Tracking using Kalman filter in ALICE Framework

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https://agenda.infn.it/event/1096/contributions/6159/attachments/4504/4980/Rotondi_3.pdf

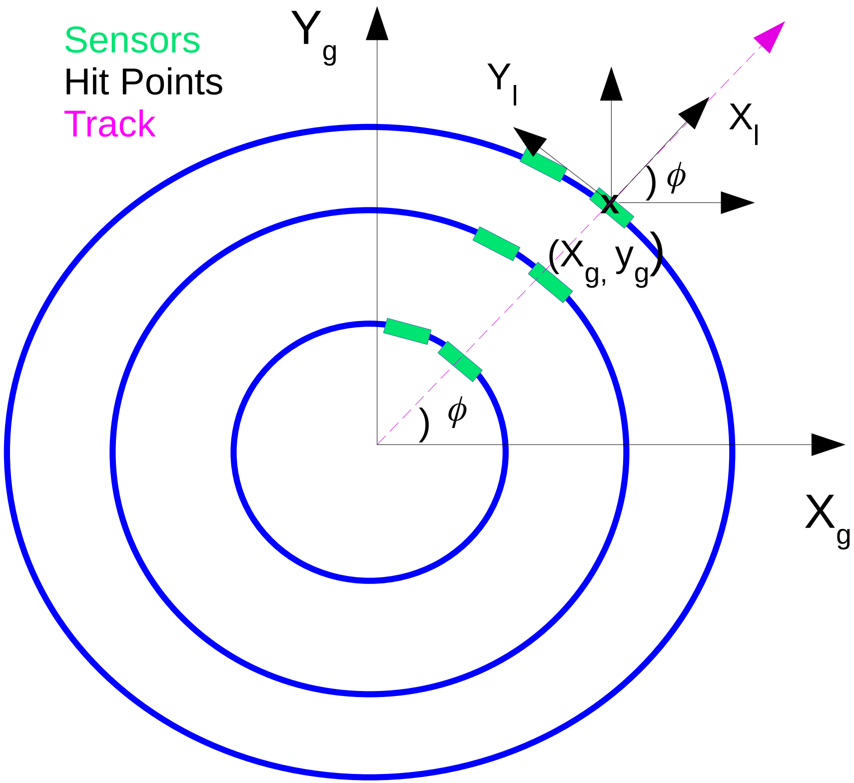
http://www.le.infn.it/lhcschool/talks/Ragusa.pdf

Local Coordinate ALICE

X local is normal to the sensor

Y local is on the sensor

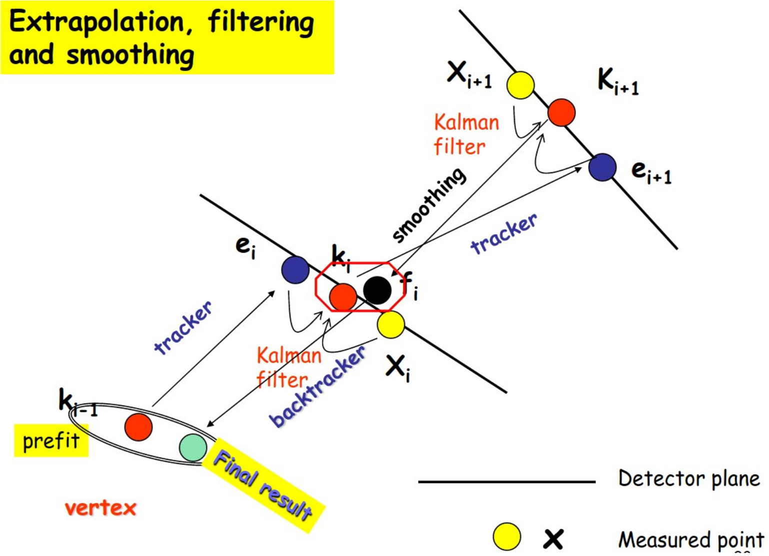
$$\phi = \tan^{-1}\left(\frac{y_g}{x_g}\right)$$



In this coordinate system: ylocal and zlocal will describes the sensor

Uncertainties in ylocal is σ(rφ) and on zlocal is σ(z)
x_l is describing the radius

Track Parameters (Local): $(y_l, z_l, \sin\phi, \tan\lambda, q/p_T)$



Kalmn Filter Method:

1. Extrapolation: Extrapolation (e_i) on the next plane with M.S. effect
2. Filtering: Weighted average of extrapolated value (e_i) and measured value (x_i), known as Kalman filtered value (k_i)
3. Smoothing: Estimation of parameters to extrapolate

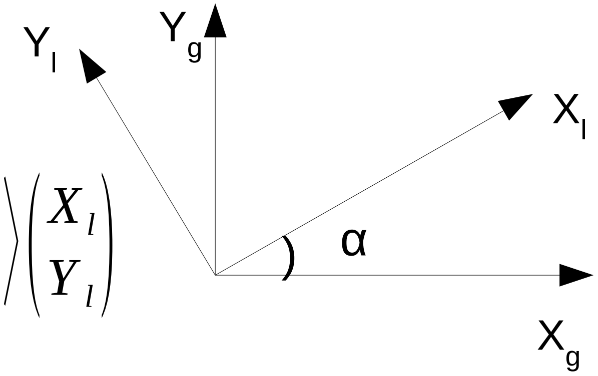
ALICE Track Parameters

Track Parameters (Local): $(y\,, z\,, \sin\phi, \tan\lambda\,, q/p_T)$

On cylindrical surface:

$$x^2 + y^2 = R^2$$

$$\begin{pmatrix} X_g \\ Y_g \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} X_l \\ Y_l \end{pmatrix}$$



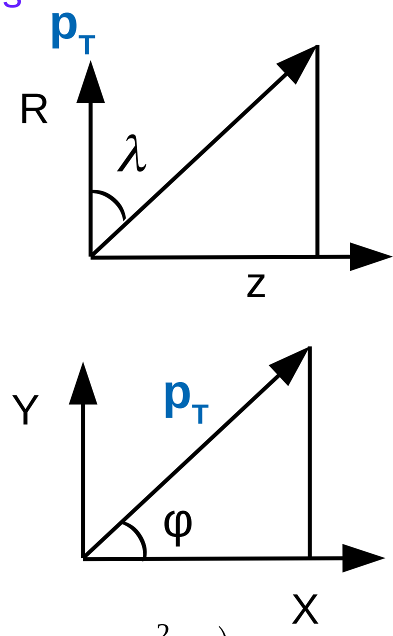
Local coordinates rotated by α w.r.t. global coordinates

$$\boxed{\frac{\sigma_{1/p_t}}{(1/p_t)} = \frac{1/p_t^2 * \sigma_{pt}}{(1/p_t)} = \frac{\sigma_{pt}}{p_t} \qquad \sigma^2_{DCA_{r\phi}} = \sigma^2_y \qquad \sigma^2_{DCA_z} = \sigma^2_z}$$

Parameter Covariance

Track Parameters

$$\begin{pmatrix} y \\ z \\ \sin\phi \\ \tan\lambda \\ 1/p_T \end{pmatrix} \qquad W = \begin{pmatrix} \boxed{\sigma^2_y} & \sigma_{yz} & \sigma_{y\sin\phi} & \sigma_{y\tan\lambda} & \sigma_{y.1/p_T} \\ \sigma_{zy} & \boxed{\sigma^2_z} & \sigma_{z\sin\phi} & \sigma_{z\tan\lambda} & \sigma_{z.1/p_T} \\ \sigma_{\sin\phi\, y} & \sigma_{\sin\phi\, z} & \sigma^2_{\sin\phi} & \sigma_{\sin\phi\tan\lambda} & \sigma_{\sin\phi.1/p_T} \\ \sigma_{\tan\lambda\, y} & \sigma_{\tan\lambda\, z} & \sigma_{\tan\lambda\sin\phi} & \sigma^2_{\tan\lambda} & \sigma_{\tan\lambda.1/p_T} \\ \sigma_{1/p_T.\, y} & \sigma_{1/p_T.\, z} & \sigma_{1/p_T.\sin\phi} & \sigma_{1/p_T.\tan\lambda} & \boxed{\sigma^2_{1/p_T}} \end{pmatrix}$$



Parameter Covariance matrix = 5X5 matrix; 5(5+1)/2 = 15 independent entries

$$(\sigma^2_y, \sigma_{yz}, \sigma_{ysin\,\phi}, \sigma_{yTan\,\lambda}, \sigma_{y.1/p_T}, \sigma^2_z, \sigma_{zsin\,\phi}, \sigma_{ztan\,\lambda}, \sigma_{z.1/p_T}, \sigma^2_{\sin\phi}, \sigma_{\sin\phi\tan\lambda}\ \sigma_{\sin\phi.1/p_T}, \sigma^2_{\tan\lambda}, \sigma_{\tan\lambda.1/p_T}, \sigma^2_{1/p_T})$$

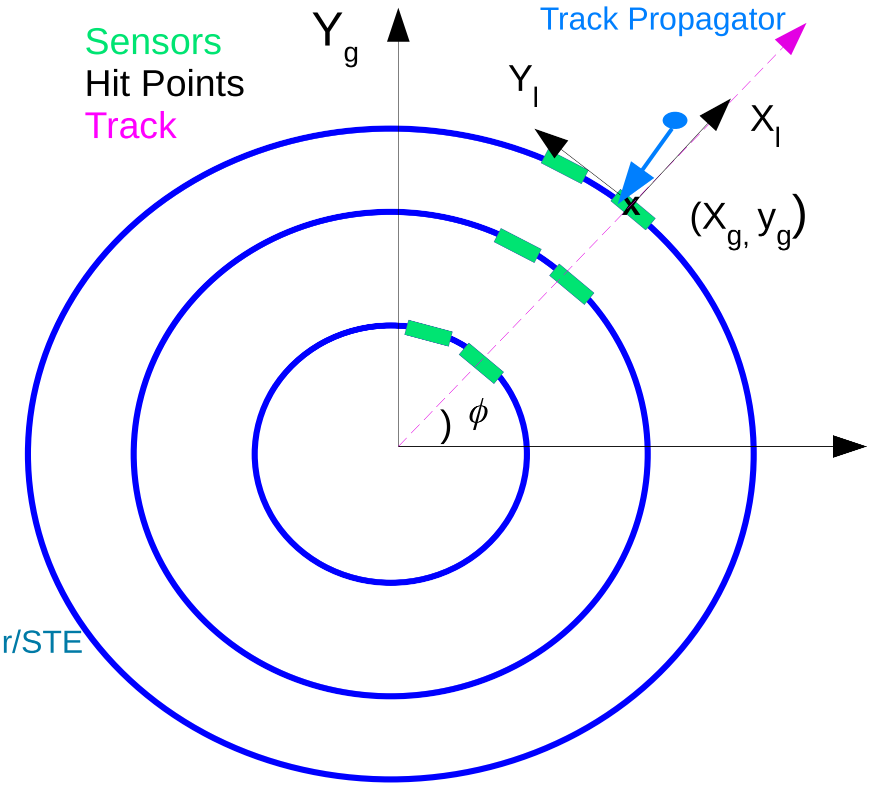
Track Fitting Method

Two things required: Extrapolation with MS and Measured Points

Common Steps:

- Track Model initialize with the last point x+Margin (0.1)
- Convert last point to local coordinate system rotated by phi.
- Extrapolate track to the last layer
- Rotate Track to local coordinate system
- Update extraolation and measurement (weight average of position and errors
- Multiple scattering correction

Repeat above steps for each layer up to layer 1 and then extrapolate to the vertex



The filtering is nothing but a **weighted average** of the

new measurement y_n

the prediction y_p

$$y_f = \frac{\frac{1}{\sigma_p^2} y_p + \frac{1}{\sigma_n^2} y_n}{\frac{1}{\sigma_p^2} + \frac{1}{\sigma_n^2}}$$

$$\boxed{y_f = \frac{\sigma_n^2}{\sigma_p^2 + \sigma_n^2} y_p + \frac{\sigma_p^2}{\sigma_p^2 + \sigma_n^2} y_n}$$

Clearly if the **new measurement** has a **very large error**

$\sigma_n \to \infty \qquad y_f \to y_p$ measurement ignored

If the **prediction** has a **large error** (for example large multiple scattering)

$\sigma_p \to \infty \qquad y_f \to y_n$ prediction ignored

hub.com/alisw/AliRoot/blob/master/STE
:RBase/AliExternalTrackParam.h

Track Extrapolation

`Bool_t PropagateTo(Double_t x, Double_t b);`

Bool_t CorrectForMeanMaterialdEdx(Double_t **xOverX0Si**, Double_t 0,
Double_t mPion, Bool_t anglecorr=kTRUE);

At the vertex x_l = 0 and y_l is DCA_{xy}

Checking code for three layers as an excercise then will apply for beam test data

Multiple Scattering

Covariance matrix entries affected by multiple scattering

$$
\begin{array}{c|ccccc}
 & 1/p & \lambda & \phi & y_{\perp} & z_{\perp} \\
\hline
1/p & 0 & 0 & 0 & 0 & 0 \\
\lambda & 0 & <\theta_p^2> & 0 & 0 & -\dfrac{<\theta_p^2>\ \mathrm{d}l}{2} \\
\phi & 0 & 0 & \dfrac{<\theta_p^2>}{\cos^2\lambda} & \dfrac{<\theta_p^2>\ \mathrm{d}l}{(2\cos\lambda)} & 0 \\
y_{\perp} & 0 & 0 & \dfrac{<\theta_p^2>\ \mathrm{d}l}{(2\cos\lambda)} & \dfrac{<\theta_p^2>\ (\mathrm{d}l)^2}{3} & \\
z_{\perp} & 0 & -\dfrac{<\theta_p^2>\ \mathrm{d}l}{2} & 0 & 0 & \dfrac{<\theta_p^2>\ (\mathrm{d}l)^2}{3}
\end{array}
$$

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