Exploring Momentum Fraction in Hadrons through Lattice Quantum Chromodynamics Simulations

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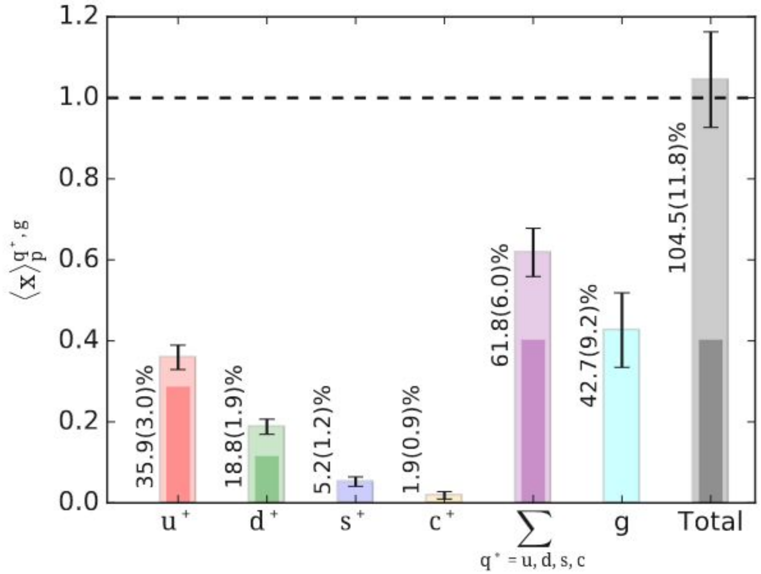
Why analyse the structure of hadrons?

- ➔ The European Muon Collaboration, at CERN, found that about half of the proton spin is carried by the valence quarks, this came to be known as the proton spin puzzle [J. Ashman et al. Nucl. Phys. B328, 1 (1989)]

- ➔ This puzzle and the fact that the structure of protons is more accessible led them to be studied extensively

- ➔ Recent lattice QCD studies have shown that sea quark contributions and gluon provide the missing component to the spin of the proton



C. Alexandrou, et al. Phys. Rev. D 101, 94513

Why analyse the structure of hadrons?

- ➔ To compute the proton spin, we need to evaluate the proton matrix elements of the euclidean energy momentum tensor

- ➔ This can be decompose into three generalized form factors

$$\langle N(p',s')|T^{\mu\nu;q,g}|N(p,s)\rangle = \bar{u}_N(p',s')\left[\, A_{20}^{q,g}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{q,g}(q^2)\frac{i\sigma^{\{\mu\rho}q_\rho P^{\nu\}}}{2m_N} + C_{20}^{q,g}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m_N}\right]u_N(p,s)$$

- ➔ $A^{g,q}_{20}(q^2)$ in the zero momentum transfer limit, gives the average momentum fraction carried by quarks and gluon [X.-D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249.]

Why analyse the structure of pion and kaon?

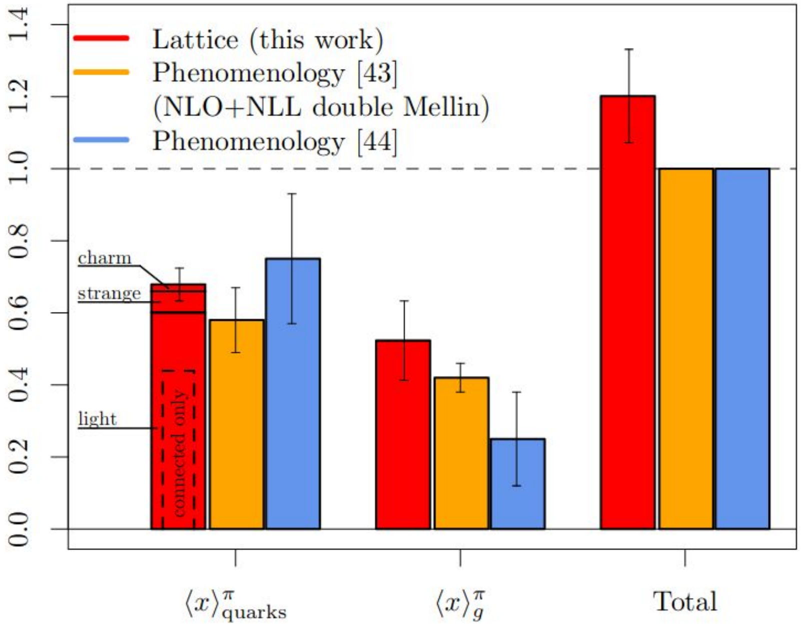
- ➔ On the other hand, particles as pion and kaon are more experimentally challenging
- ➔ There is not much work on the structure of these particles
- ➔ New experimental data will come from COMPASS++/Amber, related with quark and gluon dynamics [B. Adams et al., “Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER),” (2018)]

Can we theoretically calculate the quark and gluon momentum fraction contributions to the mesons?

$$\langle x \rangle_g^X \qquad \langle x \rangle_q^X \qquad X \in K, \pi$$

State of the art

- There are studies about the composition of pion and proton
- Around 60% of the momentum fraction come from the quarks
- The rest comes from the gluon which is only disconnected
- This calculations are done using one gauge ensemble cB211.072.64 [C. Alexandrou et al. Phys. Rev. D 98, 054518]



- [43] Authors of Ref. [44] communicating their results at 2 GeV in the MS scheme.
- [44] P.C. Barry et al, arXiv:2108.05822 [hep-ph].

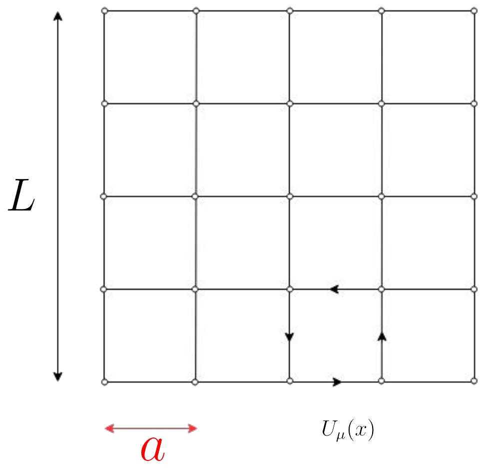
C. Alexandrou et al. Phys. Rev. Lett. 127, 252001

What is a LQCD ensemble?

- Reminding the expectation value of an operator

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D[U] D[\bar{\psi}, \psi] \mathcal{O} e^{-S[U,\bar{\psi},\psi]} \qquad Z = \int D[U] D[\bar{\psi}, \psi] e^{-S[U,\bar{\psi},\psi]}$$

- Fermions can be integrated out because they are quadratic. Remaining the integral over the gauge field
- There are different methods to do simulations such as Hybrid Monte Carlo [M. Lusher, arXiv:1002.4232 [hep-lat]]
- We would like to generate a representative number of field configurations for a certain discretization scheme of S
- The resulting set of configurations is called an ensemble
- Each ensemble has different fixed parameters as gauge coupling, masses, number of quark flavors, etc...



What is novel in what we are doing?

- We use three ensembles with different lattice spacing, in order to take the continuum limit

$Ensemble$	V/a^4	$L[fm]$	$a[fm]$
cB211.072.64	$64^3 \times 128$	5.1	0.07957
cC211.060.80	$80^3 \times 160$	5.44	0.06821
cD211.054.96	$96^3 \times 192$	5.47	0.05692

- In this work beyond the pion we include the kaon to study the effects of having a strange quark

- The momentum fraction is calculated using three $N_f = 2+1+1$ twisted mass fermions ensembles generated by the Extended Twisted Mass Collaboration

- Errors are computed using jackknife analysis

What is novel in what we are doing?

- The gluon field strength tensor is computed using clover field definition

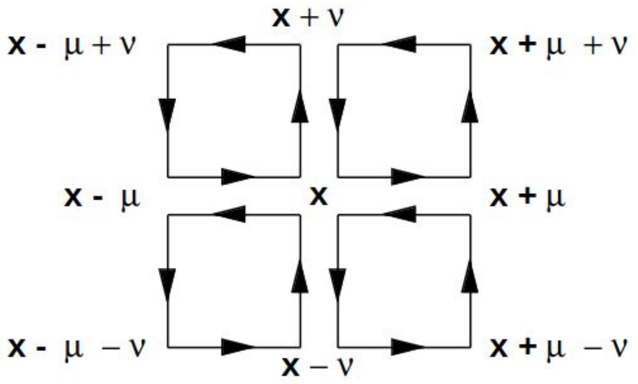
- We apply stout smearing to reduce ultraviolet fluctuations [Colin Morningstar and Mike Peardon, Phys. Rev. D 69, 054501]

- Before, 10 steps of stout smearing were used

- We use an average between stout steps 5-10

- We use model averaging based on the Akaike Information Criterion [William I. Jay and Ethan T. Neil, Phys. Rev. D 103, 114502]

$$P_i = \frac{1}{Z} e^{-\frac{1}{2}\chi_i^2 + N_{dof,i}} \qquad Z = \sum_i e^{-\frac{1}{2}\chi_i^2 + N_{dof,i}}$$



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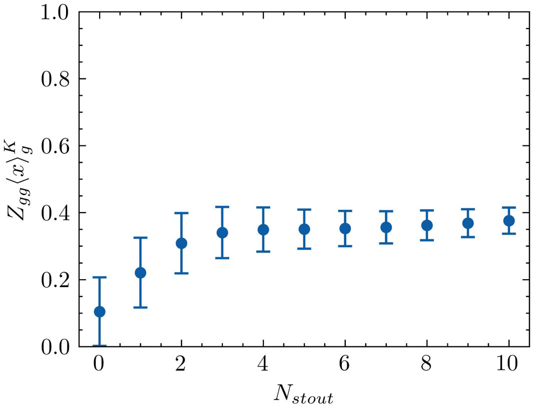
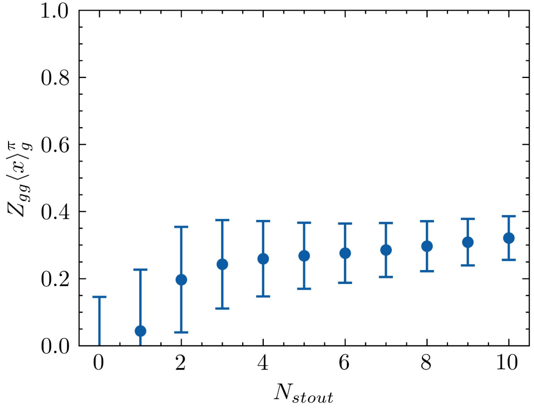


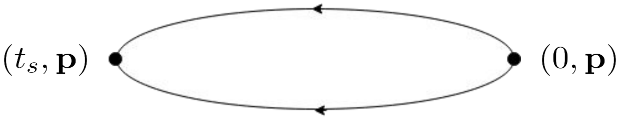


Two- and three- point functions

- Two-point function

$$C^X(t_s, \mathbf{p}) = \langle X(t_s, \mathbf{p}) X(0, \mathbf{p}) \rangle$$

$$X \in K, \pi$$

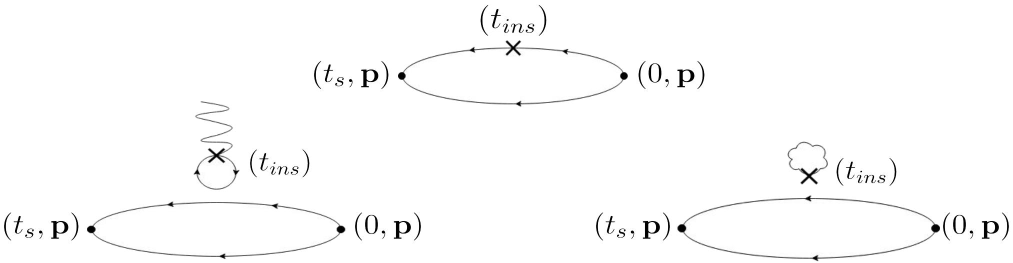


- Three-point function

$$C^{f,X}_{\mu\nu}(t_s, t_{ins}, \mathbf{p}) = \langle X(t_s, \mathbf{p}) T^f_{\mu\nu}(t_{ins}) X(0, \mathbf{p}) \rangle$$

$$T^f_{\mu\nu} = \text{Euclidian energy momentum tensor}$$

$$f \in u, d, s, c \text{ and } g$$

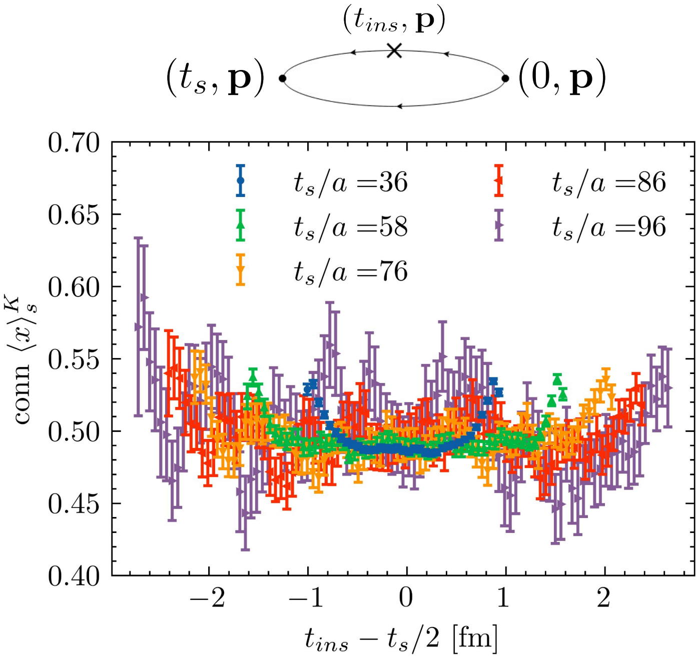


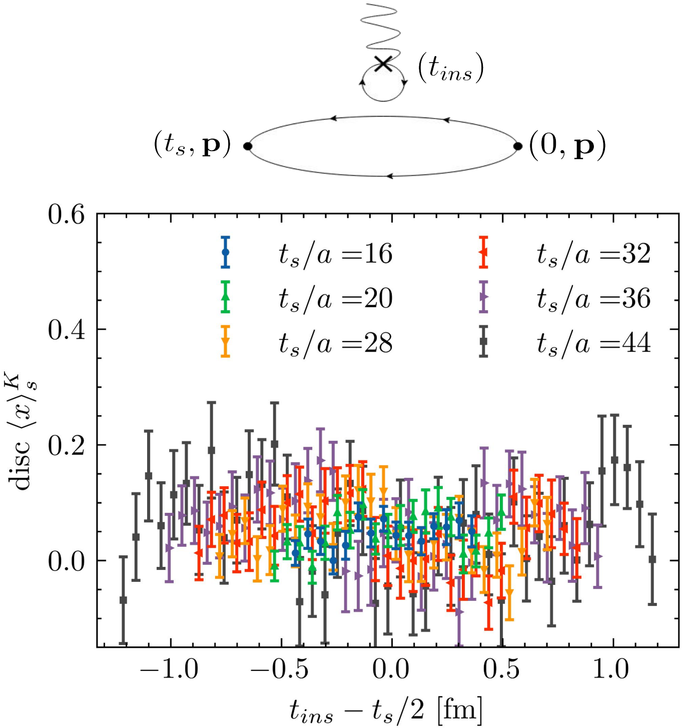
- In order to extract the matrix elements, we can calculate the ratio between the three- and two-point function. In the large enough time limit is proportional to the average momentum fraction

$$R^{f,X}_{\mu\nu}(t_s, t_{ins}, \mathbf{p}) = \frac{C^{f,X}_{\mu\nu}(t_s, t_{ins}, \mathbf{p})}{C^X(t_s, \mathbf{p})} \xrightarrow[t_s \to \infty]{t_s - t_{ins} \to \infty} \langle x \rangle^X_f$$

How does the ratio $R^{f,X}_{\mu\nu}$ look like?

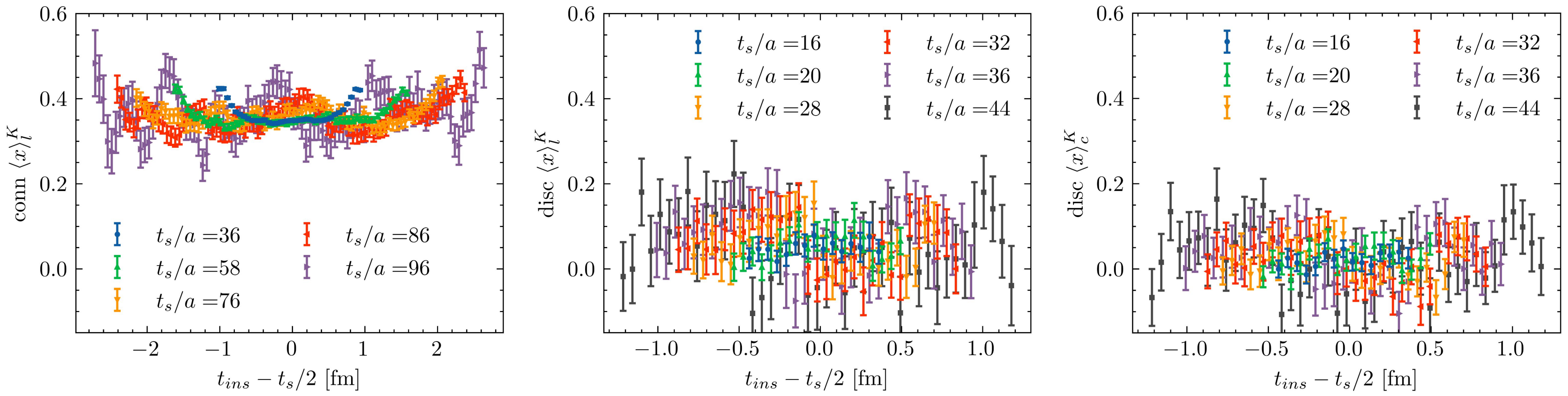
$$\bar{T}^q_{\mu\nu} = -\frac{(i)^{\kappa_{\mu\nu}}}{4}\,\bar{q}\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu - \delta_{\mu\nu}\frac{1}{2}\gamma_\rho \overleftrightarrow{D}_\rho\right) q$$





Ratio for quark connected and disconnected contributions, for different values of t_s

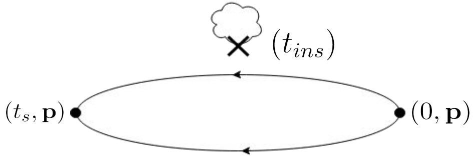
And for the rest of quarks!

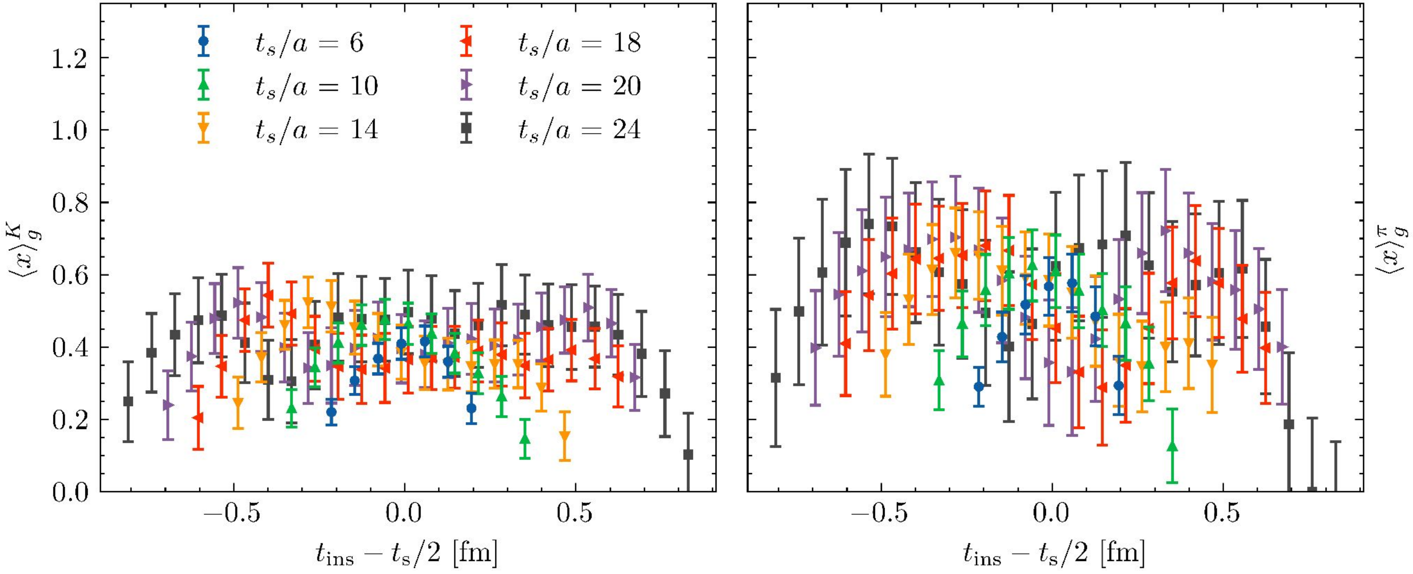


Ratio for quark connected and disconnected contributions, for different values of t_s

And for the gluon?

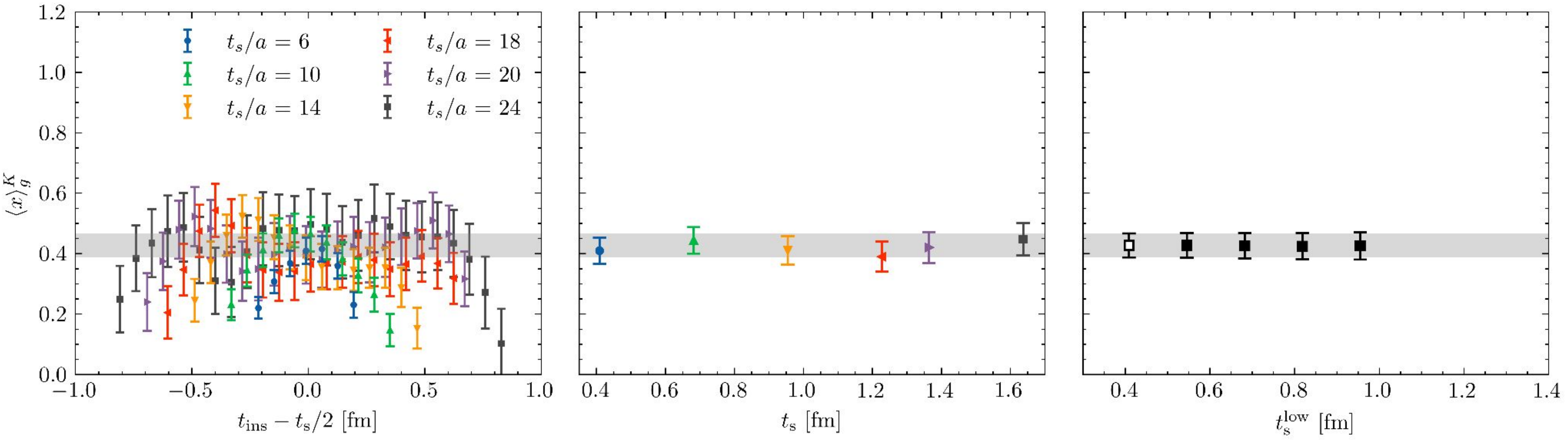
$$\bar{T}^g_{\mu\nu} = (i)^{\kappa_{\mu\nu}} \left(F_{\mu\rho}F_{\nu\rho} + F_{\nu\rho}F_{\mu\rho} - \delta_{\mu\nu}\frac{1}{2}F_{\rho\sigma}\ F_{\rho\sigma}\right)$$





Ratio for gluon disconnected contributions. As before, plotted for different values of t_s.

Plateau fit



Plateau fit. In the left panel is shown the bare ratio. In the middle panel, is shown the plateau fit per t_s . In the right panel is shown the fit per t_s^{low} . The band is the value after model averaging. The open symbol is the value with the highest probability.

Renormalization procedure

- Renormalization is done non-perturbatively in the RI-MOM scheme for diagonal contributions [Dimitra Pefkou talk on Tuesday 31/10]

- Mixing term is calculated perturbatively [George Panagopoulos, Haralambos Panagopoulos, and Gregoris Spanoudes, Phys. Rev. D 103, 014515]

Re-normalize the results, including mixing

$$\langle x \rangle_g^R = Z_{gg} \langle x \rangle_g + Z_{gq} \langle x \rangle_q$$

$$\langle x \rangle_q^R = Z_{qq} \langle x \rangle_q + Z_{qg} \langle x \rangle_g$$

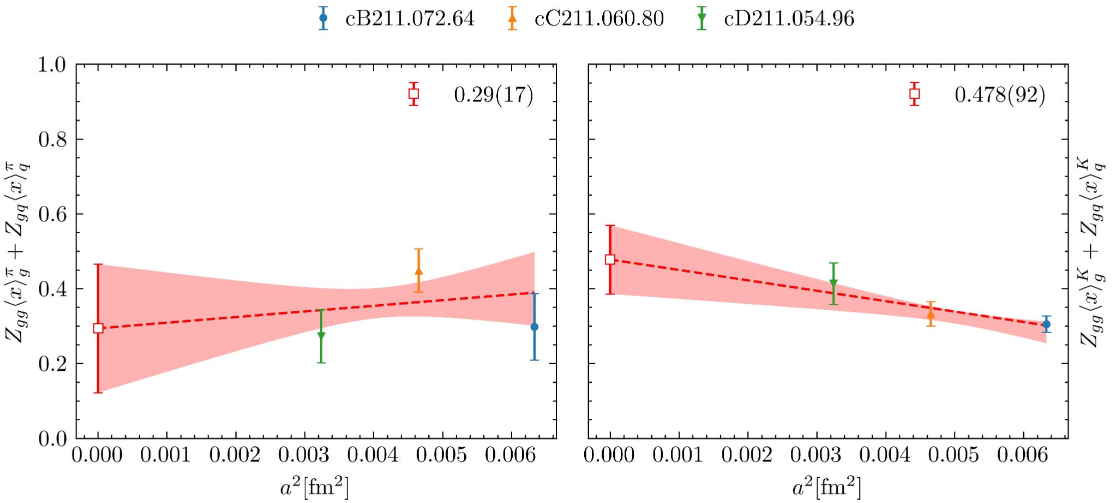
Test the momentum sum rule

$$\langle x \rangle_g^R + \langle x \rangle_q^R = 1$$

Continuum limit

- We can take the continuum limit as a linear fit in the square of the lattice spacing

$$f(a^2) = c_0 + c_1 a^2$$

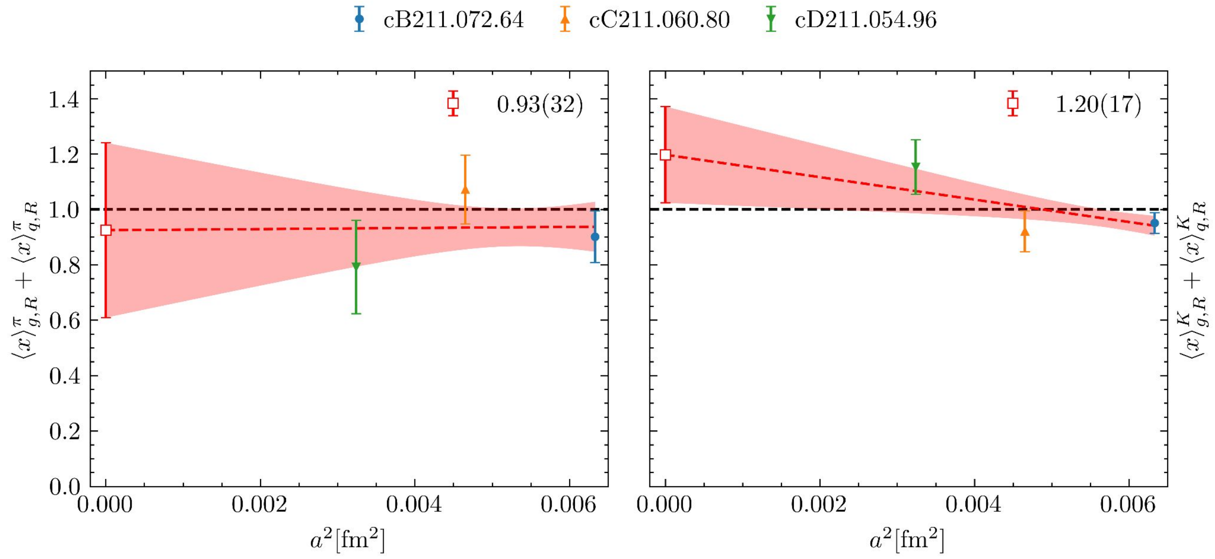


Green, yellow and blue symbols are the values per ensemble. Red symbol with open symbol is the continuum limit value. The red band is the error propagation of the fit. The dotted line is the value of the fitted function.

Sum rule

The renormalized contributions of gluon and quarks should obey the sum rule

$$\langle x \rangle_g^R + \langle x \rangle_q^R = 1?$$



Continuum limit for the sum of all components of the mesons. The results are compatible with 1.

Summary and future work

	$\langle x \rangle_g^{\pi,R}$	$\langle x \rangle_q^{\pi,R}$	$\langle x \rangle_g^{K,R}$	$\langle x \rangle_q^{K,R}$
cB211.072.64	0.298(89)	0.604(29)	0.305(22)	0.646(29)
cC211.060.80	0.448(58)	0.62(11)	0.332(32)	0.589(66)
cD211.054.96	0.273(71)	0.52(15)	0.414(56)	0.740(82)

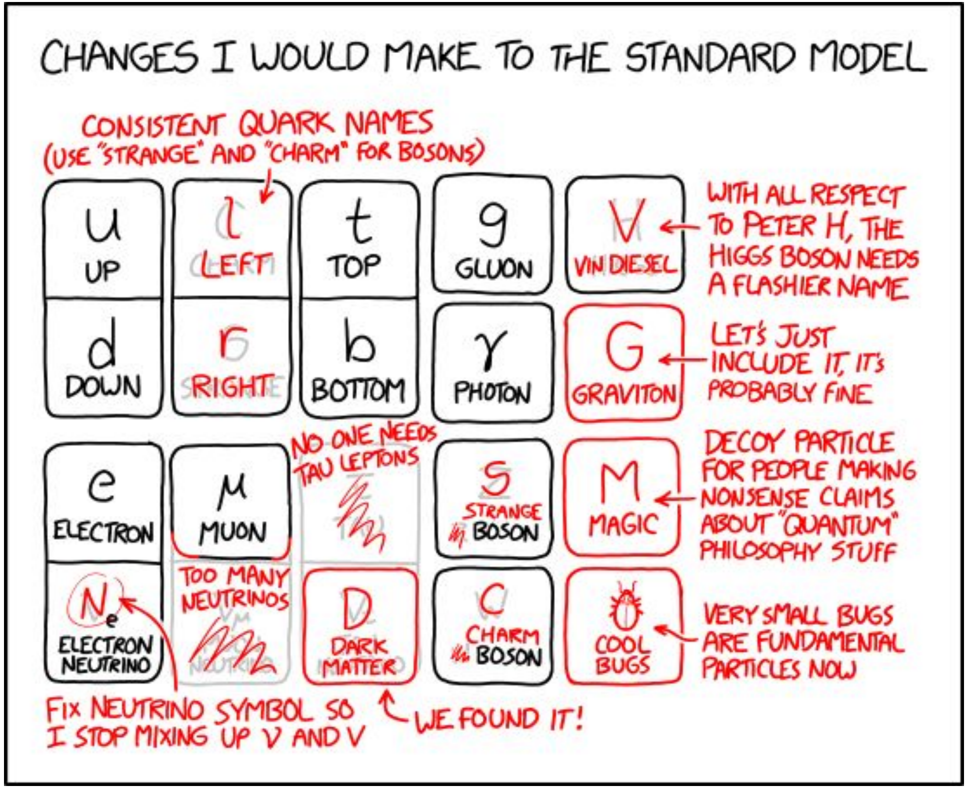
Preliminary

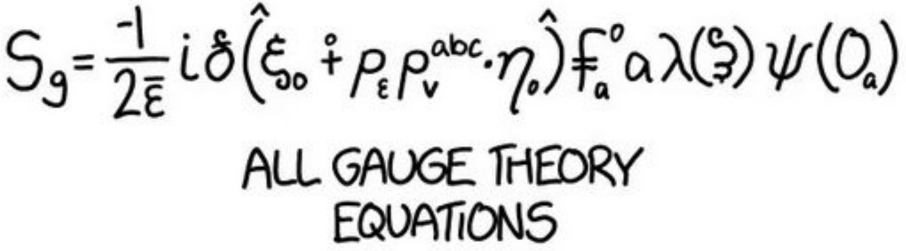
Summary of every contribution from quarks and gluon for three ensembles at 2 GeV.

$$\langle x \rangle_g^{\pi,R} + \langle x \rangle_q^{\pi,R} = 0.93(32) \qquad \langle x \rangle_g^{K,R} + \langle x \rangle_q^{K,R} = 1.20(17)$$

- This work agrees with previous results
- Sum rule is satisfied
- Increase statistics for each ensemble
- Add one more ensemble for the continuum limit
- To do a similar analysis for nucleon

Thanks!


CHANGES I WOULD MAKE TO THE STANDARD MODEL
CONSISTENT QUARK NAMES
(USE "STRANGE" AND "CHARM" FOR BOSONS)
u
UP
l
LEFT
t
TOP
g
GLUON
V
VIN DIESEL
WITH ALL RESPECT TO PETER H, THE HIGGS BOSON NEEDS A FLASHIER NAME
d
DOWN
r
RIGHT
b
BOTTOM
γ
PHOTON
G
GRAVITON
LET'S JUST INCLUDE IT, IT'S PROBABLY FINE
e
ELECTRON
μ
MUON
NO ONE NEEDS TAU LEPTONS
S
STRANGE BOSON
M
MAGIC
DECOY PARTICLE FOR PEOPLE MAKING NONSENSE CLAIMS ABOUT "QUANTUM" PHILOSOPHY STUFF
N
ELECTRON NEUTRINO
TOO MANY NEUTRINOS
D
DARK MATTER
C
CHARM BOSON
COOL BUGS
VERY SMALL BUGS ARE FUNDAMENTAL PARTICLES NOW
FIX NEUTRINO SYMBOL SO I STOP MIXING UP ν AND V
WE FOUND IT!


ALL GAUGE THEORY EQUATIONS