



EXCLUSIVE $B \rightarrow K^* \ell \ell$
WITH HIGH STATISTICS

Will Reece (CERN)

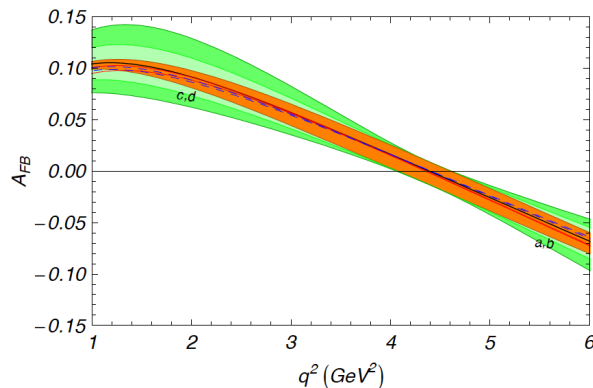
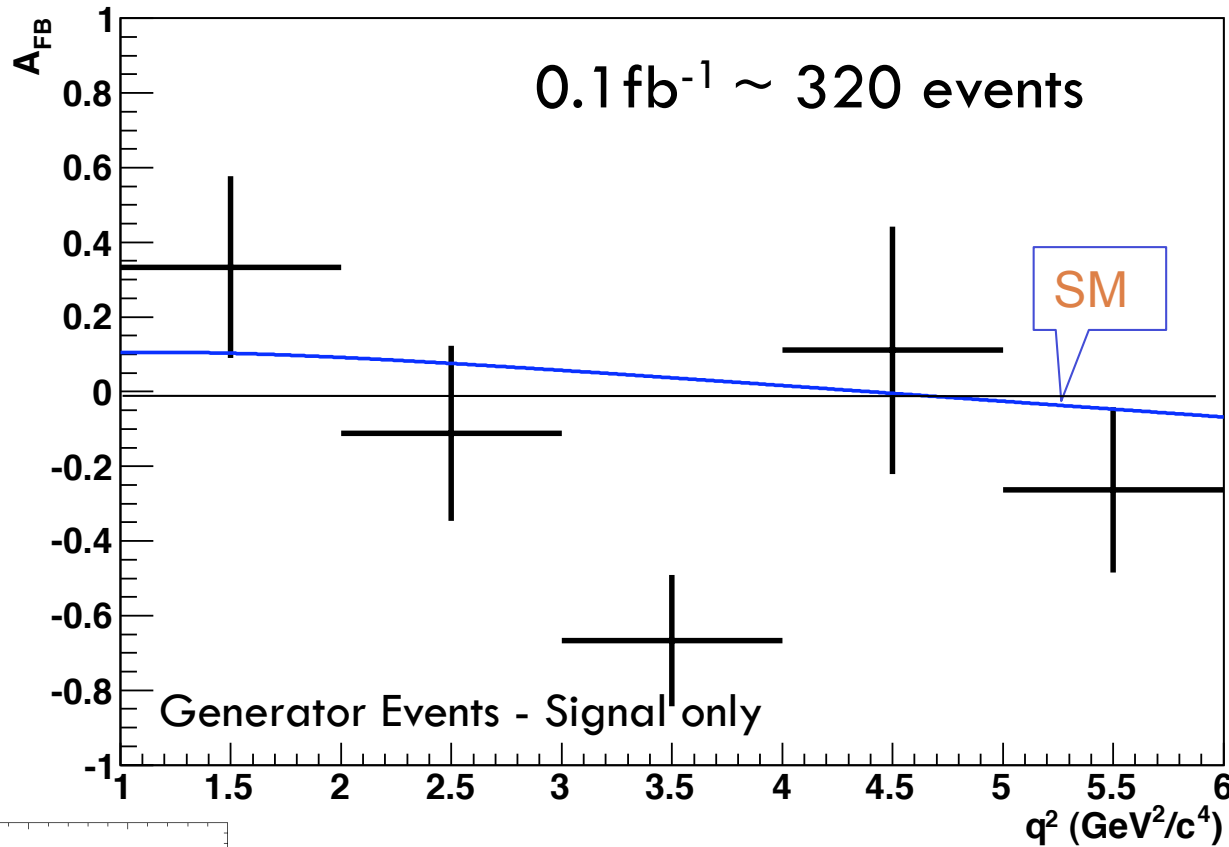
Introduction

2

- Rare decays a hot topic at the LHC
 - ▣ Core part of LHCb physics program ([arXiv:0912.4179](https://arxiv.org/abs/0912.4179))
 - ▣ Sensitive to new physics in $b \rightarrow s$ loops
- Two targets of particular interest
 - ▣ $B_s \rightarrow \mu\mu$ and $B_d \rightarrow K^* \ell\ell$
 - ▣ Orthogonal and complementary views on NP
 - ▣ Gives access to $C_7, C_9, C_{10}, C_S, C_P$ & Primes
- Will focus on $B_d \rightarrow K^* \mu\mu$ at LHCb
 - ▣ Not a member of collaboration so will concentrate on phenomenology results and interesting measurements

A Possible (Near) Future at the LHC

3

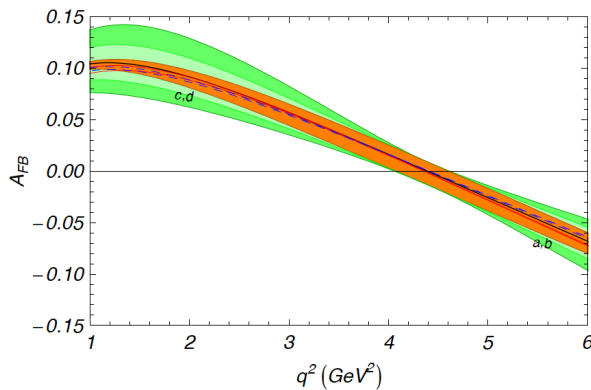
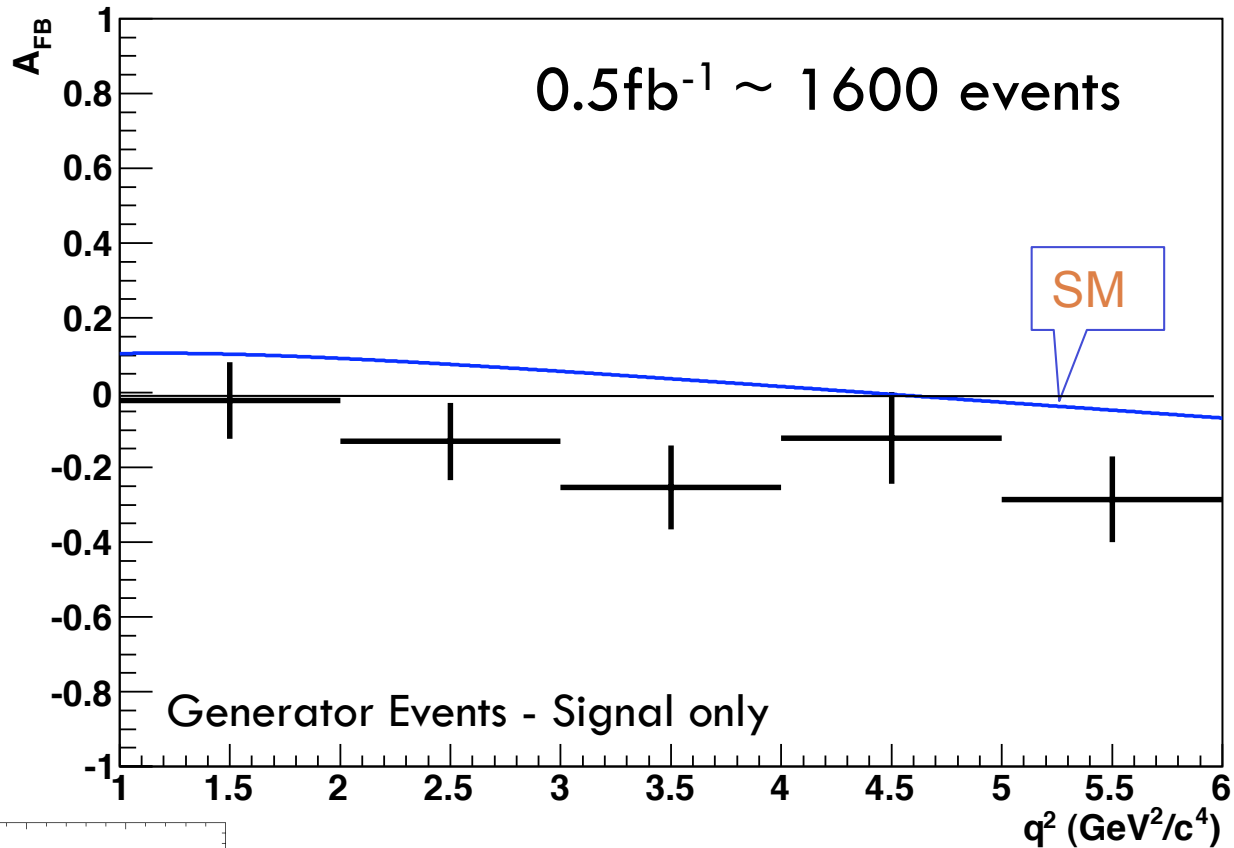


$$A_{FB} = \frac{\int_0^1 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l - \int_{-1}^0 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l}{\int_0^1 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l + \int_{-1}^0 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l}$$

William Reece - CERN

A Possible (Near) Future at the LHC

4

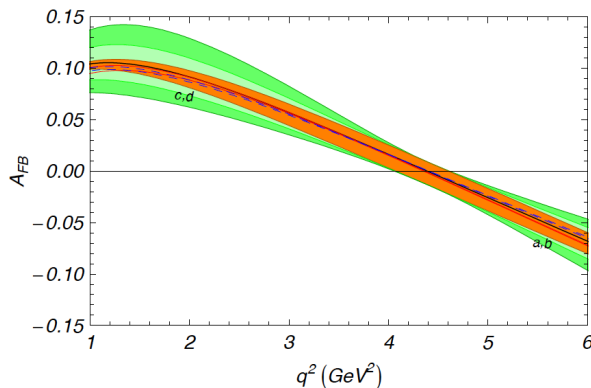
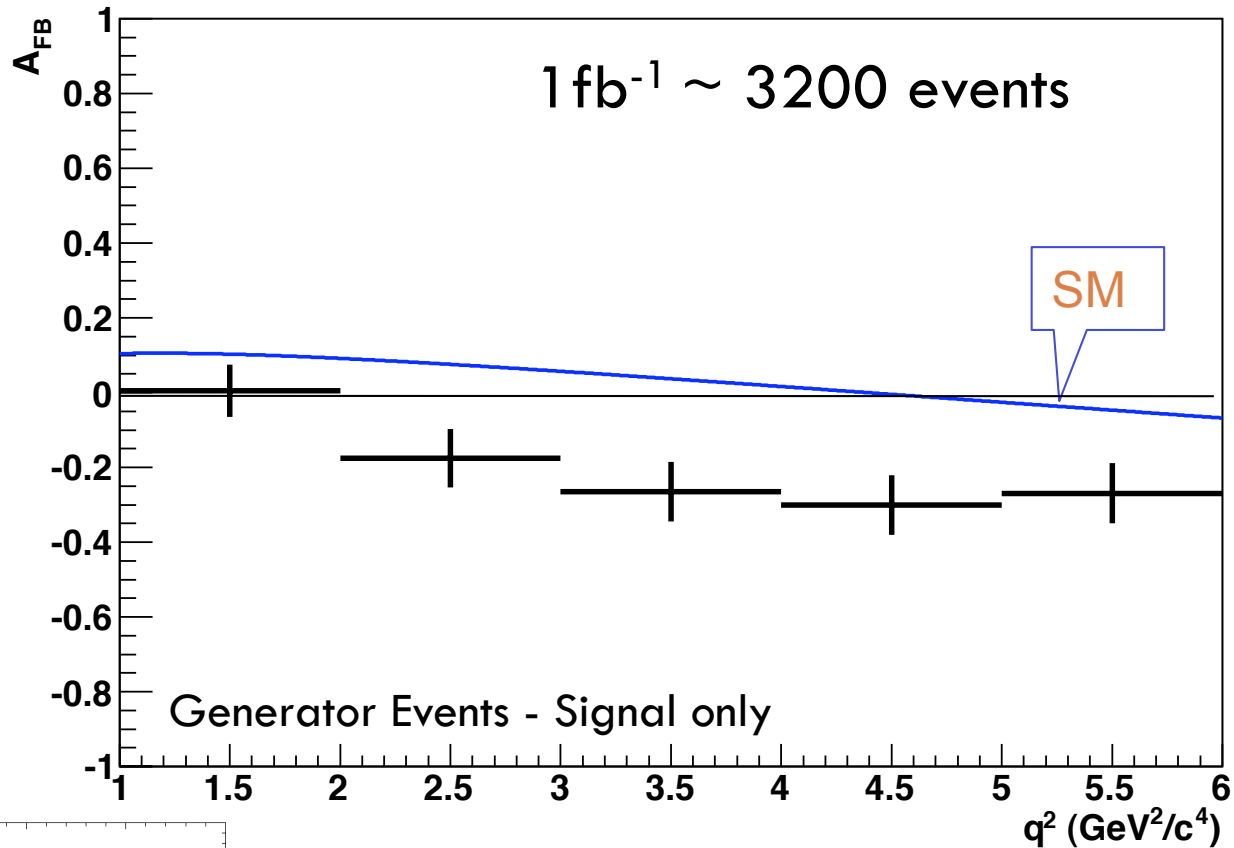


$$A_{FB} = \frac{\int_0^1 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l - \int_{-1}^0 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l}{\int_0^1 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l + \int_{-1}^0 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l}$$

William Reece - CERN

A Possible (Near) Future at the LHC

5

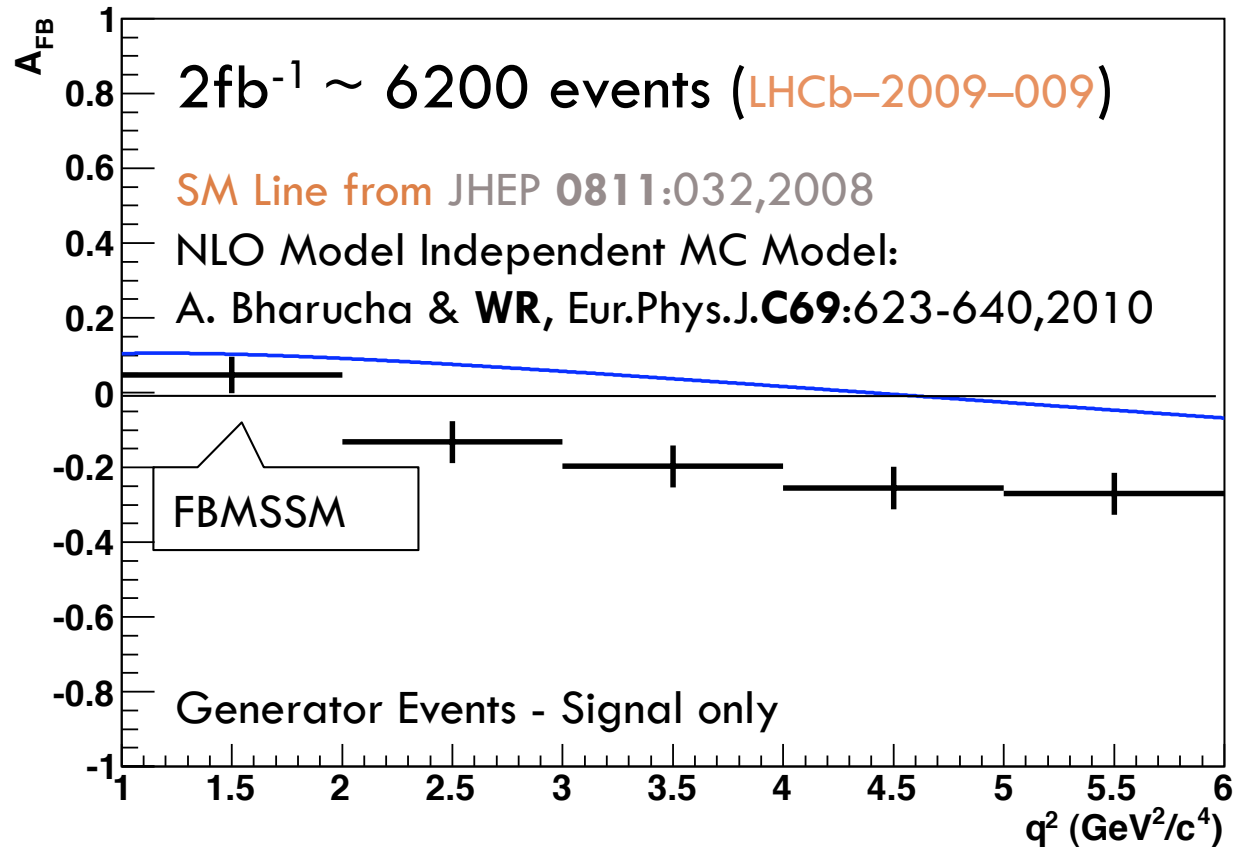


$$A_{FB} = \frac{\int_0^1 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l - \int_{-1}^0 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l}{\int_0^1 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l + \int_{-1}^0 \frac{\partial^2 \Gamma}{\partial q^2 \cos \theta_l} d \cos \theta_l}$$

William Reece - CERN

A Possible (Near) Future at the LHC

6



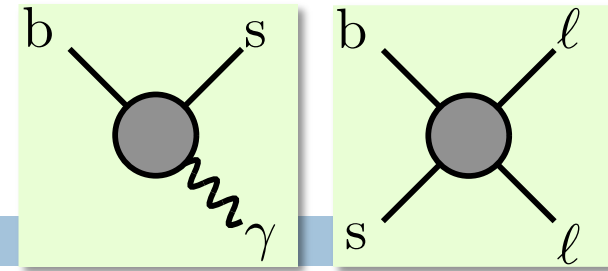
Generated with FBMSSM

[Altmannshofer, Ball, Bharucha, Buras, Straub, Wick, (JHEP 0901:019,2009)]

(LHCb would have already observed $B_s \rightarrow \mu\mu$ in this scenario)

$$B_d \rightarrow K^* \mu^+ \mu^-$$

7



$O_{7\gamma}$

$O_{9,10}$

□ First observed at Belle

□ $Br(B_d \rightarrow K^* \mu^+ \mu^-) = (11.5^{+0.16}_{-0.15}) \times 10^{-7}$

□ Particles in Loop

□ Both neutral and charged NP
(replace $W^\pm, Z^0/\gamma, u/c/t$)

□ Sensitive to NP in loops

□ Use OPE: Model independent

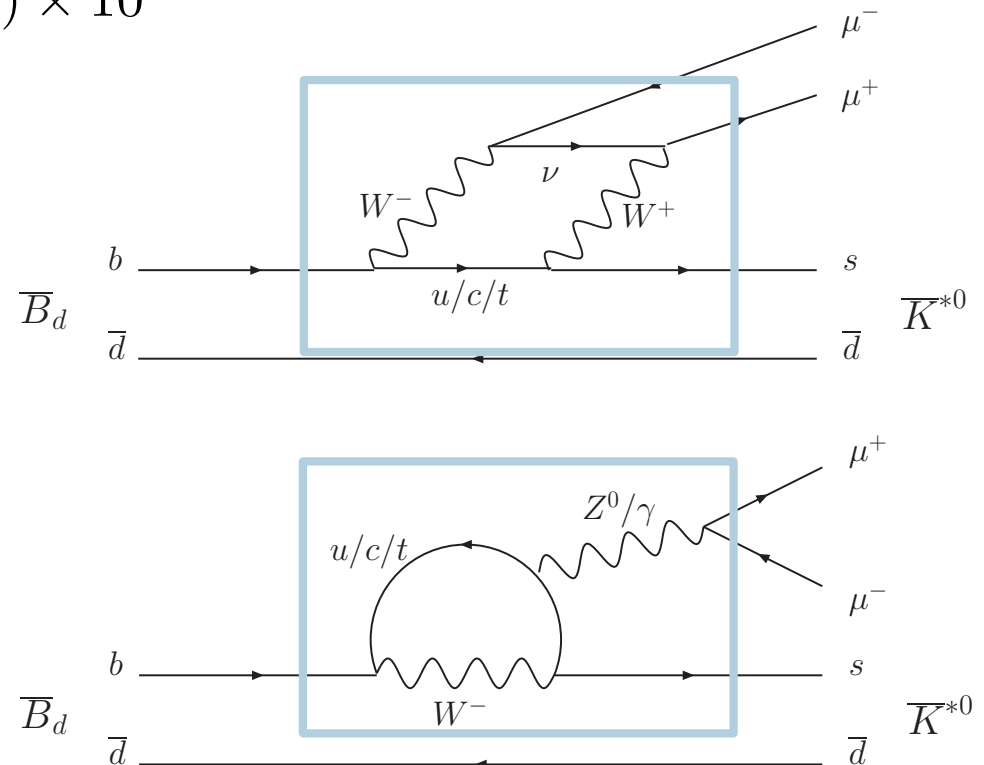
$$\mathcal{H}_{\text{eff}} \propto \sum_{i=1}^{10} [C_i \mathcal{O}_i + C'_i \mathcal{O}'_i]$$

□ Dominated by C_7, C_9, C_{10} in SM

□ Enhance other operators with NP

□ Measure Wilson coefficients

□ Discover or limit NP in $b \rightarrow s$ loop



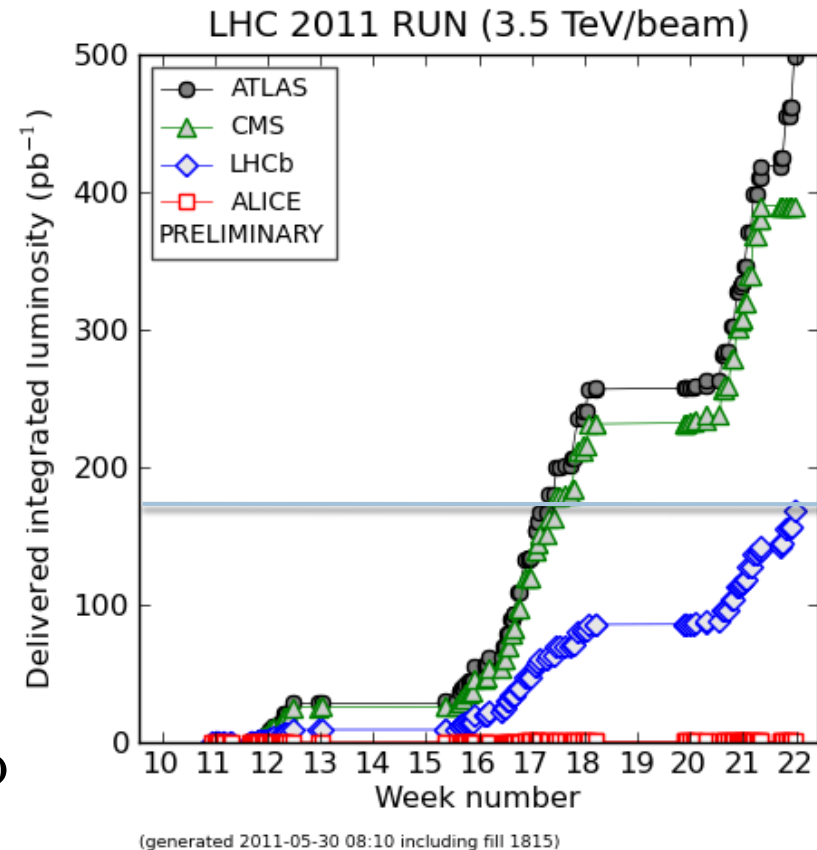
Note on Yields

8

- Public LHCb MC studies:
 - Full simulation at 14 TeV
 - Use $\sigma_{b\bar{b}} = 500 \mu\text{b}$
 - 2fb^{-1} per nominal year
- 2011 LHC Run:
 - 7TeV with pile-up
 - Measure ([arXiv:1009.2731](https://arxiv.org/abs/1009.2731)):

$$\sigma(pp \rightarrow b\bar{b}X) = (284 \pm 20 \pm 49)\mu\text{b}$$

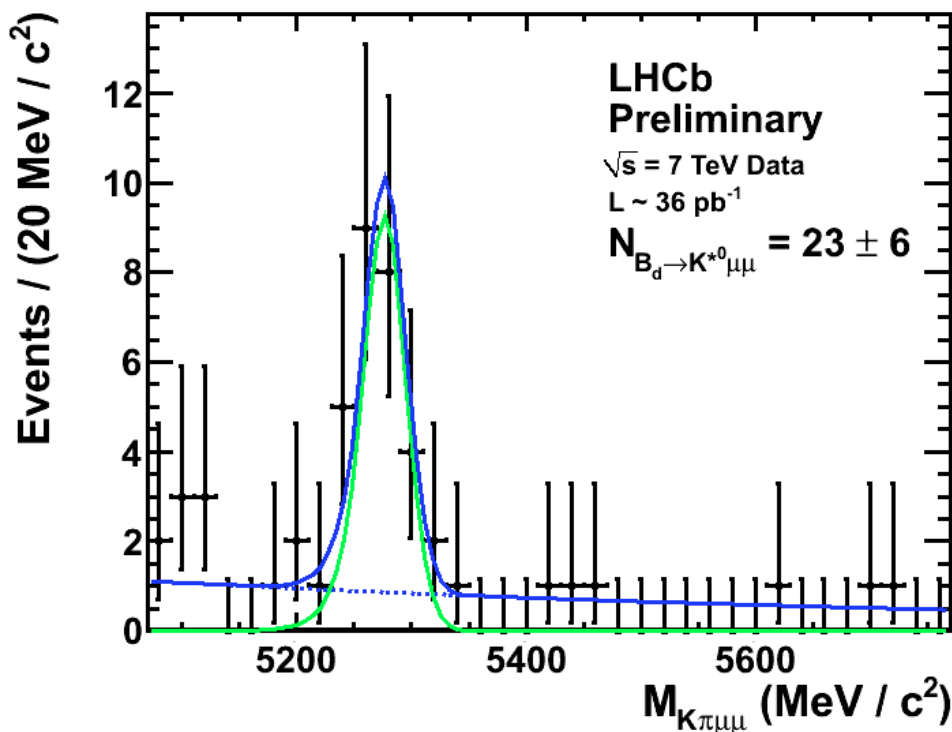
- Stick to official MC yields here:
 - Mentally scale by ~ 0.6 !



LHCb Analysis Status

9

- Selection tuned on $B \rightarrow K^* J/\psi$ events
- Clean signal sample
 - ▣ Results only for 36pb^{-1}
- 23 ± 6 evts; $B/S = 0.2$
 - ▣ BaBar: 60 0.3
 - ▣ Belle: 230 0.25
 - ▣ CDF: 100 0.4
- LHCb at planned lumi now
 - ▣ $\sim 200\text{pb}^{-1}$ on tape



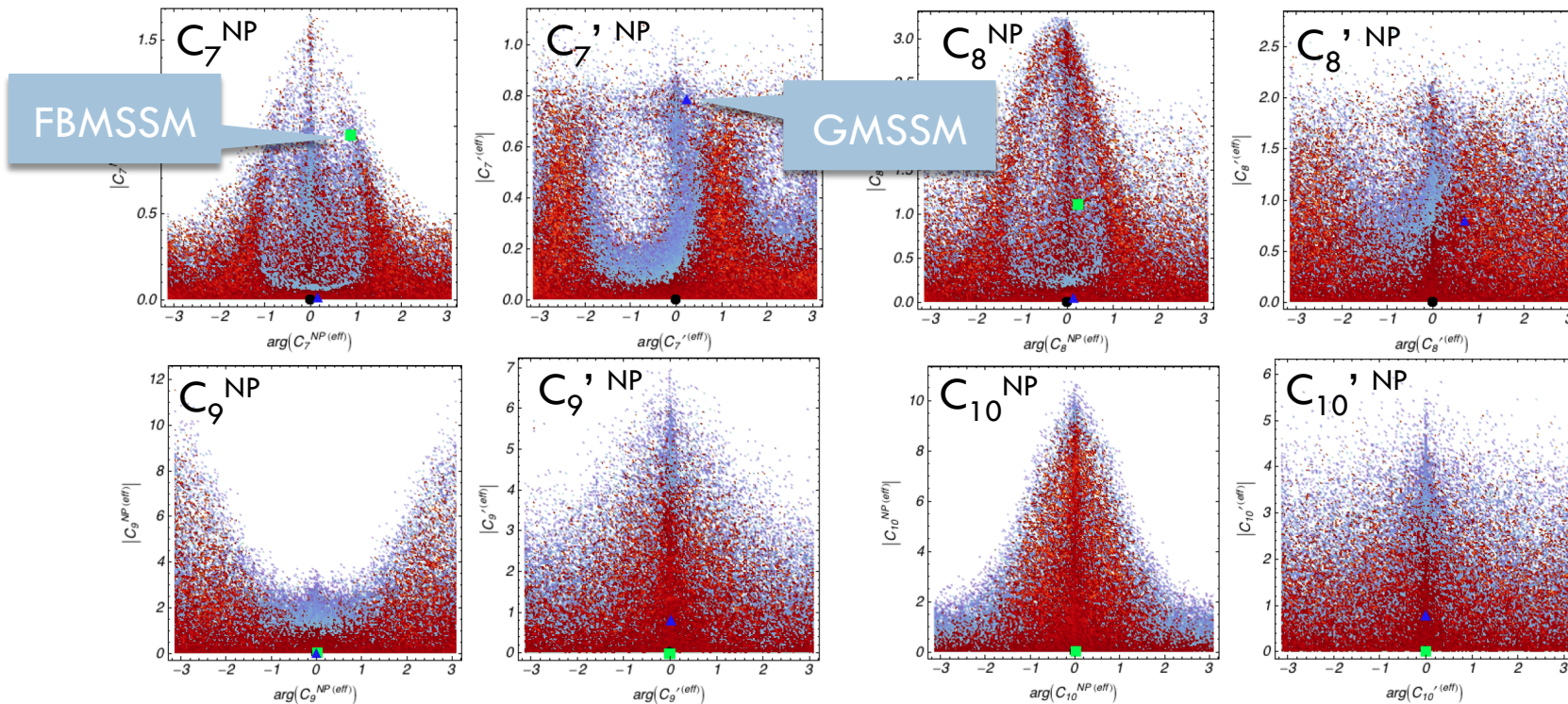
LHCb 2010 LHC Run data:
 $B/S = 0.2$; Selected with MVA
 $B/S = 1$ gives 50% more events

Wilson Coefficients

Red: 68%; Blue 95%
 Values shown at M_W
 SM at (0,0)

A. Bharucha & WR, Eur.Phys.J.**C69**:623-640,2010

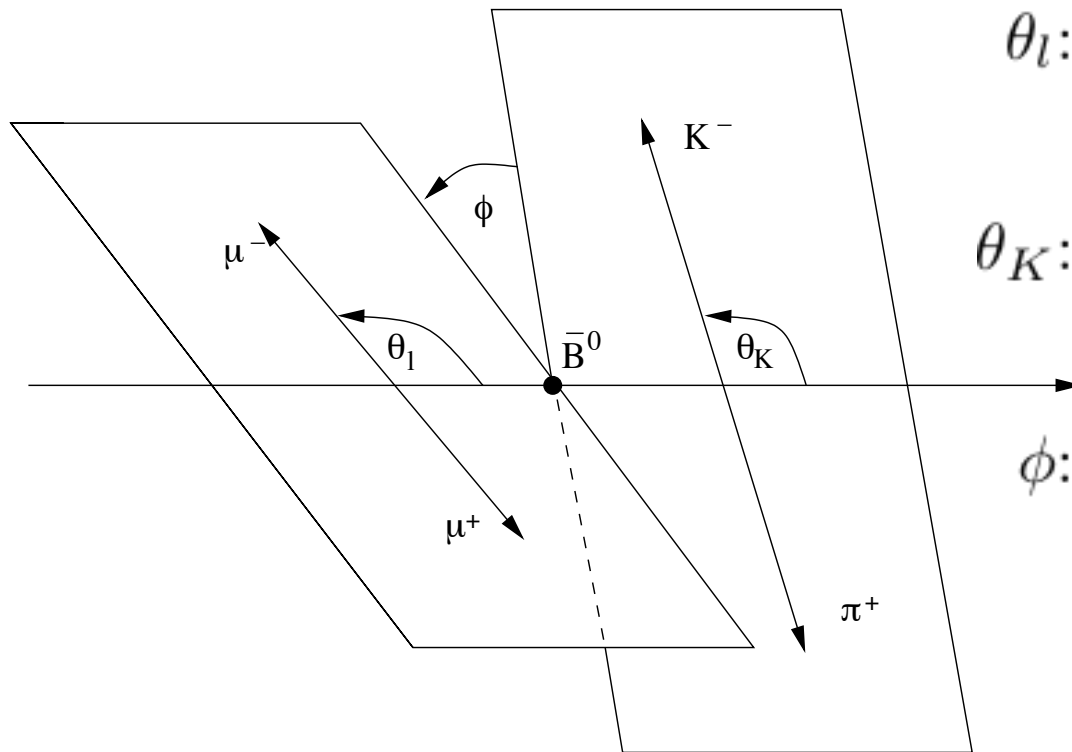
Page 10



- MCMC scan over $C_7, C_8, C_9, C_{10}, C_S, C_P$ + primes
- Phases left free, uses 2010 constraints (see backup)
 - ▣ High- q^2 constraints in A_{FB}, F_L not considered (affects C_{10})

Decay Kinematics

11



θ_l : Angle between μ^- and \bar{B} in $\mu\mu$ rest frame

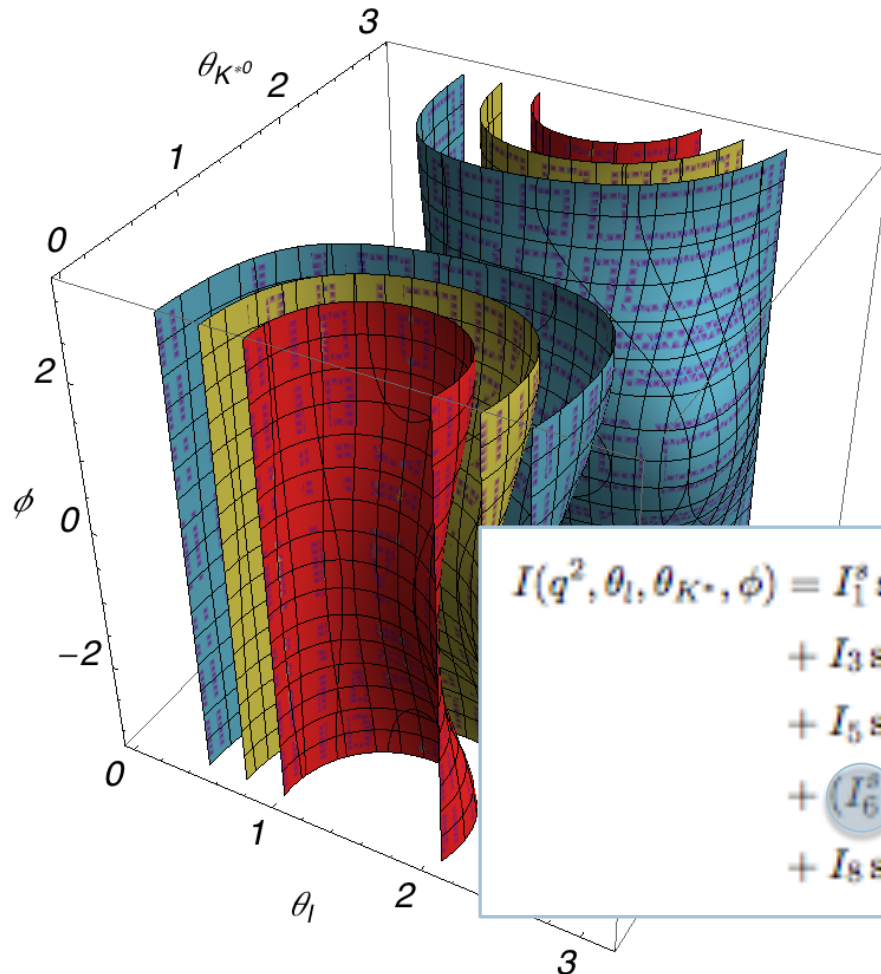
θ_K : Angle between K^- and the \bar{B} in the \bar{K}^{*0} rest frame

ϕ : Angle between the \bar{K}^{*0} and $\mu\mu$ decay planes

- Decay described in terms of 3 Angles and 1 Invariant Mass
 - ▣ θ_l, θ_K, ϕ and q^2 , the invariant mass squared of μ pair

Angular Distribution

$$\frac{d^4\Gamma}{dq^2 d \cos \theta_l d \cos \theta_{K^*} d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_{K^*}, \phi)$$



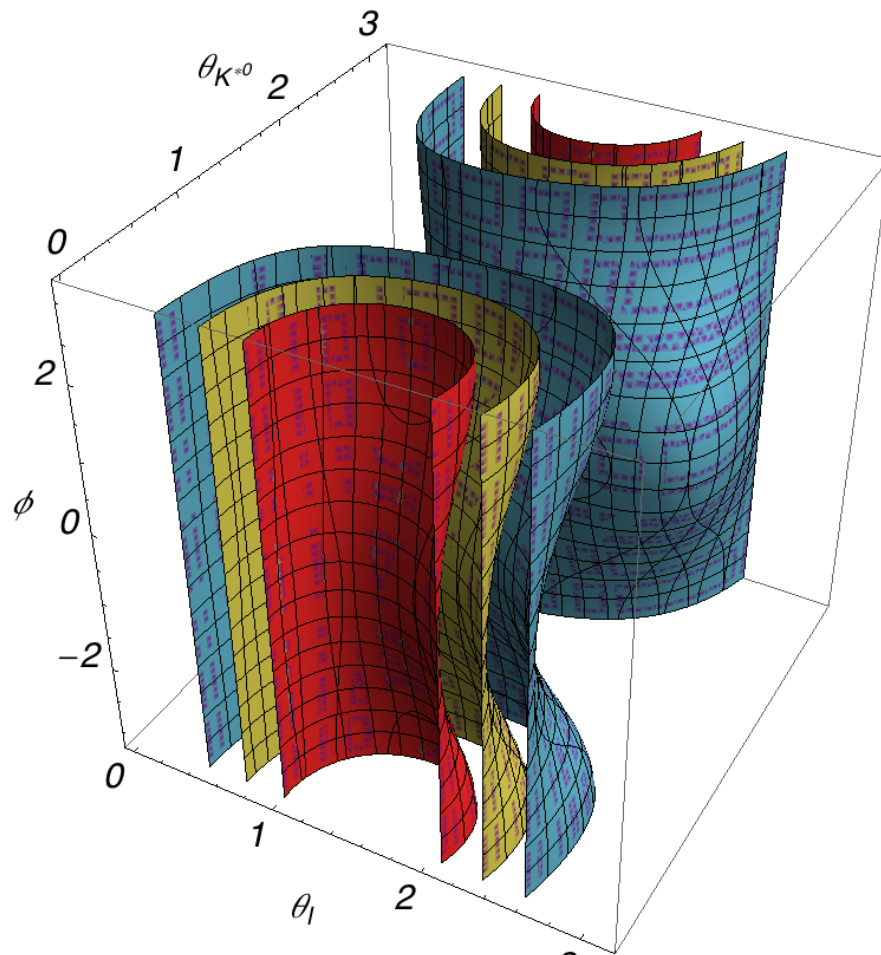
- SM agreement with BF OK
 - ▣ 2009 update pulls more
- $P \rightarrow VV$ Final state:
 - ▣ More observables to study
 - ▣ Angular variables interesting

$$\begin{aligned}
 I(q^2, \theta_l, \theta_{K^*}, \phi) = & I_1^s \sin^2 \theta_{K^*} + I_1^c \cos^2 \theta_{K^*} + (I_2^s \sin^2 \theta_{K^*} + I_2^c \cos^2 \theta_{K^*}) \cos 2\theta_l \\
 & + I_3 \sin^2 \theta_{K^*} \sin^2 \theta_l \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos \phi \\
 & + I_5 \sin 2\theta_{K^*} \sin \theta_l \cos \phi \\
 & + (I_6^s \sin^2 \theta_{K^*} + I_6^c \cos^2 \theta_{K^*}) \cos \theta_l + I_7 \sin 2\theta_{K^*} \sin \theta_l \sin \phi \\
 & + I_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin \phi + I_9 \sin^2 \theta_{K^*} \sin^2 \theta_l \sin 2\phi.
 \end{aligned}$$

SM Distribution for $q^2 = 3.5 \text{ GeV}^2$

Angular Distribution (Experimental)

Page 13



SM Distribution for $q^2 = 3.5 \text{ GeV}^2$

- Resolution: $q^2, \theta_l, \theta_{K^*}, \phi$ good
- Decay distribution symmetries:
 - Important for fitting

$$\frac{d^4\Gamma}{dq^2 d \cos \theta_l d \cos \theta_{K^*} d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_{K^*}, \phi)$$

- Number events in sample
 - Proportional to decay amplitude
 - Larger coefficients \rightarrow more events \rightarrow smaller uncertainties on given I_n
- Balance with theory errors
 - Many nice observables proposed

Landscape

14

$$S_i^{(a)} = \left(I_i^{(a)} + \bar{I}_i^{(a)} \right) / \frac{d(\Gamma + \bar{\Gamma})}{dq^2}$$

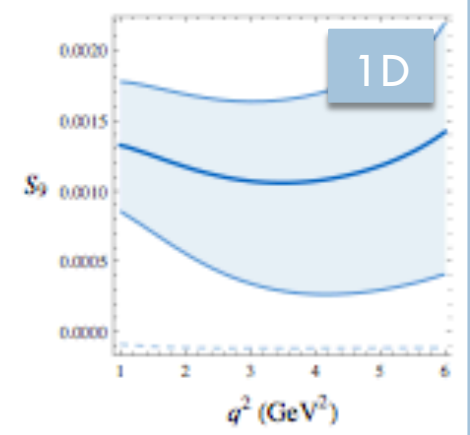
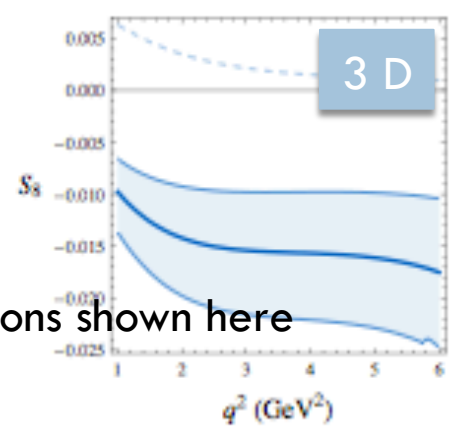
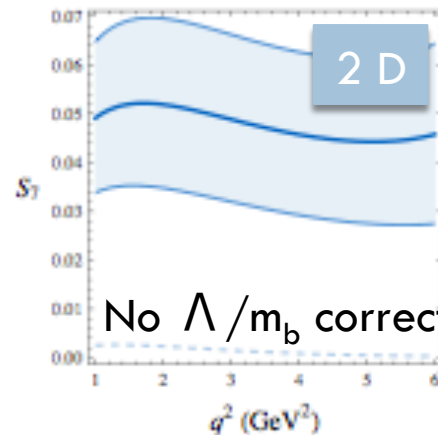
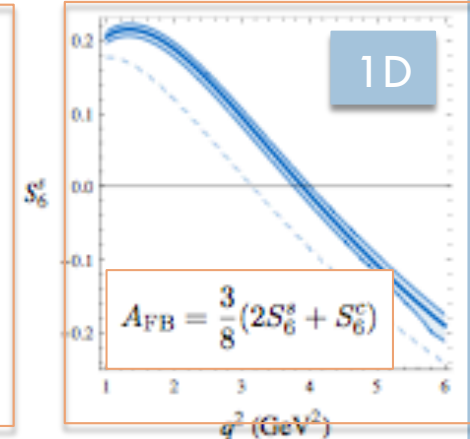
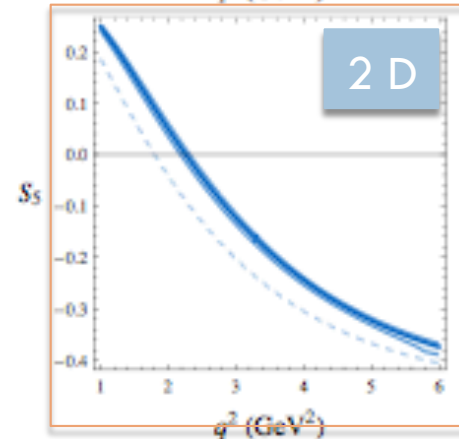
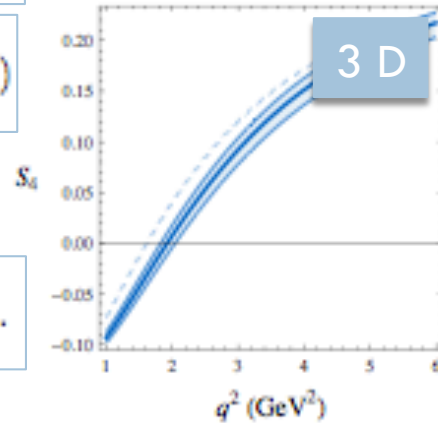
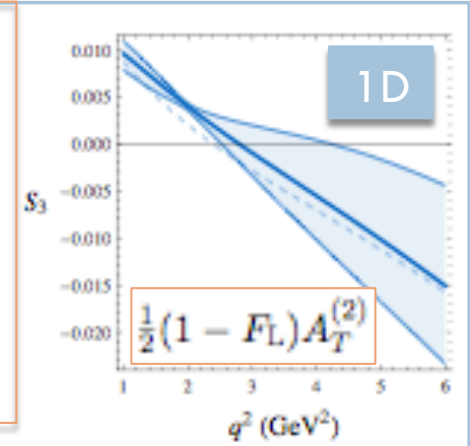
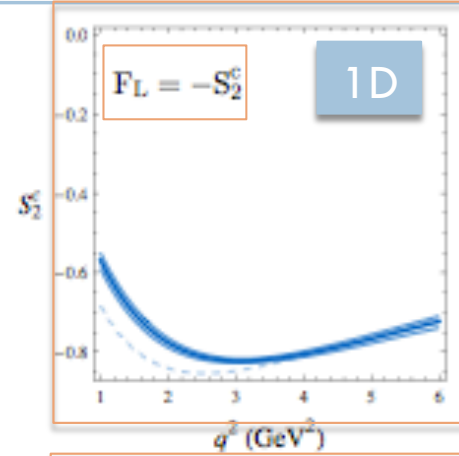
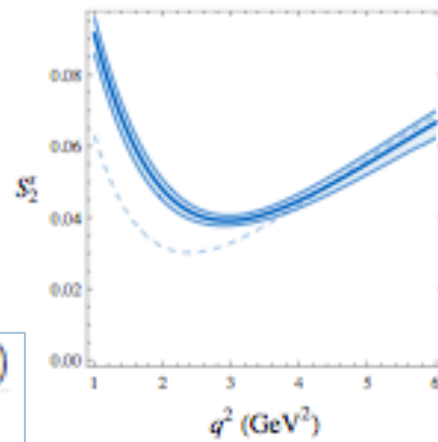
$$\frac{d\Gamma}{dq^2} = \frac{3}{4}(2I_1^s + I_1^c) - \frac{1}{4}(2I_2^s + I_2^c)$$

$$\frac{3}{4}(2S_1^s + S_1^c) - \frac{1}{4}(2S_2^s + S_2^c) = 1.$$

$$A_T^{(2)} = \frac{S_3}{2S_2^s},$$

$$A_T^{(3)} = \left(\frac{4S_4^2 + S_7^2}{-2S_2^c(2S_2^s + S_3)} \right)^{1/2}$$

$$A_T^{(4)} = \left(\frac{S_5^2 + 4S_8^2}{4S_4^2 + S_7^2} \right)^{1/2}.$$

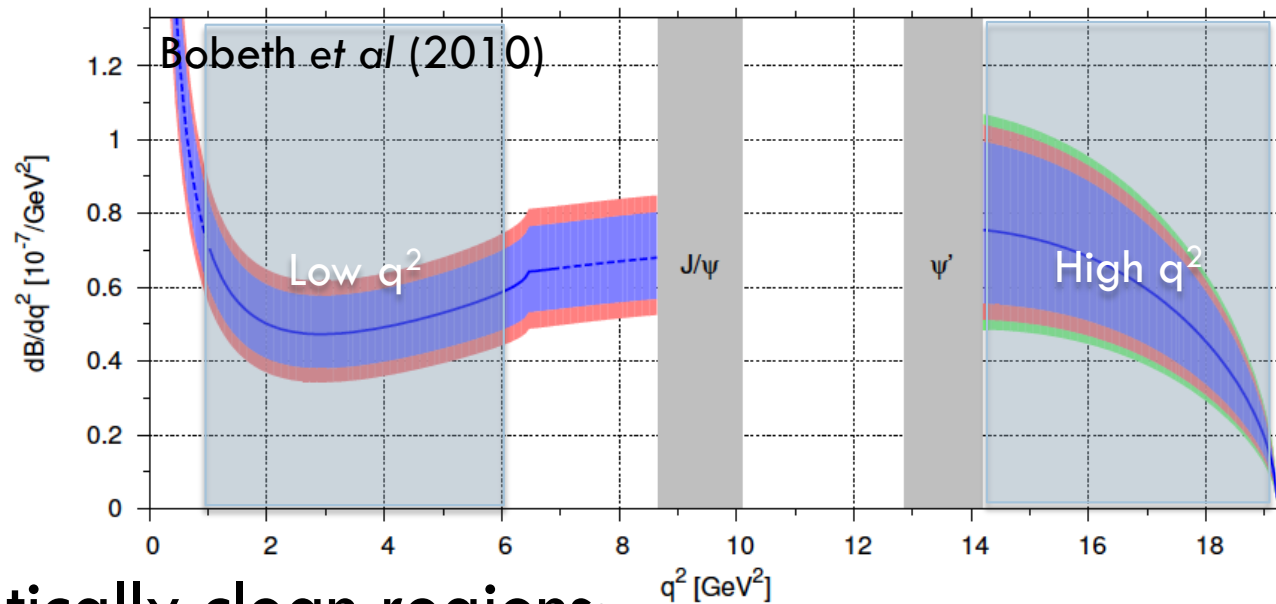


No Λ/m_b corrections shown here

$$A_T^{(5)} \Big|_{m_c=0} = \frac{\sqrt{16J_1^{s2} - 9J_6^{s2} - 36(J_3^2 + J_9^2)}}{8J_1^s}$$

Landscape – q^2

15



- Theoretically clean regions:
 - ▣ Large recoil ($1 < q^2 < 6 \text{ GeV}^2$) – e.g. Kruger, Matias, Phys.Rev.**D71**:094009,2005
 - ▣ Low recoil ($14 < q^2 < 19.2 \text{ GeV}^2$) - Bobeth et al, JHEP **1007**:098,2010
- Belle measures signal yields in both regions
 - ▣ Ratio low/high: 0.35 ± 0.1

What to measure?

16

- Consider three phases: low, medium, high statistics
- Low statistics (~ 100 s): 1D projections
 - ▣ $A_{\text{FB}}, F_L, \text{BF}, (A_{\text{T}}^{(2)}, S_9)$
- Medium statistics (~ 1000 s): Can use 2D projections
 - ▣ $A_{\text{FB}} (+ \text{Zero}), S_5 (+ \text{Zero}), A_{\text{T}}^{(2)}, \text{CP asymmetries}$
- High statistics (> 5000): Full-angular analysis
 - ▣ Can measure everything!
 - ▣ Many theoretically clean variables for low and high q^2
 - ▣ May be able to possibly to reduce required yields?

Low Statistics

$$H_T^{(3)} = \frac{\text{Re}(A_{\parallel}^L A_{\perp}^{L*} - A_{\parallel}^{R*} A_{\perp}^R)}{\sqrt{(|A_{\parallel}^L|^2 + |A_{\parallel}^R|^2)(|A_{\perp}^L|^2 + |A_{\perp}^R|^2)}} = \frac{\beta_1 J_6}{2\sqrt{(2J_2^s)^2 - J_3^2}}$$

17

- Limited to 1D distributions – project over angles

$$\frac{d\Gamma'}{d\phi} = \frac{\Gamma'}{2\pi} \left(1 + \frac{1}{2}(1 - F_L)A_T^{(2)} \cos 2\phi + A_{\text{Im}} \sin 2\phi \right) \quad A_{\text{Im}} = S_9$$

$$\frac{d\Gamma'}{d \cos \theta_1} = \Gamma' \left(\frac{3}{4}F_L \sin^2 \theta_1 + \frac{3}{8}(1 - F_L)(1 + \cos^2 \theta_1) + A_{\text{FB}} \cos \theta_1 \right)$$

$$\frac{d\Gamma'}{d \cos \theta_K} = \frac{3\Gamma'}{4} (2F_L \cos^2 \theta_K + (1 - F_L) \sin^2 \theta_K)$$

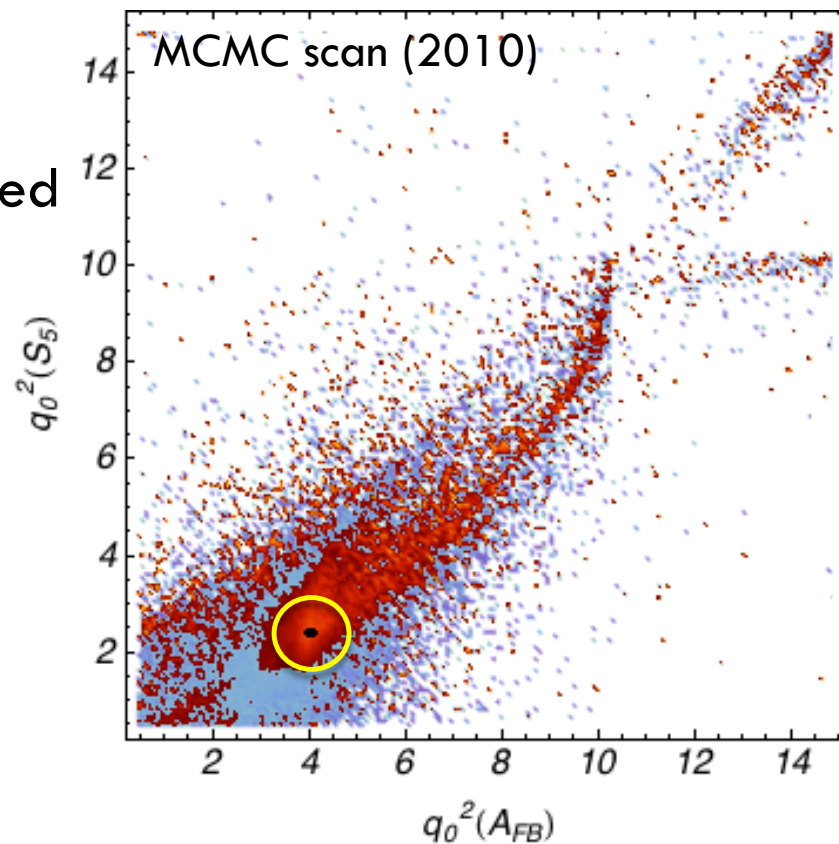
- Try simultaneous fit – 3x1D (LHCb-PUB-2007-057)
- Sensitivity to $A_T^{(2)}$ poor in low q^2 region (F_L large)
 - ▣ In high q^2 region, F_L smaller so expect much better results
 - C.F. Very low q^2 $B \rightarrow K^* e e$ analysis (LHCb-PUB-2009-008)
- Excellent prospects for $H_T^{(3)}$ at this stage

Medium Statistics

$$H_T^{(2)} = \frac{\text{Re}(A_0^L A_{\perp}^{L*} - A_0^{R*} A_{\perp}^R)}{\sqrt{(|A_0^L|^2 + |A_0^R|^2)(|A_{\perp}^L|^2 + |A_{\perp}^R|^2)}} = \frac{\beta_1 J_5}{\sqrt{-2J_2^c(2J_2^s + J_3)}}$$

18

- 2D distributions + zeros
 - ▣ S_5, S_7 become available
 - ▣ CP asym (A_9), $A_T^{(2)}$ more constrained
- Zeros experimental uncertainty:
 - ▣ Proportional to gradient
- A_{FB} and S_5 zeros clean
 - ▣ Form factors cancel at LO
- Gradient for S_5 often large
 - ▣ Get ~ 2 times sensitivity
- Ratio $A_{\text{FB}}^{\{1,2\}}/A_{\text{FB}}^{\{4,6\}}$ may help
 - ▣ See [Lunghi, Soni, JHEP 1011:121,2010](#)
- $H_T^{(2)}$ could prove powerful here



$$\frac{G_0(S_5)}{G_0(A_{\text{FB}})} \approx 1.75$$

Comparing Observables

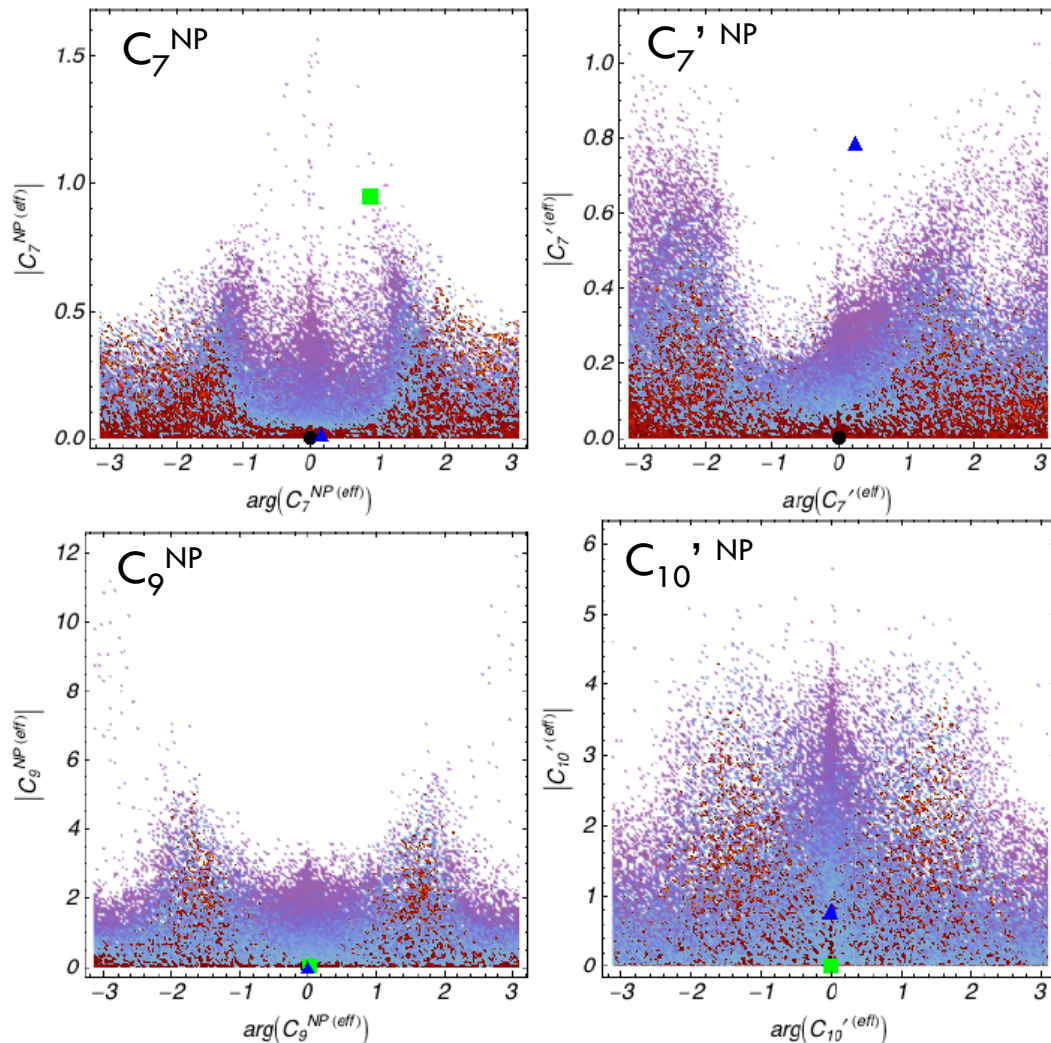
19

Observable	2 fb^{-1}	1 fb^{-1}	0.5 fb^{-1}	LHCb 2 fb^{-1}	Ref.
$q_0^2(A_{\text{FB}})$	+0.56 -0.94	+1.27 -0.97	—	0.42	[128]
$q_0^2(S_5)$	+0.27 -0.25	+0.53 -0.40	—	—	
$\langle A_{\text{FB}} \rangle_{1-6 \text{ GeV}^2}$	+0.03 -0.04	+0.05 -0.03	+0.08 -0.06	0.020	[107]
$\langle F_{\text{L}} \rangle_{1-6 \text{ GeV}^2}$	+0.02 -0.02	+0.04 -0.03	+0.04 -0.06	0.016	[107]
$\langle S_5 \rangle_{1-6 \text{ GeV}^2}$	+0.07 -0.08	+0.09 -0.11	+0.16 -0.15	—	
$\langle A_9 \rangle_{1-6 \text{ GeV}^2}$	+0.08 -0.07	+0.11 -0.11	+0.22 -0.14	0.015	[2] ^a

- Use simple counting experiments (non-optimal)
 - ▣ Compare sensitivities (official LHCb numbers shown in box)
 - ▣ See [CERN-THESIS-2010-095](#) for more details

Effects on Parameter Space (2fb^{-1})

20



- What if LHCb 2fb^{-1} results were at SM?
 - See previous table
 - A_9 not considered
- Parameter space much reduced at **68%**
 - High- q^2 will help further
 - CP asymmetries

High q^2 Estimates: ($q^2 > 14\text{GeV}^2$)

21

Observable	2 fb^{-1}	1 fb^{-1}	0.5 fb^{-1}	LHCb 2 fb^{-1}	Ref.
My (unofficial) high- q^2 estimates:				Official low- q^2	
A_{FB} :	± 0.01			± 0.02	
F_L :	± 0.01			± 0.016	
$A_T^{(2)}$:	± 0.2			± 0.42	
$H_T^{(3)}$:	± 0.1				
Based on CERN-LHCB-2007-057					

- Increased statistics in high- q^2 bin: Scale by $\sqrt{1/0.35}$
 - ▣ Selection efficiencies, trigger, acceptance easier?
- F_L suppression of $A_T^{(2)}$ reduced: Take 25% effect here

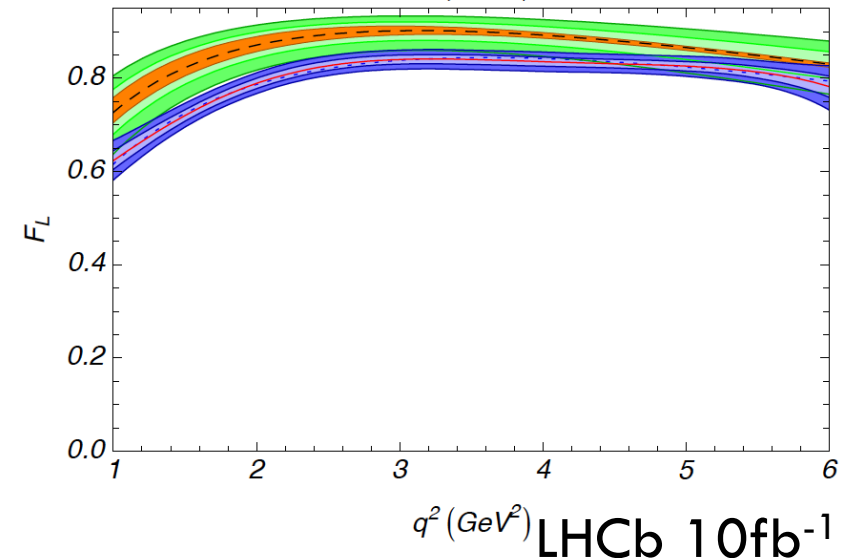
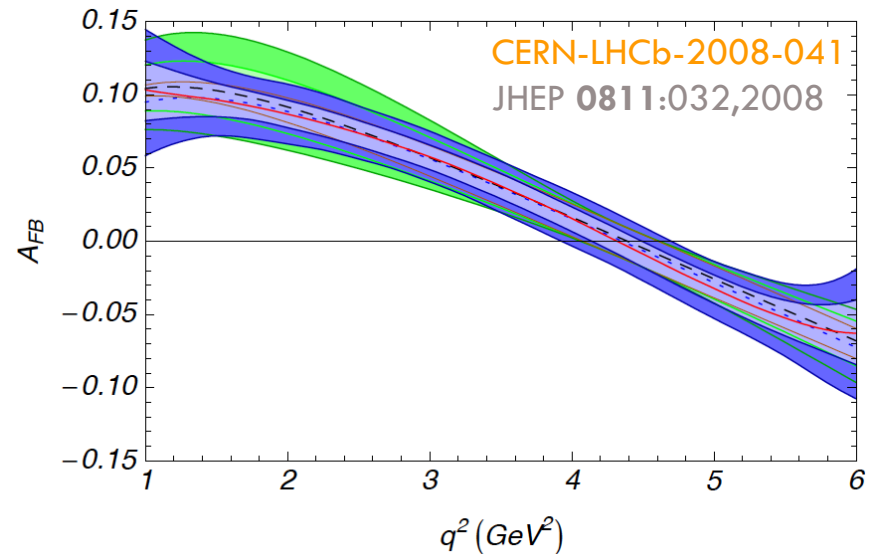
High Statistics

SM Theory Distribution

Toy fits to SUSY model b ($C_7' \neq 0$) $1, 2\sigma$

22

- Perform full-angular analysis
 - Fit for spin amplitudes
 - $A_{\perp L,R}, A_{||L,R}, A_{0L,R}$
 - Assume polynomial q^2 variation
- Calculate any observable from amplitudes
 - New observables $A_T^{(3)}, A_T^{(4)}$ optimized for C_7' sensitivity
 - 10fb^{-1} sensitivities for SUSY input
 - JHEP 0704 (2007) 058 – model ‘b’
 - Allowed by experimental constraints
- MC Fits converge with 2fb^{-1}
- Separate fit in high- q^2 region $\rightarrow H_T^{(1)}$
- All CP asymmetries now available



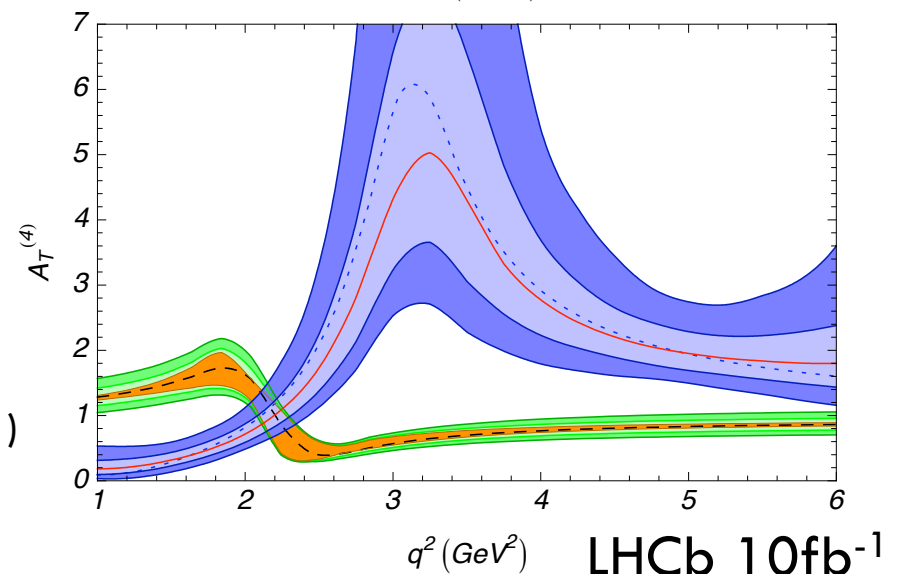
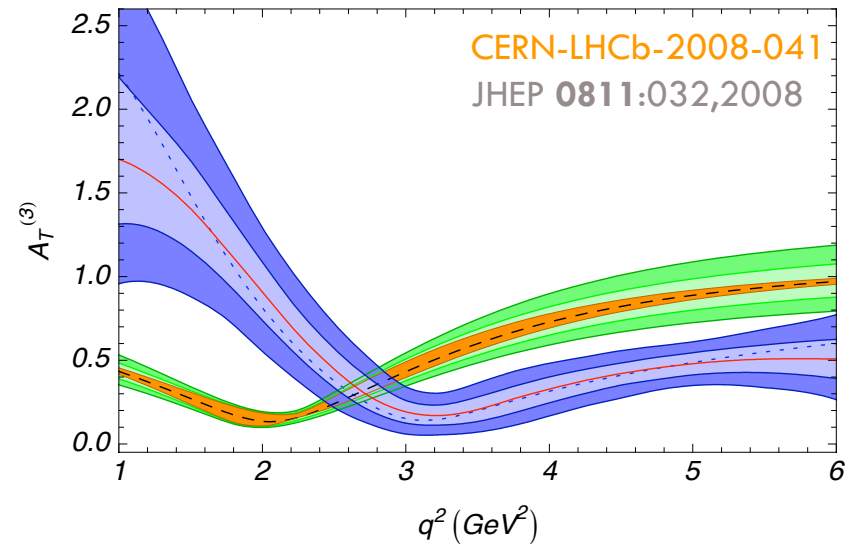
High Statistics

SM Theory Distribution

Toy fits to SUSY model b ($C_7' \neq 0$) $1, 2\sigma$

23

- Perform full-angular analysis
 - Fit for spin amplitudes
 - $A_{\perp L,R}, A_{\parallel L,R}, A_{0L,R}$
 - Assume polynomial q^2 variation
- Calculate any observable from amplitudes
 - New observables $A_T^{(3)}, A_T^{(4)}$ optimized for C_7' sensitivity
 - 10fb^{-1} sensitivities for SUSY input
 - JHEP 0704 (2007) 058 – model ‘b’
 - Allowed by experimental constraints
- MC Fits converge with 2fb^{-1}
- Separate fit in high- q^2 region $\rightarrow H_T^{(1)}$
- All CP asymmetries now available



Note on Symmetries

24

Case	Coefficients	Dependencies	Amplitudes	Symmetries
$m_\ell = 0, A_S = 0$	11	3	6	4
$m_\ell = 0$	11	2	7	5
$m_\ell > 0, A_S = 0$	11	1	7	4
$m_\ell > 0$	12	0	8	4

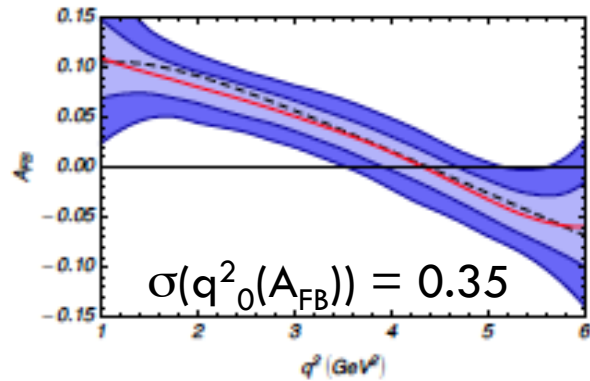
- Angular distribution has symmetries
 - ▣ Must be removed before fitting (under-constrained)
- All observables must be invariant
 - ▣ Many in literature are not! E.g. $A_T^{(1)}$
- Massless leptons case: 3 trivial + 1 non-trivial
 - ▣ Independent L and R phase rotations
 - ▣ Two L-R rotations: One real, one complex
 - ▣ See [Egede et al, JHEP 1010:056,2010](#)

Full Angular Fit Sensitivities (SM)

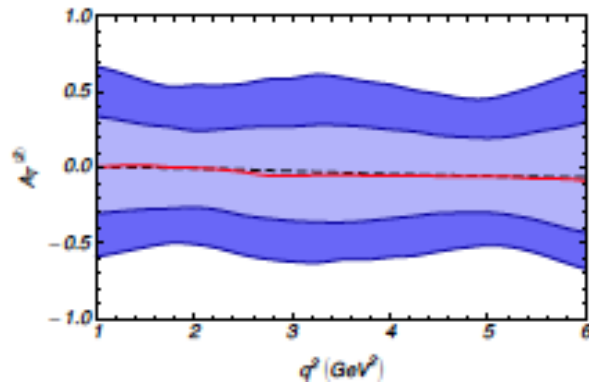
25

LHCb 2fb⁻¹

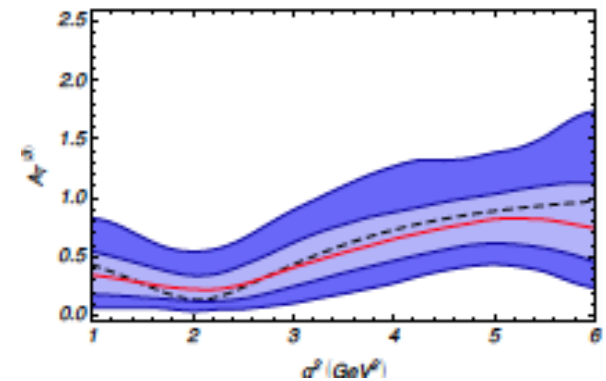
A_{FB}



$A_T^{(2)}$

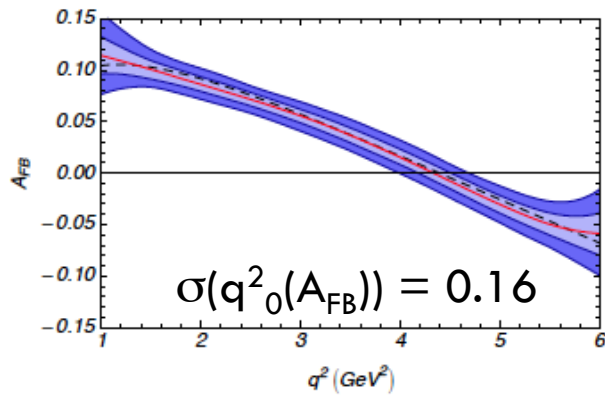


$A_T^{(3)}$

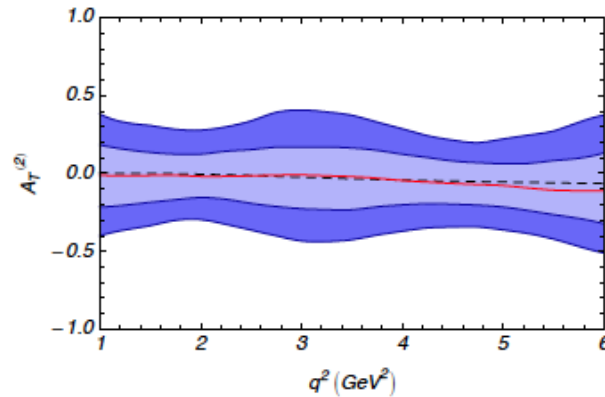


LHCb 10fb⁻¹

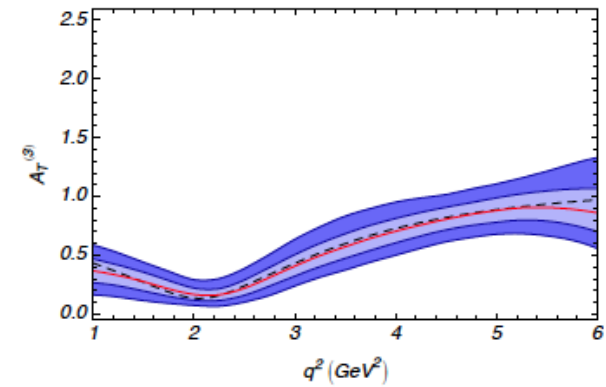
A_{FB}



$A_T^{(2)}$

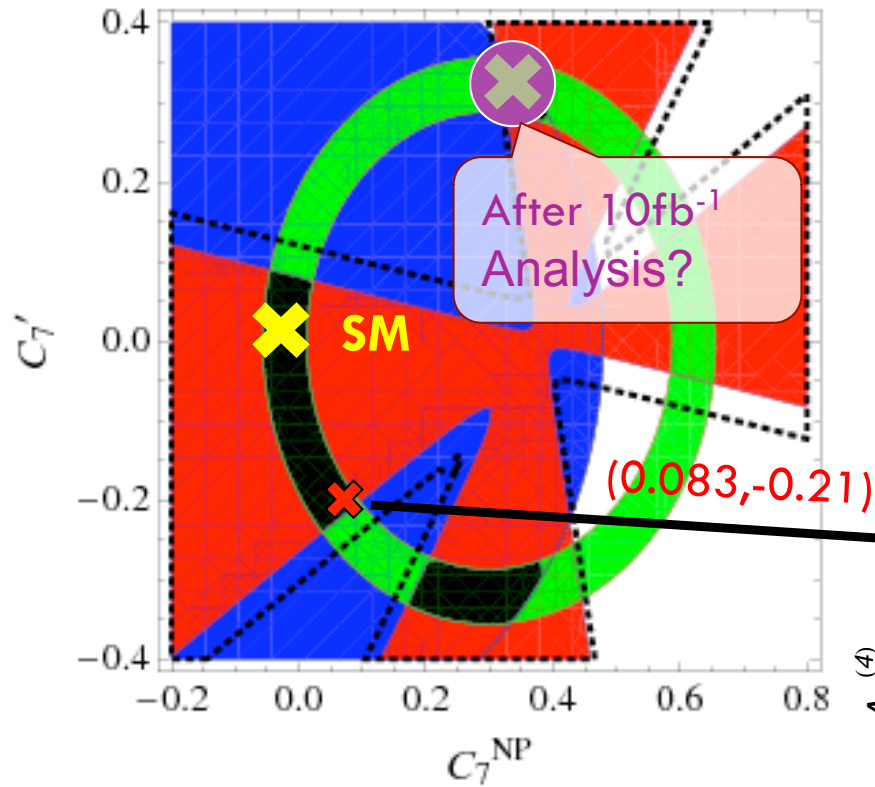


$A_T^{(3)}$



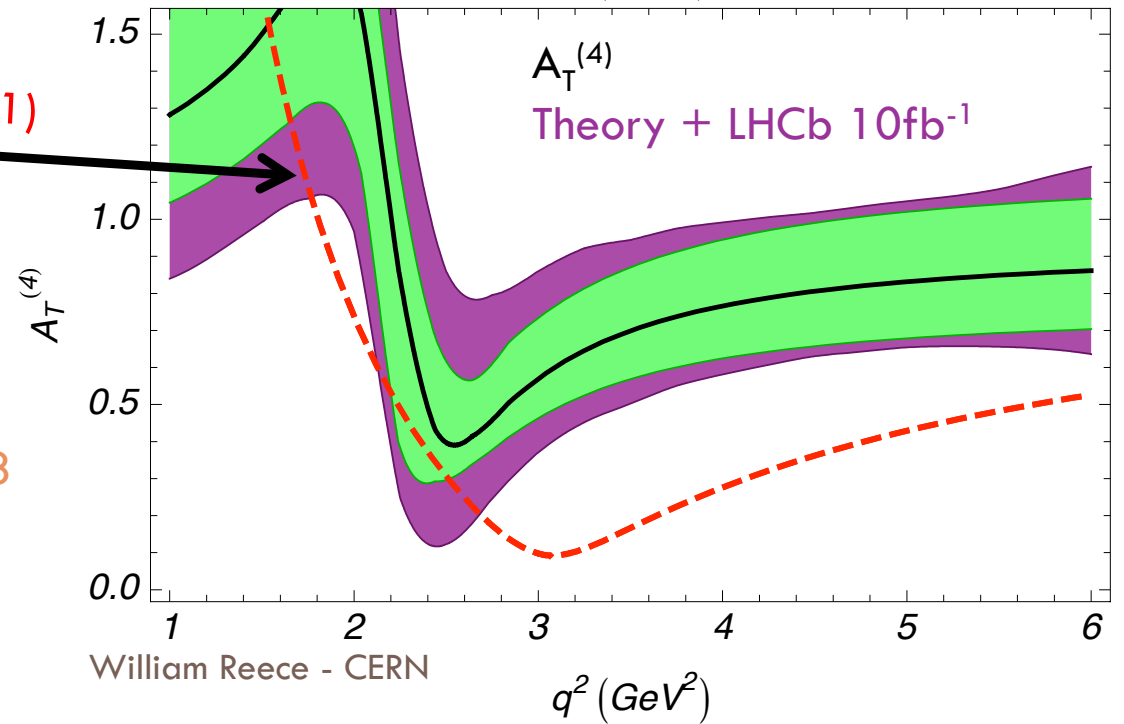
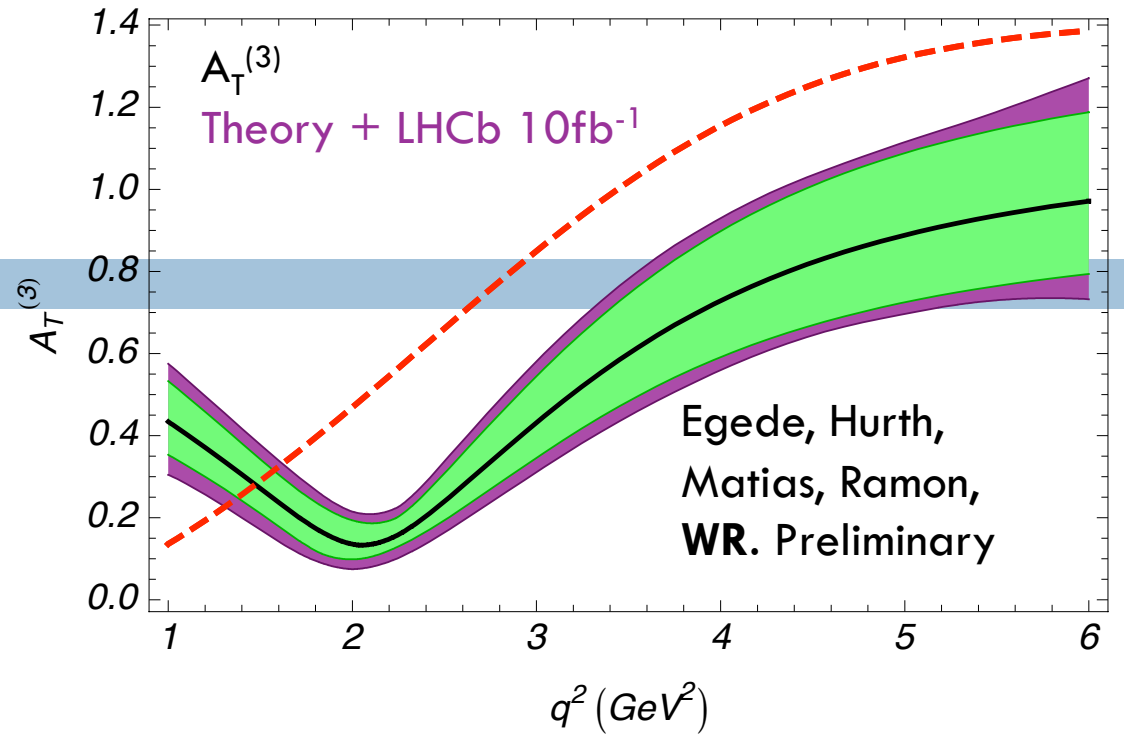
NP in C_7

26



Bobeth *et al*, JHEP 0807:106,2008

$$C_7 = C_7^{\text{SM}} + C_7^{\text{NP}}$$



Summary

27

- Three eras of $B \rightarrow K^* \mu\mu$ measurements at LHCb
 - ▣ Each has interesting observables to study
 - ▣ Must balance experimental and theoretical uncertainties
- Should really cut into allowed regions
 - ▣ Severe limits or explore structure of NP
- High- q^2 region little studied so far by experimentalists
 - ▣ Looks pretty promising
- Full-angular analysis will be key
 - ▣ Most interesting area for comparison with SuperB

BACKUP



Comparisons with Theory (2010)

Belle Update (2009)
 $0.99 \pm 0.2 \pm 0.9$
[Hiroyuki Nakayama, \(Tokyo U.\) . Dec 2009. 142pp.](#)
[Ph.D. Thesis](#)

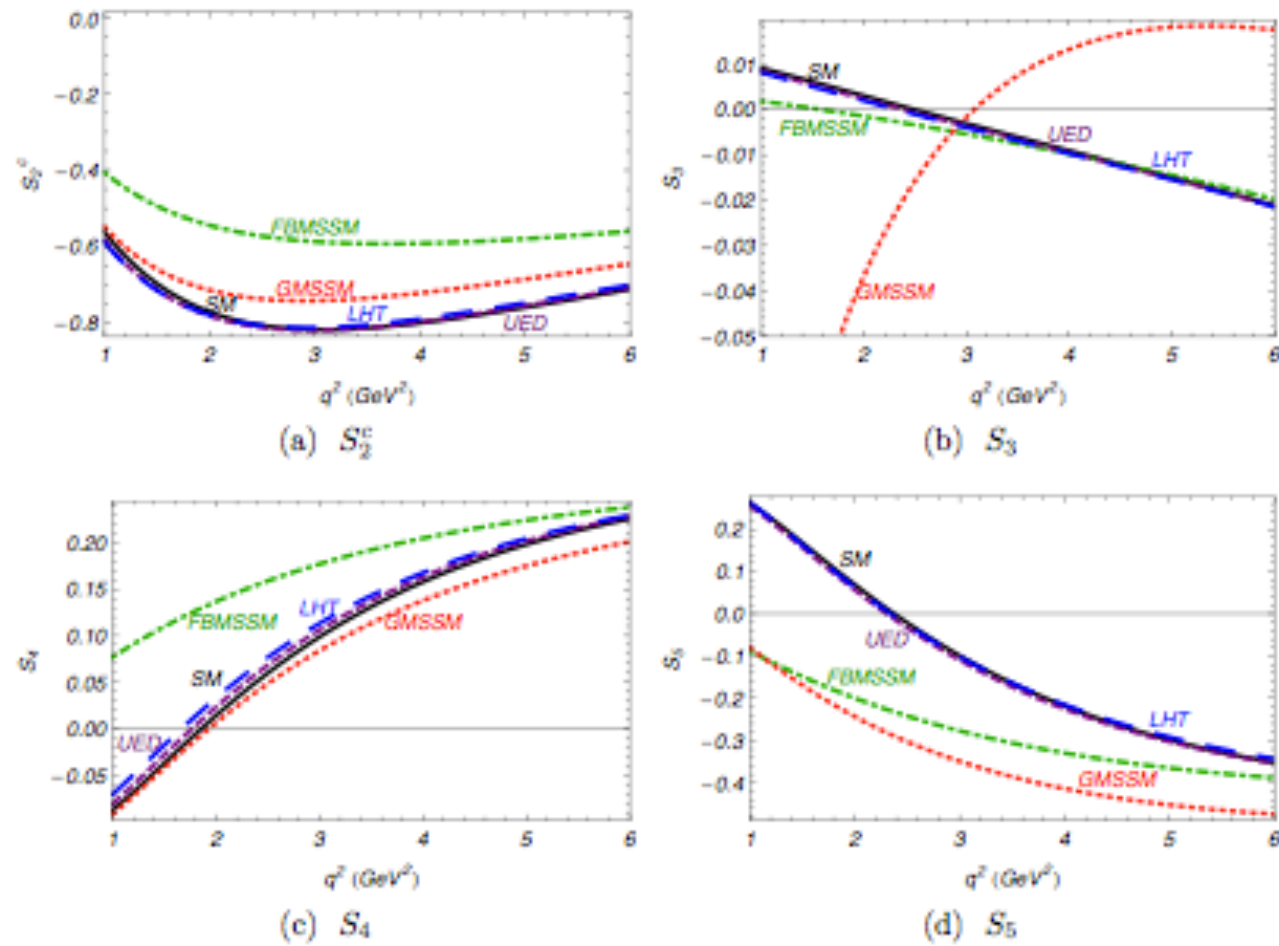
Observable	Experiment	SM Theory
$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$	$3.6 \cdot 10^{-8}$ [57]	$(3.70 \pm 0.31) \cdot 10^{-9}$
$\mathcal{B}(B \rightarrow X_s l^+ l^-)_{1-6 \text{ GeV}^2}$	$(1.60 \pm 0.51) \cdot 10^{-6}$ [14]	$(1.97 \pm 0.11) \cdot 10^{-6}$
$\mathcal{B}(B \rightarrow X_s \gamma)$	$(3.52 \pm 0.23 \pm 0.09) \cdot 10^{-4}$ [56]	$(3.28 \pm 0.25) \cdot 10^{-4}$
$S(B \rightarrow K^* \gamma)$	$(-1.6 \pm 2.2) \cdot 10^{-1}$ [56]	$(-0.26 \pm 0.05) \cdot 10^{-1}$
$\langle A_{\text{FB}} \rangle_{1-6 \text{ GeV}^2}$	-0.26 ± 0.29 [8]	0.04 ± 0.03
$\langle F_L \rangle_{1-6 \text{ GeV}^2}$	0.67 ± 0.24 [8]	0.76 ± 0.08

Table 8: Experimental measurements used as constraints, along with theoretical predictions in the SM.

A. Bharucha & WR, Eur.Phys.J.**C69**:623-640,2010

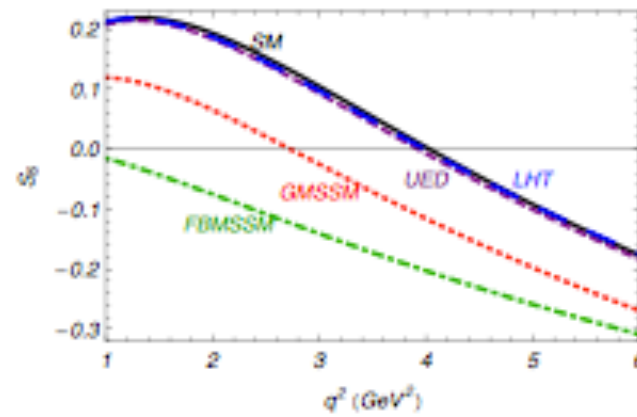
Landscape with NP

30

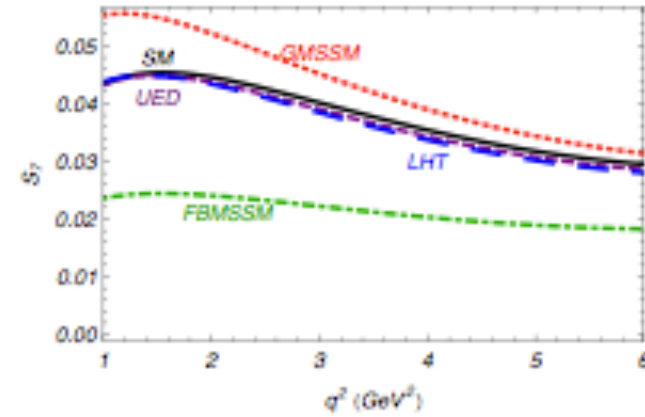


Landscape with NP

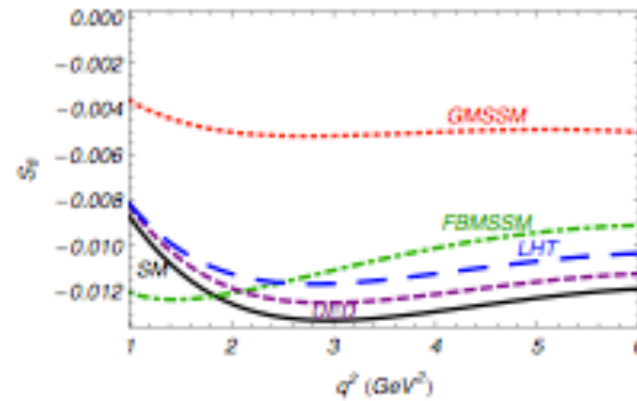
31



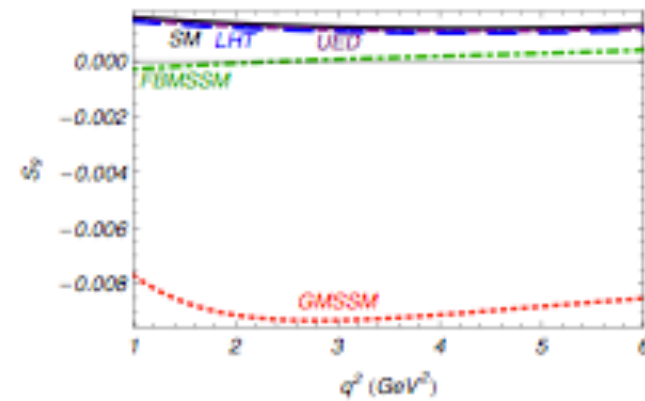
(e) S_6



(f) S_7



(g) S_8



(h) S_9

Massless symmetries

32

Left-handed phase rotation

$$A'_{\perp L} = e^{i\phi_L} A_{\perp L}, \quad A'_{\parallel L} = e^{i\phi_L} A_{\parallel L}, \quad A'_{0L} = e^{i\phi_L} A_{0L}$$

Right-handed phase rotation

$$A'_{\perp R} = e^{i\phi_R} A_{\perp R}, \quad A'_{\parallel R} = e^{i\phi_R} A_{\parallel R}, \quad A'_{0R} = e^{i\phi_R} A_{0R}$$

Continuous L-R global rotation

$$A'_{\perp L} = +\cos\theta A_{\perp L} + \sin\theta A_{\perp R}^*$$

$$A'_{\perp R} = -\sin\theta A_{\perp L}^* + \cos\theta A_{\perp R}$$

$$A'_{0L} = +\cos\theta A_{0L} - \sin\theta A_{0R}^*$$

$$A'_{0R} = +\sin\theta A_{0L}^* + \cos\theta A_{0R}$$

$$A'_{\parallel L} = +\cos\theta A_{\parallel L} - \sin\theta A_{\parallel R}^*$$

$$A'_{\parallel R} = +\sin\theta A_{\parallel L}^* + \cos\theta A_{\parallel R}$$

Continuous L-R global rotation

$$A''_{\perp L} = +\cosh i\phi A_{\perp L} + \sinh i\phi A_{\perp R}^*$$

$$A''_{\perp R} = +\sinh i\phi A_{\perp L}^* + \cosh i\phi A_{\perp R}$$

$$A''_{0L} = +\cosh i\phi A_{0L} - \sinh i\phi A_{0R}^*$$

$$A''_{0R} = -\sinh i\phi A_{0L}^* + \cosh i\phi A_{0R}$$

$$A''_{\parallel L} = +\cosh i\phi A_{\parallel L} - \sinh i\phi A_{\parallel R}^*$$

$$A''_{\parallel R} = -\sinh i\phi A_{\parallel L}^* + \cosh i\phi A_{\parallel R}$$