4D symplectic tracking in the final doublet

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Tracking method
introduction

4D symplectic integrator for the particle tracking through the final quadrupole of the SuperB final focus.

- investigate and correct multipole components;
- study of the aberrations;
- study of the dynamical aperture.

Relativistic hamiltonian:

$$\mathcal{H} = c\sqrt{m^2c^2 + (p_x - eA_x)^2 + (p_y - eA_y)^2 + (p_z - eA_z)^2} \tag{1}$$

We need to separate the square root in the exponential:

$$e^{-h:\mathcal{H}:} \tag{2}$$

$$\mathcal{H}' = \frac{(p_x - eA_x)^2 + (p_y - eA_y)^2 + (p_z - eA_z)^2}{2M'} \tag{3}$$

where
$M' = \sqrt{m^2c^2 + (p_x - eA_x)^2 + (p_y - eA_y)^2 + (p_z - eA_z)^2}$
gives the same Hamilton equations of (1)

* E. Chacon-Golcher, F. Neri. A symplectic integrator with arbitrary vector and scalar potential, Physics Letters A 372 (2008)

Tracking method

Generalized leapfrog method 2

$$e^{-h:\mathcal{H}':} = e^{-\frac{h}{2}:\frac{(p_x - eA_x)^2}{2M'}:} e^{-\frac{h}{2}:\frac{(p_y - eA_y)^2}{2M'}:} e^{-h:\frac{(p_z - eA_z)^2}{2M'}:} e^{-\frac{h}{2}:\frac{(p_y - eA_y)^2}{2M'}:} e^{-\frac{h}{2}:\frac{(p_x - eA_x)^2}{2M'}:} + O(h^3) \tag{4}$$

the x component can be calculated:

$$x^f = x^i + \frac{p_x^i - eA_x(x^i, y^i, z^i)}{M'}\frac{h}{2} \qquad y^f = y^i \qquad z^f = z^i \tag{5}$$

$$p_x^f = p_x^i + eA_x(x^f, y^f, z^f) - eA_x(x^i, y^i, z^i) \tag{6}$$

$$p_y^f = p_y^i + \int_{x^i}^{x^f} e\frac{\partial}{\partial y^i} A_x(x^i, y^i, z^i)\mathrm{d}x' \tag{7}$$

$$p_z^f = p_z^i + \int_{x^i}^{x^f} e\frac{\partial}{\partial z^i} A_x(x^i, y^i, z^i)\mathrm{d}x' \tag{8}$$

integrals of equations (7) and (8) are evaluated with a second order gaussian quadrature.

Integration

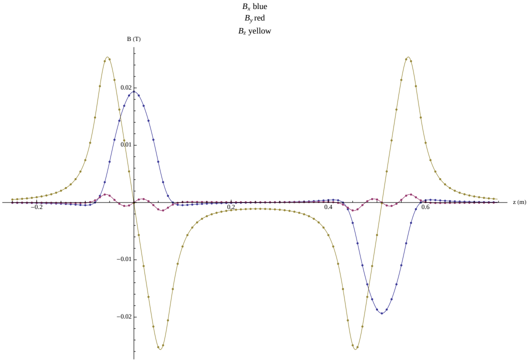
I obtain the vector potential and its derivatives integrating the Biot-Savart law.

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \oint_{l'} \frac{I \mathrm{d}\vec{l'}}{|\vec{r} - \vec{r'}|} \tag{9}$$

The numerical integral is done with an 11^{th} order gaussian method. Every coil was divided in 12 arcs.

Test
magnetic field

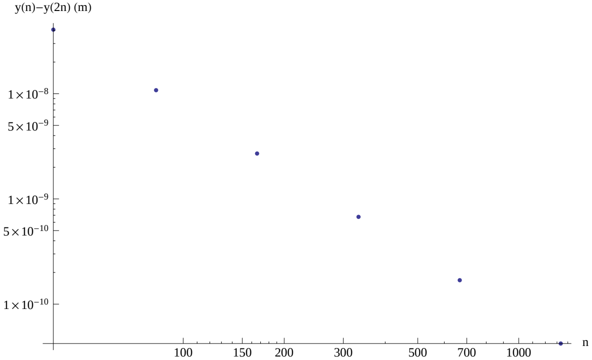
Magnetic field on the magnet axis calculated with C program and with Mathematica



The order of the gaussian integral and the number of intervals are choosen to have an accuracy of 10^{-6} on fields values.

Test
Algorithm convergence

The rest of the algorithm on a single time step Δt is $O(\Delta t^3)$.
With n time steps, the rest is $O(n^{-2})$.



Characteristics of tracking code

With the time step $\Delta t = 4\,\mathrm{ps}$:

- execution time: about $50\,\mathrm{s}/particle$ (from $15\,\mathrm{cm}$ before the magnet to the focus point).
- accuracy of results: $< 1\,\mathrm{nm}$ on three coordinates

With this code, the simulation of all the final focus will be possible.

Magnet model

The simulated magnet is a $30\,\mathrm{cm}$ quadrupole with a field gradient of $103\,\frac{\mathrm{T}}{\mathrm{m}}$. The internal radius is $19.5\,\mathrm{mm}$.

internal coils

- $x_1 = r_1 \cos(\phi)$;
- $y_1 = -r_1 \,\mathrm{sen}(\phi)$;
- $z_1 = -A_1 \sin(2\phi) - \frac{\phi \Delta z}{2\pi} + N\Delta z$;

external coils

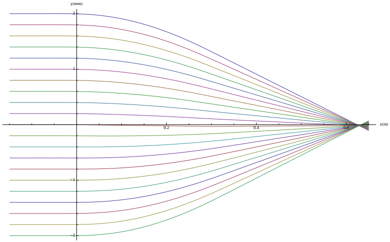
- $x_2 = r_2 \cos(\phi)$;
- $y_2 = r_2 \,\mathrm{sen}(\phi)$;
- $z_2 = -A_2 \sin(2\phi) + \frac{\phi \Delta z}{2\pi}$;

$$0 < \phi < 2N\pi$$

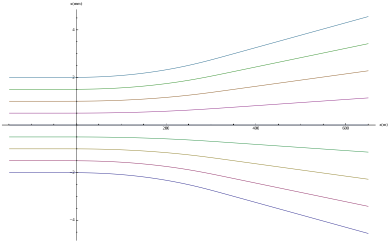
$\Delta z = 6.4\,\mathrm{mm}$
$N = 47$
$I = 2500\,\mathrm{A}$

Tracking

$x = 0$

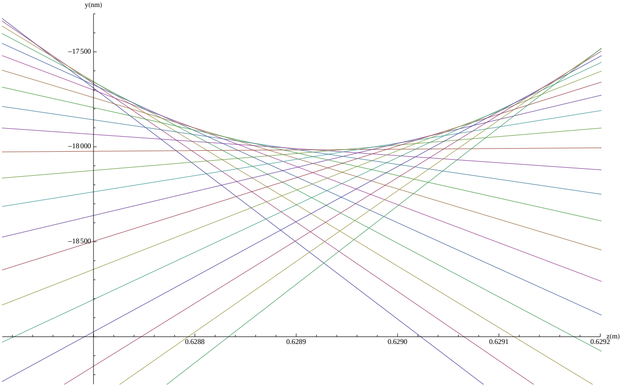


$y = 0$



Focus point is not in the magnet axis.

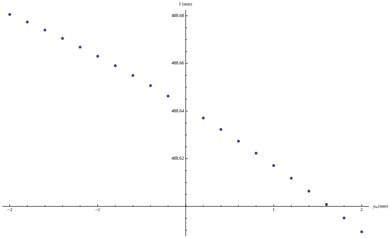
$$F_y \simeq -18\,\mu\text{m} \tag{10}$$

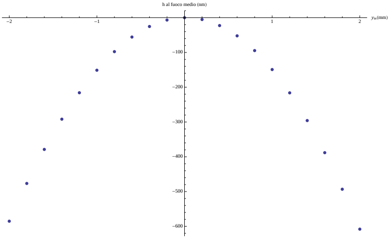


Tracking
sextupole component

The focus is not a point: there is a sextupole component.
The distance f between the principal plane and the focus:

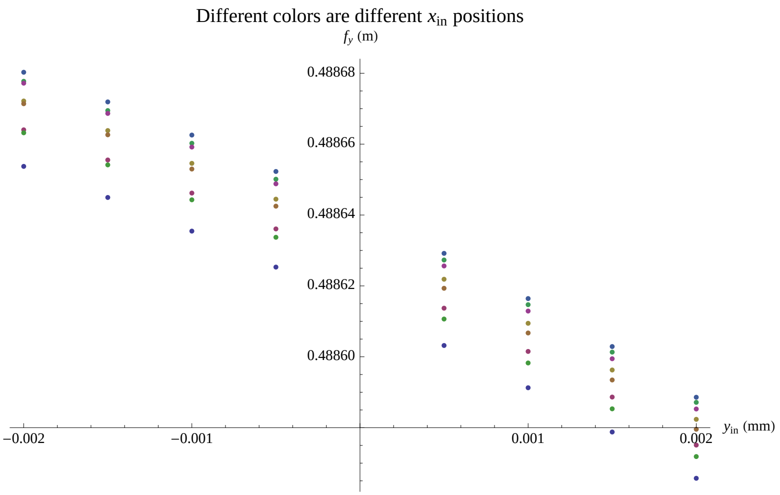
$$f_j = \frac{{y^{in}}_j}{y'_j} \tag{11}$$





The results are consistent with a small sextupole effect.

Tracking
focus for different x positions



Conclusions

The code will be used to simulate the QD0 doublet, to investigate the value of the multipole components, and to simulate all the final focus magnets.