the isospin-averaged nucleon-nucleon total cross section at the energy of interest



$$r^2=b^2+z^2.$$

Nuclei cannot be modelled as uniform spheres. It is fundamental to make use of a reasonable model for the shape of the nuclear matter density distribution.

A possible choice is to assume (as an approximation) a simple Gaussian shape. Its parameters can be deduced semi-empirically to reproduce the tails of the nuclear matter distribution.

Notice that σ_R variations are mainly governed by the most peripheral collisions and thus they are related to the shape of the nuclear surface. For this simple approximation:

$$T_0(b)=\exp\left[-\frac{\pi^2\sigma_T^{\overline{NN}}\rho_t(0)\rho_p(0)a_t^3a_p^3}{a_t^2+a_p^2}\times\exp\left(-\frac{b^2}{a_t^2+a_p^2}\right)\right].$$

Here a_p and a_t represent the Gaussian shape distributions of projectile and target

Other refinements important for low and intermediate energies

The impact parameter b must be modified to take into account the effective trajectory of the projectile as determined by both Coulomb and nuclear potential.

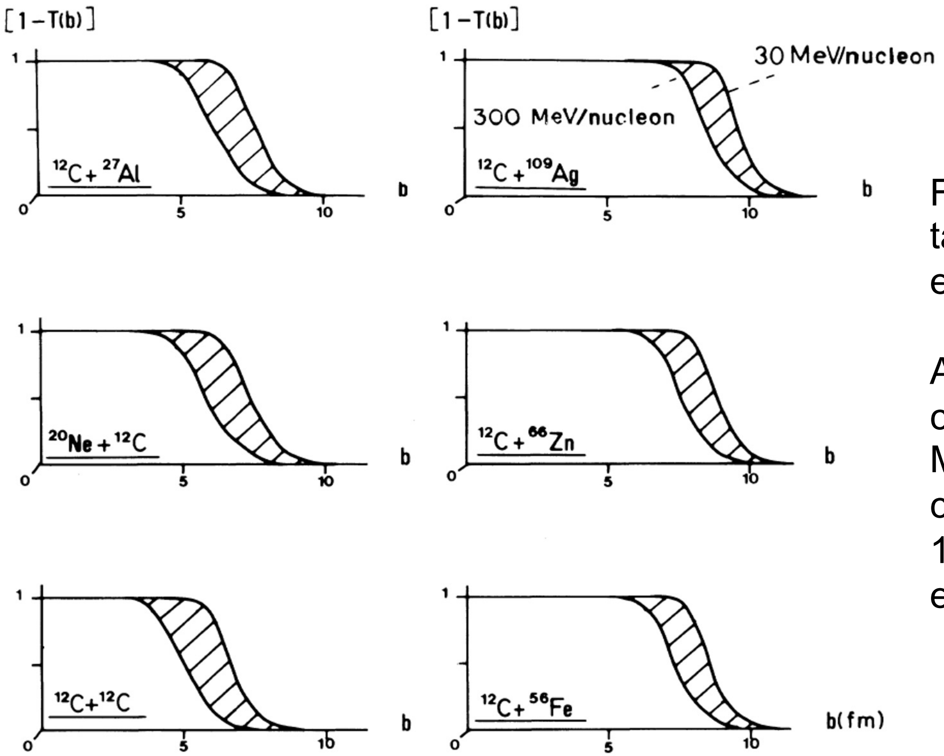
A modified b' can be obtained for each impact parameter in the framework of classical trajectory calculations, to to make use of an effective $T_0(b')$ function:

$$\sigma_R = \int_0^\infty 2\pi b[1 - T_0(b')]db$$

In these calculations the nuclear potential can be obtained from the real part of an optical potential extracted by an analysis of elastic scattering.

Coulomb repulsion usually dominates the projectile trajectory.

For the lightest projectiles the effect of the nuclear potential is important and can significantly counterbalance the Coulomb repulsion.



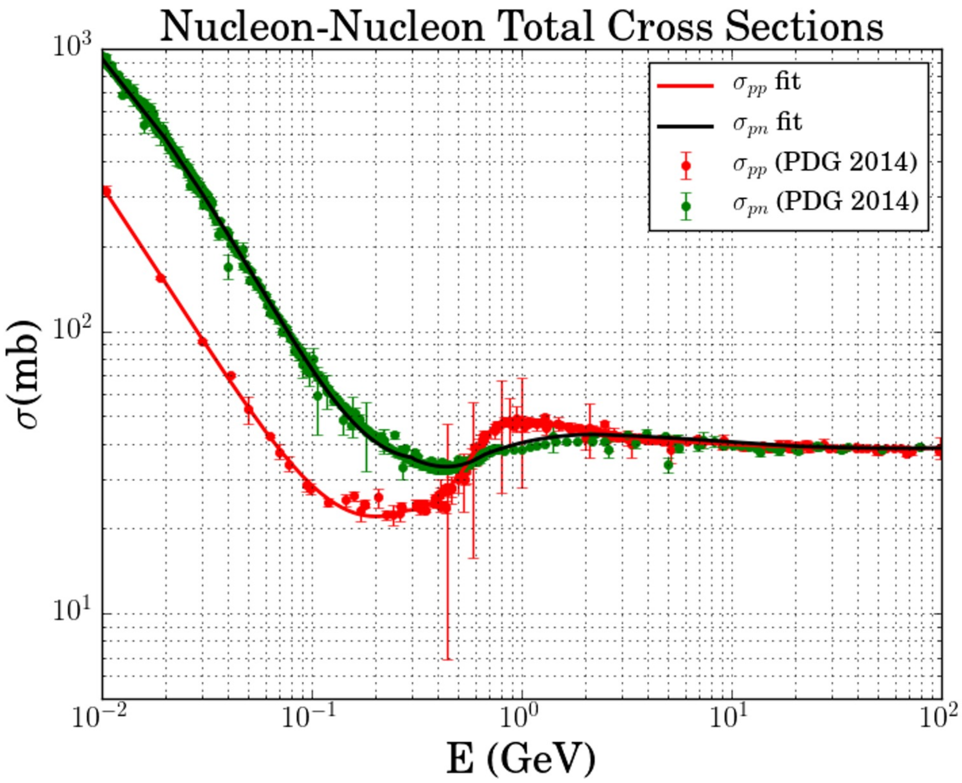
For a given projectile, the larger is the target, the higher is the relevant extension of impact parameter ***b***

At 30 MeV/u the nucleon-nucleon cross section is higher that at 300 MeV/u, therefore, for a given combination of projectile and target, 1-T(b) will remain high for a larger extension of impact parameter ***b***

Modification of the transmission function [1-T_0(b')] with projectile energy, from 30 to 300 MeV/nucleon.

The predicted change of transparency is confined to a 1.5 fm band in the impact parameter on the surface of the reaction sphere: peripheral interactions will dominate Nucleus-Nucleus cross sections (intuitive geometrical reasons).

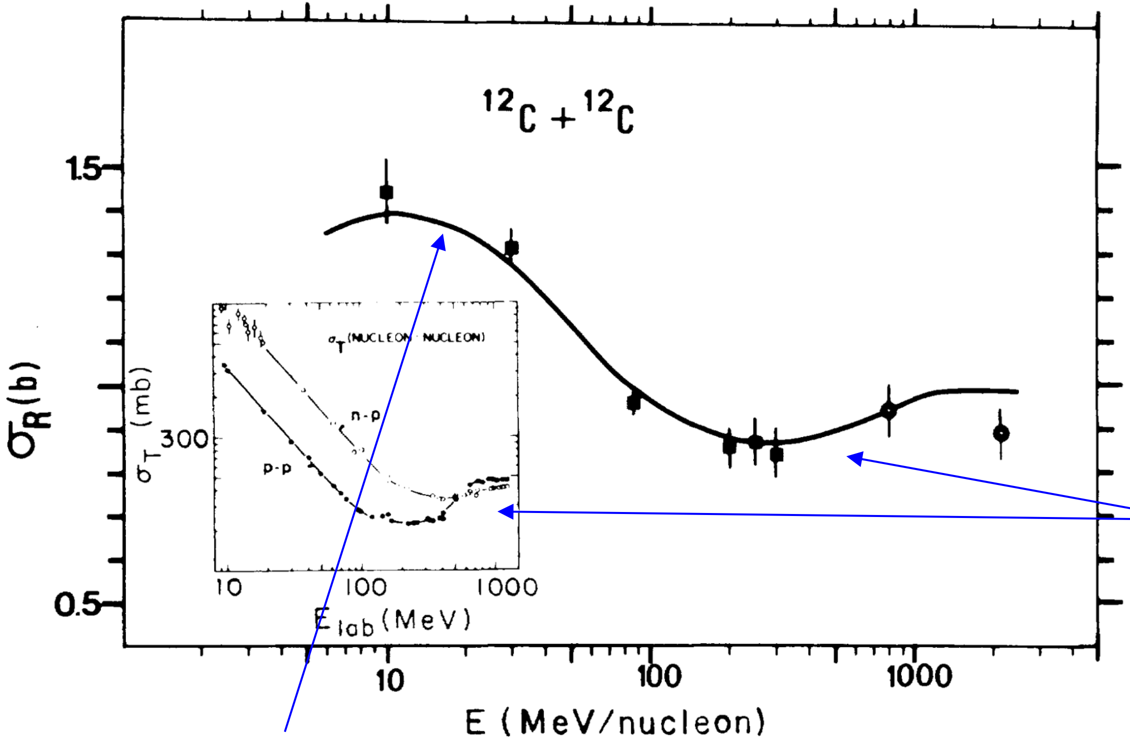
Total p-n and p-p cross sections



It is fundamental in these energies to distinguish between p-n and p-p collisions.

This distinction becomes almost irrelevant for E>few GeVs

An example:

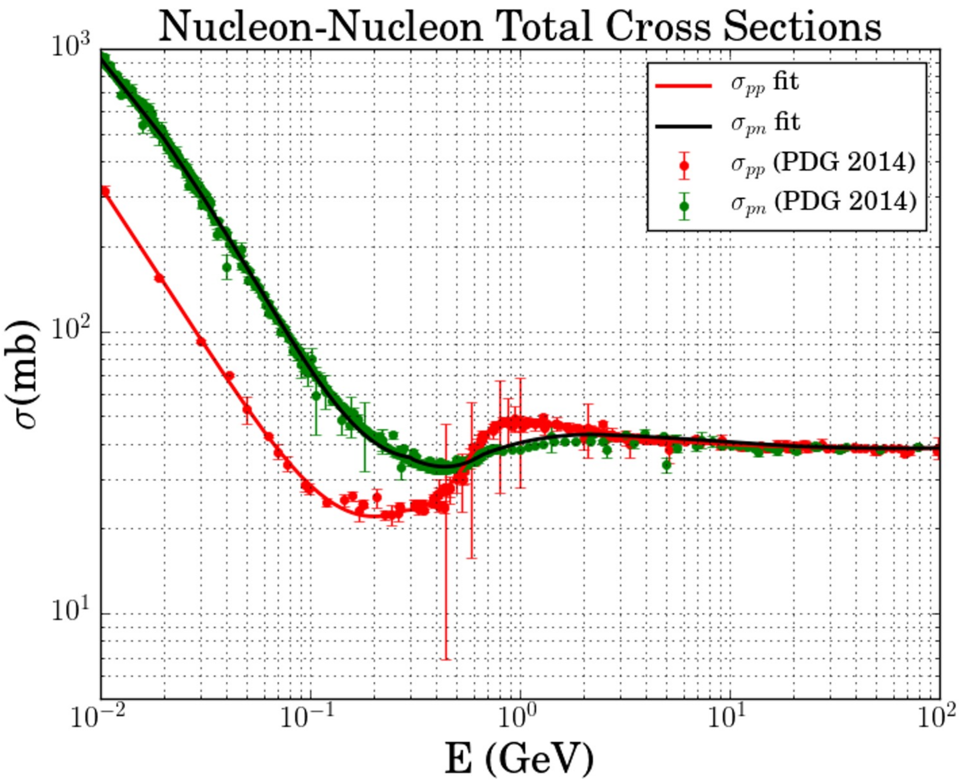


Measurements of the $^{12}C + ^{12}C$ total σ_R (data points) compared to the prediction from a microscopic model like the one summarized here (solid line) *[S. Cox et al., Phys. Rev. C 35 no.5 (1987) 1687]*
This paper does not include Pauli blocking and Fermi motion.
Nucleon-nucleon total cross sections used in the model are shown in the inset.
Their behaviour as a function of energy determines the shape of nucleus-nucleus σ_R

The dip in Nucleus-Nucleus (and also in nucleon-Nucleus) cross section <u>is strongly connected to the dip in nucleon-nucleon cross sections</u>
This example demonstrates the important role of these elementary collisions at intermediate energies.

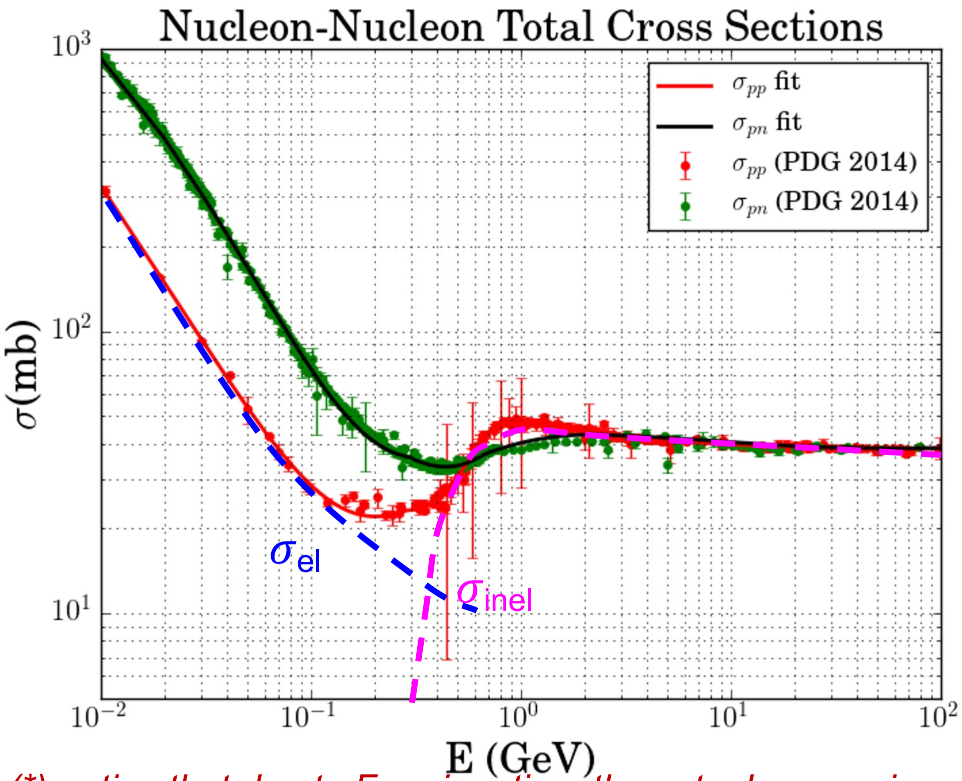
The decrease for σ_R below 15 MeV/nucleon is dominated by Coulomb repulsion and then we enter a regime where the description in terms of single nucleon-nucleon collisions is no more valid

Then the question is: why total p-n and p-p cross sections have this shape?



The ultimate answer can be given only by non-perturbative QCD

However it is possible to summarize some important basic facts that help in understanding the gross features in their behaviour



The total cross section is the sum of elastic and inelastic cross sections:

$\sigma_T = \sigma_{el} + \sigma_{inel}$

They have 2 very different behaviours.

Notice that the *threshold for π production in p-p collision is 290 MeV(*)*

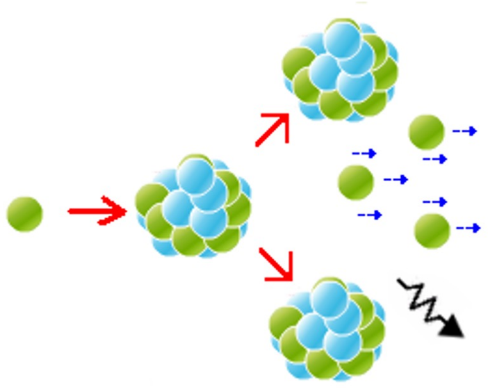
The dip around 200 MeV in p-p (and above for p-n) collisions is determined by the combined behaviour of elastic and inelastic cross sections

(*) notice that due to Fermi motion, the actual energy in collisions inside the nuclei will cover all the cases between these 2 extremes:

Therefore in the lab system the threshold for π production will fluctuate from E<290 to E>290 MeV

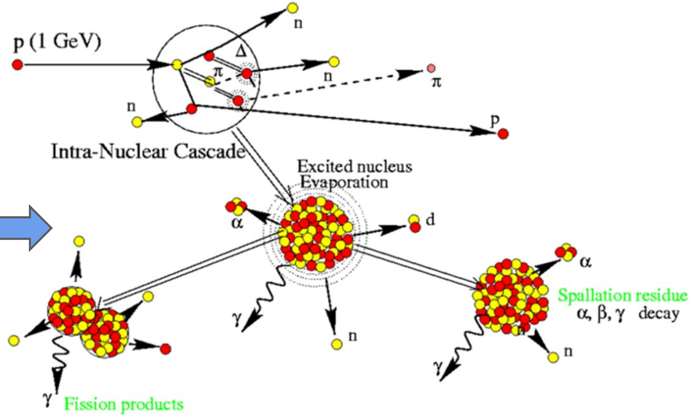
*We also can see that the proton-Nucleus and Nucleus-Nucleus cross sections relevant for the FOOT experiment and hadrontherapy (200-400 MeV/u) are regulated by **total** nucleon-nucleon cross sections which are **dominated by the** nucleon-nucleon **elastic scattering!***

*Nevertheless, although these are mostly elastic scatterings, they are in any case able to excite the nuclei so to produce breakup, leading to the inelastic process that we define as **fragmentation***



[actually we are interested to a mix of different reactions: those which nuclear physicists define as "direct reactions" (essentially peripheral collisions) and those which they classify as "multifragmentation" (mostly deep-inelastic central collisions)]"

*For higher energies, like those which are more relevant for radioprotection in space (>700 MeV/u), the inelastic nucleon-nucleon collisions start to become important (and consequently also the elementary meson-nucleon collisions, in principle, should be considered): **«Intranuclear Cascade»***



How many nucleon-nucleon collisions in a N-N collision?

It is obviously a function of impact parameter value ***b***

When 2 nuclei, respectively of mass number A and B, collide, the maximum number of nucleon-nucleon collisions will be $A \cdot B$ but the probability of having n collisions is given by a binomial distribution

Let's call $q(b)$ the probability that 2 nucleons at impact parameter b do not collide

Then the probability of having n nucleon-nucleon collisions is:

$$P(n,b) = \binom{A \cdot B}{n} [1 - q(b)]^n \, [q(b)]^{A \cdot B}$$

We can also say how this is linked to the cross section:

$$\frac{d\sigma}{d\,b} = \sum_{n=1}^{A \cdot B} P(n,b) = 1 - q(b)^{A \cdot B} \quad \Longrightarrow \quad \sigma = 2\pi \int b\, db\, \left(1 - q(b)^{A \cdot B}\right)$$

How many nucleon-nucleon collisions in a N-N collision?

The other way round:

In proton-Nucleus A: $\langle n \rangle \sim \frac{Z\,\sigma_{pp} + N\,\sigma_{pn}}{\sigma_{pA}} \sim \frac{A\,\sigma_{pp}}{\sigma_{pA}}$ (neglegting p-n differences)

In Nucleus A – Nucleus B: $\langle n \rangle \sim \frac{Z_A\,\sigma_{pB} + N_A\,\sigma_{pB}}{\sigma_{AB}} + \frac{Z_B\,\sigma_{pA} + N_B\,\sigma_{pA}}{\sigma_{AB}}$

$\sim \frac{A\,\sigma_{pA}}{\sigma_{AB}} + \frac{B\,\sigma_{pB}}{\sigma_{AB}}$ (neglegting p-n differences)

This average number is much less than A·B

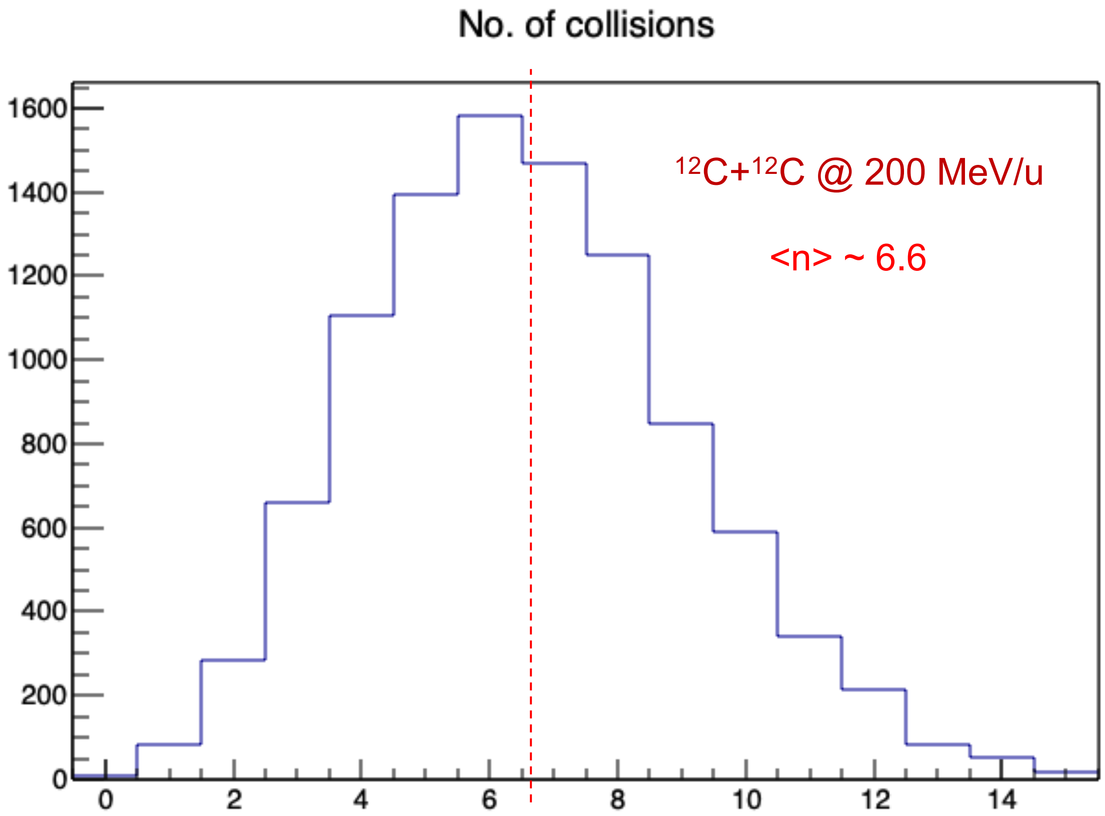
Example:

p+^{12}C @ 200 MeV/u ➔ $\sigma_{pp} \sim$ 20 mb, $\sigma_{pn} \sim$ 40 mb, $\sigma_{pC} \sim$220 mb ➔ $\langle n \rangle \sim 1.6$

^{12}C+^{12}C @ 200 MeV/u ➔ A=12, $\sigma_{pC} \sim \sigma_{nC} \sim$220 mb, $\sigma_{CC} \sim$ 800 mb ➔ $\langle n \rangle \sim 6.6$

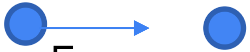
^{16}O+^{12}C @ 200 MeV/u ➔ A=16, $\sigma_{pO} \sim \sigma_{nO} \sim$270 mb, $\sigma_{OC} \sim$ 920 mb ➔ $\langle n \rangle \sim 7.3$

Sampling the number of collisions event-by-event



C.M. energy available in a single nucleon-nucleon collision within a N-N reaction

Lab. system average case:



E_0

$$E_{kin}^{cm} = \sqrt{2\,m^2 + 2\,mE_0} - 2\,m$$

For E_0 = 200 MeV + m ➔ E_{kin} (cm) ~ 98 MeV

Consider Fermi mom. in 2 cases:

$$E_{kin}^{cm} = \sqrt{2\,m^2 + 2\,(E_1 E_2 - P_1 P_2\,cos\theta)} - 2\,m$$

ε_F(kin) ~ 33 MeV; P_F ~ 250 MeV

$E_1 = E_0 - E_F$ $E_2 = E_F$

θ=0°; For E_0 = 200 MeV + m ➔ E_{kin} (cm) ~ 70 MeV

$E_1 = E_0 + E_F$ $E_2 = E_F$

θ=180°; For E_0 = 200 MeV + m ➔ E_{kin} (cm) ~ 218 MeV

Is this enough?

Yes!

^{12}C Break-up Thresholds

$\alpha + {}^{8}Be$	(7.3 MeV)
$p + {}^{11}B$	(~16 MeV)
$n + {}^{11}C$	(18.7) MeV
${}^{9}B + {}^{3}H$	(27.4 MeV)
${}^{10}B + {}^{2}H$	(27.7 MeV)
…	

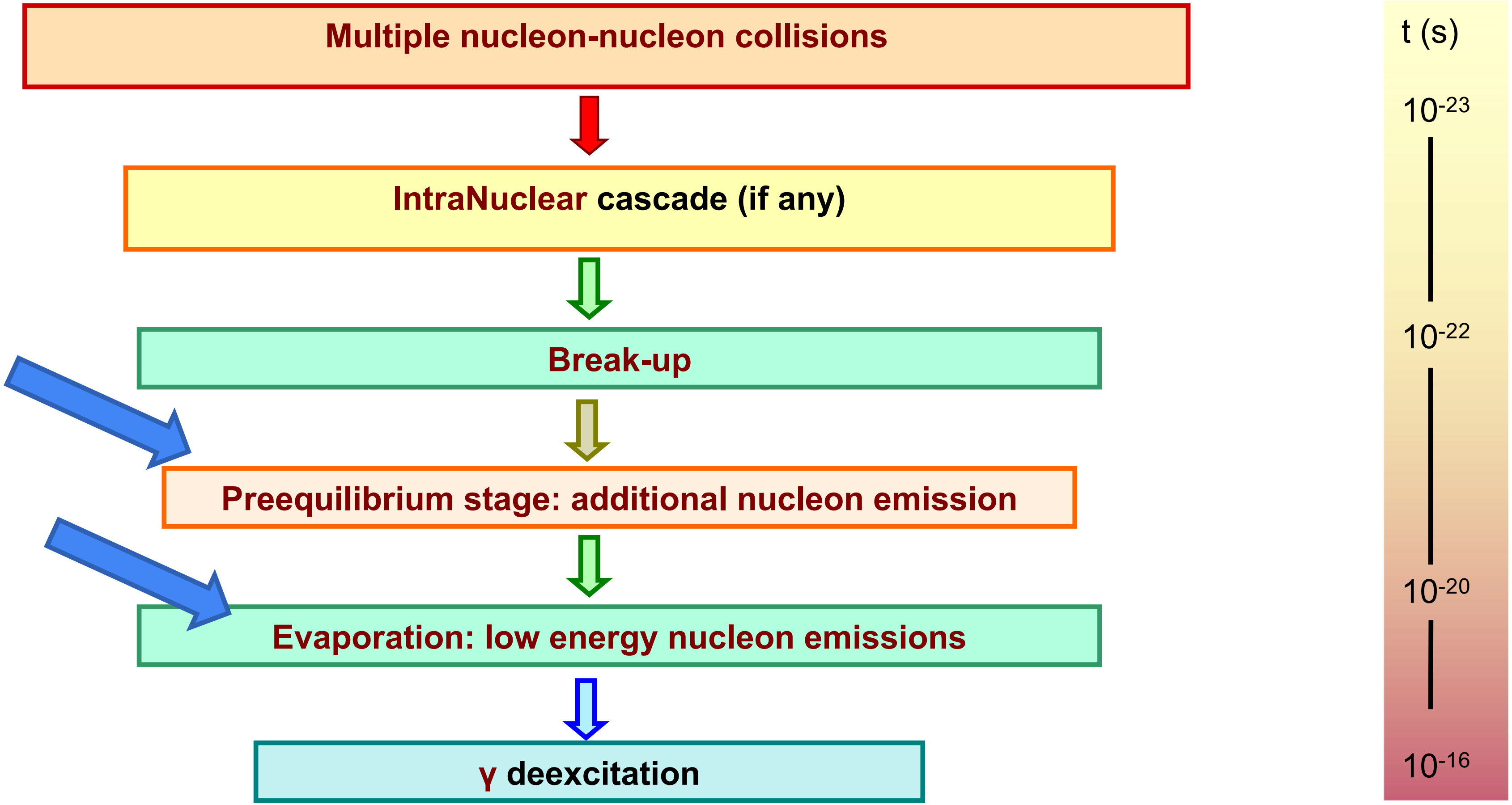
^{16}O Break-up Thresholds

$\alpha + {}^{12}C$	(7.2 MeV)
$p + {}^{15}N$	(12.1 MeV)
${}^{8}Be + {}^{8}Be$	(14.6 MeV)
$n + {}^{15}O$	(15.7 MeV)
….	

^{4}He Break-up Thresholds

$p + {}^{3}H$	(19.8 MeV)
$n + {}^{3}He$	(20.6 MeV)
$d + {}^{2}H$	(23.8 MeV)
$d + {}^{2}H$	(26.1 MeV)
$2n + 2p$	(28.3 MeV)

What happens after the (fast) break-up?

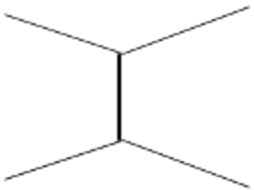


The difference between p-p and p-n collisions at low energy: - 1

Elastic scattering

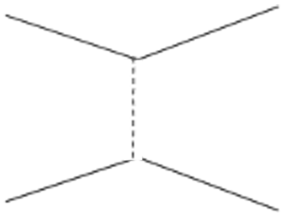
Diagrams representation of nucleon-nucleon elastic scattering amplitude: in principle 2 contributions

Warning: these are not true Feynman graphs, they just represent a process which is intrinsically non-perturbative



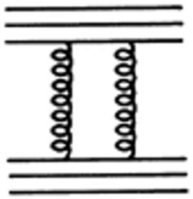
flavor exchange

+



“Pomeron” exchange

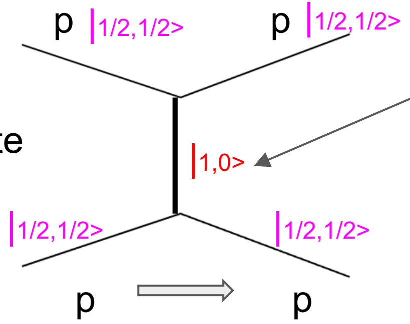
(exchange having the quantum numbers of vacuum)



The only dominant channel at low/intermediate energies of our interest

This contribution is null or negligible at low/intermediate energies

The difference between p-p and p-n collisions at low energy - 2: isospin considerations Elastic process



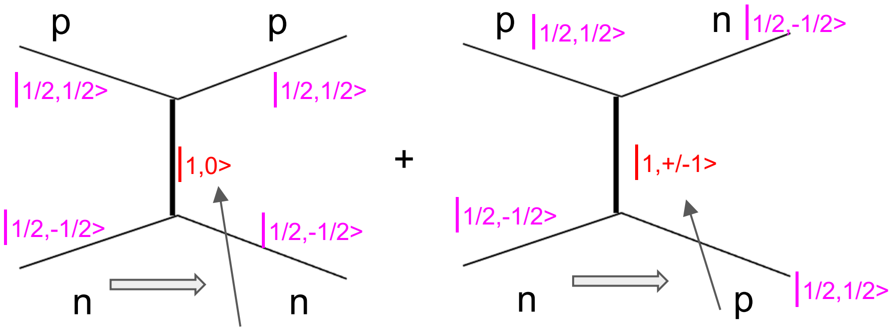
(same quantum numbers as π^0)

In terms of isospin the p-p system is a state $|I,I_3> = |1,1>$

There is only 1 diagram in flavor exchange

the p-n system is a superposition of $|1,0>$ and $|1,1>$ states

There are 2 diagrams in flavor exchange (addition of charge exch. process)



(same quantum numbers as π^0)

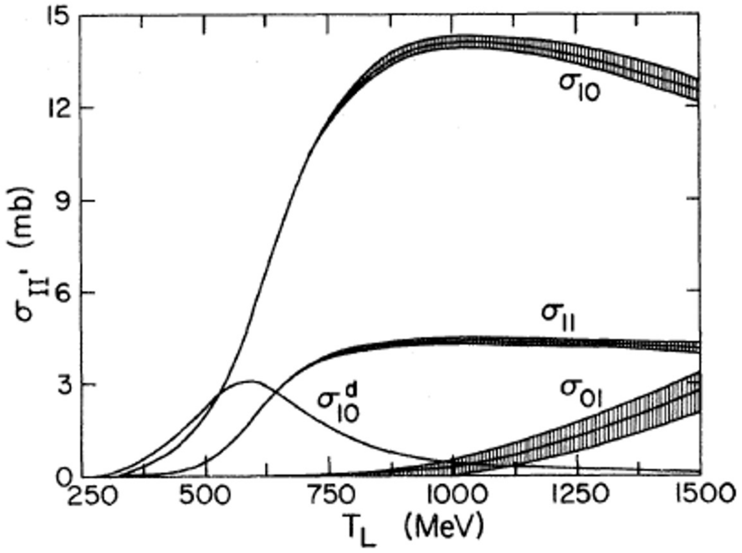
(same quantum numbers as charged π)

The difference between p-p and p-n collisions at low energy - 3: isospin considerations Inelastic processes

Let's take the "simplest" inelastic process: single π production in nucleon-nucleon collisions.

The cross section for this process can be decomposed into isospin components: σ_{11} , σ_{10},...

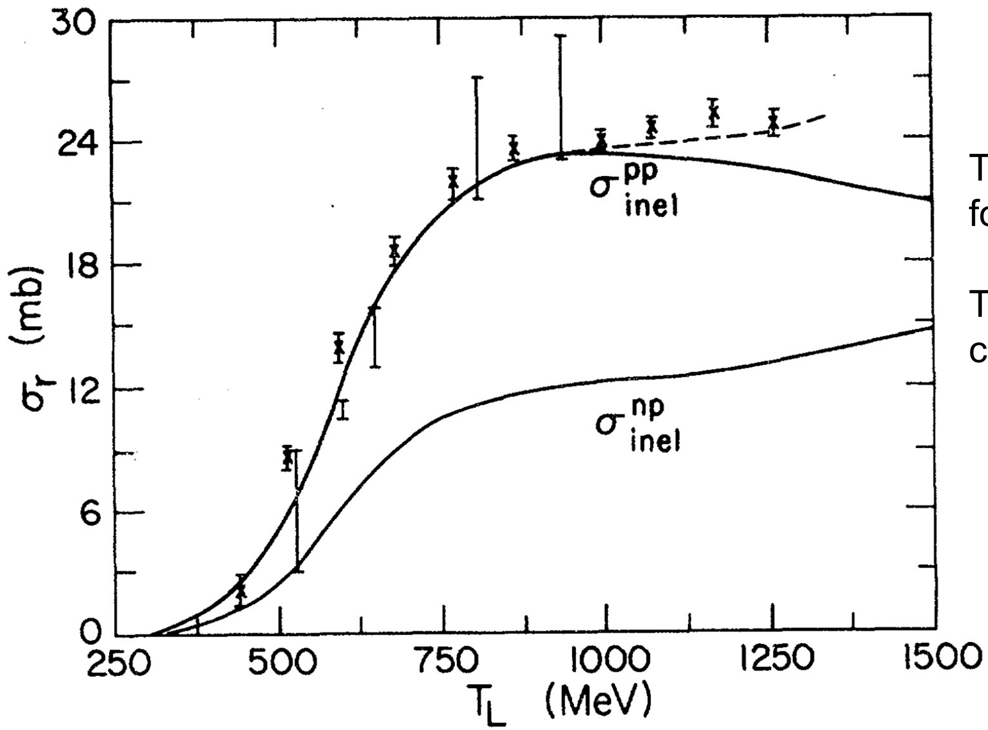
Reactions	Cross Section Decomposition
$pp \to d\pi^+$	σ_{10}^d
$pp \to pp\pi^0$	σ_{11}
$pp \to pn\pi^+$	$\sigma_{10}+\sigma_{11}$
$np \to d\pi^0$	$(\frac{1}{2})\sigma_{10}^d$
$np \to np\pi^0$	$(\frac{1}{2})[\sigma_{10}+\sigma_{01}]$
$np \to nn\pi^+$	$(\frac{1}{2})[\sigma_{11}+\sigma_{01}]$
$np \to pp\pi^-$	$(\frac{1}{2})[\sigma_{11}+\sigma_{01}]$
$pp \to$ inelastic	$\sigma_{10}^d+\sigma_{10}+2\sigma_{11}=\sigma_{I=1}=\sigma_{\text{inel}}^{pp}$
$np \to$ inelastic	$(\frac{1}{2})[\sigma_{10}^d+\sigma_{10}+2\sigma_{11}+3\sigma_{01}]$ $=(\frac{1}{2})[\sigma_{I=1}+\sigma_{I=0}]=\sigma_{\text{inel}}^{np}$ where $\sigma_{I=0}=3\sigma_{01}$.



Isospin reaction cross sections as a function of nucleon incident energy

[B.J. VerWest and R.A. Arndt, Phys. Rev. C 25 no.4 (1982) 1979]

The difference between p-p and p-n collisions at low energy - 4: isospin considerations Inelastic processes



Total reaction cross sections for single π production for pp and np scattering.

The dashed curve indicates the result of adding the cross section for 2 π production

[B.J. VerWest and R.A. Arndt, Phys. Rev. C 25 no.4 (1982) 1979]

It can be seen that $\sigma_{pp}^{inel} > \sigma_{pn}^{inel}$

Conclusions

- At intermediate energies the basic features of total nucleon-Nucleus and Nucleus-Nucleus reaction cross sections can be understood by means of simple microscopic, Glauber-like, models

- The fundamental ingredients to determine the general behaviour of Nucleus-Nucleus cross sections are the *total cross sections of the elementary nucleon-nucleon collisions.* Models of nuclear density and potential, including Coulomb interactions, are the other essential ingredients.

- At intermediate energy it is important to consider the differences in p-p and p-n collisions, which are determined by the isospin structure of strong interactions.

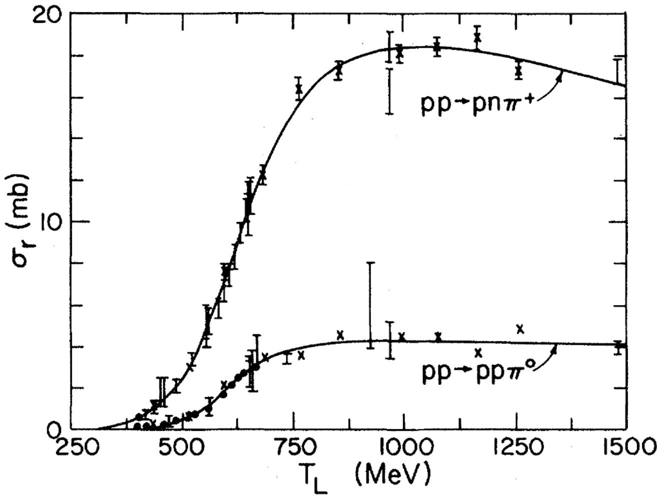
- The proton-Nucleus and Nucleus-Nucleus cross sections relevant for the FOOT experiment and hadrontherapy (200-400 MeV/u) are regulated by total nucleon-nucleon cross sections which are dominated by the elastic scattering

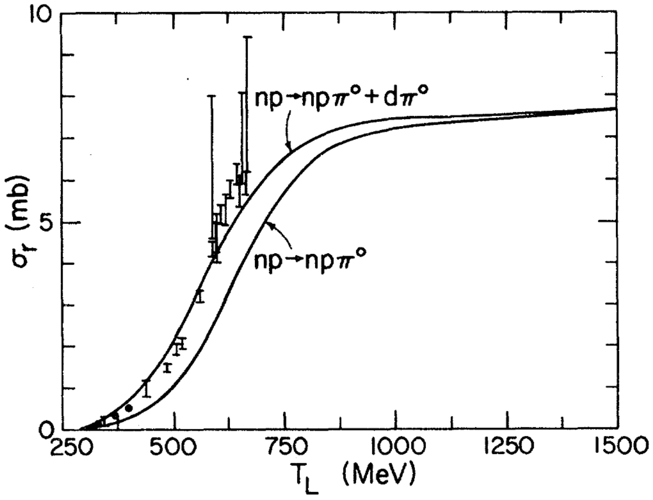
- In the collisions of Light Nuclei, a few nucleon-nucleon collisions are involved. The c.m. kinetic energy released in these collisions is in general enough to induce the break-up of both target and projectile

- Final remark: even more fundamental and refined microscopic models, like those adopted in general purpose MC codes, or in specialized codes (QMD, AMD…), in order to build interactions, need in any case to incorporate the elementary nucleon-nucleon cross sections. In general, fits to existing data are used for this purpose.

Appendix

The particular case of single π^0 production



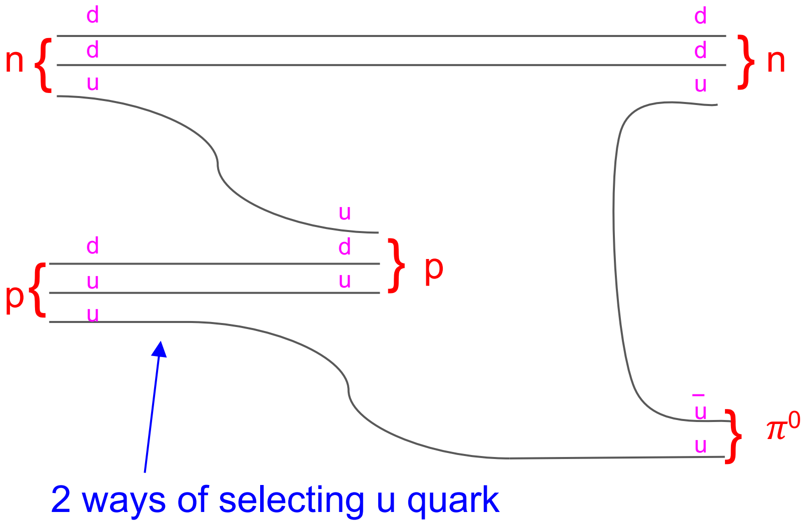


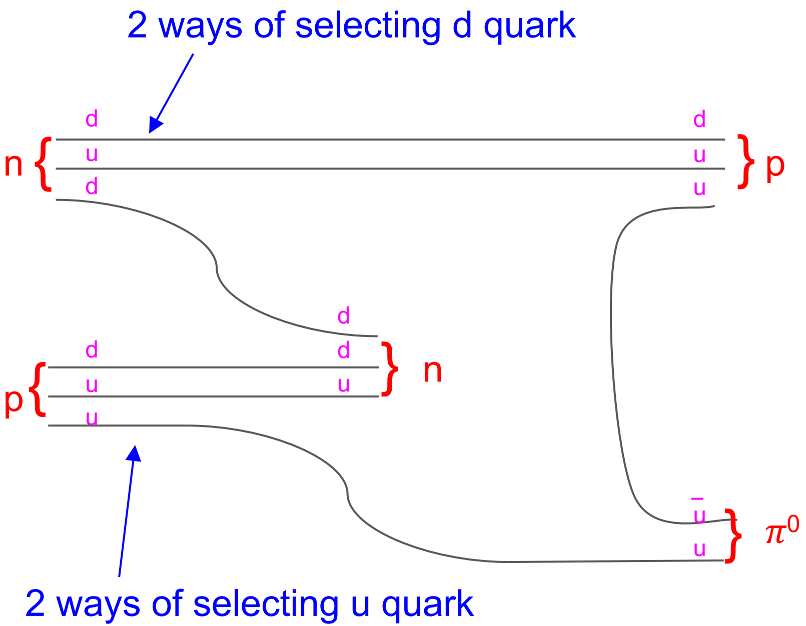
Notice that in this particular case $\sigma_{pp}^{inel} < \sigma_{pn}^{inel}$ since $\sigma_{11} < (\sigma_{10}+\sigma_{01})/2$

Another way (qualitative) of looking at inelastic processes

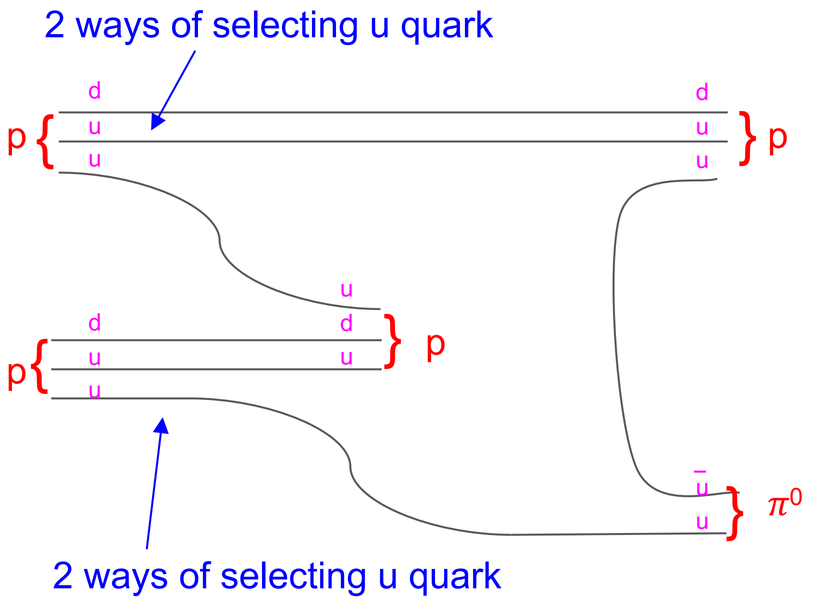
for example let's take the particular case of

a) $np \rightarrow np\pi^0$





b) $pp \rightarrow pp\pi^0$



less contributions with respect to $np \rightarrow np\pi^0$

Exercise: build the diagrams for $np \rightarrow pp\pi^-$ $np \rightarrow nn\pi^+$ $pp \rightarrow pn\pi^+$