Coating Brownian Thermal Noise

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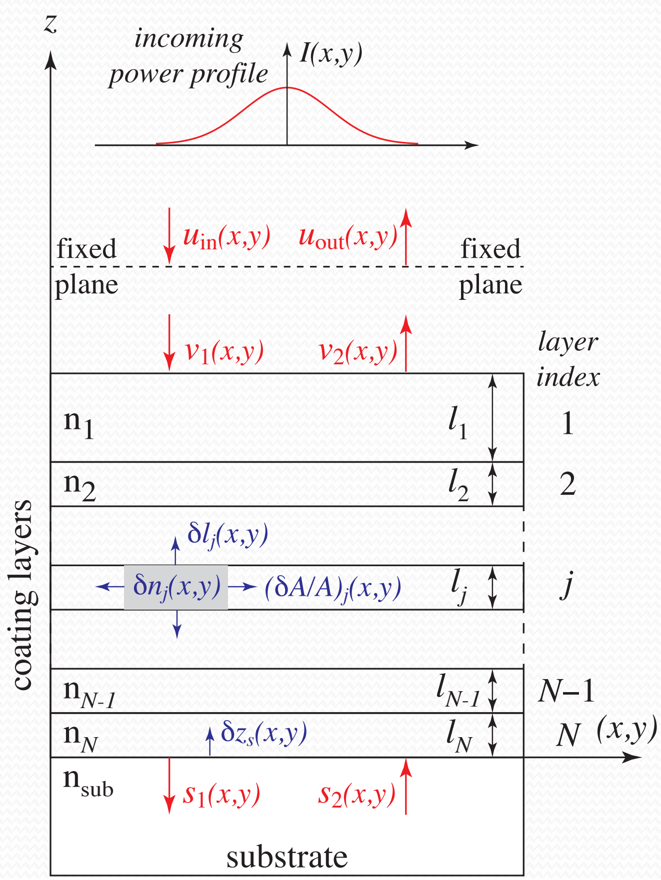
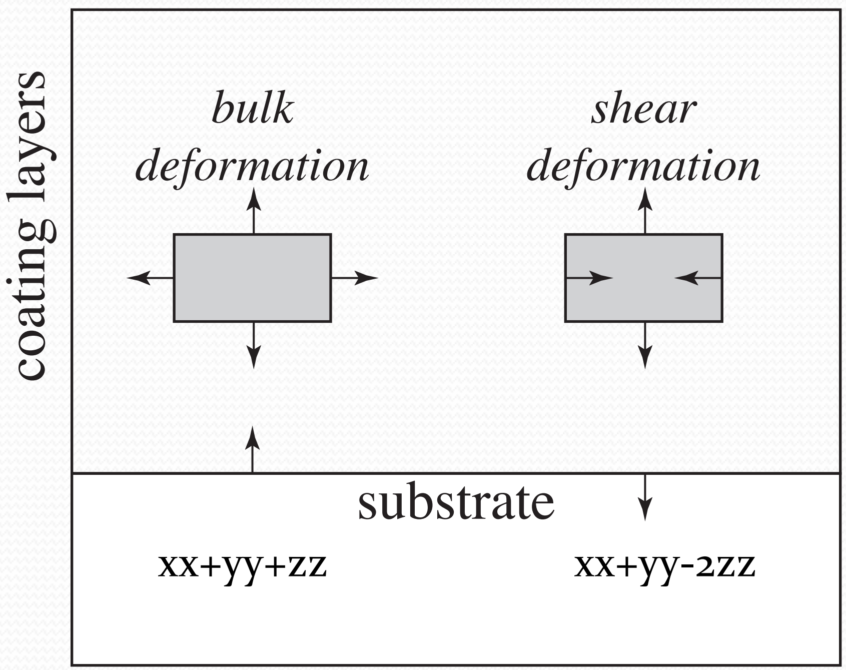
Coating Brownian Noise Components



1. Thickness fluctuations of the coating layers
2. Height fluctuation of coating-substrate interface
3. Refractive index fluctuations of the coating layers

Fluctuations induced by Bulk and Shear Losses



$$\phi_\Theta \qquad \phi_\Sigma$$

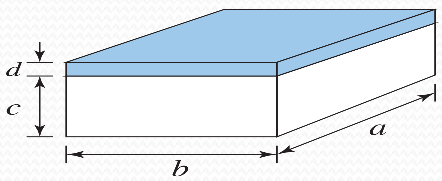
We assign a separate loss angle for bulk and shear energy.

$$U = \frac{1}{2} K\Theta^2 + \mu\Sigma^2$$

$$W_{diss}(per\_cycle) = \phi_\Theta \frac{1}{2} K\Theta^2 + \phi_\Sigma \mu\Sigma^2$$

- Bulk Noise:
  - xx+yy+zz — Coating thickness and interface
- Shear Noise:
  - xz+zx and yz+zy — No influence
  - xx-yy and xy+yx — Coating-Substrate Interface
  - xx+yy-2zz — Coating thickness and Interface

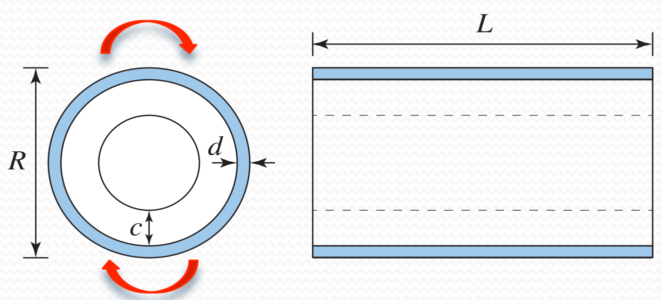
Ringdown Measurement of Loss angles



$$\phi = \frac{d}{c}\frac{Y_c}{Y_s}\frac{(1-\sigma_s^2)}{(1-\sigma_c)^2}\left[\phi_B(1-2\sigma_c)+2\phi_s\frac{1-\sigma_c+\sigma_c^2}{1+\sigma_c}\right]+\phi_{sub}$$

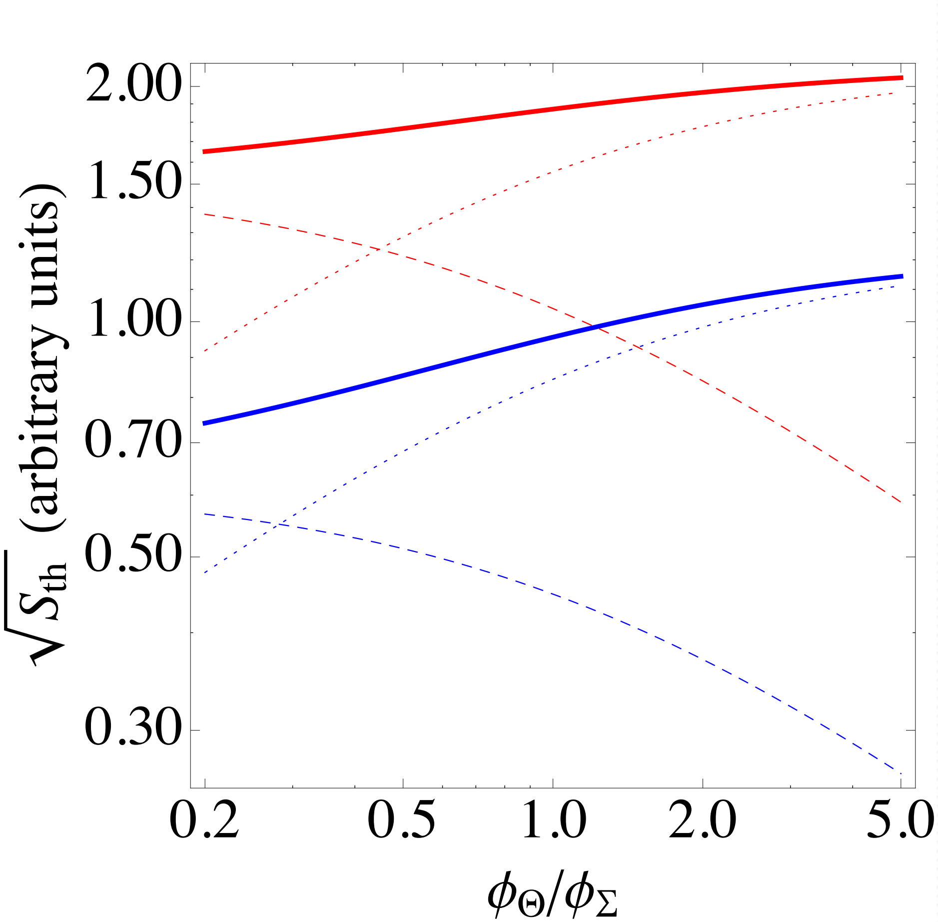
Oscillation

$$\phi = \frac{1}{Q}$$



$$\phi = \phi_{sub} + \frac{d}{c}\frac{Y_c}{Y_s}\frac{(1+\sigma_s)}{(1+\sigma_c)}\phi_S$$

Effect of Uncertainties in Loss Angles



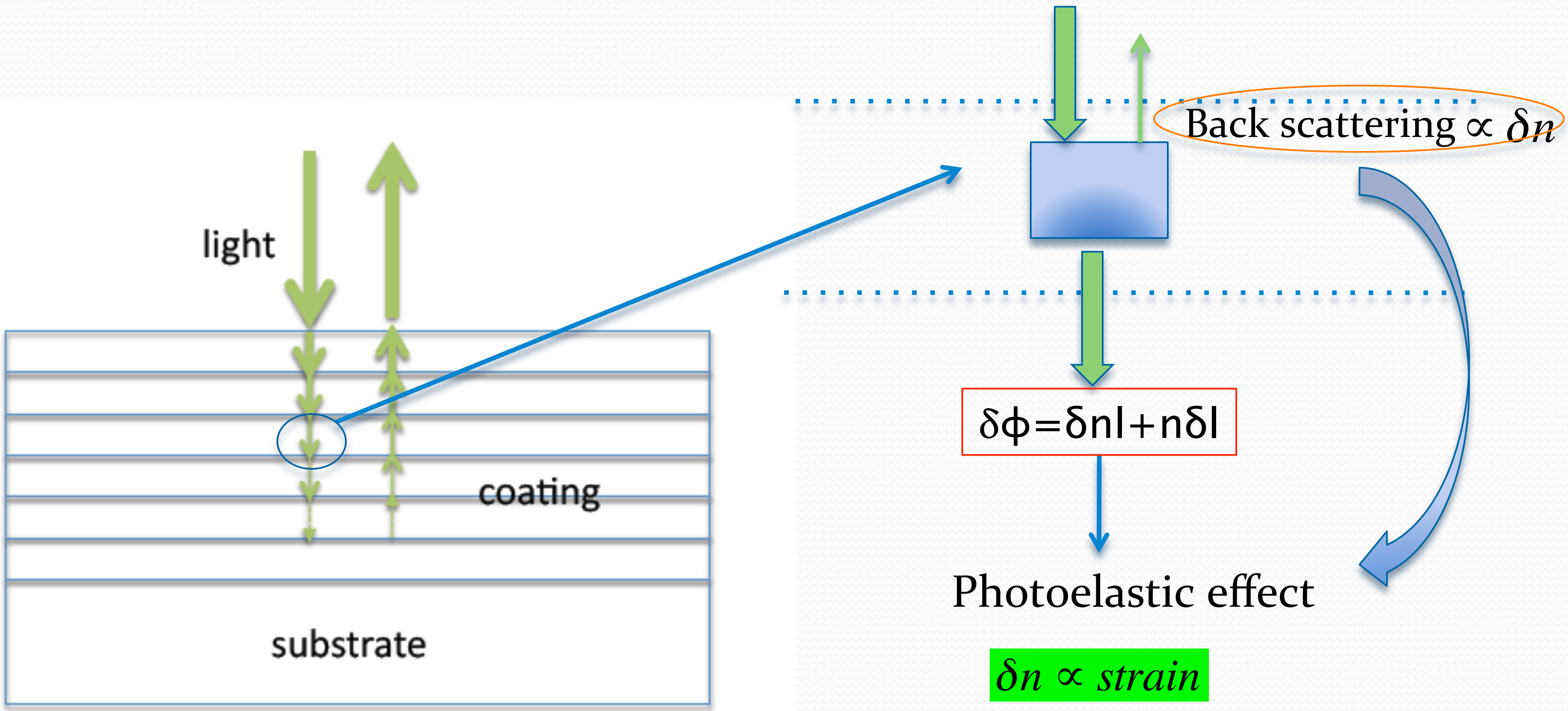
Baseline Parameters used for Coating Materials

	Ti_2O_5	SiO_2
Refractive index	2.07	1.45
Poisson Ratio	0.23	0.17
Young's Modulus (Pa)	1.4×10^{11}	7×10^{10}
Loss Angle ($\phi_B=\phi_S$)	2×10^{-4}	4×10^{-5}

G. M.Harry et al., Class. Quantum Grav. 19, 897 (2002)

Red: Tantala. Blue:Silica. Bulk in dotted lines and shear in dashed lines

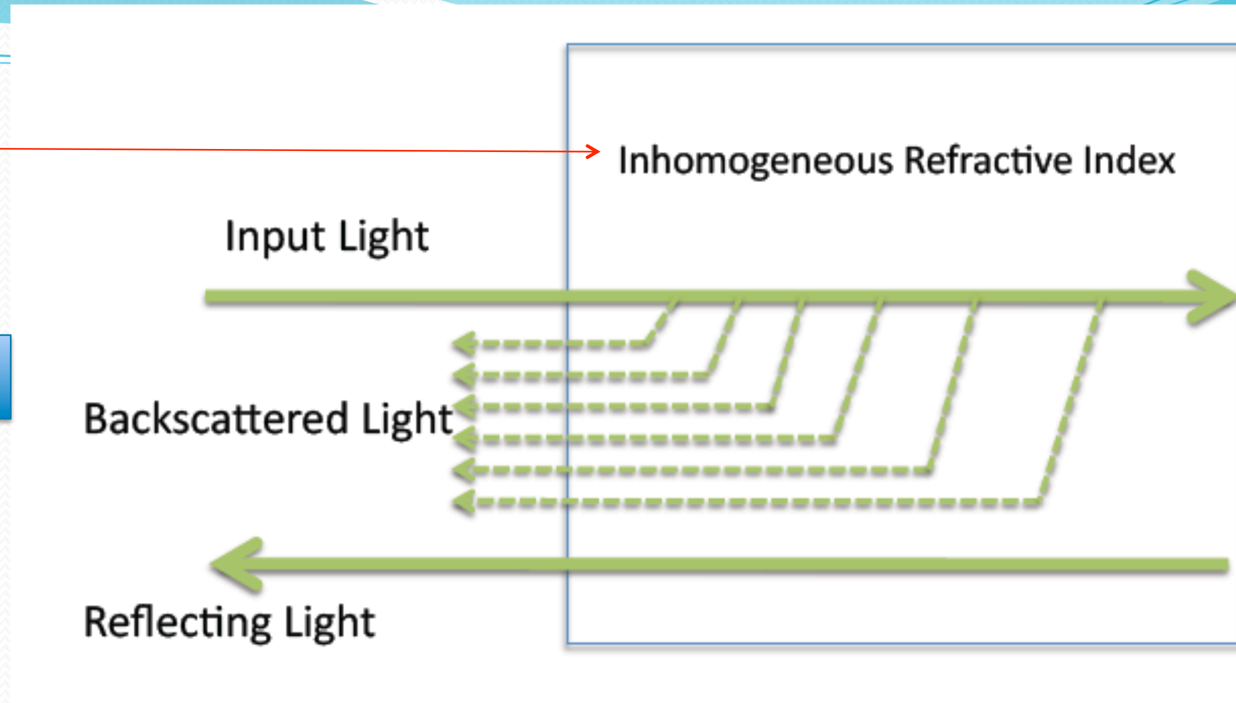
Thermal Noise Considering Light Penetration



$$\Delta n = -\frac{1}{2} n^3 p_{ij} a_j$$

Photoelastic Effect

Strain Fluctuation

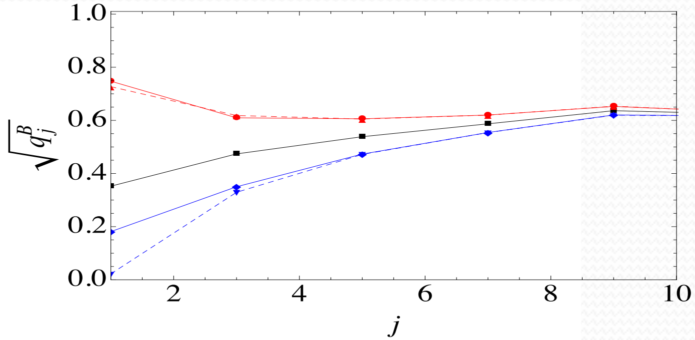


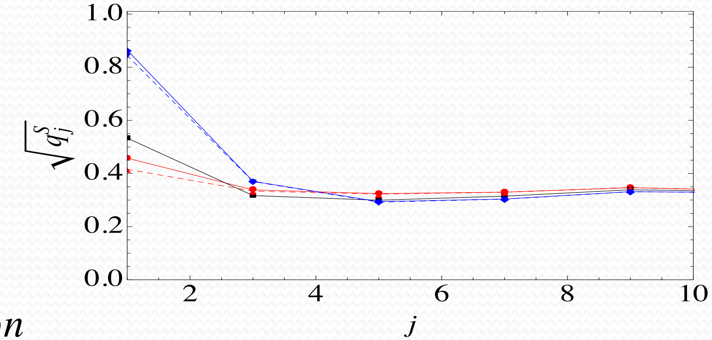
$$\delta\phi_j = k_0 \left[(n_j + \beta_j)\delta l_j + \frac{1 - r_j^2}{2r_j}\beta_j \delta l_j^c - \frac{1 + r_{j-1}^2}{2r_{j-1}}\beta_{j-1}\delta l_{j-1}^c \right], \quad \delta r_j = k_0 t_j^2 \beta_j \delta l_j^s$$

$$\begin{matrix} \delta l_j^c \\ \delta l_j^c \end{matrix} = \int_{\text{layer } j} \begin{matrix} \cos(2k_0 n_j z') \\ \sin(2k_0 n_j z') \end{matrix} \, u_{zz}(z')dz' \qquad \beta_j = \frac{\partial n}{\partial u_{zz}} = -\frac{n_j^3}{2} p_{12}^{(j)}$$

Coating Brownian Noise: Full Calculation

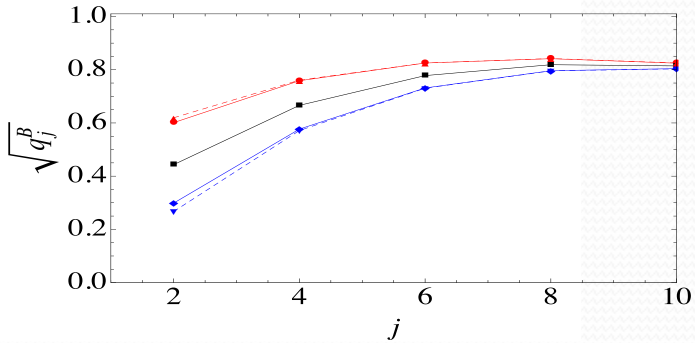
Silica

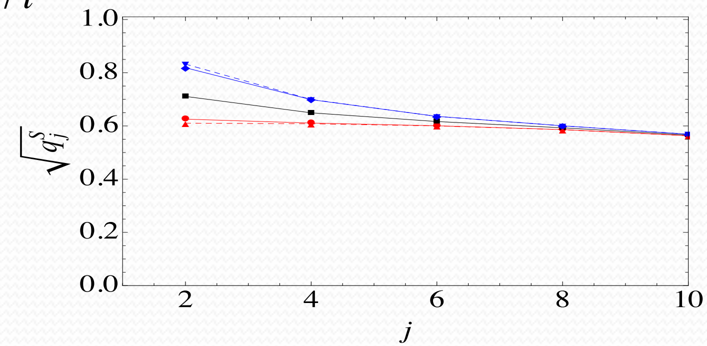




$$\beta = \frac{\delta n}{\delta l / l}$$

Tantal
a





Bulk

Shear

Red for $\beta=1$, blue for $\beta=-1$, *dashed* for ignoring back-scattering terms

$$S = \sum_j (q_j^B \phi_B^j + q_j^S \phi_S^j) S_j$$

$$S_j \equiv \frac{4 k_B T \lambda_j (1 - \sigma_j - 2\sigma_j^2)}{3\pi f Y_j (1 - \sigma_j)^2 A_{eff}}$$

Conclusion

- Different components of Coating Brownian Noise
  - Coating-substrate bending is important
  - Back-scattering of light is important when considering light penetration
- Bulk & shear loss angle and their measurement
- Reduce Bending effect: Stiffer Substrate
  - Sapphire? (Reduce noise by factor of 3)
- Optimized coating design based on bulk and shear loss angle measurements