Thermal-noise reduction ideas

Elba
May 26, 2011

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K.Somiya

Ideas

(1) Sub-wavelength layered coatings [I.Pinto]

→ R.DeSalvo

(2) Multi-probe thermal noise analysis [K.Somiya]

(3) Stress-optic noise [S.Ballmer]

(4) TBA [R.Adhikari]

(5) Discussions

Sub-wavelength layered coatings

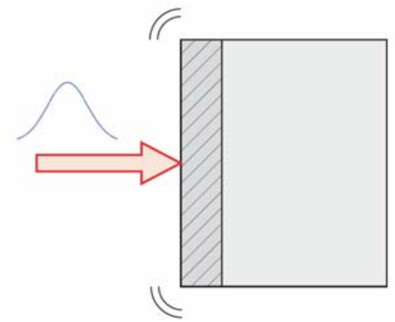
I.Pinto

→ R.DeSalvo

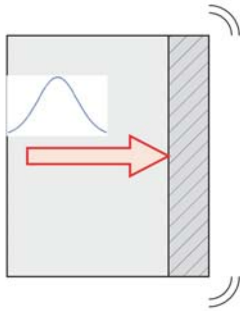
Multi-probe TN analysis

K.Somiya

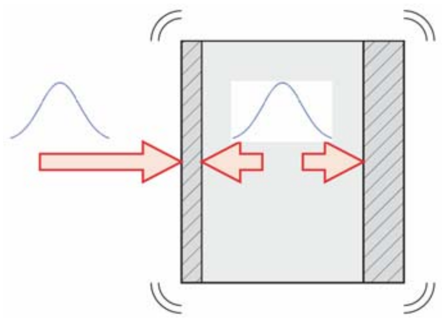
Contents of the talk



- Harry's method of coating TN (review)



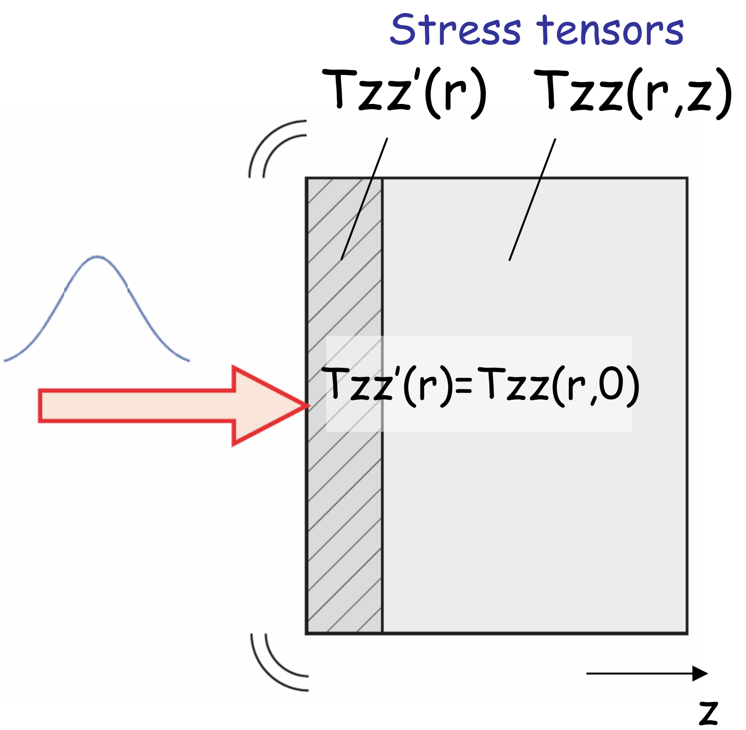
- Yanbei's idea of flipping a mirror
- Khalili's Etalon for TN reduction



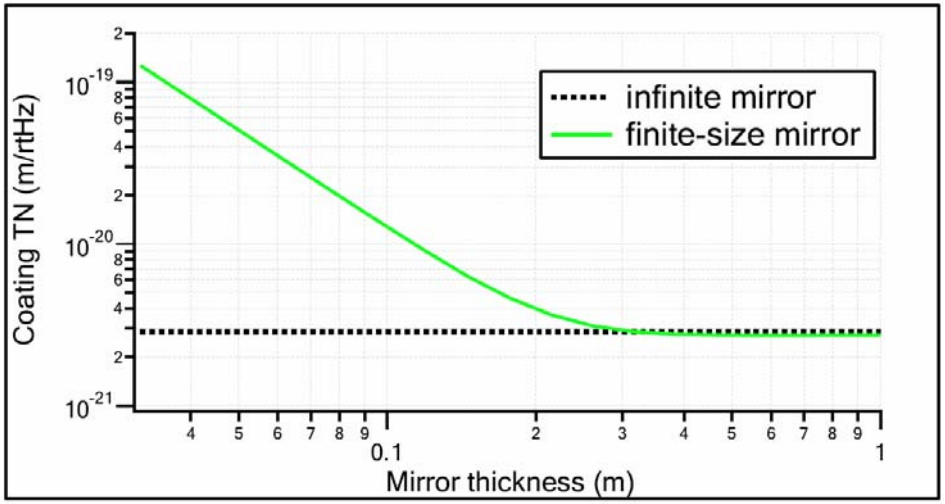
- Multi-layer correction

Coating TN of a conventional mirror

- Solve the Elastic equation of a cylinder
- Extend it with a mono-layer thin coating
- Apply imaginary force on front surface
- Sum up the dissipation in the coating
- TN spectrum is given by FdT

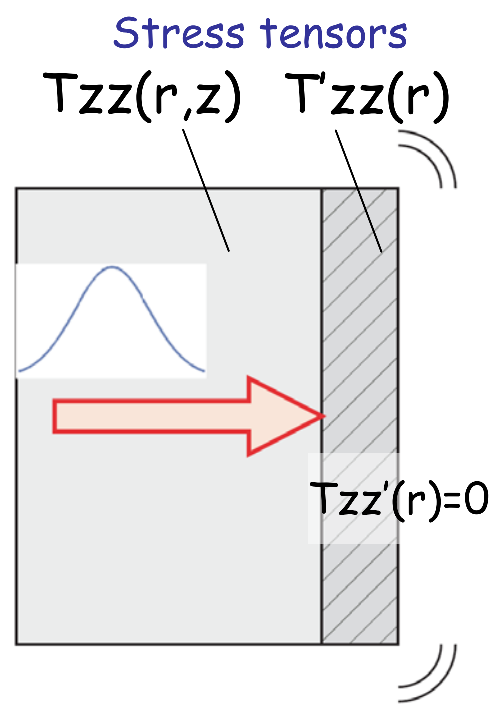


Ex) ET mirror [mirror radius=31cm]



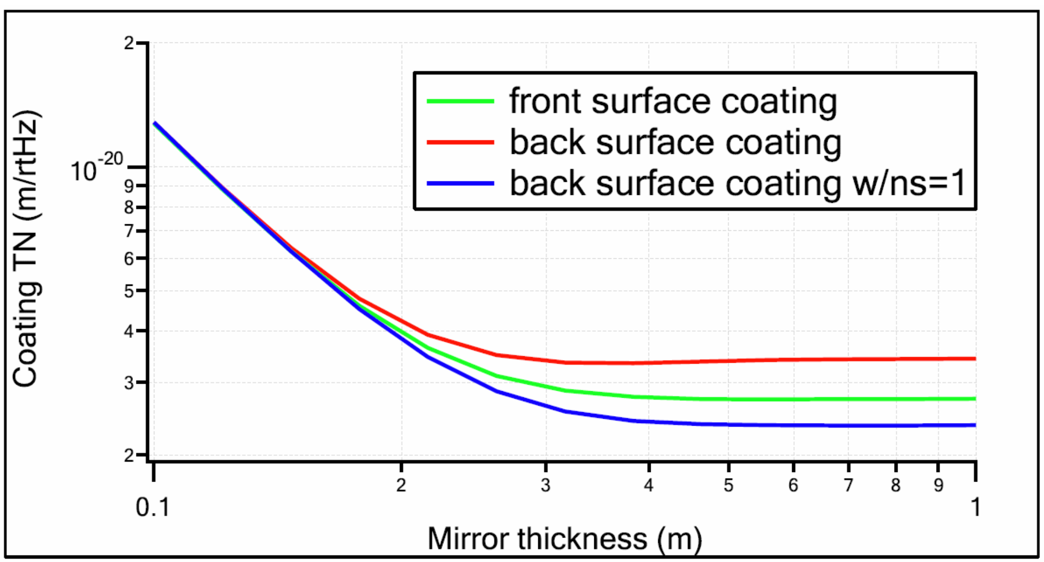
- Coating TN depends on the aspect ratio of the mirror
- Namely, the shear motion is dominant (mirror deformation)

Flipping the mirror



- No contribution of thickness change
- Shear motion still remains
- Elastic eq solution is the same for substrate
- Tzz' is zero

Ex) ET mirror [mirror radius=31cm]

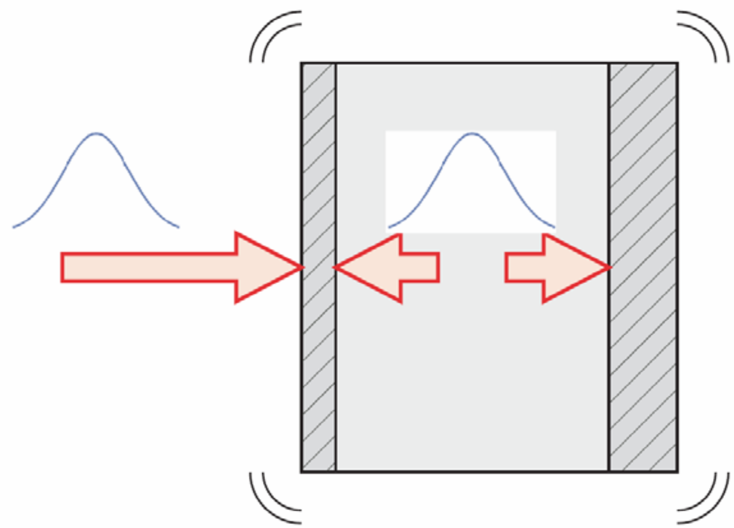


- It'd decrease by 14% if substrate ref-index were 1 (thickness change contribution = 14%)
- In reality, TN increases by 25% (more sensitive to phase shift)
- 2 imaginary forces necessary for the rigorous calculation

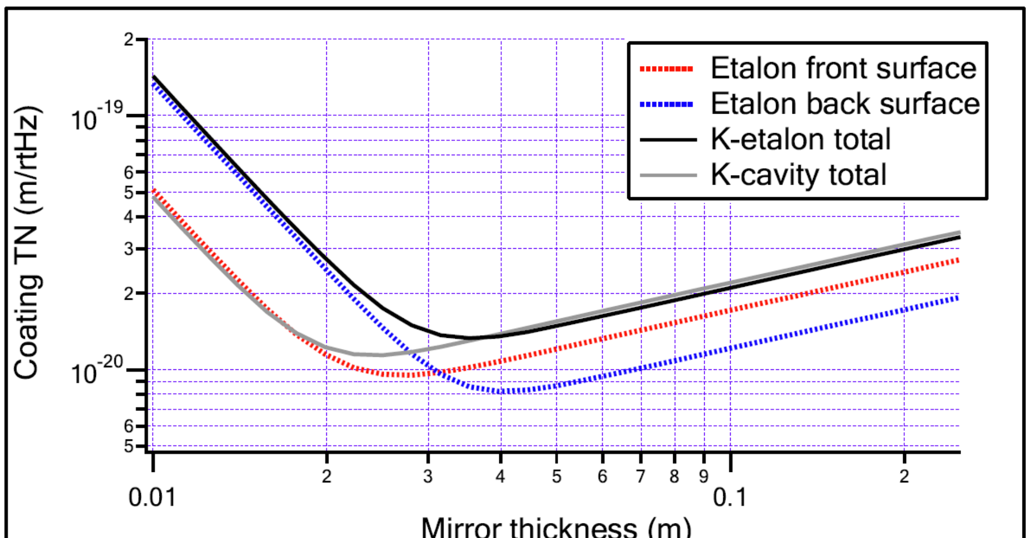
Khalili Etalon

- Etalon version of Khalili cavity
- 2 layers on front, 15 layers on back
- high reflectivity, lower TN in total
- 3 imaginary forces (probes)

  [calculation complexity is same as 2 probes]



Ex) AEI10m sub-SQL IFO ETM [mass=100g]



2-layer coating is interesting

- Mono-layer approximation breaks
- more TE-TR noise cancellation
- BR-dn/dV noise cancellation?

[-> Stefan's talk]

Multi-layer treatment

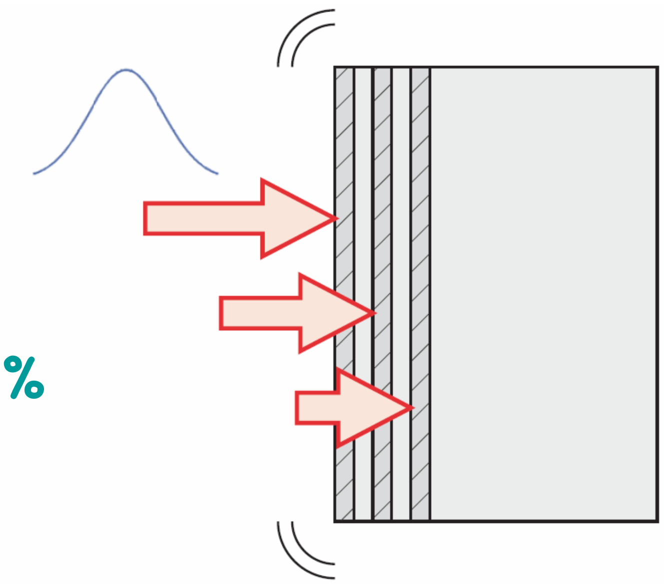
Gurkovsky PLA 2010

- N imaginary forces (N=num of layers)

  [calculation complexity is same as 2 probes]

- Correction factor for aLIGO is 2.8%

- Correction factor for ET with KE is 8.3%

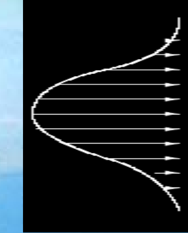


Summary

- Interesting TN analysis with multi-probes
- Interesting feature with a few layer coatings

Stress-optic noise

S.Ballmer



Stress-optic noise?

GWADW
La Biodola,
Isola d'Elba, Italy

May 2011
Stefan Ballmer

Outline

- Generalized Fluctuation-Dissipation Theorem
  - And the choice of TD variables
- Stress-Optic effect
  - and a simple example
- What we ought to do

Fluctuation-Dissipation, general

- A 2nd set of conjugate TD variables: S & T

$$dF(T,\vec{\varepsilon}) = -SdT + \vec{\sigma}d\vec{\varepsilon}$$

- F-D theorem applicable:
  - Thermal Impedance Z
  - Drive the system with

$$\begin{pmatrix} \vec{F} \\ \vec{S} \end{pmatrix} = G_0 \begin{pmatrix} \vec{f}_x \\ \vec{f}_T \end{pmatrix} \cos(\omega t)$$ where $$e = \vec{f}_x^{\,+} \vec{x} + \vec{f}_T^{\,+} \vec{T}$$

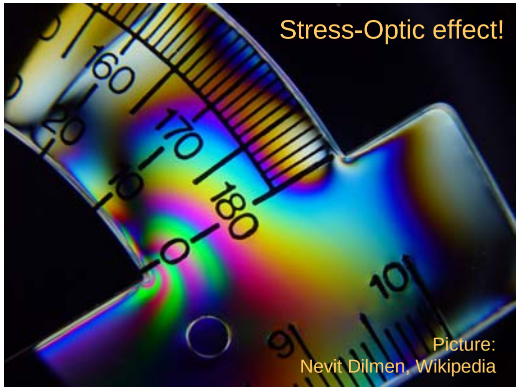
➔ $$S_{ee}(f) = \frac{8k_B T}{\omega^2} \frac{\mathrm{P}_{\mathrm{diss}}}{\mathrm{G}_0^2}$$

Levin, Physics Letters A 372 (2008) 1941–1944

Choice of your TD variables

- Hooke's law: $\vec{\sigma} = \overleftrightarrow{H} \bullet \left(\vec{\varepsilon} - \vec{\alpha} T \right)$ | $\vec{\sigma} = \overleftrightarrow{H} \bullet \left(\vec{\varepsilon}_\sigma \right)$
- Lin. expansion: $\delta d = \varepsilon_1 \cdot d$ | $\delta d = \left(\vec{\varepsilon}_\sigma + \vec{\alpha} T \right)_1$
- TD variables: $T \text{ and } \vec{\varepsilon}$ | $T \text{ and } \vec{\varepsilon}_\sigma$
- TE noise from: adiabatic heating (Fejer et. al.) | explicit T dependence (Levin)
- Opt. thickness: $\frac{\delta(nd)}{nd} = \delta\varepsilon_1 + \frac{\beta}{n}\delta T$ | $\frac{\delta(nd)}{nd} = \delta\varepsilon_{\sigma 1} + \left(\frac{\beta}{n} + \alpha_1 \right) \delta T$

  (one layer)
- So far missing in GW literature:



Stress-Optic Effect

- n depends on both T *and* σ
  - Typically measured:

$$\beta = \left.\frac{\partial n}{\partial T}\right|_{\sigma \text{ const}}$$

$$C_p = -\frac{2}{n^3}\left.\frac{\partial n}{\partial \sigma_p}\right|_{T \text{ const}} \qquad C_s = -\frac{2}{n^3}\left.\frac{\partial n}{\partial \sigma_s}\right|_{T \text{ const}}$$

    - σ_p & σ_s are parallel and orthogonal strain
  - Values for Silica:

$$\frac{C_p}{C_s} = \frac{4.22}{0.65} \times 10^{-12}\,\frac{m^2}{N}$$

Stone, Journal of Lightwave Technology Vol 6 No 7 July1988

  - Convert to T & ε dependence using TD id's.
- Small effect for mirror coatings, but…

Example: Propagation in bulk

- For simplicity:
  - Uniform, 3-D model, Silica
- Brownian noise:
  - without Stress-optic effect:

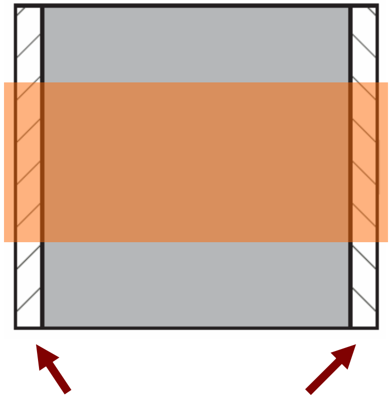
$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \frac{(n-1)^2}{E}$$

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \times 2.81 \times 10^{-12} \frac{m^2}{N}$$

  - including Stress-optic effect:

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \times 0.81 \times 10^{-12} \frac{m^2}{N}$$

- Relevant e.g. for Khalili cavities…



Silica:

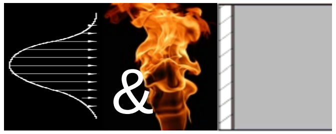
$$n = 1.45$$

$$E = 7.2 \times 10^{10}\, pa$$

$$\frac{C_p}{C_s} = \frac{4.22}{0.65} \times 10^{-12} \frac{m^2}{N}$$

What we ought to do

- More realistic calculations get complicated
- We ought to
  - Use an FEM model to calculate P_{diss}
  - Calc. Brownian & Thermo-optic noise in one run
  - Optimize for mirror / coating geometries



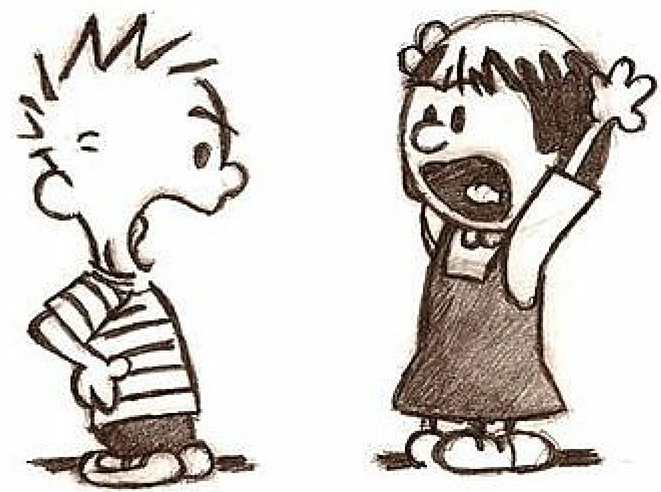
- I advertised this before
  - No progress yet – man power needed


That's it…
Adirondack Park
New York

Title:TBA

R.Adhikari

Discussion



Supplementary slides

Example: Propagation through bulk

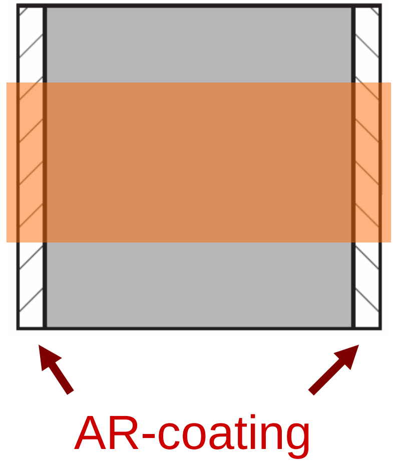
- For simplicity:
  - Uniform, 3-D model, Silica
- Brownian noise:
  - without Stress-optic effect:

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \frac{(n-1)^2}{E}$$

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \times 2.81 \times 10^{-12} \frac{m^2}{N}$$

  - including Stress-optic effect:

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \left[(n-1)\delta_1^T - \frac{n^3}{2} C^T H \right] H^{-1} \left[(n-1)\delta_1 - \frac{n^3}{2} HC \right]$$



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