What determines the resolution of a crystal calorimeter ?

David Hitlin

Super$B$ Workshop XVII - Elba
May 29, 2011

What is in the GEANT4 Monte Carlo?

- Our GEANT4 expectation for the energy resolution for the LYSO beam test calorimeter is based on photoelectron statistics:

- Method:
  - Use deposited energy/crystal
  - Add Poisson smearing with 450 P.E./MeV
  - Add noise effect
    - Simple sinusoidal
    - From measured power spectrum

Stefano Germani, Nov. 10, 2011 EMC meeting

Plus a linear correction for light collection non-uniformity

- As we know, the situation is actually somewhat more complicated

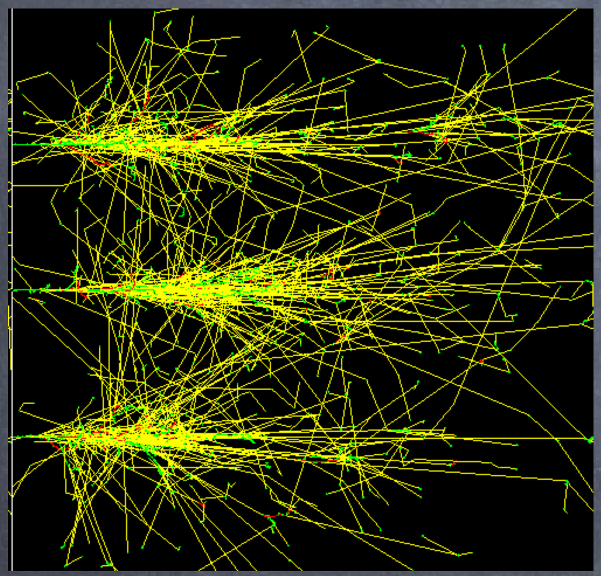
There are many contributions to the energy resolution:

$$\sigma_{\mathrm{exp}}^{2} = \sigma_{\mathrm{shower}}^{2} + \sigma_{\mathrm{leakage}}^{2} + \sigma_{\mathrm{photoelectrons}}^{2} + \sigma_{\mathrm{electronics}}^{2} + \sigma_{\mathrm{light\,collection}}^{2} + \sigma_{\mathrm{intrinsic}}^{2} + \sigma_{\mathrm{beam}}^{2}$$

Contributions to the energy resolution

$$\sigma_{\mathrm{exp}}^{2}=\sigma_{\mathrm{shower}}^{2}+\sigma_{\mathrm{leakage}}^{2}+\sigma_{\mathrm{photoelectrons}}^{2}+\sigma_{\mathrm{electronics}}^{2}+\sigma_{\mathrm{light\ collection}}^{2}+\sigma_{\mathrm{intrinsic}}^{2}+\sigma_{\mathrm{beam}}^{2}$$

GEANT4



APD area
+
QE *vs* λ
+
self-absorption

preamp noise
+
crosstalk
+
dark current fluctuation

geometry
+
surface
+
wrapping

non-proportionality of scintilllation process
+
Ce_3^+ doping inhomogenity

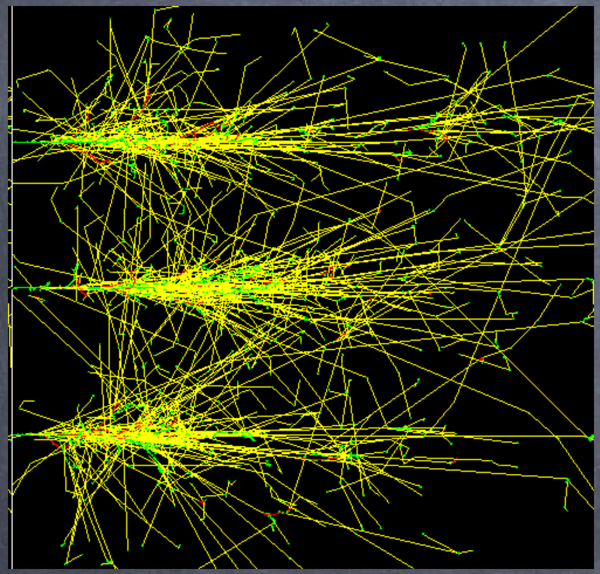
CERN
LNF
MAMI
…

rms 5.5% LO 1500 sqrt = 2.6% as received
SIPAT rms 3.3% LO 1200 sqrt = 2.9% after 15mm stripe

Contributions to the energy resolution

$$\sigma_{exp}^2 = \sigma_{shower}^2 + \sigma_{leakage}^2 + \sigma_{photoelectrons}^2 + \sigma_{electronics}^2 + \sigma_{light\,collection}^2 + \sigma_{intrinsic}^2 + \sigma_{beam}^2$$

GEANT4



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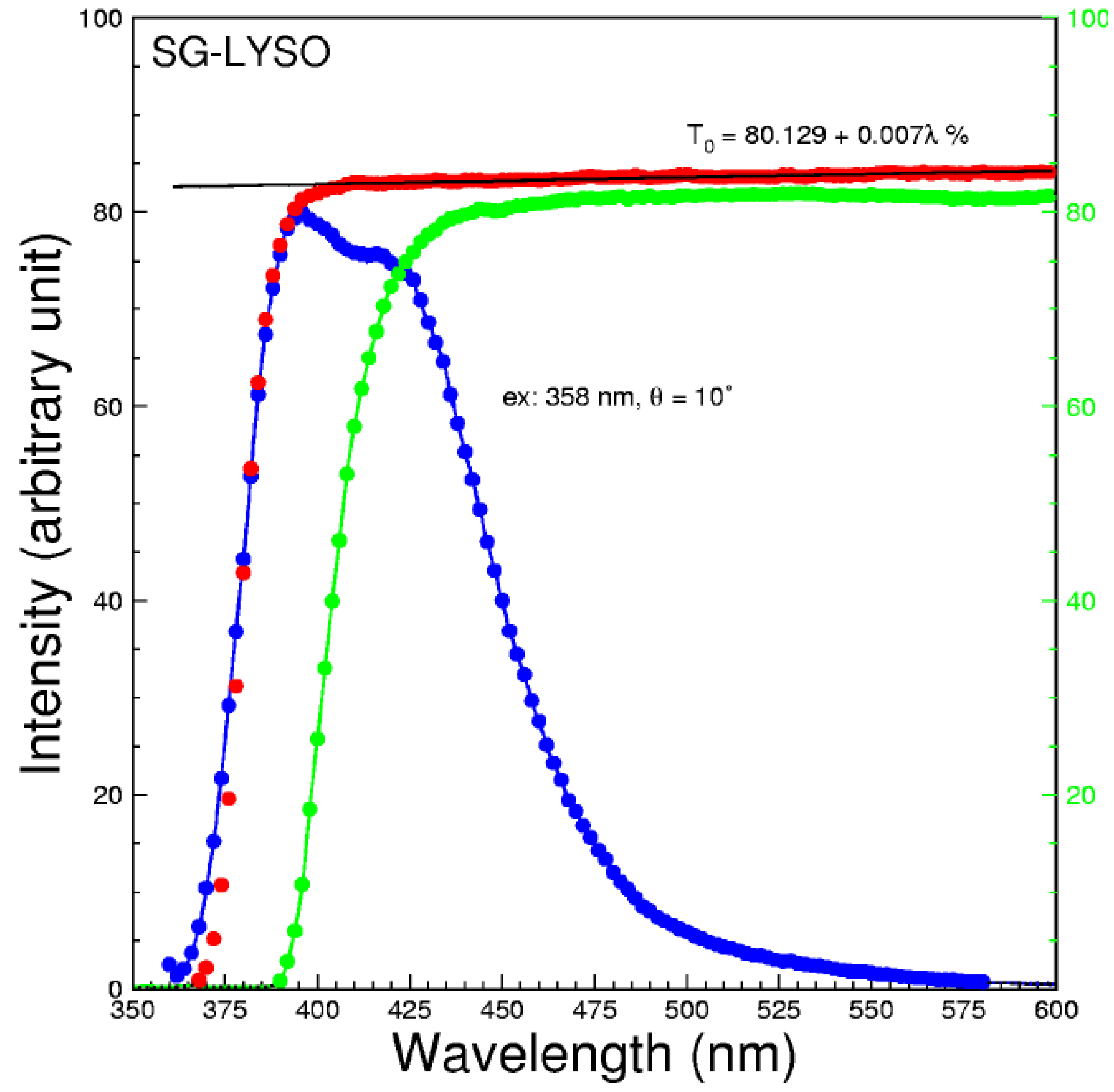
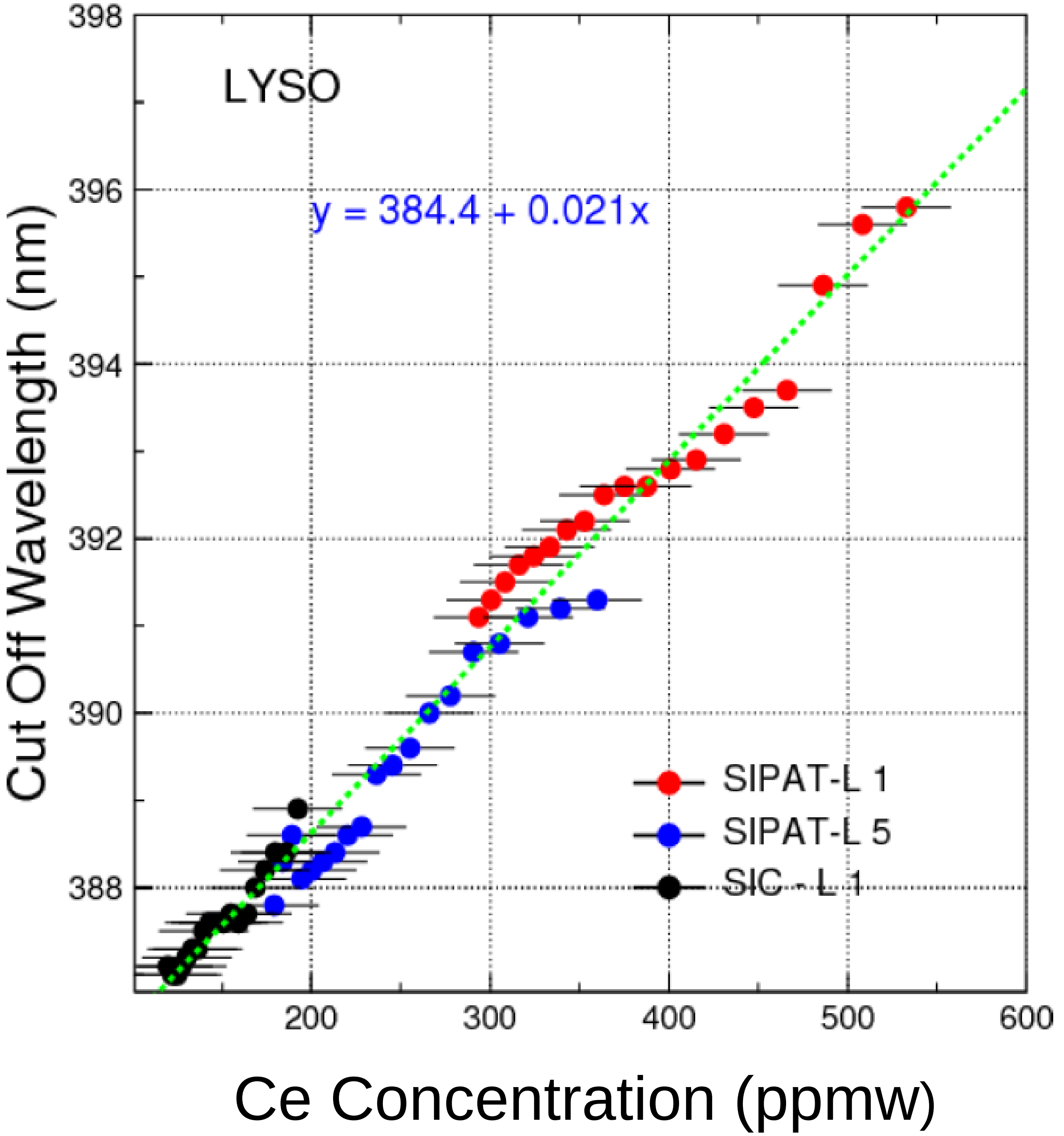
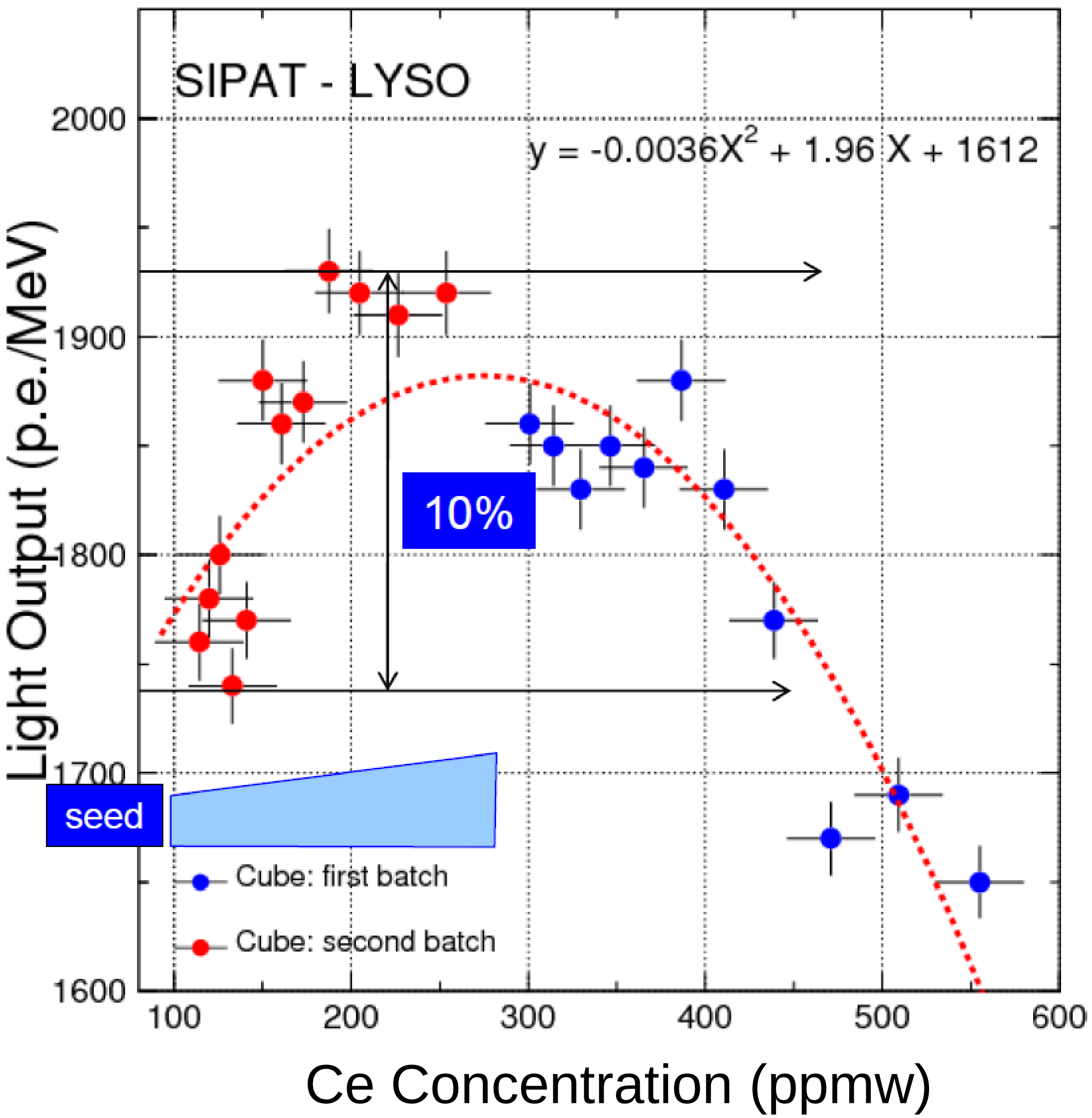
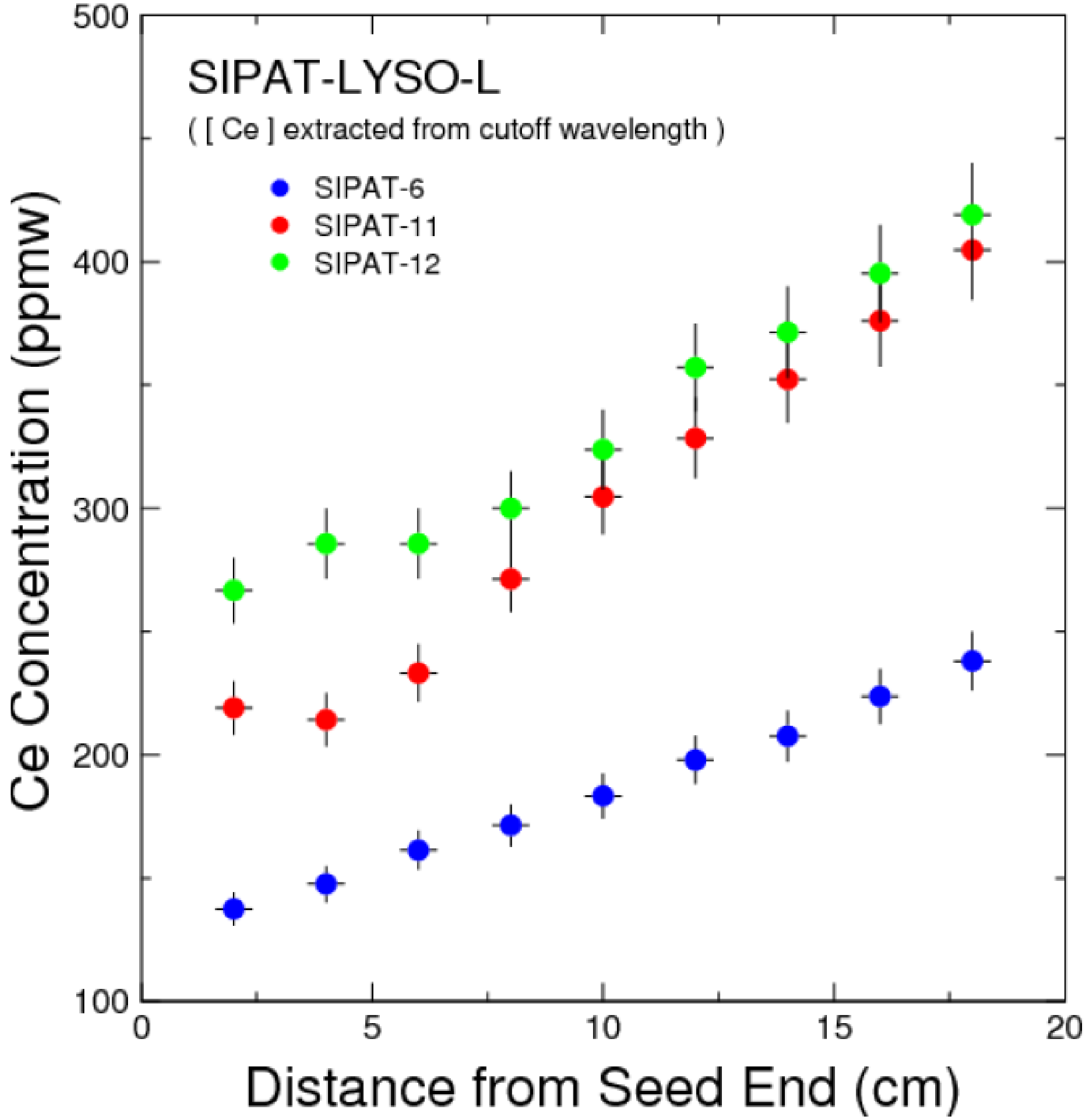
Senesitivity of cutoff to cerium concentration





Effect of cerium concentration





Summary of SuperB Test Beam Crystals

Caltech-ID	Vendor-ID	Test-Beam-Position	Type	LT @ 420 nm (%)	LY, ER & Uniformity by PMT* (% of candel 1), (FWHM, %), (δ, %)	LY, ER & Uniformity by APD (As)* (p.e./MeV), (σ, %), (δ, %) (rms, %)	LY, ER & Uniformity by APD (Uni)* (p.e./MeV), (σ, %) , (δ, %) (rms, %)	LO Loss %
SIPAT-11	02_08_08	ring 8-3	8	82.3	47.6, 10.7, 5.3	1420, 15.5, 12.9, 6.5	1190, 21.4, 7.2, 3.8	16.2
SIPAT-12	02_08_08	ring 8-1	8	82.2	46.5, 10.4, 3.9	1440, 15.1, 14.2, 7.1	1210, 20.7, 10.0, 5.1	15.9
SIPAT-13		ring 6-1	6	82.6	52.5, 11.5, 2.7	1440, 14.9, 6.8, 3.6	1220, 20.4, 3.4, 2.0	15.3
SIPAT-14		ring 6-2	6	82.7	53.7, 10.9, 3.2	1500, 14.9, 14.4, 7.4	1200, 20.4, 9.0, 4.6	20.0
SIPAT-15		ring 6-4	6	80.7	52.8, 10.5, 3.4	1580, 13.7, 11.9, 6.0	1310, 19.1, 6.1, 3.4	17.1
SIPAT-16		ring 6-5	6	81.1	51.8, 10.1, -0.8	1570, 13.5, 9.7, 5.0	1100, 19.6, 5.3, 2.7	29.9
SIPAT-17		ring 6-3	6	82.1	53.0, 12.2, 3.5	1260, 17.1, 9.8, 4.9	1080, 24.1, 4.9, 2.7	14.3
SIPAT-20	07_10_02	ring 7-2	7	79.8	56.4, 10.0, 5.6	1670, 14.6, 8.7, 4.4	1340, 18.2, 5.1, 2.6	19.8
SIPAT-21	02_10_23	ring 7-5	7	81.6	48.8, 10.9, 3.0	1550, 15.8, 10.7, 5.6	1190, 20.7, 6.1,3.2	23.2
SIPAT-22	07_10_02	ring 7-1	7	81.4	52.6, 11.0, 2.7	1600, 15.2, 9.2, 4.8	1180, 20.3, 5.2, 3.0	26.3
Average				81.7	51.5, 10.8, 3.3	1500, 15.0, 10.8, 5.5	1200, 20.5, 6.2, 3.3	19.8
SG-S1			8	80.5	52.2, 9.8, 1.0	1370,14.5,9.6,5.0	1040,19.7,5.4,2.8	24.1
SG-S2			8	79.5	54.2, 9.6, 1.4	1400,14.3,9.0,4.7	1040,19.5,6.6,3.4	25.7
SG-S3			9	79.1	56.0, 9.8, 1.0	1370,14.7,8.0,4.2	1000,19.7,6.1,3.2	27.0
SG-S4			9	80.1	56.5, 9.7, 0.1	1310,15.4,9.6,5.0	970,20.5,7.0,3.6	26.0
SG-S5			9	80.9	54.5, 9.9, 3.6	1330,15.0,11.4,5.9	961,20.8,9.8,5.0	27.8
SG-S6			9	79.7	57.6, 9.7, 1.8	1290,15.5,8.3,4.6	980,20.3,5.9,3.1	24.0
SG-S7			9	79.3	55.2, 9.7, 0.5	1350,14.7,5.9,3.5	970,20.7,3.9,2.1	28.1
SG-S8			10	80.7	54.3, 9.8, 1.9	1350,15.2,8.1,4.3	1040,19.6,5.6,2.8	23.0
SG-S9			10	81.4	54.1, 9.8, -1.4	1320,15.0,6.3,3.3	960,20.0,4.9,2.5	27.3
SG-S10			10	79.5	54.3, 9.6, 3.4	1350,14.8,10.8,5.7	990,20.3,5.5,2.8	26.7
SG-S11			10	80.6	51.6, 10.0, 1.4	1330,15.0,6.9,3.7	980,20.4,5.6,2.9	26.3
SG-S12			10	81.2	53.4, 10.0, 0.6	1350,14.7,9.3,4.9	930,20.8,6.0,3.2	31.1
Average				80.2	54.5, 9.8, 1.3	1340,14.9,8.6,4.6	1000, 20.2,6.0,3.1	26.4
SIC-3			8	80.5	54.8, 10.9, 6.6	1380, 18.0, 15.1, 7.8	1020, 23.8, 10.9, 5.6	26.1
SIC-4			7	77.5	58.7, 11.9, -2.1	1170, 16.8, 9.3, 5.1	880, 23.2, 5.4, 2.9	24.8
SIC-5			7	78.6	59.4, 10.6, -1.8	1290, 15.5, 10.9, 6.1	910, 20.1, 5.4, 2.9	29.5
Average				78.9	57.6, 11.1, 0.9	1280, 16.8, 11.8, 6.3	940, 22.4, 7.2, 3.8	26.8

Test beam location is unknown

* Light Yield (LY) and Energy Resolution (ER) are the average of the seven points measured along the crysals.

Note 1 Light Yield (LY) for the APD readout is measured with a quartz plate between the crystal and the APDs.

Note 2 Width of the black band at the small end on the smallest side surface: 15 mm

With radioactive sources, the measured resolution is always worse than photoelectron statistics

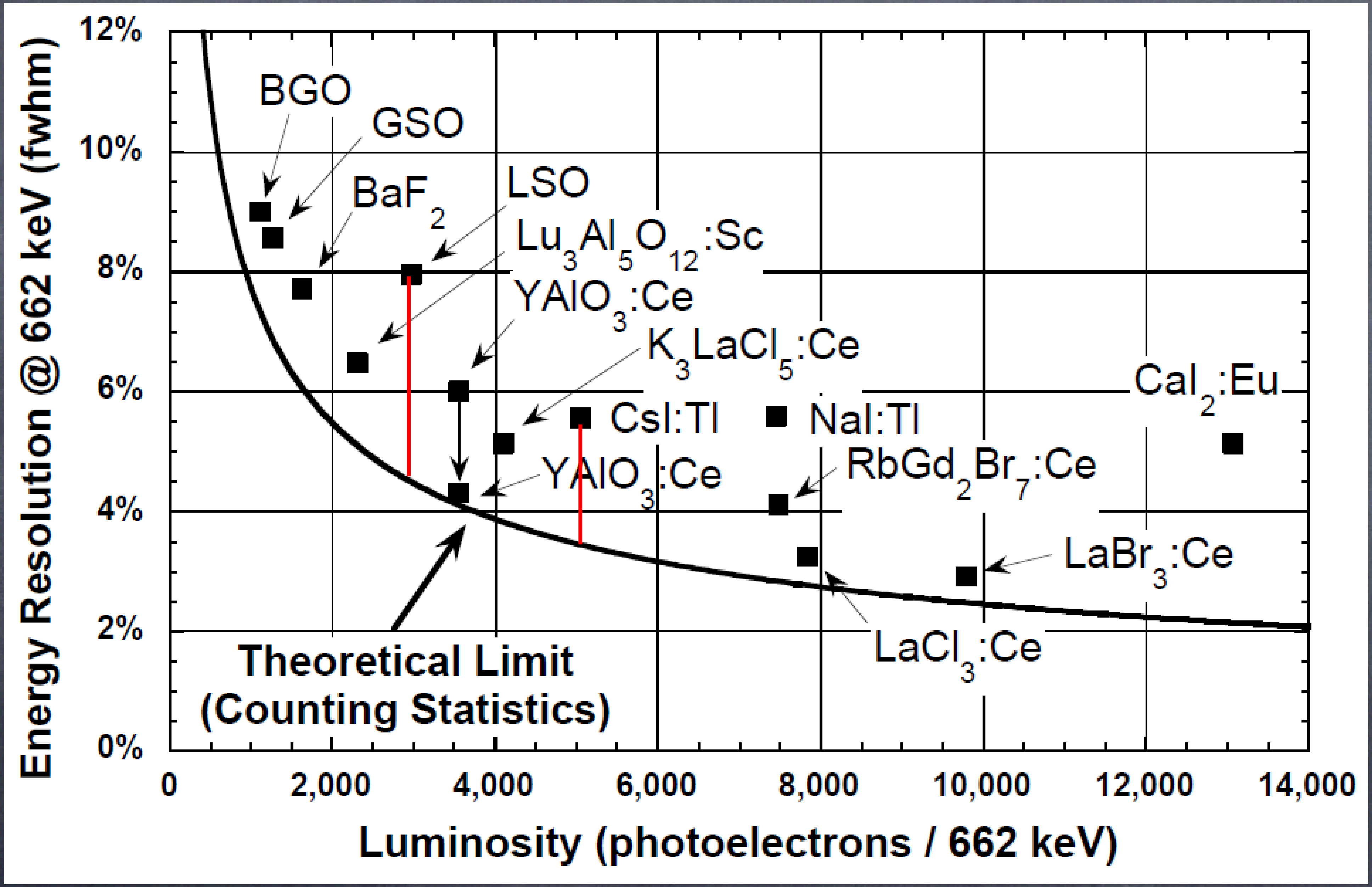
- RYZ:
  - rms 5.5% LO 1500 sqrt = 2.6% as received
- SIPAT
  - rms 3.3% LO 1200 sqrt = 2.9% after 15mm stripe

This example shows: 1) measured resolution is worse that implied by N_{pe}
2) surface treatment can correct for geometrically caused non-uniformity in light collection efficiency

Small crystal studies can help separate effects

The concept of "intrinsic resolution"

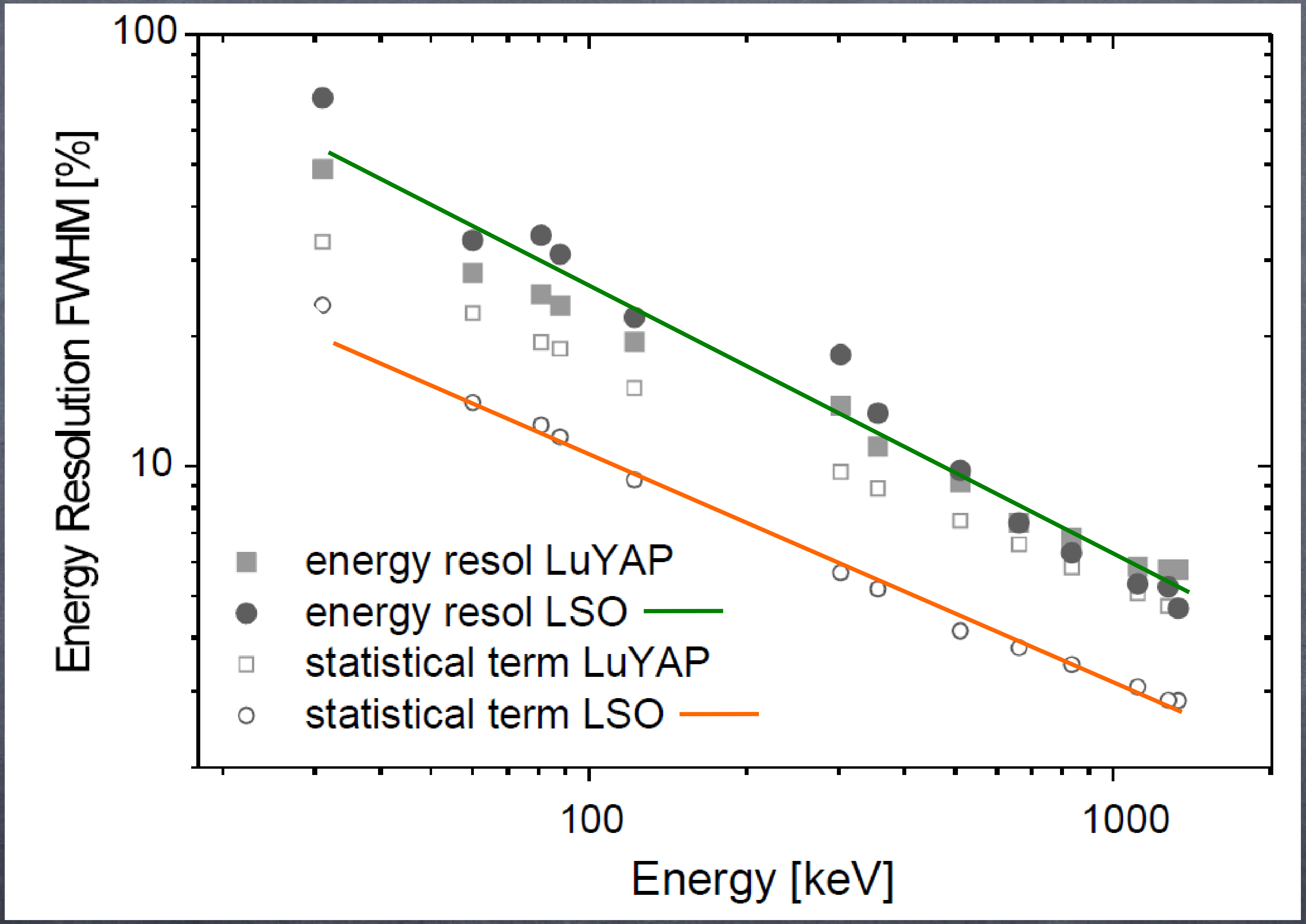
- No inorganic scintillator achieves the naively expected energy resolution based on photoelectron statistics
  - Measured small sample resolution on ^{137}Cs (662 keV) with PMT



W.W. Moses, NIM A487, 123 (2002)

The intrinsic contribution for LSO

Measured with radioactive sources : 0.31 to 1.33 MeV



Source	Energy
^{133}Ba	31
^{241}Am	60
^{133}Ba	81
^{109}Cd	88
^{57}Co	122
^{133}Ba	302
^{133}Ba	356
^{22}Na	511
^{137}Cs	662
^{54}Mn	835
^{65}Zn	1115
^{60}Co	1173
^{22}Na	1275
^{60}Co	1333

C. Kuntner et al., NIM A493, 131 (2002)

TABLE I

PHOTON YIELDS (N), PHOTOELECTRON YIELDS ($N\overline{p}$), AND ENERGY RESOLUTIONS (R) DETERMINED AT 662 keV GAMMA EXCITATION OF SCINTILLATORS OPTICALLY COUPLED TO PHOTOMULTIPLIER TUBES WITH BIALKALI PHOTOCATHODES. R_M AND R_s ARE THE PHOTOMULTIPLIER AND SCINTILLATOR RESOLUTIONS CALCULATED FROM THE PHOTOELECTRON YIELD AND RESOLUTION COMPILED IN THE THIRD AND FOURTH COLUMNS, RESPECTIVELY. THE REFERENCE IN COLUMN NINE PERTAINS TO THE DATA IN COLUMNS THREE THROUGH EIGHT. SEE THE TEXT FOR A DISCUSSION ON THE PHOTON YIELDS COMPILED IN THE SECOND COLUMN

R=2.35σ

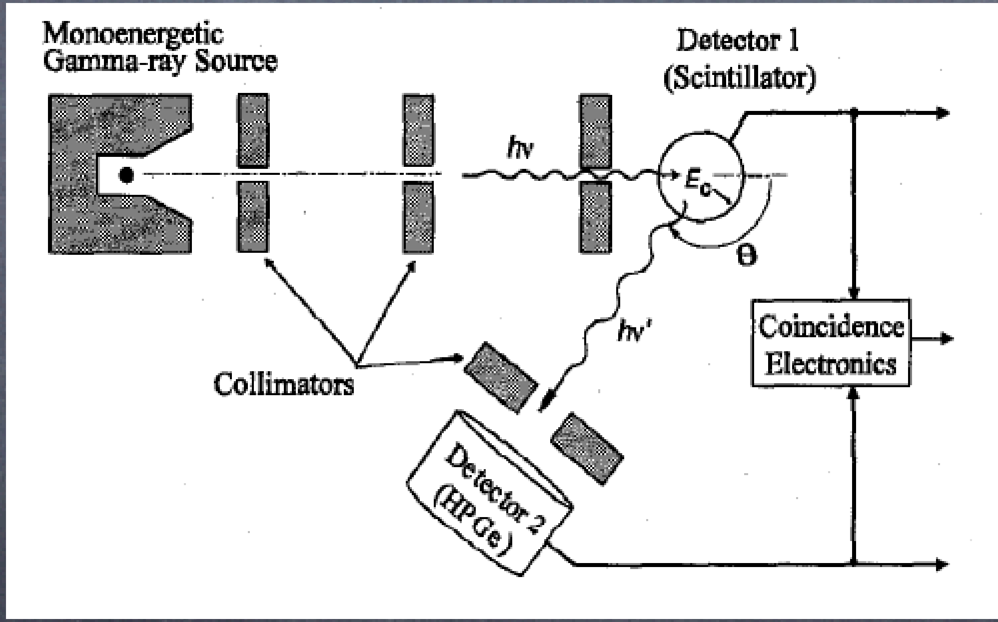
crystal	N 10^3 ph/MeV	$N\overline{p}$ phe/662 keV	R %	R_M %	R_s %	dimension mm^3	PMT	
$NaI(Tl^+)$	38-43	6,230	6.5	3.1	5.7±0.2	Ø25×12.5	R1306	[50]
		4,900	7.1	3.7	6.1±0.2	10×10×20	R1306	[44]
$CsI(Tl^+)$	65	3,240	7.3	4.3	5.9±0.3	Ø18×18	XP2020Q	[53]
		3,640	5.7	4.2	3.9±0.4	10×10×7	R1306	[44]
$CsI(Na^+)$	42	5,740	7.4	3.3	6.6±0.3	10×10×7	R1306	[44]
$CaF_2(Eu^{2+})$	24							
$CdWO_4$	≈28	2,380	6.8	5.2	4.4±0.4	Ø25×12.5	R1306	[50]
		1,200	8.0	7.3	3.3±0.6	10×10×20	R1306	[44]
$Bi_4Ge_3O_{12}$	9	960	9.3	8.1	4.2±0.6	24×24×15	R1306	[57]
		960	9.0	8.1	3.9±0.7	10×10×10	R1306	[44]
BaF_2	11	1,590	7-8	6.2	4±1	Ø24×10	XP2020Q	[58]
		1,590	7.7	6.2	4.6±0.5	Ø20×10	XP2020Q	[59]
$Gd_2SiO_5(Ce^{3+})$	8.5-10	1,250	7.8	7.3	2.7±1.0	10×10×10	R878	[61, 62]
		1,480	7.8	6.6	4.2±0.5	10×10×10	R1306	[44]
$YAlO_3(Ce^{3+})$	14,3	1,900	7.2	5.7	4.4±0.5	21×21×21	R2059	[69]
		1,740	7.5	5.9	4.6±0.5	Ø10×1	XP2020Q	[67]
$Lu_2SiO_5(Ce^{3+})$	23	3,360	7.9	4.4	6.6±0.4	10×10×10	R878	[42]
$Lu_3Al_5O_{12}(Sc^{3+})$	23	2,320	6.5	5.1	4.0±0.4	11×5×1	XP2020Q	[17, 18]
$K_2LaCl_5(Ce^{3+})$	28	3,900	5.1	3.9	3.3±0.4	Ø7×2.5	XP2020Q	[19]

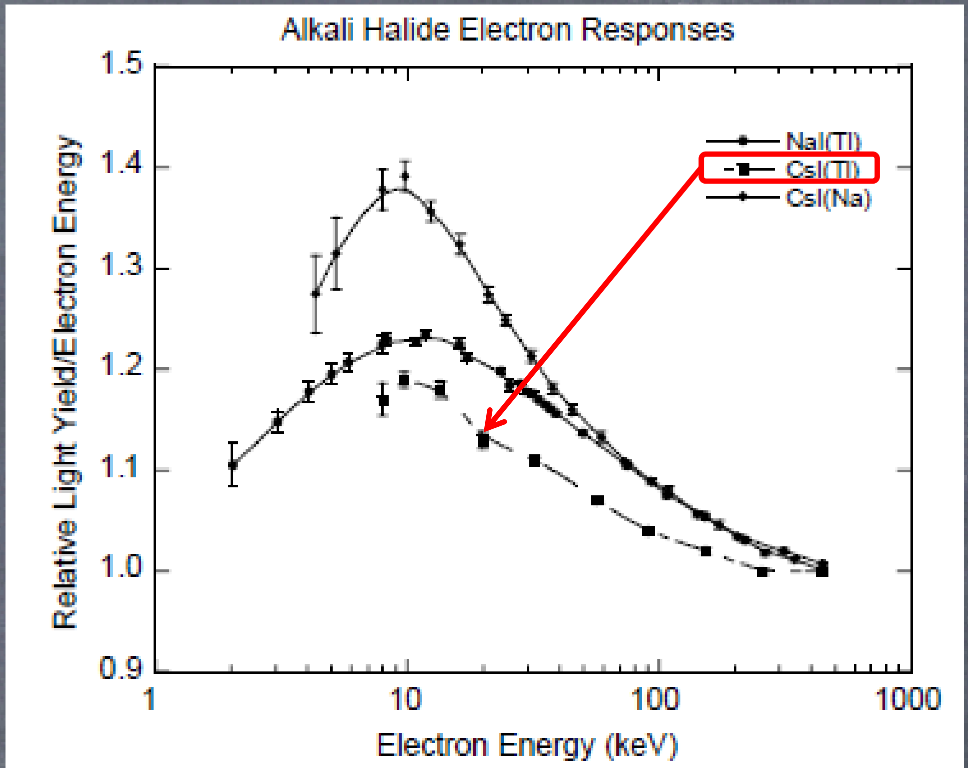
P. Dorenbos et al., IEEE Trans. Nucl Sci. 42, 2190 (1995)

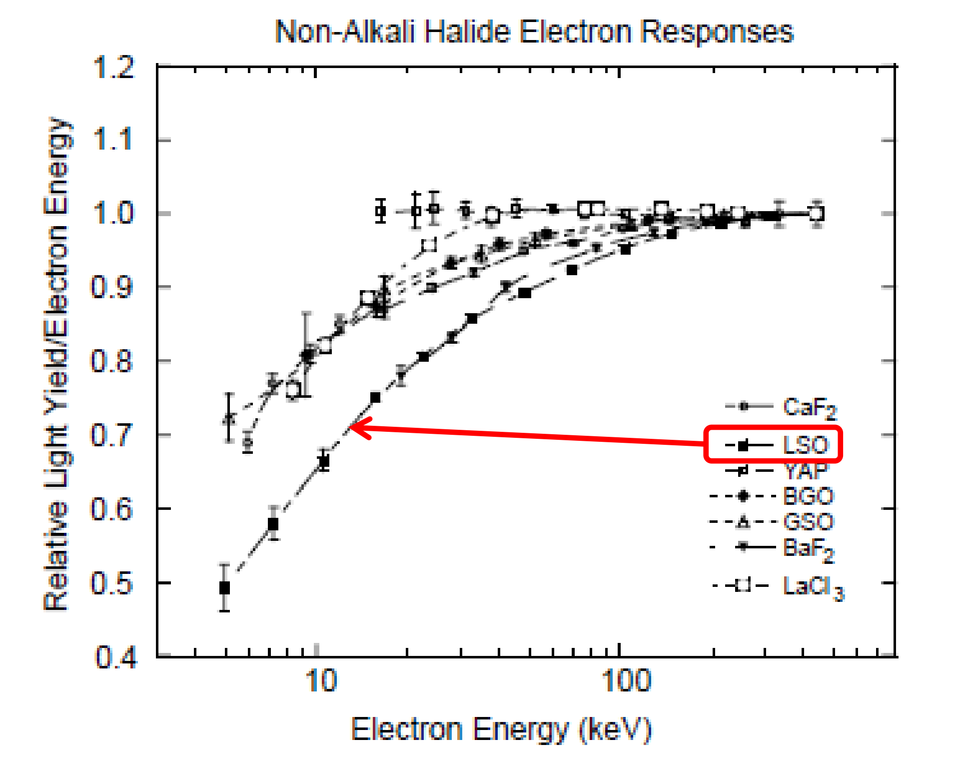
Relative light output is a function of electron energy

- Crystal light output depends on details of the energy loss mechanism
  - Delta rays
  - Compton scattering
  - Photoelectric effect

  and on the energy transfer mechanism to the scintillator active sites







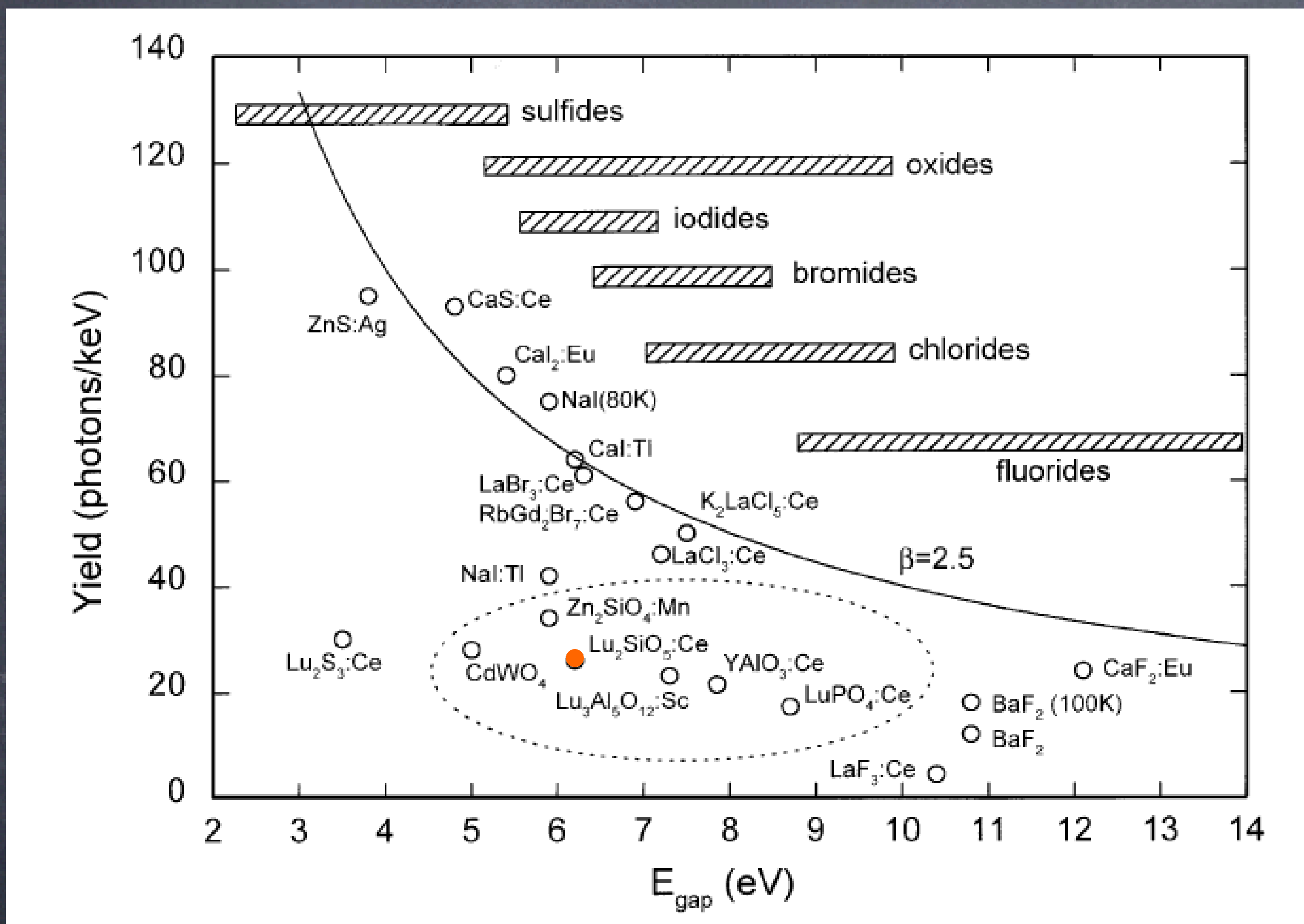
Behavior varies widely in different types of crystals

W.W. Moses, NIM A487, 123 (2002)

The fundamental limit on light output of scintillation crystals is the number of electron-hole pairs created by ionization tracks

$$n_{e-h} = \frac{E_\gamma}{\beta E_{gap}}$$

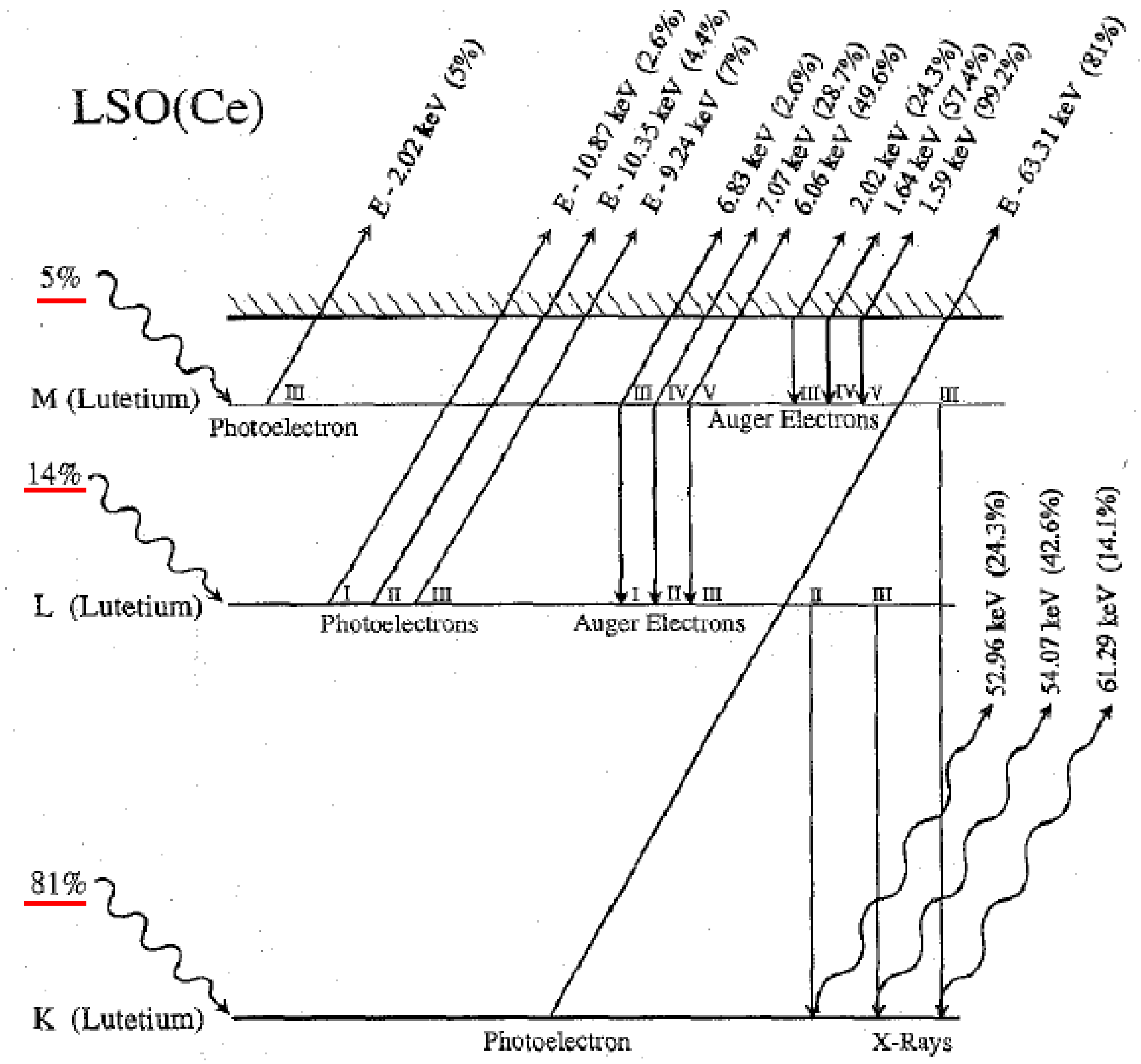
efficiency factor

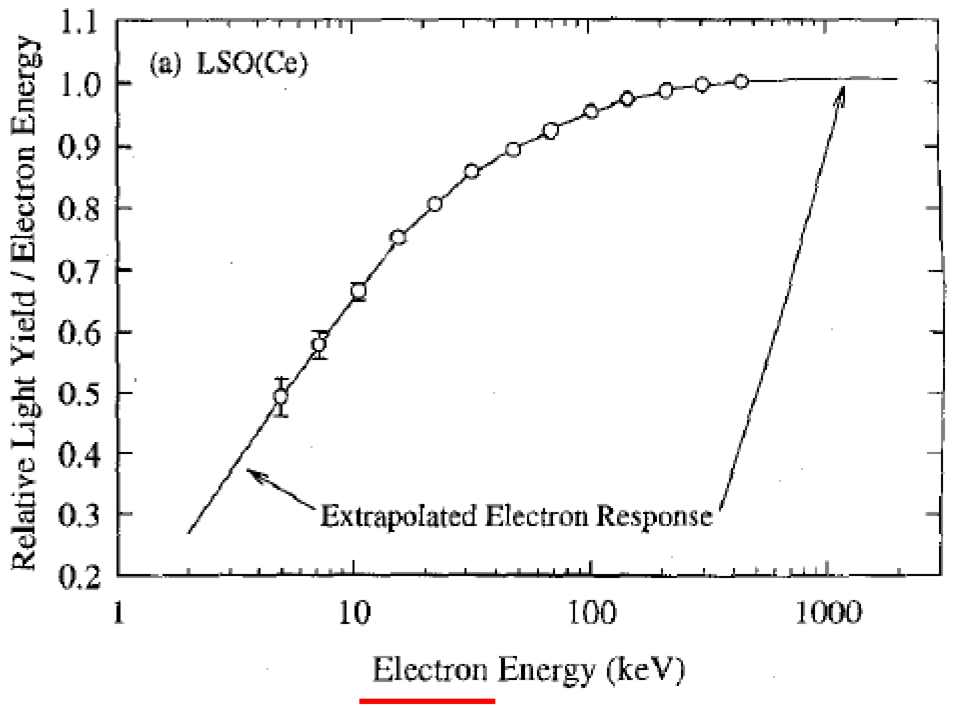


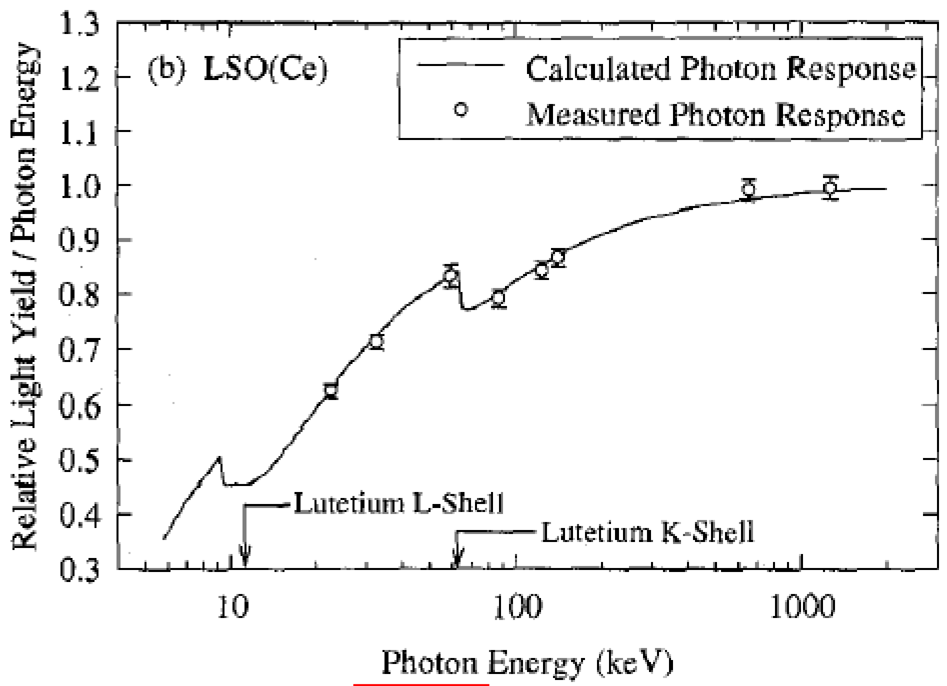
This assumes that all charge carriers are transferred to activator ions

P. Dorenbos, NIM A486, 208 (2002)

LYSO : electron, photon deposition *vs* energy



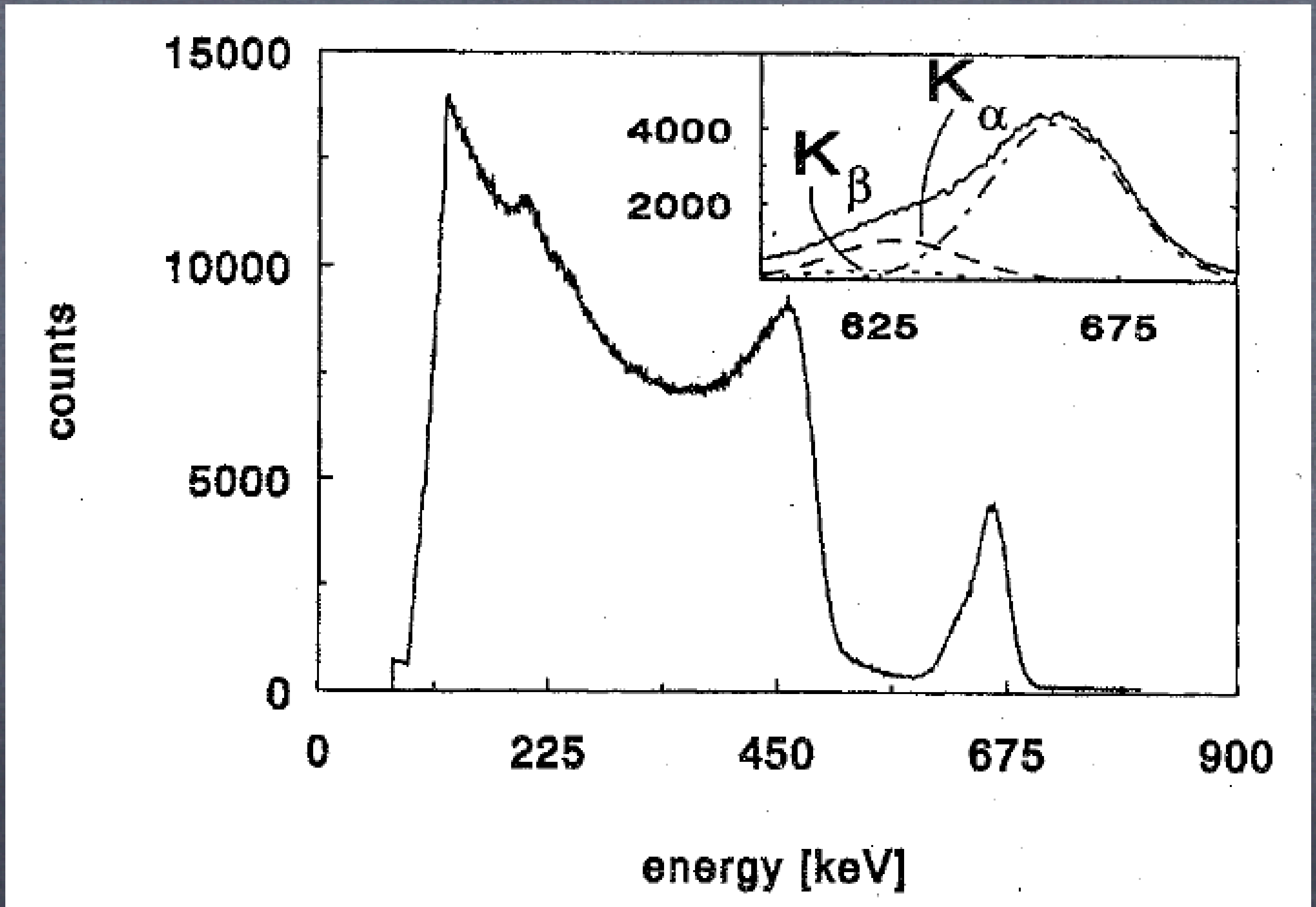




B.D. Rooney and J.D. Valentine, IEEE Trans. Nucl Sci. 44, 509 (1997)

With a small crystal, the absorption edges can be seen directly

- ^{137}Cs 662 keV line in a 7 mm dia x 2,5 mm length crystal of K_2LaCl_5 (Ce^{3+}) clearly shows K_α, K_β structure

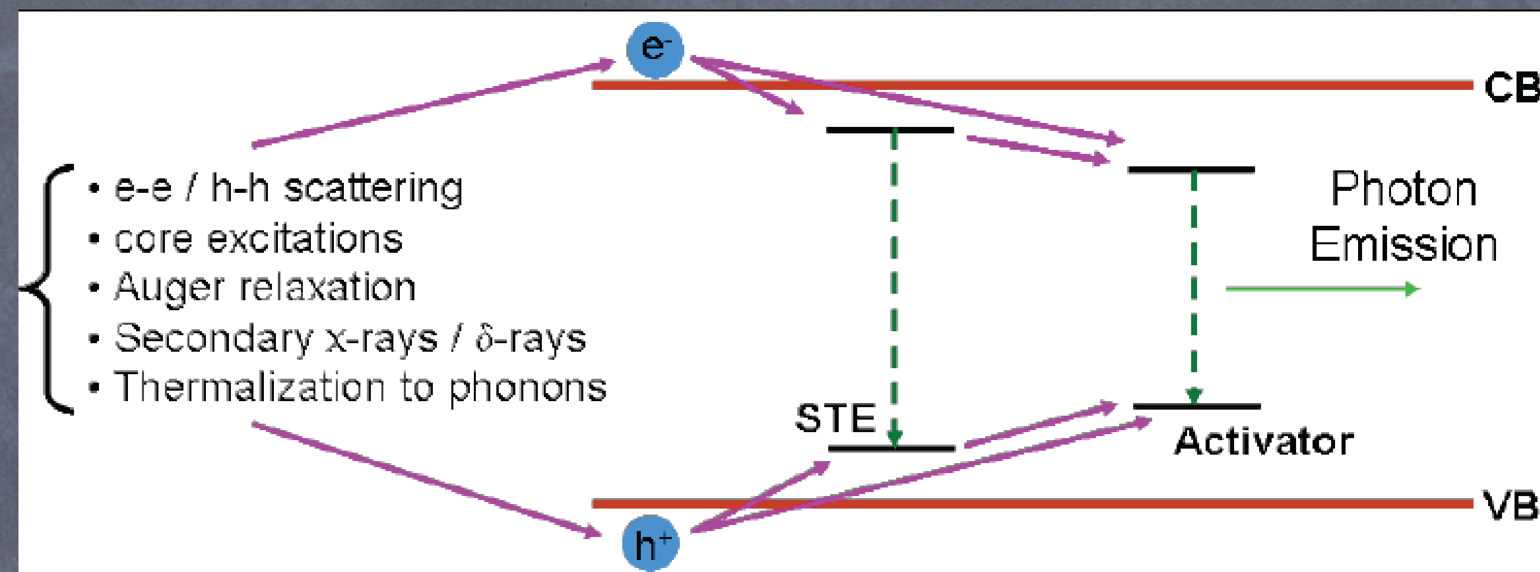


P. Dorenbos et al., IEEE Trans. Nucl Sci. 42, 2190 (1995)

Non-linearity of energy deposition processes

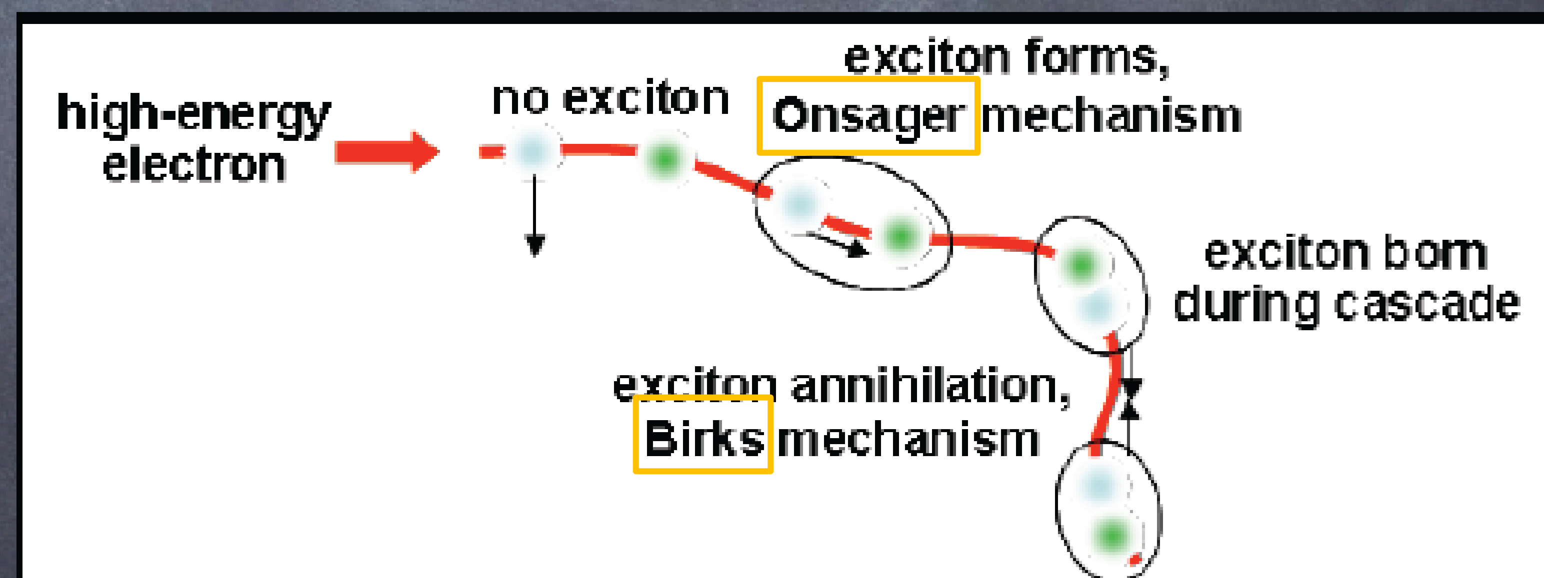
- The effect of non-proportionality of energy deposition process on the resolution of scintillating calorimeters has been modeled:
  S.Payne et al., IEEE Trans. Nucl Sci. 56,, 2506 (2009)

Electrons: Bethe/Bloch/Landau:
delta rays
Positrons: Bethe/Bloch/Landau:
delta rays, annihilation
Photons: Compton scattering,
photoelectric effect



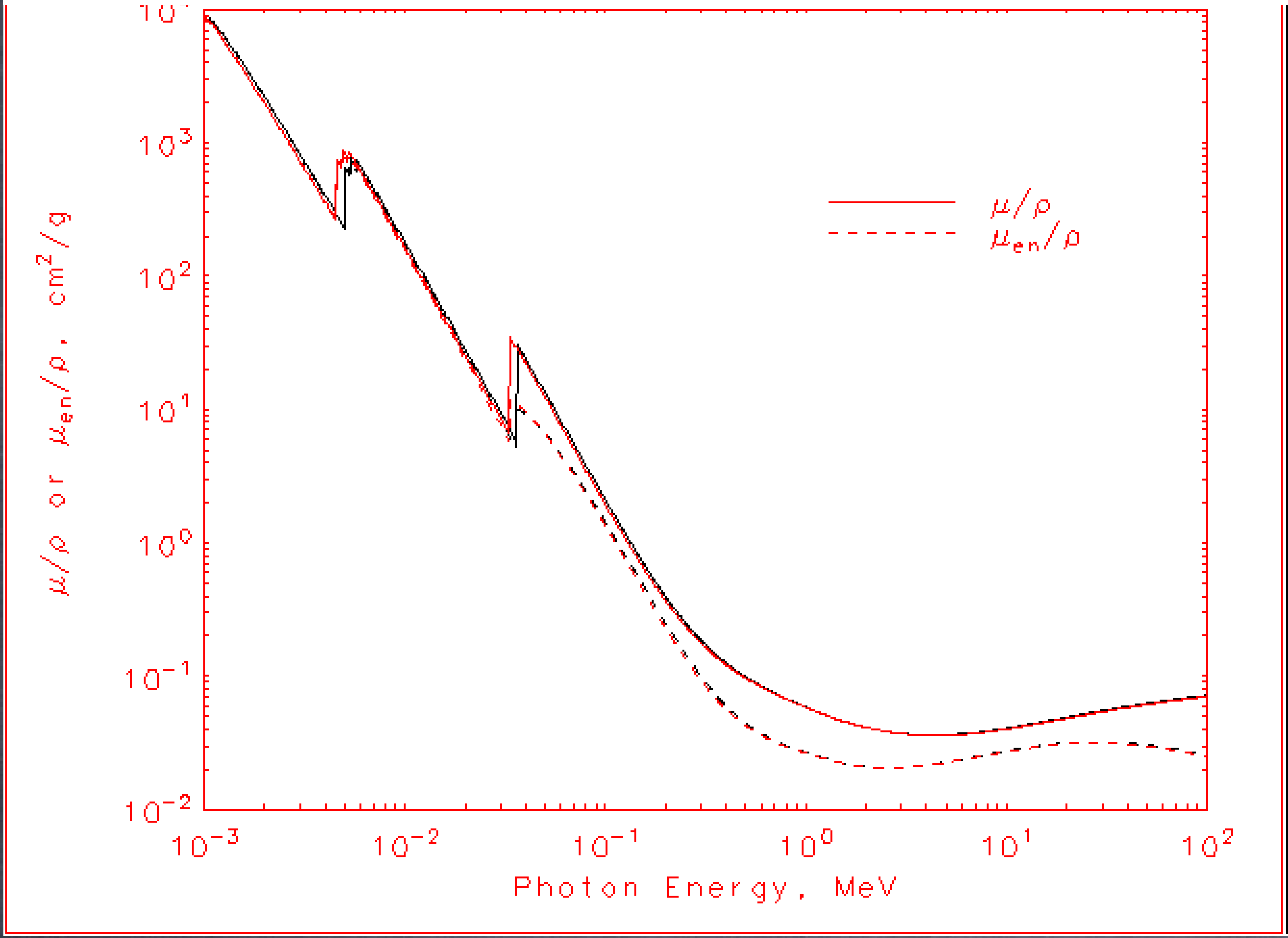
All create electron-hole pairs depending on the band gap

These form excitons, and with differing efficiencies transfer energy to the acitvators, which then generate scintillation light

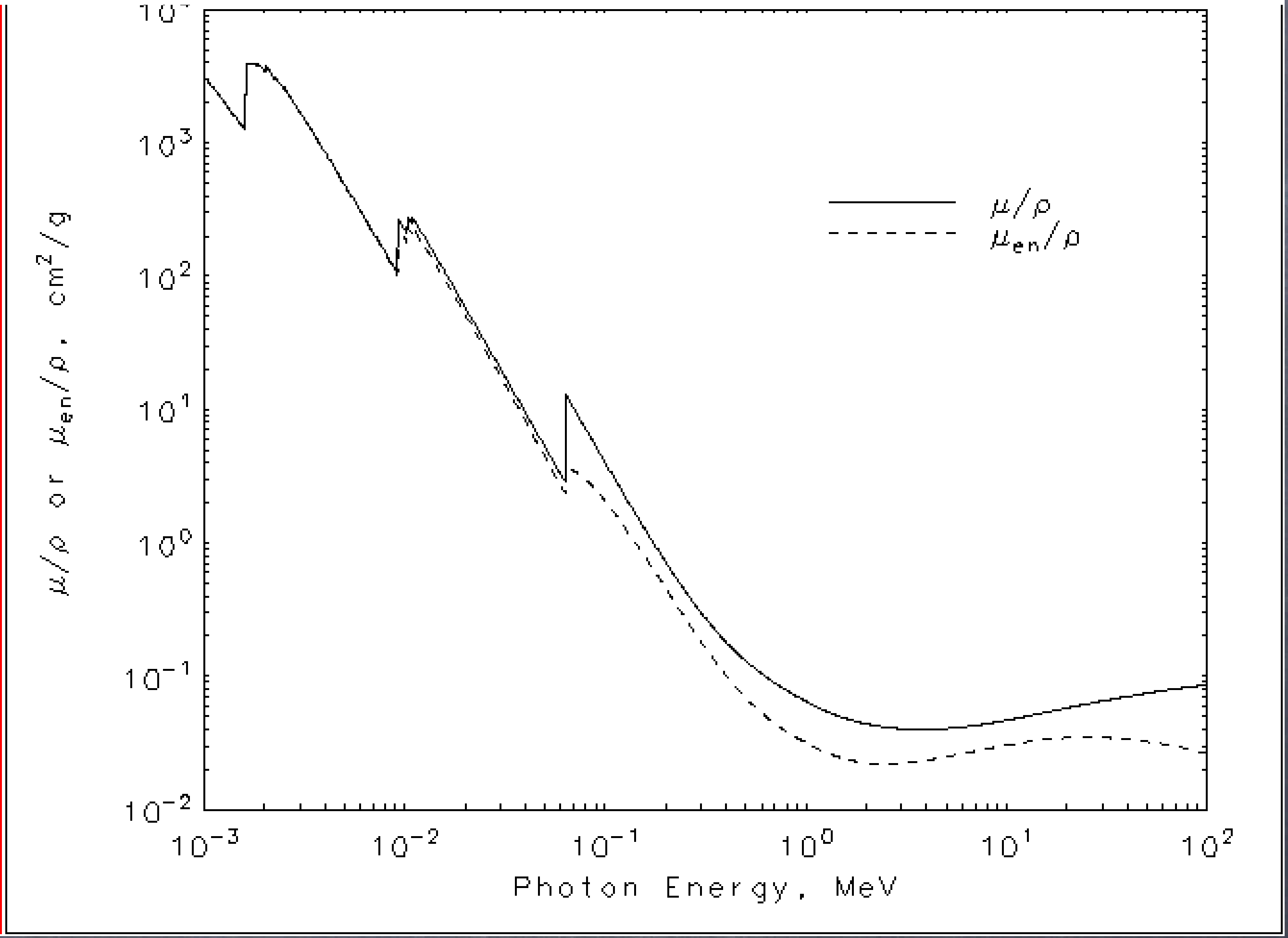


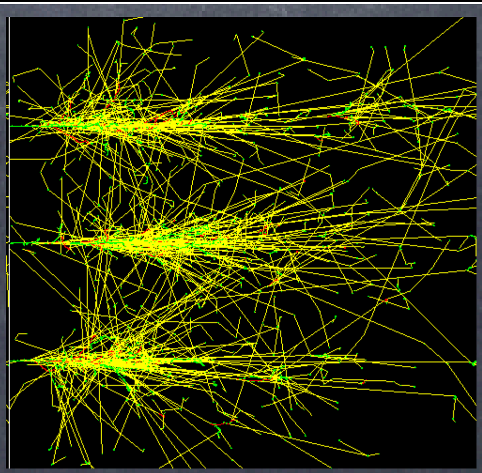
Mass attenuation coefficients Cs, I, Lu

Z=53 Iodine Z=55 Cesium



Z=71 Lutetium



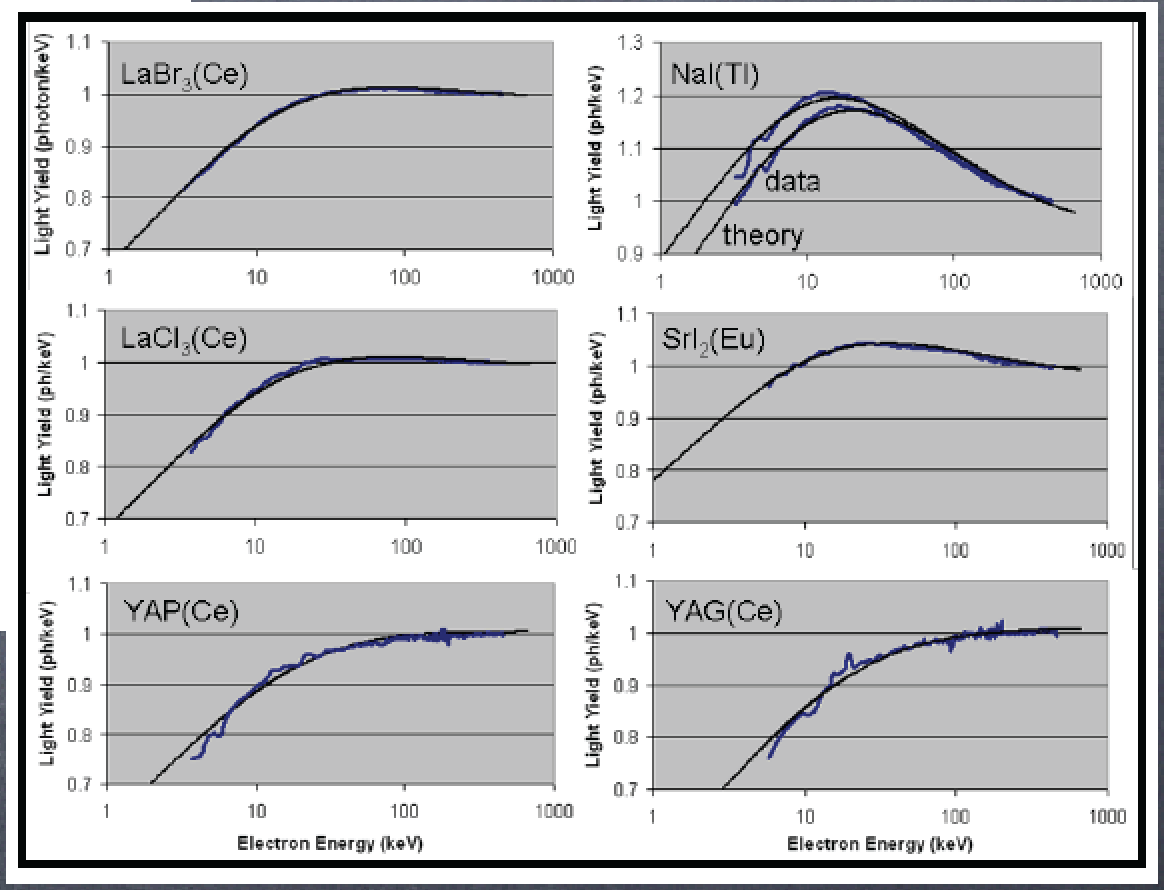


Degree of non-proportionality can be modeled reasonably well

TABLE II
CALCULATED AND MEASURED VALUES OF THE NONPROPORTIONALITY AT 662 KEV, BASED ON REFS. [14], [15], [29]-[33]

Scintillator	R_{NonP} (%) Calculated	R_{NonP} (%) Literature
$LaBr_3$(Ce)	2.52	1.6
$LaCl_3$(Ce)	2.27	1.4
NaI(Tl) #1	4.64	5.9
NaI(Tl) #2	4.44	
SrI_2(Eu)	2.30	<2.0
YAP(Ce)	2.43	2.5 +/-1
YAG(Ce)	2.85	Unknown
LSO(Ce)	5.93	6.6
BGO	3.21	4.1

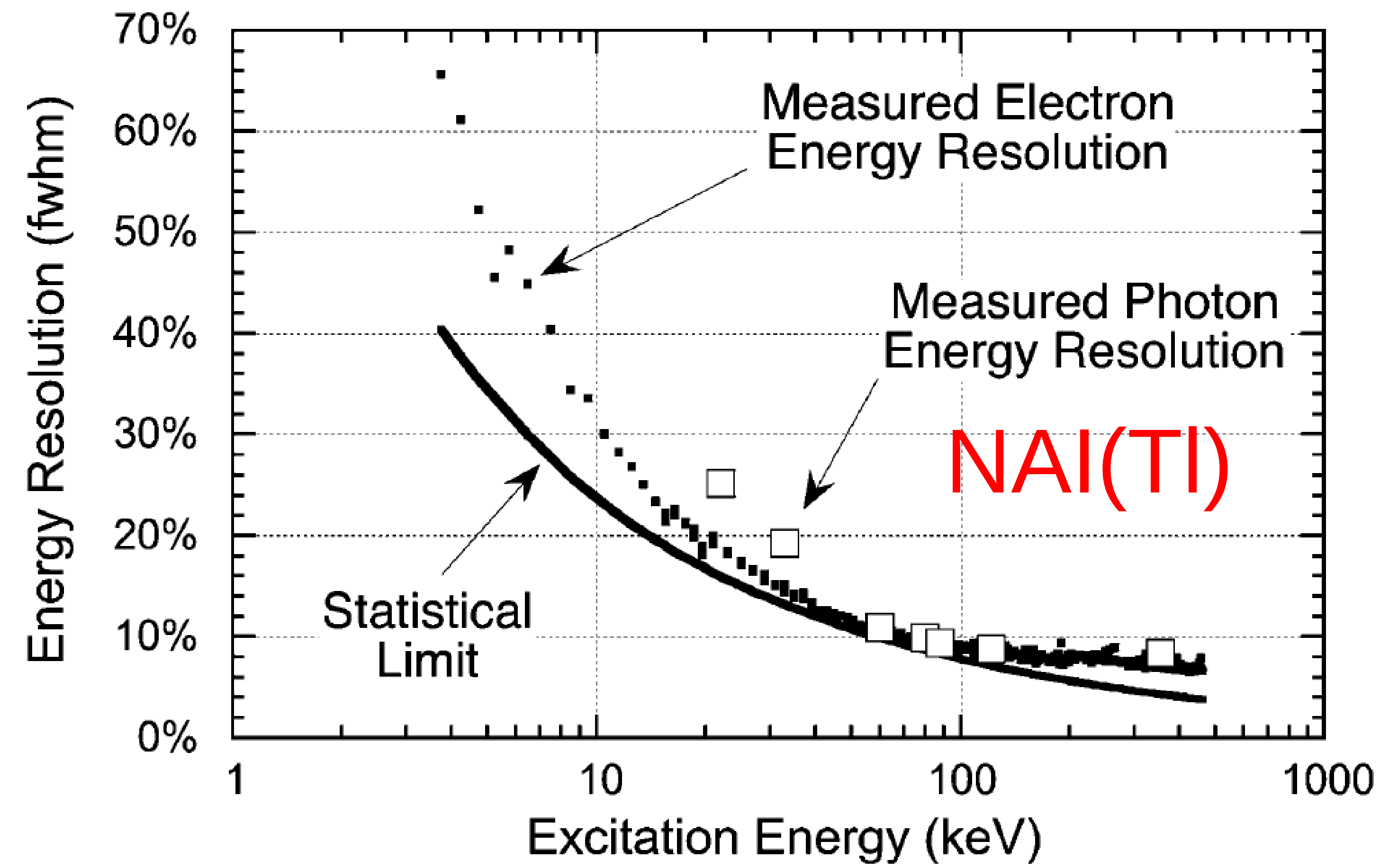
$R=2.35\sigma$



S.Payne et al., IEEE Trans. Nucl Sci. 56, 2506 (2009)

What is the effect at high energies?

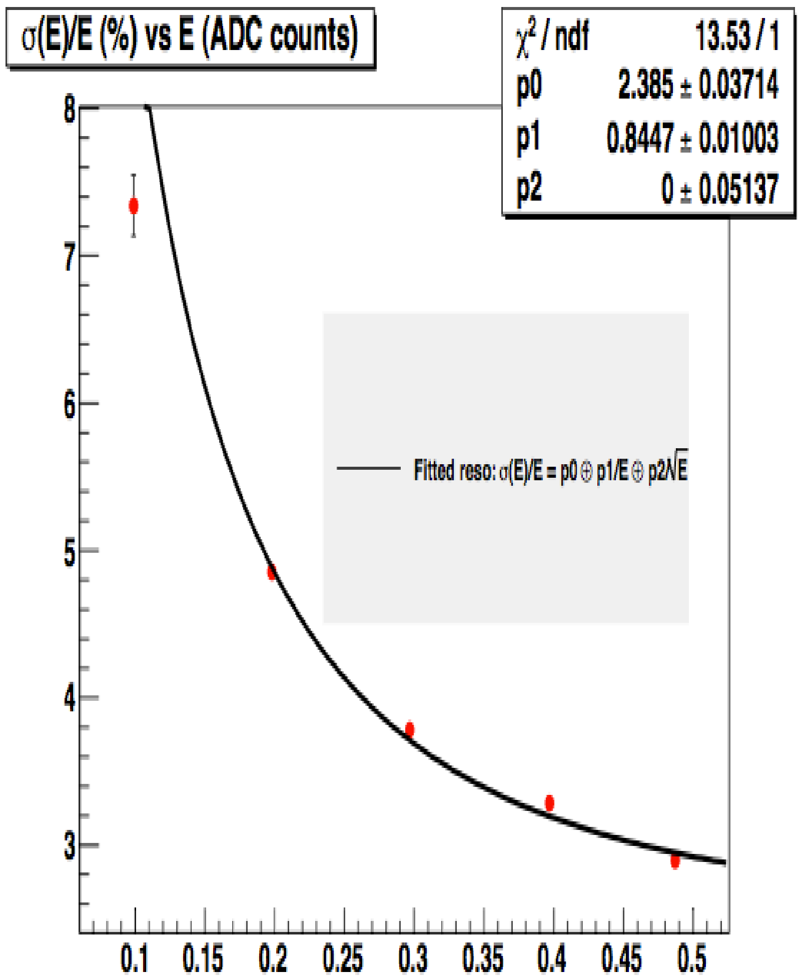
- Whatever the energy of the incident electron or photon, the bulk of the energy that goes into scintillation light from excitons transferring energy to the activators, is deposited by low energy components of the electromagnetic shower in the crystal
- Therefore, the intrinsic term, which differs from crystal to crystal, and is measured with 100 keV to 1 MeV sources should apply at the 100 MeV – 5 GeV scale of our beam tests
- The variance due to the non-proportionality of scintillation light to deposited energy results in an intrinsic term that must be included in our estimation of the target energy resolution as expressed in the GEANT4 Monte Carlo



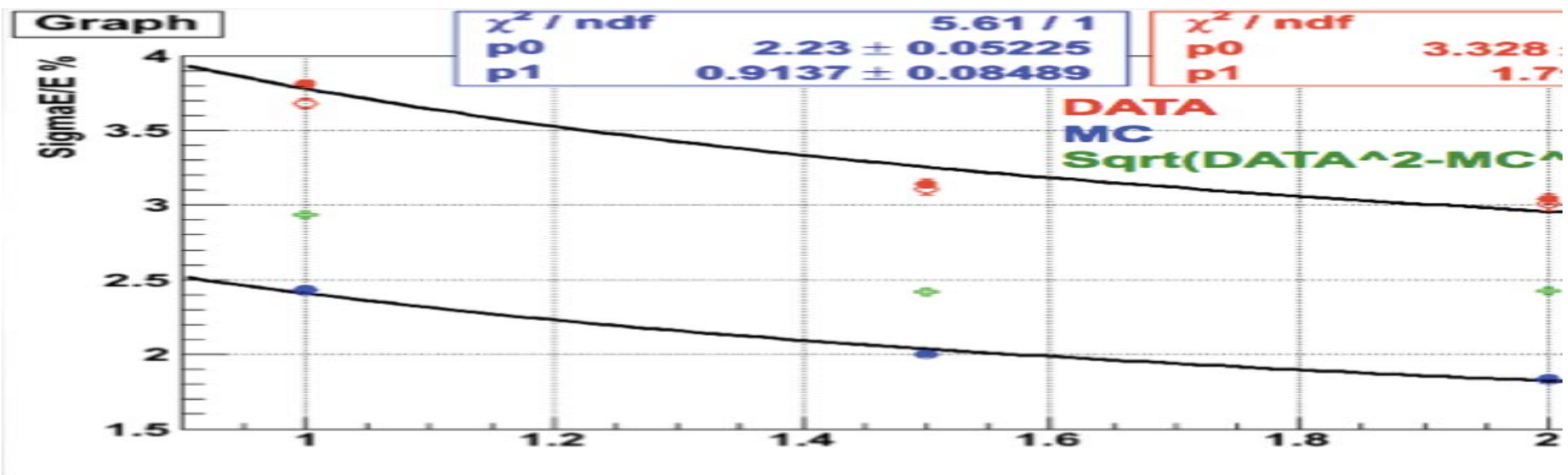
For LSO/LYSO the intrinsic term is expected and is not negligible

Energy resolution fit - two Super$B$ beam tests

LNF

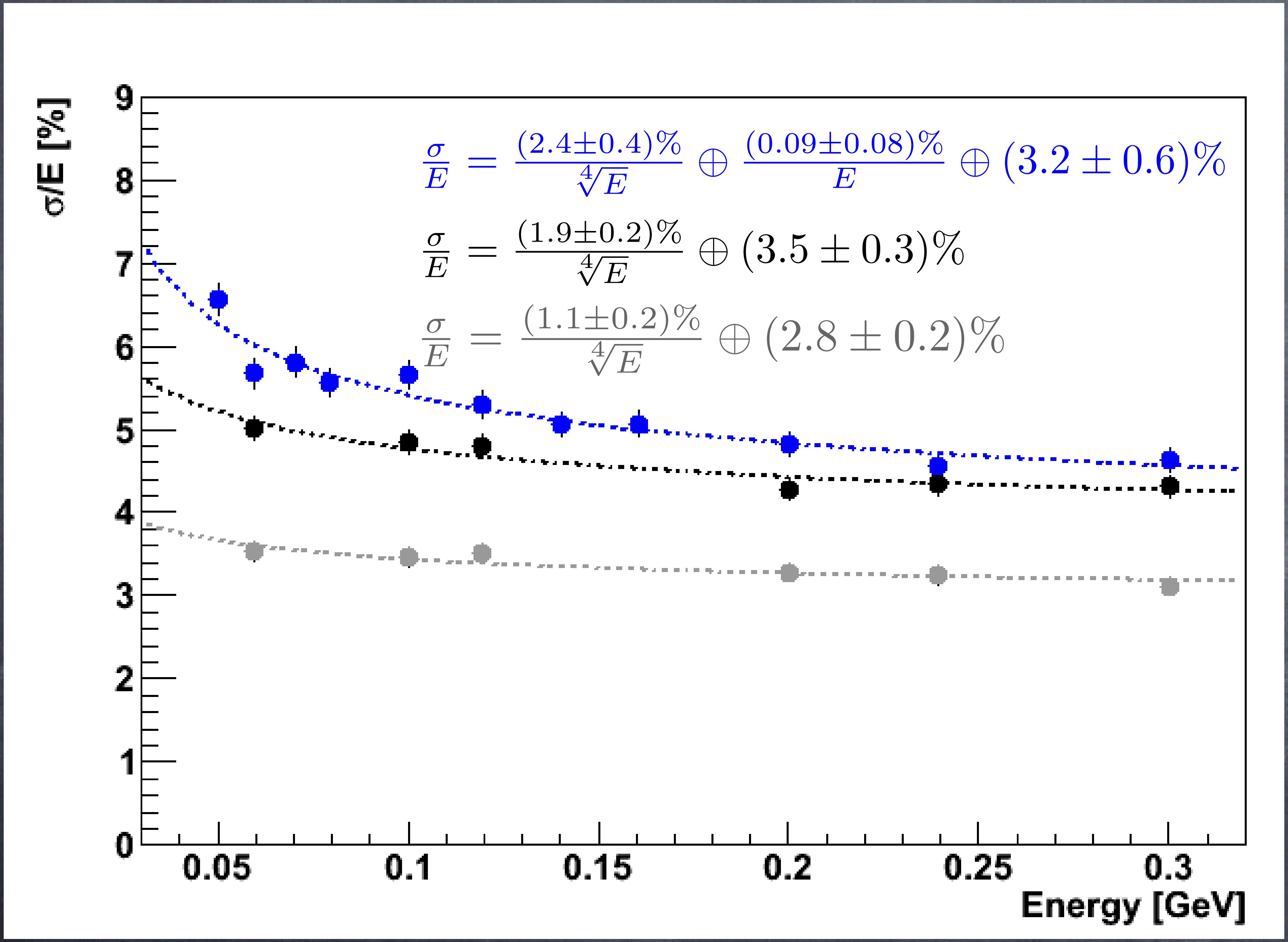


CERN



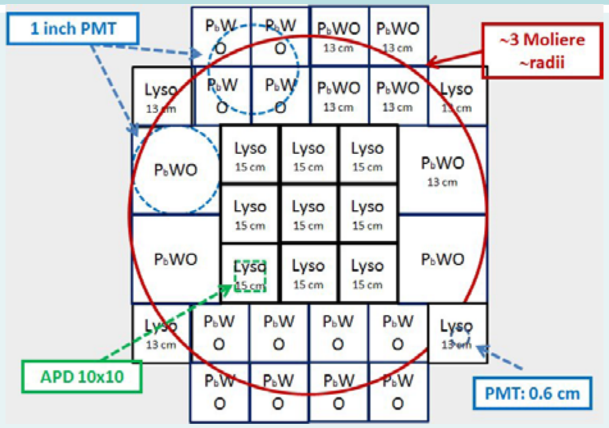
Energy resolution of Mu2e LYSO+PbWO$_4$ matrix

Test at MAMI - Preliminary – courtesy of the KLOE-2/Mu2e group



- data
- MC with 2% corr
- MC no corr.

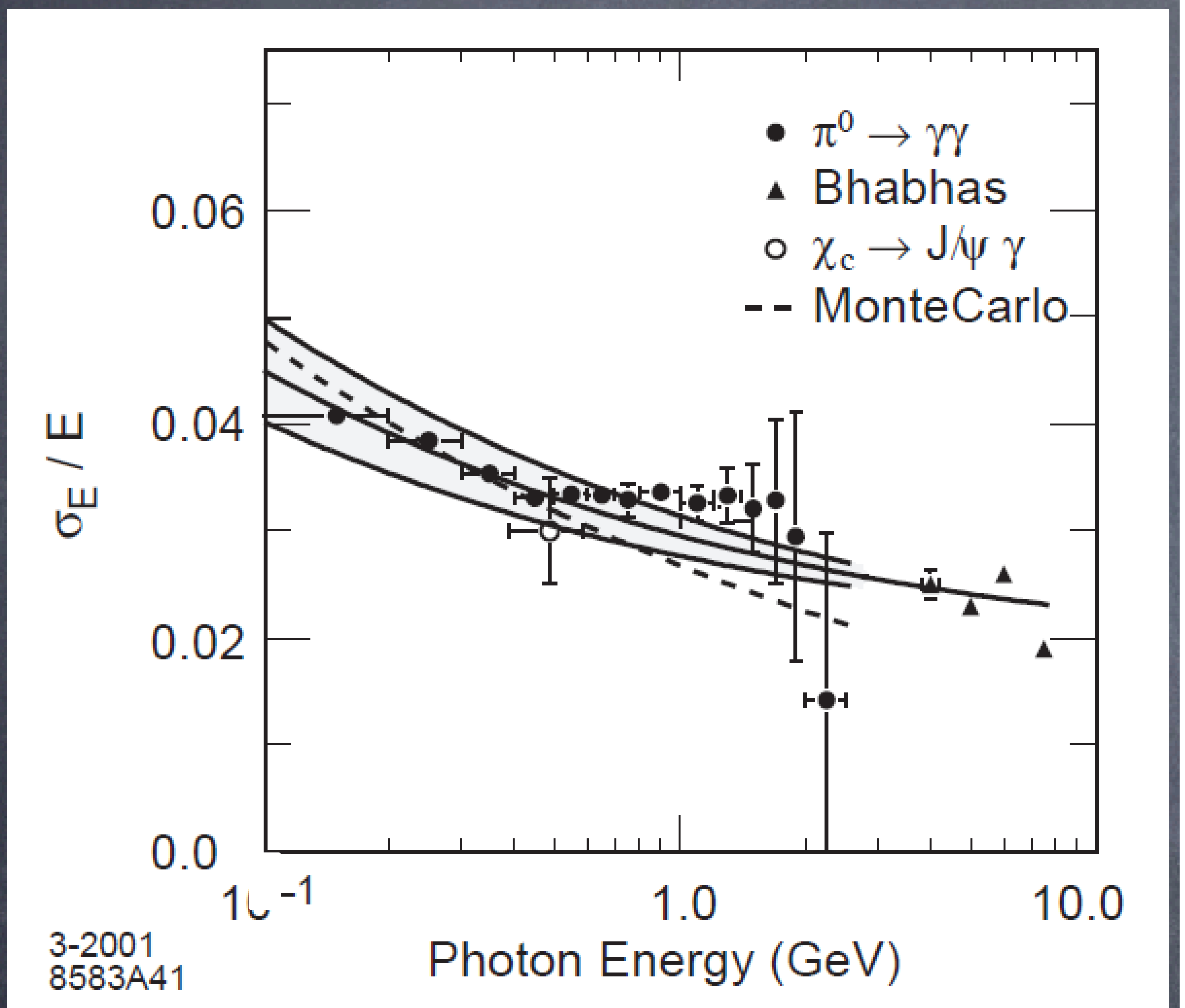
What are the differences in the SuperB and Mu2e tests?

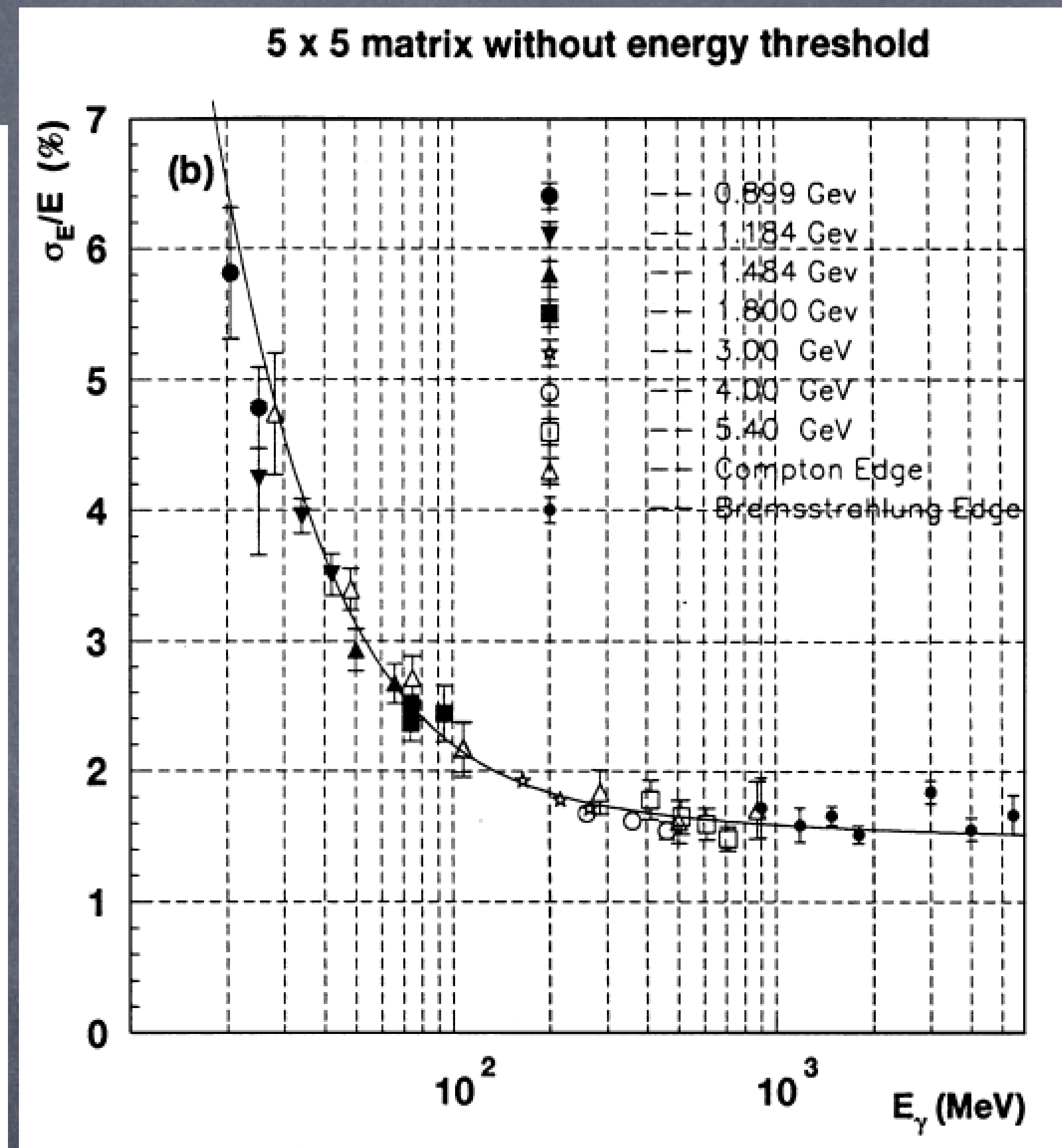
	SuperB	Mu2e
Array	LYSO 5x5	
Crystal dimensions (cm)	25@2.5x2.5x20	9@2x2x15 + PWO
Vendor	SG, SIPAT	SICCAS
Geometry	Projective	Rectangular
Beam	CERN e^-/LNF e^-	LNF e^-/MAMI tagged γ
Readout APD (mm)	5x4+5x1 (PD) / 2@5x5	10x10

Questions/Differences

- Beam quality
- Shower containment
- Photoelectron statistics - light collection area
- Electronic noise
- Light collection uniformity (Ce doping + geometry + surface)
- Intrinsic term – non-linearities

BABAR/Belle CsI(Tl) calorimeter resolution





$$\frac{\sigma_E}{E} = \frac{(2.32 \pm 0.30)\%}{\sqrt[4]{E(\mathrm{GeV})}} \oplus (1.85 \pm 0.12)\%$$

$$\frac{\sigma}{E} = \frac{0.066(\%)}{E} \oplus \frac{0.81(\%)}{E^{1/4}} \oplus 1.34(\%)$$

E in GeV.