Leading hadronic contribution to the muon magnetic moment from lattice QCD

Z. Fodor

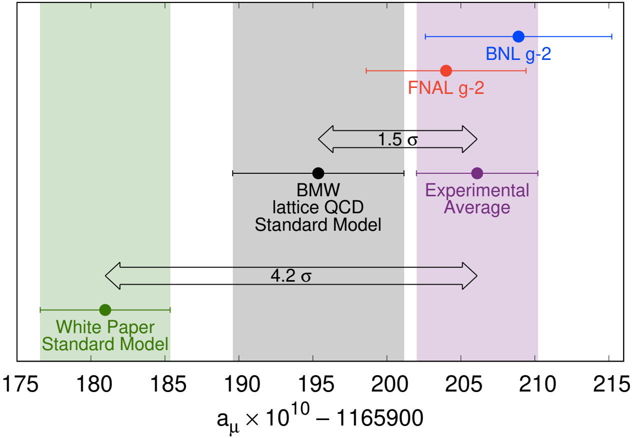
Penn State, Univ. Wuppertal, FZ Juelich, Univ. Budapest, UCSD

Budapest–Marseille–Wuppertal collaboration (BMW)

Borsanyi, Fodor, Guenther, Hoelbling, Katz, Lellouch, Lippert,
Miura, Parato, Szabo, Stokes, Toth, Torok, Varnhorst

EINN2023, Paphos, Cyprus, October 31, 2023

Tensions in $(g\text{-}2)_\mu$: take-home message (before 10/23)



[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

[Muon g-2 coll., Phys. Rev. Lett. 126, 141801 (2021)]

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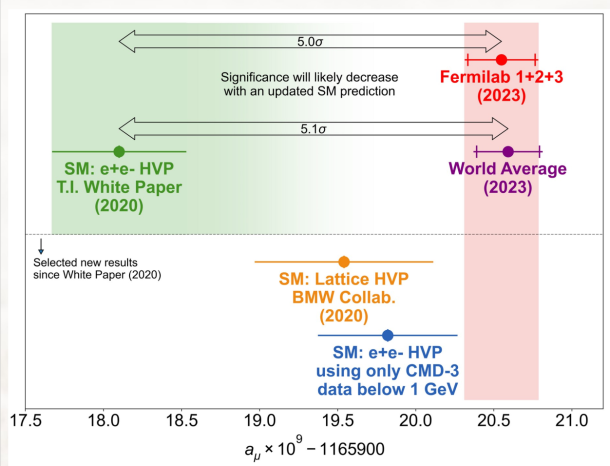

[Phys. Rev. Lett. 131, 161802 (17 October 2023)]

Outline

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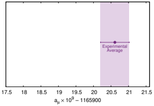
1. 
Experimental
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$a_\mu \times 10^9 - 1165900$

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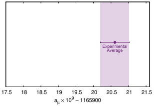
1. 
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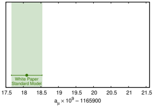
2. 
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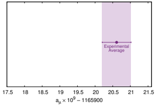
1. 
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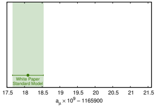
2. 
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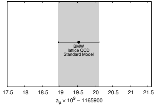
3. 
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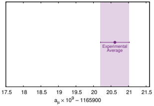
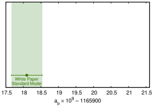
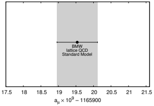
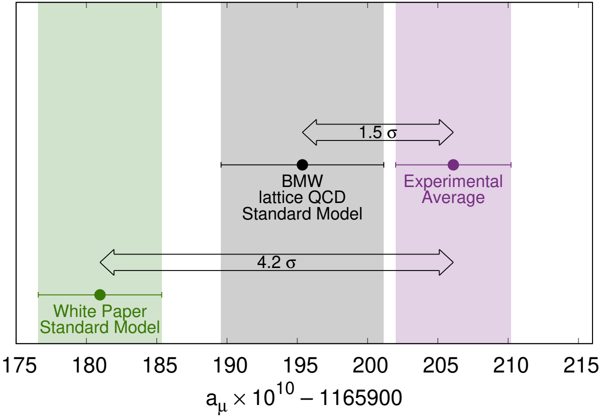
1. 

2. 

3. 

4. Summary



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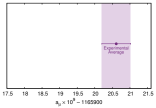
1. 
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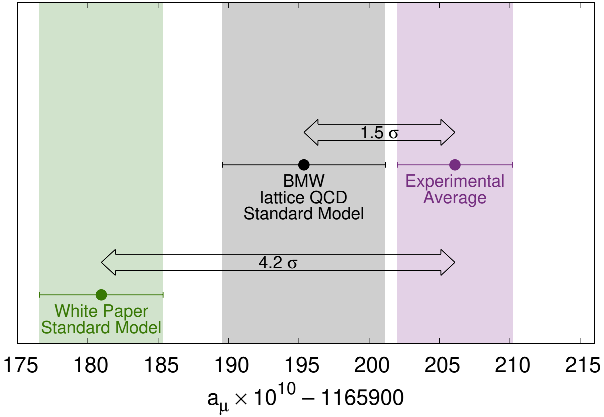
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- J-PARC experiment very different systematics but same accuracy (2027)

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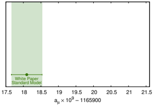
2. 
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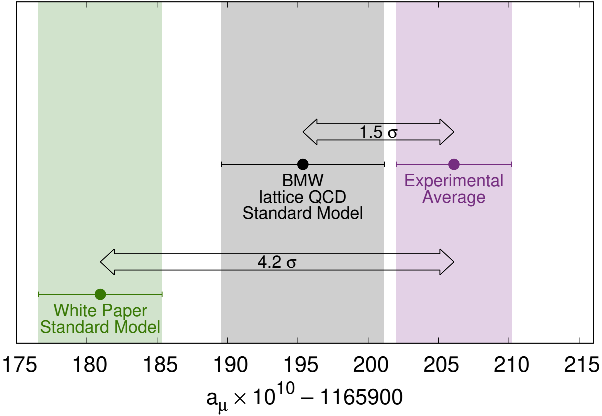
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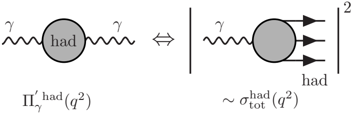
HVP from R-ratio

- Optical theorem



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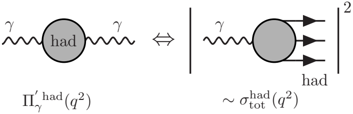

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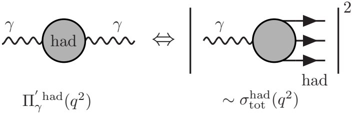

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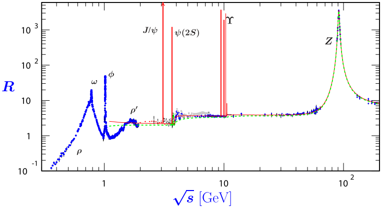

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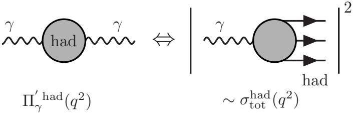

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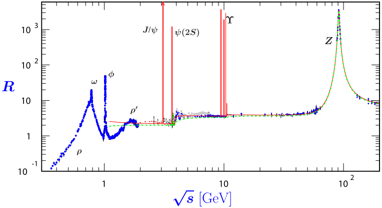

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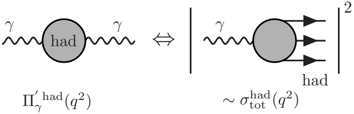

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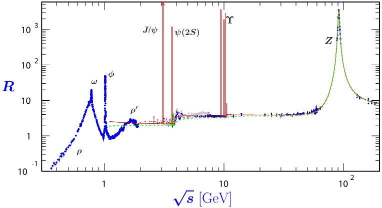

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Systematic uncertainty: ≈4 times larger than the statistical error (e.g. Davier et al.)

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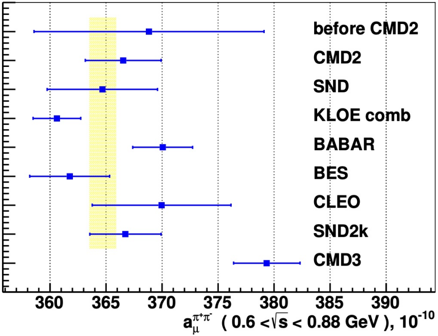

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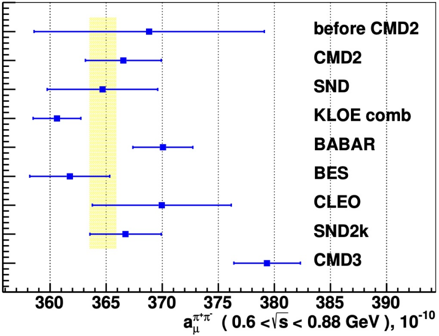

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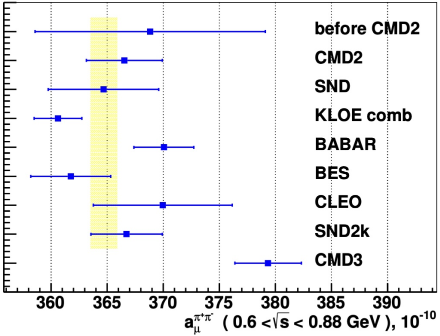

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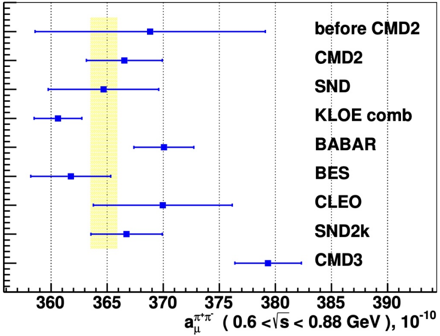

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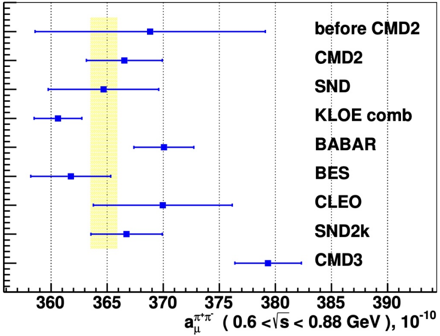

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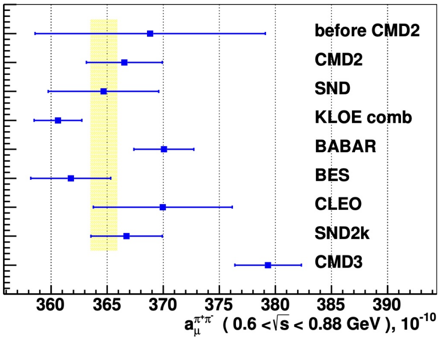

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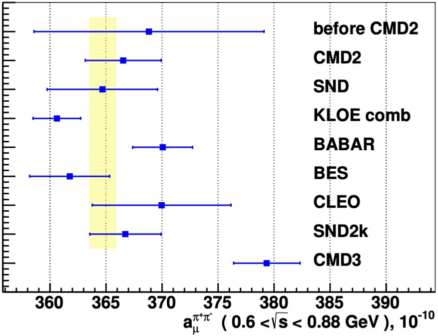

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Experimental error: halved on 8/10 or 10/17/2023/publ. (interpret $\gtrsim 5\sigma$)

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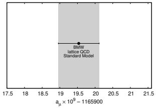
3. 
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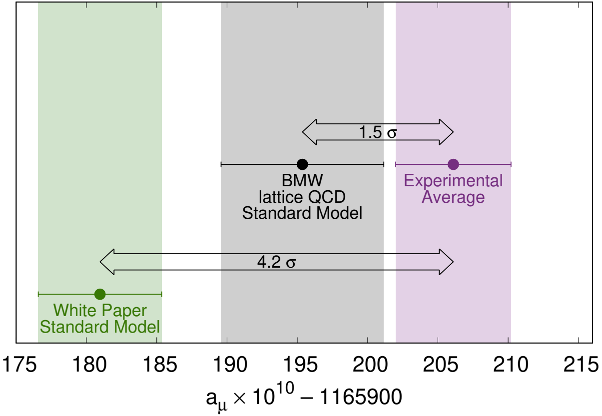
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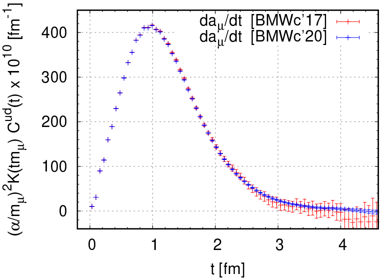
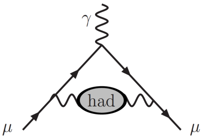
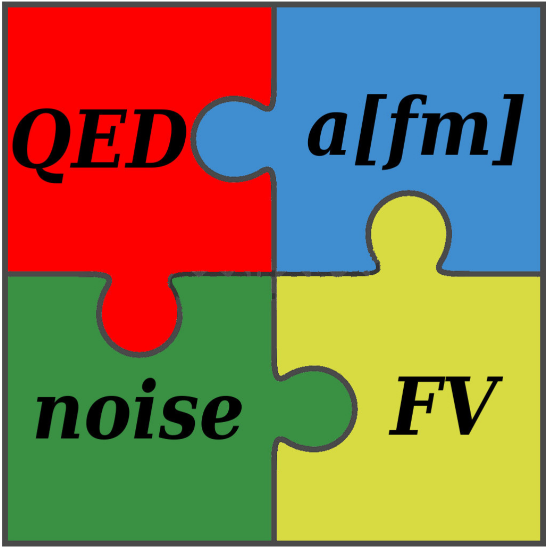


$K(t)$ describes the leptonic part of diagram



New challenges



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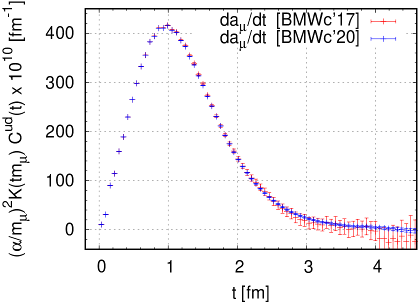
2. For separation of isospin breaking effects: w_0 scale setting
   - Moderate m_q dependence
   - Can be precisely determined on the lattice
   - No experimental value
     $\longrightarrow$ Determine value of w_0 from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



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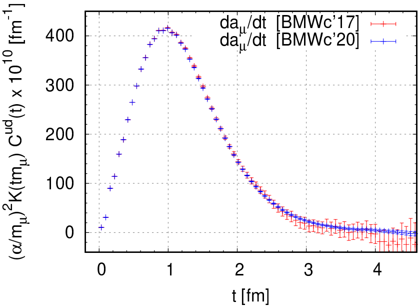

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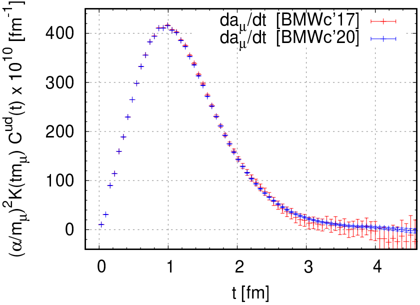

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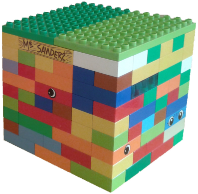

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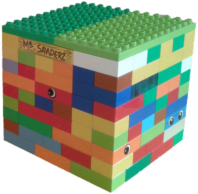

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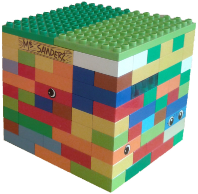

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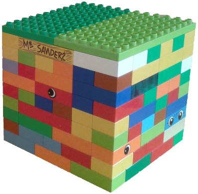

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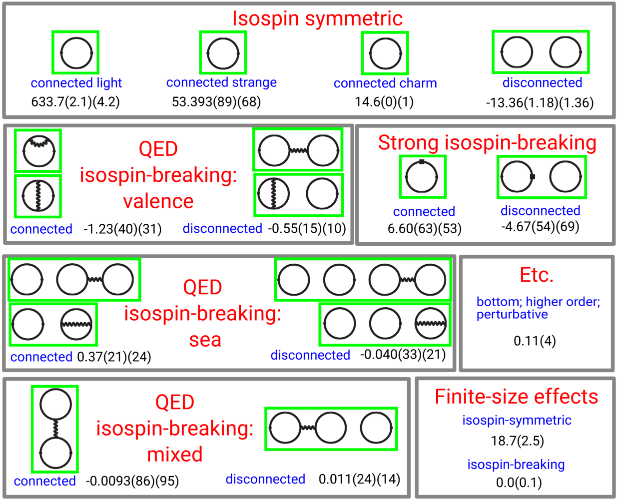
Isospin breaking effects

- Include leading order IB effects: $\mathcal{O}(e^2)$, $\mathcal{O}(\delta m)$



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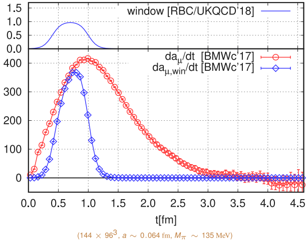
[RBC/UKQCD'18]



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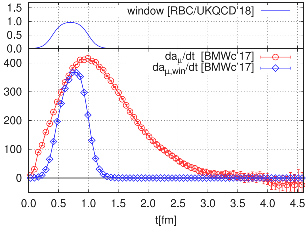

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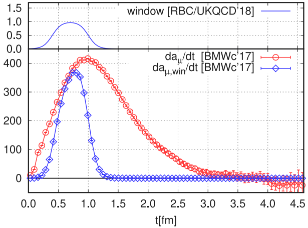

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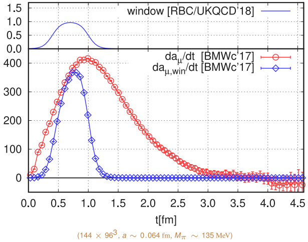

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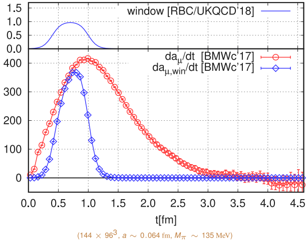

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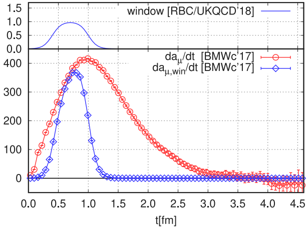

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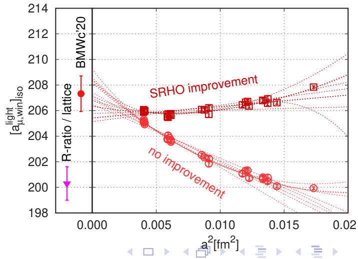

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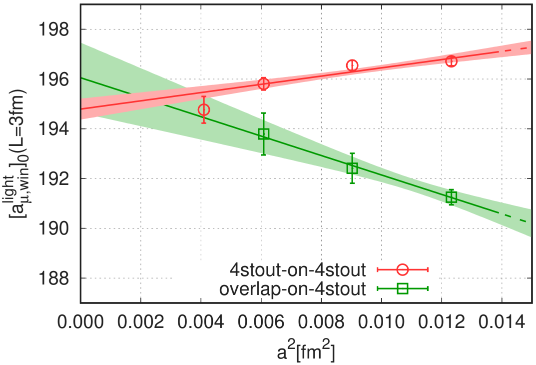

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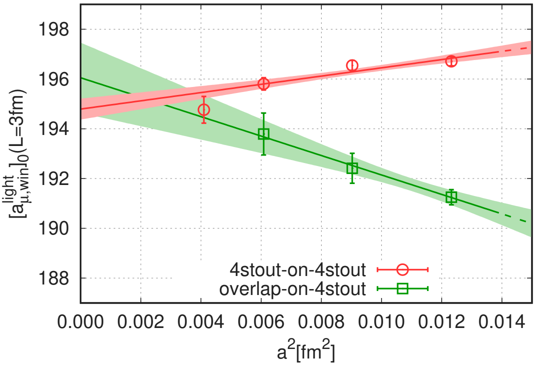
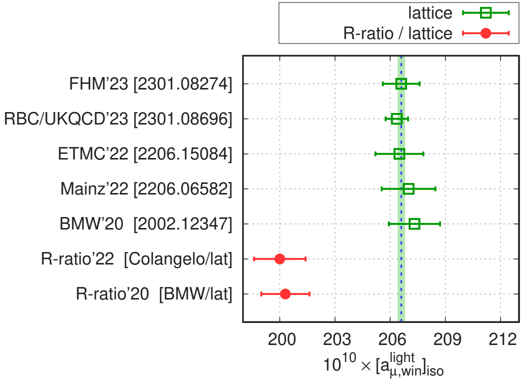


- compute $a_{\mu,\mathrm{win}}$ with overlap valence
- local current instead of conserved $\longrightarrow$ had to compute Z_V
- cont. limit in $L = 3\,\mathrm{fm}$ box consistent with staggered valence

Tension in the window observables



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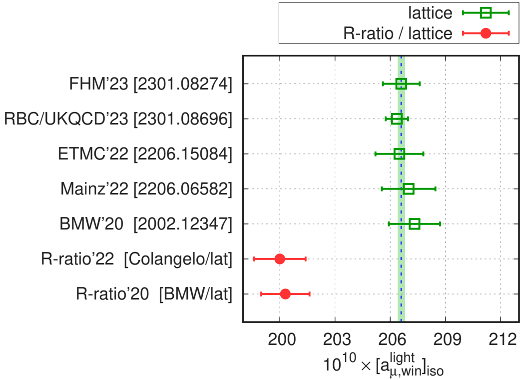

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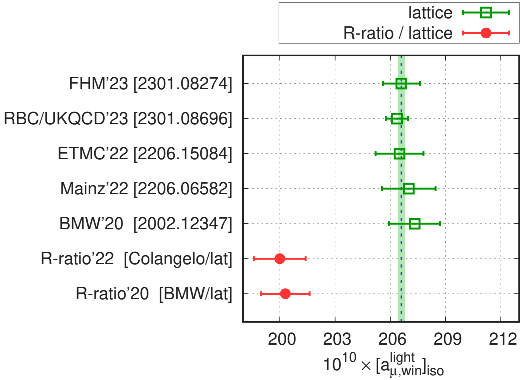

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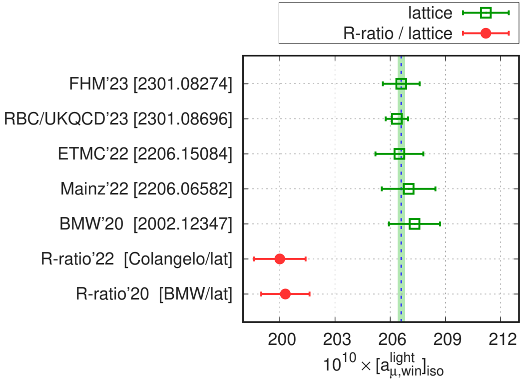

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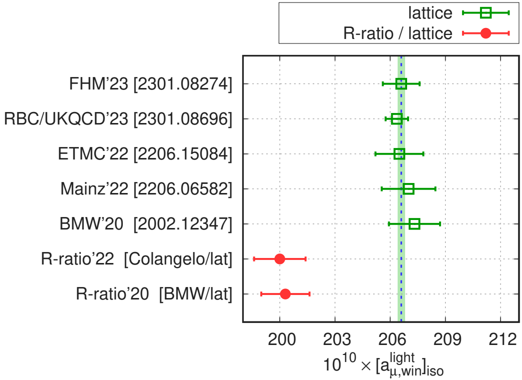

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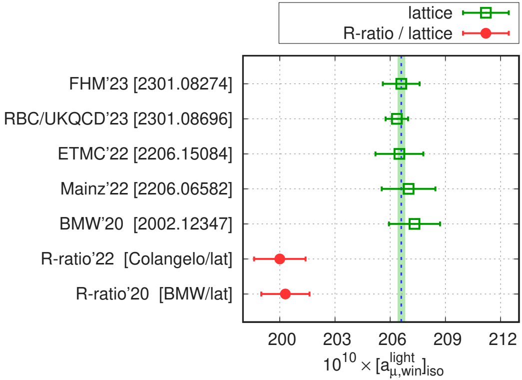

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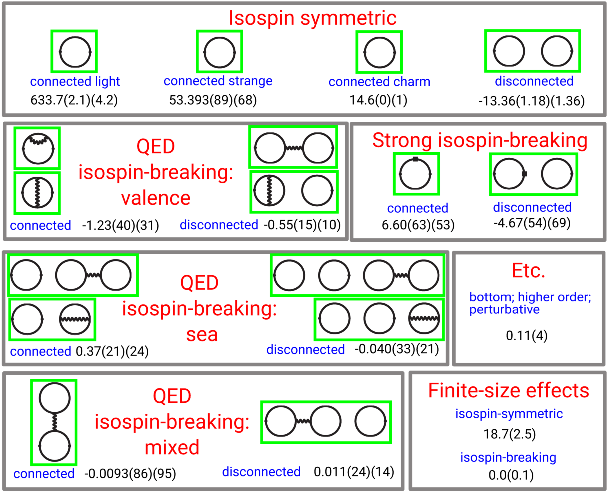
either new physics
or underestimated errors

Outline

5. Summary

Final result



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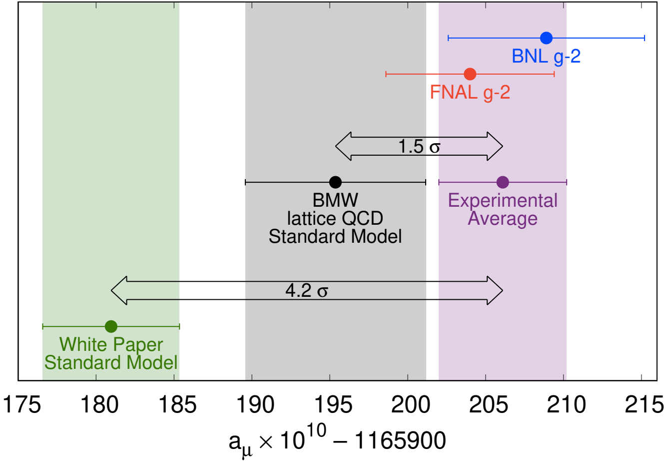

Tension: take-home message #1 g-2 (after 10/2023)

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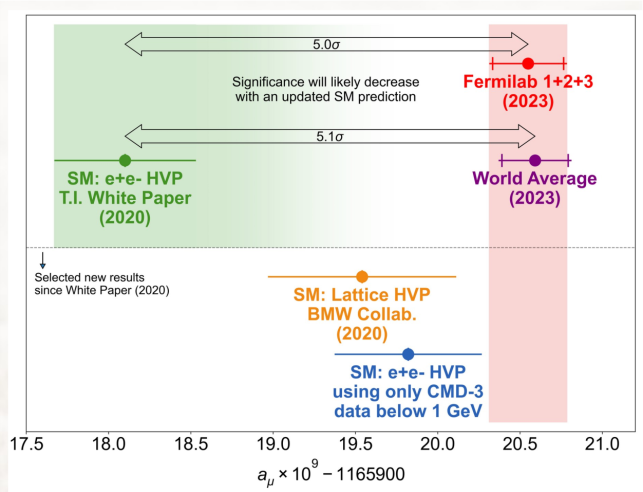

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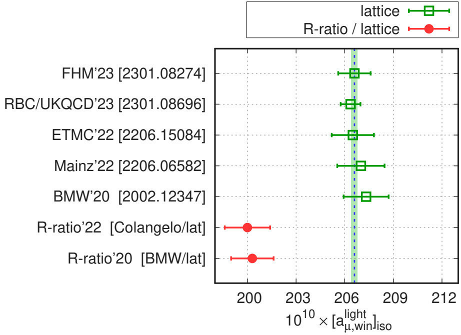

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