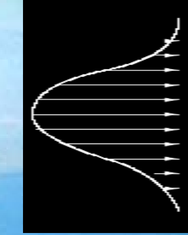
# Stress-optic noise?



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LIGO-G1100571

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# Outline

- Generalized Fluctuation-Dissipation Theorem
  - And the choice of TD variables
- Stress-Optic effect
  - and a simple example
- What we ought to do

# Fluctuation-Dissipation, general

- A 2nd set of conjugate TD variables: S & T

$$dF(T,\vec{\varepsilon}) = -SdT + \vec{\sigma}d\vec{\varepsilon}$$

- F-D theorem applicable:
  - Thermal Impedance Z
  - Drive the system with

$$\begin{pmatrix} \vec{F} \\ \vec{S} \end{pmatrix} = G_0 \begin{pmatrix} \vec{f}_x \\ \vec{f}_T \end{pmatrix} \cos(\omega t)$$ where $$e = \vec{f}_x^{\,+} \vec{x} + \vec{f}_T^{\,+} \vec{T}$$

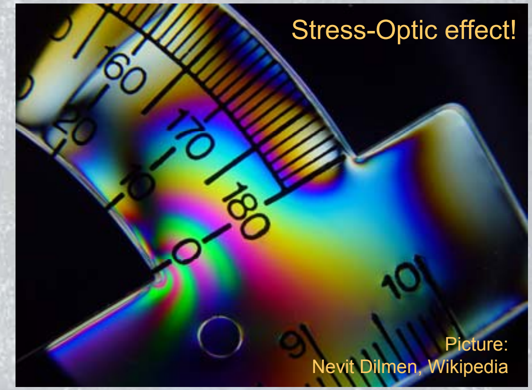
$$\rightarrow \quad S_{ee}(f) = \frac{8k_B T}{\omega^2} \frac{\mathrm{P_{diss}}}{\mathrm{G}_0^2}$$


Levin, Physics Letters A 372 (2008) 1941–1944

# Choice of your TD variables

- Hooke's law: $\vec{\sigma} = \overleftrightarrow{H} \bullet (\vec{\varepsilon} - \vec{\alpha} T)$ | $\vec{\sigma} = \overleftrightarrow{H} \bullet (\vec{\varepsilon}_\sigma)$
- Lin. expansion: $\delta d = \varepsilon_1 \cdot d$ | $\delta d = (\vec{\varepsilon}_\sigma + \vec{\alpha} T)_1$
- TD variables: $T$ and $\vec{\varepsilon}$ | $T$ and $\vec{\varepsilon}_\sigma$
- TE noise from: adiabatic heating (Fejer et. al.) | explicit T dependence (Levin)
- Opt. thickness: $\frac{\delta(nd)}{nd} = \delta\varepsilon_1 + \frac{\beta}{n}\delta T$ | $\frac{\delta(nd)}{nd} = \delta\varepsilon_{\sigma 1} + \left(\frac{\beta}{n} + \alpha_1\right)\delta T$

  (one layer)
- So far missing in GW literature:



# Stress-Optic Effect

- n depends on both T *and* σ
  - Typically measured:

$$\beta = \left.\frac{\partial n}{\partial T}\right|_{\sigma \text{ const}}$$

$$C_p = -\frac{2}{n^3}\left.\frac{\partial n}{\partial \sigma_p}\right|_{T \text{ const}} \qquad C_s = -\frac{2}{n^3}\left.\frac{\partial n}{\partial \sigma_s}\right|_{T \text{ const}}$$

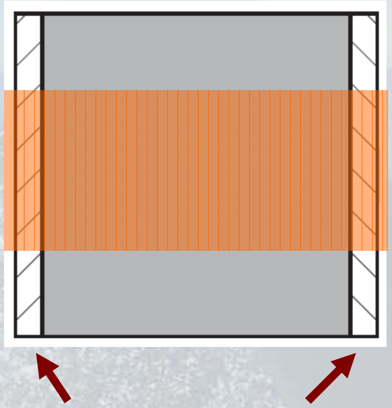
    - $\sigma_p$ & $\sigma_s$ are parallel and orthogonal strain
  - Values for Silica:

$$\begin{matrix} C_p \\ C_s \end{matrix} = \begin{matrix} 4.22 \\ 0.65 \end{matrix} \times 10^{-12} \frac{m^2}{N}$$

Stone, Journal of Lightwave Technology Vol 6 No 7 July1988

  - Convert to T & ε dependence using TD id's.
- Small effect for mirror coatings, but…

# Example: Propagation in bulk




- For simplicity:
  - Uniform, 3-D model, Silica
- Brownian noise:
  - without Stress-optic effect:

$$S_{xx}(f) = \frac{4k_B T}{\omega}\Phi\frac{\mathrm{L}}{\mathrm{A}}\frac{(n-1)^2}{E}$$

$$S_{xx}(f) = \frac{4k_B T}{\omega}\Phi\frac{\mathrm{L}}{\mathrm{A}}\times 2.81\times 10^{-12}\frac{m^2}{N}$$

  - including Stress-optic effect:

$$S_{xx}(f) = \frac{4k_B T}{\omega}\Phi\frac{\mathrm{L}}{\mathrm{A}}\times 0.81\times 10^{-12}\frac{m^2}{N}$$

- Relevant e.g. for Khalili cavities…

Silica:

$$n = 1.45$$

$$E = 7.2\times 10^{10}\,pa$$

$$\begin{matrix} C_p \\ C_s \end{matrix} = \begin{matrix} 4.22 \\ 0.65 \end{matrix}\times 10^{-12}\frac{m^2}{N}$$

# What we ought to do

- More realistic calculations get complicated
- We ought to
  - Use an FEM model to calculate $P_{diss}$
  - Calc. Brownian & Thermo-optic noise in one run
  - Optimize for mirror / coating geometries


- I advertised this before
  - No progress yet – man power needed


That's it. . .
Adirondack Park
New York

# Example: Propagation through bulk

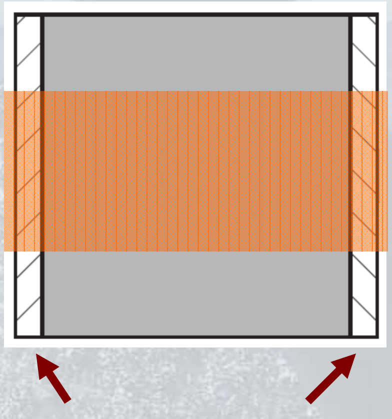
- For simplicity:
  - Uniform, 3-D model, Silica
- Brownian noise:
  - without Stress-optic effect:

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \frac{(n-1)^2}{E}$$

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \times 2.81 \times 10^{-12} \frac{m^2}{N}$$

  - including Stress-optic effect:

$$S_{xx}(f) = \frac{4k_B T}{\omega} \Phi \frac{\mathrm{L}}{\mathrm{A}} \left[ (n-1)\delta_1^T - \frac{n^3}{2} C^T H \right] H^{-1} \left[ (n-1)\delta_1 - \frac{n^3}{2} HC \right]$$




Silica:

$$n = 1.45$$

$$E = 7.2 \times 10^{10}\ pa$$

$$\frac{C_p}{C_s} = \frac{4.22}{0.65} \times 10^{-12} \frac{m^2}{N}$$