

News on polarized PDFs

Werner Vogelsang
Univ. of Tübingen

EINN 2023, 10/31/2023



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* thanks to my collaborators

I. Borsa, D. de Florian, R. Sassot, M. Stratmann



$$\Delta q(x) = \text{[Red sphere with white dot and right-pointing yellow arrow]} \rightarrow \text{[Red sphere with white dot and left-pointing yellow arrow]}$$

$$\Delta g(x) = \text{[Red sphere with 'eee' and right-pointing yellow arrow]} \rightarrow \text{[Red sphere with 'eee' and left-pointing yellow arrow]}$$

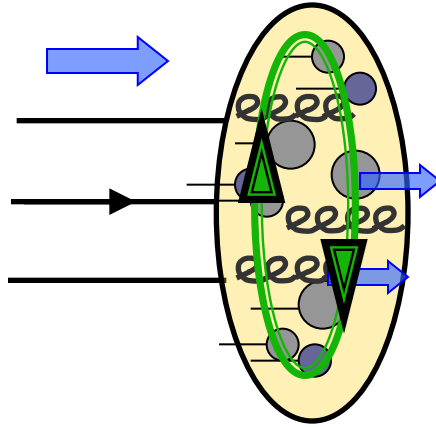
$$\Delta q(x) = \text{[Diagram: Red circle with white dot and right-pointing yellow arrow]} - \text{[Diagram: Red circle with white dot and left-pointing yellow arrow]}$$

$$\Delta g(x) = \text{[Diagram: Red circle with 'eee' and right-pointing yellow arrow]} - \text{[Diagram: Red circle with 'eee' and left-pointing yellow arrow]}$$

- in QCD: dependence on “resolution” scale μ

proton spin:

Jaffe, Manohar; Chen et al;
Wakamatsu; Hatta; ...



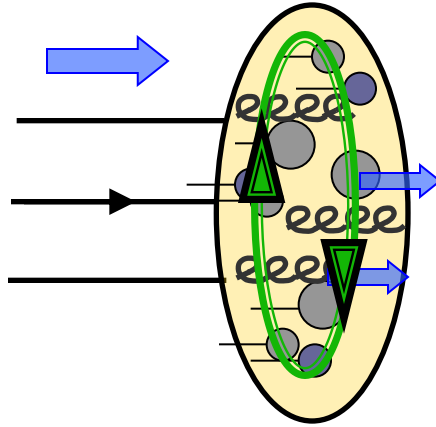
$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \int_0^1 dx [\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}](x)$$

$$\Delta G = \int_0^1 dx \Delta g(x)$$

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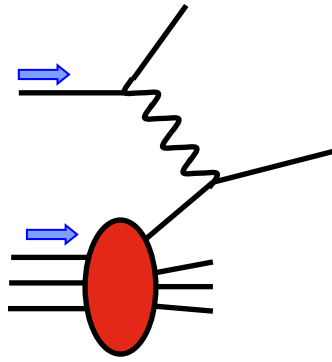
Outline:

- Global analysis of helicity PDFs: status
- Theory advances
- Conclusions

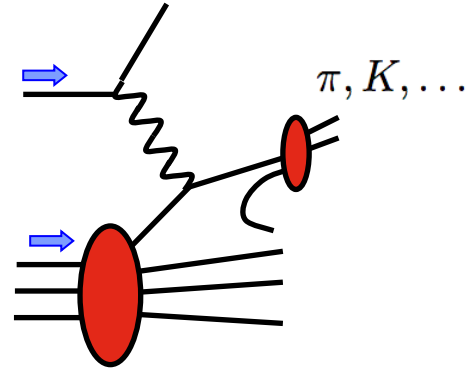
Global analysis: status

Helicity PDFs accessible in polarized high-energy scattering:

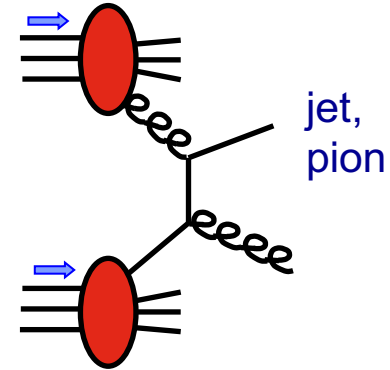
EMC, SMC,
COMPASS,
E142, E143, E154,
E155, HERMES,
CLAS, HALL-A



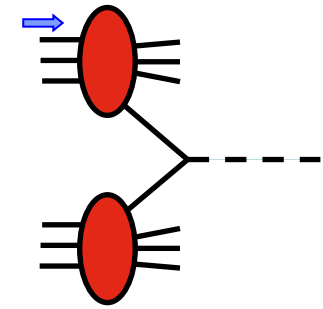
DIS



SIDIS



high- p_T

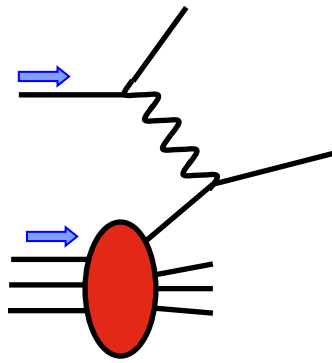


W bosons

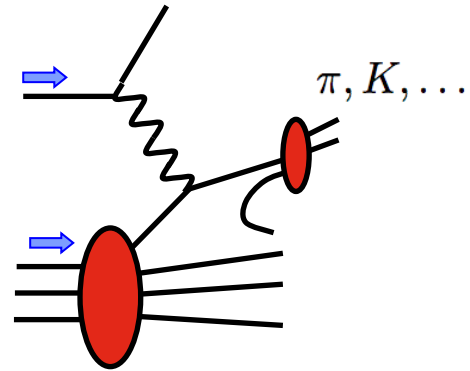
RHIC:
PHENIX,
STAR

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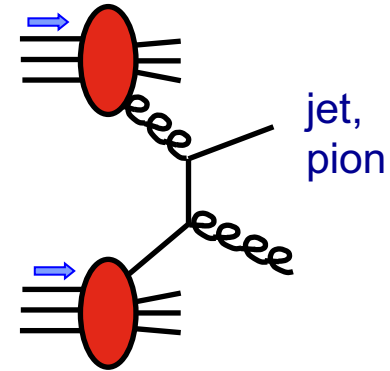
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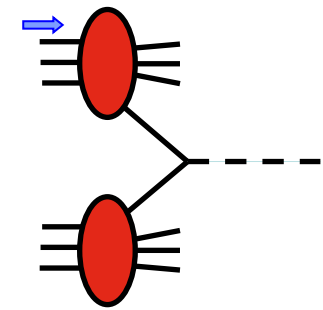
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high- p_T



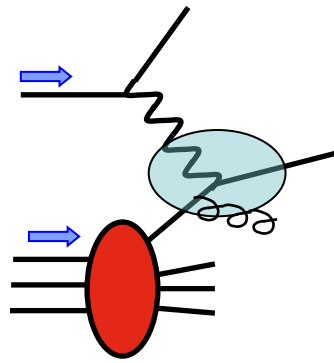
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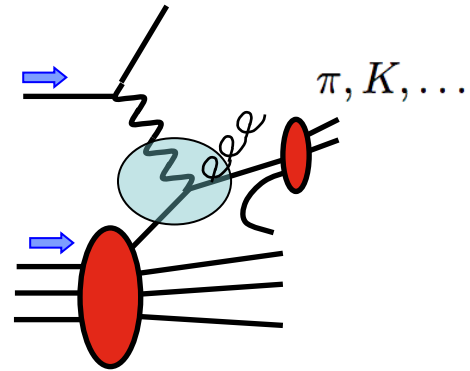
e.g.
$$\Delta\sigma_{pp}^{\text{jet}} = \sum_{a,b=q,\bar{q},g} \Delta f_a(x_a, \mu) \otimes \Delta f_b(x_b, \mu) \otimes \Delta\hat{\sigma}_{ab}$$

Helicity PDFs accessible in polarized high-energy scattering:

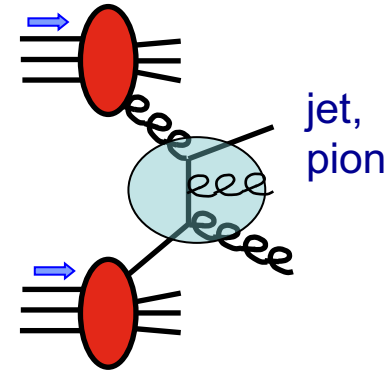
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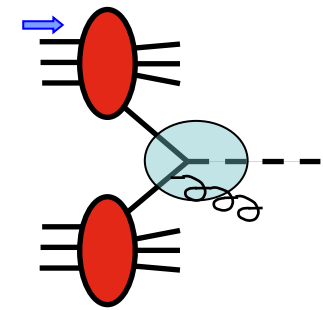
DIS



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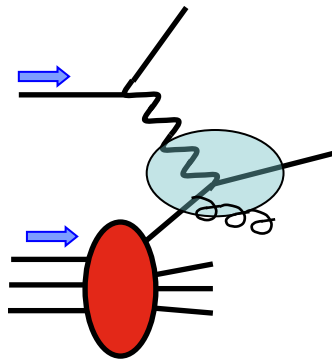
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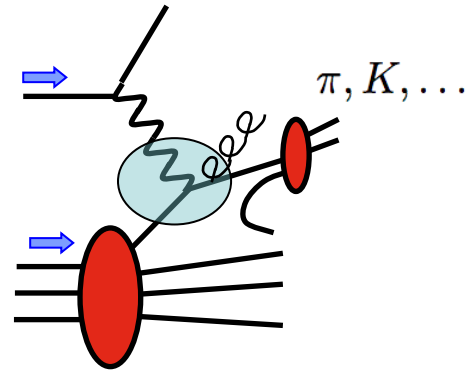
$$\Delta\hat{\sigma}_{ab} = \Delta\hat{\sigma}_{ab}^{\text{LO}} + \frac{\alpha_s}{\pi} \Delta\hat{\sigma}_{ab}^{\text{NLO}} + \dots$$

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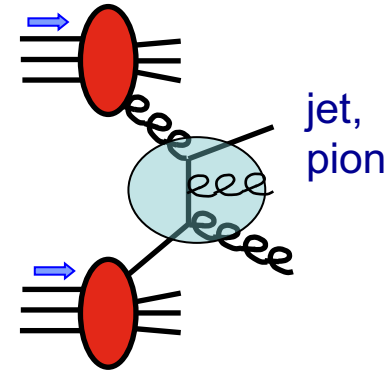
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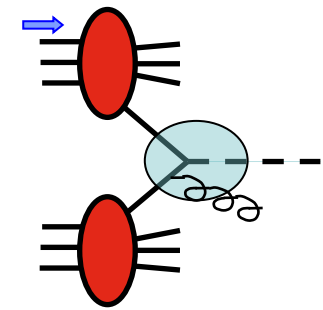
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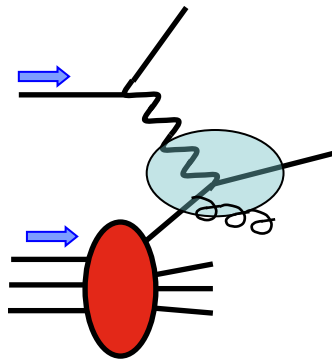
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$$\frac{d}{d \log \mu^2} \Delta f_i(x, \mu) = \sum_j \Delta \mathcal{P}_{ij} \otimes \Delta f_j(x, \mu)$$

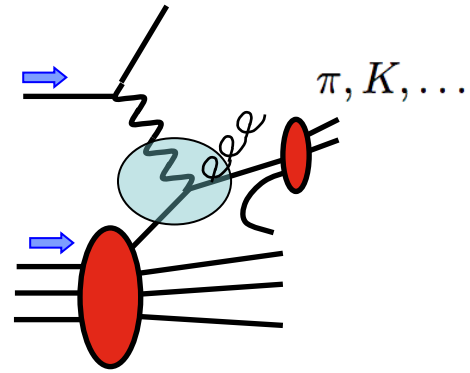
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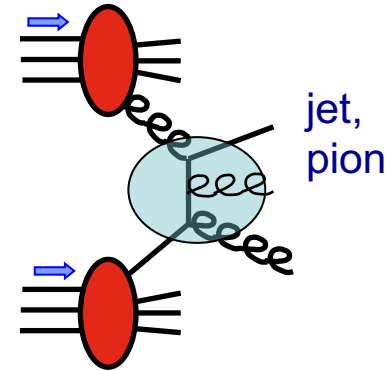
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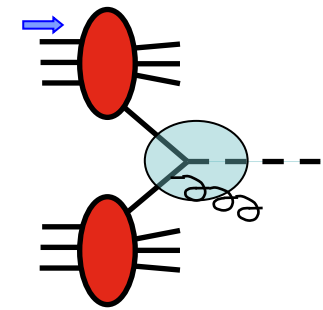
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Key players over recent years (all at **NLO** ($\overline{\text{MS}}$):

DSSV (2008 –)

de Florian, Sassot,
Stratmann, WV

NNPDF (2013 –)

Nocera, Ball, Forte,
Ridolfi, Rojo, ...

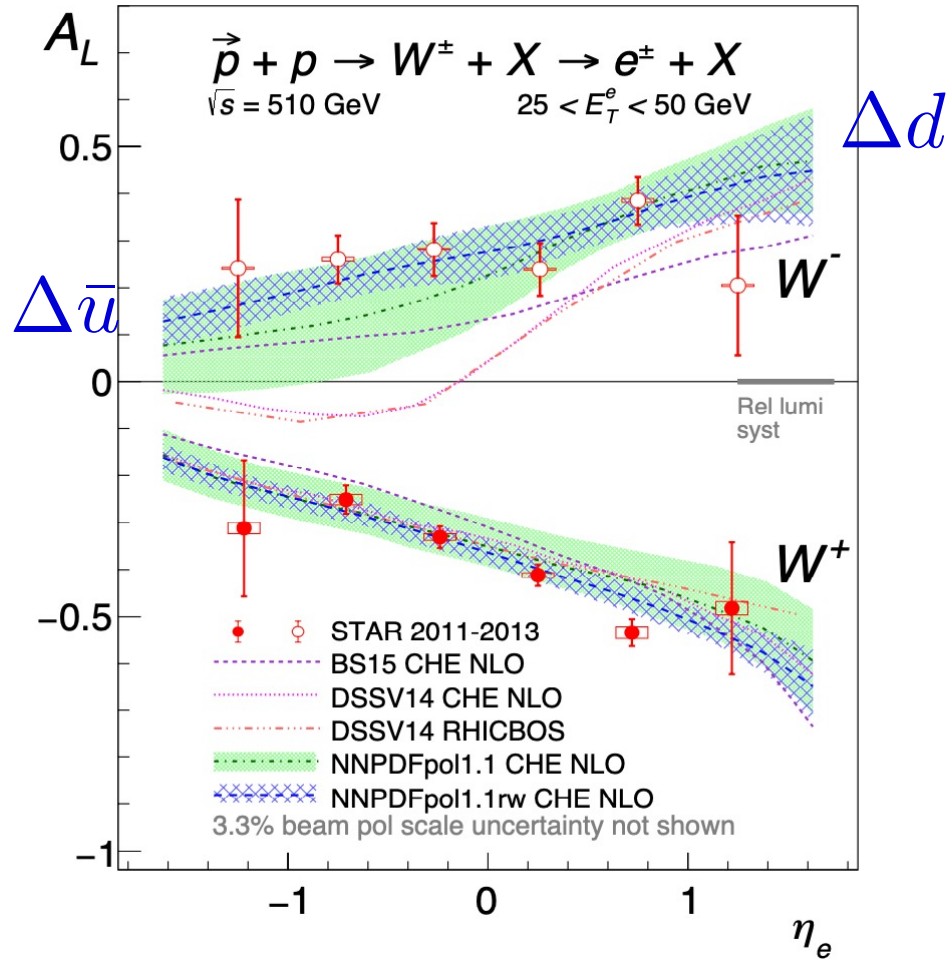
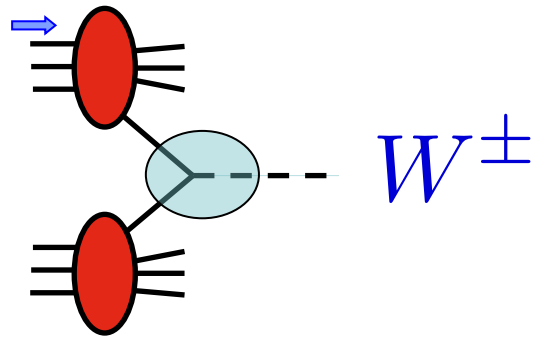


(2013 –)

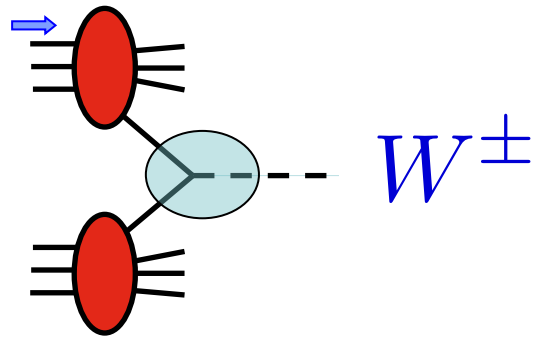
Sato, Cocuzza, Ethier,
Melnitchouk, ...

- differences in methodology:
 - DSSV** and **JAM**: Mellin moment techniques
 - NNPDF**: neural-network technique, x-space
- mature analysis frameworks with robust assessment of uncertainties

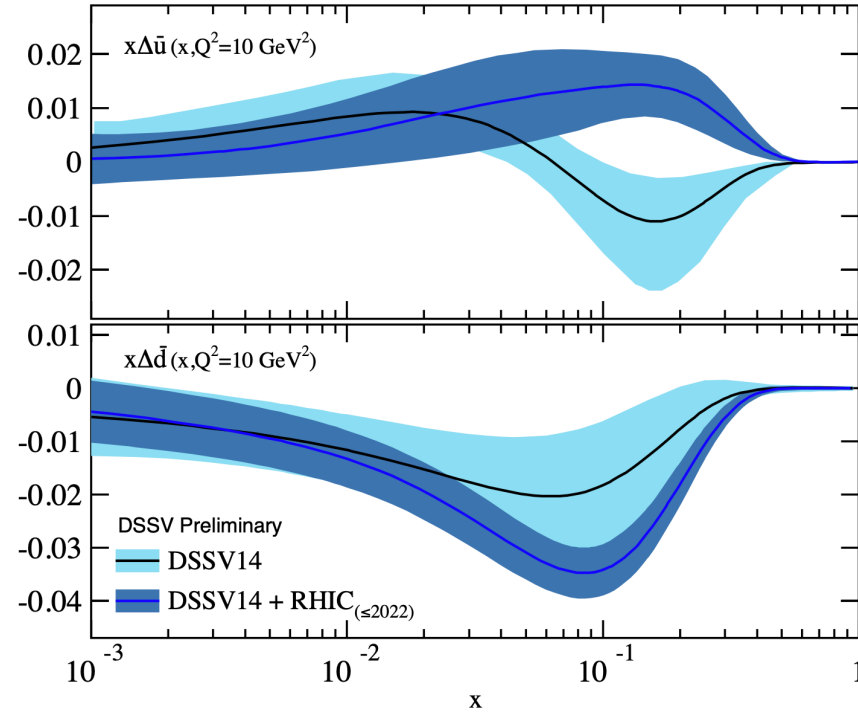
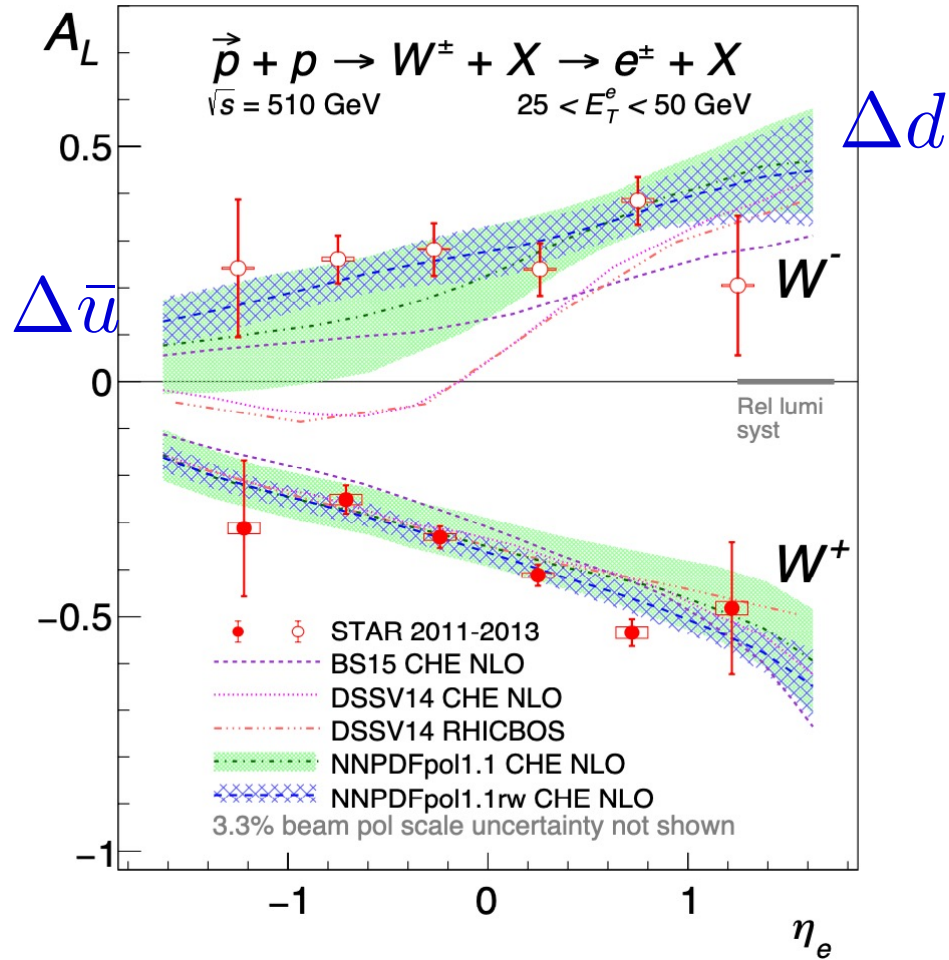
Light sea quarks:



Light sea quarks:



(from 2302.00605)



(consistent w/ SIDIS)

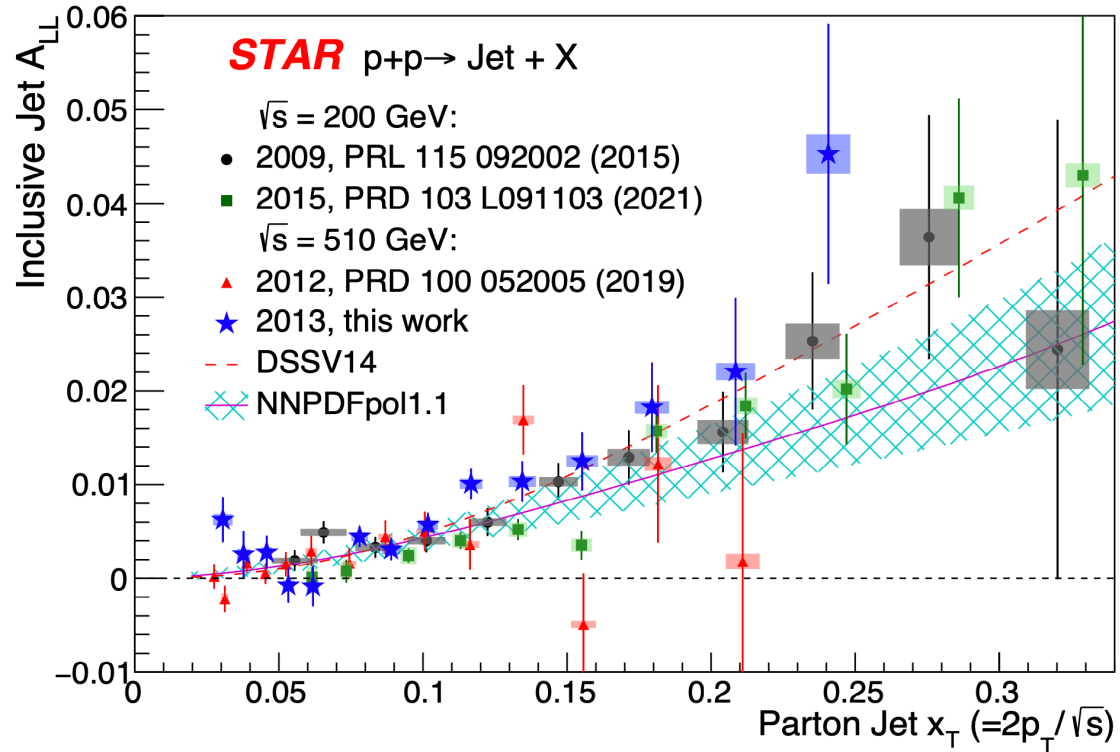
also NNPDF, JAM
 (Cocuzza, Melnitchouk, Metz, Sato)

Gluon polarization: 2014 RHIC-discovery that $\Delta g > 0$

→ improvement and consolidation !

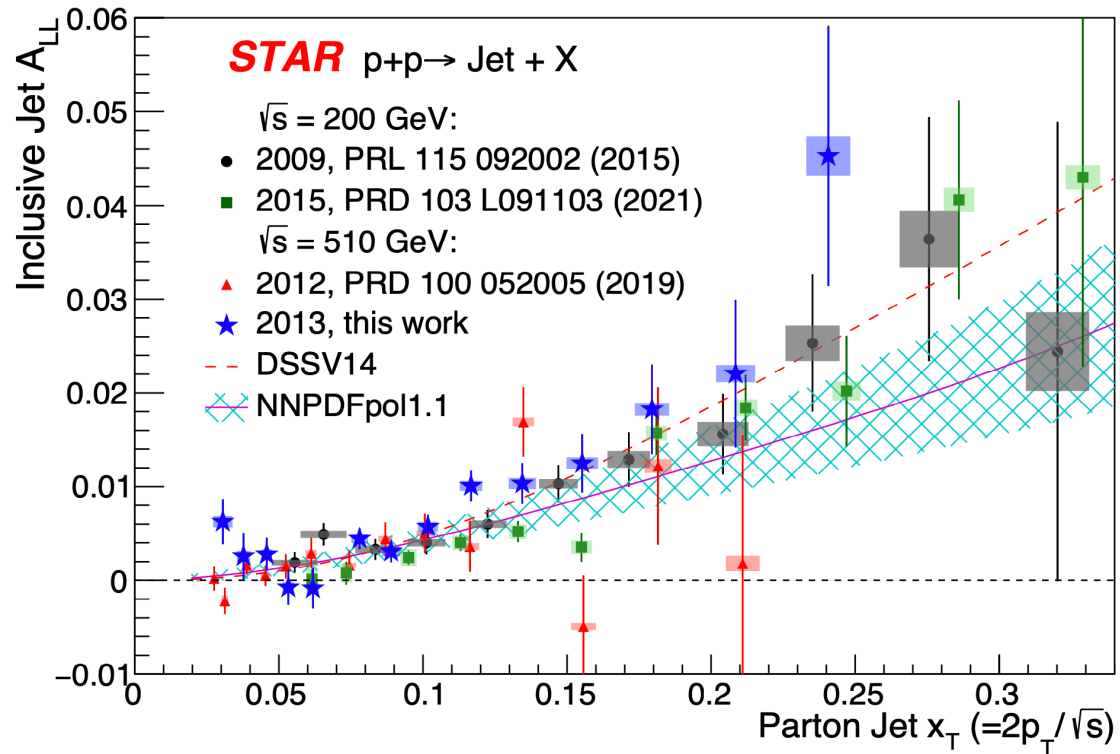
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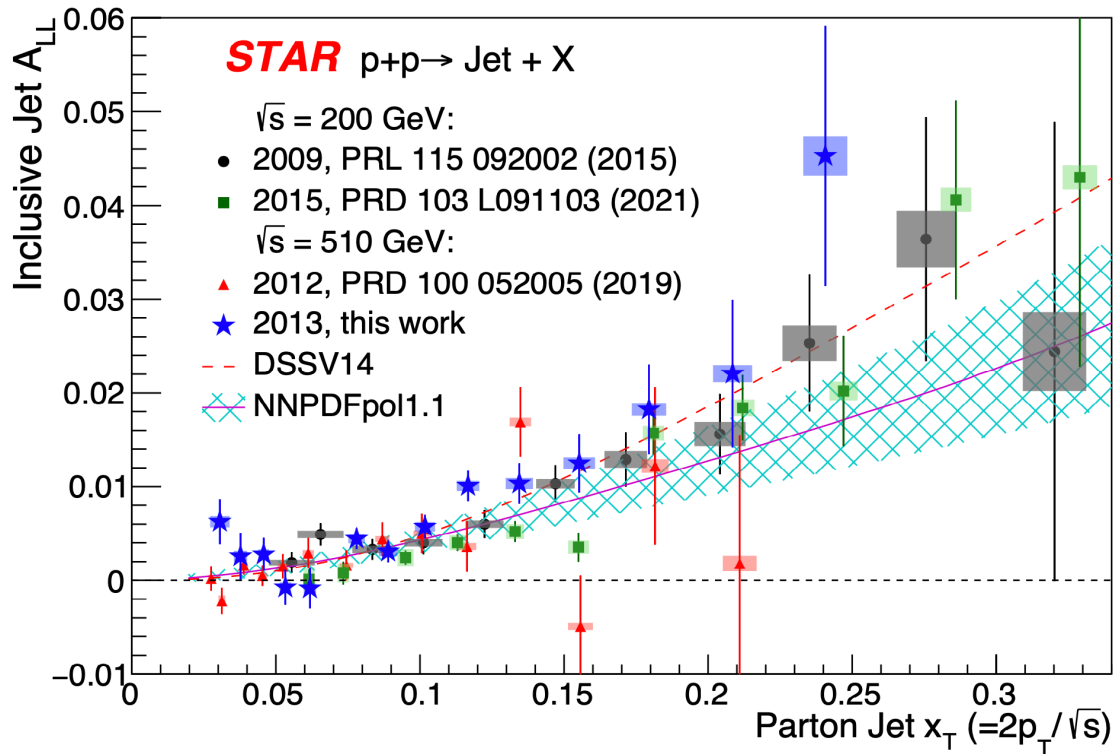
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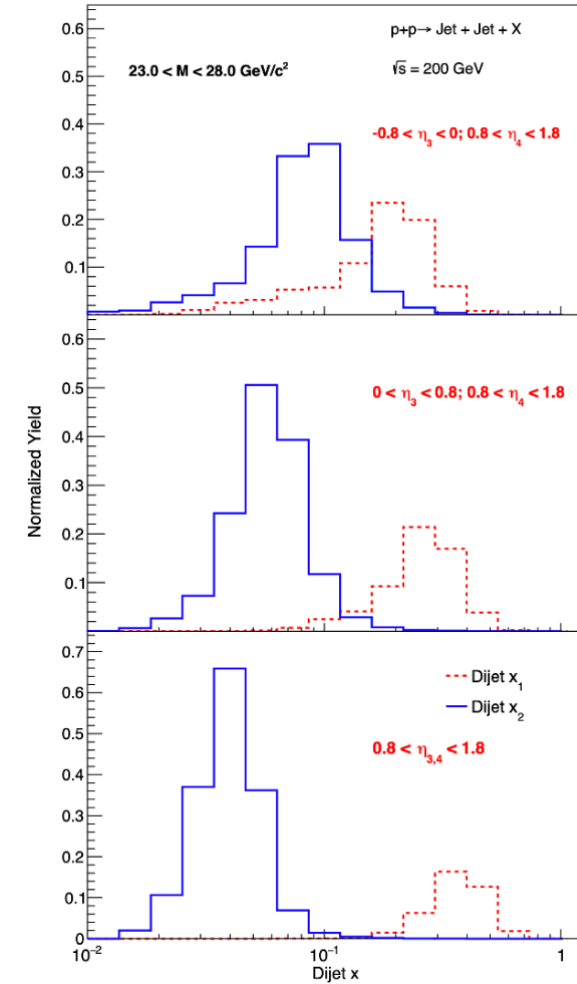
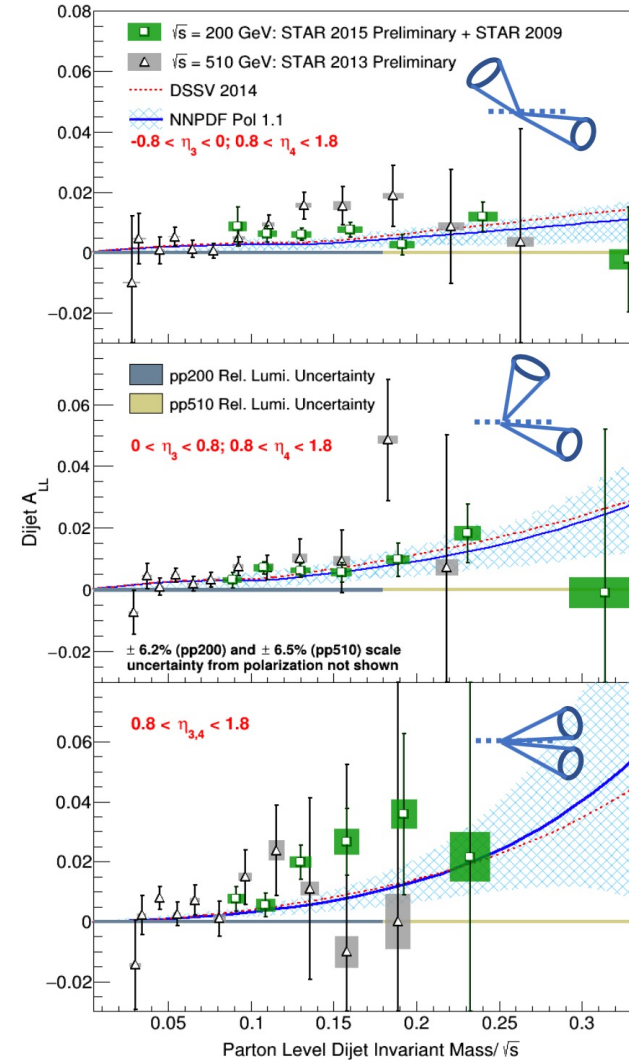
+ Phenix $\pi^{0,\pm}$

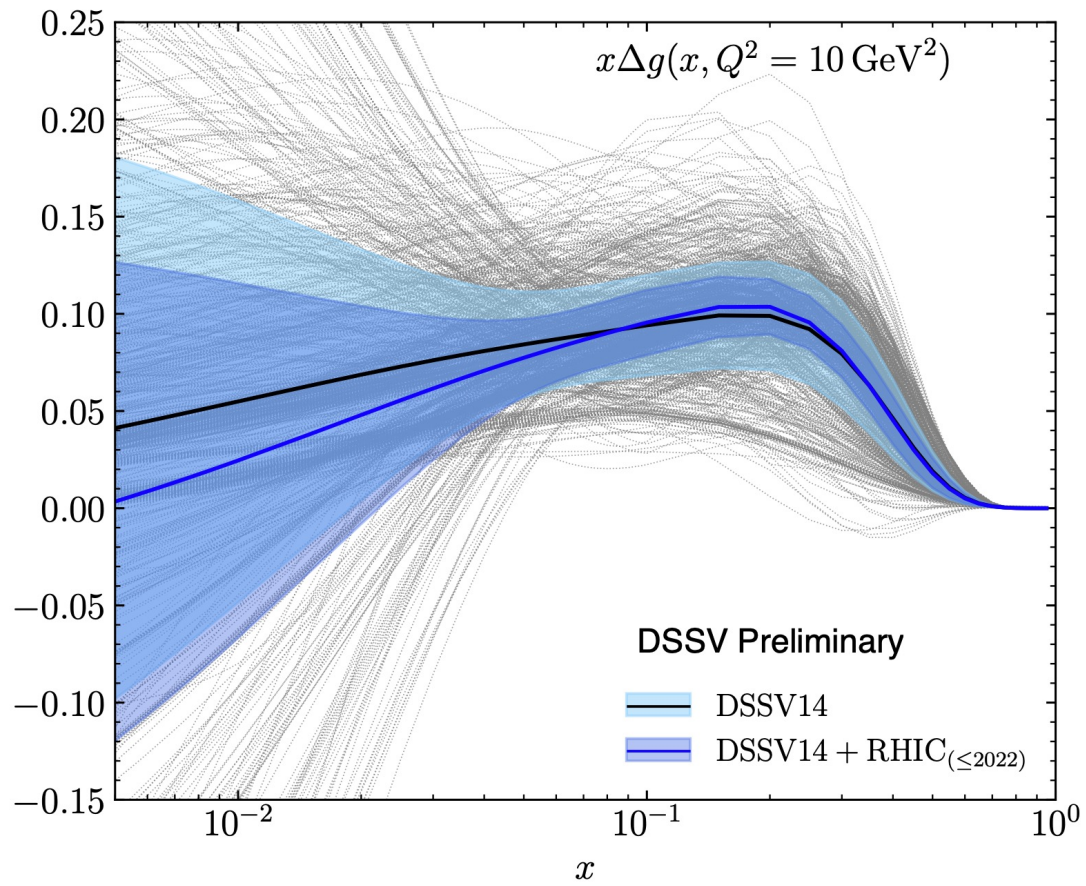
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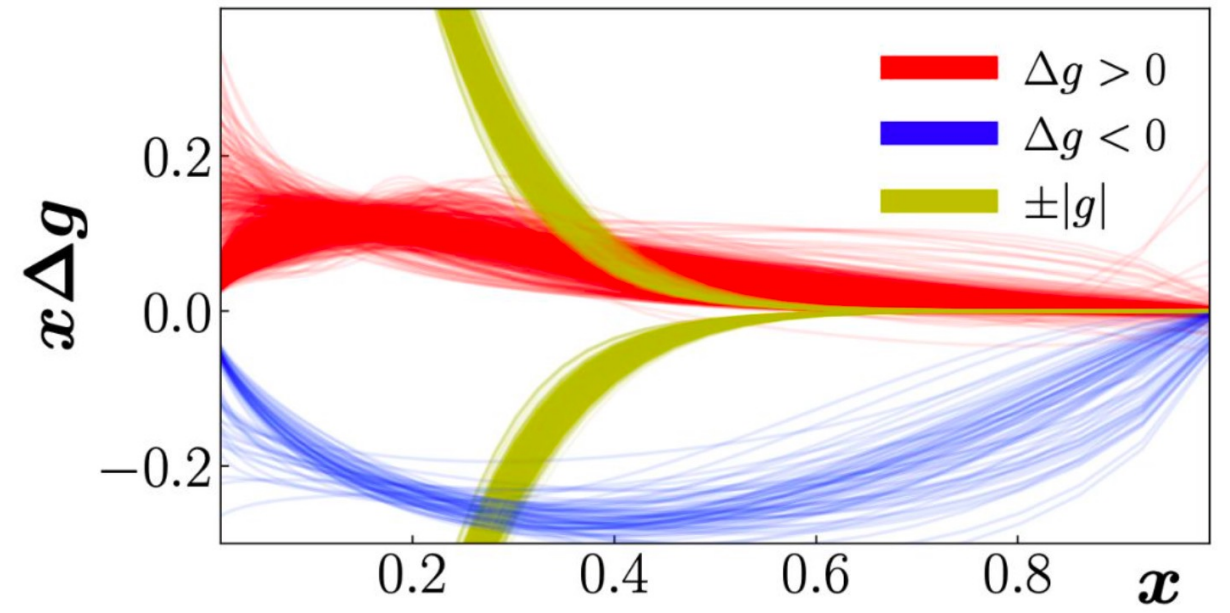
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(from 2302.00605)

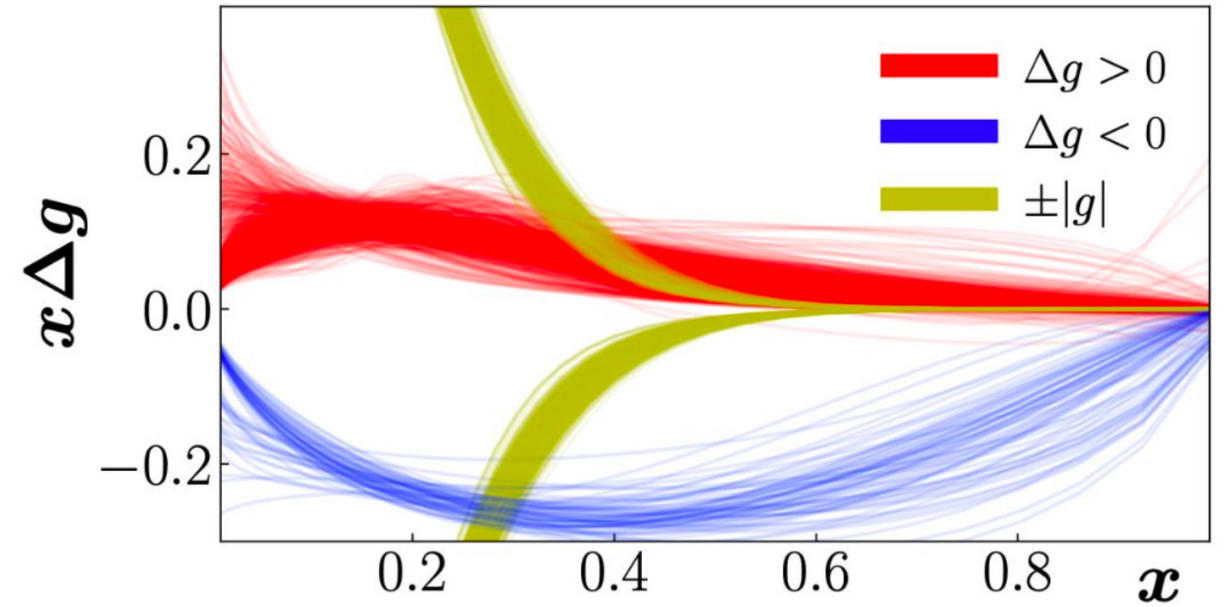
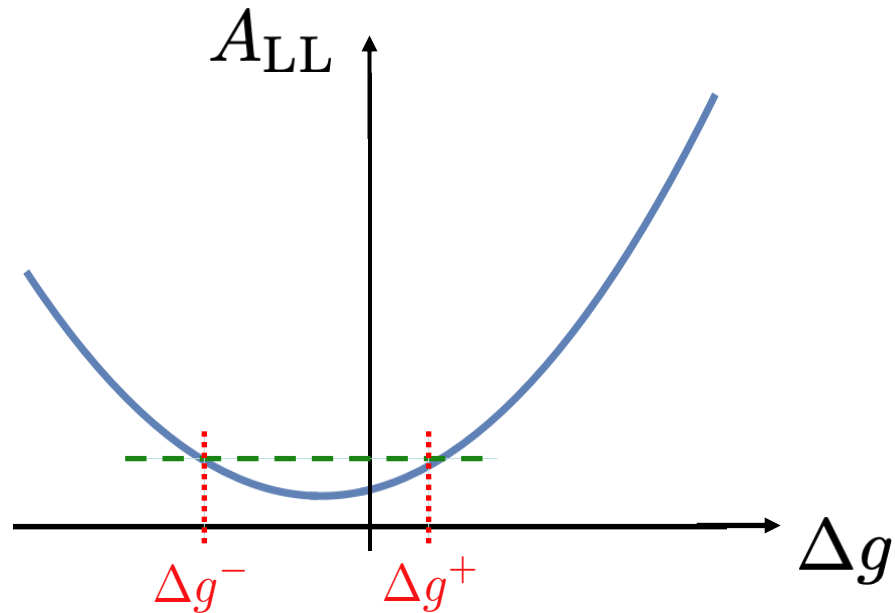
JAM 2022



JAM 2022

$$“ A_{LL} = a_{gg} (\Delta g)^2 + a_{qg} \Delta g + a_{qq} ”$$

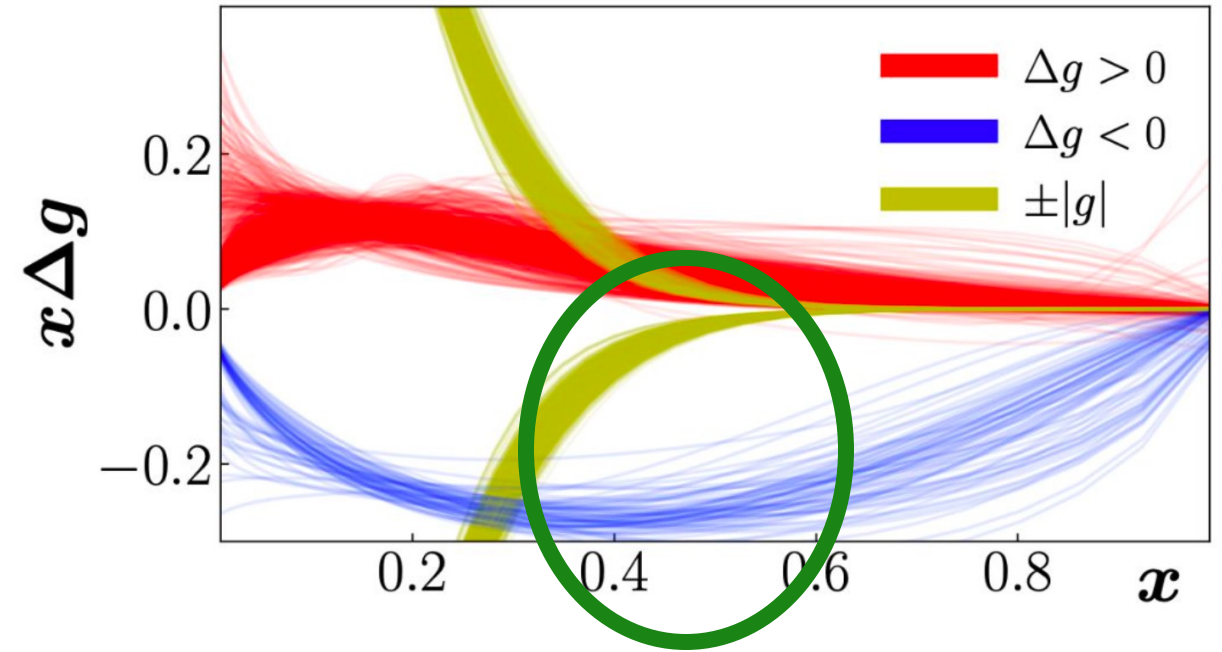
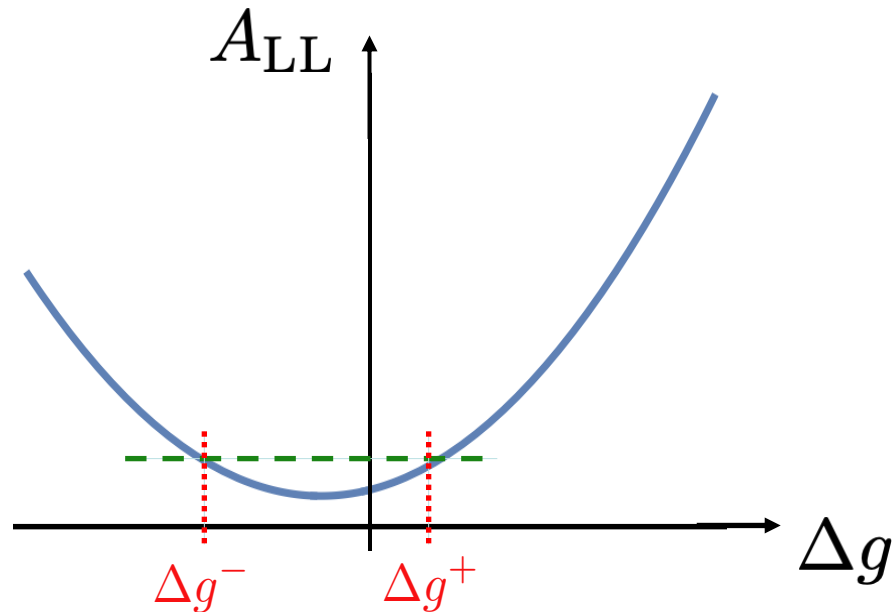
Jäger, Kretzer,
Stratmann,
WV 2004



JAM 2022

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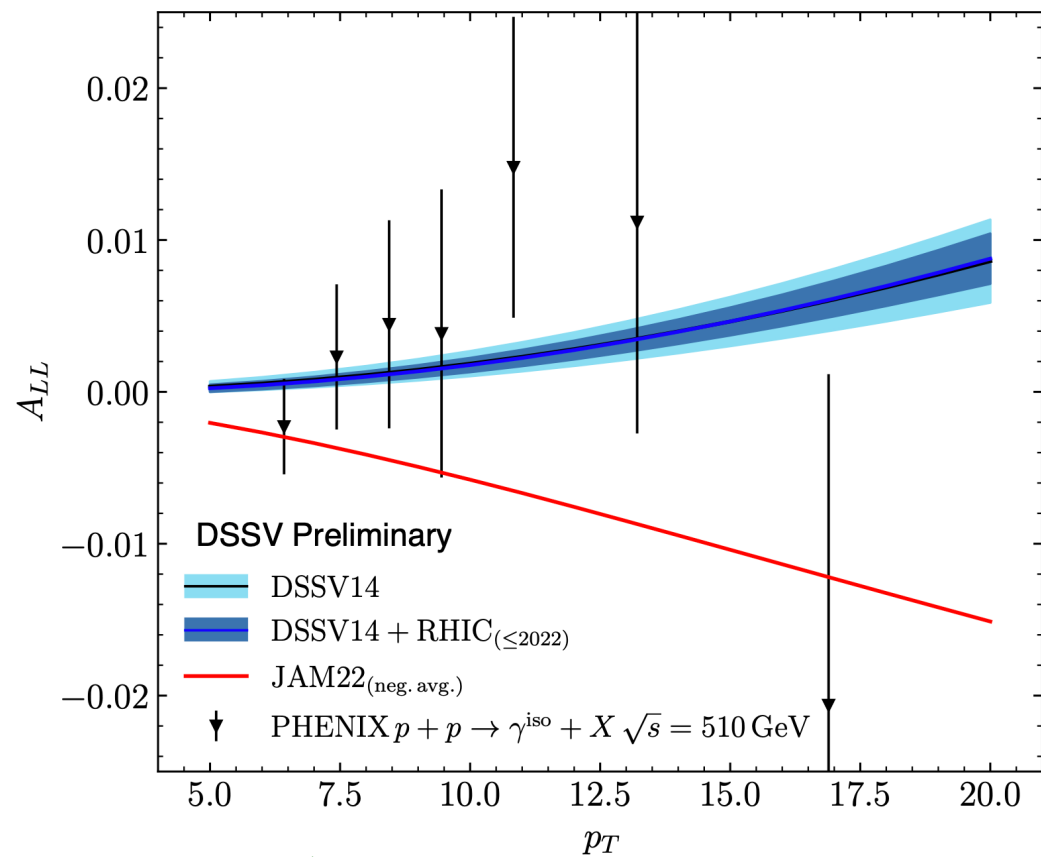
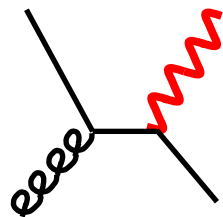
$$|\Delta g| > g \quad !?$$

$$\Delta g = g^+ - g^-$$

$$g = g^+ + g^-$$

In any case, not favored by data:

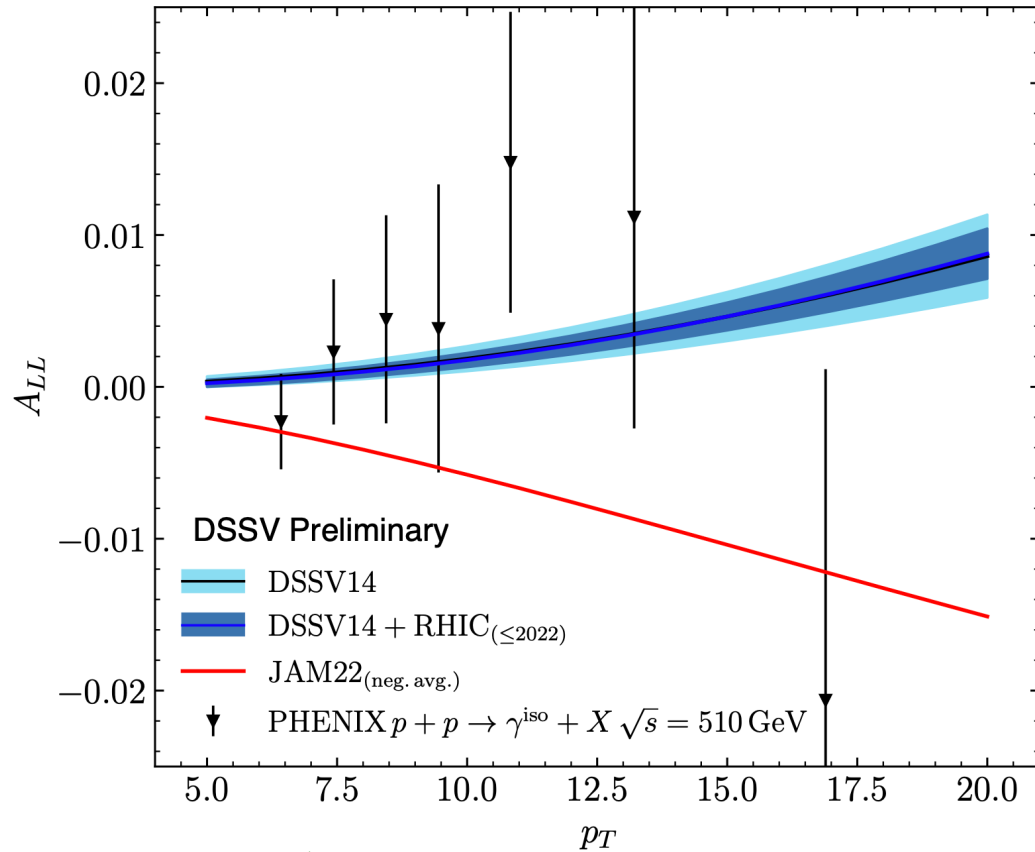
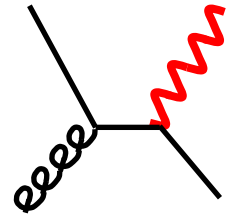
Phenix direct photons:



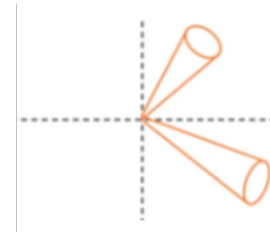
(from 2302.00605)

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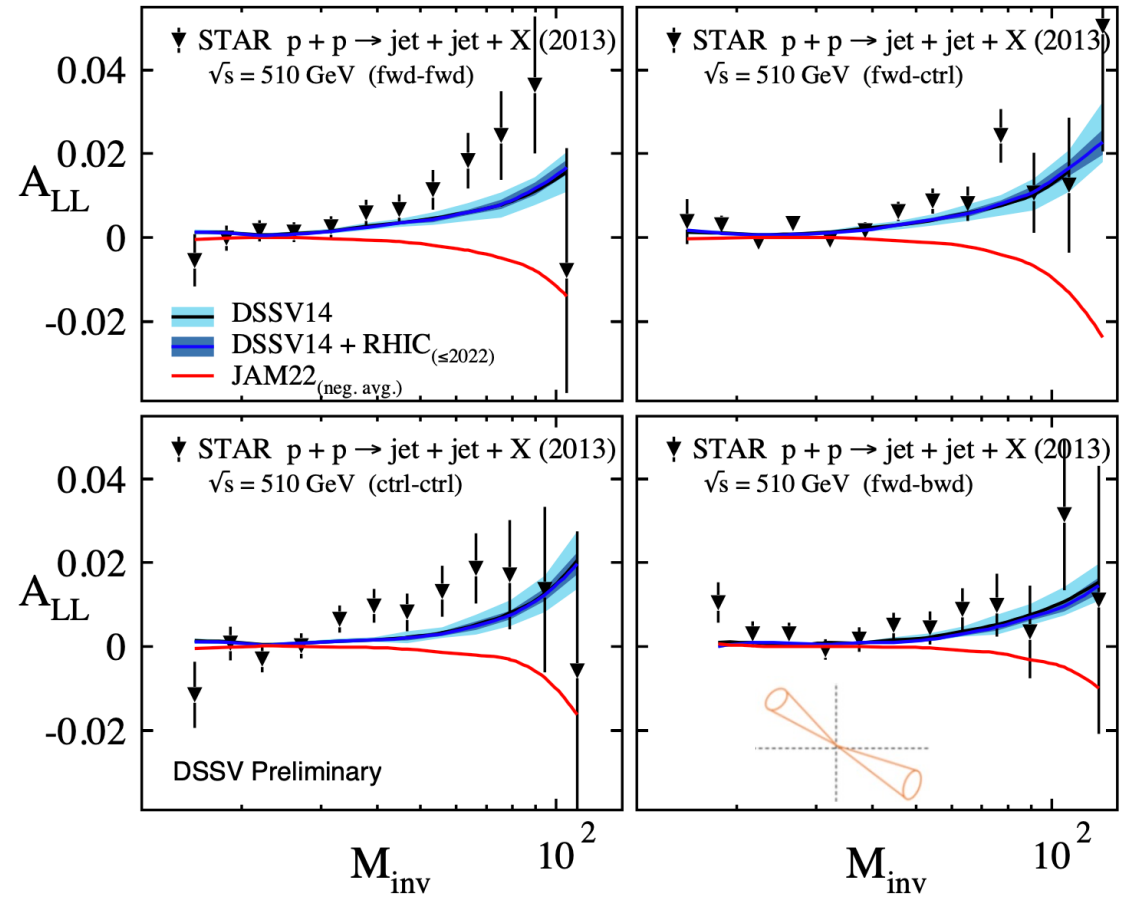
Phenix direct photons:



(from 2302.00605)



STAR dijets



First ever NLO “global” analysis of nucleon helicity (GRSV, 1996)

PHYSICAL REVIEW D

VOLUME 53, NUMBER 9

1 MAY 1996

Next-to-leading-order radiative parton model analysis of polarized deep inelastic lepton-nucleon scattering

M. Glück, E. Reya, and M. Stratmann

Universität Dortmund, Institut für Physik, D-44221 Dortmund, Germany

W. Vogelsang

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon OX11 0QX, England

(Received 28 August 1995; revised manuscript received 10 January 1996)

A next-to-leading-order QCD analysis of spin asymmetries and structure functions in polarized deep inelastic lepton-nucleon scattering is presented within the framework of the radiative parton model. A consistent NLO formulation of the Q^2 evolution of polarized parton distributions yields two sets of plausible NLO spin-dependent parton distributions in the conventional $\overline{\text{MS}}$ factorization scheme. They respect the fundamental positivity constraints down to the low resolution scale $Q^2 = \mu_{\text{NLO}}^2 = 0.34 \text{ GeV}^2$. The Q^2 dependence of the spin asymmetries $A_{p,n,d}(x, Q^2)$ is similar to the leading-order (LO) one in the range $1 \leq Q^2 \leq 20 \text{ GeV}^2$ and is

Nucleon helicity structure at NNLO



Borsa, de Florian, Sassot, Stratmann, WV

BDSSV

Why NNLO ?

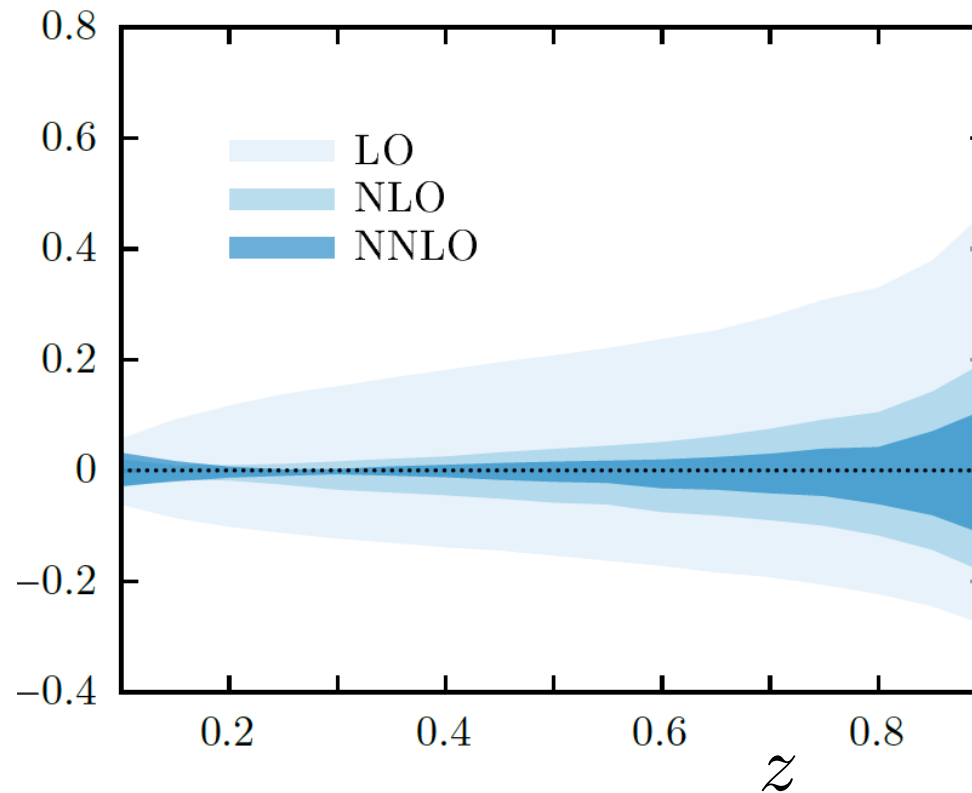
- need per cent accuracy for EIC and JLab (cf. LHC experience)

Why NNLO ?

- need per cent accuracy for EIC and JLab (cf. LHC experience)
- reduce theory uncertainty

$$\frac{\sigma(\mu) - \sigma(Q)}{\sigma(Q)}$$

$$Q/2 \leq \mu_{R,F} \leq 2Q$$



$$Q^2 > 5 \text{ GeV}^2$$

SIDIS@EIC

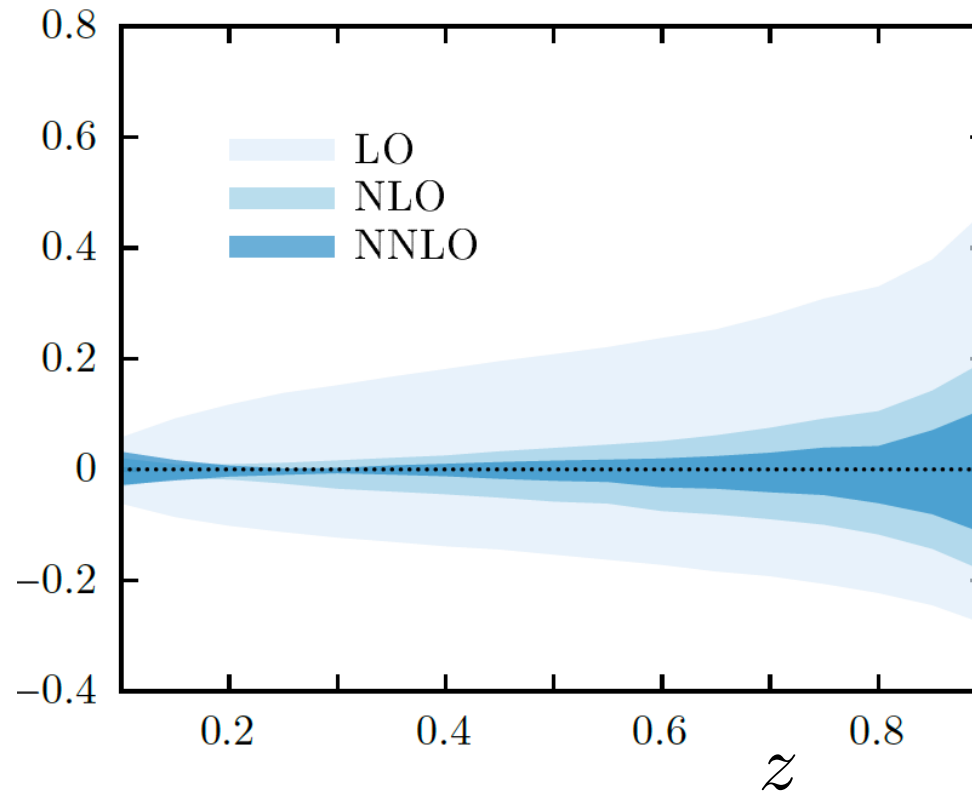
Abele, De Florian, WV

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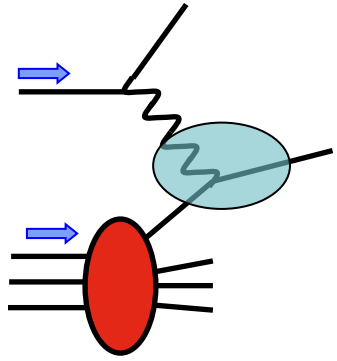
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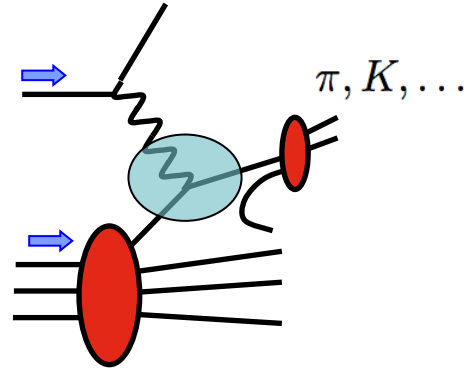
Abele, De Florian, WV

- progress in lattice computations

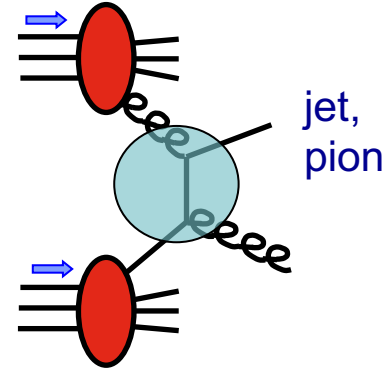
How to get to NNLO ?



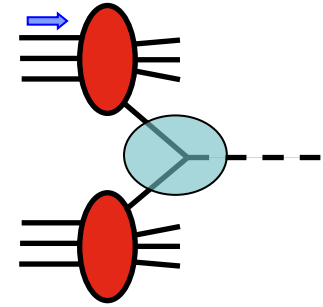
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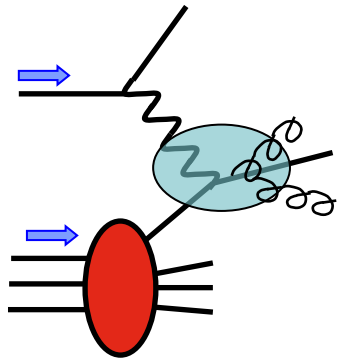


pp high- p_T

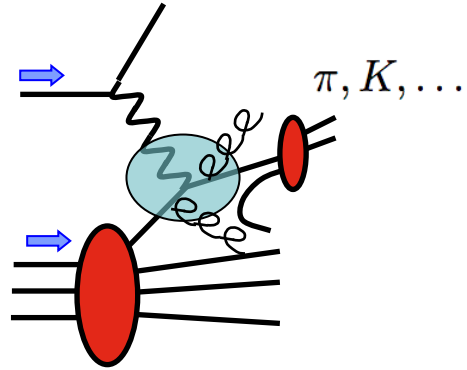


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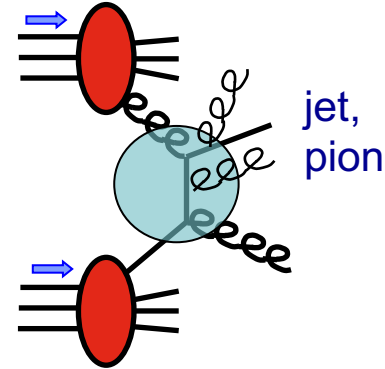
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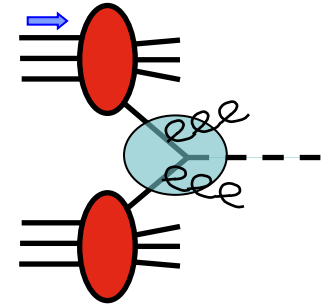
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pp high- p_T

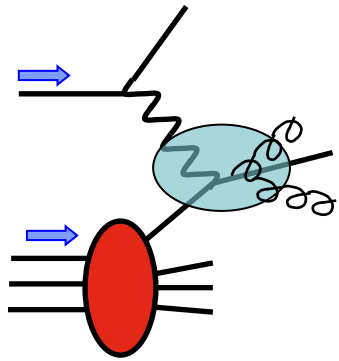


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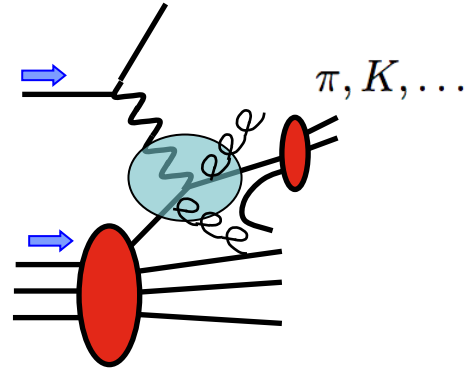
Ingredients for NNLO:

- Partonic hard scattering:
$$\Delta \hat{\sigma}_{ab} = \Delta \hat{\sigma}_{ab}^{\text{LO}} + \frac{\alpha_s}{\pi} \Delta \hat{\sigma}_{ab}^{\text{NLO}} + \left(\frac{\alpha_s}{\pi} \right)^2 \Delta \hat{\sigma}_{ab}^{\text{NNLO}} + \dots$$

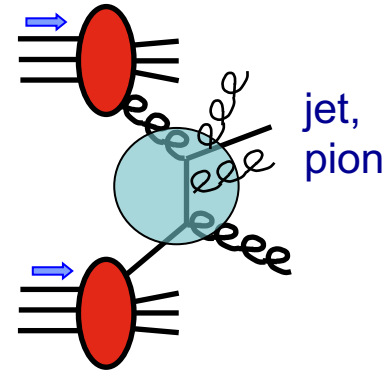
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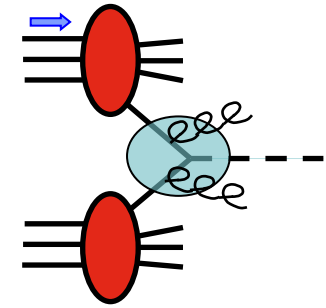
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- PDF evolution:
$$\Delta \mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \Delta P_{ij}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi} \right)^2 \Delta P_{ij}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi} \right)^3 \Delta P_{ij}^{\text{NNLO}} + \dots$$

NNLO PDF evolution known:

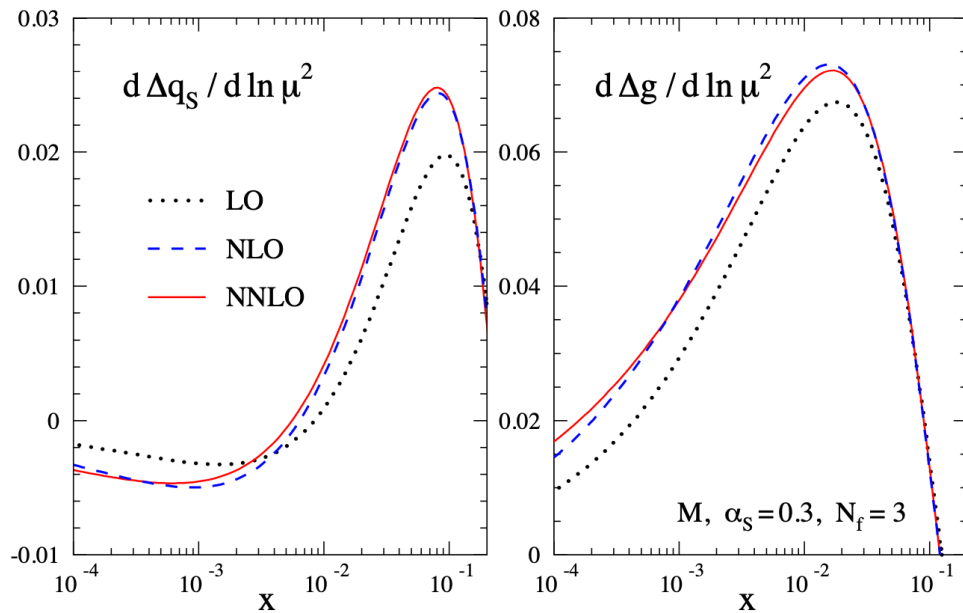
$$\Delta\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi} \Delta P_{ij}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{\text{NNLO}} + \dots$$



Moch, Vogt, Vermaseren 2008, 2014, 2015
Blümlein, Marquard, Schneider, Schönwald 2022

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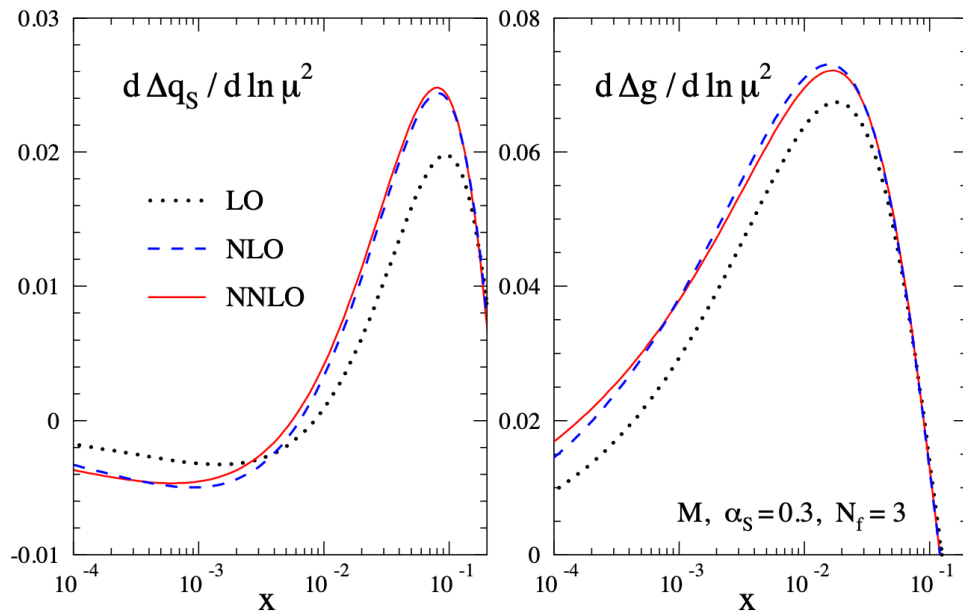
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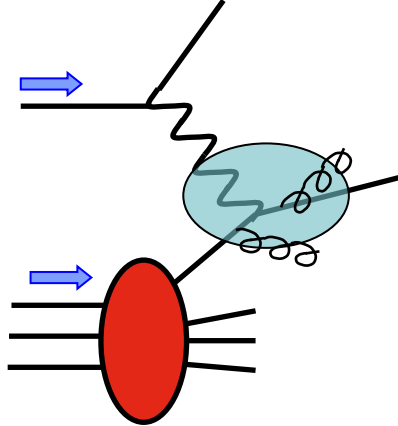
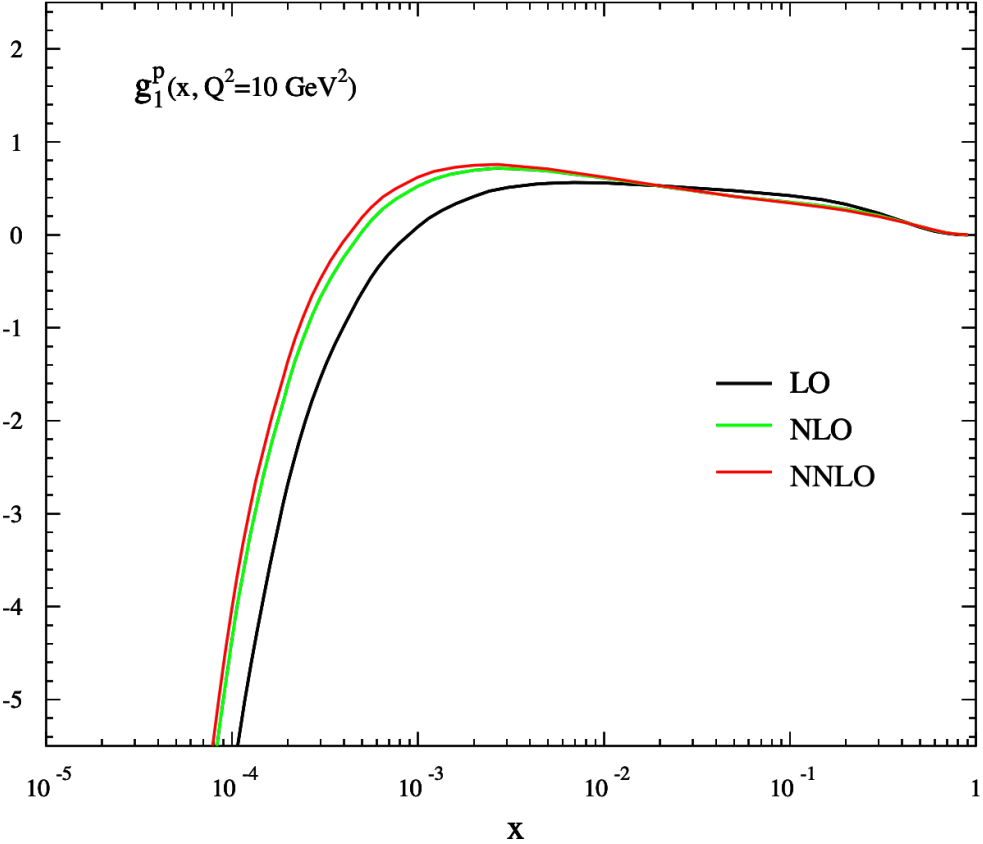
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Blümlein, Marquard, Schneider, Schönwald 2022

implemented in QCD-PEGASUS
framework

A. Vogt

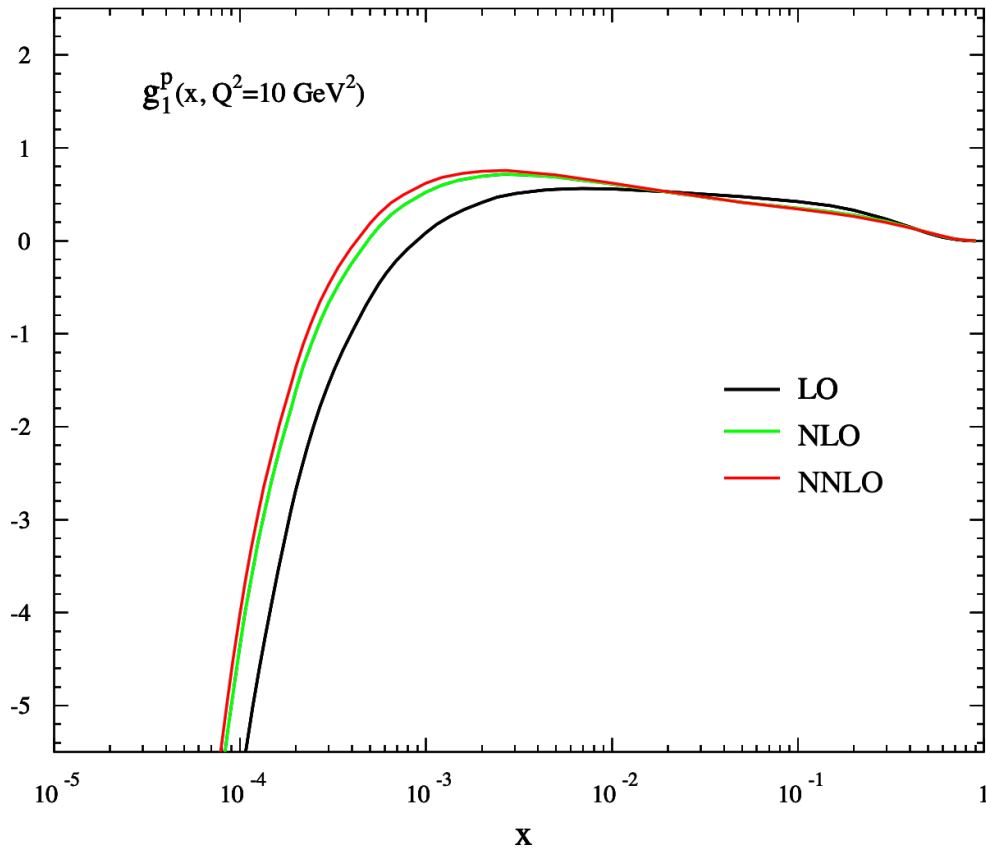
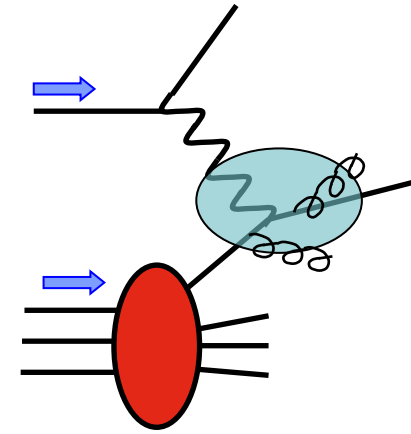
NNLO corrections to DIS (g_1 structure function):

Zijlstra, van Neerven 1994



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We use Mellin moments:

program MT:

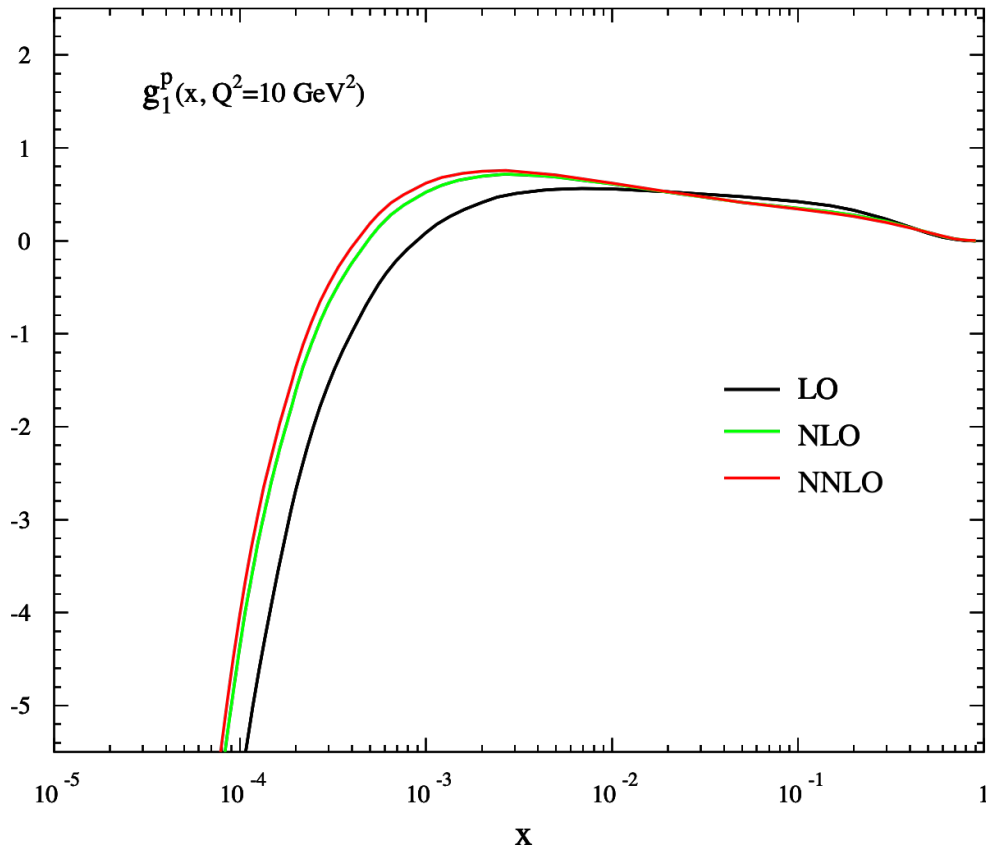
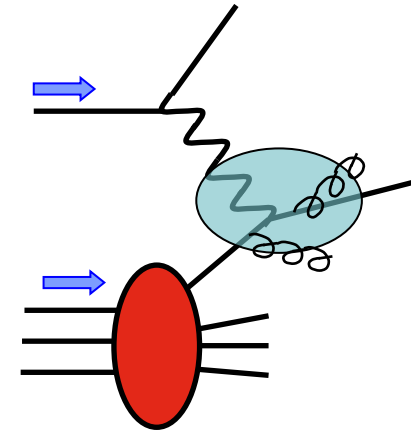
Höchele, Hoff, Pak, Steinhauser, Ueda 2013

program ANCONT: Blümlein 2000

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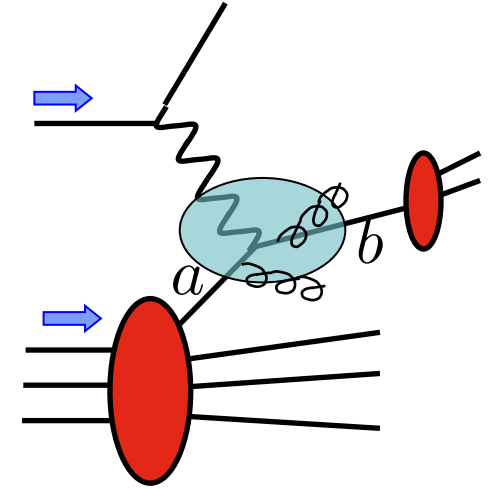
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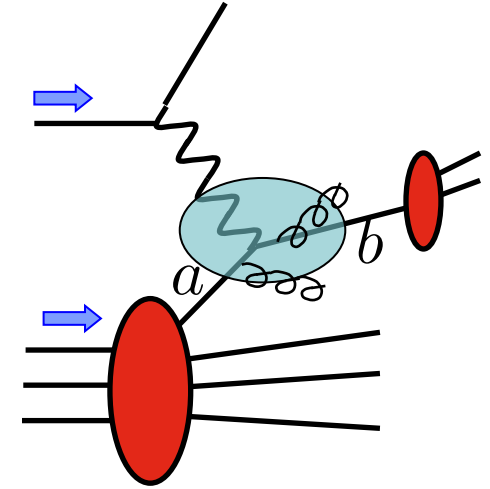
- DIS-only analysis: Taghavi-Shahri et al., 2016

NNLO corrections to SIDIS:
not fully available yet...



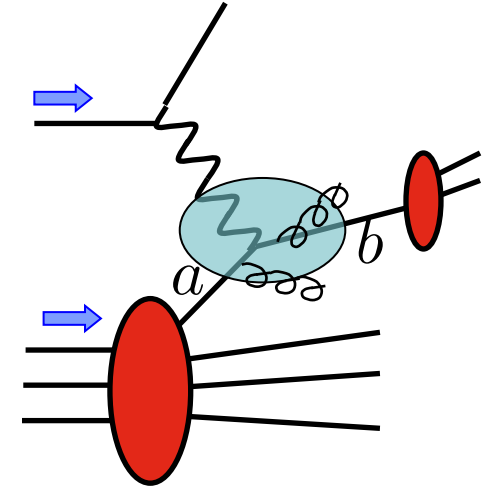
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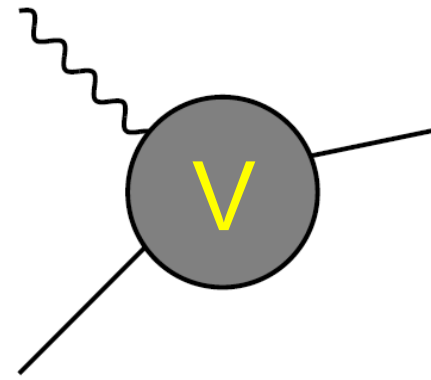
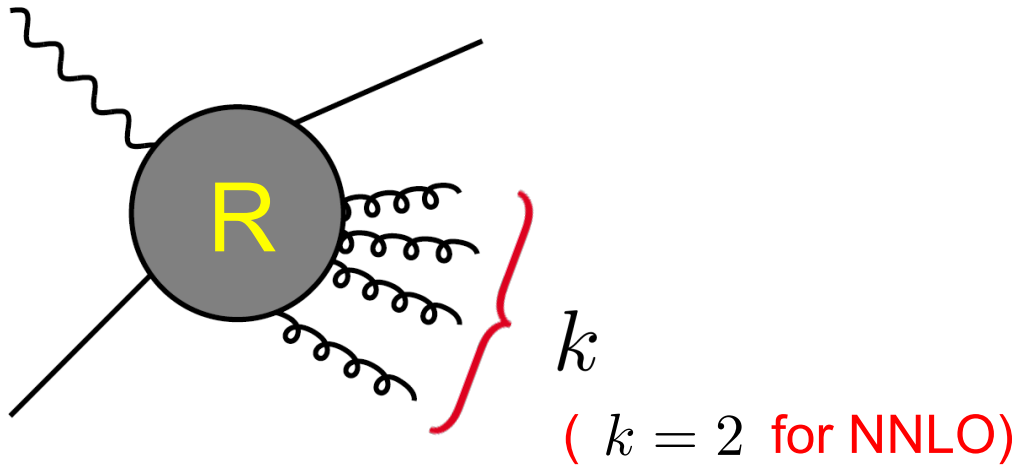


$$\hat{x} = \frac{Q^2}{2p_a \cdot q}$$

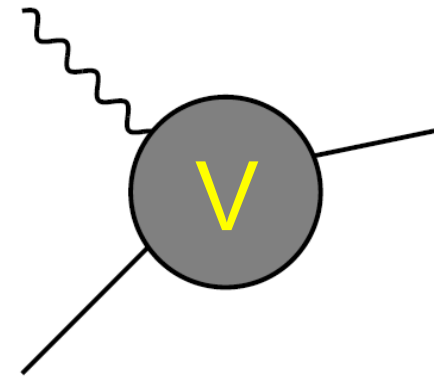
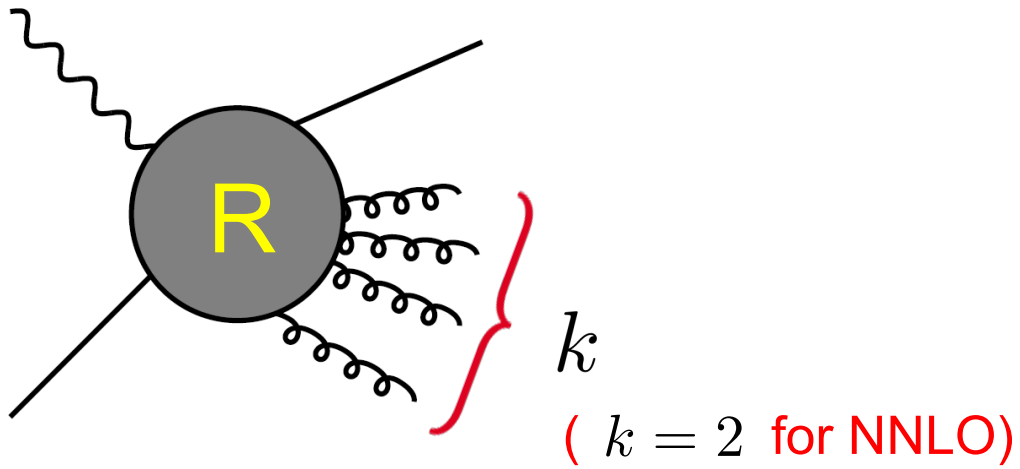
k^{th} order of perturbation theory:

$$\hat{z} = \frac{p_a \cdot p_b}{p_a \cdot q}$$

$$\Delta \hat{\sigma}_{qq}^{\text{N}^k\text{LO}}(\hat{x}, \hat{z}) \sim \alpha_s^k \left[\delta(1 - \hat{x}) \left(\frac{\ln^{2k-1}(1 - \hat{z})}{1 - \hat{z}} \right)_+ + \delta(1 - \hat{z}) \left(\frac{\ln^{2k-1}(1 - \hat{x})}{1 - \hat{x}} \right)_+ \right. \\ \left. + \frac{1}{(1 - \hat{x})_+} \left(\frac{\ln^{2k-2}(1 - \hat{z})}{1 - \hat{z}} \right)_+ + \frac{1}{(1 - \hat{z})_+} \left(\frac{\ln^{2k-2}(1 - \hat{x})}{1 - \hat{x}} \right)_+ + \dots \right]$$



$$\hat{x} = \hat{z} = 1$$



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- logs can be resummed to all orders: **threshold resummation**

Anderle, Ringer, WV
Abele, de Florian, WV

Fixed Order						
Resummation	LO	1				
	NLO	$\alpha_s L^2$	$\alpha_s L$	α_s		
	NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2

	N ^k LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$
						...

↓

LL

↓

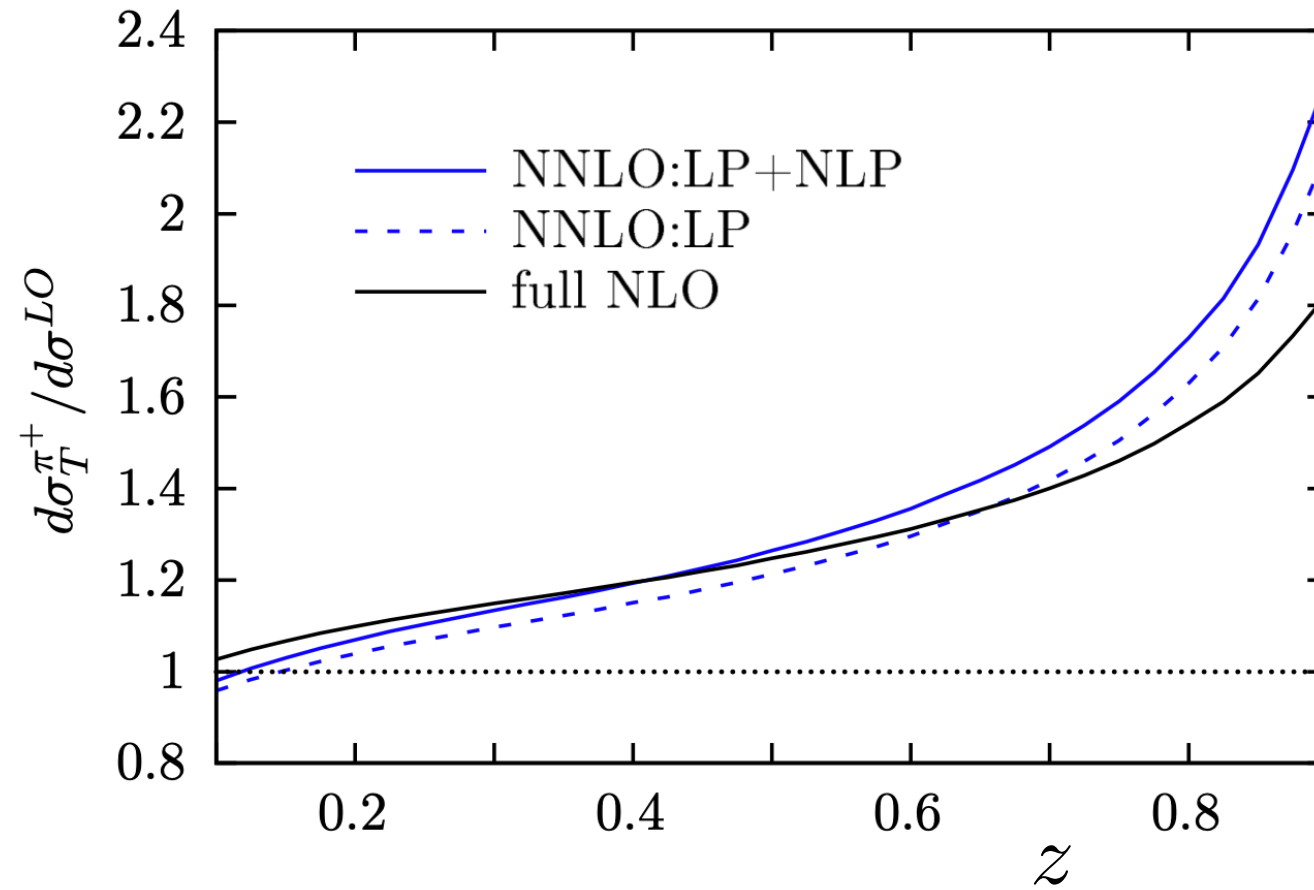
NLL

↓

NNLL

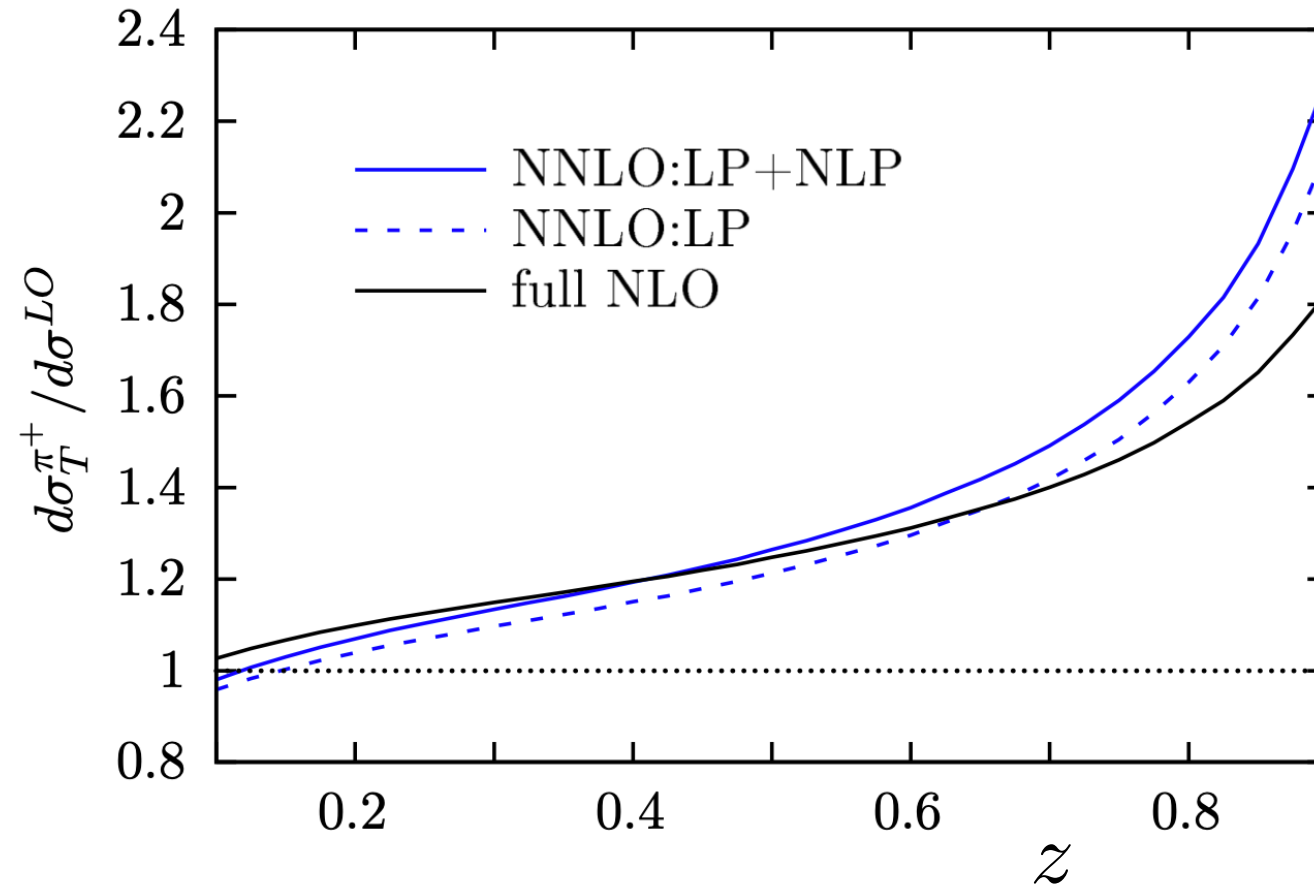
$\mu p \rightarrow \mu \pi^+ X$

COMPASS



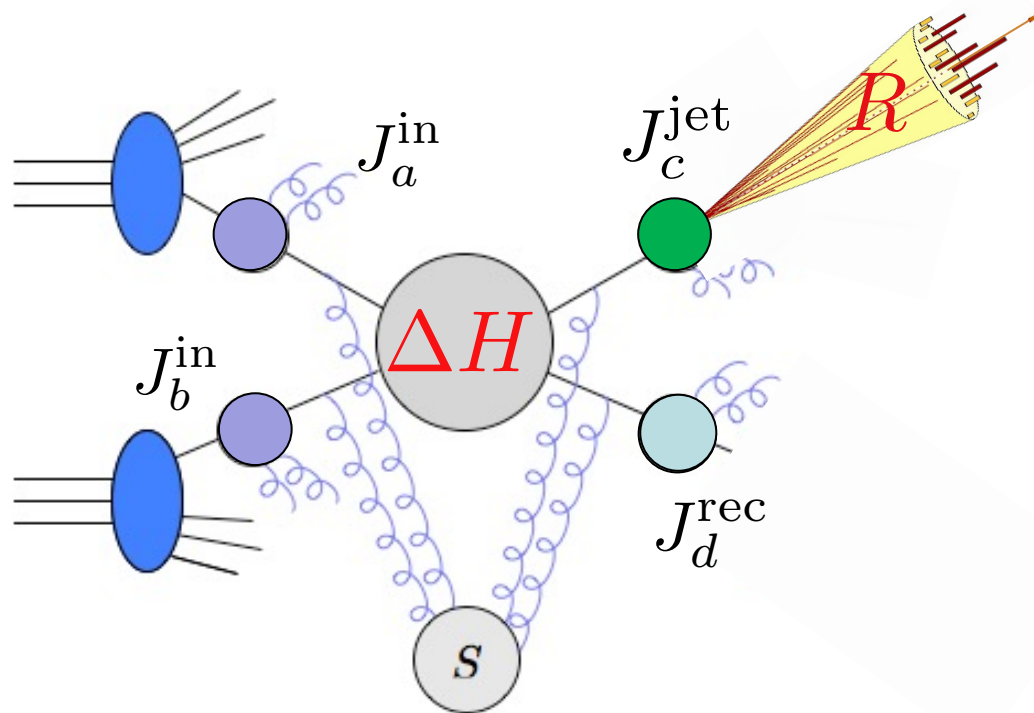
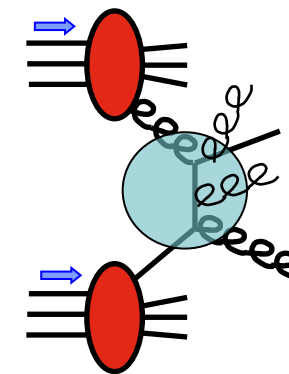
$\mu p \rightarrow \mu \pi^+ X$

COMPASS



Framework used for first global NNLO analysis of **fragm. fcts.**

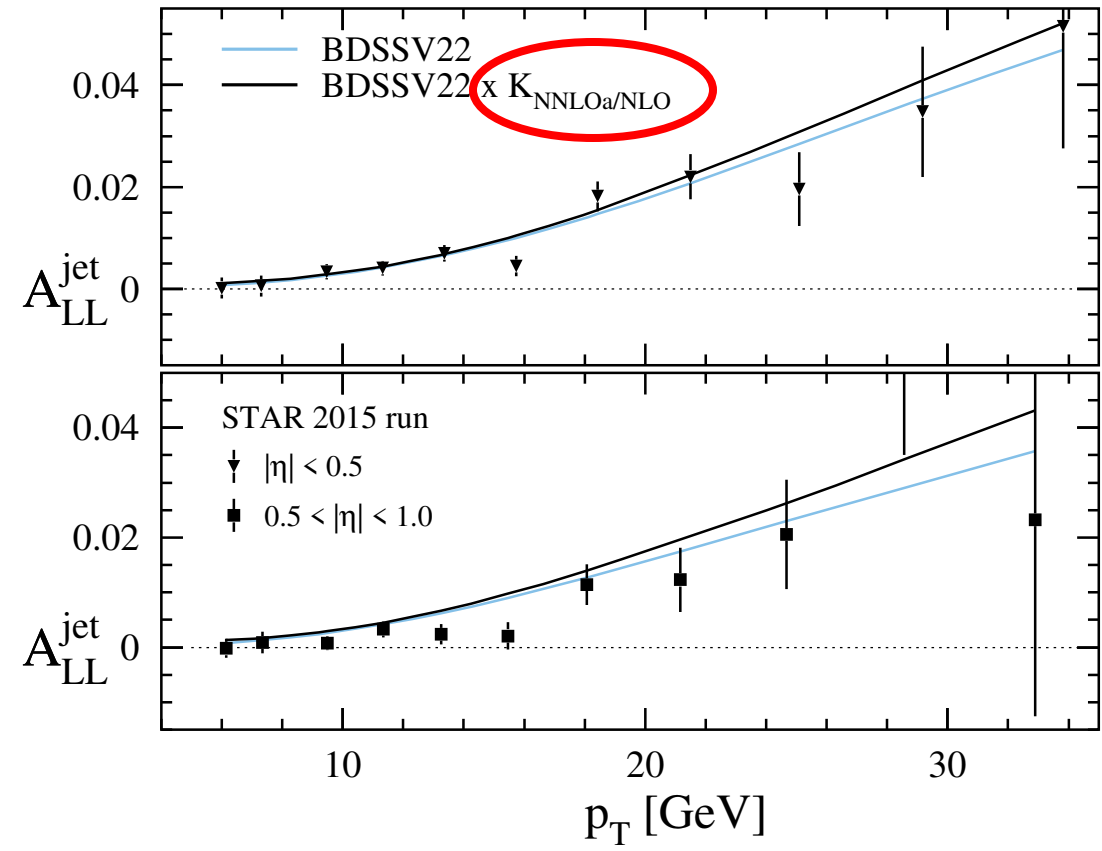
Approximate NNLO corrections for $pp \rightarrow \text{jet}+X$ at RHIC:



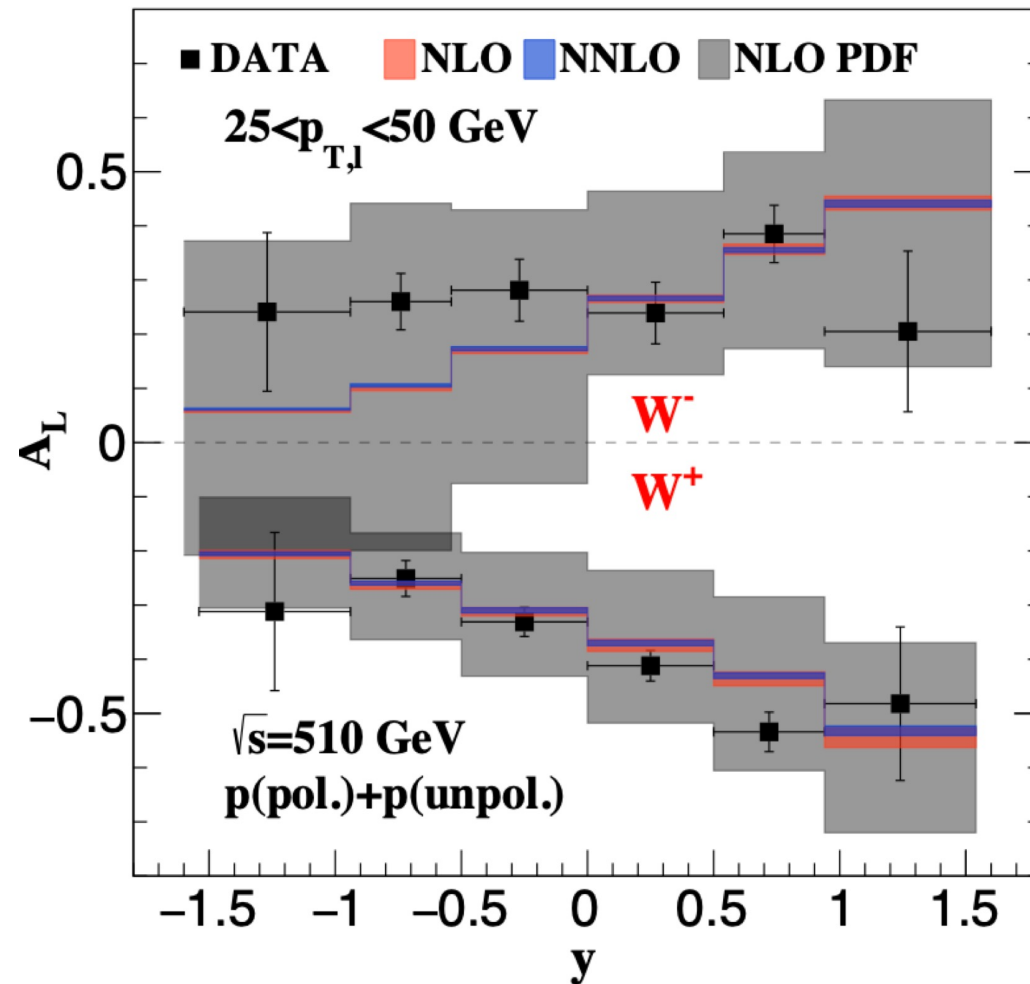
Kidonakis, Oderda, Sterman
de Florian, WV
Hinderer, Ringer, Sterman, WV,

threshold logs $\ln \left(1 - \frac{s_{\text{rad}}}{s} \right)$

$$\Delta \hat{\sigma}^{ab \rightarrow cd} \sim J_a^{\text{in}} \times J_b^{\text{in}} \times J_c^{\text{jet}} \times J_d^{\text{rec}} \times \text{Tr} \left[\Delta H S^\dagger S S \right]_{ab \rightarrow cd}$$



NNLO corrections for W bosons at RHIC:



Boughezal, Li, Petriello

Our 1st global NNLO fit !

Data:

DIS: EMC,SMC,E142,E143,E154,E155, HERMES, COMPASS, HALL-A,CLAS (p,n,d,He)	378
SIDIS: HERMES, COMPASS (p- π^\pm ,d- π^\pm)	80
PP-JETS: STAR run 5,6,9,12,13,15 ($\sqrt{s} = 200, 510 GeV$) (no dijets yet)	91
PP-π^0/π^\pm: PHENIX, STAR	82
PP W^\pm: PHENIX, STAR	22
Total:	653

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Parameterizations:

$$(\Delta q + \Delta \bar{q})(x, Q_0^2) = N_q x^{\alpha_q} (1-x)^{\beta_q} \left(1 + \gamma_q x^{\delta_q} + \eta_q x \right)$$

$$\Delta \bar{q}(x, Q_0^2) = N_{\bar{q}} x^{\alpha_{\bar{q}}} (1-x)^{\beta_{\bar{q}}} \left(1 + \gamma_{\bar{q}} x^{\delta_{\bar{q}}} \right)$$

$$\Delta g(x, Q_0^2) = N_g x^{\alpha_g} (1-x)^{\beta_g} \left(1 + \gamma_g x^{\delta_g} \right)$$

$$Q_0^2 = 1 GeV^2 \quad (32 \text{ parameters})$$

χ^2

	<i>NNLO</i>	<i>NLO</i>
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<i>DIS</i> (378)	302	298
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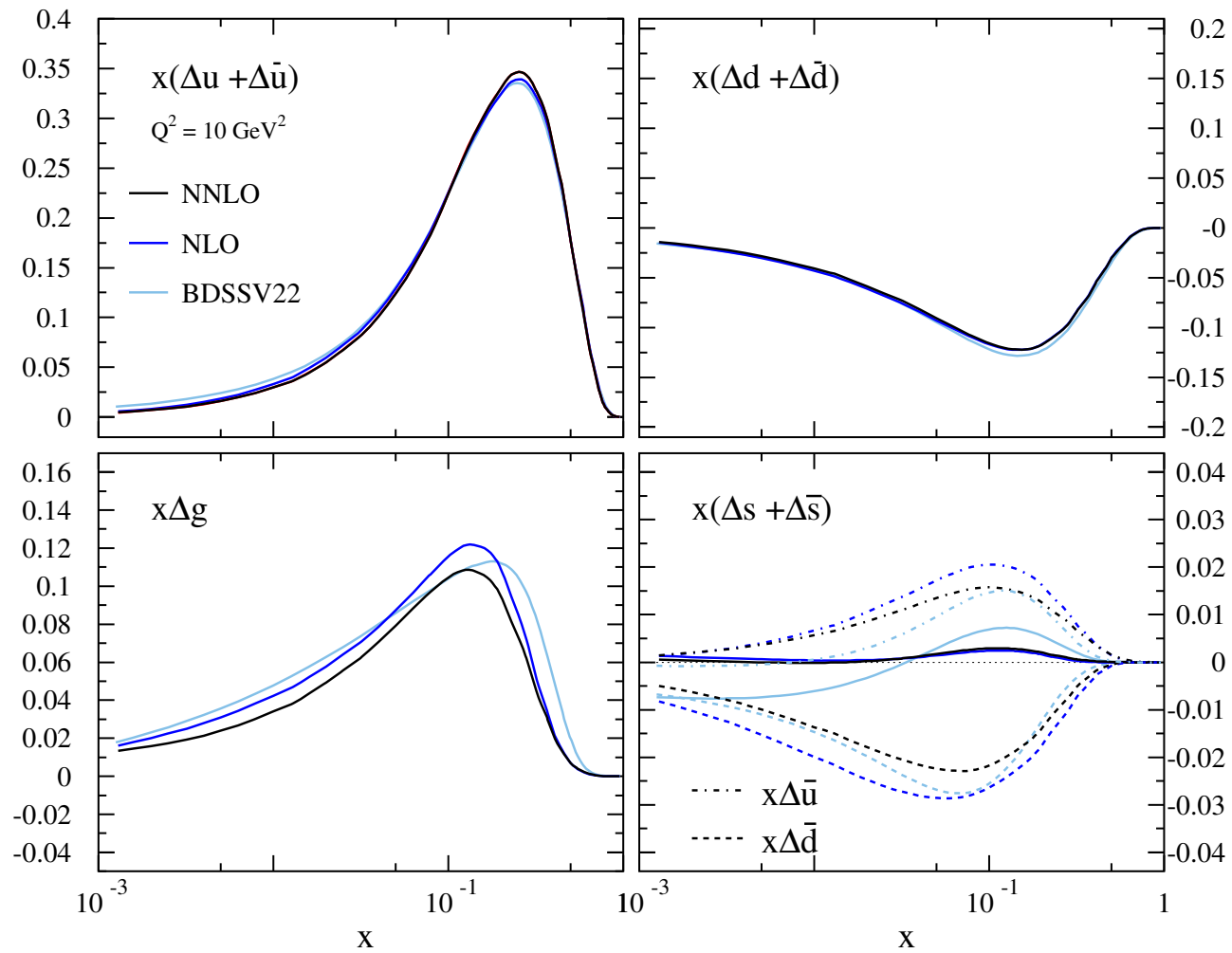
<i>SIDIS</i> (80)	99	93
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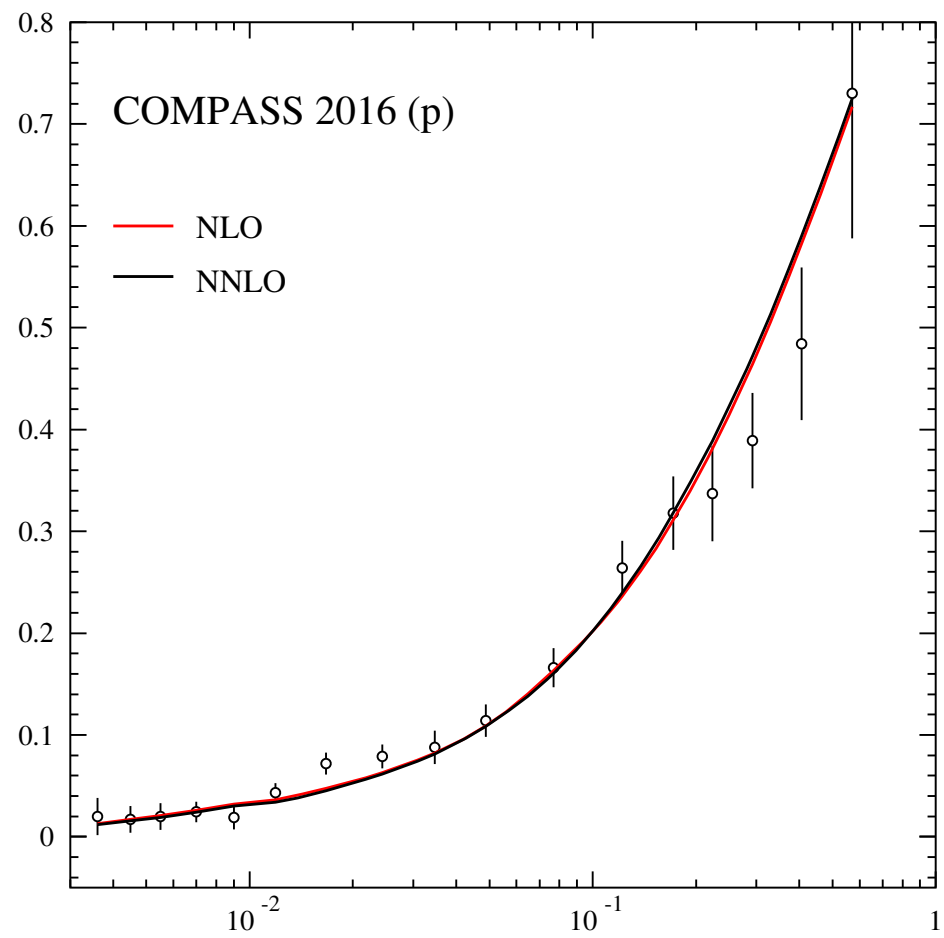
<i>JETS</i> (91)	103	111
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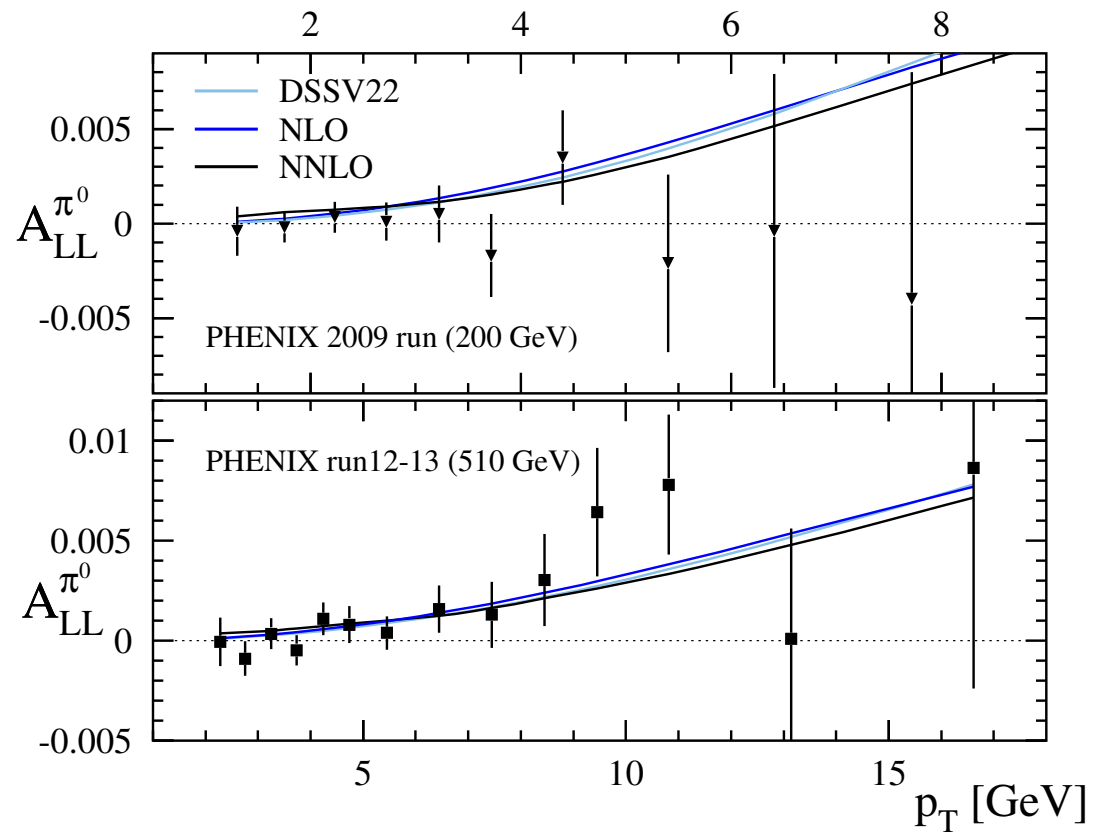
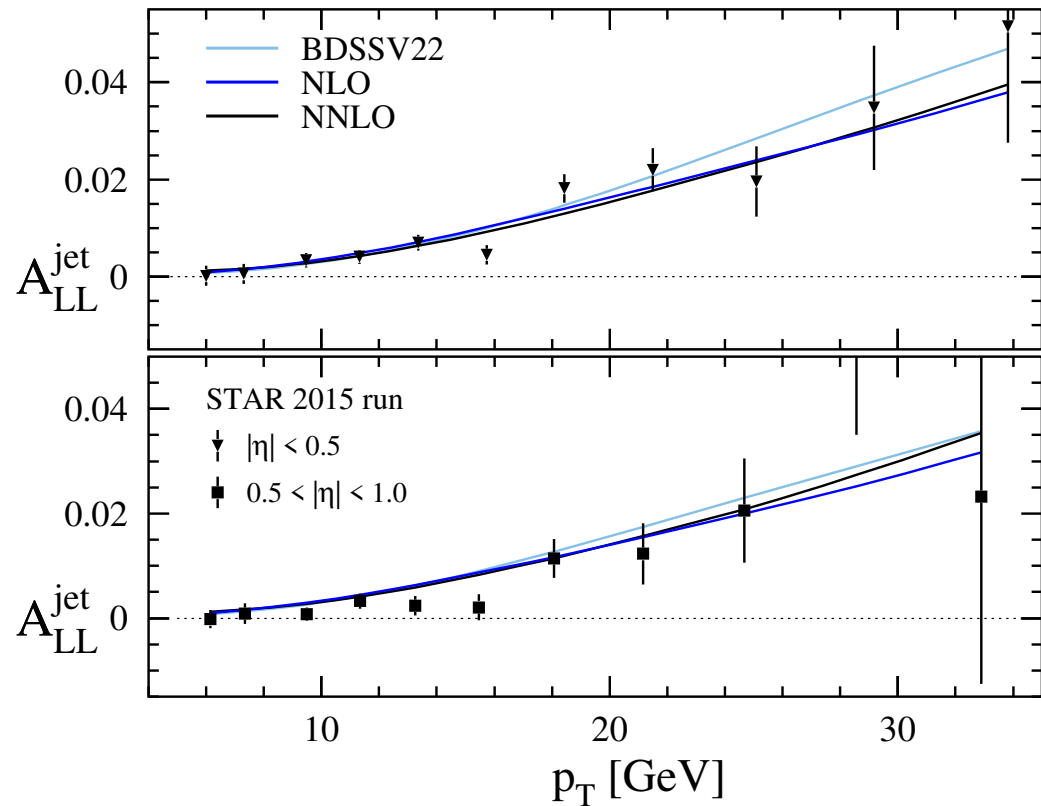
$\pi^{0,\pm}$ (82)	65	65
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W^\pm (22)	22	21
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	χ^2	
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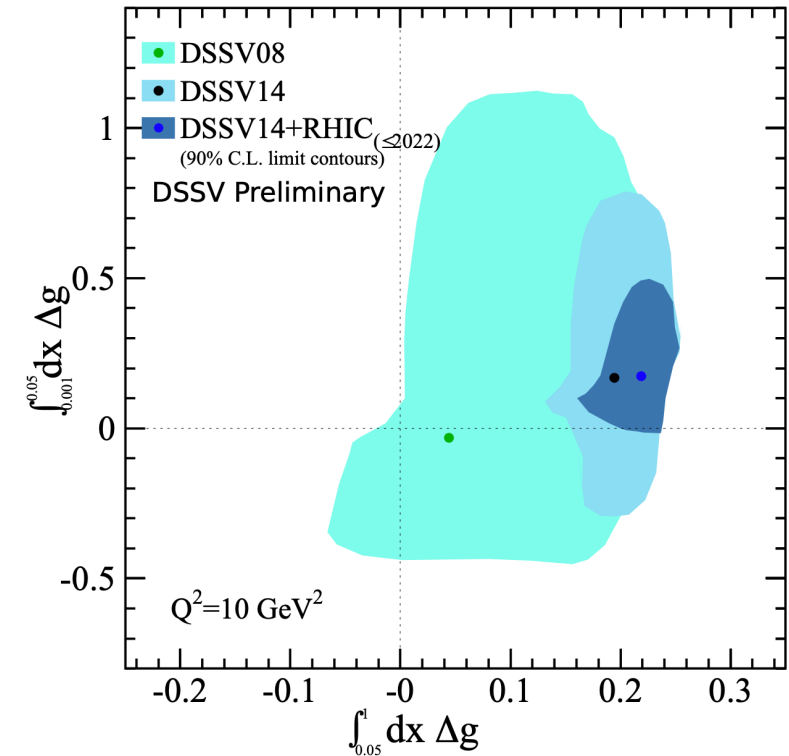


Concluding remarks:

- enormous recent progress on helicity PDFs

$$\int_{0.01}^1 dx \Delta\Sigma(x, Q^2) = 0.43 \pm 0.08$$

$$\int_{0.01}^1 dx \Delta g(x, Q^2) = 0.3 \pm 0.1$$



- qualitative step forward to NNLO: pQCD analysis “in good shape”
- work on improving NNLO analysis ongoing
- numerous outstanding issues: low-x, power corrections, synergies with lattice