Correlated D decays at the $\Psi(3770)$

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We want to calculate the correlated amplitude for the D and the $\overline{D}$ to decay to the states α and β at times t_1 and t_2 respectively, where the times are measured in the center-of-mass (CM) system and $t = 0$ is the time of the $e^+e^- \to c\overline{c}$ production. Because the $\Psi(3770)$ is $J^{PC} = 1^{--}$ state, we antisymmetrize the amplitude with respect to charge conjugation.

$$\mathcal{M} = \frac{1}{\sqrt{2}}\left[\langle\alpha|\mathcal{H}|D^0(t_1)\rangle\langle\beta|\mathcal{H}|\overline{D}^0(t_2)\rangle - \langle\beta|\mathcal{H}|D^0(t_2)\rangle\langle\alpha|\mathcal{H}|\overline{D}^0(t_1)\rangle\right] \tag{1}$$

The time evolution of the D^0–$\overline{D}^0$ system is described by the Schrödinger equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(\mathrm{M} - \frac{i}{2}\Gamma\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}, \tag{2}$$

where the M and Γ matrices are Hermitian, and CPT invariance requires $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$.

The two eigenstates D_1 and D_2 of the effective Hamiltonian are

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle \ , \qquad |p|^2 + |q|^2 = 1\,. \tag{3}$$

The corresponding eigenvalues are

$$\lambda_{1,2} \equiv m_{1,2} - \frac{i}{2}\Gamma_{1,2} = \left(M - \frac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right), \tag{4}$$

where $m_{1,2}$, $\Gamma_{1,2}$ are the masses and decay widths and

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \quad \left(\rightarrow \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \text{ for } B_d\right)\,. \tag{5}$$

The proper time evolution of the eigenstates of Eq. 2 is

$$|D_{1,2}(t)\rangle = e_{1,2}(t)|D_{1,2}\rangle,\ e_{1,2}(t) = e^{[-i(m_{1,2} - \frac{i\Gamma_{1,2}}{2})t]}. \tag{6}$$

A state that is prepared as a flavor eigenstate $|D^0\rangle$ or $|\overline{D}^0\rangle$ at $t = 0$ will evolve according to

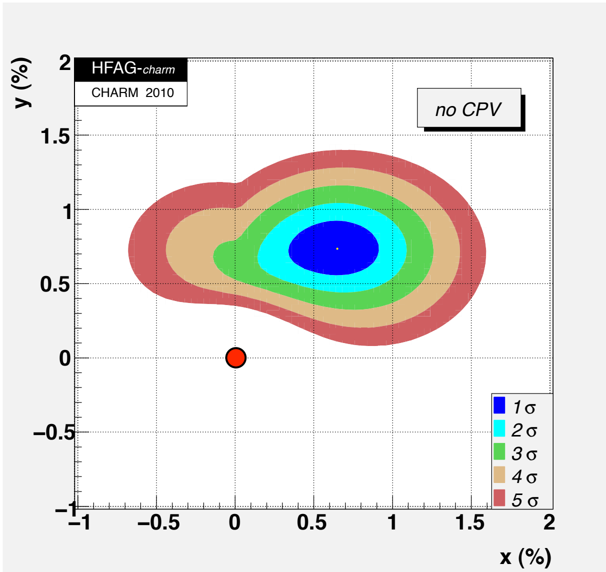
$$|D^0(t)\rangle = \frac{1}{2p}\Big[p(e_1(t)+e_2(t))|D^0\rangle + q(e_1(t)-e_2(t))|\overline{D}^0\rangle\Big] \tag{7}$$

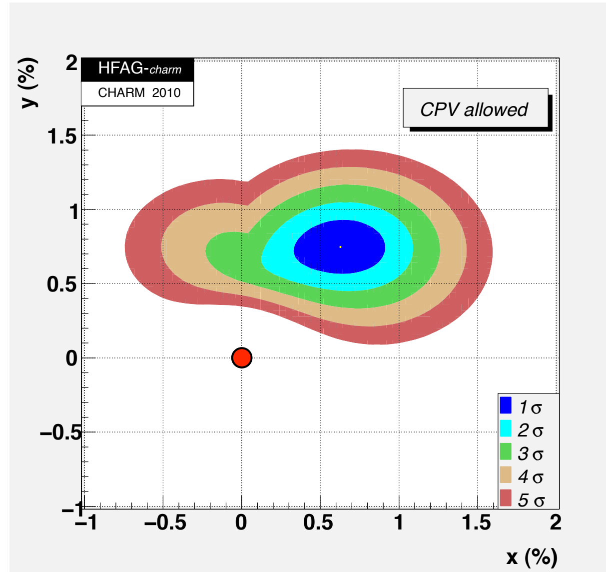
$$|\overline{D}^0(t)\rangle = \frac{1}{2q}\Big[p(e_1(t)-e_2(t))|D^0\rangle + q(e_1(t)+e_2(t))|\overline{D}^0\rangle\Big]\,. \tag{8}$$

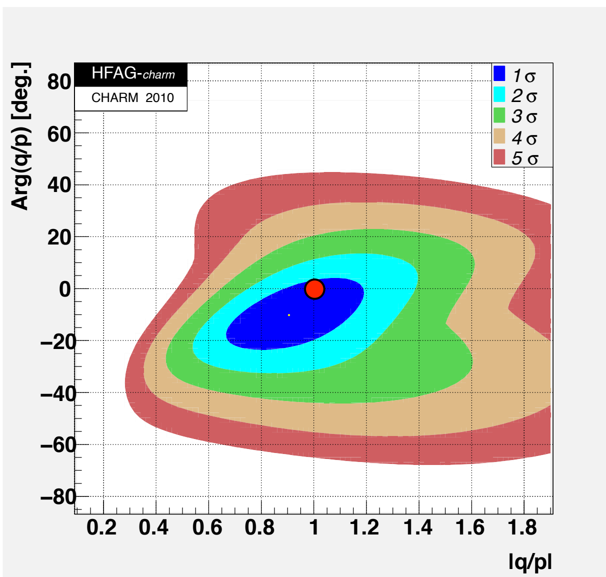
We adopt a version of the standard notation

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}, \qquad x = \frac{m_1 - m_2}{\Gamma}, \qquad y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}\,. \tag{9}$$

HFAG Summary from October, 2010







Fit to all time-dependent CPV measurements.

CPV-allowed plot, no mixing (x,y) = (0,0) point: $\Delta\chi^2 = 109.6$, CL = 1.56 x 10^{-24}, no mixing excluded at 10.2σ

No CPV (|q/p|, φ) = (1,0) point: $\Delta\chi 2 = 1.218$, CL = 0.456 , consistent with CP conservation

Forms of $\mathcal{M}$ and $|\mathcal{M}|^2$

After a bit of algebra we can write the matrix element as

$$2\sqrt{2}\,\mathcal{M} = \left(\frac{q}{p}\overline{\mathcal{A}}_\alpha\overline{\mathcal{A}}_\beta - \frac{p}{q}\mathcal{A}_\alpha\mathcal{A}_\beta\right)[e_1(t_1)e_2(t_2) - e_1(t_2)e_2(t_1)] + (\mathcal{A}_\alpha\overline{\mathcal{A}}_\beta - \overline{\mathcal{A}}_\alpha\mathcal{A}_\beta)[e_1(t_1)e_2(t_2) + e_1(t_2)e_2(t_1)] \qquad (10)$$

which has the form

$$2\sqrt{2}\,\mathcal{M} = X(e_{11}e_{22} - e_{12}e_{21}) + Y(e_{11}e_{22} + e_{12}e_{21})\,. \qquad (11)$$

From this one calculates

$$8|\mathcal{M}|^2 = e^{-\Gamma(t_1+t_2)} \times \{ XX^*(\cosh y\Gamma\Delta t - \cos x\Gamma\Delta t) - 2\,\Re(XY^*)\sinh y\Gamma\Delta t + 2\,\Im(XY^*)\sin x\Gamma\Delta t + YY^*(\cosh y\Gamma\Delta t + \cos x\Gamma\Delta t\,\} \qquad (12)$$

For $x\Gamma\Delta t$, $y\Gamma\Delta t \ll 1$ this can be approximated by

$$4|\mathcal{M}|^2 = e^{-\Gamma(t_1+t_2)} \times \left\{ XX^*\left[\frac{(x^2+y^2)}{4}(\Gamma\Delta t)^2\right] - \Re(XY^*)\,y\Gamma\Delta t + \Im(XY^*)\,x\Gamma\Delta t + YY^*\left[1 + \frac{(y^2-x^2)}{4}(\Gamma\Delta t)^2\right]\right\} \qquad (13)$$

- Y is the unmixed amplitude
- X is the mixing amplitude
- XY^* controls the interference terms in the mixing rate

Some General Observations

- Each of X and Y is the difference of two products of amplitudes; the difference reflects the charge conjugation symmetry of the initial $D^0\overline{D}^0$ state.
- The components of the decay rate proportional to the real and imaginary parts of XY^* corresponds to the interference of the direct and mixing amplitudes to a common final state.
- The relative time-dependence dependence of the interference term is proportional to $y' \Gamma \Delta t$ where $y' = y \cos\delta + x \sin\delta$ with $XY^* = Ce^{i\delta}$ (C and δ real).
- The phase δ depends upon the phase of p/q and also on both the final state α and the final state β.
- The interference term is odd in $\Gamma\Delta t$ while the pure mixing and unmixed terms are even in $\Gamma\Delta t$. Thus, the interference term disappears when considering only time-integrated decay rates.

We make some back-of-the envelope calculations of sensitivity to mixing and CP violation making a number of assumptions. The numbers must be refined to be considered more than rough estimates. However, they can guide thinking about which channels warrant detailed study. We will assume that

- we can scale from CLEO-c's 281 fb^{-1} sample to a Super$B$ sample using a factor of 1500. This corresponds about 500 fb^{-1} of data with a somewhat lower efficiency for tighter cuts related to vertex resolution.
- we measure time-dependent asymmetries for $|\Delta t| > 2\tau_{D^0}$ perfectly and we have no sensitivity to asymmetries for lower values of $|\Delta t|$.
- we sometimes estimate the fraction of events with $|\Delta t| > 2\tau_{D^0}$ to be $1/e^2$ and the average value of $|\Gamma\Delta t|$ for these events to be 3.

CP even versus CP even

For two CP-even eigenstates α and β,

$$\begin{aligned} Y &= 0 \\ X &= \left(\frac{q}{p} - \frac{p}{q}\right) \mathcal{A}_\alpha \mathcal{A}_\beta \,. \end{aligned} \tag{14}$$

so the rate is

$$|\mathcal{M}|^2 = e^{-\Gamma(t_1+t_2)} \times \left|\frac{q}{p} - \frac{p}{q}\right|^2 |A_\alpha|^2 \, |A_\beta|^2 \left(\frac{x^2+y^2}{4}\right) (\Gamma \Delta t)^2 \,. \tag{15}$$

In the limit that CP is a good symmetry, this rate goes to zero. To estimate what might be possible at Super$B$, we take the numbers of $K^{\mp}\pi^{\pm}$ versus CP even events observed by CLEO-c (605), scale by the approximate ratio of K^-K^+ plus $\pi^-\pi^+$ events observed (≈ 0.13) [to account for the value of $|A_\alpha|^2 \, |A_\beta|$], and scale by the nominal relative luminosity. This procedure gives approximately 120K as the coefficient of $(x^2+y^2)\,(\Gamma\Delta t)^2/4$. Using $(x^2+y^2)\,(\Gamma\Delta t)^2/2$ as an estimate of the time integral, and taking $x^2+y^2 = 10^{-4}$, the integrated signal will be about

$$\left|\frac{q}{p} - \frac{p}{q}\right|^2 \times 6 \text{ events} \,. \tag{16}$$

$K^-\pi^+$ versus $K^-\pi$

A similar result obtains for common final states such as $K^-\pi^+$. If $\alpha = \beta$ then $\mathcal{A}_\beta = \mathcal{A}_\alpha$ and $\overline{\mathcal{A}}_\beta = \overline{\mathcal{A}}_\alpha$. Again, the unmixed amplitude goes to zero. However, the pure mixing term does not require CP violation to be non-zero.

$$\begin{aligned} Y &= 0 \\ X &= \left(\frac{q}{p}\overline{\mathcal{A}}_\alpha\overline{\mathcal{A}}_\alpha - \frac{p}{q}\mathcal{A}_\alpha\mathcal{A}_\alpha\right). \end{aligned} \tag{17}$$

In this case, $\mathcal{A}_\alpha$ corresponds to the Cabibbo-favored amplitude and $\overline{\mathcal{A}}_\alpha$ to the doubly Cabibbo-suppressed amplitude. With $\overline{\mathcal{A}}_\alpha = ke^{i\delta}\mathcal{A}_\alpha$ the rate can be written

$$|\mathcal{M}|^2 = e^{-\Gamma(t_1+t_2)} \times \left|\frac{q}{p}k^2e^{i2\delta} - \frac{p}{q}\right|^2 |A_\alpha|^2 |A_\alpha|^2 \left(\frac{x^2+y^2}{4}\right)(\Gamma\Delta t)^2 . \tag{18}$$

As a first approximation, we can ignore both the doubly Cabibbo-suppressed amplitude and CP violation. In this case

$$|\mathcal{M}|^2 \approx e^{-\Gamma(t_1+t_2)} \times |A_\alpha|^2 |A_\alpha|^2 \left(\frac{x^2+y^2}{4}\right)(\Gamma\Delta t)^2 . \tag{19}$$

CLEO-c observes 600 $K^-\pi^+$, $K^+\pi^-$ events, which corresponds to $2\,|A_\alpha|^2\,|A_\alpha|^2$. Scaling by relative luminosities, and again using 10^{-4} for (x^2+y^2), we can project a mixing signal of 23 events in this channel and a similar number in $K^+\pi^-$ versus $K^+\pi^-$. While differences nominally can be due to direct CP violation, indirect CP violation, or statistical fluctuation, given the existing HFAG bounds on direct and indirect CP violation, any variation we observe in this channel will be predominantly due to statistical fluctuations.

Opposite-sign semileptonic final states

For opposite-sign semileptonic decays we can choose $\alpha = K^-\ell^+\nu$ and $\beta = K^+\ell^-\overline{\nu}$ for which

$$\begin{aligned} Y &= \mathcal{A}_\alpha \overline{\mathcal{A}}_\beta \\ X &= 0 \end{aligned} \tag{20}$$

The rate is proportional to

$$|\mathcal{M}|^2 = e^{-\Gamma(t_1+t_2)} \times |\mathcal{A}_\alpha|^2 \, |\mathcal{A}_\beta|^2 \left[1 + \frac{(y^2 - x^2)}{4}(\Gamma\Delta t)^2\right] . \tag{21}$$

CLEO-c has not reported a signal in the corresponding opposite-sign dilepton channel, but we can optimistically estimate that the rate will be similar to that for $(K^-\pi^+)$ versus $K^+\pi^-$. This allows us to estimate 900K $K^-e^+\nu_e$ versus $K^+e^-\overline{\nu}_e$ events.

The only signature of mixing in this final state is the quadratic departure from purely exponential decay which is proportional to $(y^2 - x^2)$. This is less than one part in 10^4, significantly less than the rate of statistical fluctuations. This final state has no sensitivity to CP violation.

Same-sign semileptonic final states

For same-sign semileptonic decays we can choose $\alpha = \beta = K^-\ell^+\nu$. In this case

$$\begin{aligned} Y &= 0 \\ X &= -\frac{p}{q}\left(\mathcal{A}(D^0 \to K^- e^+ \overline{\nu}_e)\right) . \end{aligned} \tag{22}$$

The corresponding rate is

$$|\mathcal{M}|^2 = e^{-i\Gamma(t_1+t_2)} \left| \left(\frac{p}{q}\right) \mathcal{A}_\alpha \mathcal{A}_\beta \right|^2 \left(\frac{x^2+y^2}{4}\right) (\Gamma\Delta t)^2 . \tag{23}$$

Using the same assumptions as for the opposite-sign dilepton events, we again estimate 23 mixing events in each of $K^-e^+\nu_e$ versus $K^-e^+\nu_e$ and $K^+e^-\overline{\nu}_e$ versus $K^+e^-\overline{\nu}_e$.

This is a bit optimistic as the branching fraction for $Ke\nu$ is less than that for $K\pi$, and also because the efficiencies are likely to lower, the backgrounds higher, and the vertex resolutions worse.

Semileptonic versus a CP eigenstate - I

The correlated decays of $D^0\overline{D}^0$ into a CP eigenstate and and semileptonic final state are also (relatively) easy to understand. Consider $\mathcal{A}_\alpha = \mathcal{A}(D^0 \to K^- e^+ \nu_e)$ and $\mathcal{A}_\beta = \mathcal{A}(D^0 \to K^- K^+)$ as an example such a final state. In this case

$$Y = \mathcal{A}_\alpha \mathcal{A}_\beta \,; \qquad X = -\frac{p}{q} \mathcal{A}_\alpha \mathcal{A}_\beta \tag{24}$$

The interference terms proptional to $y\,\Gamma\Delta t$ and $x\,\Gamma\Delta t$ in the decay rate, see Eqn. (13), are proportional to the real and imaginary parts of

$$XY^* \;=\; (-\frac{p}{q}\mathcal{A}_\alpha \mathcal{A}_\beta)\,(\mathcal{A}^*_\alpha \mathcal{A}^*_\beta) \;=\; -\frac{p}{q} |A_\alpha|^2\, |\mathcal{A}_\beta|^2 \tag{25}$$

which are directly proportional to the real and imaginary parts of p/q. There is no sensitivity to strong phase differences between decays of D^0 and $\overline{D}^0$ to the same final state in this case. If one replaces the CP even final state with a CP odd final state, the interference term changes sign

$$XY^* \;=\; (-\frac{p}{q}\mathcal{A}_\alpha \mathcal{A}_\beta)\,(-\mathcal{A}^*_\alpha \mathcal{A}^*_\beta) \;=\; +\frac{p}{q} |A_\alpha|^2\, |\mathcal{A}_\beta|^2 \,. \tag{26}$$

The (small $y\Gamma\Delta t$, small $x\Gamma\Delta t$) limit for $D^0 \to K^- \ell^+ X$ opposite CP eigenstates is

$$|\mathcal{M}|^2 \;=\; e^{-\Gamma(t_1+t_2)}\, |\mathcal{A}_\alpha|^2\, |\mathcal{A}_\beta|^2 \;\times \tag{27}$$
$$\left\{ 1 \mp \; \Re(\frac{p}{q})\, y\Gamma\Delta t \;\pm\; \Im(\frac{p}{q})\, x\Gamma\Delta t + \frac{y^2}{2}(\Gamma\Delta t)^2 \right\} .$$

Semileptonic versus a CP eigenstate - II

For $\overline{D}^0 \to K^+\ell^- X$ detected in conjunction with a CP even final state, $(-p/q)$ in XY^* becomes $(+q/p)$ and $\mathcal{A}_\alpha = \mathcal{A}(\overline{D}^0 \to K^+\ell^- X)$. As a first approximation, the difference between positive and negative decay time distributions will be proportional to

$$\left(\Re(\frac{p}{q})\, y - \Im(\frac{p}{q})\, x\right) \times \Gamma\, |\Delta t| \;=\; y'\Gamma\, |\Delta t| \tag{28}$$

for $D^0 \to K^-\ell^+ X$ and to

$$\left(\Re(\frac{q}{p})\, y - \Im(\frac{q}{p})\, x\right) \times \Gamma\, |\Delta t| \;=\; y''\Gamma\, |\Delta t| \tag{29}$$

for $\overline{D}^0 \to K^+\ell^- X$. For each sign of $Ke\nu$ we estimate $1500 \times 150 = 225$K reconstructed events based on CLEO-c's observed rates of $Xe\nu$ versus K^-K^+ and $\pi^-\pi^+$. Of these we estimate that $1/e^2$ (30K) will be produced with $|\Gamma\Delta t| > 2$ with $\langle|\Gamma\Delta t|\rangle = 3$. Assuming $y' = 0.01$, the $y'^{(\prime)}\Gamma\,|\Delta t|$ term will create a surplus of 913 events for $\Delta t < 0$ and a deficit of 913 events for $\Delta t > 0$ out of $\approx$ 60K events with $\Gamma\,|\Delta t| > 2$ for an asymmetry of 1827 ± 247 events.

Same-sign semileptonic versus hadronic - I

The correlated decays to a semileptonic final state and a hadronic non-CP eigenstate are somewhat more complicated. For the final state $(K^-\pi^+, K^-e^+\nu_e)$ we can write

$$\begin{aligned}
\mathcal{A}_\alpha &= \mathcal{A}(D^0 \to K^-\pi^+) = ae^{i(\delta+\phi)} \\
\overline{\mathcal{A}}_\alpha &= ke^{i\delta_{K\pi}}\mathcal{A}_\alpha \\
\mathcal{A}_\beta &= \mathcal{A}(D^0 \to K^-e^+\nu_e) \\
\overline{\mathcal{A}}_\beta &= 0
\end{aligned}$$

where a, δ, ϕ, k and $\delta_{K\pi}$ are real numbers. Writing $\mathcal{A}_\alpha$ in the form $ae^{i(\delta+\phi)}$ will be useful when we consider final states including a $K^+\pi^-$. The factor $k \approx \tan^2\theta_C$ is the ratio of the magnitudes of the doubly Cabibbo-suppressed (DCS) and Cabibbo-favored (CF) amplitudes. The angle $\delta_{K\pi}$ is the relative strong phase between the CF and DCS amplitudes to the same final state. The mixing and direct amplitudes for $(K^-\pi^+, K^-e^+\nu_e)$ are

$$\begin{aligned}
X &= -\frac{p}{q}\mathcal{A}_{\alpha\mathcal{A}\beta} \\
Y &= ke^{i\delta_{k\pi}}\mathcal{A}_\alpha\mathcal{A}_\beta
\end{aligned}$$

The mixing, interference, and direct terms in the decay rate are

$$\begin{aligned}
XX^* &= \left|\frac{p}{q}\right|^2 |\mathcal{A}_\alpha|^2|\mathcal{A}_\beta|^2 \\
XY^* &= \frac{p}{q}e^{-i\delta_{K\pi}}k|\mathcal{A}_\alpha|^2|\mathcal{A}_\beta|^2 \\
YY^* &= k^2|\mathcal{A}_\alpha|^2|\mathcal{A}_\beta|^2
\end{aligned}$$

Same-sign semileptonic versus hadronic - II

The (small $y\Gamma\Delta t$, small $x\Gamma\Delta t$) limit for the $(K^-\ell^+X, K^-\pi^+)$ decay rate is

$$
\begin{aligned}
|\mathcal{M}|^2 \;=\; & \frac{1}{4} e^{-\Gamma(t_1+t_2)} \,|\mathcal{A}_\alpha|^2\,|\mathcal{A}_\beta|^2 \,\times \left\{ \left|\frac{p}{q}\right|^2 \left(\frac{x^2+y^2}{4}\right) (\Gamma\Delta t)^2 \right. \\
& - \left(\Re(\frac{p}{q}) \cos\delta_{K\pi} + \Im(\frac{p}{q}) \sin\delta_{K\pi} \right) k\, y\Gamma\Delta t \\
& + \left(\Im(\frac{p}{q}) \cos\delta_{K\pi} - \Re(\frac{p}{q}) \sin\delta_{K\pi} \right) k\, x\Gamma\Delta t \\
& \left. + k^2 \left[1 + \left(\frac{y^2 - x^2}{4}\right) (\Gamma\Delta t)^2 \right] \right\} .
\end{aligned}
\tag{30}
$$

To make a back-of-the envelope sensitivity estimate, we consider

- the limit $p = q$ and $\cos\delta_{K\pi} = 1$
- wth $x = 0$, $y = 0.01$ and $k^2 = 0.003$.

The rate now has the form

$$
|\mathcal{M}|^2 \propto k^2 - ky(\Gamma\Delta t) + \frac{y^2(1+k^2)}{4}(\Gamma\Delta t)^2 \,. \tag{31}
$$

Same-sign semileptonic versus hadronic - III

For the $(K^-\ell^+X, K^-\pi^+)$ with

- the limit $p = q$ and $\cos\delta_{K\pi} = 1$
- and assuming $x = 0$, $y = 0.01$, $k^2 = 0.003$.

the rate now has the form

$$|\mathcal{M}|^2 \propto k^2 - ky(\Gamma\Delta t) + \frac{y^2(1+k^2)}{4}(\Gamma\Delta t)^2\,, \tag{32}$$

We have used Mathematica to compute the total rate and the rates for $|\Gamma\Delta t| > 2$ in terms of the corresponding opposite-sign rate. As a good approximation,

- the total rate just the doubly-Cabibbo suppressed rate, 0.003,
- the integrated rate for $\Gamma\Delta t < -2$ is $\approx 3.3\times 10^{-4}$, and
- the integrated rate for $\Gamma\Delta t > 2$ is $\approx 1.1\times 10^{-4}$.

CLEO-c observes $\approx$ 1175 events in each of $(X^+e^-\overline{\nu}_e, K^-\pi^+)$ and $(X^-e^+\nu_e, K^+\pi^-)$. In Super$B$ we therefore expect

- 1.76×10^6 events for each opposite-sign combination,
- $\approx$ 5300 events for each same-sign combination,
- 584 observed with $\Gamma\Delta t < -2$ and 191 observed with $\Gamma\Delta t > 2$
- for a summed asymmetry of 800 ± 40 events.

CP eigenstates versus hadronic non-CP - I

The correlated decays to a CP eigenstate and a hadronic non-CP eigenstate are somewhat more complicated. Consider, as a first example, the final state $(K^-\pi^+, K^-K^+)$. We can write

$$\begin{aligned}
\mathcal{A}_\alpha &= \mathcal{A}(D^0 \to K^-\pi^+) = ae^{i(\delta+\phi)} \\
\overline{\mathcal{A}}_\alpha &= ke^{i\delta_{K\pi}}\mathcal{A}_\alpha \\
\mathcal{A}_\beta &= \mathcal{A}(D^0 \to K^-K^+) \\
\overline{\mathcal{A}}_\beta &= \mathcal{A}_\beta
\end{aligned}$$

The mixing and direct amplitudes for $(K^-\pi^+, K^-K^+)$ are

$$\begin{aligned}
X &= \left(\frac{q}{p}ke^{i\delta_{K\pi}} - \frac{p}{q}\right)\mathcal{A}_\alpha\mathcal{A}_\beta \\
Y &= (1 - ke^{i\delta_{K\pi}})\mathcal{A}_\alpha\mathcal{A}_\beta
\end{aligned}$$

As is well-known, the time-integrated rate is dominated by the term

$$YY^* = (1 - 2k\cos\delta_{K\pi} + k^2)\mathcal{A}_\alpha\overline{\mathcal{A}}^*_\alpha\mathcal{A}_\beta\overline{\mathcal{A}}^*_\beta \tag{33}$$

which depends linearly on $\cos\delta_{K\pi}$. CLEO-c observes about 60 events in each sign of $(K^\mp\pi^\pm, K^-K^+$.

- assuming $2k\cos_{\delta_{K\pi}} \approx 2\cdot\sqrt{0.003}\cdot 1$,
- the total signal is $\approx$ 180K events $\pm$425,
- differs from the $2k\cos_{\delta_{K\pi}} = 0$ value by $\approx$ 20K $\pm$425,
- indicates we can measure $\cos\delta_{K\pi}$ with 2% precision.

CP eigenstates versus hadronic non-CP - II

The real and imaginary parts of the interference term are

$$\Re(XY^*) = k\left(1 + \left|\frac{q}{p}\right|^2\right)\left[\Re\left(\frac{p}{q}\right)\cos\delta - \Im\left(\frac{p}{q}\right)\sin\delta\right] - \Re\left(\frac{p}{q}\right)(1+k^2) \tag{34}$$

$$\Im(XY^*) = k\left(1 - \left|\frac{q}{p}\right|^2\right)\left[\Im\left(\frac{p}{q}\right)\cos\delta + \Re\left(\frac{p}{q}\right)\sin\delta\right] - \Im\left(\frac{p}{q}\right)(1+k^2)$$

Again, we estimate sensitivity to mixing

- in the limit $p = q$
- assuming we detect $1/e^2$ of the events with $|\Gamma\Delta t| > 2$ with average $|\Gamma\Delta t| = 3$,
- assuming $1 - 2k\cos\delta_{K\pi} = 0.89$ and $y = 0.01$.

We then expect to observe an asymmetry of 650 ± 156 events.

Summary of Calculations to Date

We have made rough estimates of Super$B$ sensitivity to mixing assuming

- events rates scale from CLEO-c,
- Super$B$ integrated $\mathcal{L} = 500\,\mathrm{fb}^{-1}$,
- we can cleanly separate $|\Gamma\Delta t| > 2$ from $|\Gamma\Delta t| < 2$,
- $p/q \approx 1$,
- $y \approx 0.01$

channel	type of measurement	figure of merit
$K^-K^+, \pi^-\pi^+$ v $K^-K^+, \pi^-\pi^+$	integrated	$\|q/p - p/q\|^2 \times 6$ events
$K^-\pi^+$ v $K^-\pi^+$ + cc	integrated	46 events
$K^-e^+\nu$ v $K^-e^+\nu$ + cc	integrated	46 events
$K^-e^+\nu$ v $K^-K^+, \pi^-\pi^+$ + cc	TDA	1887 ± 247 events ($\sim 7\sigma$)
$K^-e^+\nu$ v $K^-\pi^+$, + cc	TDA	800 ± 40 events ($\sim 20\sigma$)
$K^-\pi^+$ v $K^-K^+, \pi^-\pi^+$ + cc	integrated	$\cos\delta_{K\pi} \sim \pm 2\%$
$K^-\pi^+$ v $K^-K^+, \pi^-\pi^+$ + cc	TDA	650 ± 156 events ($\sim 4\sigma$)

Sensitivity to mixing (and CP violation) is greatest when the interference term is as large as possible compared to the direct correlated decay term. This requires "same-sign" decays with a DCS amplitude interfering with a CF amplitude.

Future Directions - I

The channel studied with the greatest mixing/CP violation reach is

- $(K^- e^+ \nu_e,\ K^- \pi^+) + \mathrm{cc}$

where the measurable time-dependent asymmetry is estimated to be $20\,\sigma$. Other correlated final states whose rates will be dominated by one or more DCS amplitudes and will enjoy a relatively large interference terms include

- $K^- e^+ \nu_e,\ K^- \pi^+ \pi^0 + \mathrm{cc}$
- $K^- e^+ \nu_e,\ K^- \pi^- \pi^+ \pi^+ + \mathrm{cc}$
- $K^- \pi^+,\ K^- \pi^+ \pi^0 + \mathrm{cc}$
- $K^- \pi^+,\ K^- \pi^- \pi^+ \pi^+ + \mathrm{cc}$
- $K^- \pi^+ \pi^0,\ K^- \pi^- \pi^+ \pi^+ + \mathrm{cc}$

"Same-sign" events in which both Ds are observed in the same hadronic final state, but at different points in phase space (the Dalitz plot, for three-body channels) may also manifest large time-dependent assymmetries, at least in parts of the phase space. If this is true, we may be able to exploit

- $K^- \pi^+ \pi^0,\ K^- \pi^+ \pi^0 + \mathrm{cc}$
- $K^- \pi^- \pi^+ \pi^+,\ K^- \pi^- \pi^+ \pi^+ + \mathrm{cc}$

will similar benefit.

Future Directions - II

In traditional (single-tag) analyses, the final state $K^0_S\pi^-\pi^+$ has been especially useful for studying mixing as the interference of CF and DCS amplitudes produces time-dependent rate variations as a function of position in the Dalitz plot. Because there are intermediate amplitudes which are CP eigenstates, both x and y can be extracted without confusion due to strong phase differences between CF and DCS amplitudes. This suggests the possibility that the correlated final states

- $K^-e^+\nu_e$, $K^0_S\pi^-\pi^+$ + cc
- $K^-\pi^+$, $K^0_S\pi^-\pi^+$ + cc
- $K^-\pi^+\pi^0$, $K^0_S\pi^-\pi^+$ + cc
- $K^-\pi^-\pi+\pi+$, $K^0_S\pi^-\pi^+$ + cc
- $K^0_S\pi^-\pi^+$, $K^0_S\pi^-\pi^+$ + cc

will be similarly useful.

Conclusions

We have made rough estimates of Super$B$ sensitivity to mixing assuming

- events rates scale from CLEO-c,
- Super$B$ integrated $\mathcal{L} = 500\,\text{fb}^{-1}$,
- we can cleanly separate $|\Gamma\Delta t| > 2$ from $|\Gamma\Delta t| < 2$,
- $p/q \approx 1$,
- $y \approx 0.01$

It appears that

- Sensitivity to mixing (and CP violation) is greatest when the interference term is as large as possible compared to the direct correlated decay term. This requires "same-sign" decays with a DCS amplitude interfering with a CF amplitude.
- $K^- e^+ \nu_e$, $K^- \pi^+$ + cc allows 20σ measurement of mixing
- at least 5 other same-sign channels promise similar mixing sensitivity
- 2 additional channels with same sign decays to different points in phase space are probably similarly sensitive
- correlated final states with at least one $K^0_S \pi^- \pi^+$ may also be useful
- measuring time-dependent asymmetries down to $|\Gamma\Delta t| = 1$, can increase the effective statistics substantially.

Time-dependent measurements of asymmetries in correlated decays at the $\Psi(3770)$ may allow mixing parameters to be determined with 1% - 2% precision.