Universitat Autònoma de Barcelona

Strangeness changing form factors and V_{us} from τ decay data

Diogo Boito

SuperB - Elba
31 May 2011

outline

- Inclusive τ decays (in a nut shell)
  - $\alpha_s(m_\tau^2)$ from $|\Delta S| = 0$ decays
  - V_{us} from $|\Delta S| = 1$ decays
- Decay $\tau \to K_S^0 \pi^- \nu_\tau$: form factors $\longleftrightarrow$ V_{us} from K_{l3} decays

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General discussion. (b-factories have contributed and can contribute a lot.)

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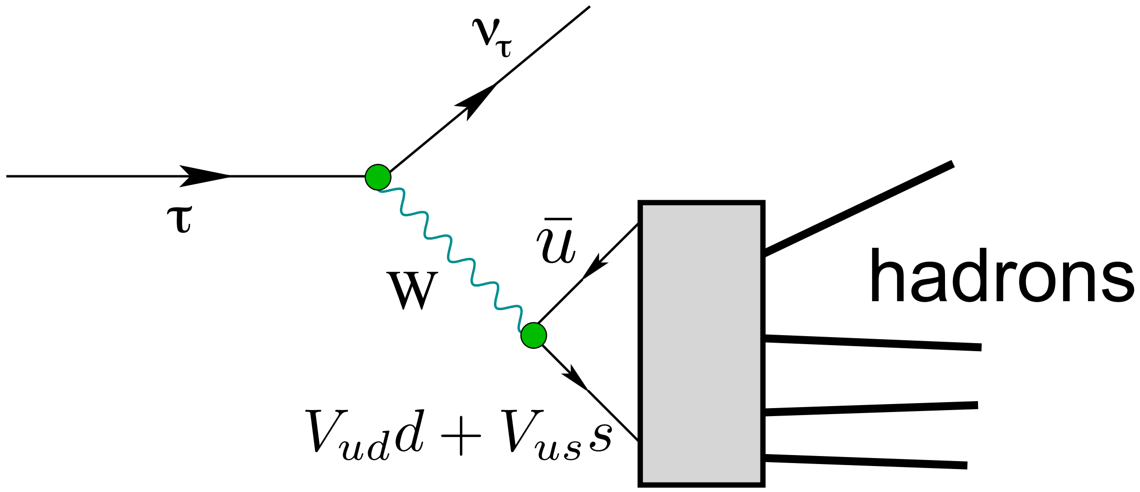
Results from an analysis of the Belle spectrum

- Data from *b*-factories:
  - Several decays with $|\Delta S| = 1$ were measured:

    $\tau \to K_S^0 \pi^- \nu_\tau$, $\tau \to \nu_\tau \phi K^-$, $\tau \to \nu_\tau K^- K^- K^+$ ...
  - Spectral functions for $|\Delta S| = 0$ still to be done.
    Indirect contributions from *b*-fac.: e.g. $e^+ e^- \to K^+ K^- \pi^0$

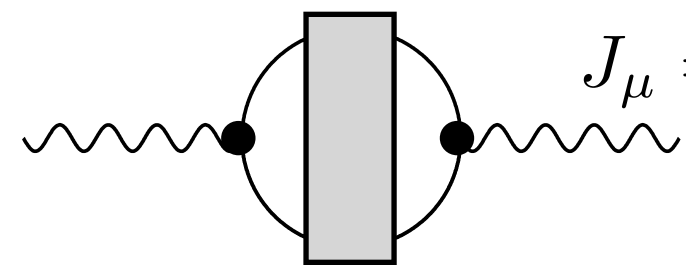


$$R_\tau = \frac{\Gamma[\tau \to \text{hadrons}\,\nu_\tau]}{\Gamma[\tau \to e^-\bar{\nu}_e\,\nu_\tau]} = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$= \frac{1 - B_e - B_\mu}{B_e} = 3.640 \pm 0.010$$

- Related to the correlators

$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0|T\{J_\mu(x) J_\nu(0)^\dagger\}|\rangle$$



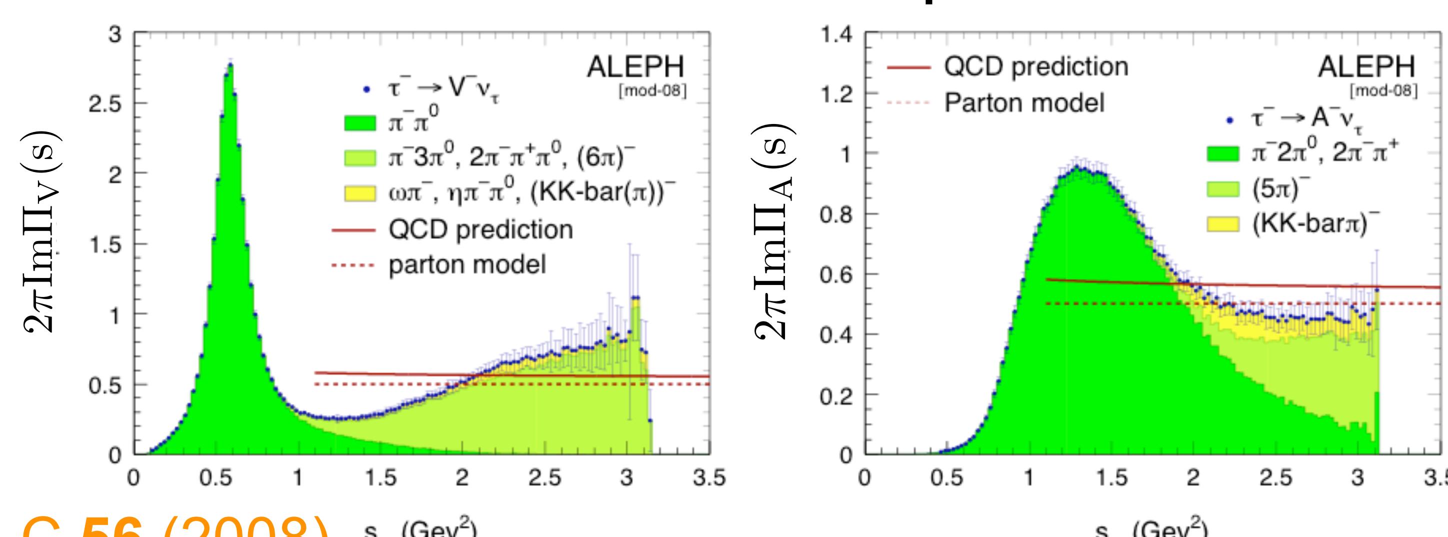
$$J_\mu = \bar{u}\gamma_\mu(\gamma_5)q(x)$$

via (optical theorem)

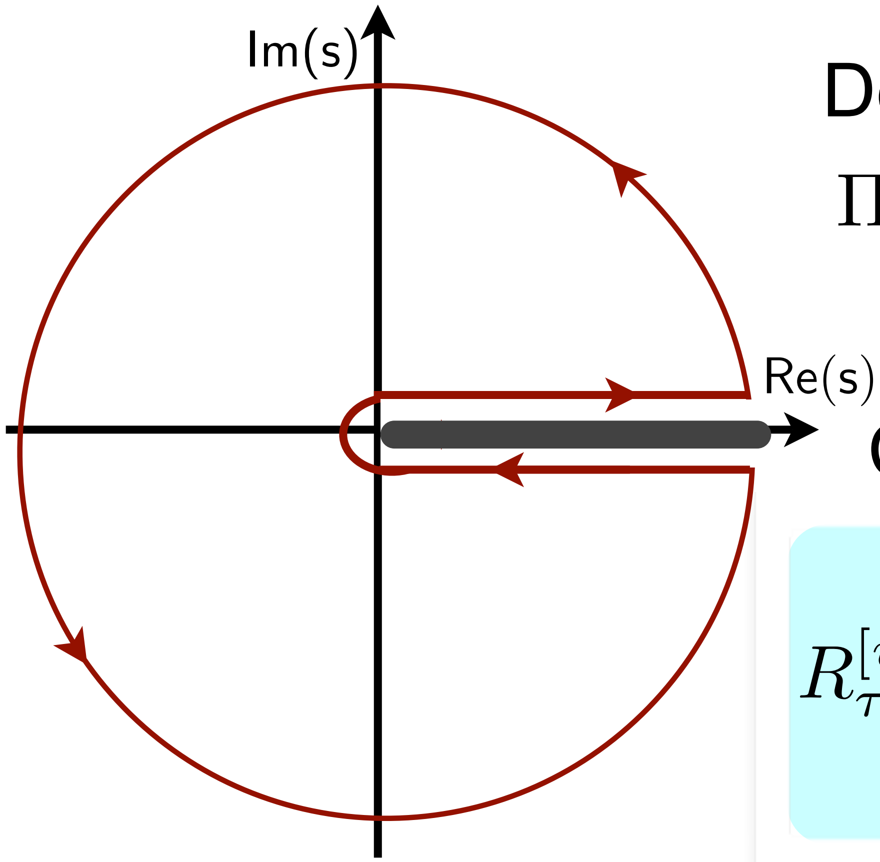
$$R_\tau = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^{(1)} + \mathrm{Im}\Pi^{(0)}\right]$$

- Imaginary parts of the correlators can be determined from experiment (ALEPH, OPAL)

Publicly available for $|\Delta S| = 0$



Davier *et al* EPJ.C **56** (2008)



Decomposition of the correlators

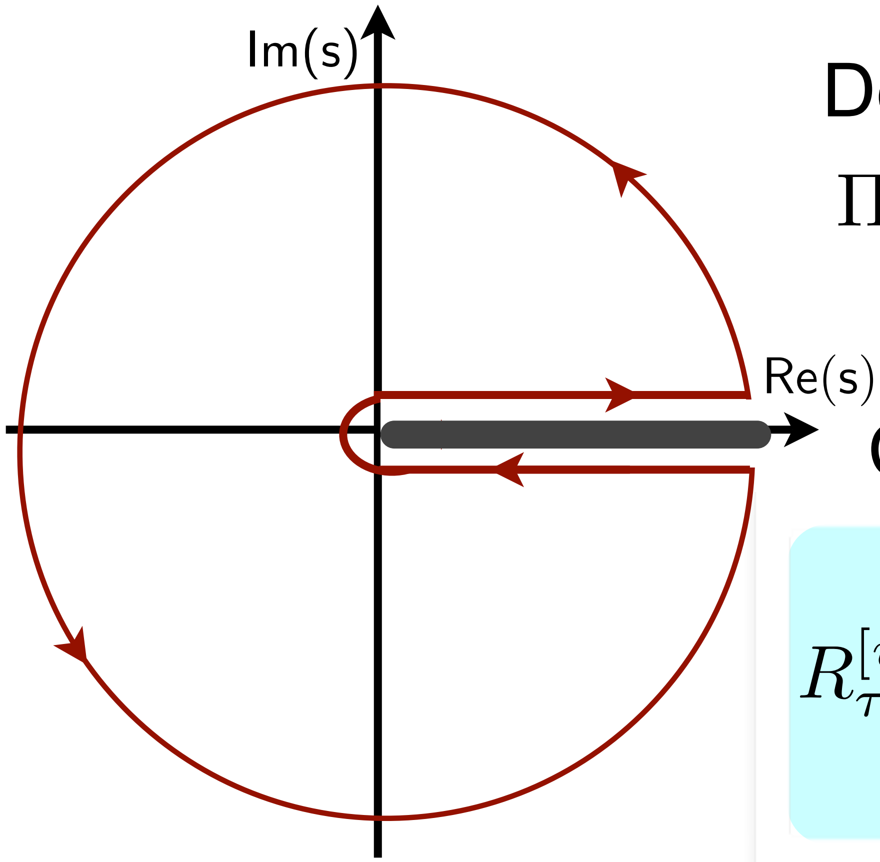
$$\Pi_{\mu\nu}(q) \;=\; (q_\mu q_\nu - g_{\mu\nu} s)\,\Pi^{(1)}(s) + q_\mu q_\nu\,\Pi^{(0)}(s)$$

Cauchy's theorem Braaten, Narison, and Pich, 1992

$$R_\tau^{[w]} = 12\pi \int_0^1 dx\, w(x)\,\mathrm{Im}\tilde{\Pi}(x\,s_0) = 6\pi i \oint_{|x=1|} w(x)\tilde{\Pi}(x\,s_0)$$

$$\tilde{\Pi}(x\,s_0) = \Pi^{(1+0)}(x\,s_0) - (1+2x)^{-1} 2x\Pi^{(0)}(x\,s_0)$$

R_τ corresponds to $w_\tau = (1-x)^2(1+2x)$ and $s_0 = m_\tau^2$



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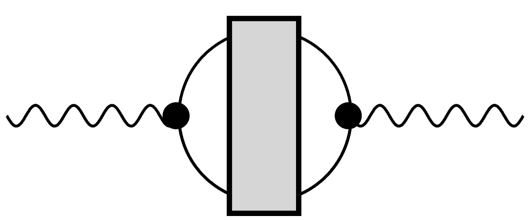
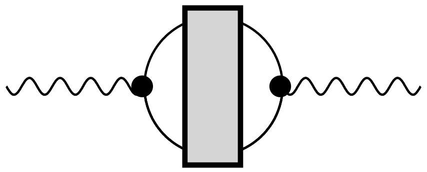
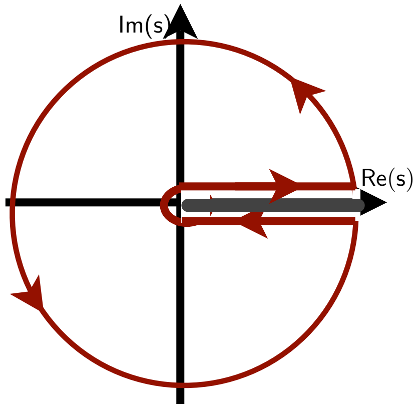
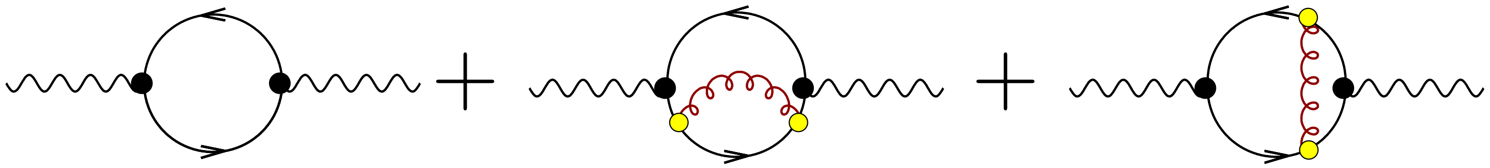




$$\Pi(s) = \Pi_{\rm OPE}(s) + \Delta_{\rm DVs}(s)$$

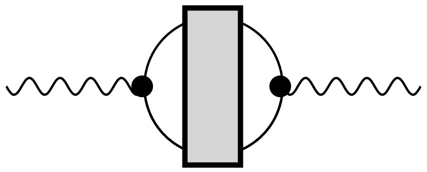


- OPE $$\Pi^{(J)}_{\rm OPE}(s) = \sum_{D=2n} \frac{C^{(J)}(s,\mu)\langle \mathcal{O}(\mu)\rangle}{(-s)^{D/2}}$$
  - Perturbative contribution (D=0, calculated in the massless limit)



$+ \begin{pmatrix}\text{3-loop}\\ \alpha_s^2\end{pmatrix} + \begin{pmatrix}\text{4-loop}\\ \alpha_s^3\end{pmatrix} + \begin{pmatrix}\text{5-loop}\\ \alpha_s^4\end{pmatrix} + \cdots$

1-loop | 2-loop α_s | 2-loop α_s | 1992



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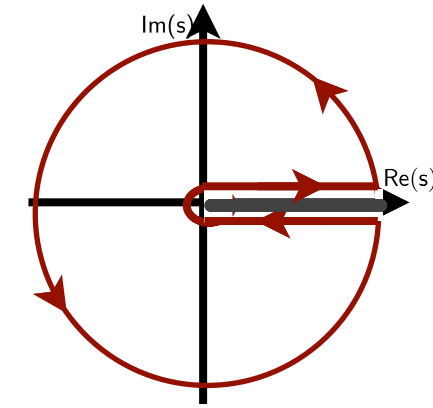

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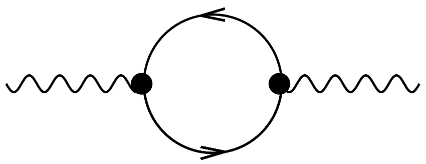
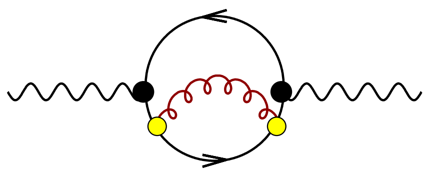
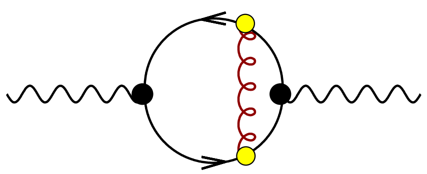
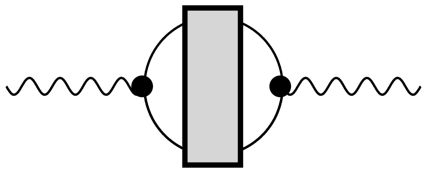
 +  +  $+\begin{pmatrix}\text{3-loop}\\ \alpha_s^2\end{pmatrix}+\begin{pmatrix}\text{4-loop}\\ \alpha_s^3\end{pmatrix}+\begin{pmatrix}\text{5-loop}\\ \alpha_s^4\end{pmatrix}+\cdots$

1-loop | 2-loop α_s | 2-loop α_s | 4-loop: 1992 | new 5-loop: 2008 (~20,000 diags.)

Baikov, Chetyrkin, and Kuhn, 2008

inclusive tau decays in a nut shell



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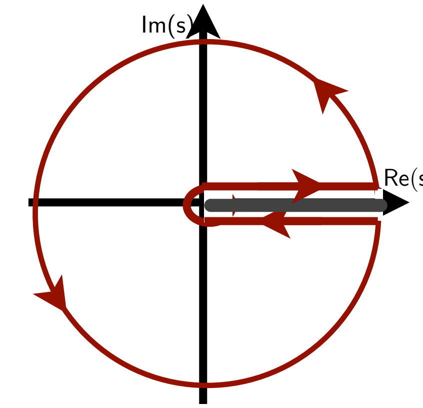

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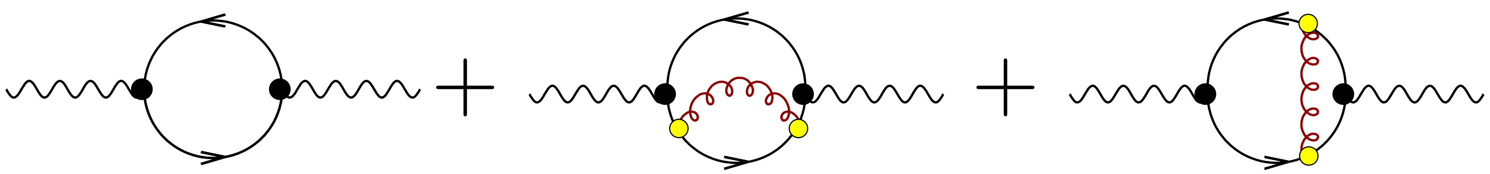


1-loop + 2-loop α_s + 2-loop α_s + (3-loop α_s^2) + (4-loop α_s^3) + (5-loop α_s^4) + $\cdots$

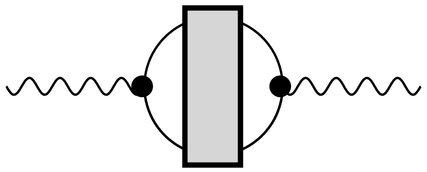
4-loop: 1992

5-loop: new, 2008 (~20,000 diags.)

Baikov, Chetyrkin, and Kuhn, 2008

Long-standing controversy: RG improvement (Contour Improved vs Fixed Order)

Pivovarov (1992); Pich and Le Diberder 1992; Jamin and Beneke, 2008; Caprini and Fischer 2009



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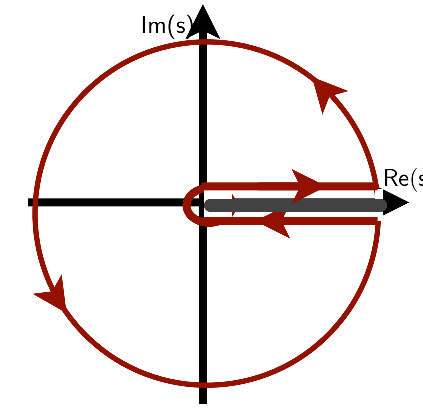

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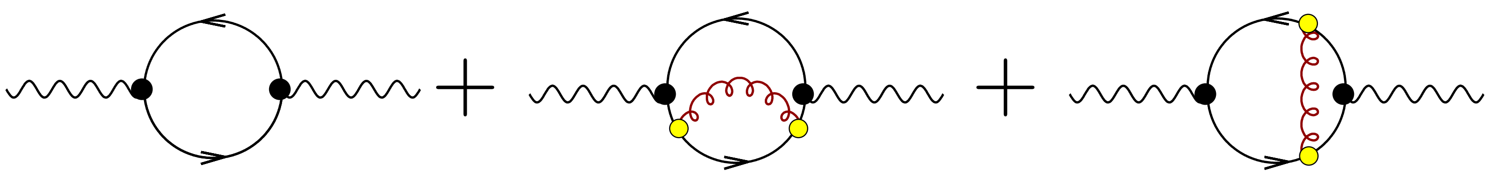

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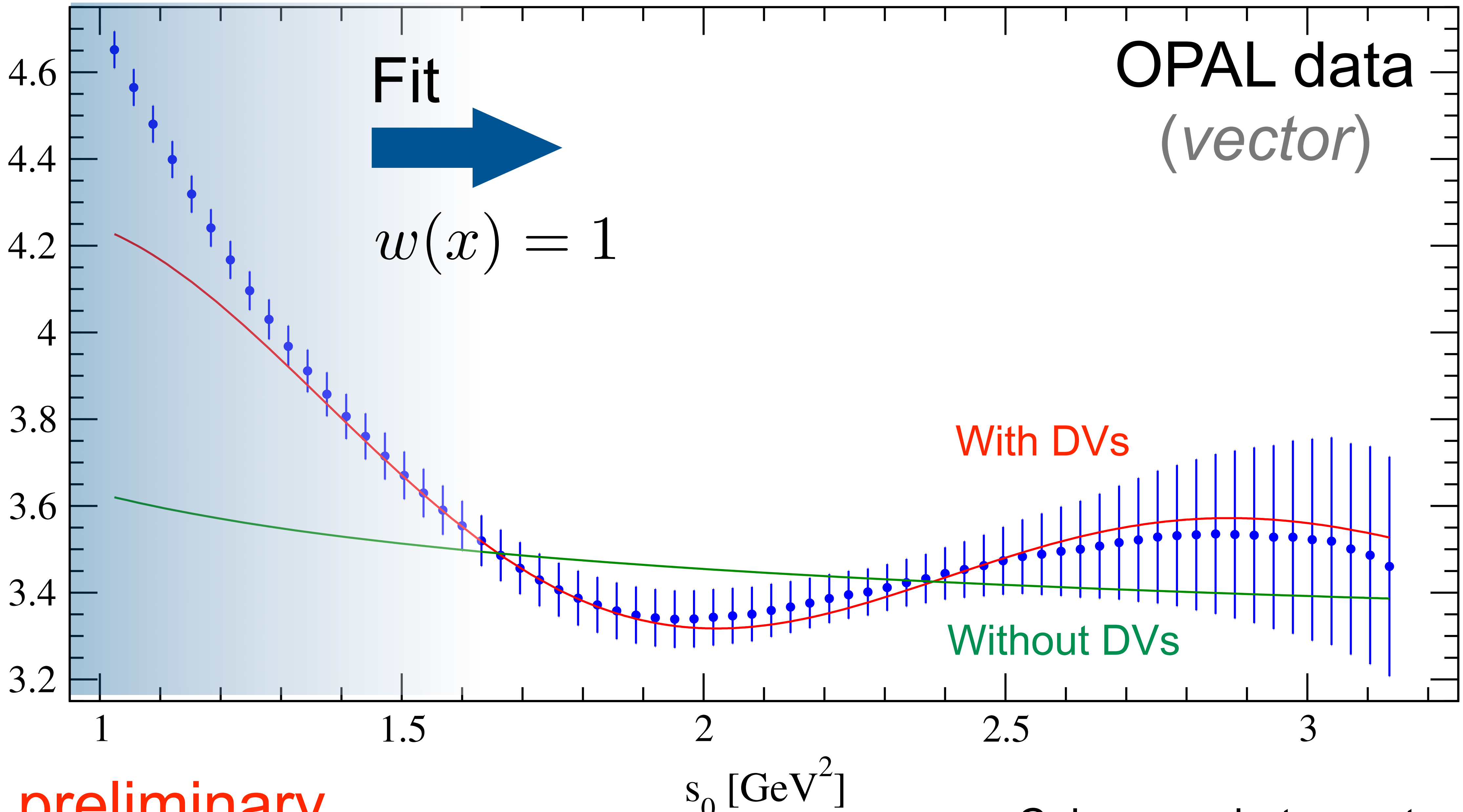
  - Higher dimensions in the OPE (mass corrections and QCD condensates)
Pich and Prades (1999)

- Duality Violations (DVs) [almost always disregarded]
Blok, Shifman, and Zhang (1998); Catà, Golterman, and Peris (2005)
  Ansatz with parameters fitted to data (for *V* and *A*)
  Catà, Golterman, and Peris (2009)
  Corrects the OPE near the real axis

$$\alpha_s(m_\tau^2)$$

- With DVs

- Fits to moments of OPAL data (problem with ALEPH correlations [see arXiv: 1011.4426])



preliminary

$$\alpha_s^{\rm FO}(m_\tau^2) = 0.307(18)_{\rm stat}(4)_{s_{\rm min}}(5)_{\alpha_s^5}$$

$$\alpha_s^{\rm CI}(m_\tau^2) = 0.322(25)_{\rm stat}(7)_{s_{\rm min}}(4)_{\alpha_s^5}$$

Coherence between truncation of the OPE and the weight function (thanks to DVs)

DB, Catà, Golterman, Jamin, Maltman, Osborne and Peris, arXiv:1103.4194

$$V_{us}$$

- Construct the following quantity

$$\delta R_\tau^{[w]} \equiv \frac{R_{\tau,V+A}^{[w]}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{[w]}}{|V_{us}|^2}$$

Gámiz et. al. PRL (2005); JHEP (2003)

that vanishes in SU(3) limit (no perturbative contribution). Contributions coming from quark mass differences. One has (taking m_s from other measurements):

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,V+A}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

Tension (~3σ) among results from tau and kaon decays. Reason?

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Tension (~3σ) among results from tau and kaon decays. Reason?

- $R_{\tau,V+A} = 3.479(11)$, V_{ud} is very well known: $\sigma\left(\frac{R_{\tau,V+A}}{|V_{ud}|^2}\right) \sim 0.3\%$

- $R_{\tau,S} = 0.1615(40)$ Dominant uncertainty (result with input from *b*-factories). Davier *et al* EPJ.C **56** (2008)

  Smaller value leads to smaller V_{us} . *b*-factories branching ratios systematically smaller.

  Pich (Tau2010), Maltman (Tau2010), Lusiani (ICHEP 2010)

- $\delta R_{\tau,\mathrm{th}} = 0.216(16)$ Small impact on the final uncertainty of V_{us}

- Stability with respect to s_0 ? Maltman PLB (2009); Maltman and Wolfe, PLB (2006)

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[see M. Antonelli's talk]

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$$\tau \to K_S^0 \pi^- \nu_\tau$$

- Spectra from b-factories

D. Epifanov et. al., PL **B65** (2007)

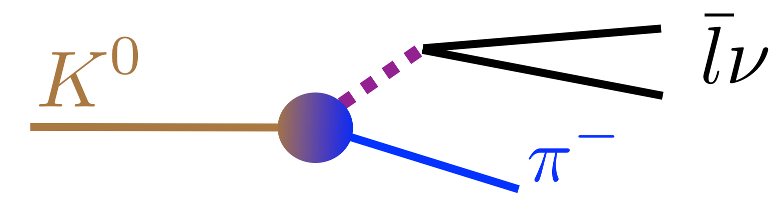
Still not published (presented in conferences e.g. Tau2010 Manchester)

- Results from our fits to Belle data

DB, Escribano, and Jamin, EPJC 59 (2009)

DB, Escribano, and Jamin, JHEP 09 (2010)

- K_{l3}: the main route towards $|V_{us}|$
  Leutwyler and Roos 1984



- Form factors: Parametrization in terms of f_+ and f_0

$$\langle \pi^-(k)|\bar{s}\gamma^\mu u|K^0(p)\rangle = \left[(p+k)^\mu - \frac{\Delta_{K\pi}}{t}(p-k)^\mu\right] f_+(t) + (p-k)^\mu \frac{\Delta_{K\pi}}{t} f_0(t)$$

vector   scalar

$$f_+(0) = f_0(0)$$

$$\Gamma_{K_{l3}} \propto |V_{us}|^2 |f_+(0)|^2 I(K_{l3})$$

$f_{+,0}(0)$ → Lattice

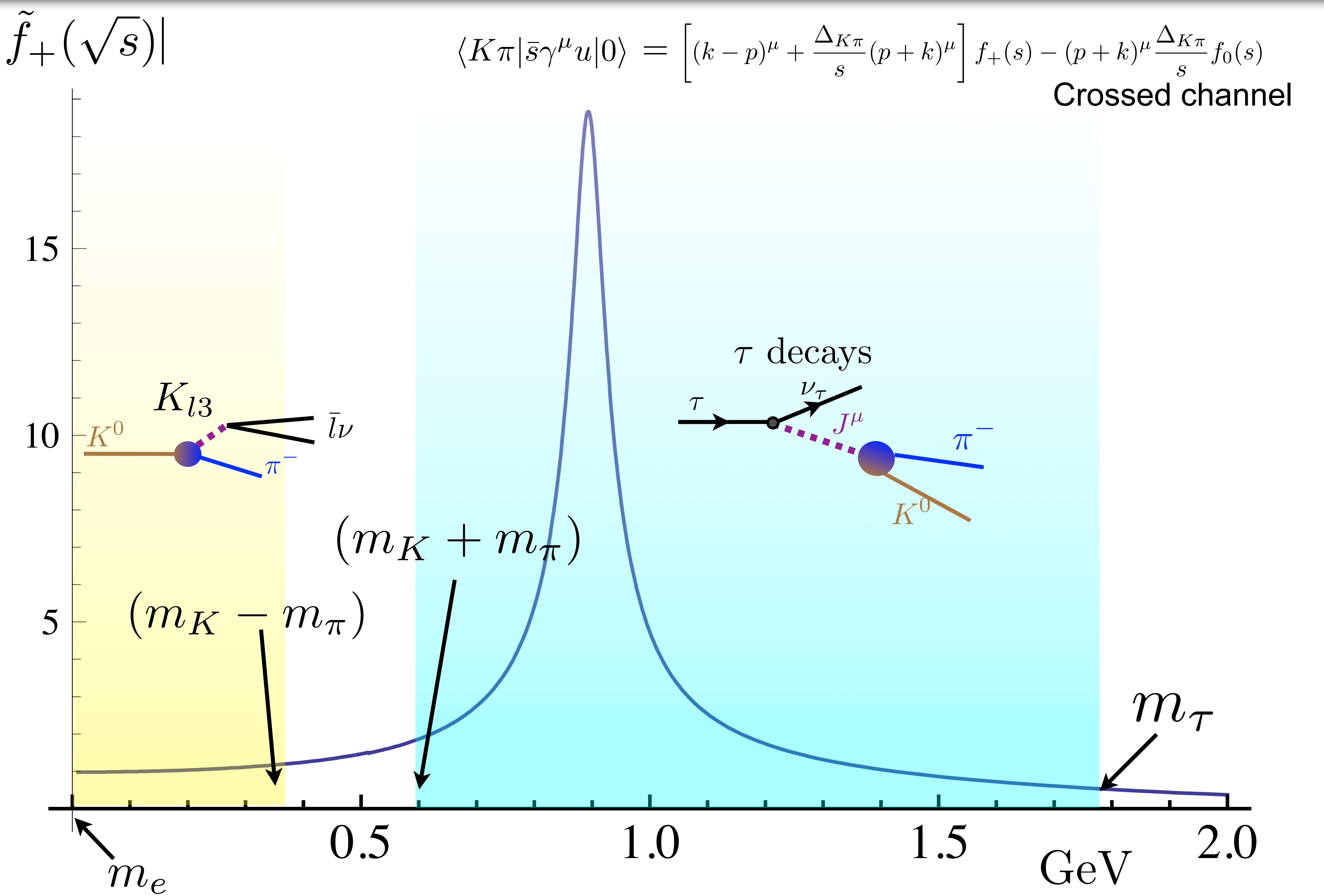
$\tilde{f}_{+,0}(s)$ → (R)ChPT, DR, Latt.

$\tilde{f}_+(t) = f_+(t)/f_+(0)$ → Energy dependence

- Phase space integrals

$$I(K_{l3}) = \frac{1}{m_K^8}\int_{m_l^2} dt\lambda^{3/2}(t)\,(\text{p.s.})\left[\tilde{f}_+^2(t) + \eta(t,m_i)\tilde{f}_0(t)^2\right]$$

$$\langle K\pi|\bar{s}\gamma^\mu u|0\rangle = \left[(k-p)^\mu + \frac{\Delta_{K\pi}}{s}(p+k)^\mu\right] f_+(s) - (p+k)^\mu \frac{\Delta_{K\pi}}{s} f_0(s)$$



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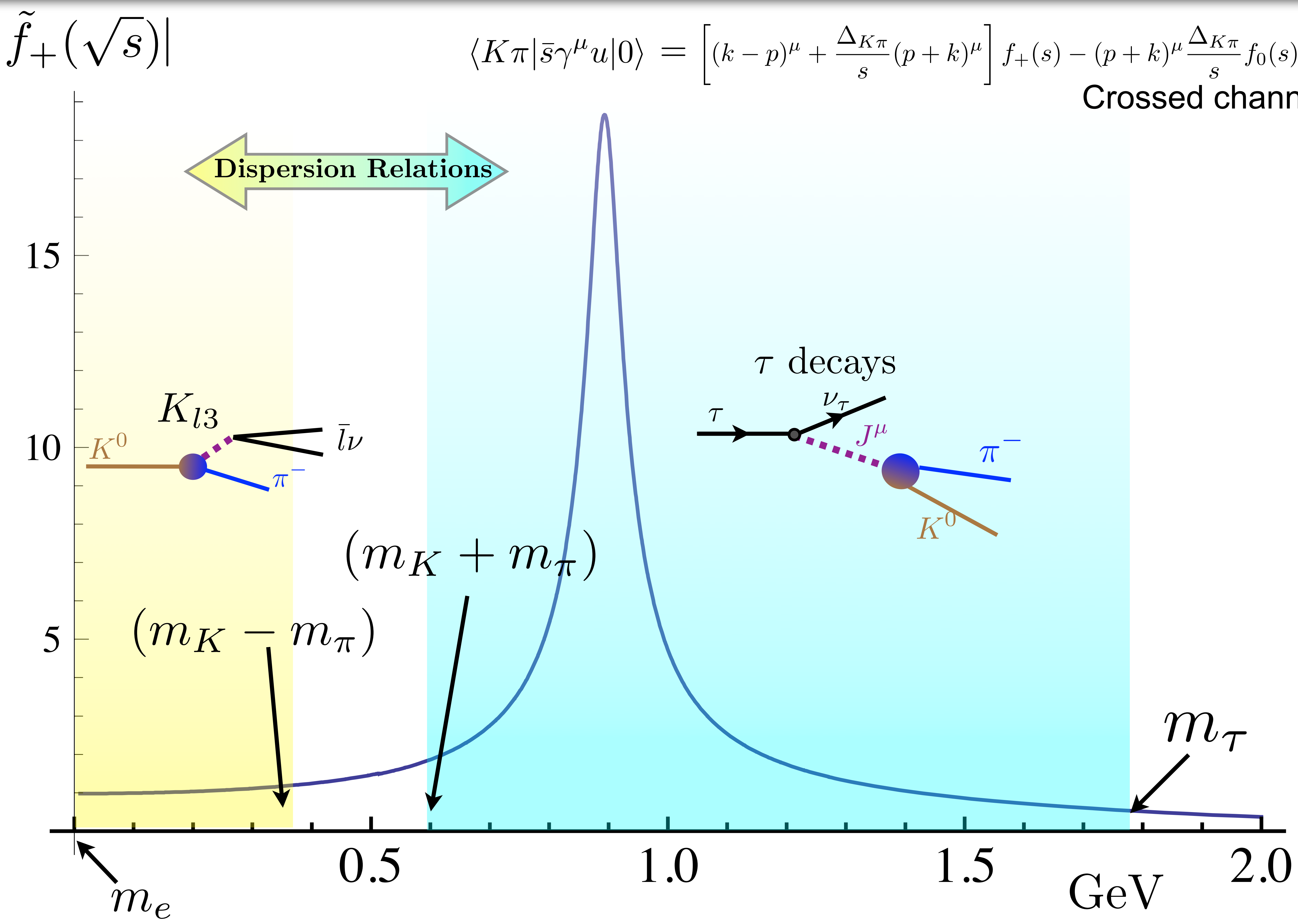
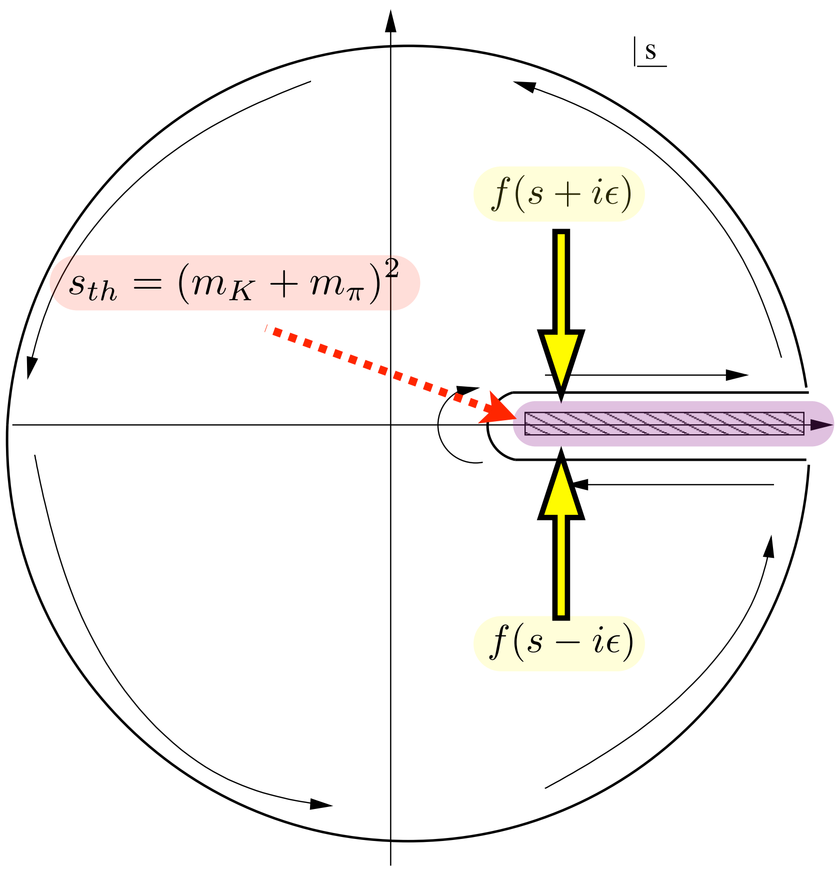
Crossed channel



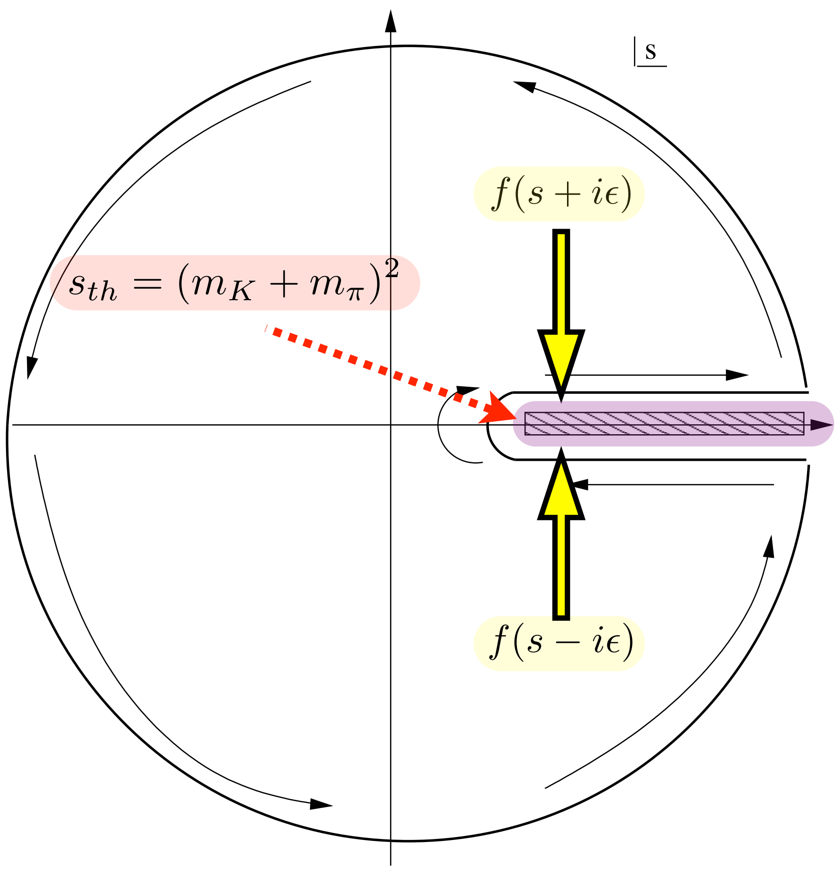


DB, Escribano, and Jamin, EPJC 59 (2009)

see also the works by Bernard, Oertel, Passemar, and Stern

Description of $f_+(s)$ with three subtractions

$$\tilde{f}_+(s) = \exp\left[\alpha_1 \frac{s}{m_\pi^2} + \frac{1}{2}\alpha_2 \frac{s^2}{m_\pi^4} + \frac{s^3}{\pi}\int_{s_{\rm th}}^{s_{\rm cut}} \frac{ds'}{s'^3}\frac{\delta(s')}{s'-s-i\epsilon}\right]$$



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Fit

Cut-off to check the stability

Pich and Portolés (2001)

$$\lambda'_+ = \alpha_1 \qquad \lambda''_+ = \alpha_2 + \alpha_1^2$$

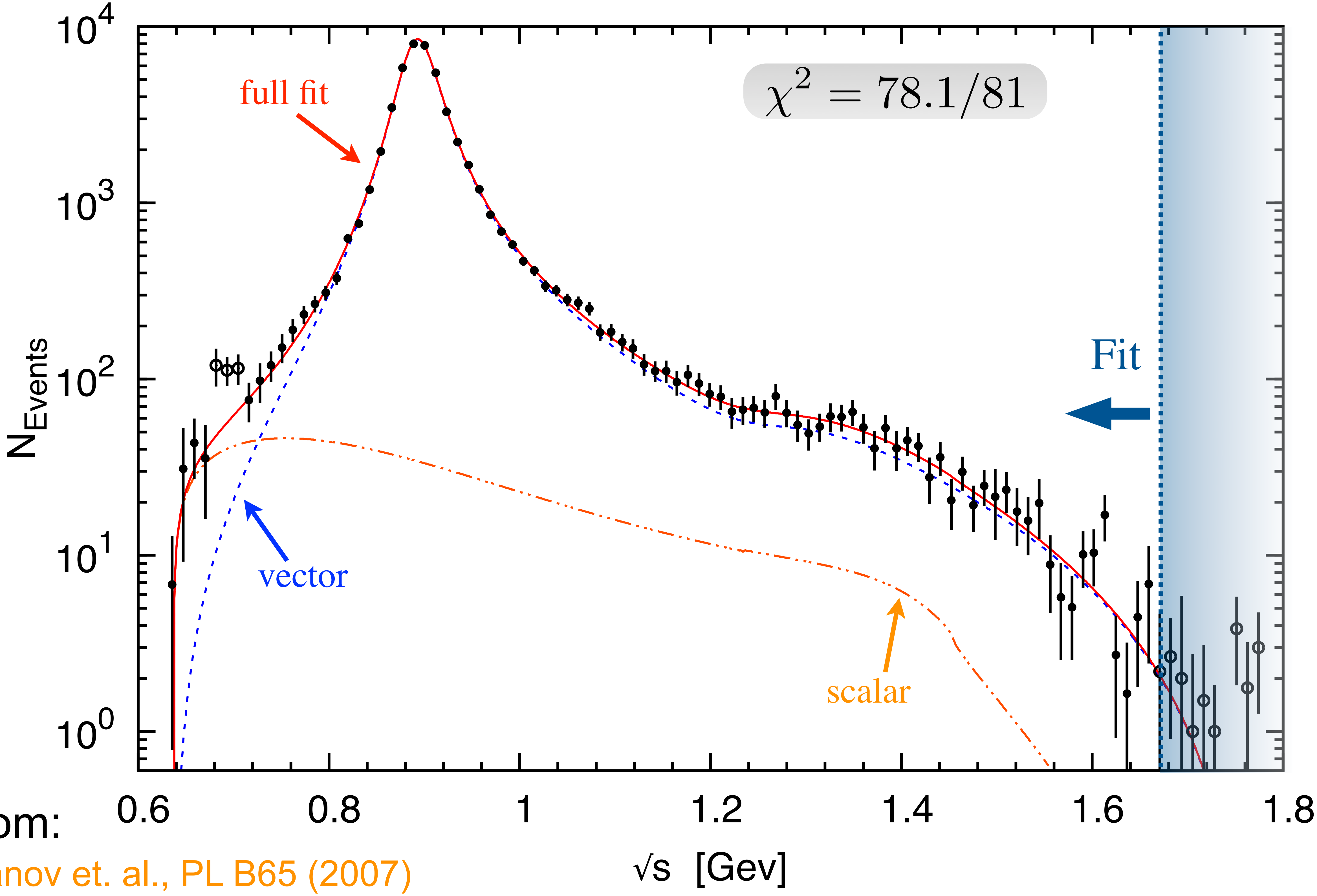
$$\tilde{f}_+(t) = 1 + \frac{\lambda'_+}{m_\pi^2}t + \frac{1}{2}\frac{\lambda''_+}{m_\pi^4}t^2 + \mathcal{O}(t^3)$$

- We employ a phase with two resonances
- Parameters of the fit: $\lambda'_+, \lambda''_+, m_1, \Gamma_1, m_2, \Gamma_2, \gamma$

Taylor coefficients (λ'_+, λ''_+) — Resonance parameters ($m_1, \Gamma_1, m_2, \Gamma_2, \gamma$)

DB, Escribano, and Jamin, JHEP 09 (2010)

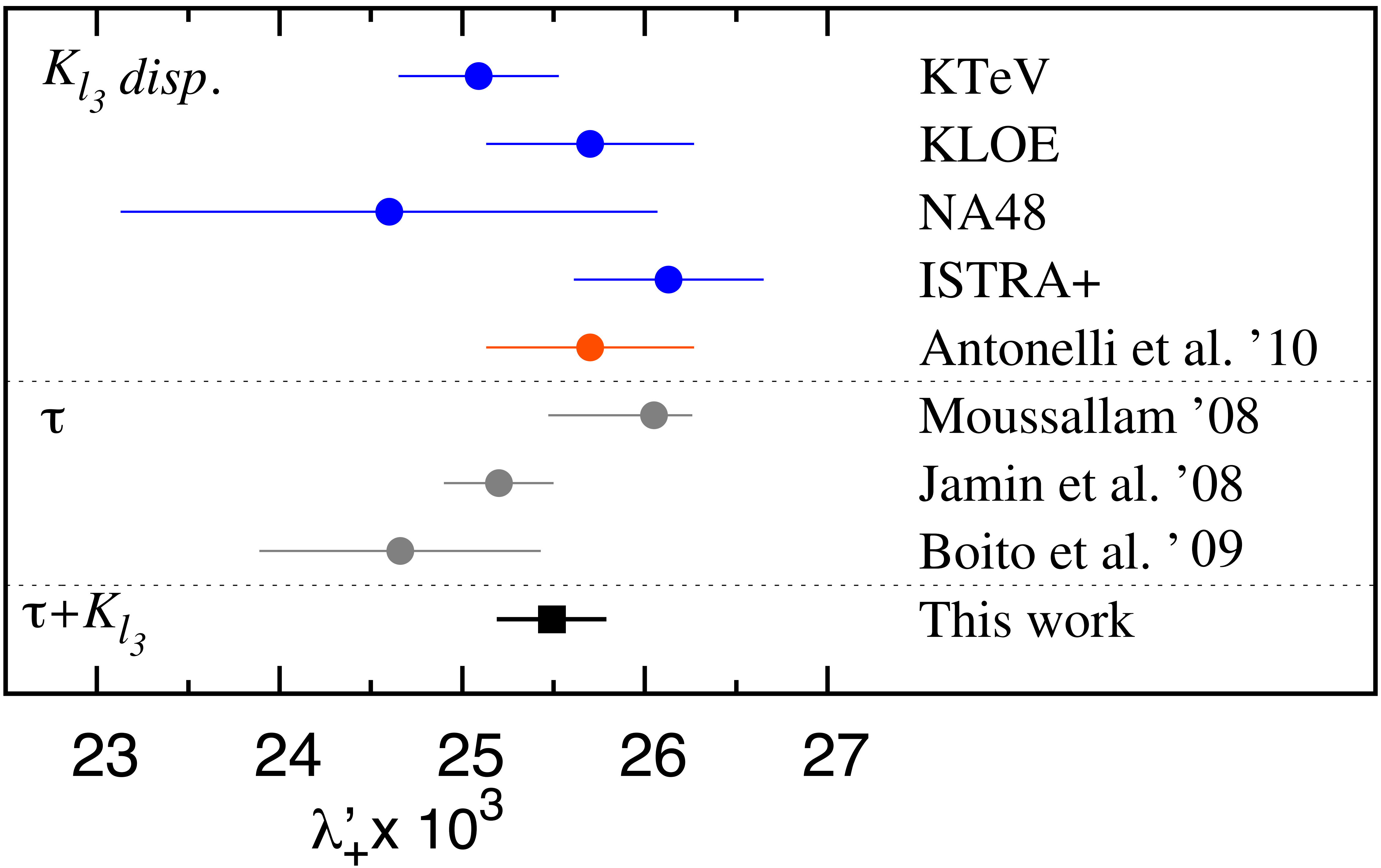
fit with constrains from K_{l3} decays *[see M. Antonelli's talk]*



Data from:

D. Epifanov et. al., PL B65 (2007)

$$\lambda'_+ \times 10^3 = 25.49 \pm (0.30)_{\rm stat} \pm (0.06)_{s_{\rm cut}}$$

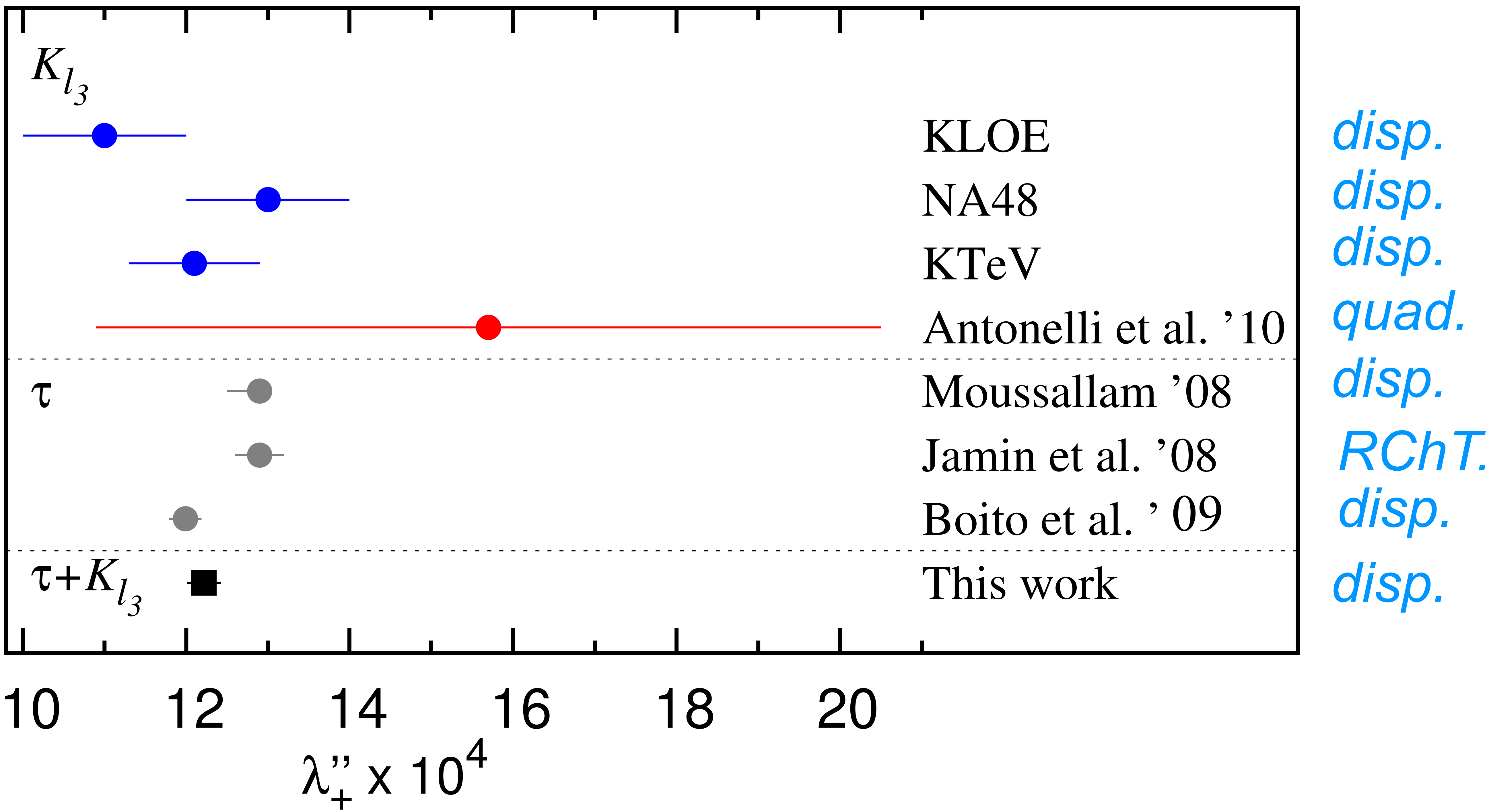


Model

disp.
disp.
disp.
disp.
quad.
RChT.
disp.
disp.
disp.

This work =DB, Escribano, and Jamin, JHEP 09(2010)

$$\lambda''_+ \times 10^4 \quad = \quad 12.22 \pm (0.10)_{\rm stat} \pm (0.10)_{s_{\rm cut}}$$



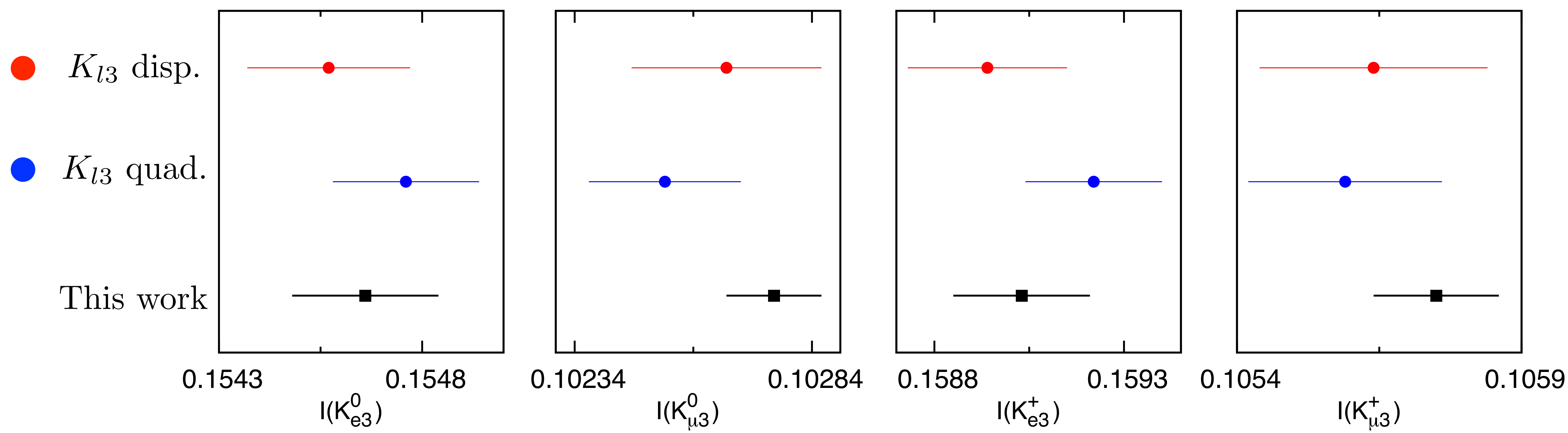
phase-space integrals (needed for V_{us} extraction from K_{l3})

$$I_{K_{l_3}} = \frac{1}{m_K^2} \int_{m_l^2}^{(m_K - m_\pi)^2} \mathrm{d}t\, \lambda(t)^{3/2} \left(1 + \frac{m_l^2}{2t}\right) \left(1 - \frac{m_l^2}{t}\right)^2 \left(|\tilde{f}_+(t)|^2 + \frac{3\, m_l^2 (m_K^2 - m_\pi^2)^2}{(2t + m_l^2)\, m_K^4\, \lambda(t)} |\tilde{f}_0(t)|^2\right)$$

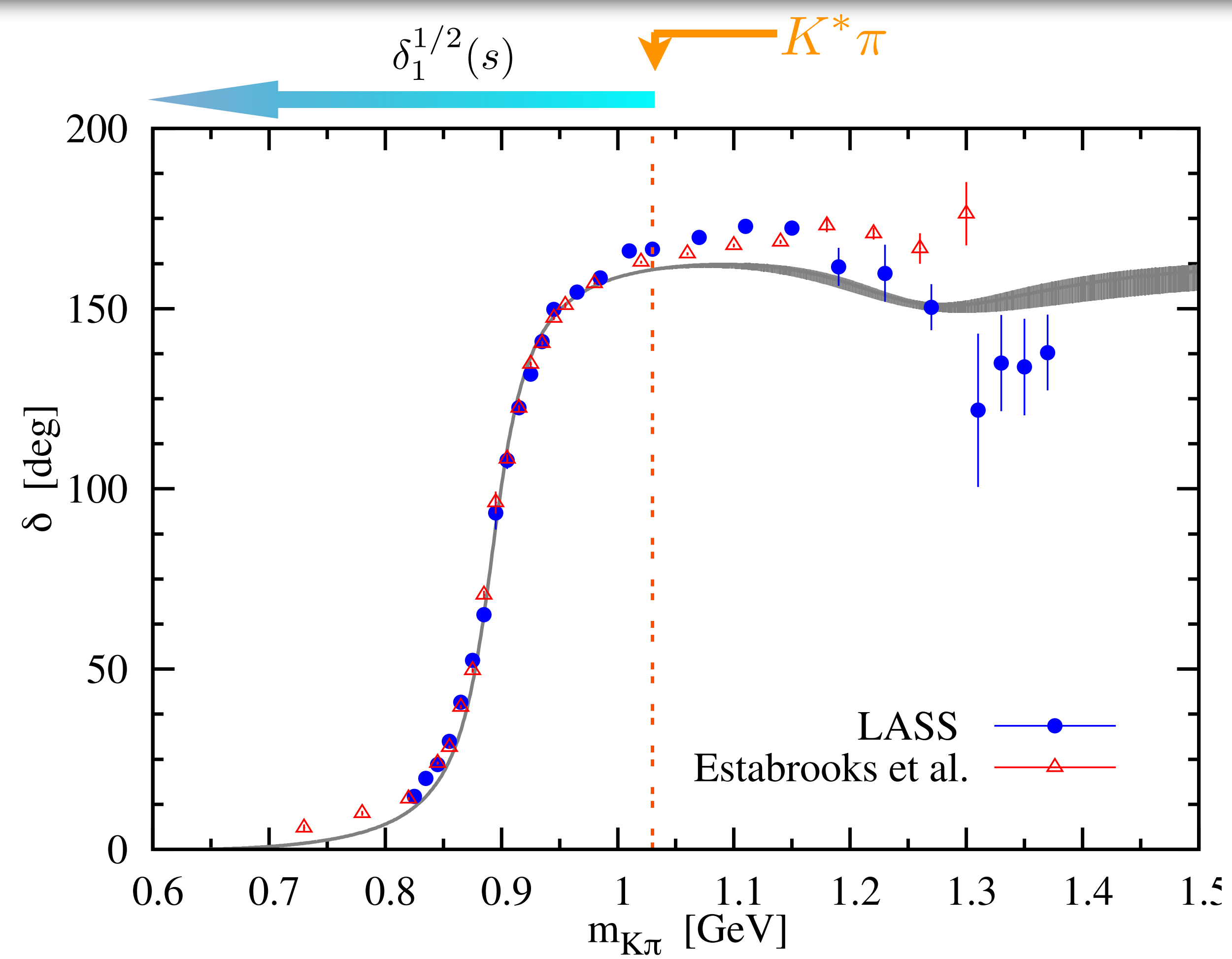
$$\lambda(t) = 1 + t^2/m_K^4 + r_\pi^4 - 2\, r_\pi^2 - 2\, r_\pi^2\, t/m_K^2 - 2\, t/m_K^2$$

	This Work	K_{l_3} disp.	K_{l_3} quad.
$I_{K^0_{e3}}$	0.15466(18)	0.15476(18)	0.15457(20)
$I_{K^0_{\mu 3}}$	0.10276(10)	0.10253(16)	0.10266(20)
$I_{K^+_{e3}}$	0.15903(18)	0.15922(18)	0.15894(21)
$I_{K^+_{\mu 3}}$	0.10575(11)	0.10559(17)	0.10564(20)

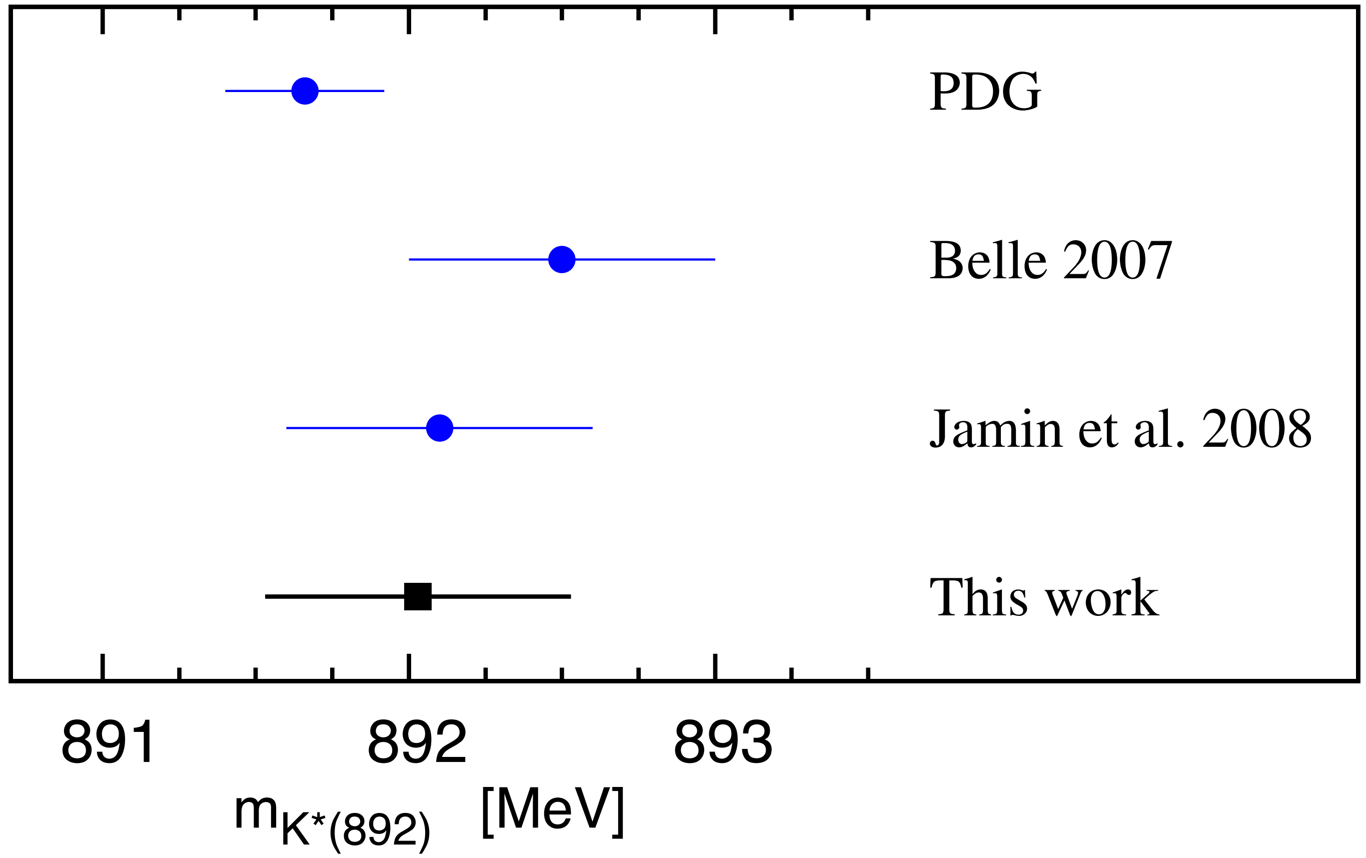
Average of K_{l3} analyses from Antonelli et al., EPJC (2010)



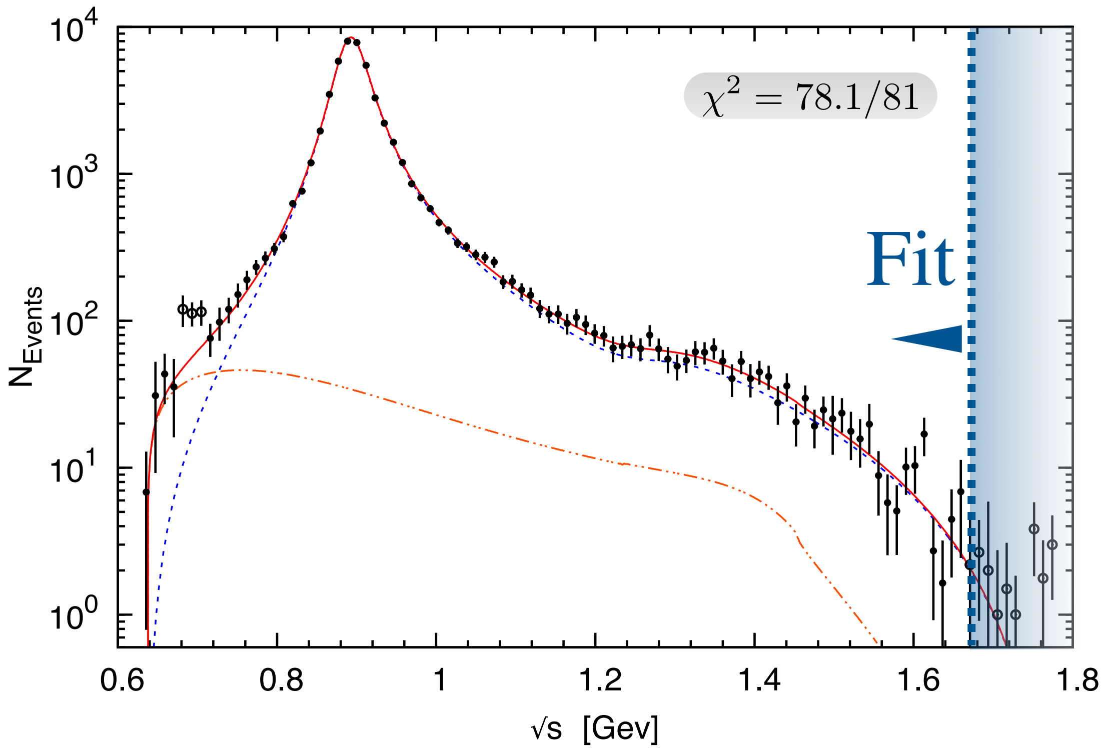
results on $K\pi$ dynamics



$m_{K^*(892)^\pm} =$
$892.03 \pm (0.19)_{\rm stat} \pm (0.44)_{\rm sys}$



- *In principle, one can obtain* V_{us} *from the fit to* $\tau \to K_S^0 \pi^- \nu_\tau$
- *With Belle spectrum the uncertainty is too large*
- *We fix in the fit* $f_+(0)^2 |V_{ud}|^2$



conclusion

- With hadronic tau decay data from *b*-factories one can improve
  - V_{us} (direct and indirect through form factors)
  - m_s (not covered here)
  - α_s
  - $K\pi$ dynamics (resonance masses, phase shifts, threshold parameters...)