

XVII SuperB Workshop and Kick Off Meeting



$\gamma\gamma$ Physics: on-going activities and Super-B



Federico Nguyen - *INFN Roma TRE*

La Biodola (LI) - May, 30th 2011

An outline (from experience at low energy...)

- main motivations and recent results:

where to improve...

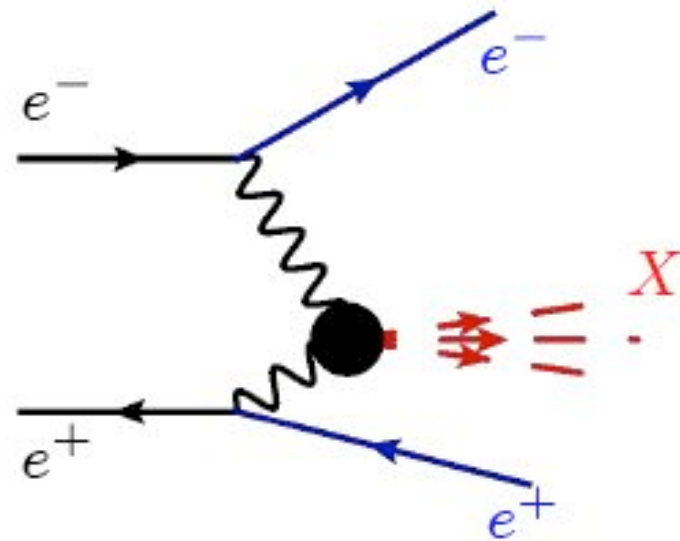
- current activities: KLOE

as an example

- opportunities @ Super-B

- conclusions

$$e^+e^- \rightarrow e^+e^- \gamma^* \gamma^* \rightarrow e^+e^- X$$



Recent results with PS mesons

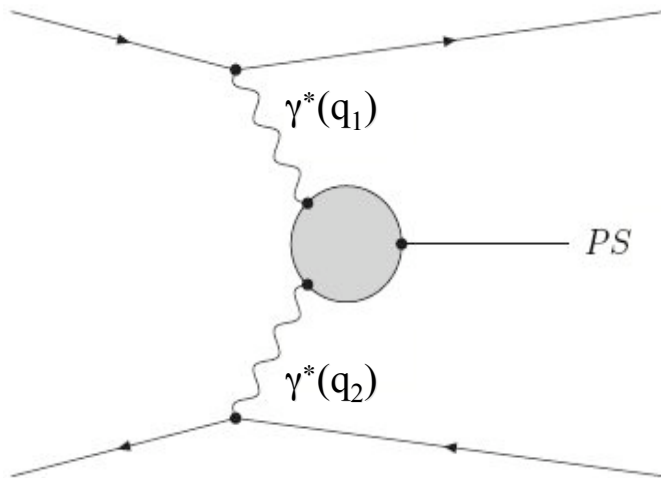
PseudoScalar mesons: $\gamma\gamma$ widths

$$N_{e^+e^- \rightarrow e^+e^- X} = L_{ee} \int \frac{dF}{dW_{\gamma\gamma}} \sigma_{\gamma\gamma \rightarrow X}(W_{\gamma\gamma}) dW_{\gamma\gamma}$$

for narrow pseudoscalar mesons (e.g. π^0 , η , η' , $\eta_c(1S)$, etc...):

$$\sigma_{\gamma\gamma \rightarrow X}(q_1, q_2) \propto \boxed{\Gamma_{X \rightarrow \gamma\gamma}} \frac{8\pi^2}{M_X} \delta((q_1 + q_2)^2 - M_X^2) |F(q_1^2, q_2^2)|^2$$

absolute measurement: either your decay channel is $X \rightarrow \gamma\gamma$ or must know $BR(X \rightarrow f)$... often the limiting factor



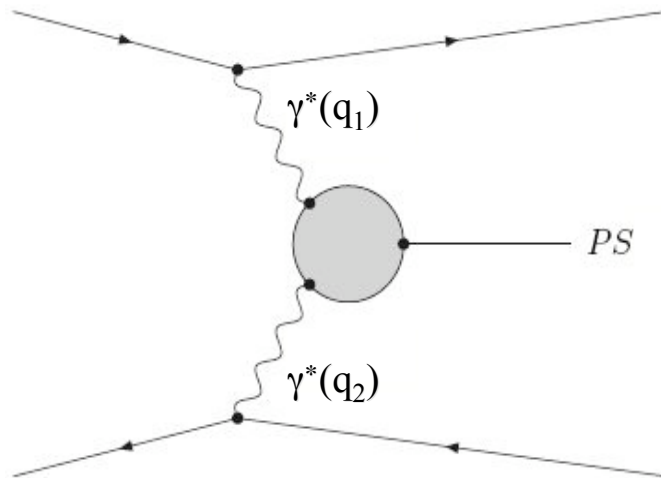
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absolute measurement: either your decay channel is $X \rightarrow \gamma\gamma$ or must know $BR(X \rightarrow f)$... often the limiting factor



spectrum measurement, as a function of a single momentum transfer, fixing or integrating over the other one
2-dim PDF not yet measured

PS mixing angle and the gluonium in η'

$$\frac{\Gamma(\eta \rightarrow \gamma\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = \left(\frac{m_\eta}{m_{\pi^0}}\right)^3 \frac{1}{9} \left(5 \cos \varphi_P - \sqrt{2} \frac{f_n}{f_s} \sin \varphi_P\right)^2$$

$$\frac{\Gamma(\eta' \rightarrow \gamma\gamma)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)} = \left(\frac{m'_{\eta}}{m_{\pi^0}}\right)^3 \frac{1}{9} \left(5 \sin \varphi_P + \sqrt{2} \frac{f_n}{f_s} \cos \varphi_P\right)^2 \cos^2 \phi_G$$

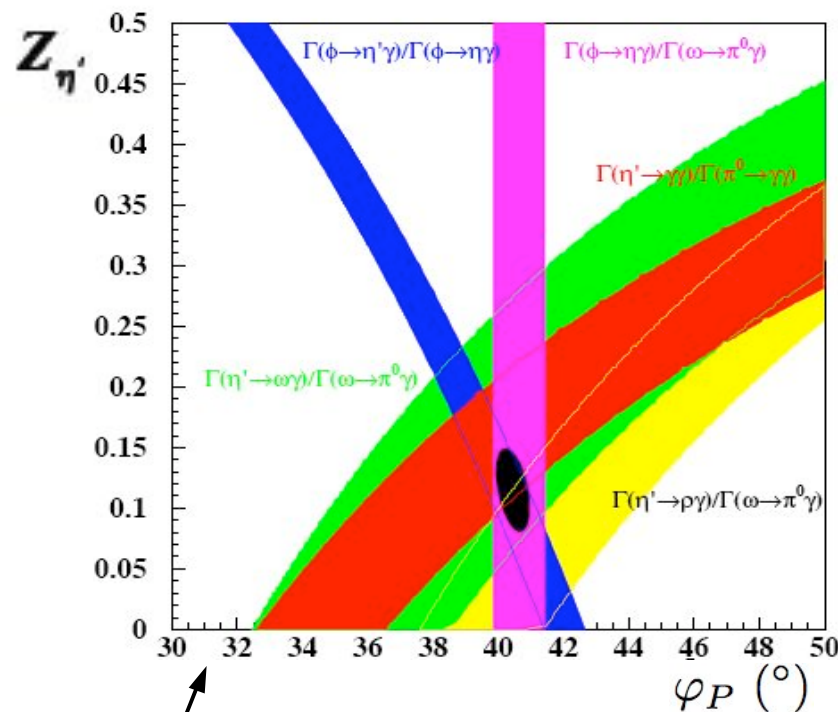
$$|\eta'\rangle = X_{\eta'} \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle + Y_{\eta'} |s\bar{s}\rangle + Z_{\eta'} |glue\rangle$$

$$|\eta\rangle = \cos \varphi_P \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle - \sin \varphi_P |s\bar{s}\rangle$$

$$X_{\eta'} = \cos \phi_G \sin \varphi_P$$

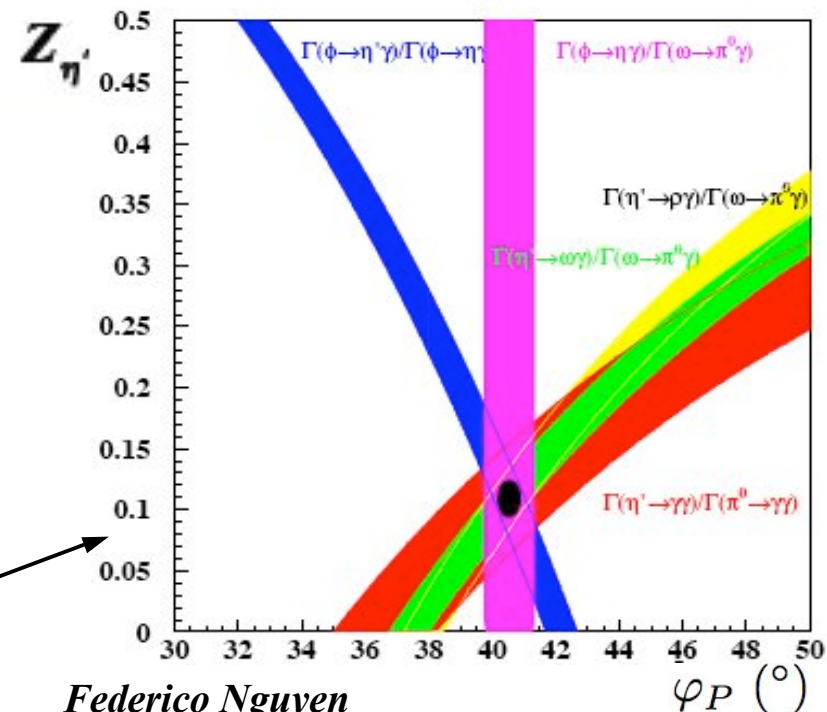
$$Y_{\eta'} = \cos \phi_G \cos \varphi_P$$

$$Z_{\eta'} = \sin \phi_G$$



present status

with dominant η' BR's to 1%



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30-05-2011

PS form factors: from models to the $(g-2)_\mu$ saga

important to test phenomenological
models, more or less QCD/ChPT
inspired..., but impacts also the $(g-2)_\mu$

e.g.

$$F(k_1^2, k_2^2) = \frac{m_\rho^2}{(m_\rho^2 - k_1^2 - k_2^2)}$$

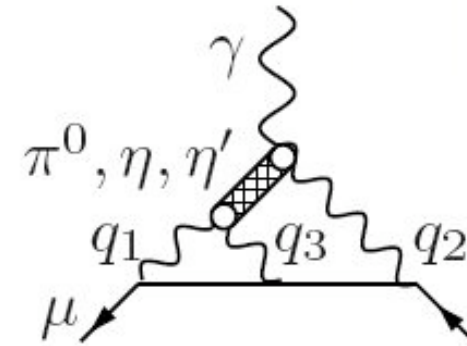
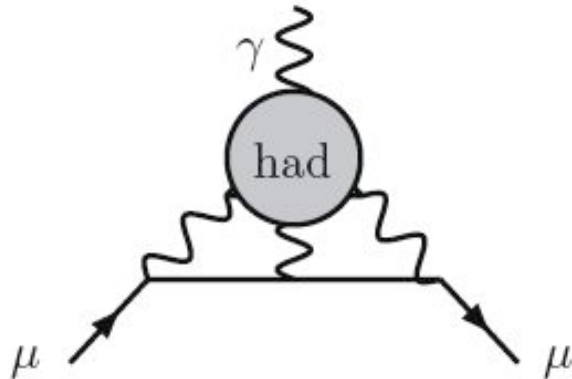
$$F(k_1^2, k_2^2) = \frac{m_\rho^4 - \frac{4\pi^2 F_\pi^2}{N_c} (k_1^2 + k_2^2)}{(m_\rho^2 - k_1^2)(m_\rho^2 - k_2^2)}$$

from *F.Jegerlehner & A.Nyffeler, Phys. Rept, 477(2009)1*

Standard model theory and experiment comparison [in units 10^{-11}].

Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116584718.1	0.2
Leading hadronic vacuum polarization	6903.0	52.6
Subleading hadronic vacuum polarization	-100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116591790.0	64.6
Experiment	116592080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3

PS transition form factors: L-by-L



$$\mathcal{F}_{\pi^0 \gamma^* \gamma^*}((q_1 + q_2)^2, q_1^2, q_2^2)$$

Contribution	N/JN
π^0, η, η'	99 ± 16
π, K loops	-19 ± 13
π, K loops + other subleading in N_c	—
axial vectors	22 ± 5
scalars	-7 ± 2
quark loops	21 ± 3
total	116 ± 39

$$a_\mu^{\text{LbL;had}} \times 10^{11}$$

- not clear how to constrain contributions from data
- pseudoscalar pole contribution dominates, many theory approaches, perhaps a cleaner case w/ only 2 independent scales, $F(m_{\text{PS}}^2, q_1^2, q_2^2)$

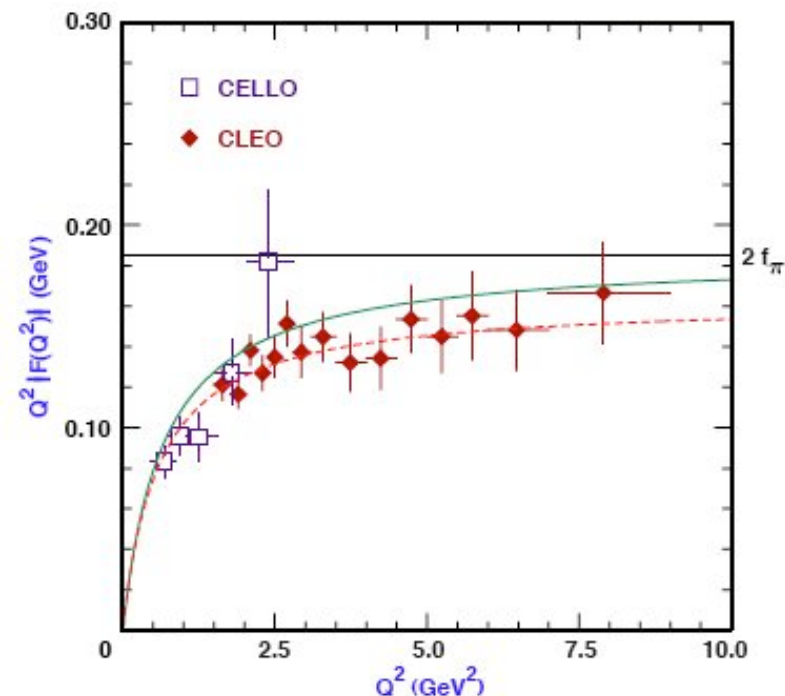
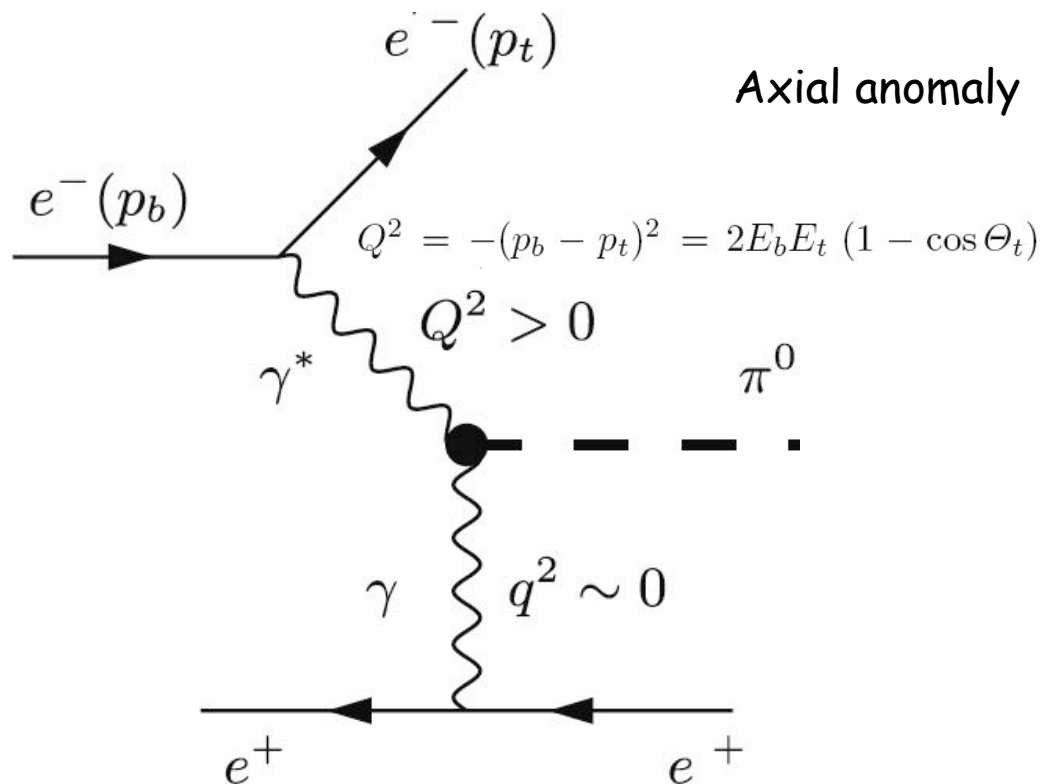
An example: π^0 transition form factor

Brodsky-Lepage

$$\lim_{Q^2 \rightarrow \infty} \mathcal{F}_{\pi^0 \gamma^* \gamma}(m_\pi^2, -Q^2, 0) \sim \frac{2F_\pi}{Q^2}$$

Axial anomaly

$$\lim_{Q^2 \rightarrow 0} \mathcal{F}_{\pi^0 \gamma^* \gamma}(m_\pi^2, -Q^2, 0) = \frac{1}{4\pi^2 F_\pi}$$



well known asymptotic limits from 1st principles,
how to interpolate? what about η , η' ?

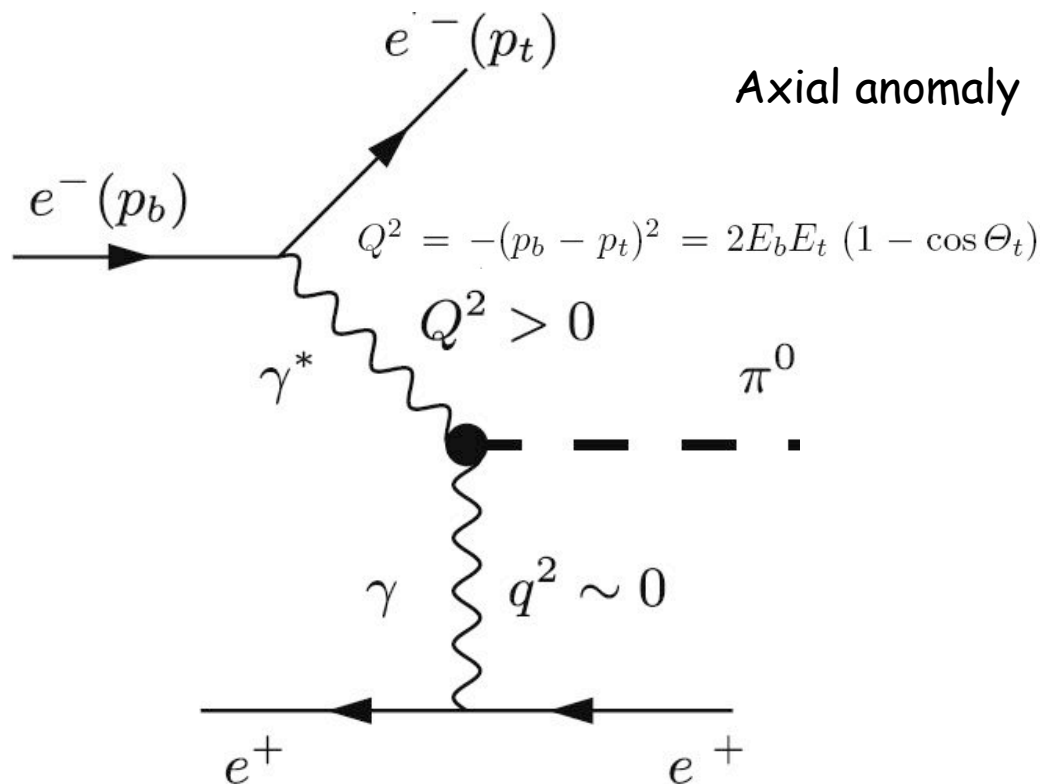
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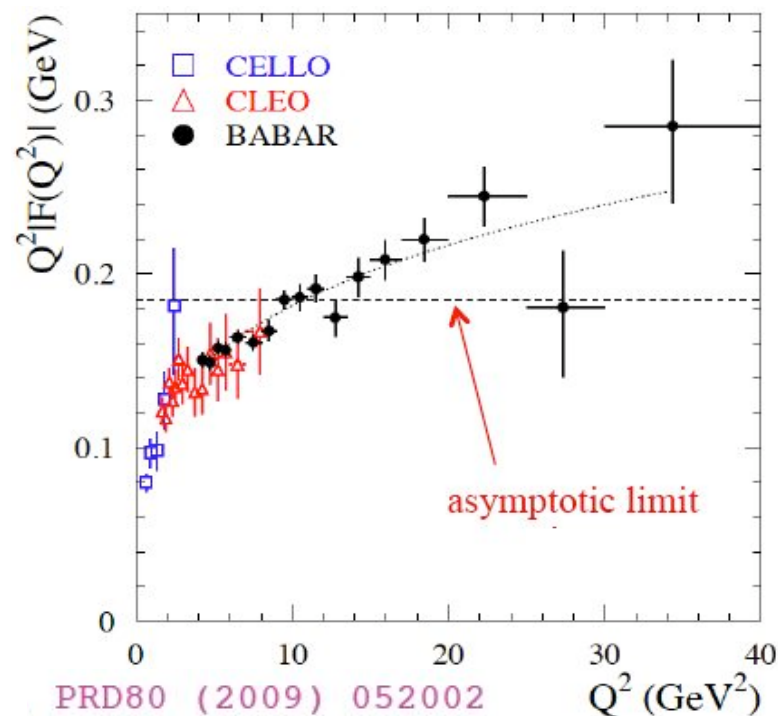
$$\lim_{Q^2 \rightarrow \infty} \mathcal{F}_{\pi^0 \gamma^* \gamma}(m_\pi^2, -Q^2, 0) \sim \frac{2F_\pi}{Q^2}$$

Axial anomaly

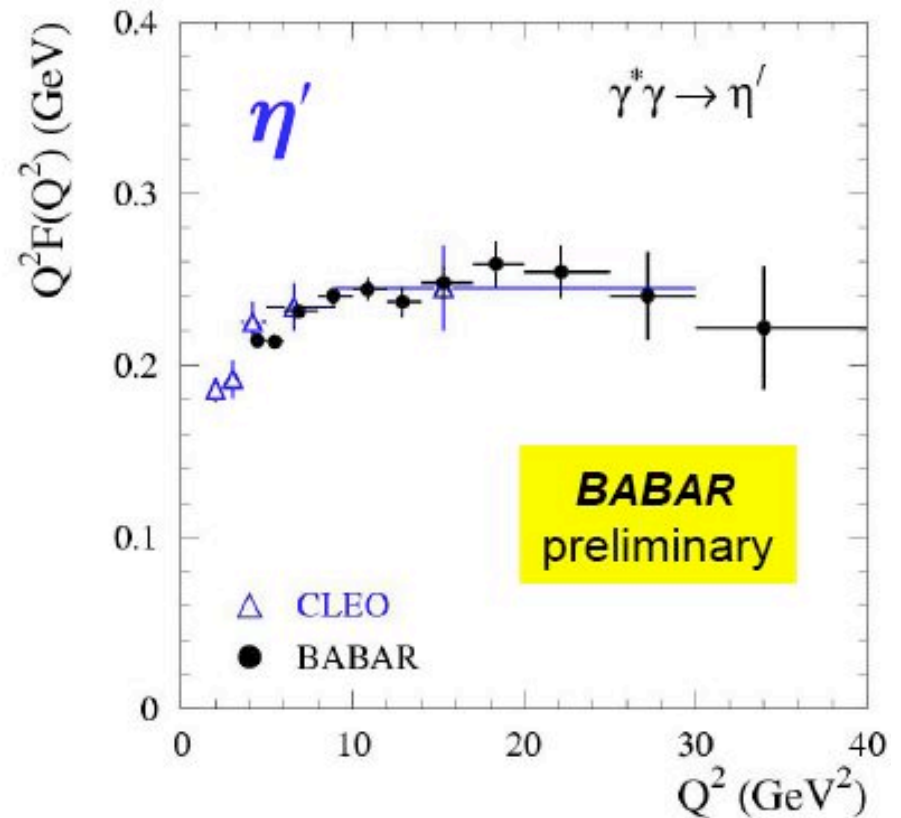
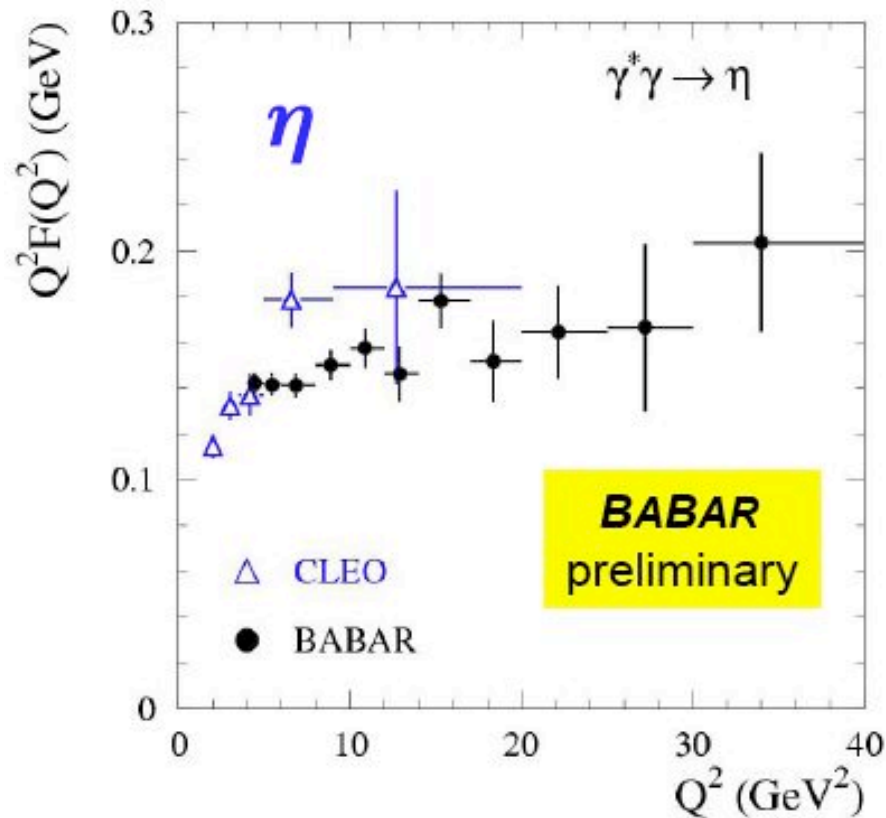
$$\lim_{Q^2 \rightarrow 0} \mathcal{F}_{\pi^0 \gamma^* \gamma}(m_\pi^2, -Q^2, 0) = \frac{1}{4\pi^2 F_\pi}$$



no longer known asymptotic limits? anything missing? what about η , η' ?



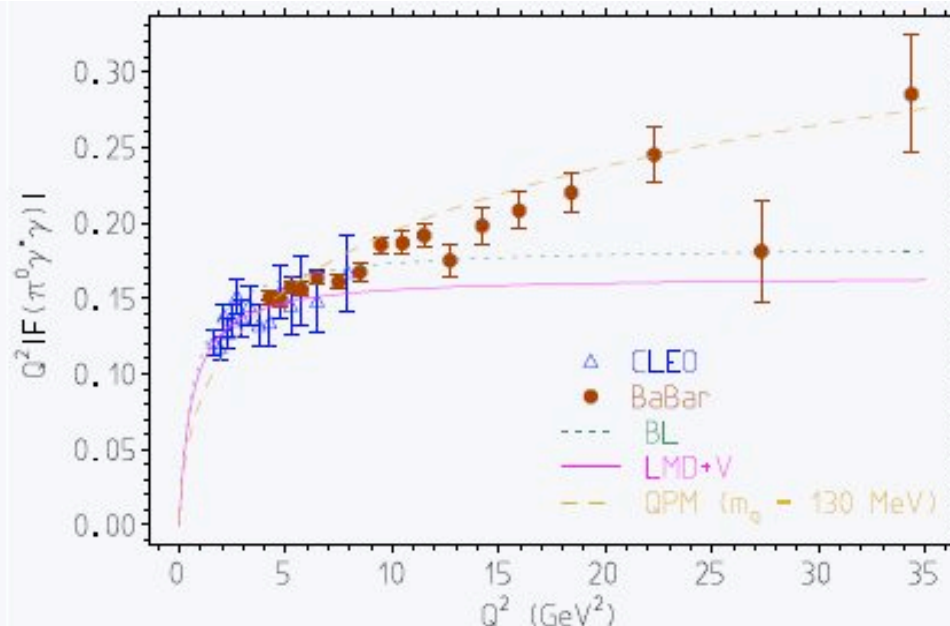
Measuring η and η' does not clarify



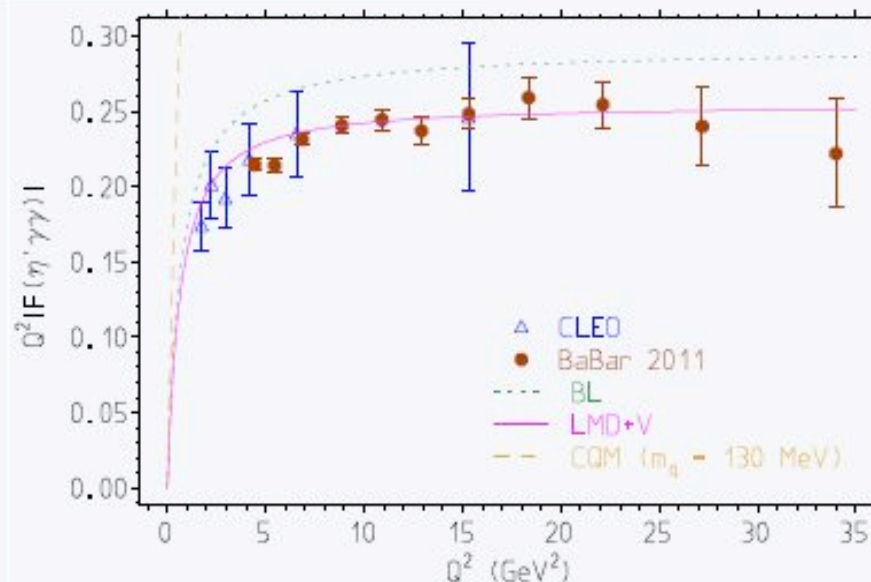
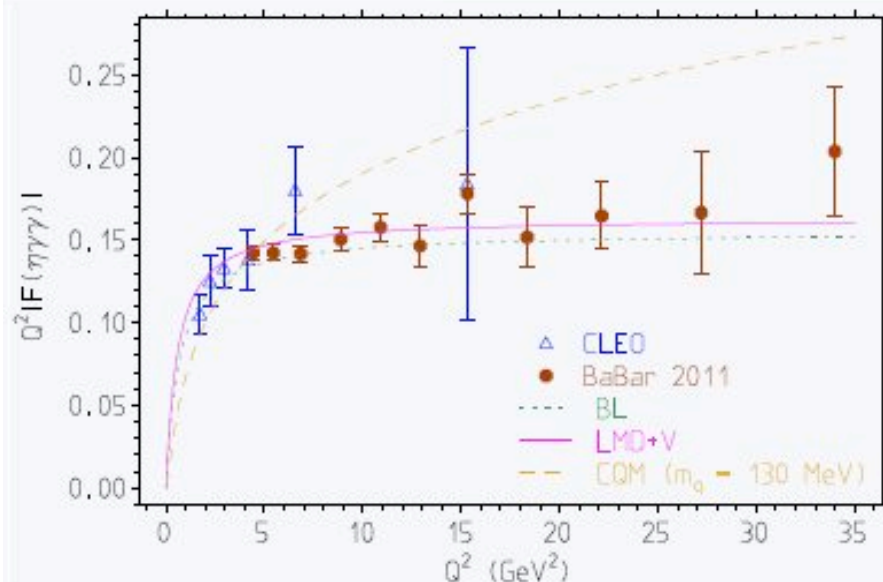
good agreement with CLEO in the overlapping regions, but...

Measuring η and η' does not clarify

...with
different
asymptotic
behaviour

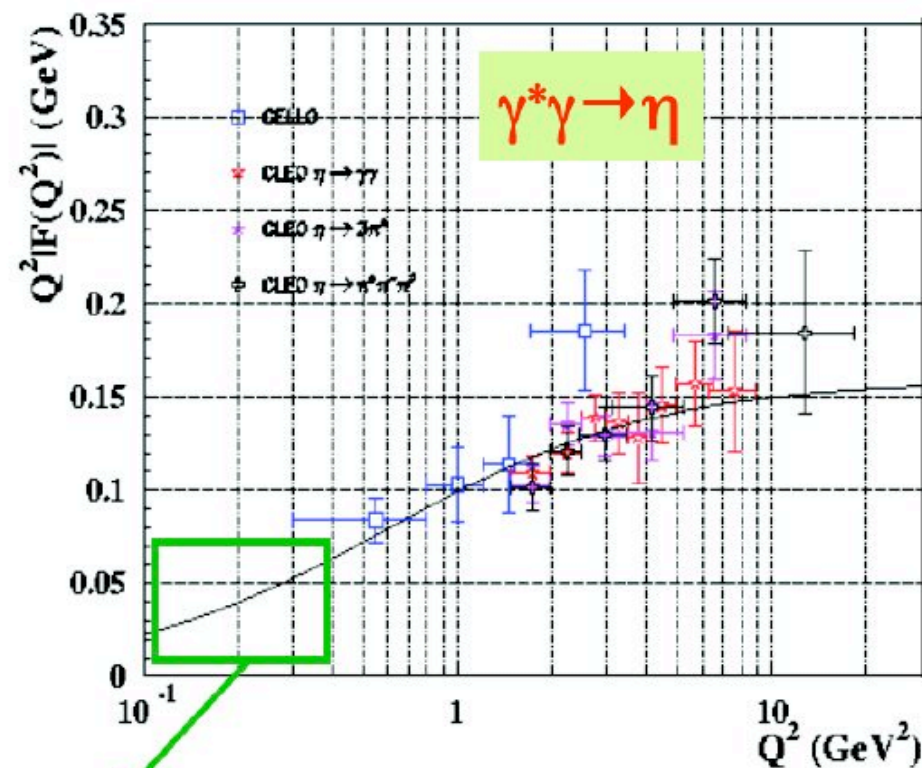
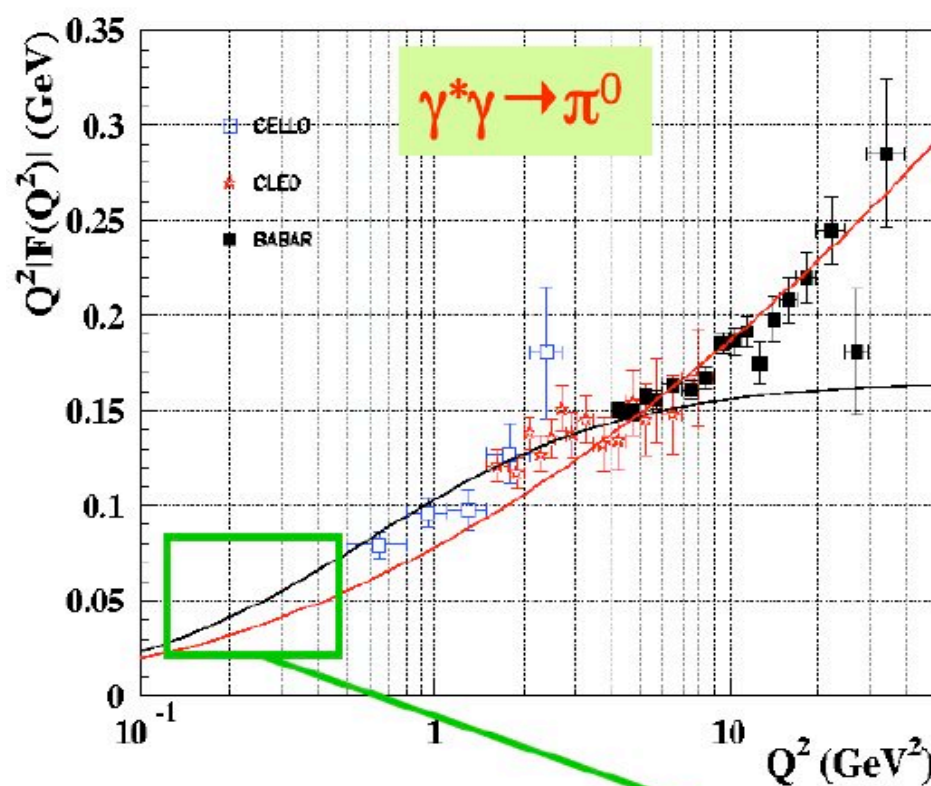


more
measurements
of the whole
spectrum are
useful



Low Q^2 region unexplored, so far

the region relevant to the $g-2$ is $Q < 1.5$ GeV for the 3 lightest PS mesons



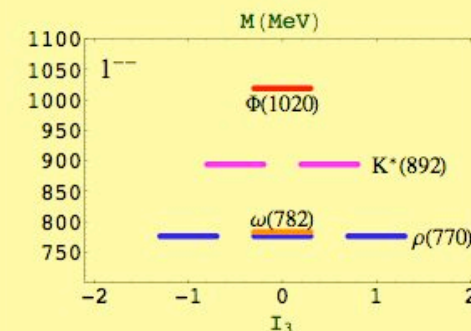
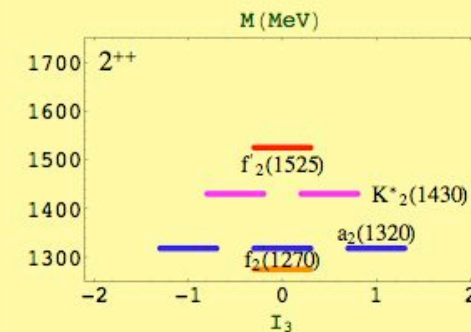
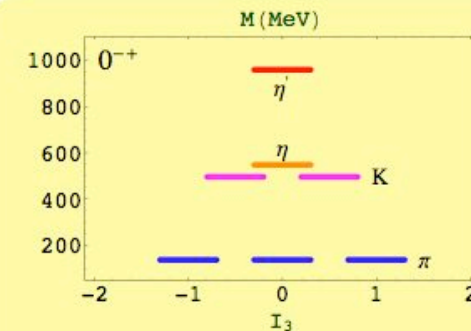
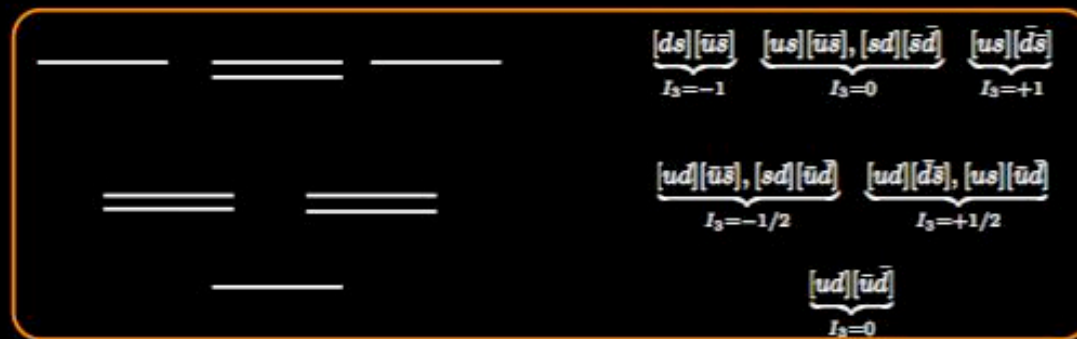
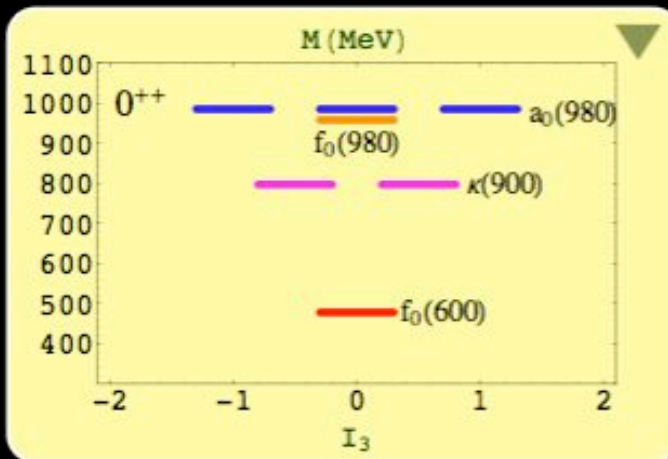
lack of
data points

Recent results with scalar mesons

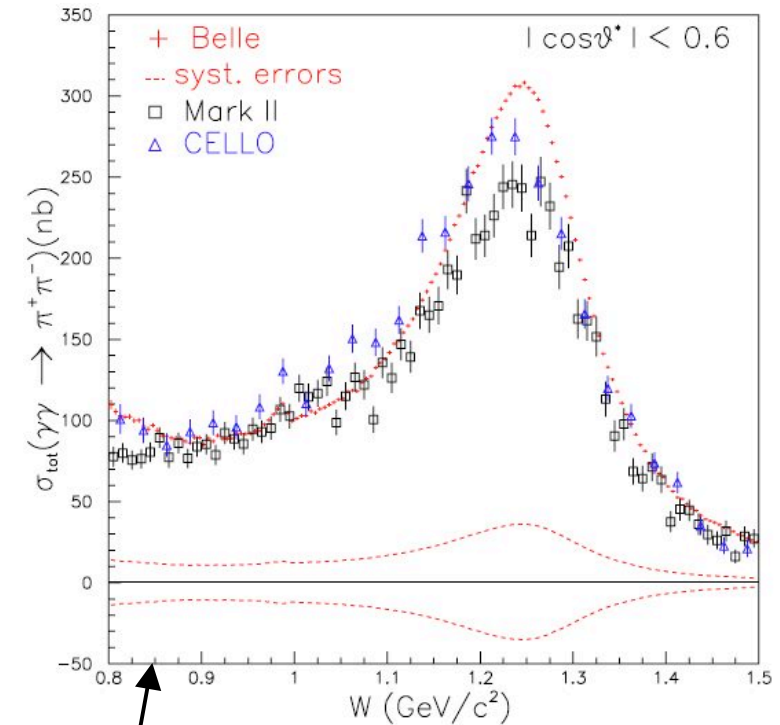
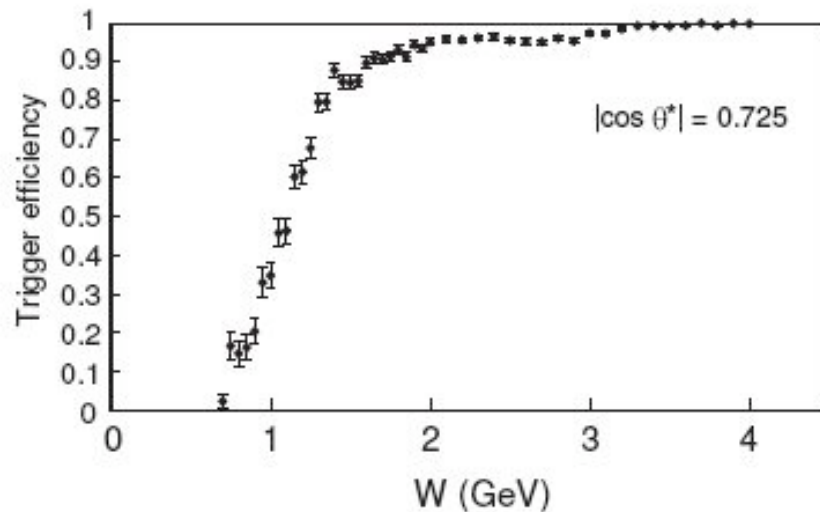
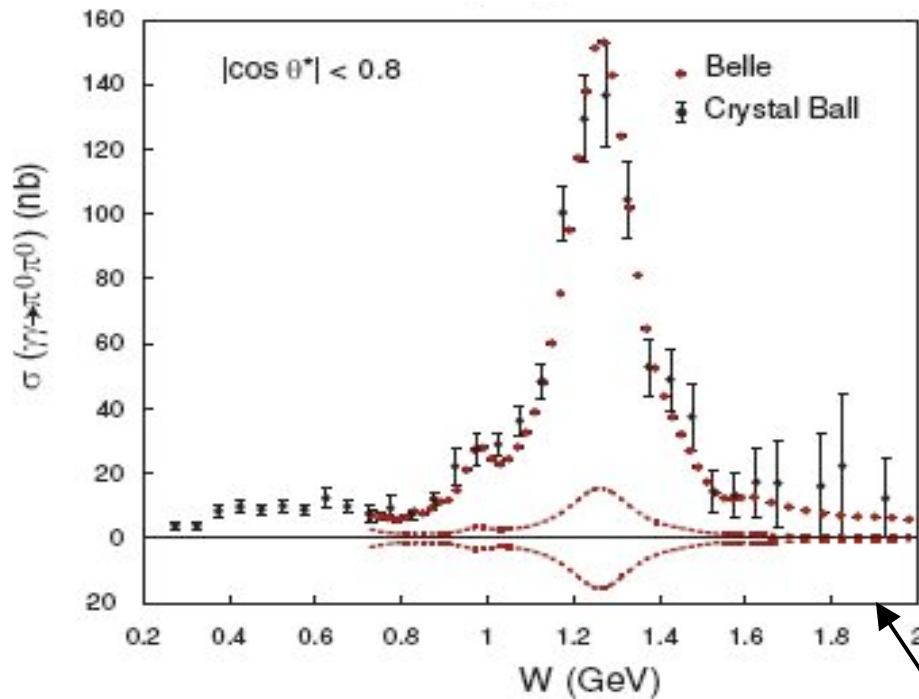
Low mass scalar mesons: puzzling since the 70's

Maiani et al. :: A new look at scalar mesons as $4q$ structures - PRL93(2004)212002
 't Hooft et al. :: A theory of scalar mesons - PLB662(2008)424

$4q$ structures explain the inverted mass spectrum (Jaffe)



Recent measurements of $\gamma\gamma \rightarrow \pi\pi$



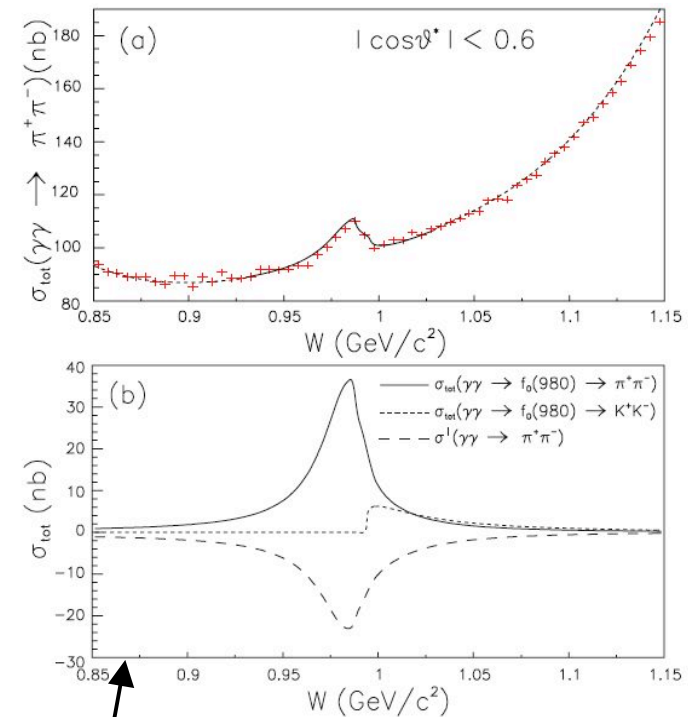
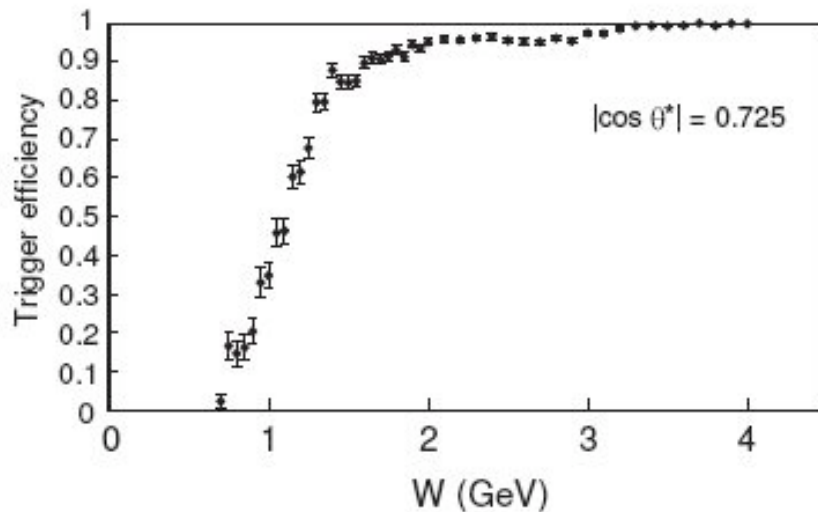
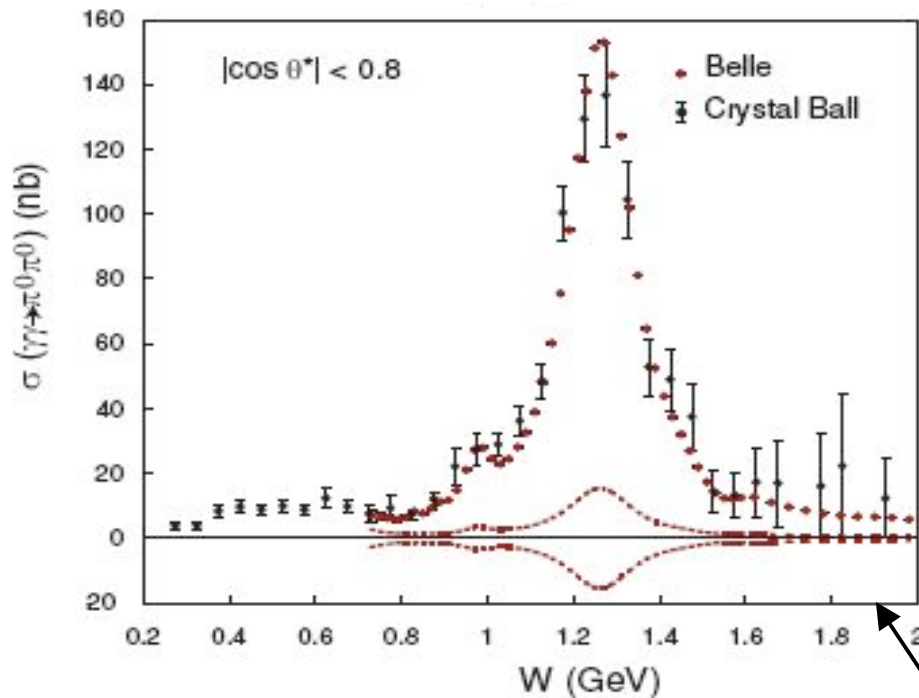
S.Uehara et al., PRD78(2008)052004

T.Mori et al., PRD75(2007)051101R

fantastic measurements that
cannot reach the low mass region
for trigger efficiency

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30-05-2011

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Searching for $\gamma\gamma \rightarrow \sigma(600) \rightarrow 2\pi^0$

$\pi^+\pi^-$ harder than $\pi^0\pi^0$ channel:

- 1) $\mu^+\mu^-$ background (need robust particle ID)
- 2) sizeable continuum $\gamma\gamma \rightarrow \pi^+\pi^-$ at tree level in QED

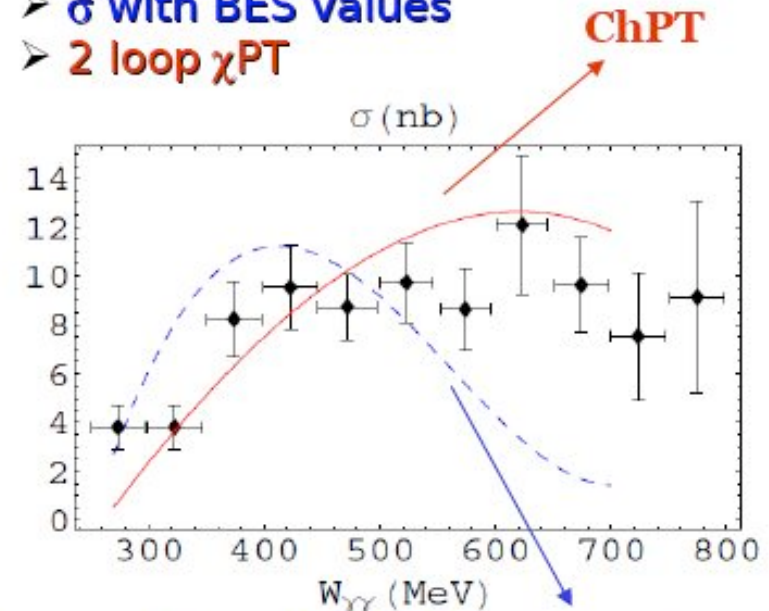
$$\sigma(\gamma\gamma \rightarrow \sigma(600)) \propto \Gamma(\sigma(600) \rightarrow \gamma\gamma)$$

$\Gamma(\gamma\gamma)$ keV		
composition	predictions	author(s)
$(\bar{u}u + \bar{d}d)/\sqrt{2}$	4.0	Babcock & Rosner ⁷³
$\bar{s}s$	0.2	Barnes ⁷⁴
$[\bar{n}s][ns], n = (u, d)$	0.27	Achasov <i>et al.</i> ⁷⁵
$\bar{K}K$	0.6 0.22	Barnes ⁷⁶ Hanhart <i>et al.</i> ⁷⁷

● Crystal Ball, PRD41 (1990) 3324

➤ σ with BES values

➤ 2 loop χ PT



Resonant contribution $\gamma\gamma \rightarrow \sigma \rightarrow \pi^0\pi^0$

Eur. Phys. J. C 47, 65–70 (2006)
F.Nguyen, F.Piccinini & A.Polosa

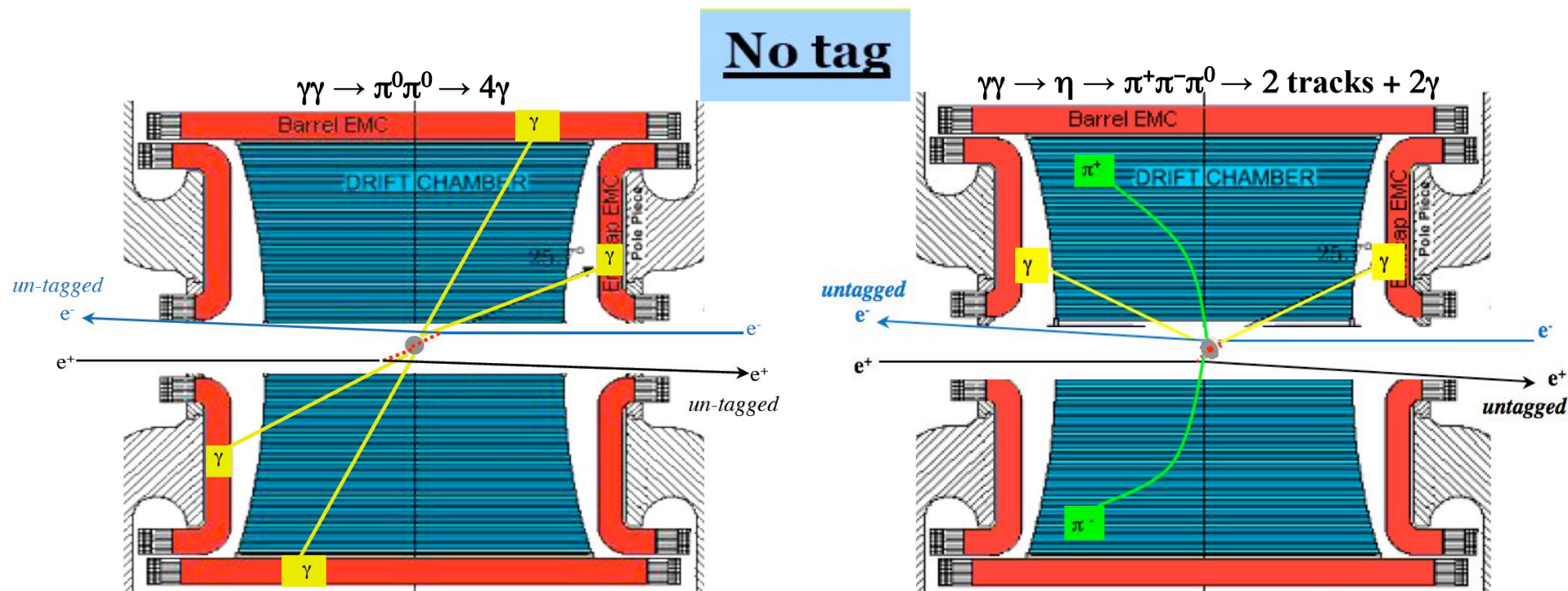
from the radiative width
→ infer the structure

Current activities: for example
KLOE/KLOE-2

Measuring $\gamma\gamma$ @ KLOE

240 pb⁻¹ taken @ $\sqrt{s} = 1$ GeV, to suppress background from ϕ decays

No tag



Calorimeter, EmC:
Pb/Scint. Fiber, 4880 PMTs
 $\sigma_E/E = 0.057/\sqrt{E}$ (GeV)
 $\sigma_t = 57$ ps/ $\sqrt{E}$ (GeV) $\oplus$ 100 ps

B=0.52 T

Drift Chamber, DC:
 90% He, 10% C₄H₁₀
 $\sigma_p/p = 0.4\%$ for $\theta > 45^\circ$
 $\sigma_{r\phi} = 0.15$ mm, $\sigma_z = 2$ mm

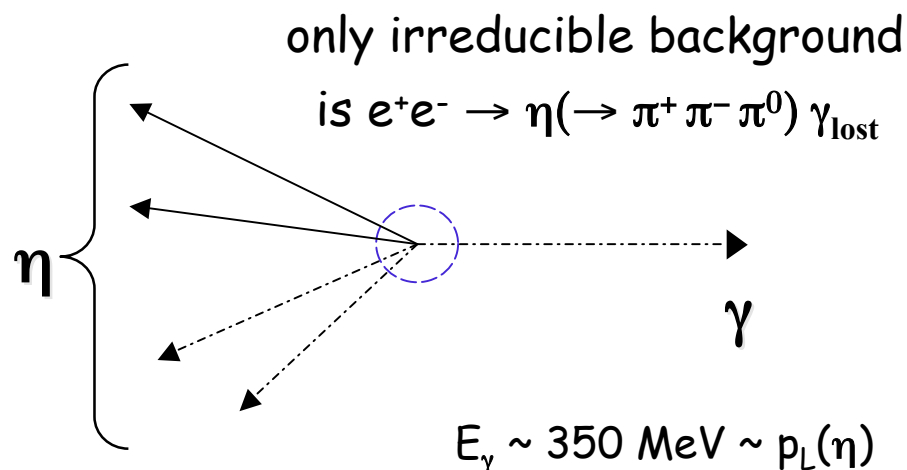
Search for $\gamma\gamma \rightarrow \eta \rightarrow \pi^+\pi^-\pi^0$ @ KLOE

BR($\eta \rightarrow \pi^+\pi^-\pi^0$) = 22.73% **2 photons + 2 tracks with opposite charge**

$$\chi^2_\eta = \sum \frac{(P_i - P_i^{meas})^2}{\sigma_i^2} + \sum \lambda_j^k C_j(P_1^k \dots P_N^k)$$

- $\gamma\gamma$ pairing
- charged pion ID
- kinematic fit, χ_η^2
- M_{miss}^2 vs p_L fits

$$\begin{aligned} m_{\gamma\gamma}^2 &= m_{\pi^0}^2 \\ m_{\pi^+\pi^-\gamma\gamma}^2 &= m_\eta^2 \\ \mathbf{t}_\gamma - |\mathbf{r}_\gamma| / c &= 0 \text{ for } 2\gamma \end{aligned}$$



$$\begin{aligned} M_{miss}^2 &= s + m_\eta^2 - 2\sqrt{s}E_T \left(1 - \frac{p_L^2}{E_T^2}\right)^{1/2} \\ &\simeq \boxed{s + m_\eta^2 - 2\sqrt{s}E_T} - \sqrt{s} \frac{p_L^2}{E_T} \end{aligned}$$

Search for $\gamma\gamma \rightarrow \eta \rightarrow \pi^+\pi^-\pi^0$ @ KLOE

$e^+e^- \rightarrow \eta\gamma \rightarrow \pi^+\pi^-\pi^0\gamma$

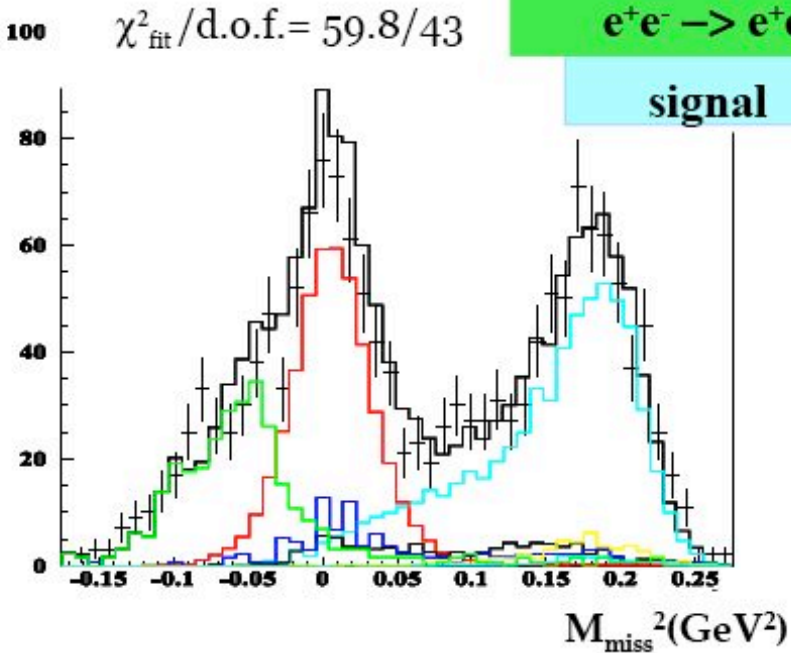
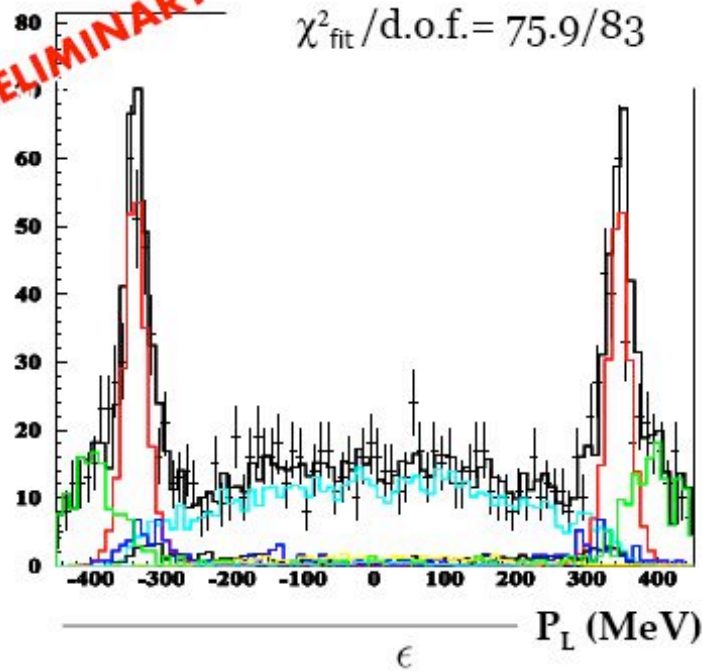
$e^+e^- \rightarrow e^+e^-\gamma$

signal

PRELIMINARY

$\chi^2_{\text{fit}}/\text{d.o.f.} = 75.9/83$

$\chi^2_{\text{fit}}/\text{d.o.f.} = 59.8/43$



Signal	η	0.196
	$\eta\gamma$	9.1×10^{-3}
	$\omega\pi^0$	6.5×10^{-5}
	$\pi^+\pi^-\pi^0$	1.5×10^{-5}
	K^+K^-	1.9×10^{-5}
	$K_S K_L$	2.6×10^{-5}
	$e^+e^-\gamma$	$\mathcal{O}(10^{-7})$

$N(\text{data})$ after cuts = 1576

$n_{\text{ev}} = 650$

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Search for $\gamma\gamma \rightarrow \eta \rightarrow 3\pi^0$ @ KLOE

$\text{BR}(\eta \rightarrow 3\pi^0) = 32.57\%$

- $\gamma\gamma$ pairing to 3 pions
- kinematic fit, χ_η^2
- most energetic γ $E < 260$ MeV

6 photons and NO tracks

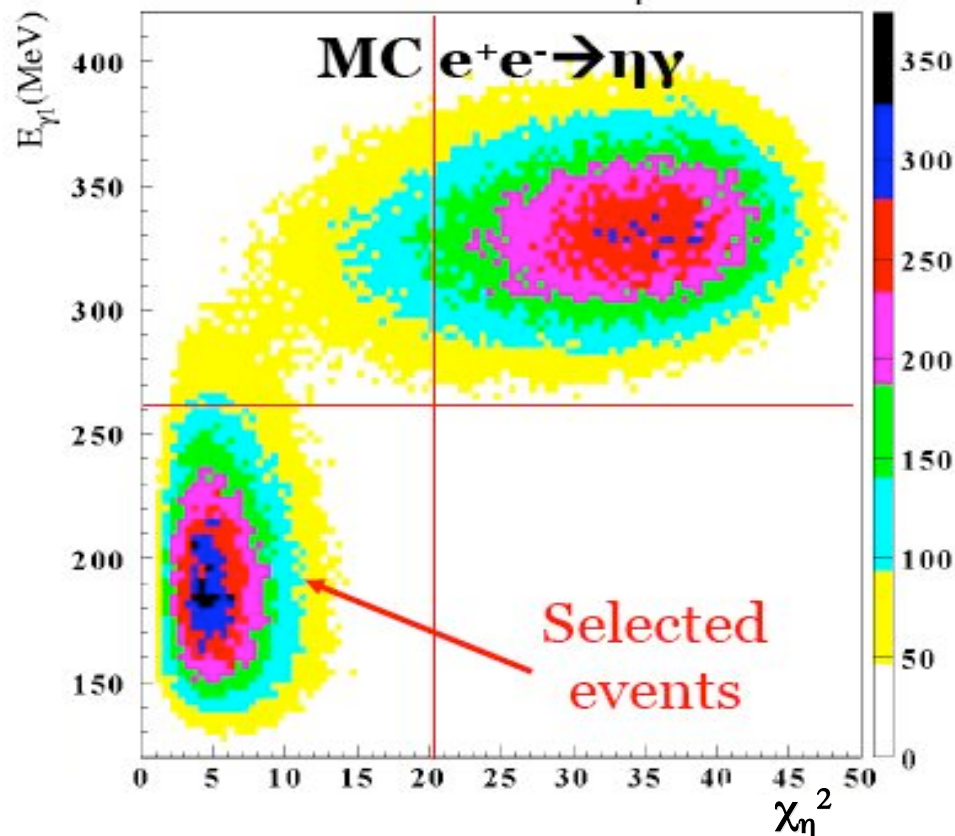
$$\chi_\eta^2 = \sum \frac{(P_i - P_i^{meas})^2}{\sigma_i^2} + \sum \lambda_j^k C_j(P_1^k \dots P_N^k)$$

$$m_{6\gamma}^2 = m_\eta^2$$

$$t_\gamma - |\underline{r}_\gamma| / c = 0 \text{ for } 6\gamma$$

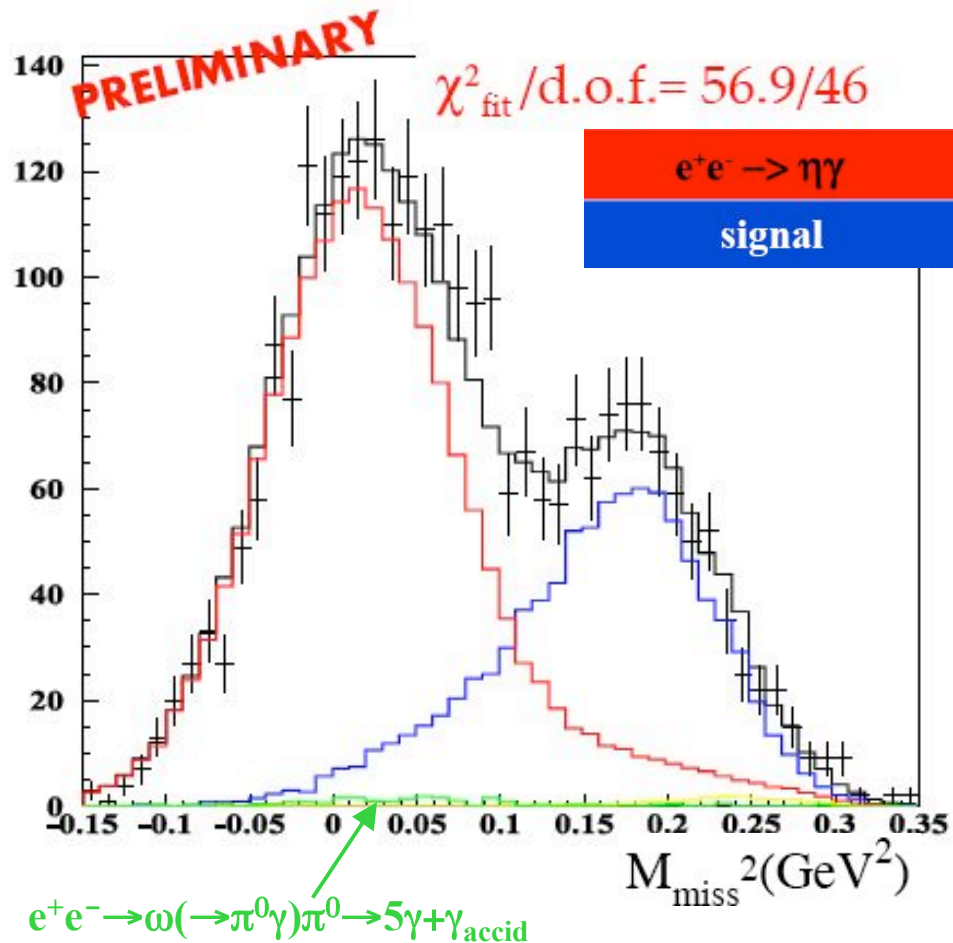
only irreducible background

is $e^+e^- \rightarrow \eta(\rightarrow \pi^0\pi^0\pi^0)\gamma_{\text{lost}}$



Search for $\gamma\gamma \rightarrow \eta \rightarrow 3\pi^0$ @ KLOE

2725 data events after all cuts



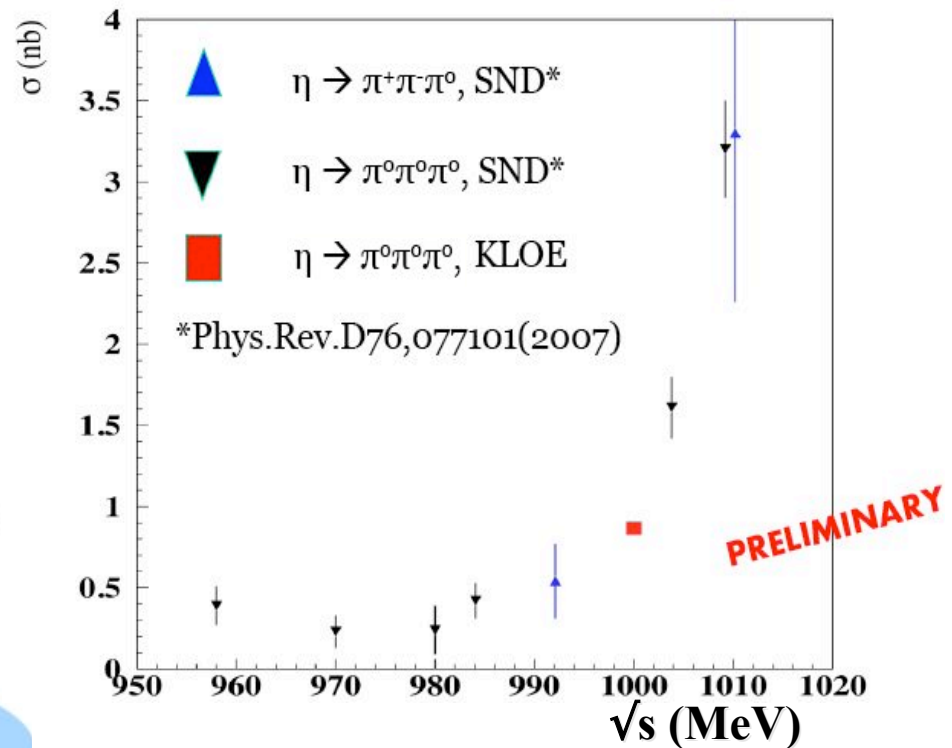
from the fit:

$$n_{\text{ev}} = 921$$

from the $\eta\gamma$ events in the fitted spectrum:

$$\sigma(e^+e^- \rightarrow \eta\gamma, 1 \text{ GeV}) = 0.875 \pm 0.009 \text{ nb}$$

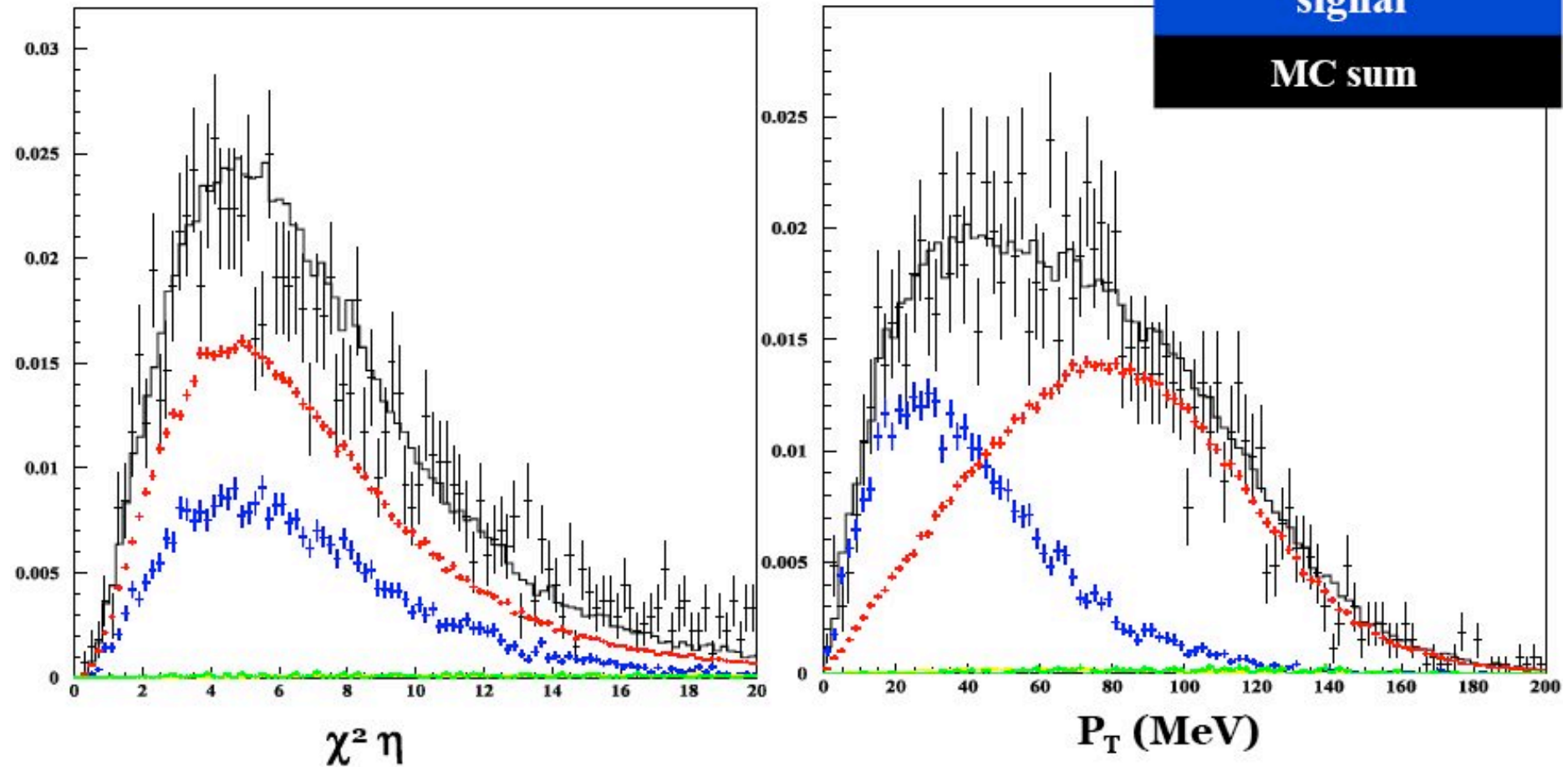
(statistical error only)



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Search for $\gamma\gamma \rightarrow \eta \rightarrow 3\pi^0$ @ KLOE

MC contributions normalized to the
cross sections obtained from the fit

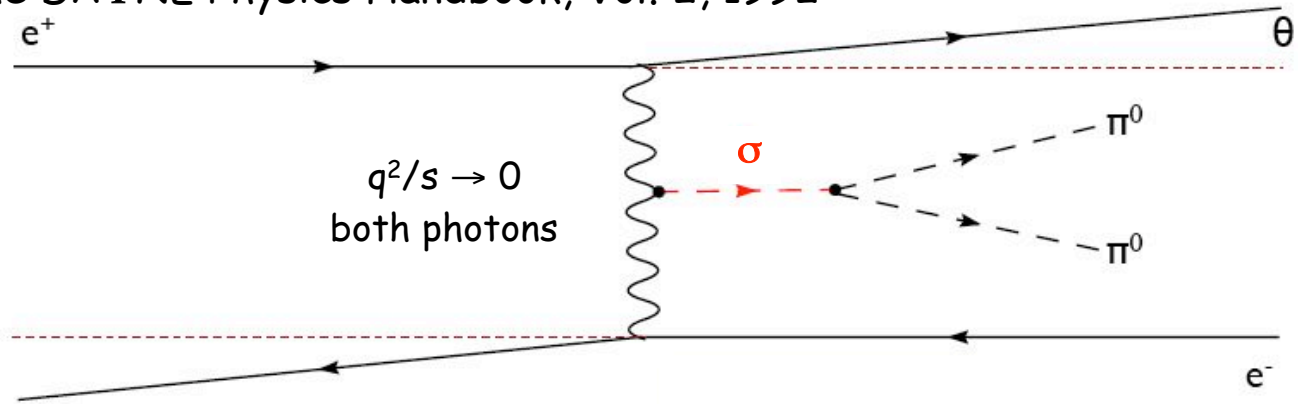


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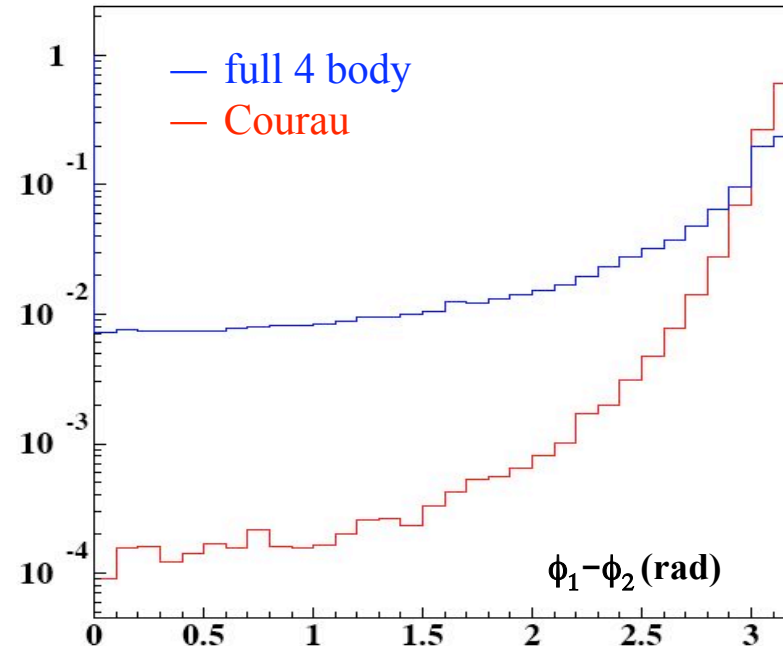
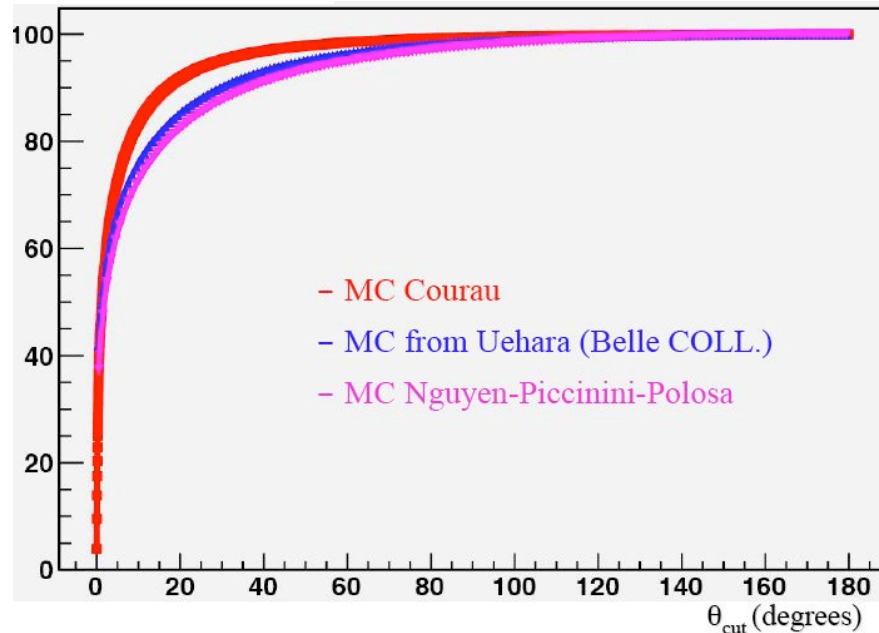
MC simulation of $\gamma\gamma \rightarrow \sigma(600) \rightarrow 2\pi^0$

the complete 4 body simulation, EPJC47 (2006) 65, is compared with the Weizsäcker-Williams approx. (head on collision of 2 quasi-real $\gamma\gamma$)

A. Courau & G. Pancheri, The DAΦNE Physics Handbook, Vol. 2, 1992

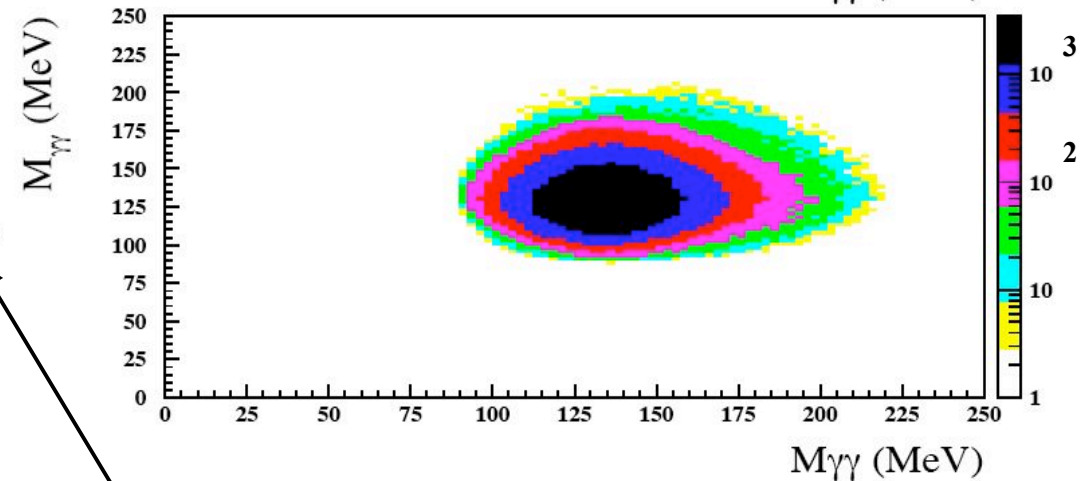
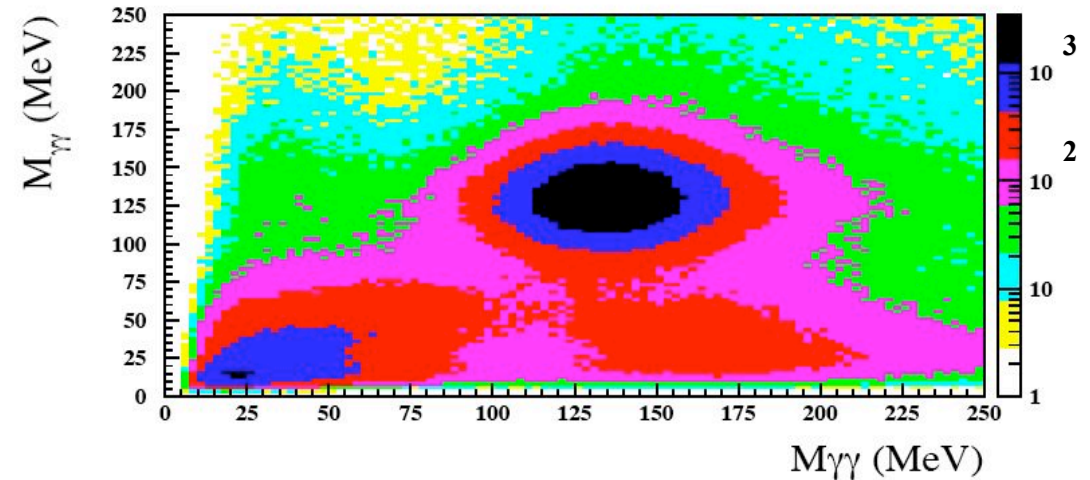
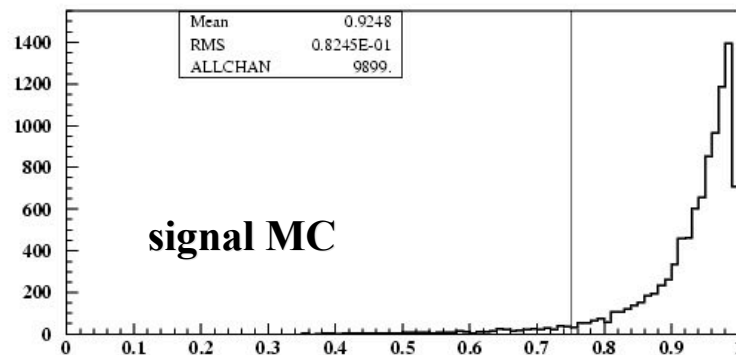
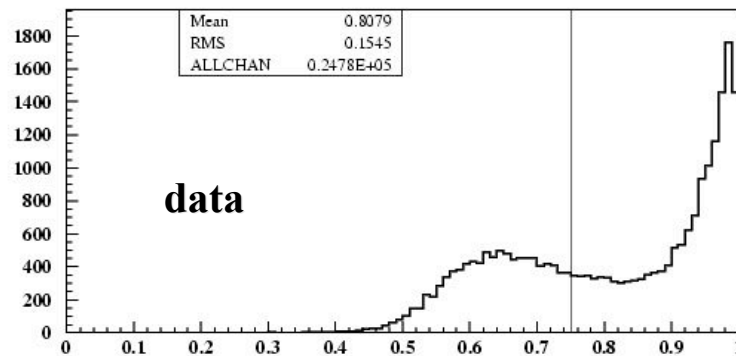


$$F(\theta_{\text{cut}}) = \int_0^{\theta_{\text{cut}}} \frac{dn}{d\theta} d\theta$$



Search for $\gamma\gamma \rightarrow \sigma(600) \rightarrow 2\pi^0$ @ KLOE

- $\gamma\gamma$ pairing to 2 pions, $\chi_{\pi\pi}^2 < 4$
- 4 photons and NO tracks
- $p_T(4\gamma) < 120$ MeV
- $\Sigma_{4\gamma} / \Sigma_{\text{CALO}} > 0.75$
- promptness enforced (t_γ cuts)

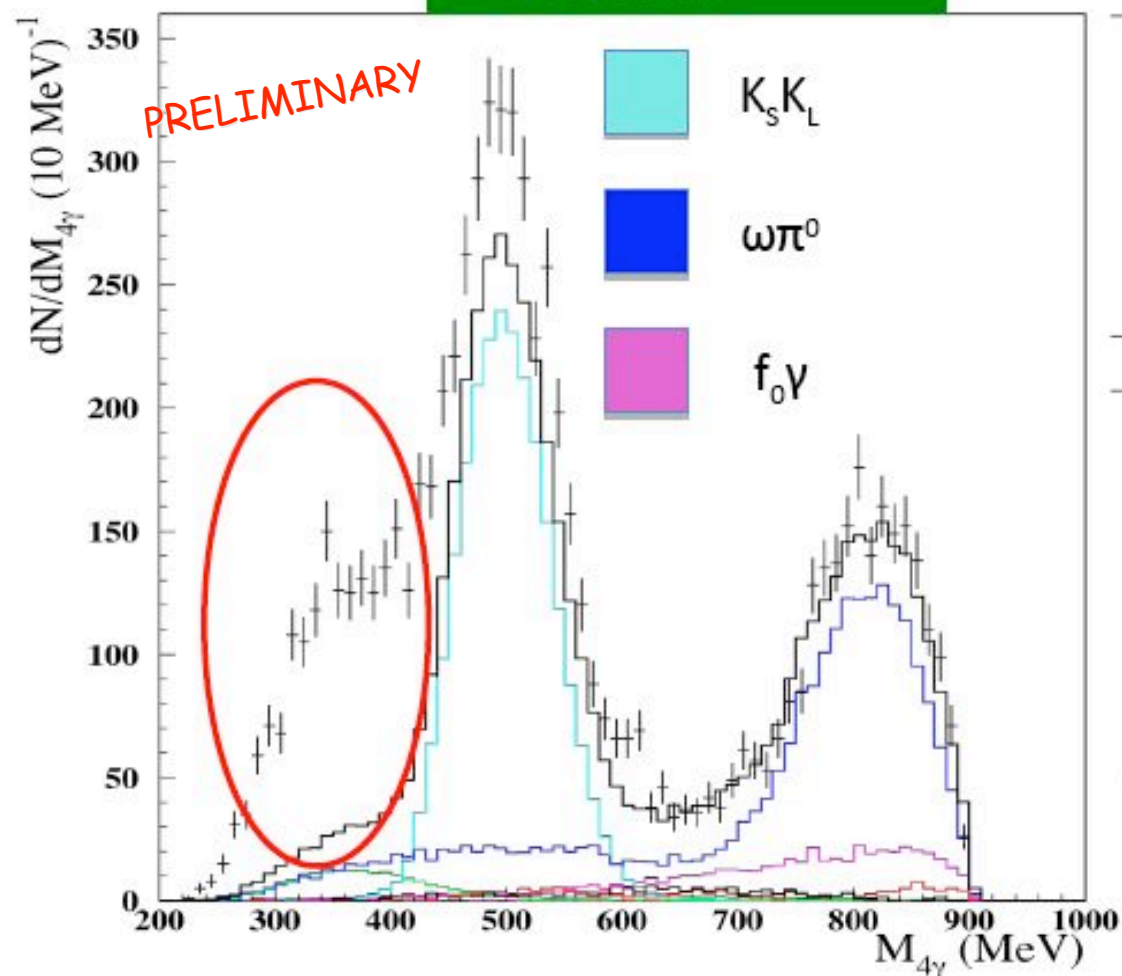


$\Sigma_{4\gamma} / \Sigma_{\text{CALO}}$

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Search for $\gamma\gamma \rightarrow \sigma(600) \rightarrow 2\pi^0$ @ KLOE

8090 events after
selections



	ϵ	σ (nb)	$n = \mathcal{L}\sigma\epsilon$
$K_S K_L$	7.5×10^{-3}	1.28	2328
$\eta \rightarrow 3\pi^0$	2.8×10^{-3}	0.284	193
$\omega \pi^0$	1.4×10^{-2}	0.55	1867
$f_0 \gamma$	2.9×10^{-2}	0.17	1204
$a_0 \gamma$	5.8×10^{-3}	0.11	155
$\gamma\gamma$	2×10^{-6}	360	175
			5922
Data			8090

Expected values

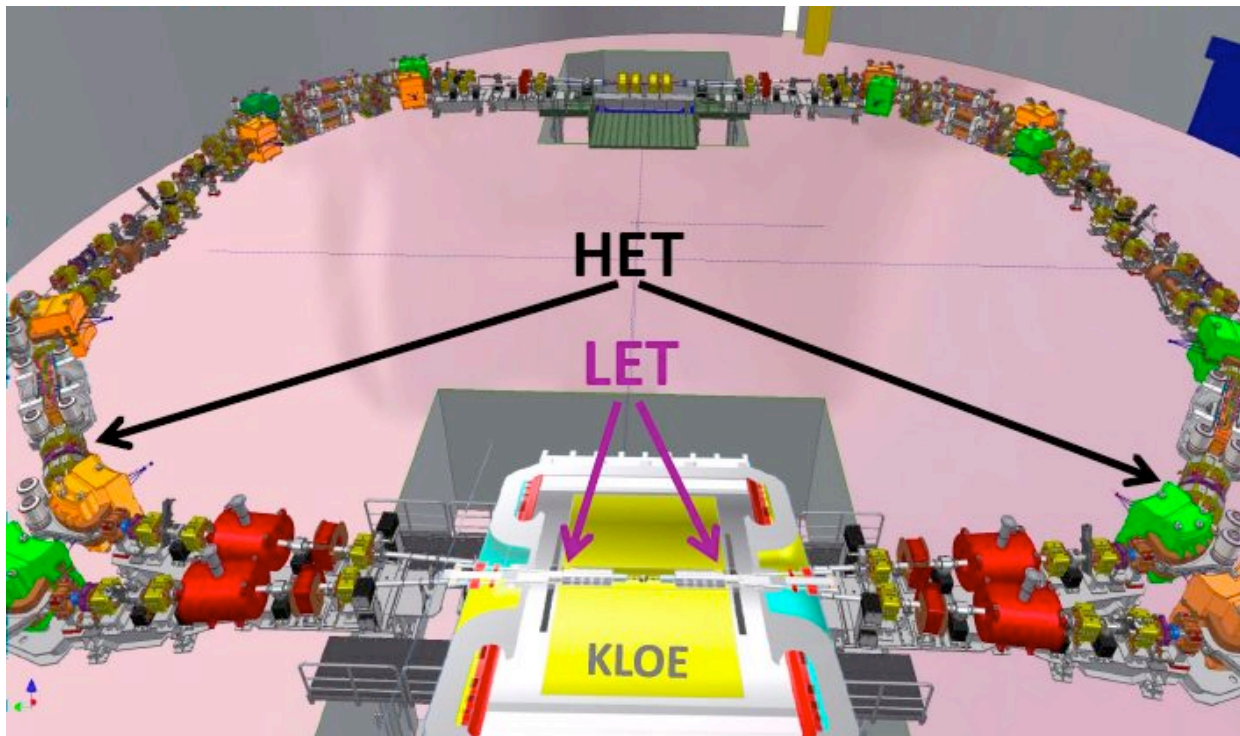
$\mathcal{O}(2000)$ candidate events of
 $\gamma\gamma \rightarrow \pi^0 \pi^0$, evaluation and
interpretation of the cross
section is in progress...

KLOE-2 plans

**Detector upgrade for the first KLOE-2 run :
2+2 detector stations for leptons in $e^+e^- \rightarrow e^+e^-\gamma^*\gamma^* \rightarrow e^+e^-X$**

LET (Low Energy Taggers) are LYSO calorimeters placed inside KLOE

HET (High Energy Taggers) are scintillator hodoscopes placed 11 m from the IP

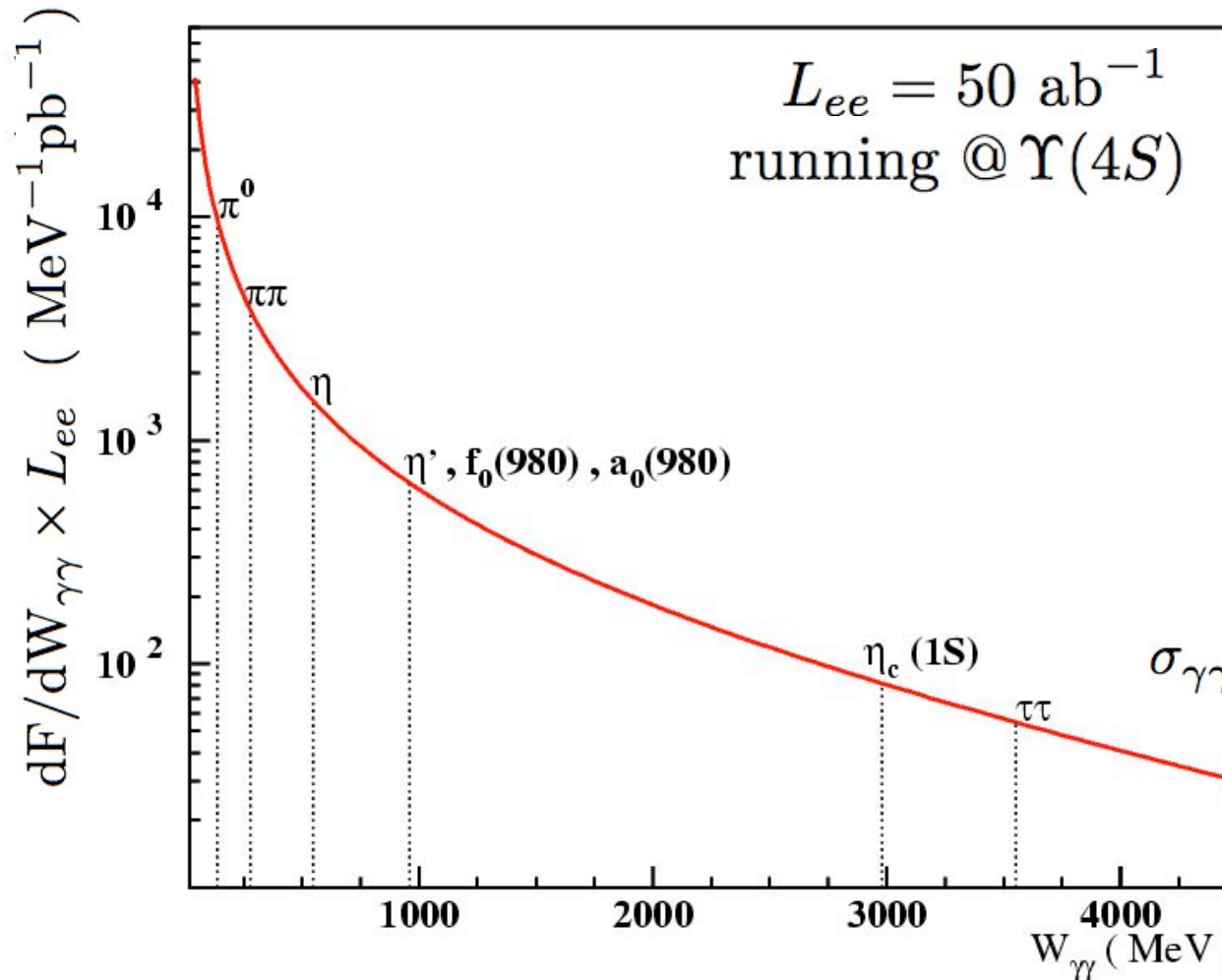


HET: $e^\pm$ of 425-490 MeV

LET: $e^\pm$ of 160-230 MeV

$\gamma\gamma$ Physics @ Super-B? so nice a
product: "incredible cross section"
× "incredible luminosity"

General considerations on yields @ Super-B



$$N_{e^+e^- \rightarrow e^+e^- X} =$$

$$L_{ee} \int \frac{dF}{dW_{\gamma\gamma}} \sigma_{\gamma\gamma \rightarrow X}(W_{\gamma\gamma}) dW_{\gamma\gamma}$$

$$\sigma_{\gamma\gamma \rightarrow \tau^+\tau^-}(\Upsilon(4S)) \sim \mathcal{O}(5 \text{ nb})$$

$$\frac{dF}{dW} = \frac{1}{W} \left(\frac{2\alpha}{\pi} \right)^2 \left(\ln \frac{E_b}{m_e} \right)^2 \left((z^2 + 2)^2 \ln \frac{1}{z} - (1 - z^2)(3 + z^2) \right) \quad z = \frac{W}{2E_b}$$

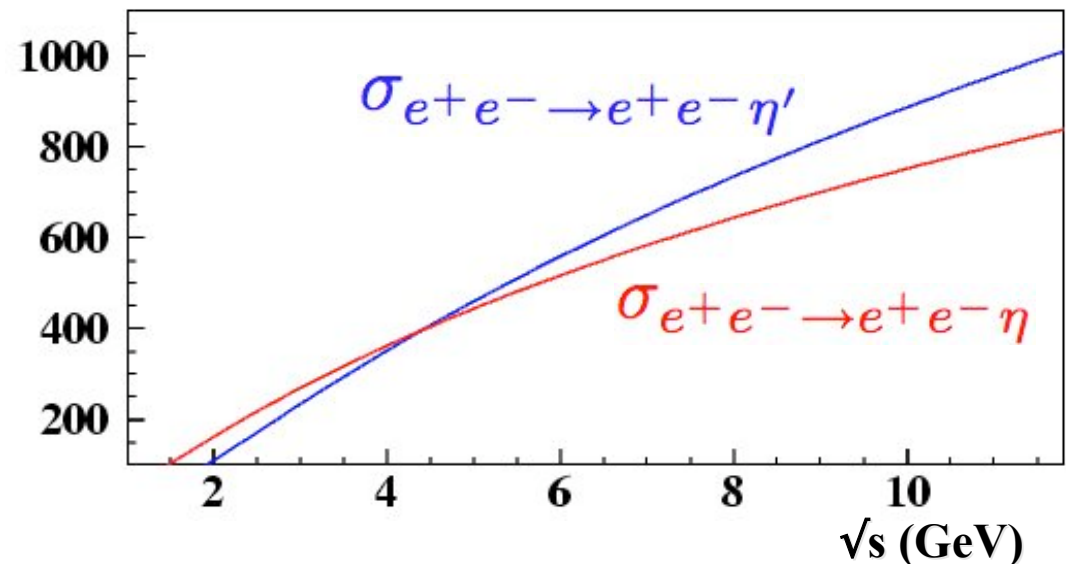
Federico Nguyen
30-05-2011

PS meson production: flavour factories comparison

$$\sigma_{e^+e^- \rightarrow e^+e^- X} = \frac{16\alpha^2 \Gamma_{X\gamma\gamma}}{m_X^3} \left(\ln \frac{E_b}{m_e} \right)^2 \left((y^2 + 2)^2 \ln \frac{1}{y} - (1 - y^2)(3 + y^2) \right) \quad y = m_X / (2E_b)$$

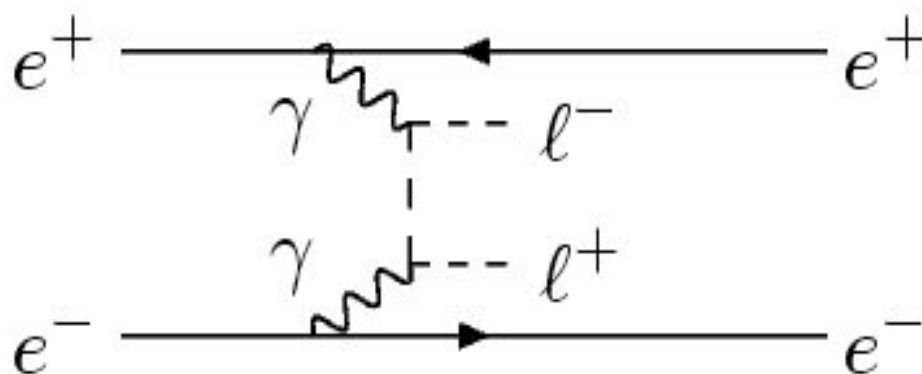
$\sigma_{e^+e^- \rightarrow e^+e^- PS} [\text{pb}]$			
$\sqrt{s}$ (GeV)	ϕ	J/ψ	$\Upsilon(4S)$
π^0	261	638	1283
η	45	279	781
η'	8	245	928
$\eta_c(1S)$	—	0.2	3.6

even at equal luminosity... high $\sqrt{s}$ matters!



flipping of the η - η' cross sections, because phase space gets marginal wrt the partial width: $\Gamma_{\eta'\gamma\gamma} \sim 10 \Gamma_{\eta\gamma\gamma}$

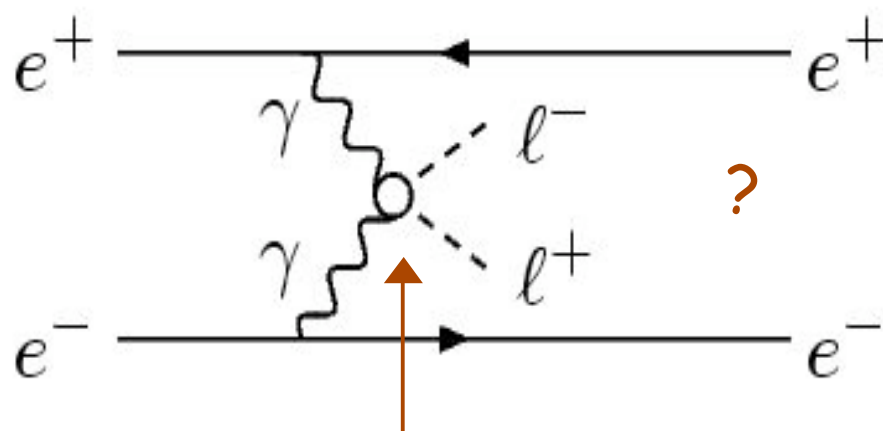
QED tests with $e^+e^- \rightarrow e^+e^- l^+l^-$ ($l = e, \mu, \tau$)



✓ $O(\alpha^2)$ tests of QED through
C,P,CP-violating asymmetries

✓ tagger providing 4-momentum of
at least 1 e^+/e^- is needed

✓ ...a way to find the "unexpected"?



light (pseudo)scalar boson

• HyperCP excess, for events $\Sigma^+ \rightarrow p\mu^+\mu^-$

<http://arxiv.org/abs/hep-ex/0501014>

• interpretation as sgoldstino S and
possible search in events

$e^+e^- \rightarrow S e^+e^- \rightarrow \mu^+\mu^- e^+e^-$,

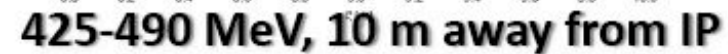
$e^+e^- \rightarrow S e^+e^- \rightarrow \gamma\gamma e^+e^-$

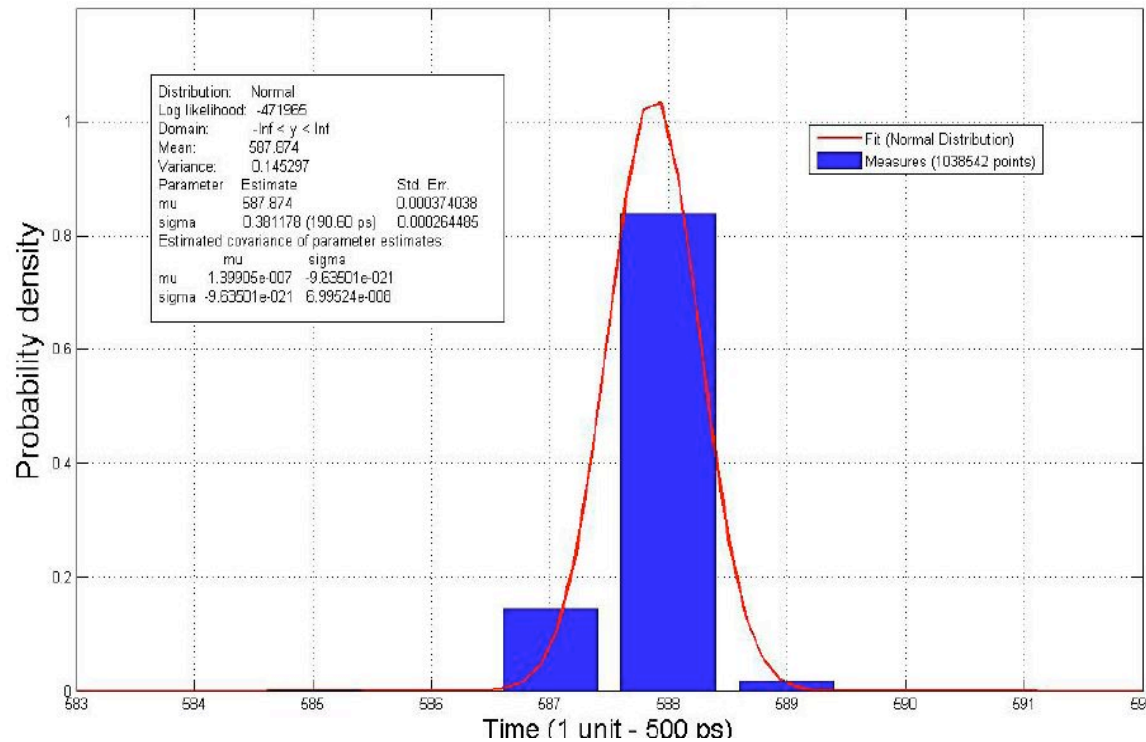
<http://arxiv.org/abs/hep-ph/0509147>

Conclusions and prospects

- ✓ important $\gamma\gamma$ measurements from the B-factories, only limit is the trigger efficiency for reaching lower momenta
- ✓ KLOE complementarity: first evidence of $\gamma\gamma \rightarrow \eta$ @ 1 GeV in 2 different channels, $O(2000)$ candidate events of $\gamma\gamma \rightarrow 2\pi^0$ at threshold
- ✓ thanks to the high luminosity, Super-B may probe the low mass region: final state $e^\pm$ taggers with trigger decision?
- ✓ unique opportunities @ Super-B: rare phenomena in $\gamma\gamma$ processes!

The top part of the figure is a schematic diagram of the SuperKEKB accelerator layout. It shows the electron (e-) and positron (e+) beams entering from the left, passing through the IP (Interaction Point) and QFI (Quadrupole Focusing Insertion) regions, and then being accelerated through a series of bending magnets and quadrupoles. The bottom part of the figure is a plot of the vertical position X[m] versus distance. The plot shows the trajectories of the electron and positron beams at different energies: 510 MeV, 327 MeV, and 327 MeV. The trajectories are labeled with 'sppst101' and 'chps101'.





- Front End
- Discriminator
- TDC con DAQ
a 368 MHz
with ~ 250 ps
resolution

*resolution which
allow clear separation between consecutive bunches.*



PS meson production: flavour factories comparison

$$\sigma_{e^+e^- \rightarrow e^+e^- X} = \frac{16\alpha^2 \Gamma_{X\gamma\gamma}}{m_X^3} \left(\ln \frac{E_b}{m_e} \right)^2 \left((y^2 + 2)^2 \ln \frac{1}{y} - (1 - y^2)(3 + y^2) \right) \quad y = m_X / (2E_b)$$

$\sigma_{e^+e^- \rightarrow e^+e^- PS} [\text{pb}]$			
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even at equal luminosity... high $\sqrt{s}$ matters!

for example $4 \times 10^{10} \eta'$ produced

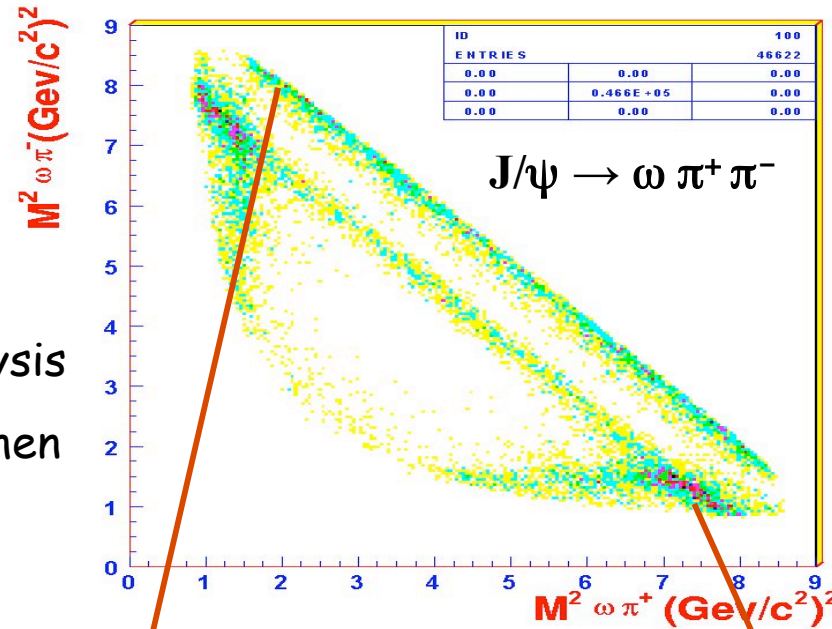
final state F	$\text{BR}(\eta' \rightarrow F) (\%)$	preferable chain	$\text{BR}_{eff} (\%)$
$\pi^+ \pi^- \eta$	44.6 ± 1.4	$\pi^+ \pi^- \eta (\rightarrow 2\gamma) \leftrightarrow \pi^+ \pi^- 2\gamma$	17.5
$\pi^+ \pi^- \gamma$	29.4 ± 0.9		
$\pi^0 \pi^0 \eta$	20.7 ± 1.2	$\pi^0 \pi^0 \eta (\rightarrow \pi^+ \pi^- \pi^0) \leftrightarrow \pi^+ \pi^- 6\gamma$	4.7
$\omega \gamma$	3.02 ± 0.31	$\omega (\rightarrow \pi^+ \pi^- \pi^0) \gamma \leftrightarrow \pi^+ \pi^- 3\gamma$	2.7
$\gamma \gamma$	2.10 ± 0.12		

flipping of the η - η' cross sections,
because phase space gets marginal
wrt the partial width: $\Gamma_{\eta' \gamma\gamma} \sim 10 \Gamma_{\eta \gamma\gamma}$

BES evidence (2004)

PLB598 (2004) 149

- ✓ partial wave analysis
- ✓ $\Delta\text{Log}\mathcal{L} = 5238$, when omitting the σ

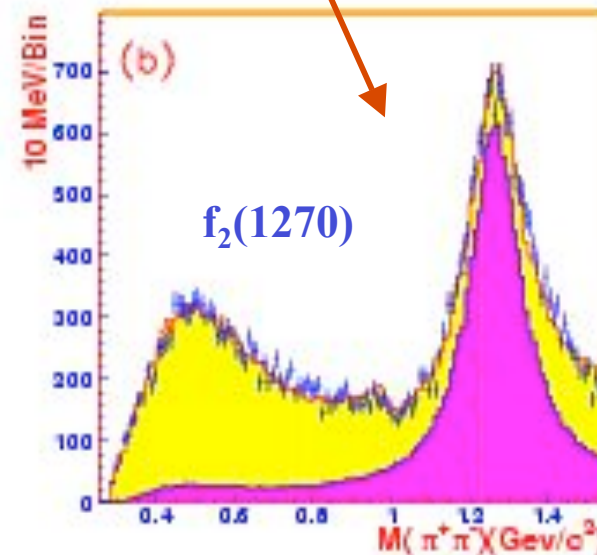
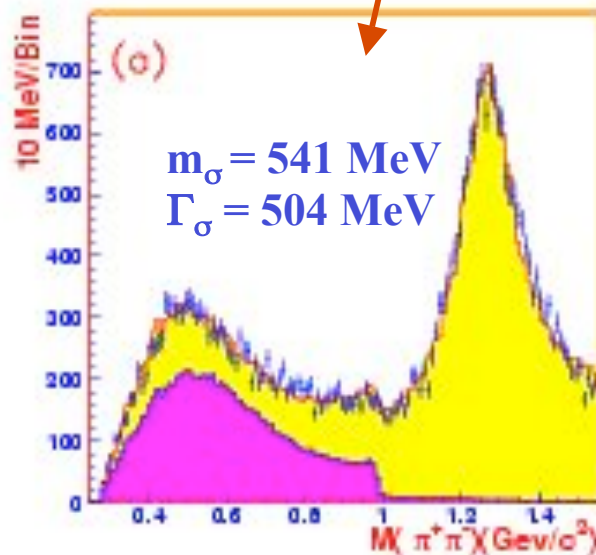


$$\left\{ \begin{array}{l} BW_\sigma = \frac{1}{m_\sigma^2 - s - im_\sigma \Gamma_\sigma} \\ \Gamma_\sigma \text{ is a constant} \end{array} \right.$$

$$\left\{ \begin{array}{l} BW_\sigma = \frac{1}{m_\sigma^2 - s - i\sqrt{s}\Gamma_\sigma(s)} \\ \Gamma_\sigma(s) = \frac{g_\sigma^2 \sqrt{\frac{s}{4} - m_\pi^2}}{8\pi s} \end{array} \right.$$

$$\left\{ \begin{array}{l} BW_\sigma = \frac{1}{m_\sigma^2 - s - i\sqrt{s}\Gamma_\sigma(s)} \\ \Gamma_\sigma(s) = \alpha \sqrt{\frac{s}{4} - m_\pi^2} \end{array} \right.$$

$$\left\{ \begin{array}{l} BW_\sigma = \frac{1}{m_\sigma^2 - s - im_\sigma(\Gamma_1(s) + \Gamma_2(s))} \\ \Gamma_1(s) = G_1 \frac{\sqrt{1 - 4m_\pi^2/s}}{\sqrt{1 - 4m_\pi^2/m_\sigma^2}} \cdot \frac{s - m_\pi^2/2}{m_\sigma^2 - m_\pi^2/2} e^{-(s - m_\sigma^2)/4\beta^2} \\ \Gamma_2(s) = G_2 \frac{\sqrt{1 - 16m_\pi^2/s}}{\sqrt{1 - 16m_\pi^2/m_\sigma^2}} \frac{1 + e^{\Lambda(s_0 - m_\sigma^2)}}{1 + e^{\Lambda(s_0 - s)}} \end{array} \right.$$



$J/\psi \rightarrow \omega f_2(1270)$
 $\omega \sigma$
 $\omega f_0(980)$
 $b_1(1235)\pi$