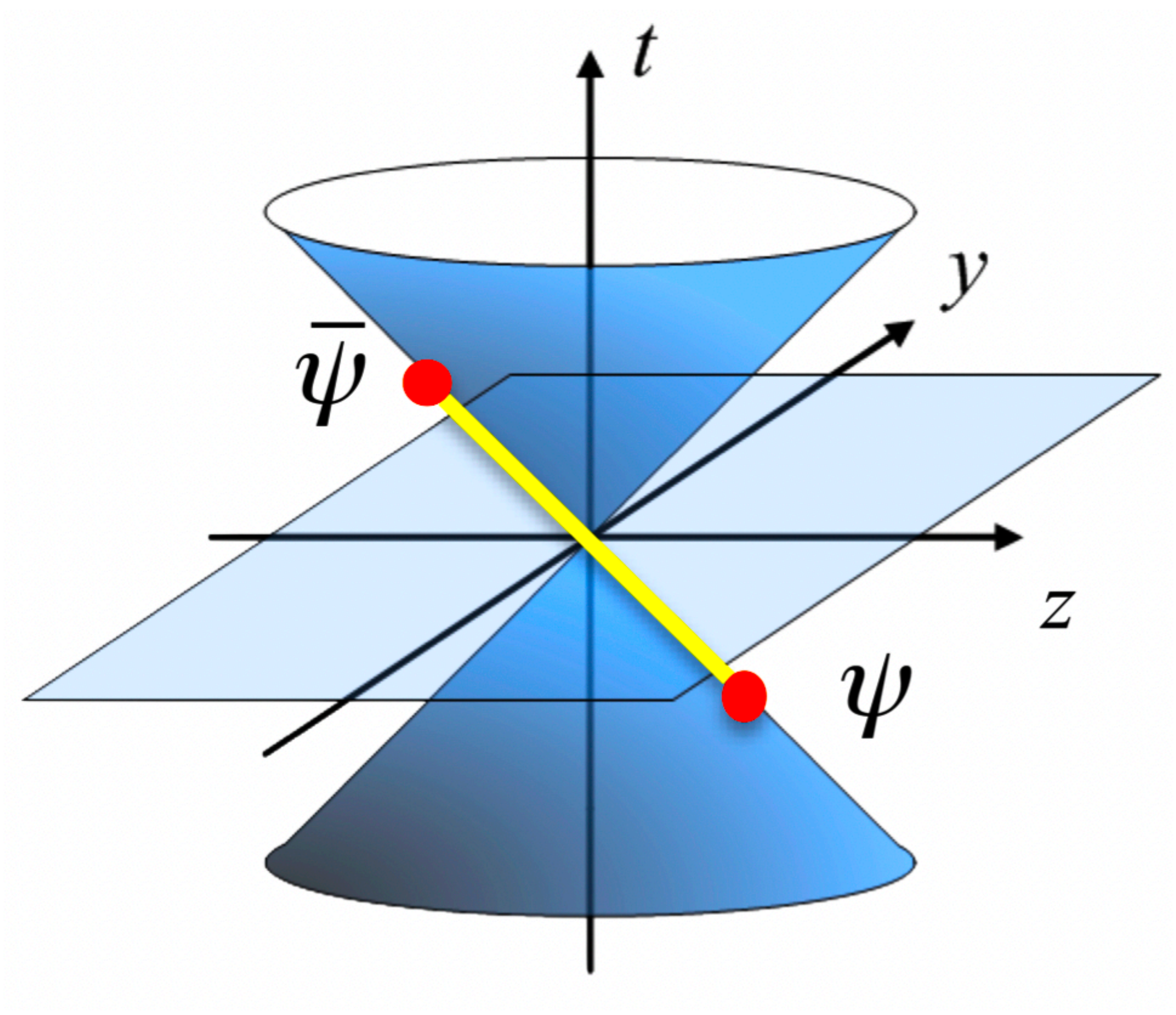
Partonic Structures from Lattice QCD: Life After NLO

November 2023,
EINN, Paphos, Cyprus

Swagato Mukherjee

Brookhaven National Laboratory

partonic structure from lattice QCD … what’s NLO & all that ?!



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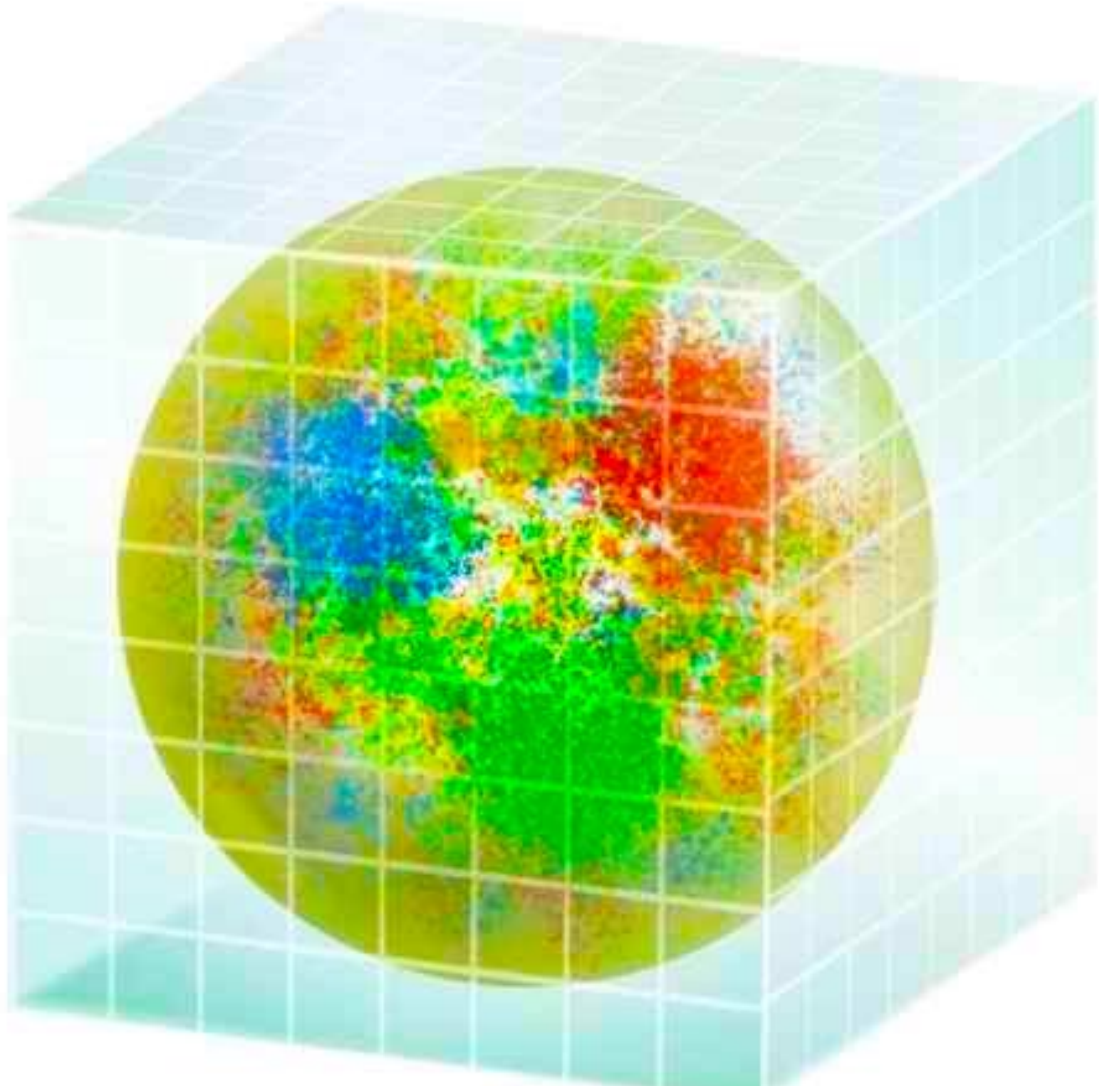
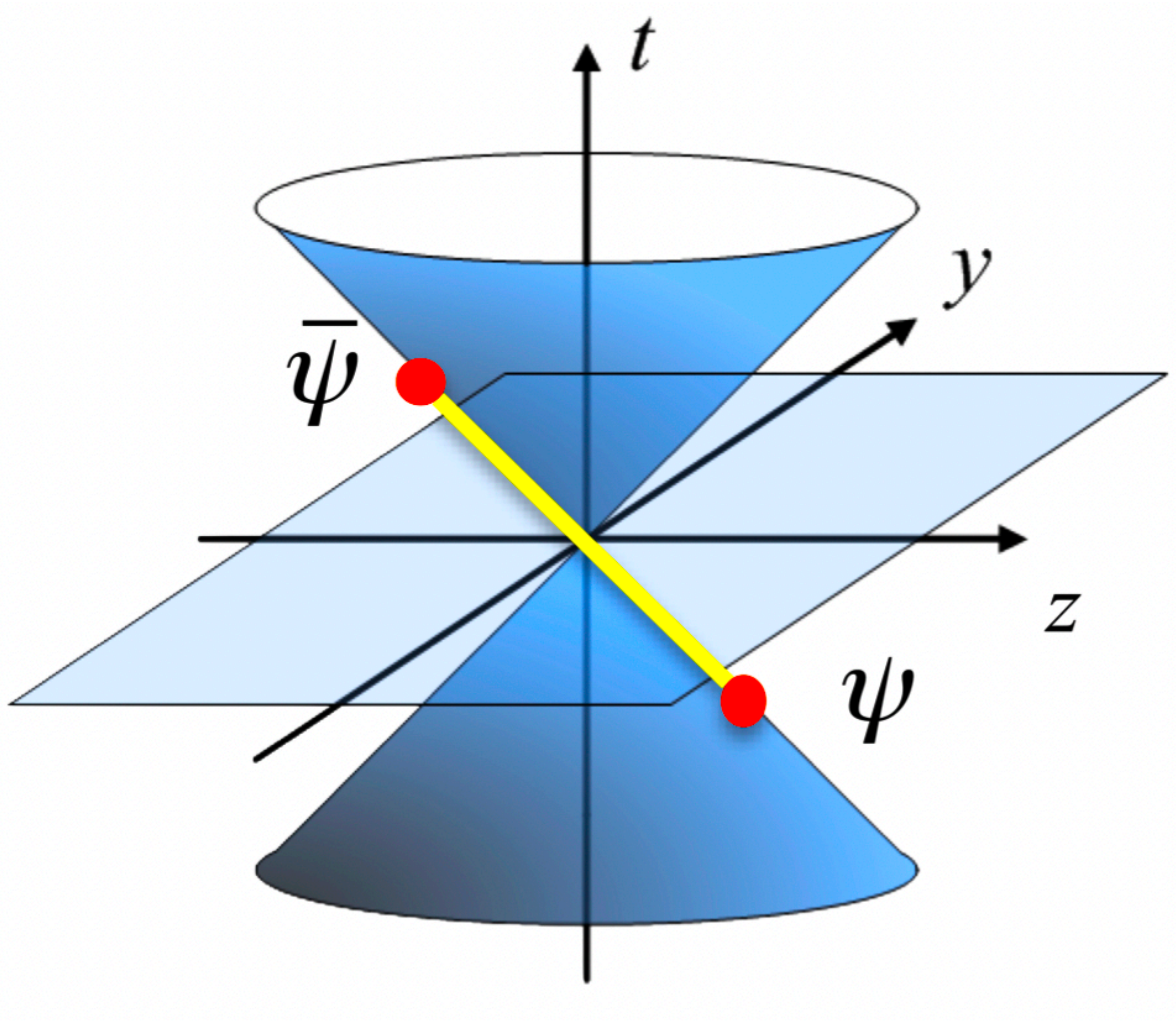
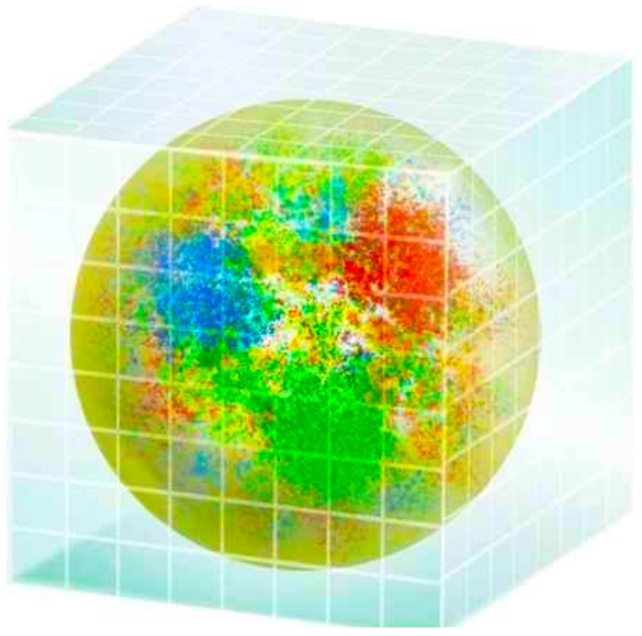


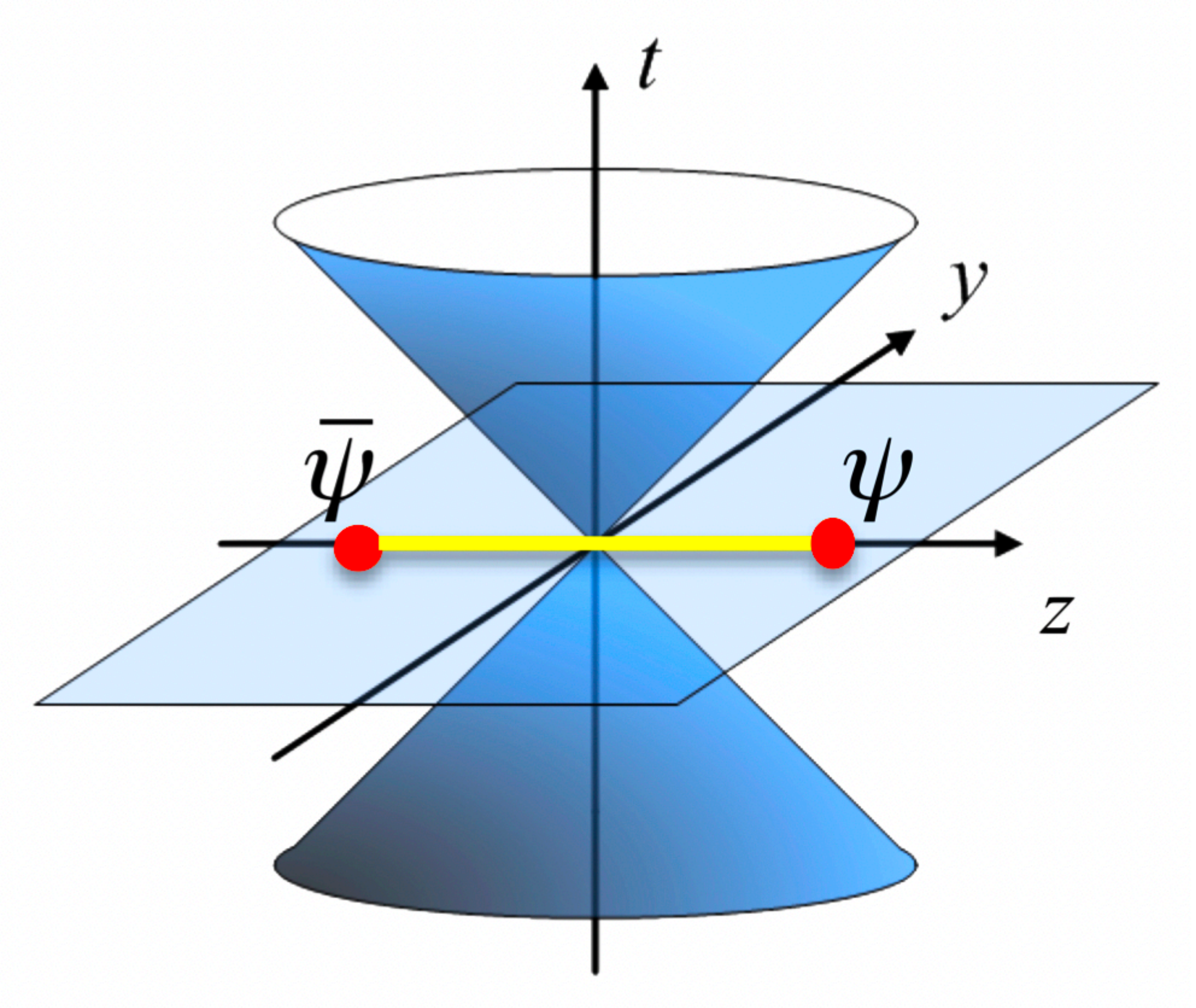


‘partonic structure’:

- effective description of QCD as observed from an infinite-momentum frame / on the lightcone
- $P_z \to \infty$ / $z^2 \to 0$ first, regularize QFT later

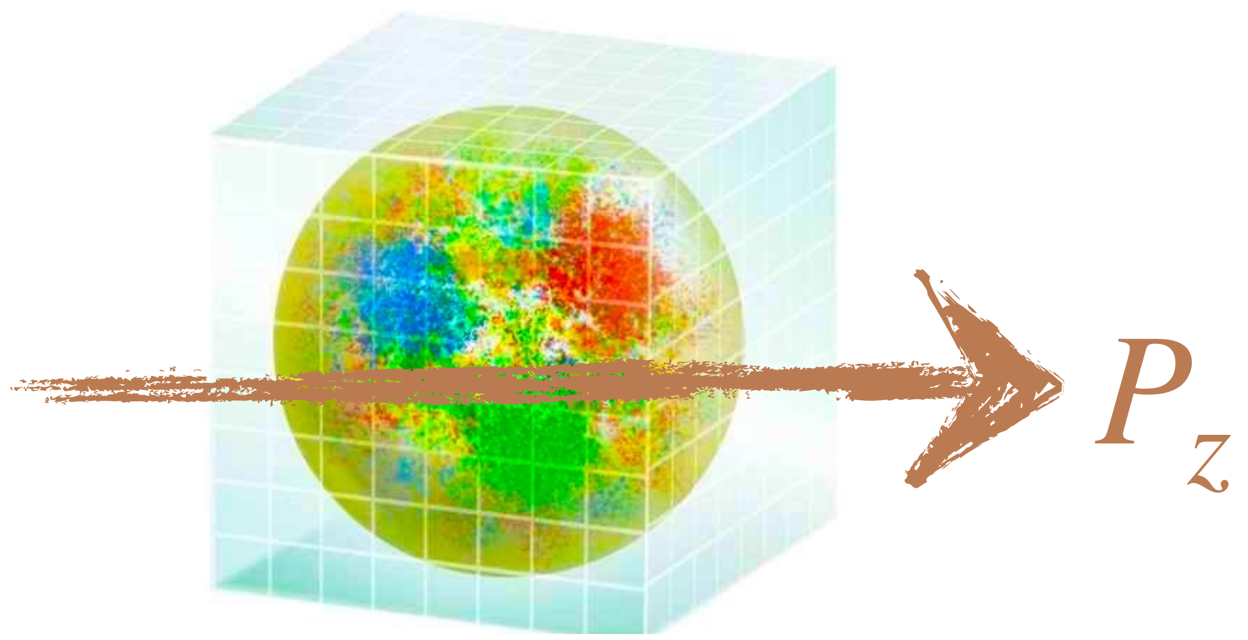
hadron at rest



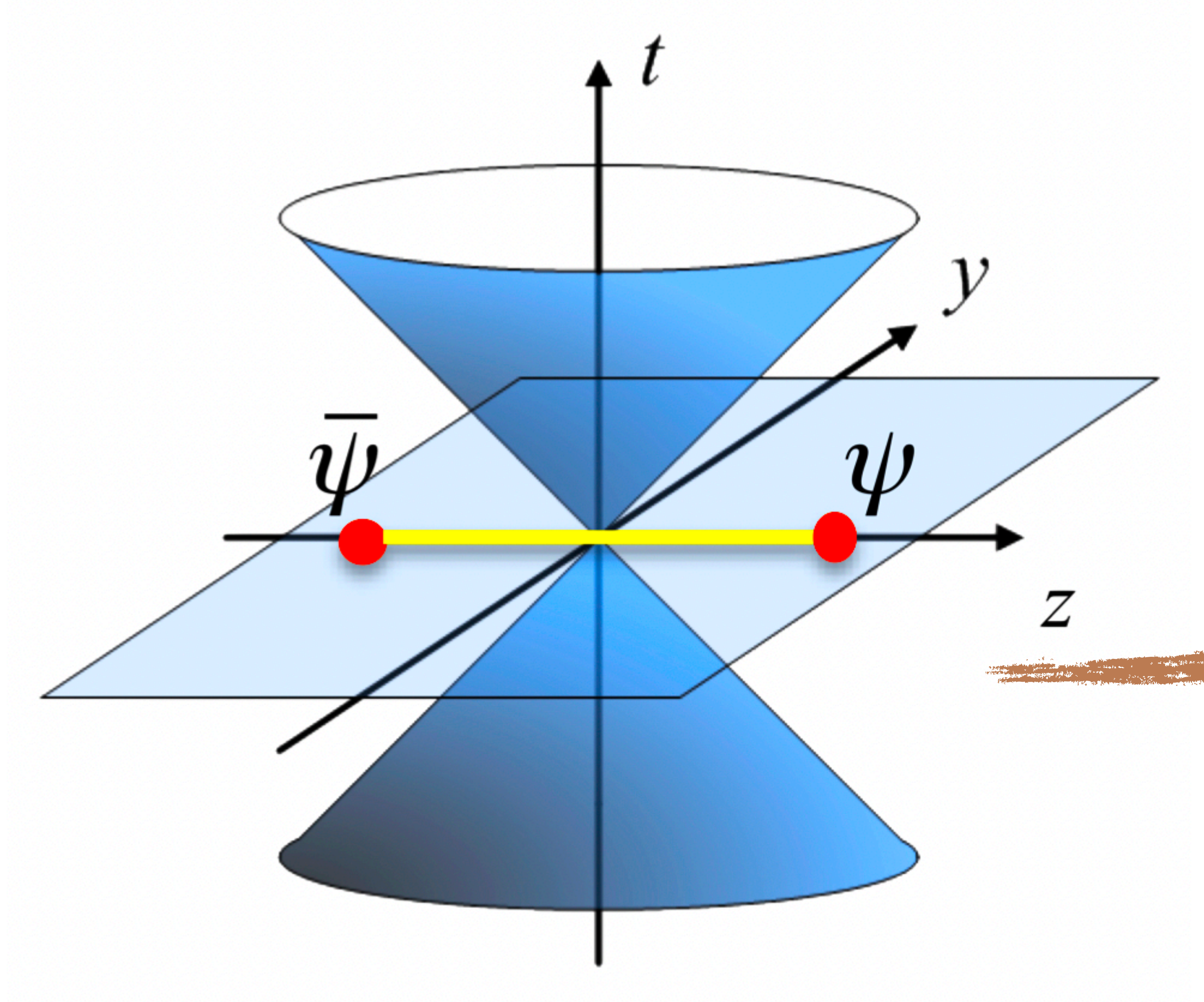


renormalize

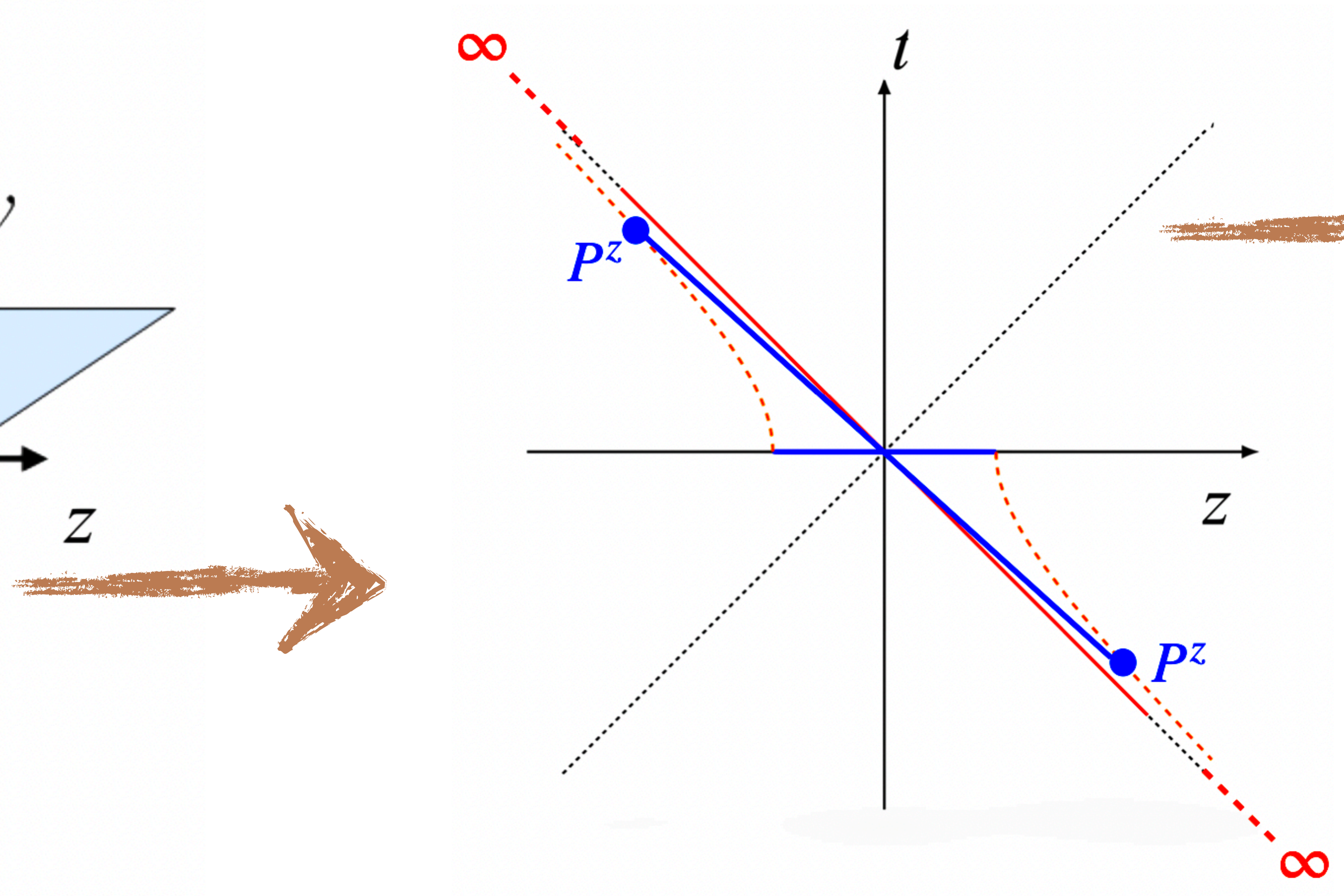
fast-moving hadron

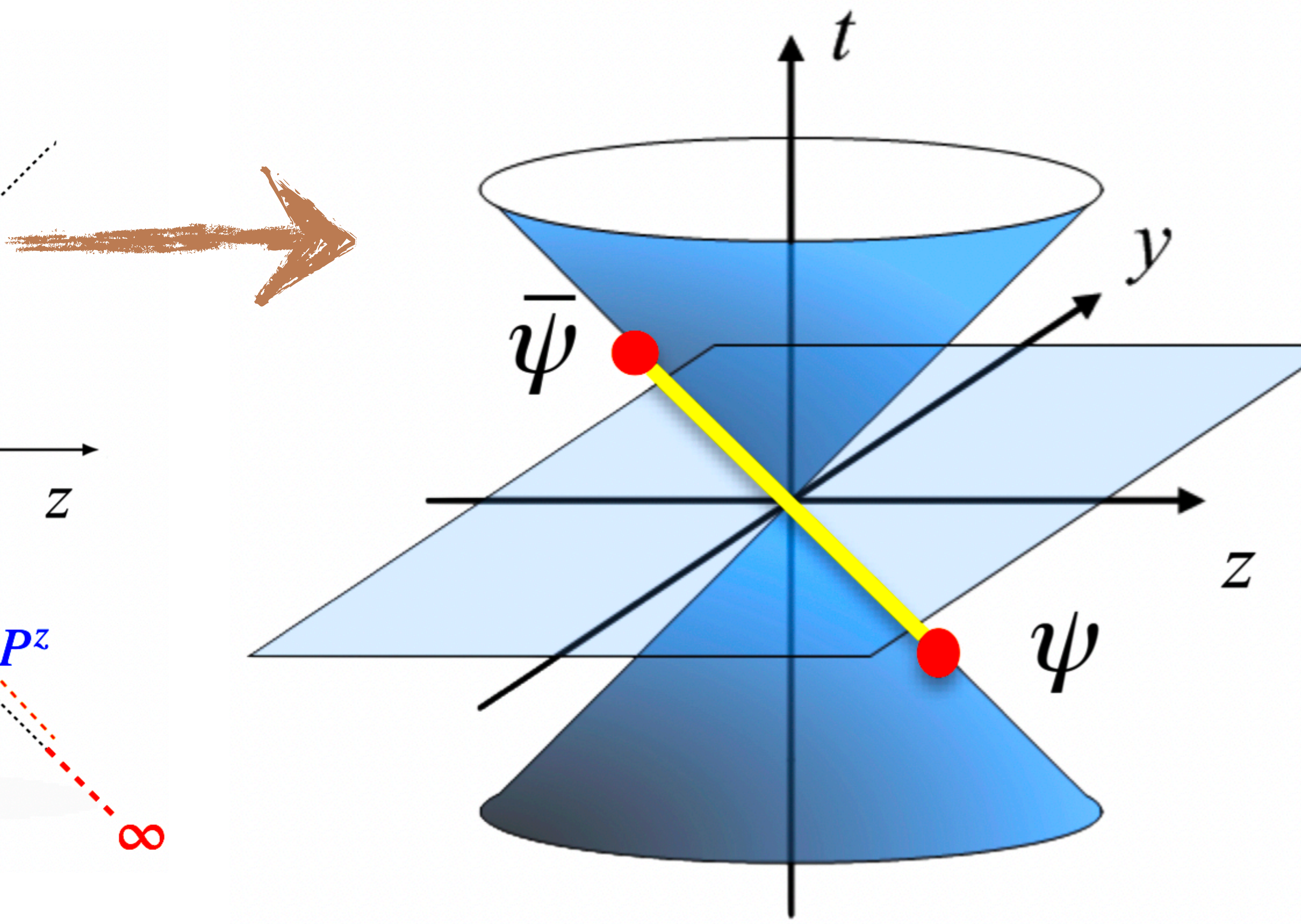


$P_z \approx E$

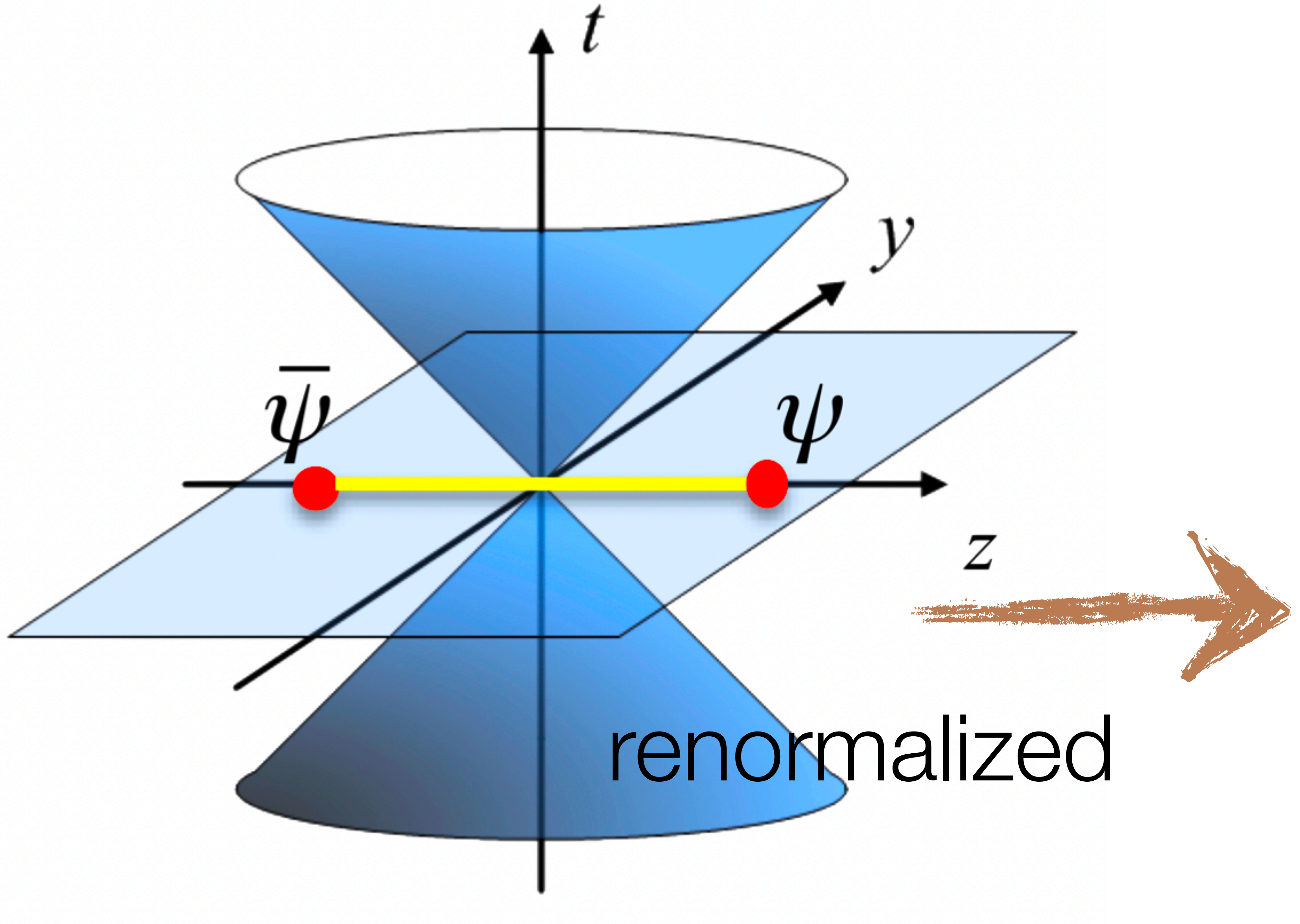


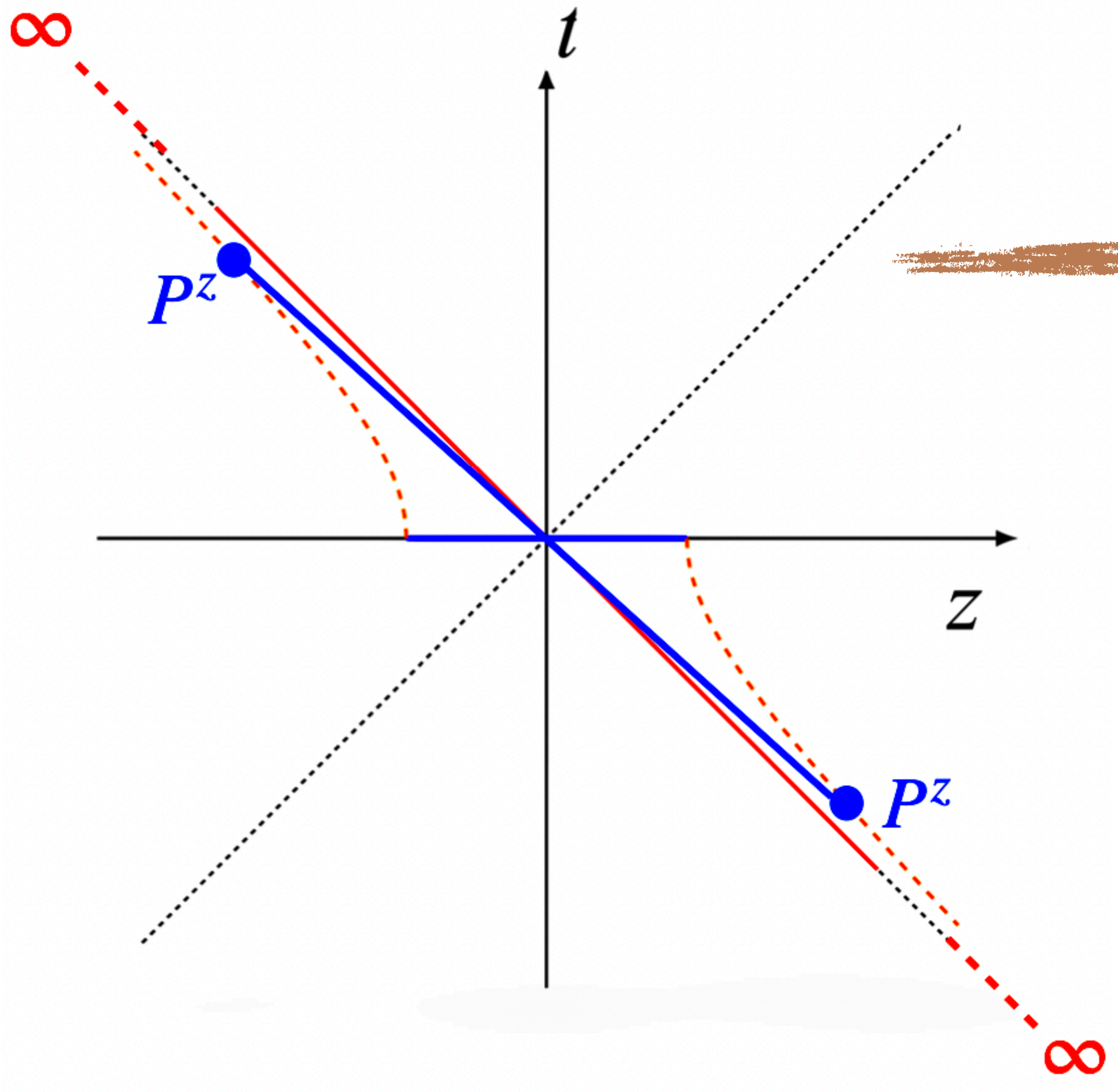
renormalize

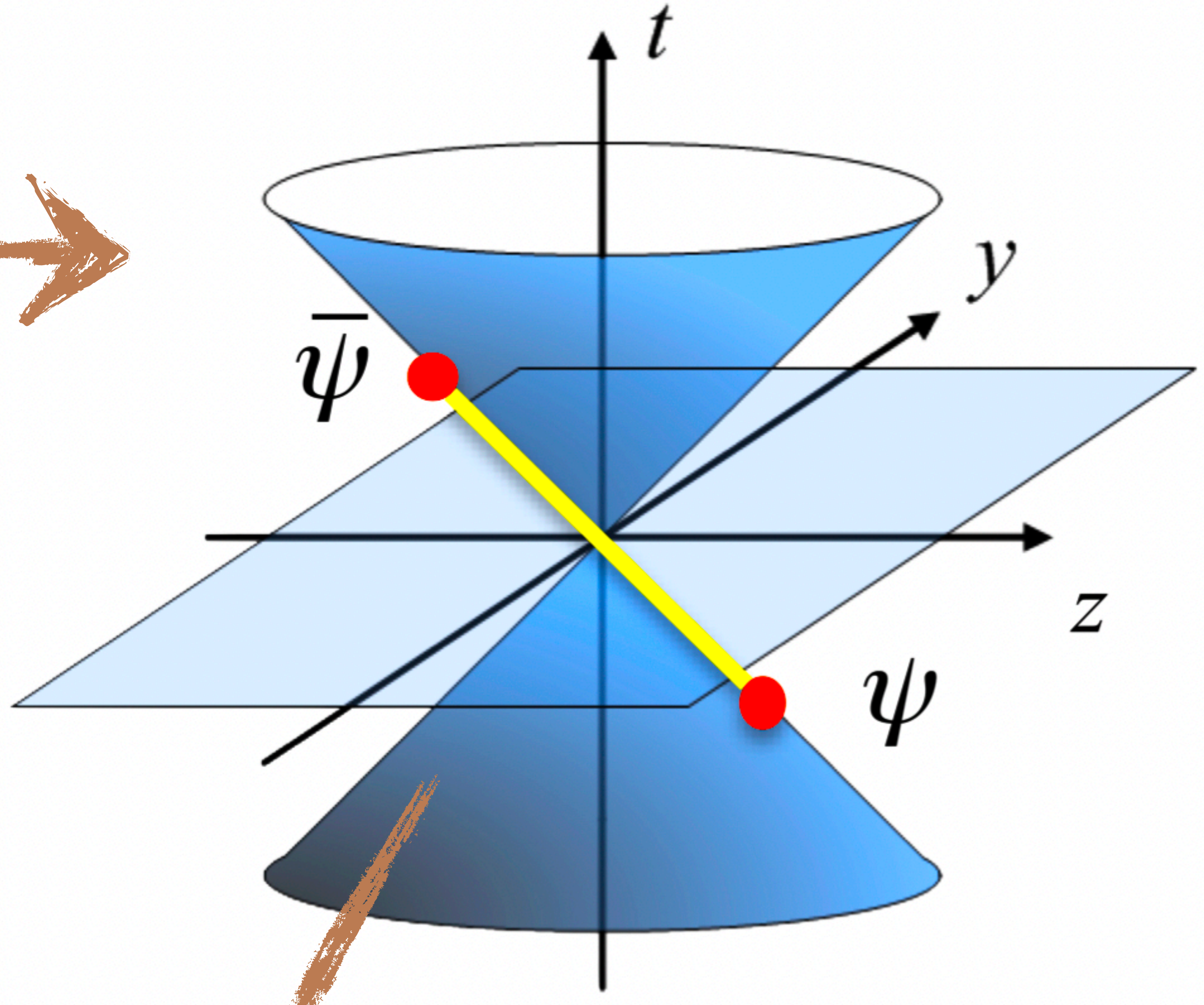




- first regularize QCD on a lattice, then $P_z \to \infty \,/\, z^2 \to 0$
- opposite order of limits for ‘seeing partonic structure’; two limits don’t commute
- difference is UV physics, can be taken care of through pQCD matching







parton physics

pQCD

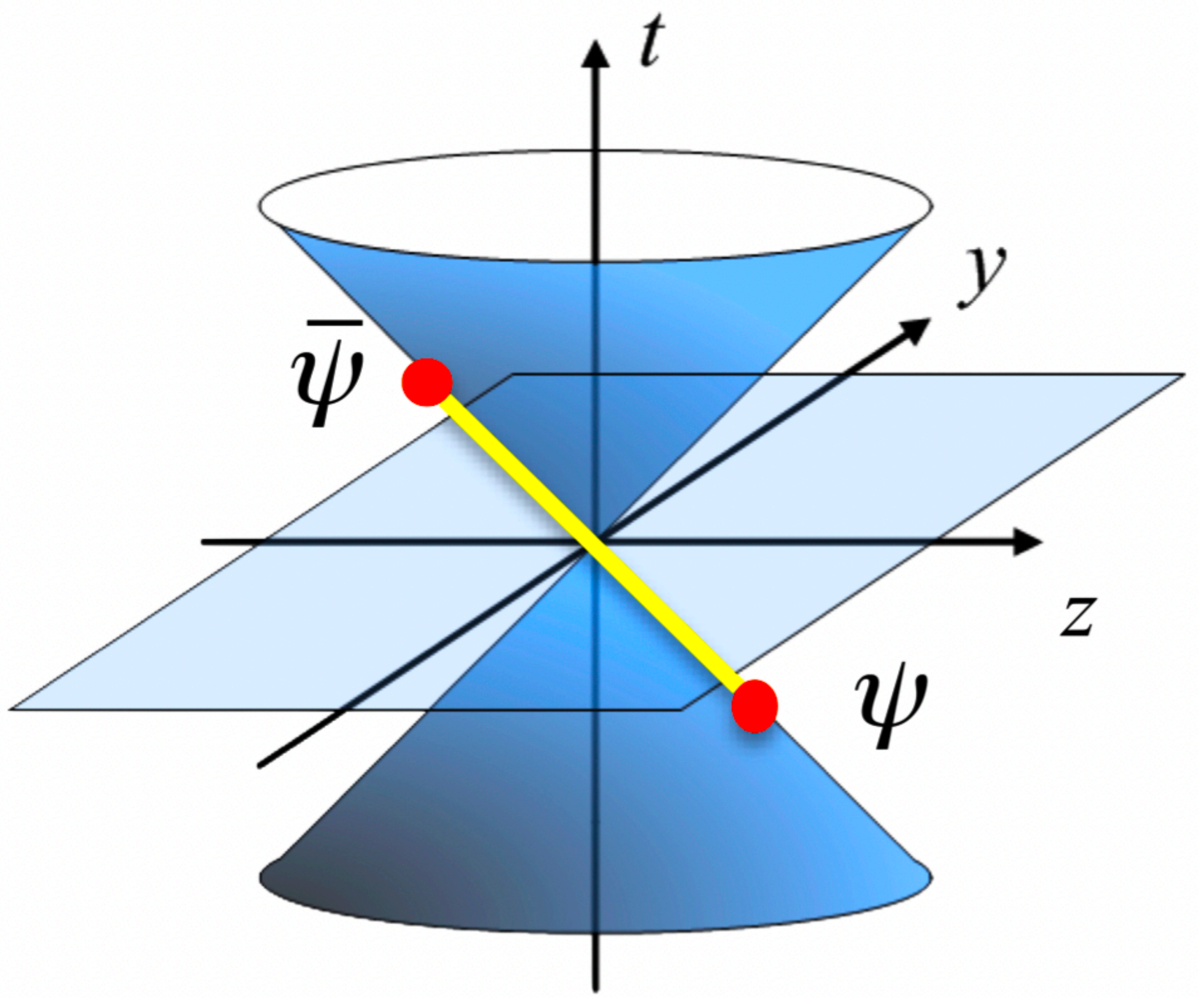
$$+\mathcal{O}\left[\frac{\Lambda^2_{\rm QCD}}{x^2P_z^2}, \frac{\Lambda_{\rm QCD}}{(1-x)P_z}, \frac{M_H^2}{P_z^2}, \ldots\right]$$

$C(x, P_z, \mu) \otimes$ momentum space

$$+\mathcal{O}\left[z^2\Lambda^2_{\rm QCD}, z^2M_H^2, \ldots\right]$$

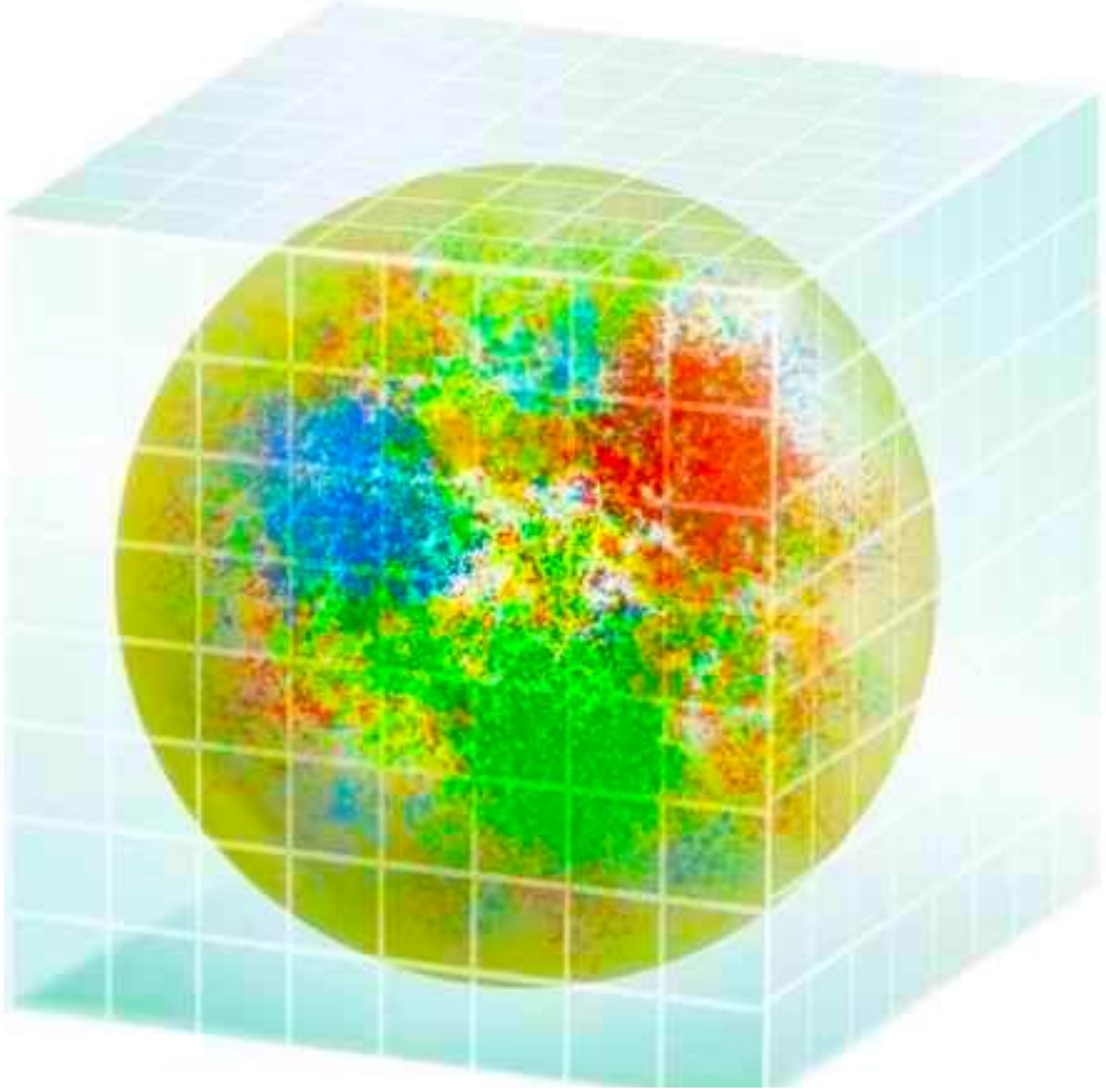
$C(\alpha, z^2, \mu) \otimes$ position space

partonic structure from lattice QCD … what's NLO & all that ?!



$C(x, P_z, \mu) \otimes$

$C(\alpha, z^2, \mu) \otimes$



LO NLO NNLO

$$C(\mathcal{S}, \mu) \;\sim\; \alpha_s^0(\mu) \;+\; \alpha_s(\mu) f\left(\ln[\mathcal{S}\mu]\right) \;+\; \alpha_s^2(\mu) f\left(\ln[\mathcal{S}\mu]\right) \;+\; \cdots$$

$$\mathcal{S} \;=\; 2xP_z \,,\; z^2$$

operator product expansion (OPE) & Mellin moments

- expansion of position space bilocal matrix elements in z^2 around $z^2 = 0$

Wilson coefficients → $C_n(z^2, \mu) \times \langle x^n \rangle$ ← n^{th} Mellin moments

$$C_n(z^2, \mu) \sim \alpha_s^0(\mu) + \alpha_s(\mu) f\left(\ln[z^2\mu]\right) + \alpha_s^2(\mu) f\left(\ln[z^2\mu]\right) + \cdots$$

LO NLO NNLO

$C_n(z^2, \mu)$: expansion of $C(\alpha, z^2, \mu)$

pion valance PDF

Yong Zhao *et al.*, *Phys.Rev.Lett.* 128 (2022) 14, 142003

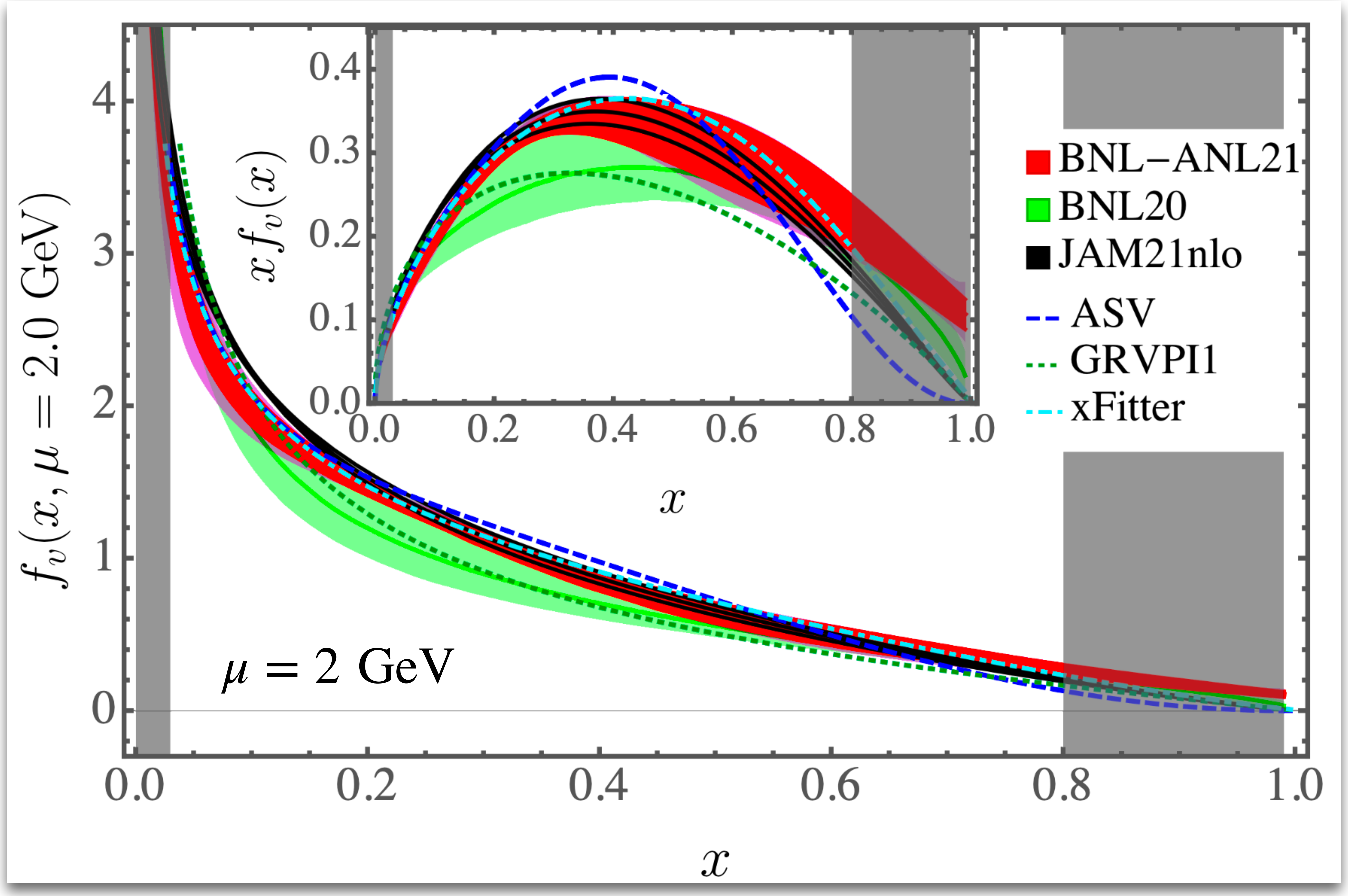
NNLO momentum matching

valance pion mass 300 MeV

lattice spacing 0.04 fm

pion momenta up to 2.4 GeV

first LQCD PDF at NNLO



pion valance PDF

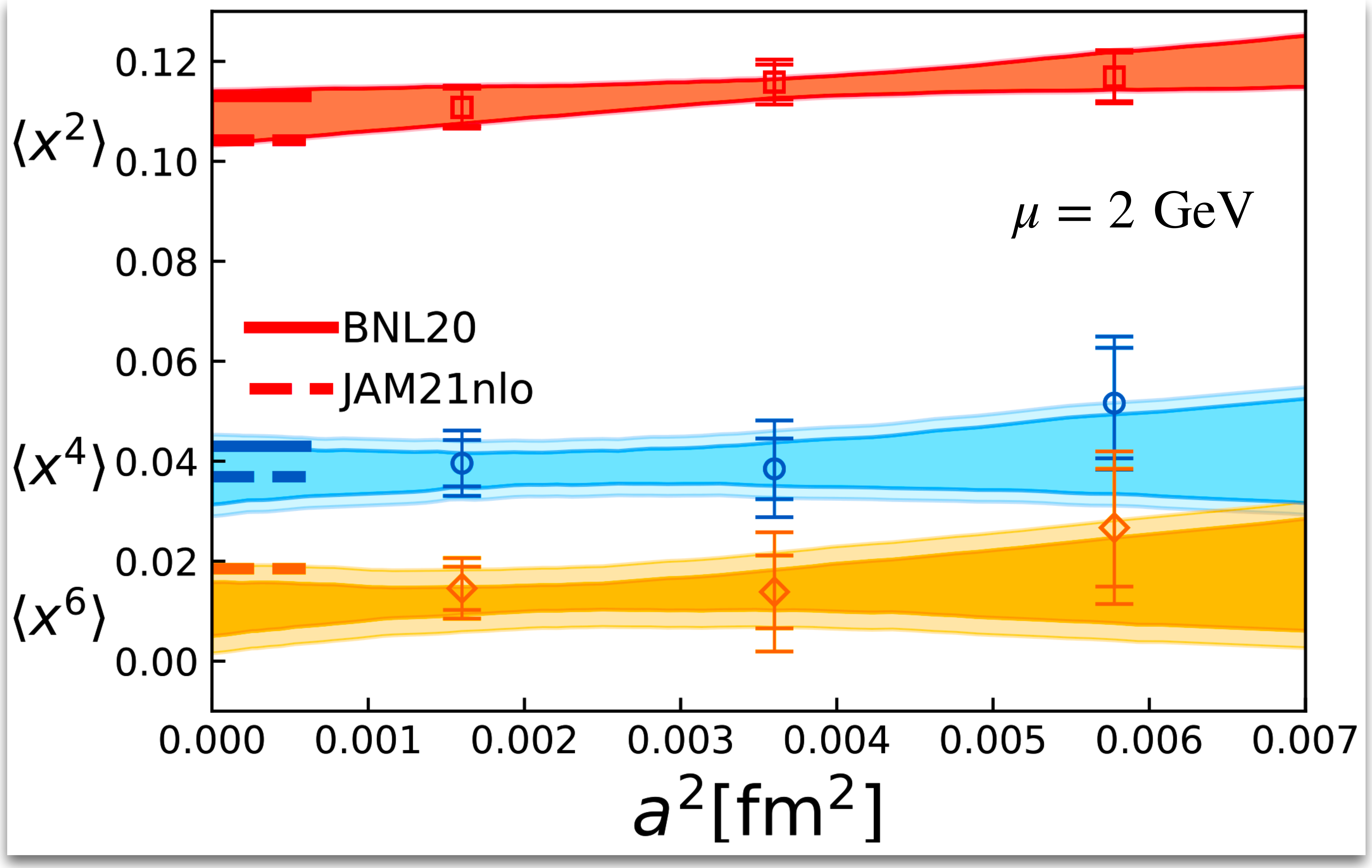
Xiang Gao *et al.*, *Phys.Rev.D* 106 (2022) 11, 114510

2nd, 4th, 6th Mellin moments

OPE: NNLO Wilson coeff

physical pion mass

continuum extrapolated



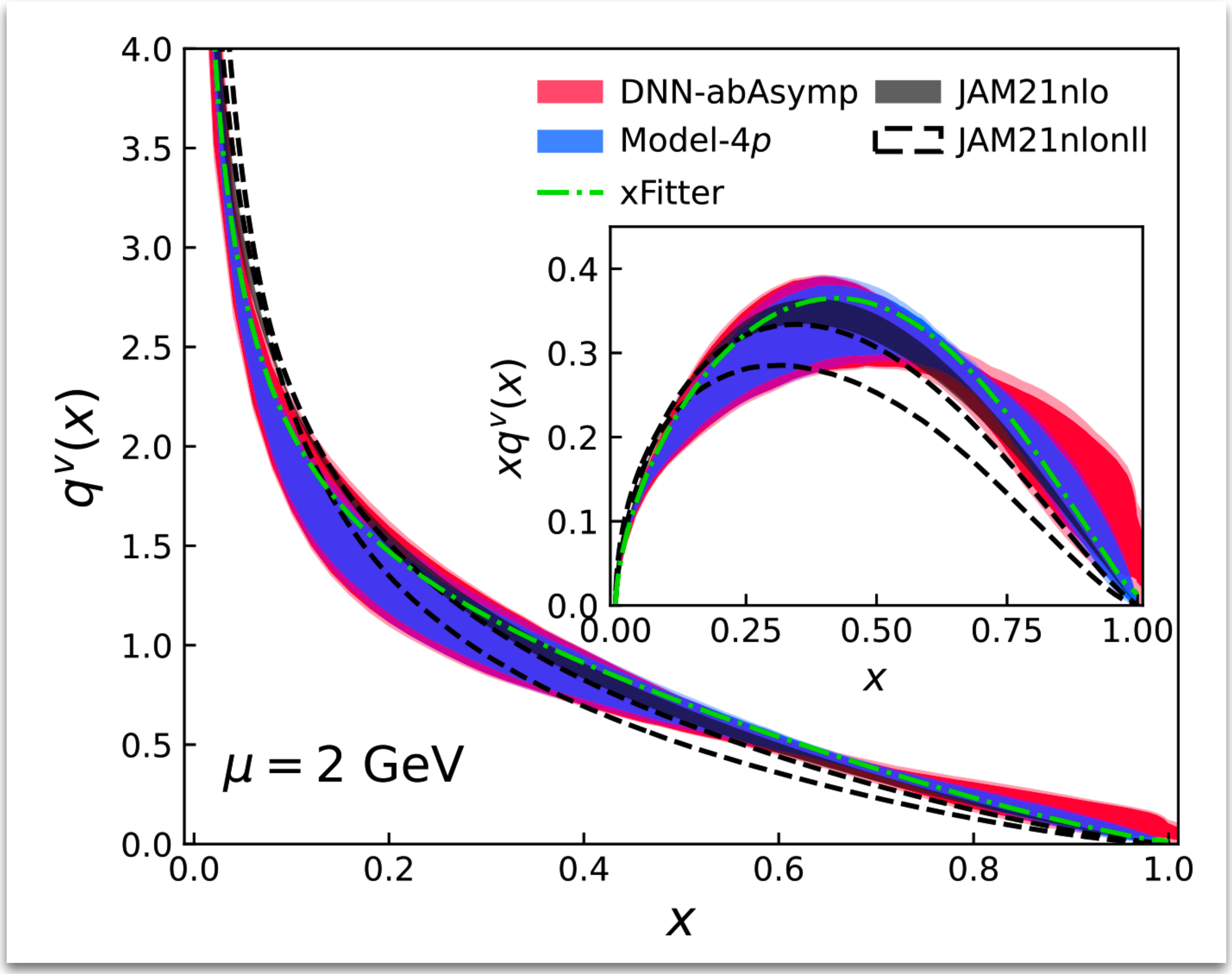
pion valance PDF

Xiang Gao *et al.*, *Phys.Rev.D* 106 (2022) 11, 114510

DNN: NNLO position matching

physical pion mass

continuum extrapolated



proton unpolarized isovector PDF

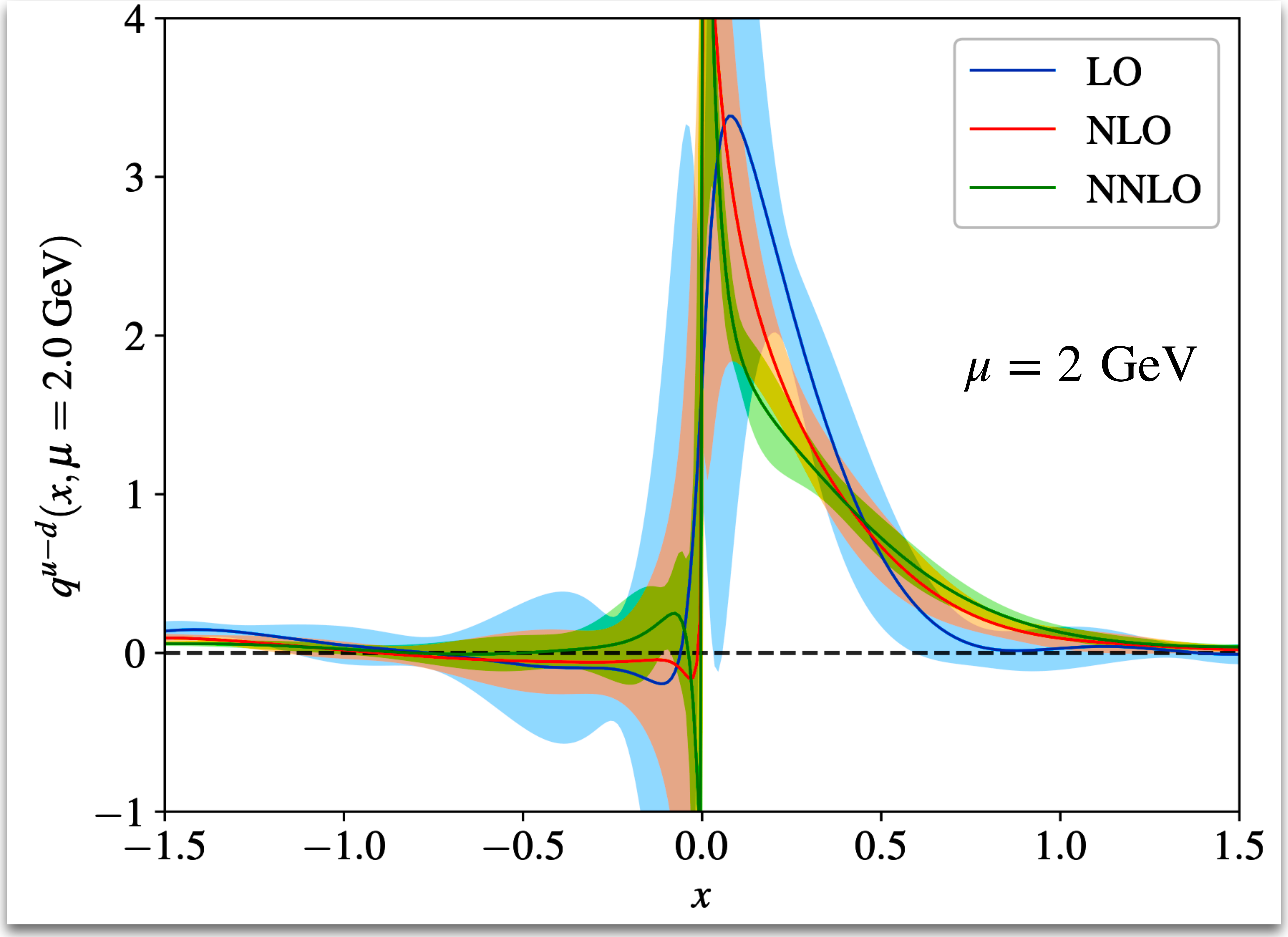
Andrew Hanlon *et al.*, *Phys.Rev.D* 107 (2023) 7, 074509

NNLO momentum matching

physical pion mass

lattice spacing 0.075 fm

proton momenta $\leq$ 1.53 GeV



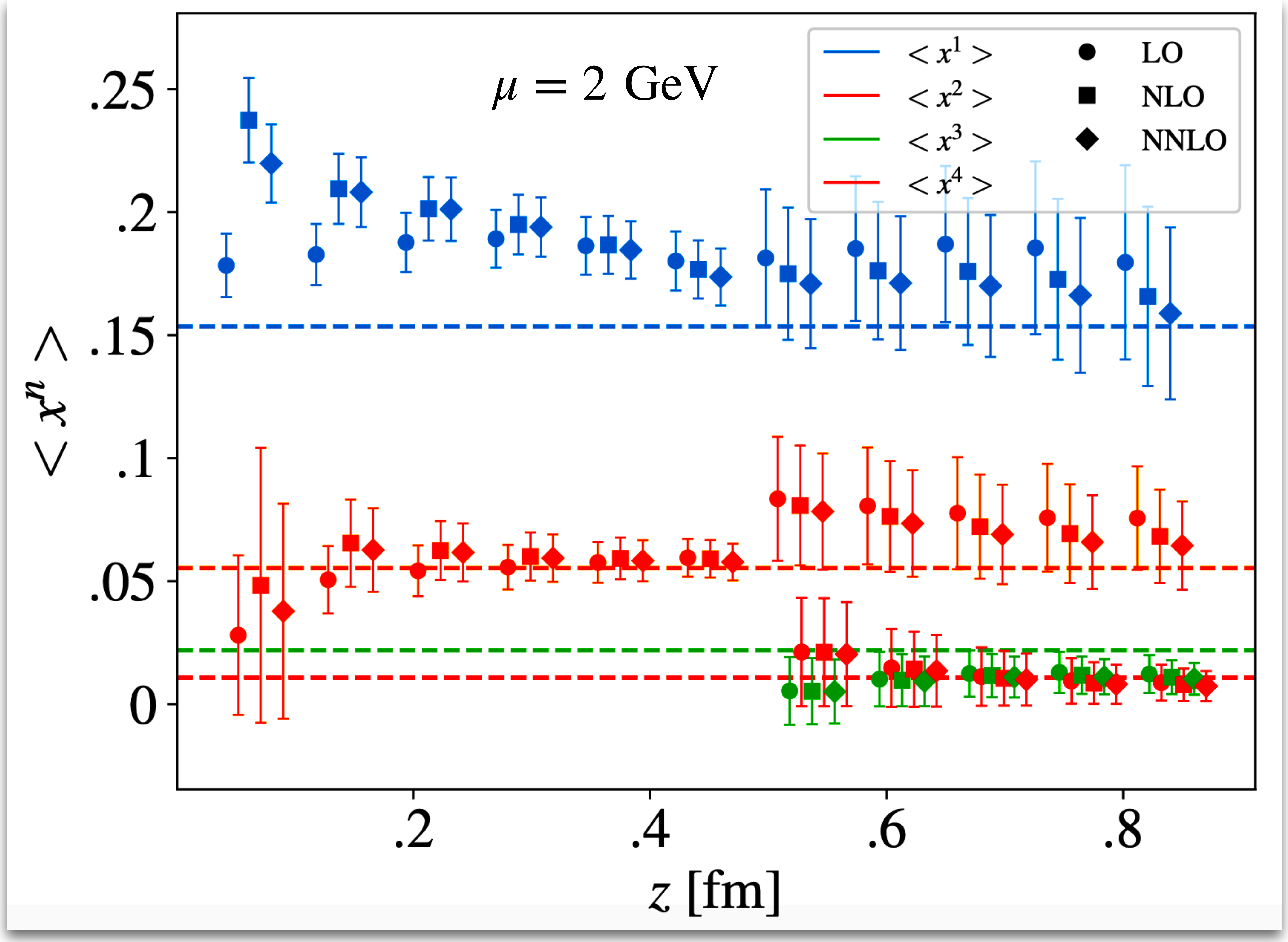
proton unpolarized isovector PDF

Andrew Hanlon *et al.*, *Phys.Rev.D* 107 (2023) 7, 074509

1^{st} — 4^{th} Mellin moments

OPE: NNLO Wilson coeff

NLO to NNLO: no significant effect within present precision

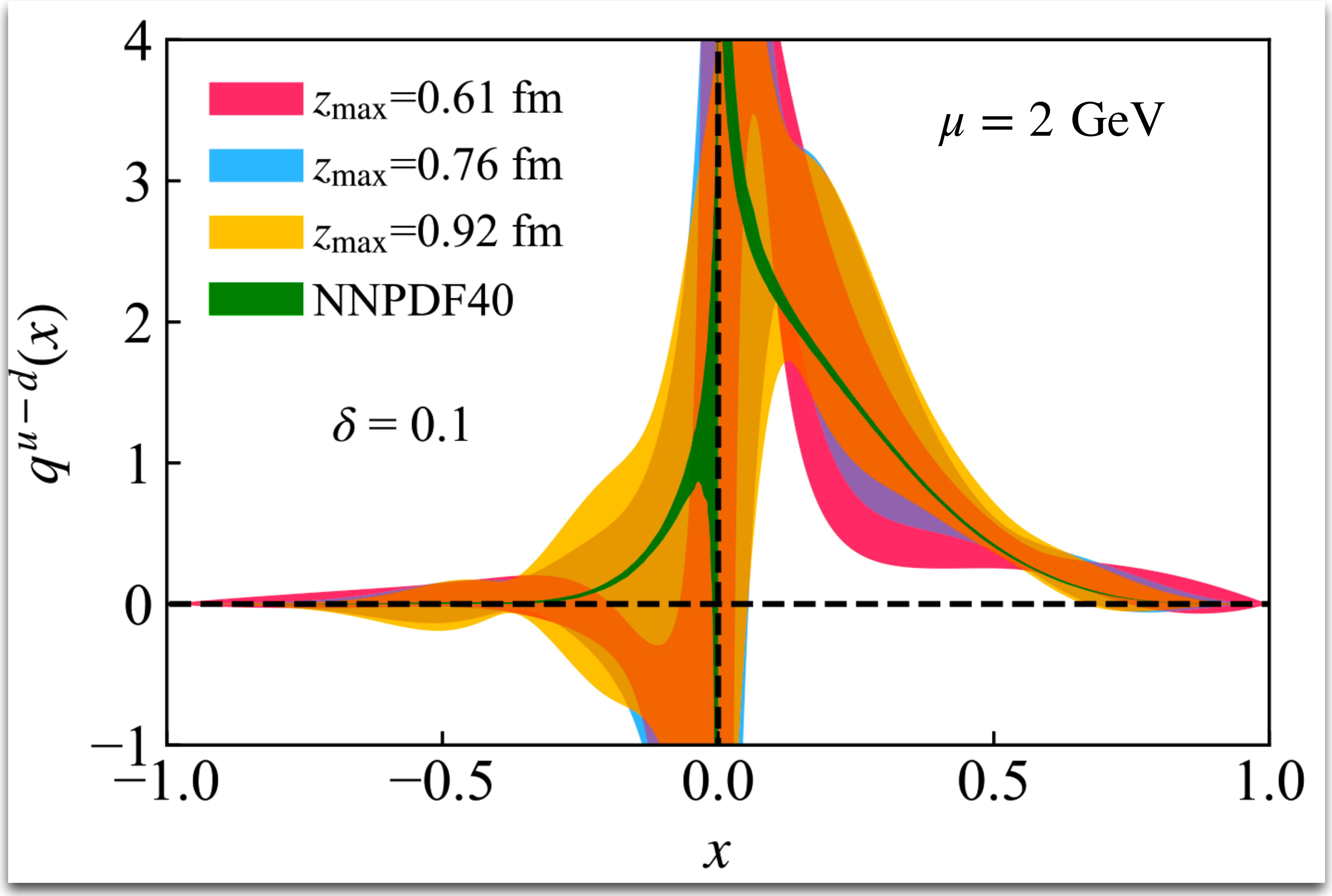


dotted lines: NNPDF4.0

proton unpolarized isovector PDF

Andrew Hanlon *et al.*, *Phys.Rev.D* 107 (2023) 7, 074509

DNN: NNLO position matching



proton GPD: Mellin moments of H and E at zero skewness

Xiang Gao *et al.*, *Phys.Rev.D* 108 (2023) 1, 014507

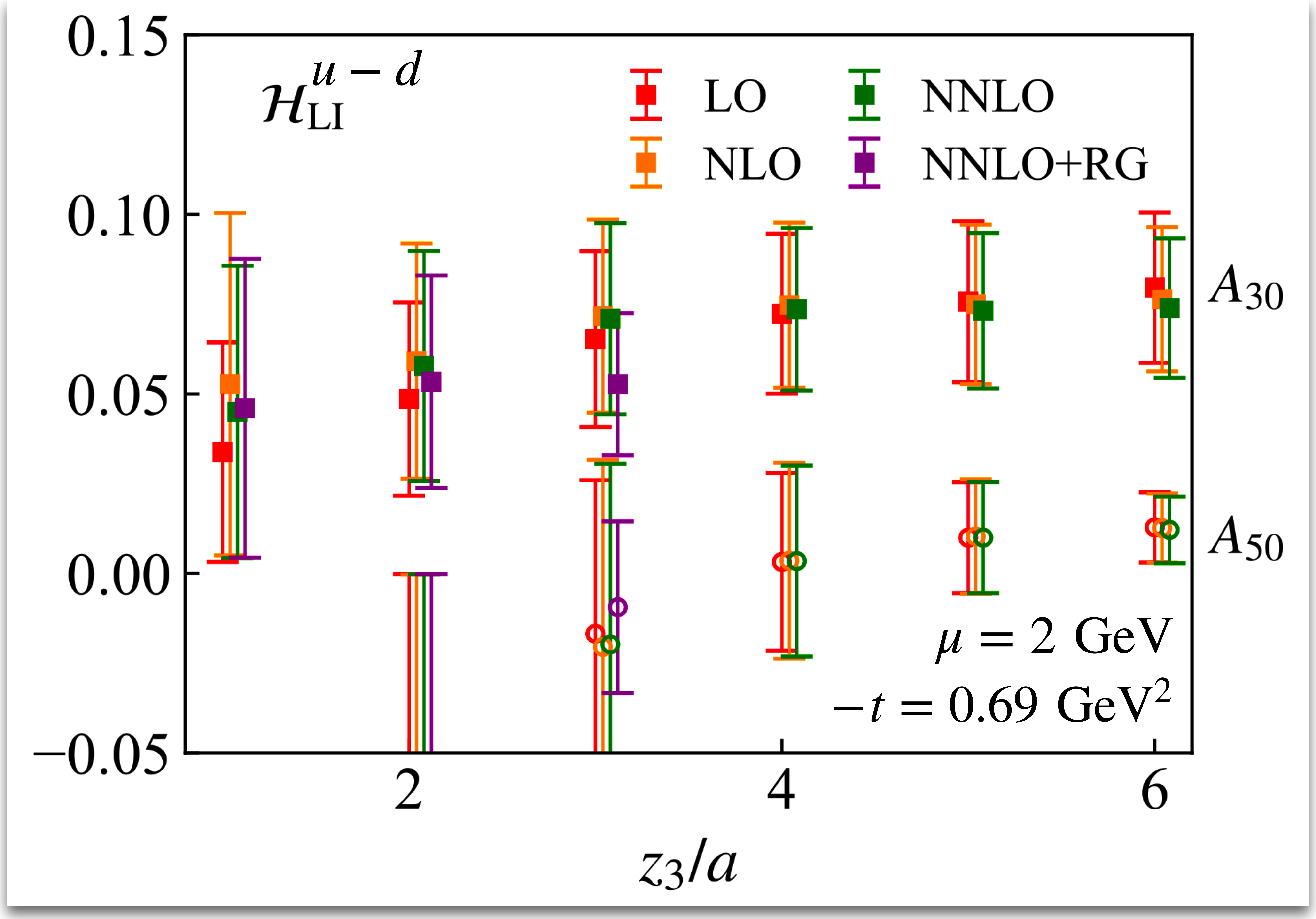
A_{n0}: nth Mellin moment of H

OPE: NNLO Wilson coeff

pion mass 260 MeV

lattice spacing 0.093 fm

proton momenta $\leq$ 1.67 GeV



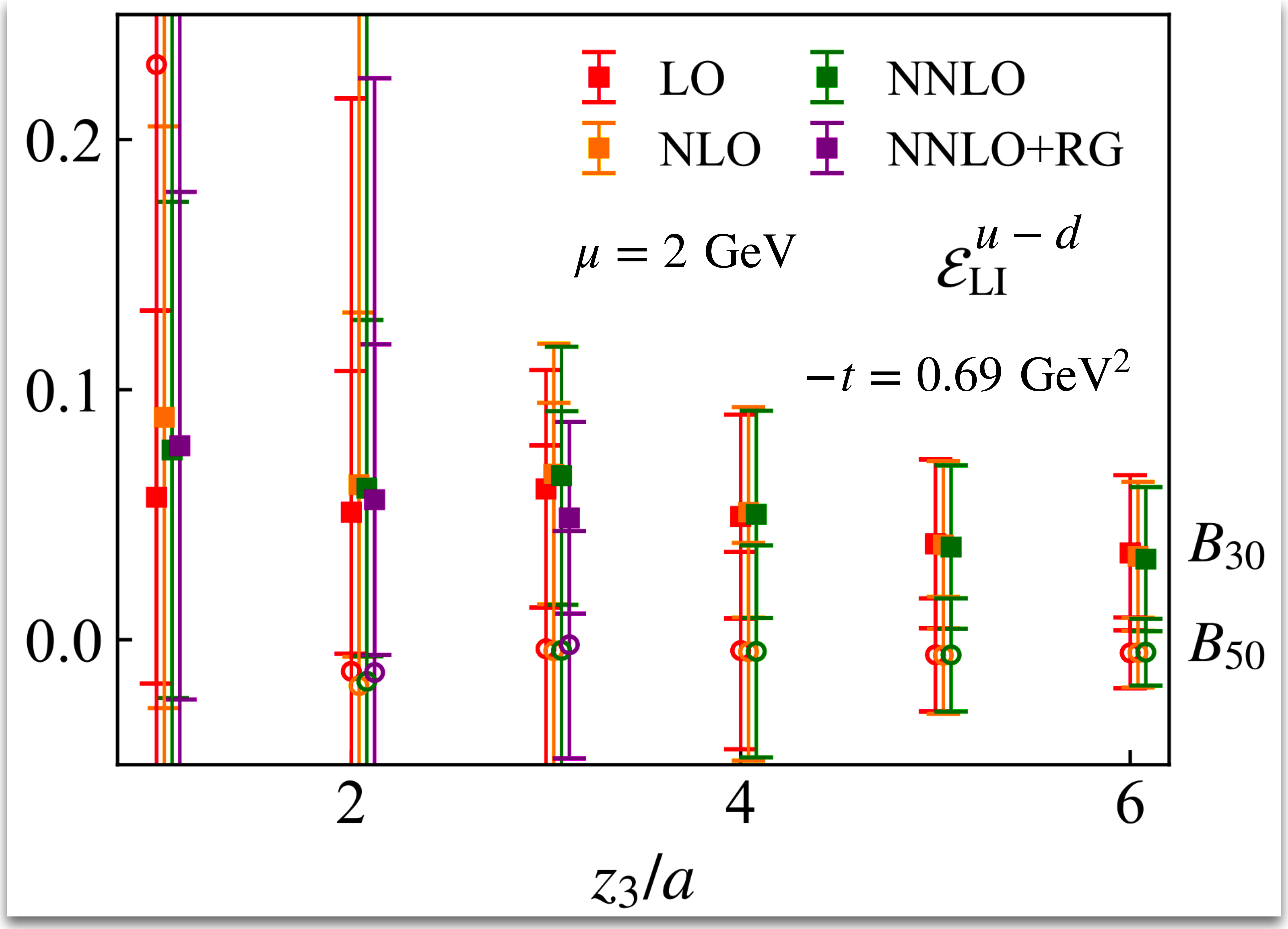
proton GPD: Mellin moments of H and E at zero skewness

Xiang Gao *et al.*, *Phys.Rev.D* 108 (2023) 1, 014507

B_{n0}: n$^{\text{th}}$ Mellin moment of E

OPE: NNLO Wilson coeff

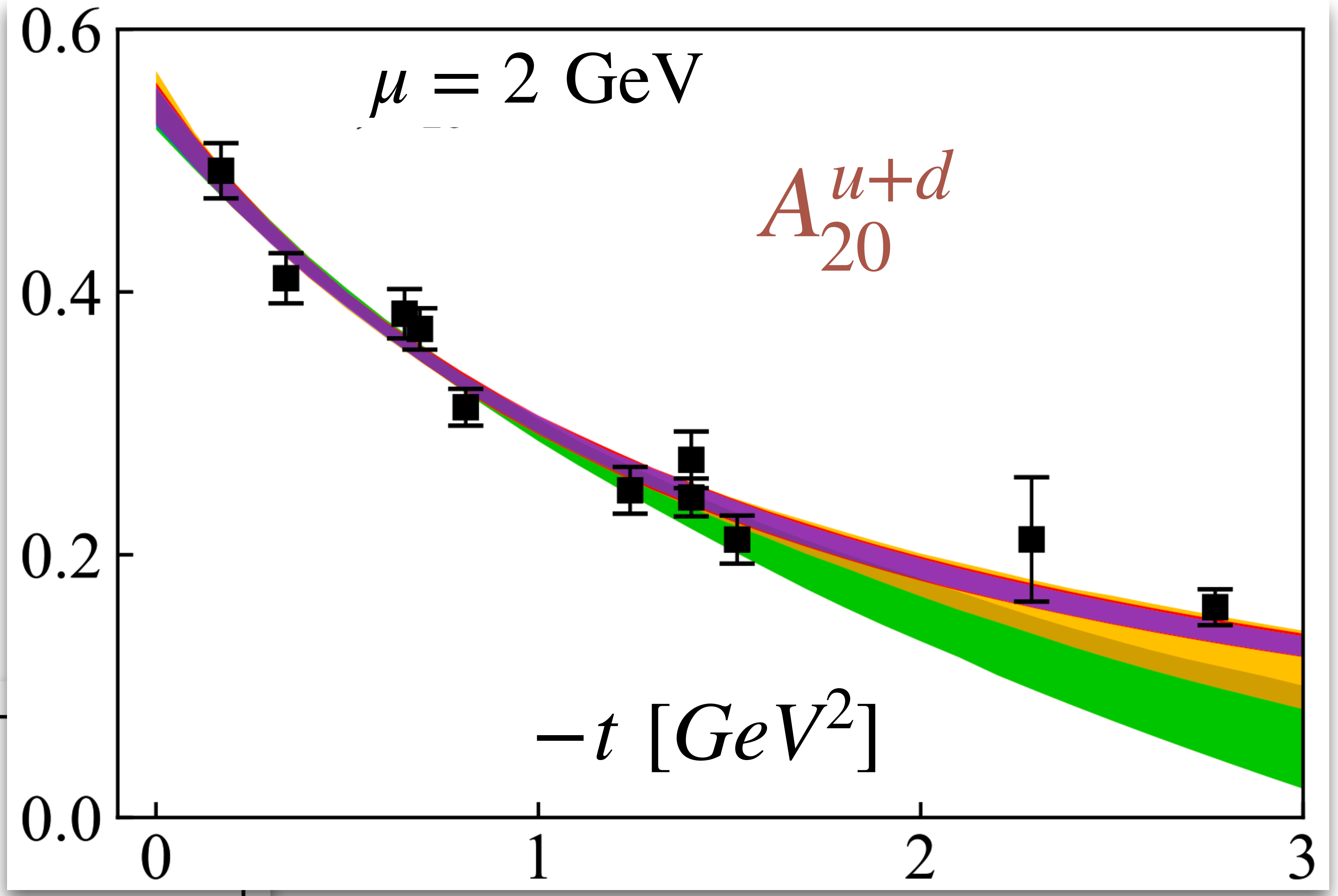
NLO to NNLO: no significant effect within present precision



quarks' angular momenta contributions to proton spin

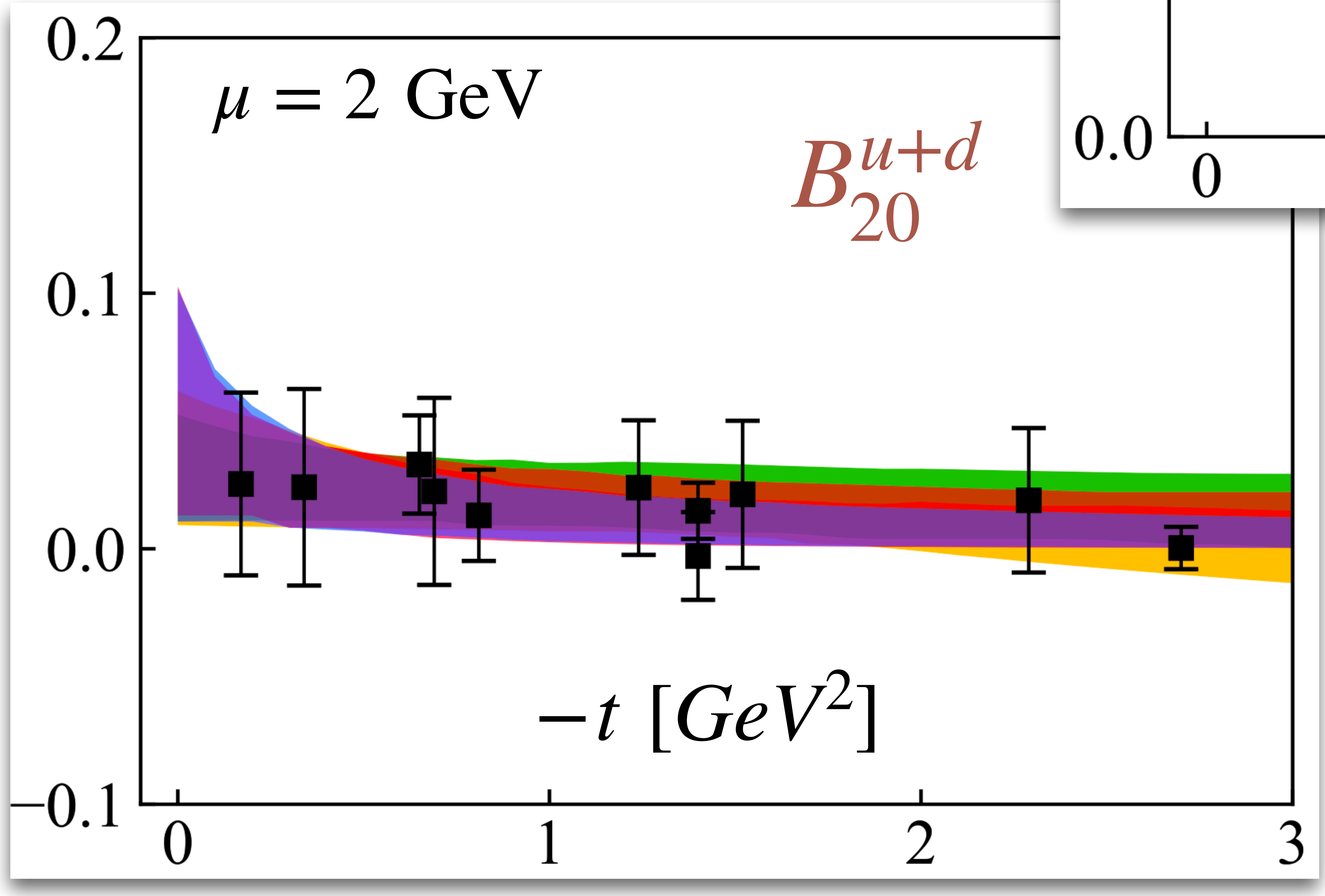
Xiang Gao *et al.*, *Phys.Rev.D* 108 (2023) 1, 014507

Ji sum rule: $J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$



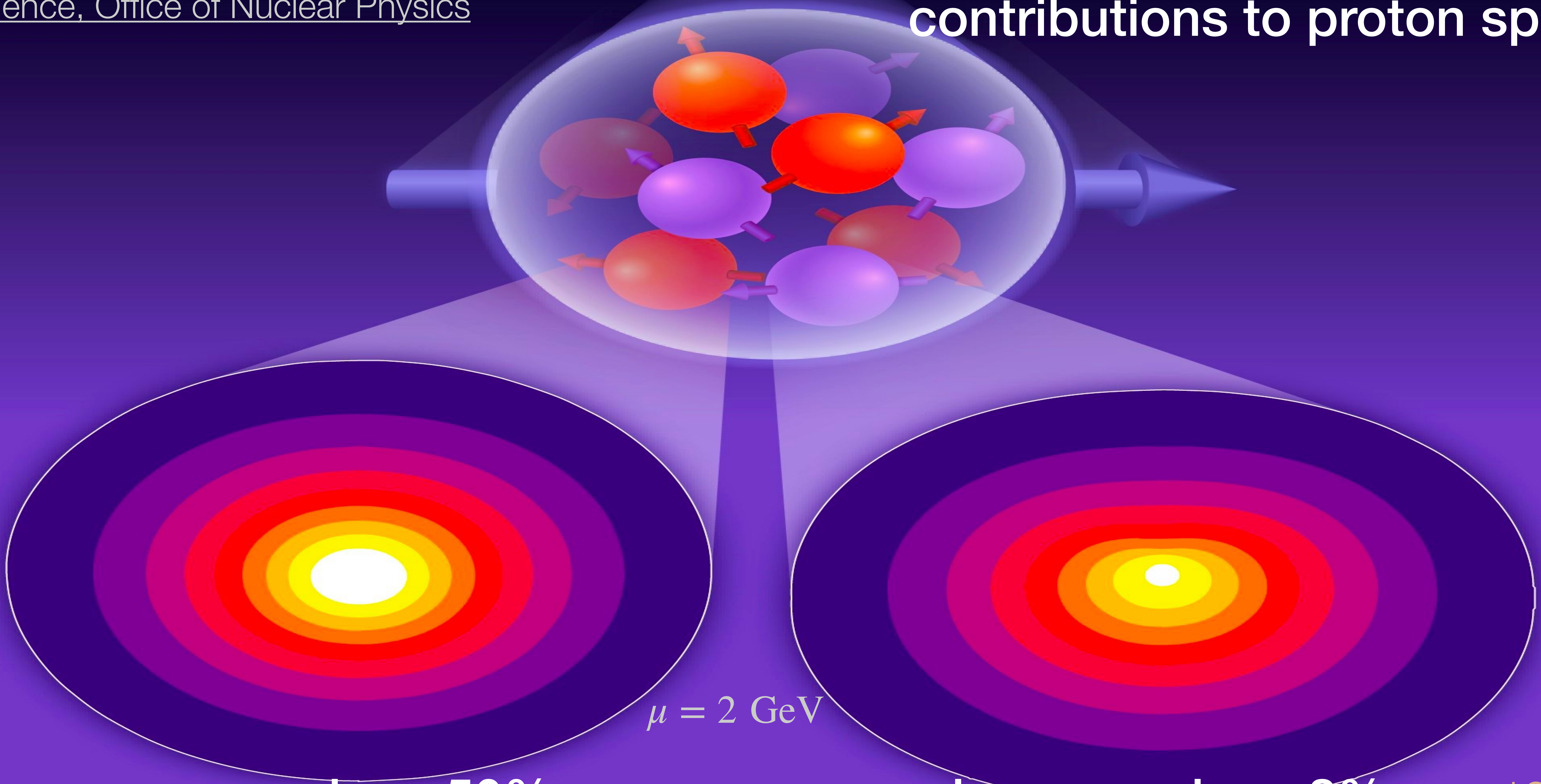
$J^{u+d} = 0.296(22)(33)$

$J^{u-d} = 0.281(21)(11)$



Science highlight of US DOE Office of Science, Office of Nuclear Physics

quarks' angular momenta contributions to proton spin

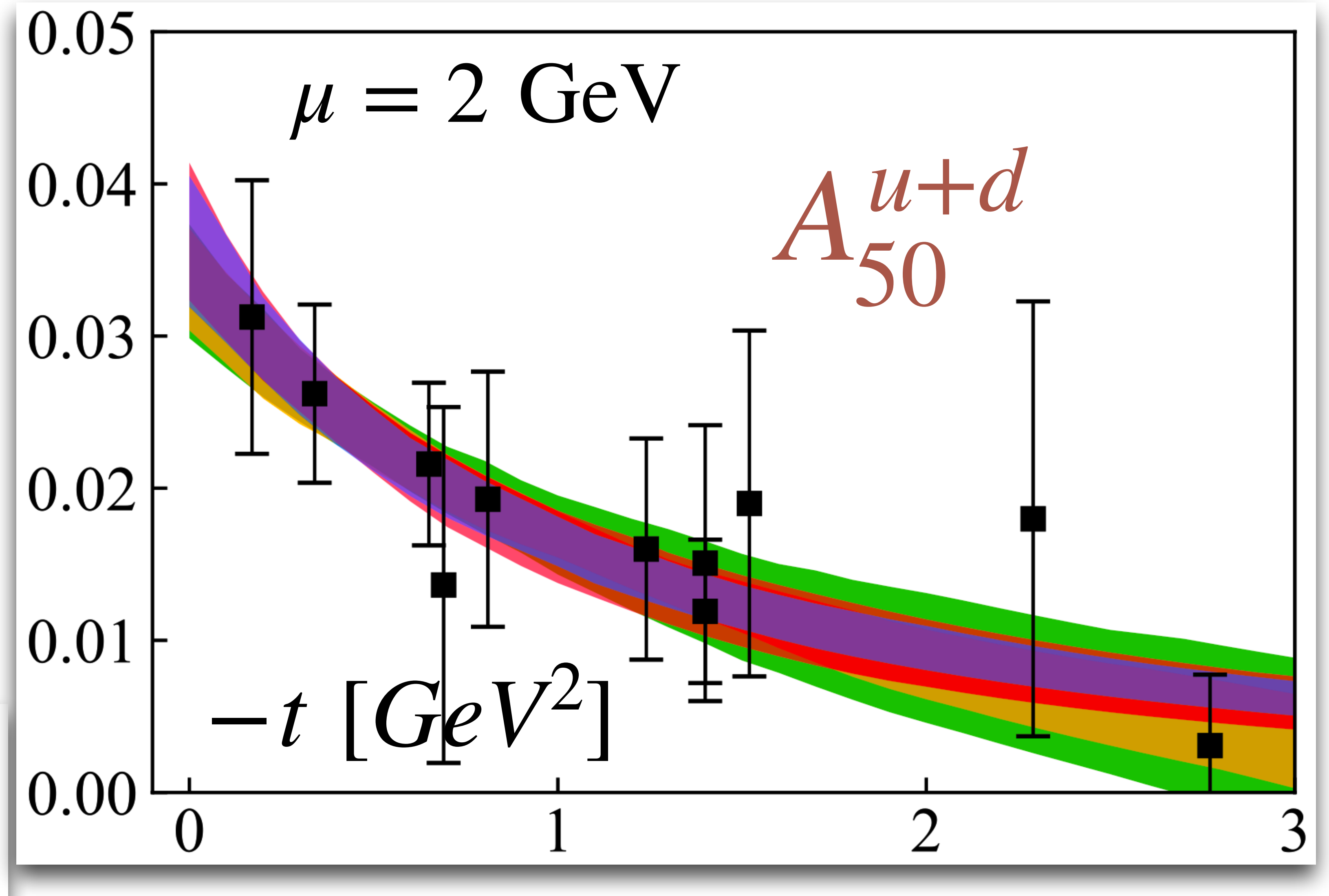


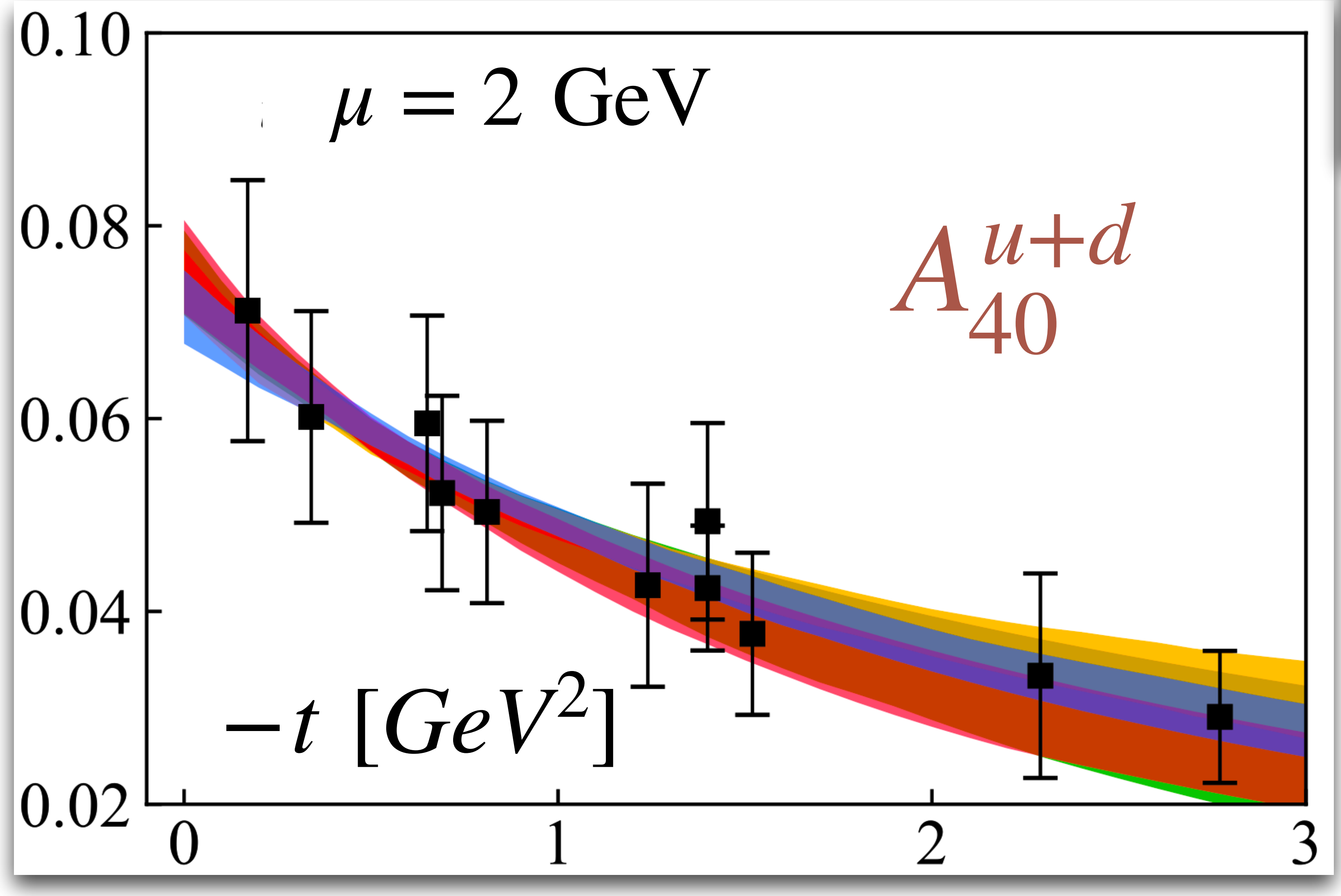
up quarks ~ 50%

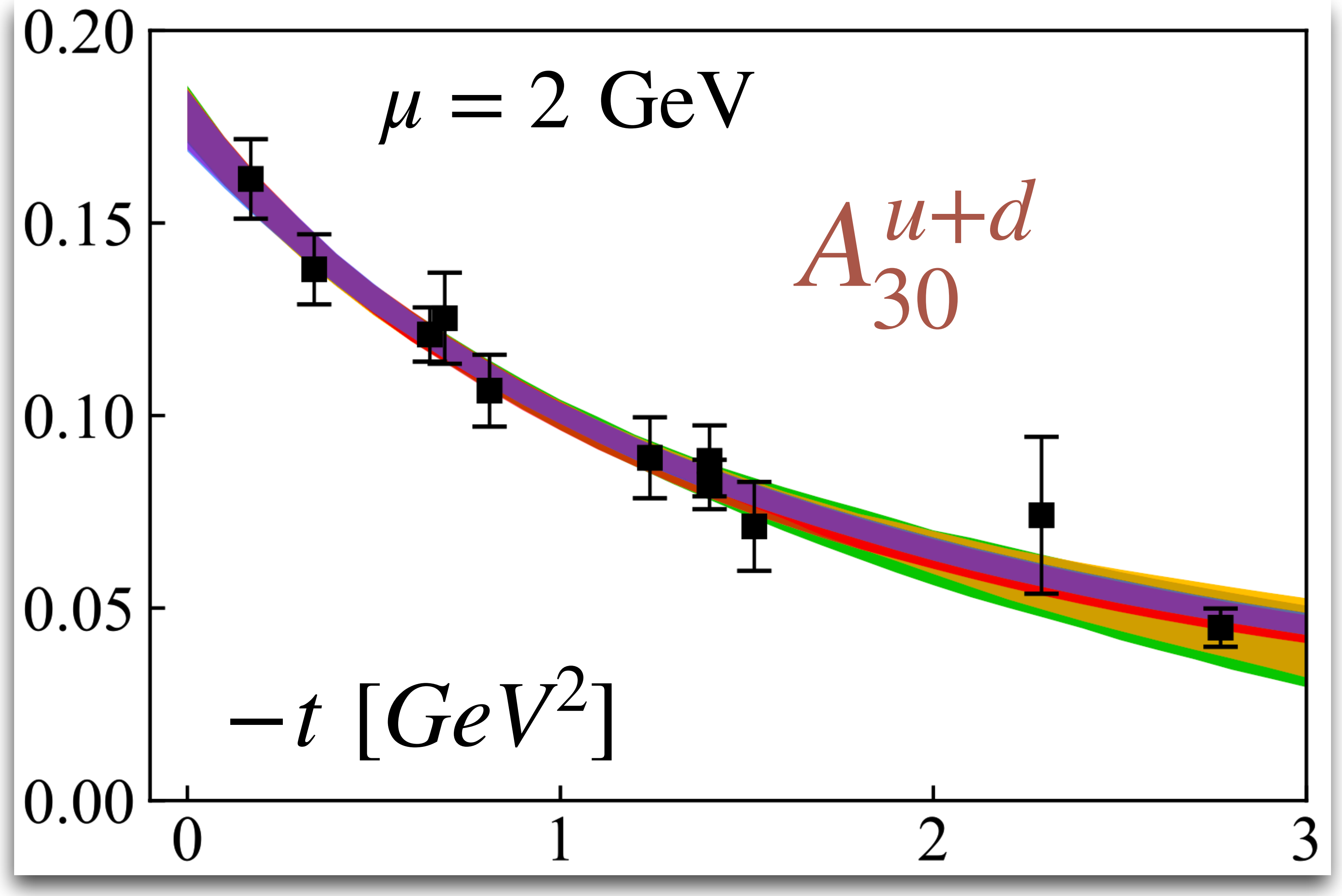
down quarks ~ 8%

proton GPD: Mellin moments of H and E

… and more





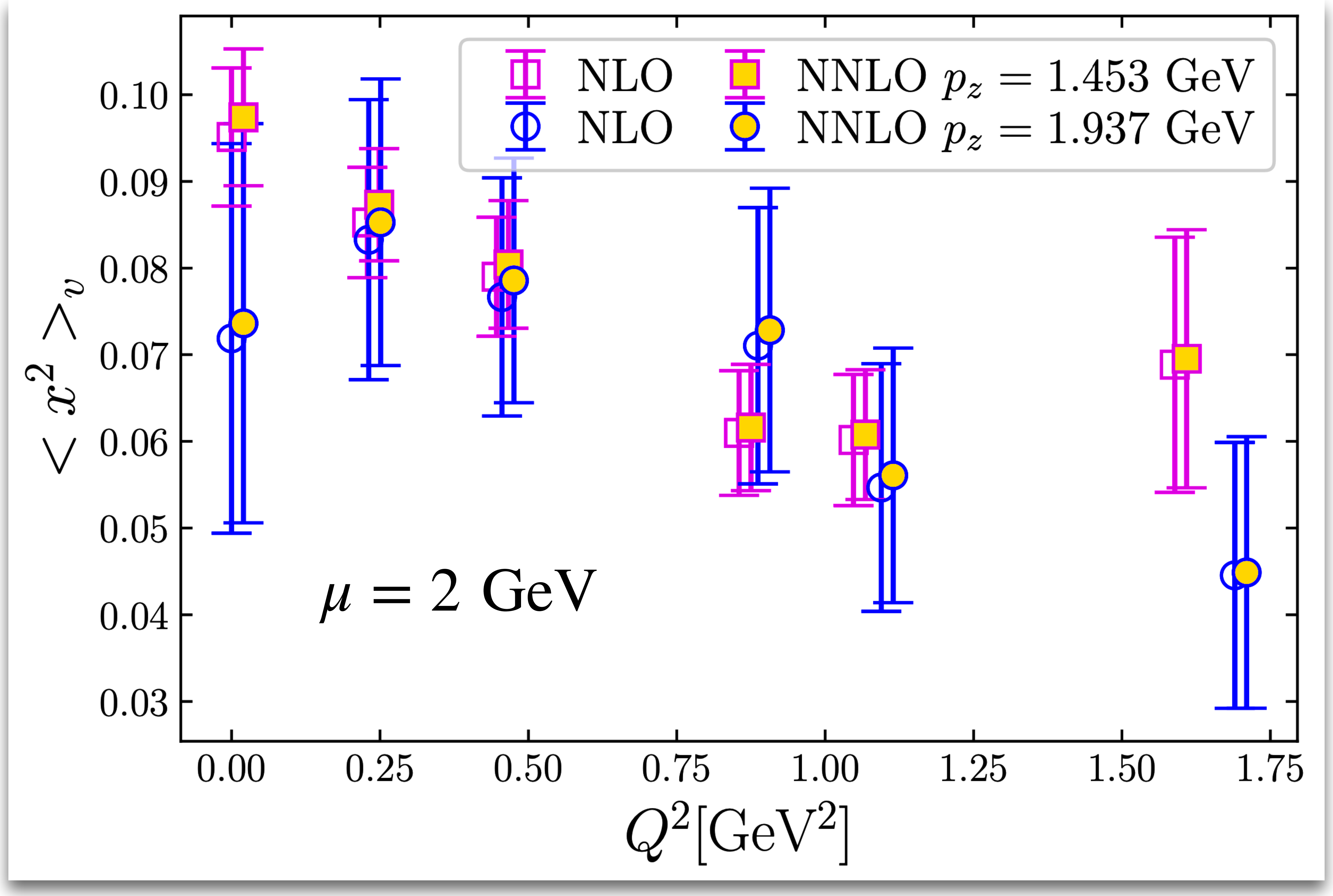


Xiang Gao *et al.*, *Phys.Rev.D* 108 (2023) 1, 014507

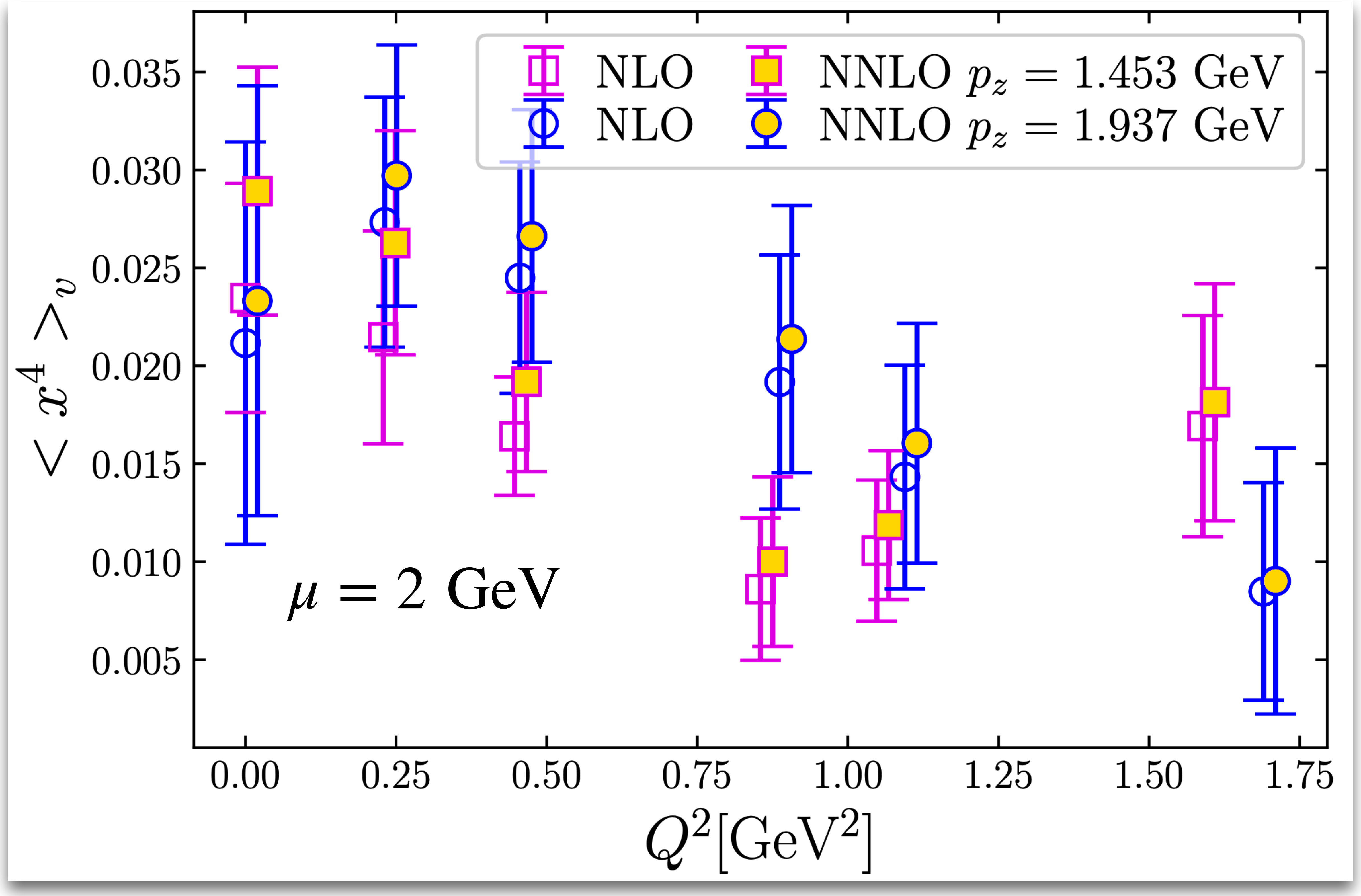
pion valance GPD: Mellin moments of H at zero skewness

Qi Shi *et al.*, in preparation

OPE: NNLO Wilson coeff

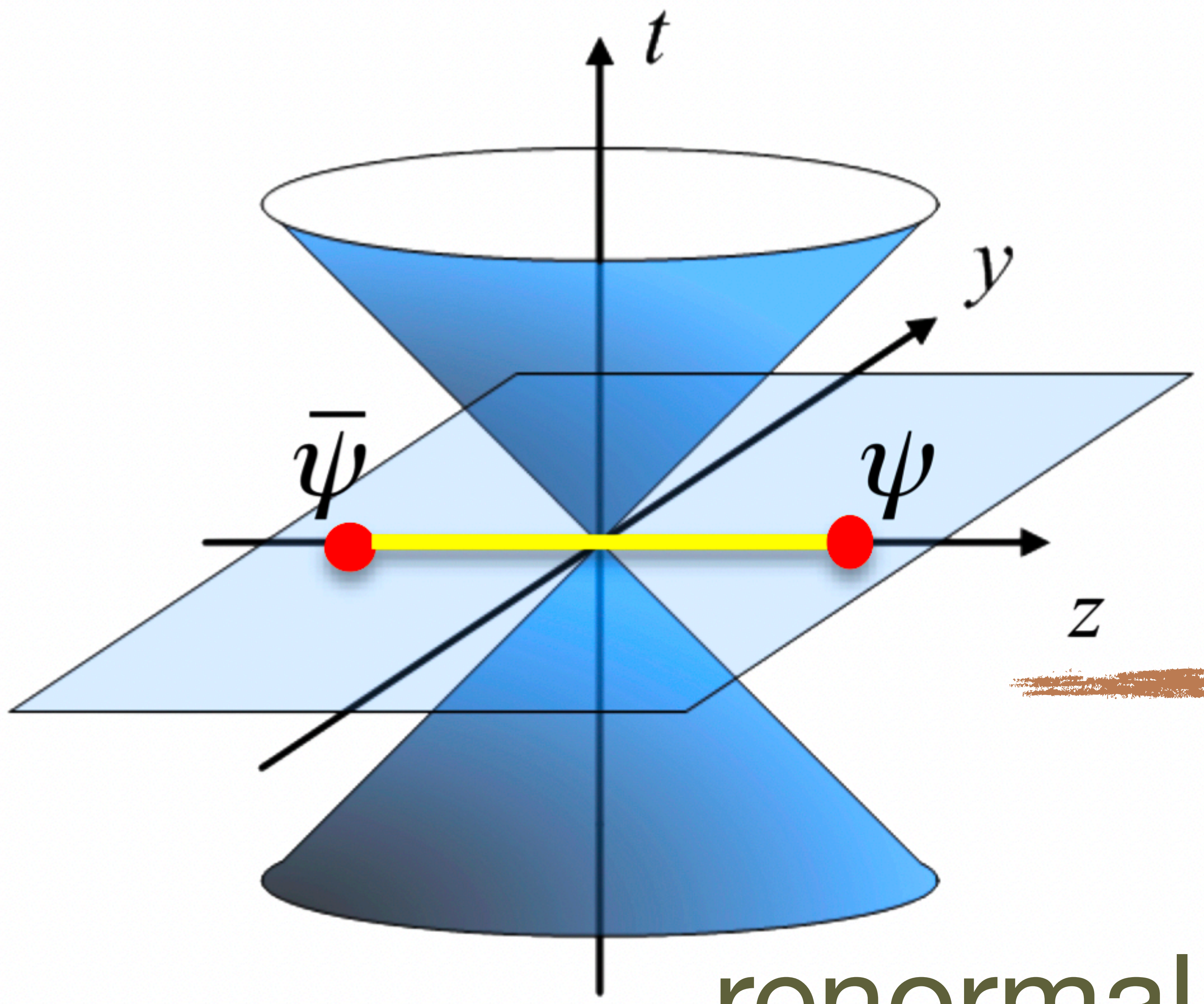


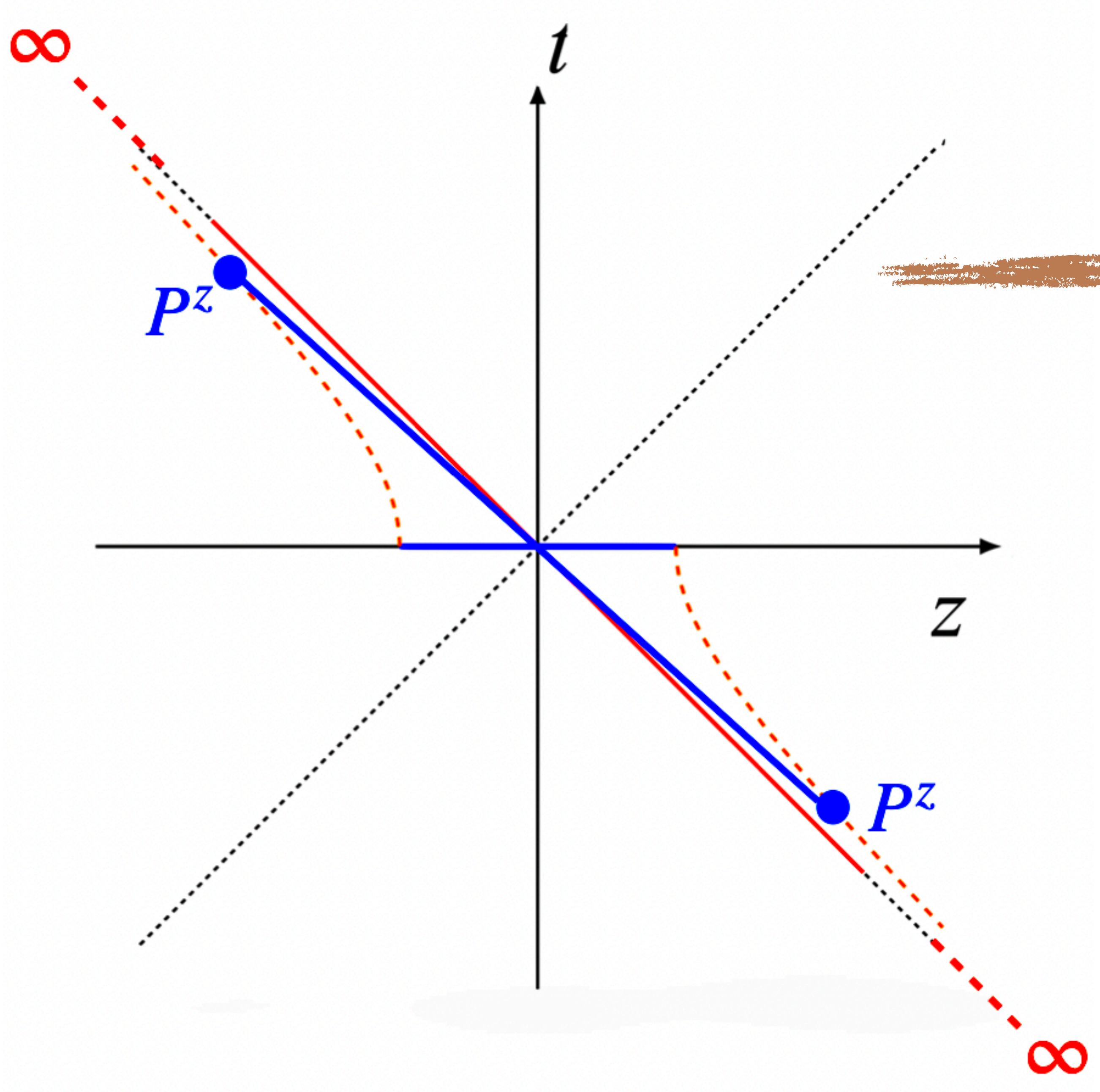
2[nd] moment

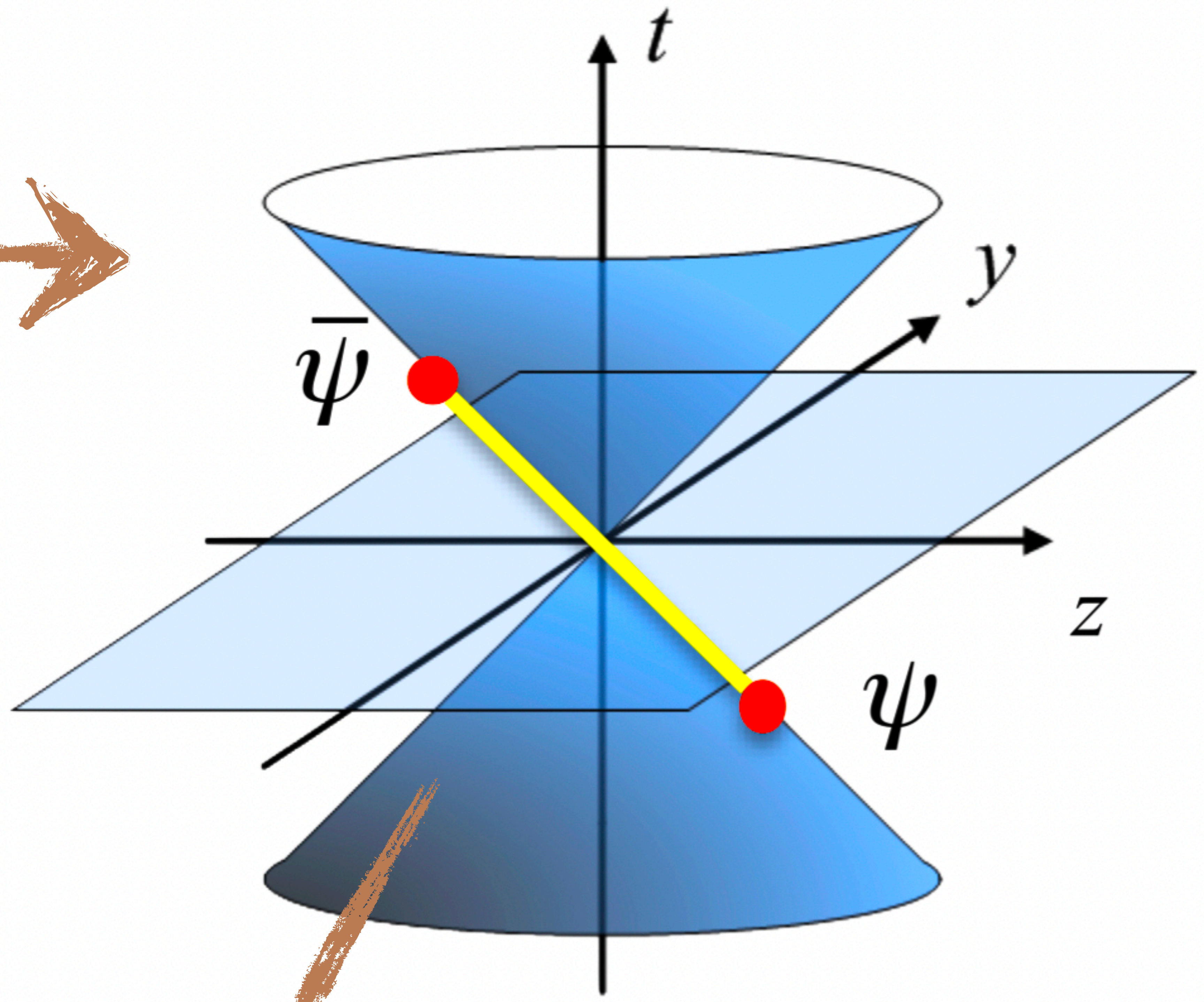


4[th] moment

leading renormalon resummation (LRR)







renormalized

parton physics

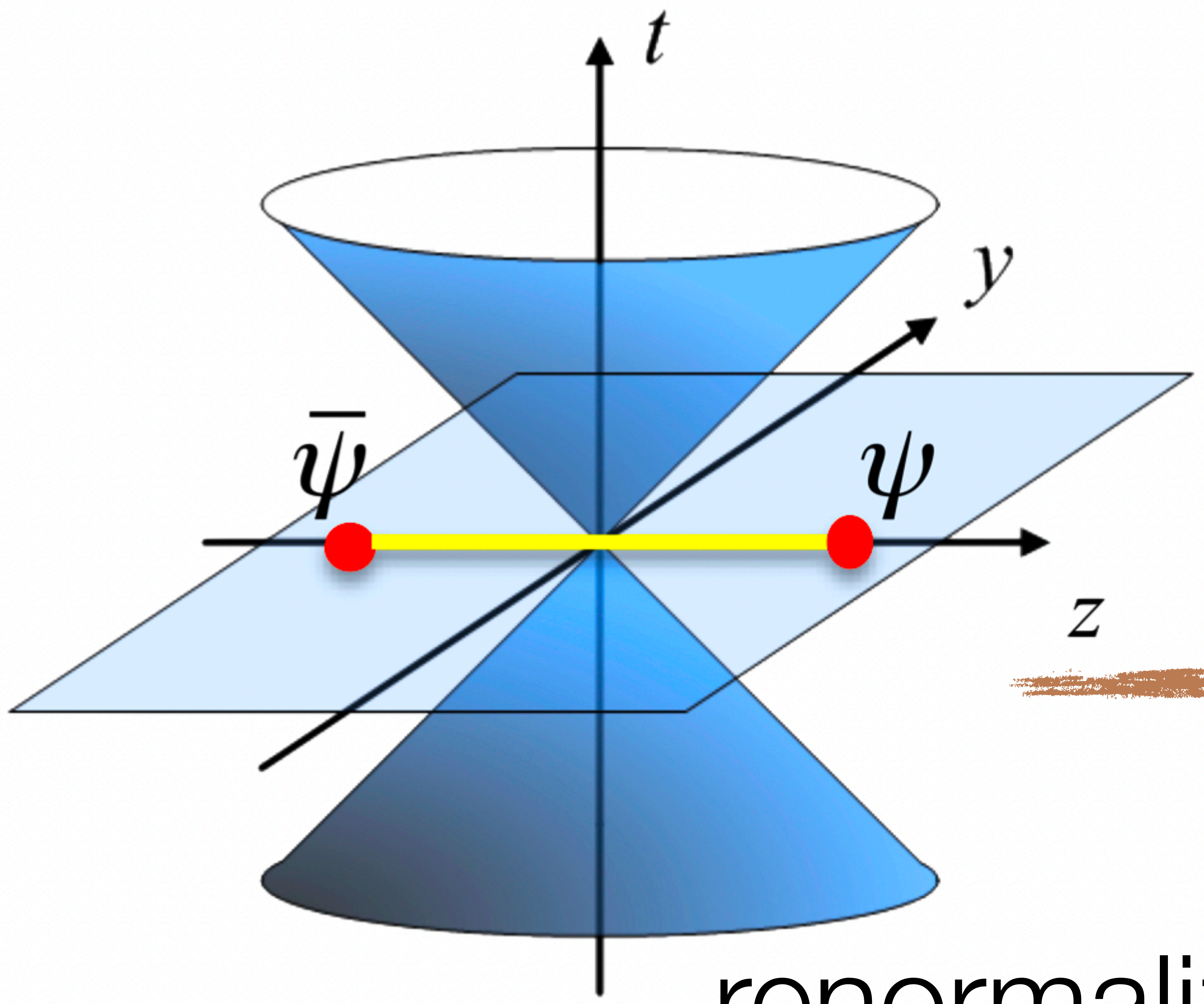
pQCD

non-perturbative ambiguity in divergent self-energy of Wilson line

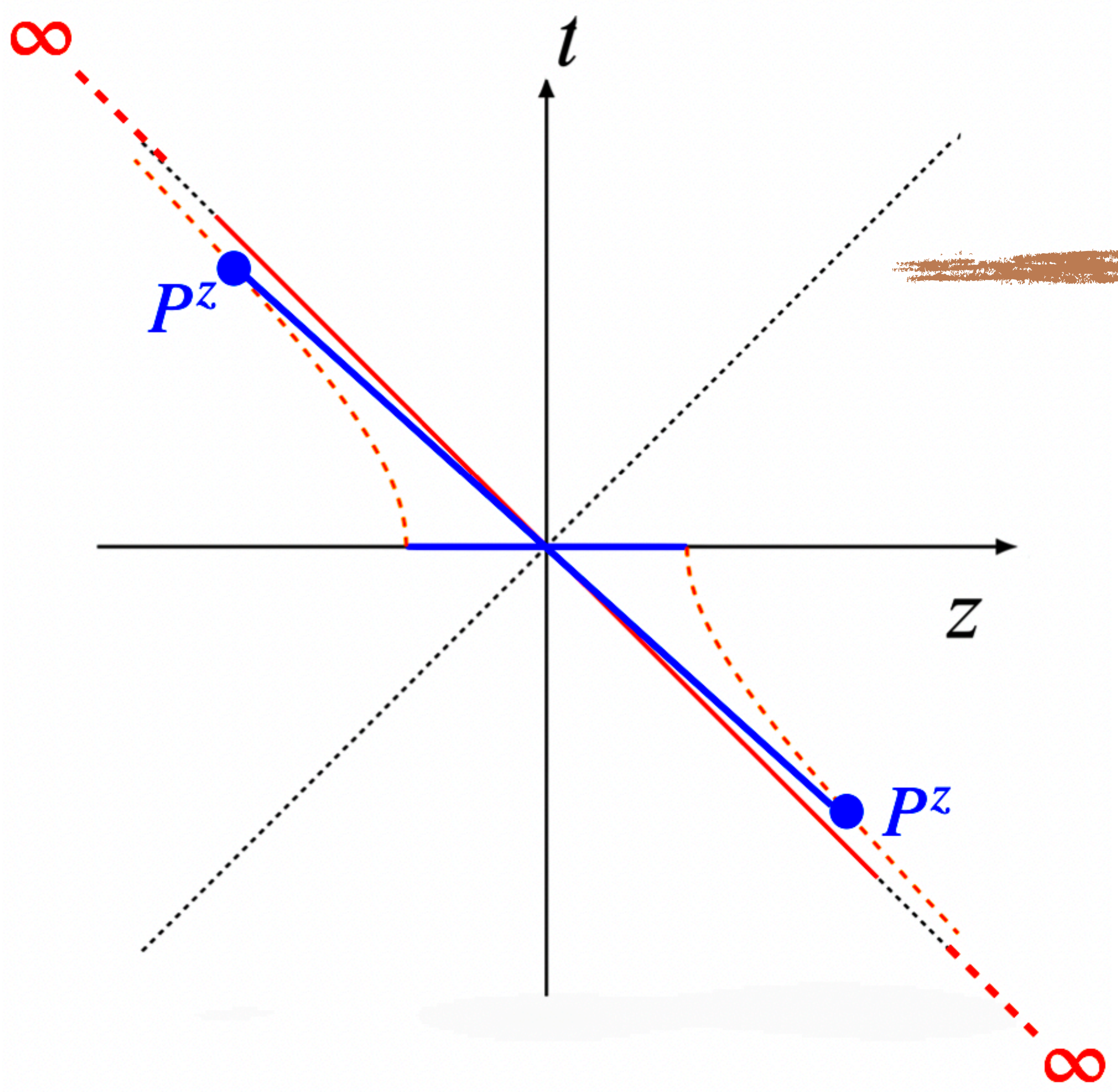
$$\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)$$

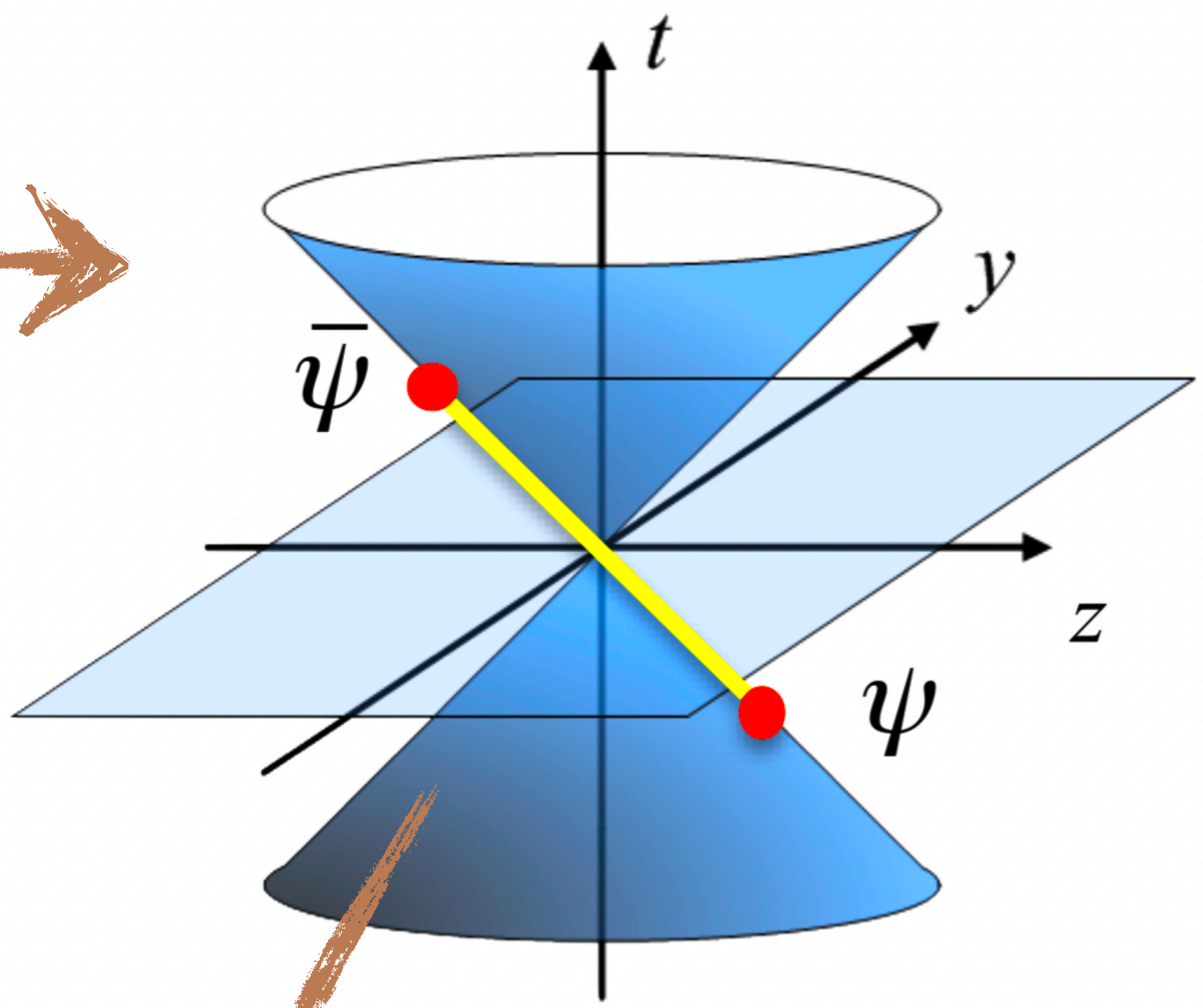
twist-3 power correction

leading renormalon resummation (LRR)



renormalized





parton physics

pQCD

non-perturbative (renormalon) ambiguity

from assymptotic series in α_s

$$\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)$$

twist-3 power correction

leading renormalon resummation (LRR)

- choose renormalization and matching schemes consistently:

$$\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right) - \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right) = 0$$

removes twist-3 power corrections from light-cone distributions

pion valance GPD: H at zero skewness

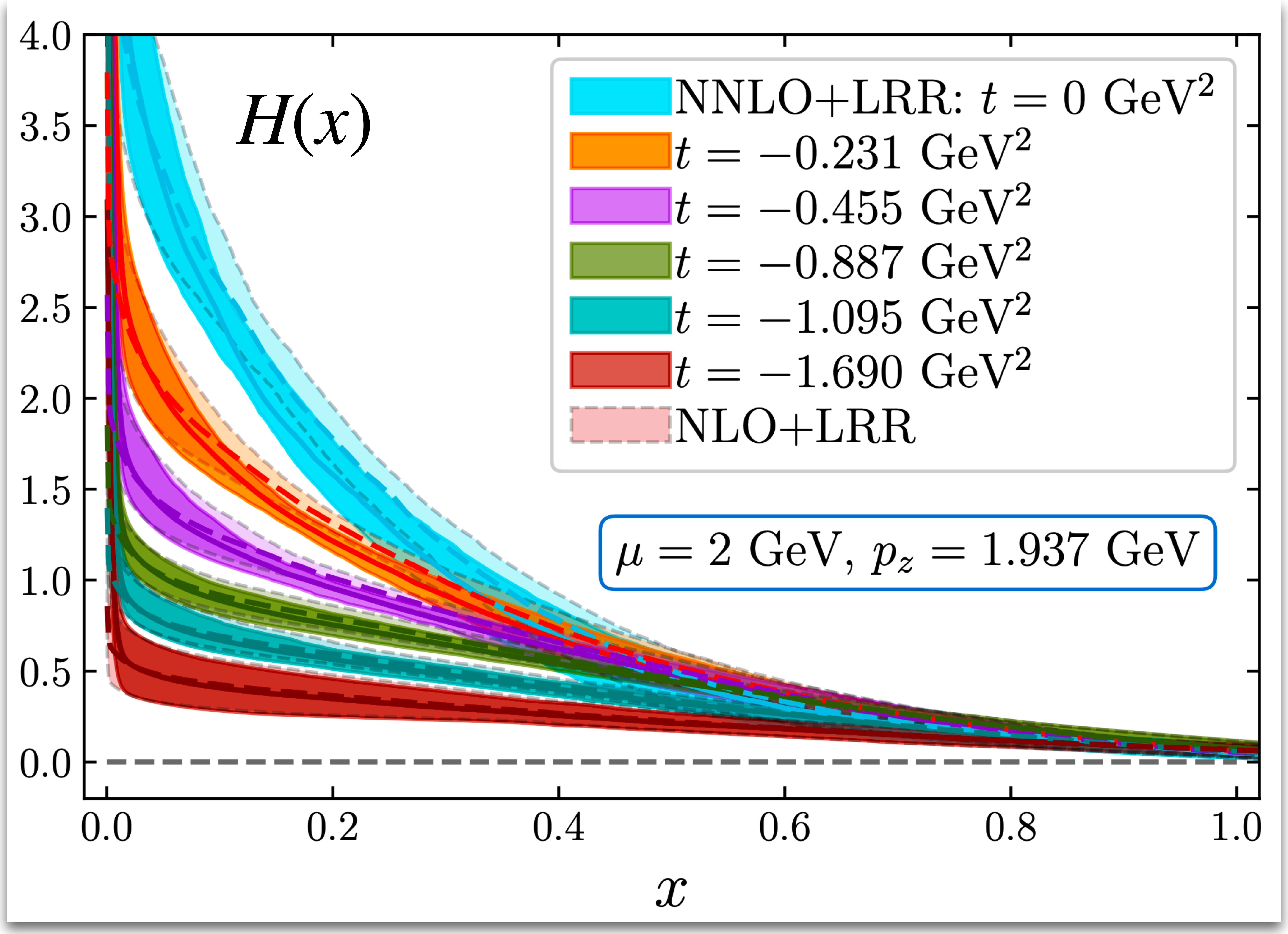
Qi Shi *et al.*, in preparation

NNLO+LRR momentum matching

valance pion mass 300 MeV

lattice spacing 0.04 fm

pion momenta up to1.94 GeV



resummation of large logs in matching

$$C(\mathcal{S},\mu) \sim \alpha_s^0(\mu) + \alpha_s(\mu) f\big(\ln[\mathcal{S}\mu]\big) + \alpha_s^2(\mu) f\big(\ln[\mathcal{S}\mu]\big) + \cdots$$

$\mathcal{S} \sim 1/\mu \longrightarrow \ln[\mathcal{S}\mu]$ large $\qquad \mathcal{S} = 2xP_z\,,\, z^2$

remove large logs by choosing: $\mu' = k/\mathcal{S}$

vary k within reasonable range to asses systematic uncertainties

C and α_s is determined at scale μ'

resummation of large logs in matching

evolve C from scale μ' to PDF scale μ

renormalization group:

$$\left[\frac{\partial}{\partial \ln \mu^2} + \beta(\alpha_s(\mu))\frac{\partial}{\partial \alpha_s} - \gamma(\alpha_s(\mu))\right] C(\mu) = 0$$

β function

operator's anomalous dimensions

renormalization group resummation (RGR)

resummation of large logs in matching

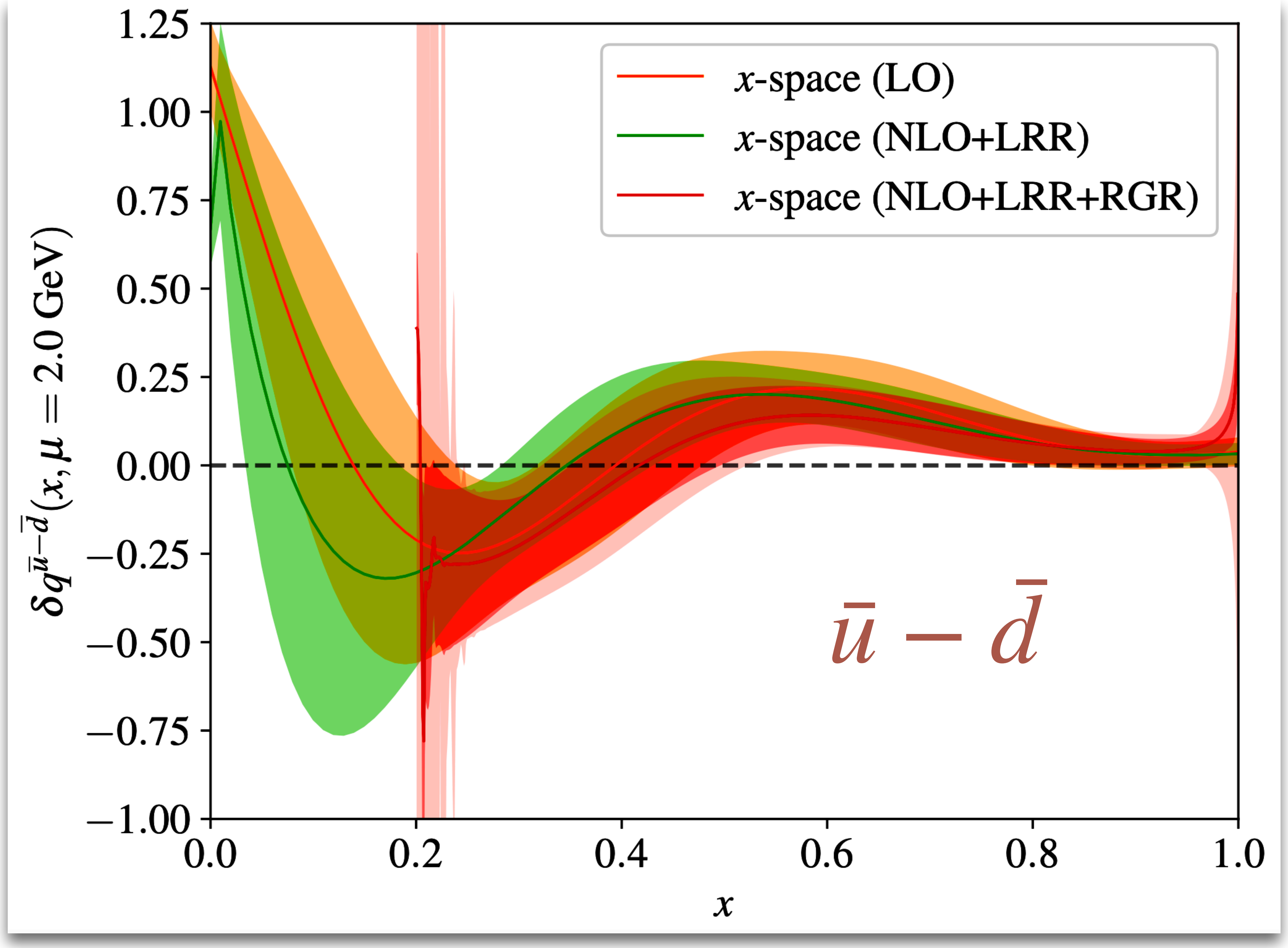
- depending on the (consistent) order of expansion in α_s used for C, β, γ one resums certain powers of the logs in C
- next example: resume next-to-leading-log (NLL), i.e RGR=NLL

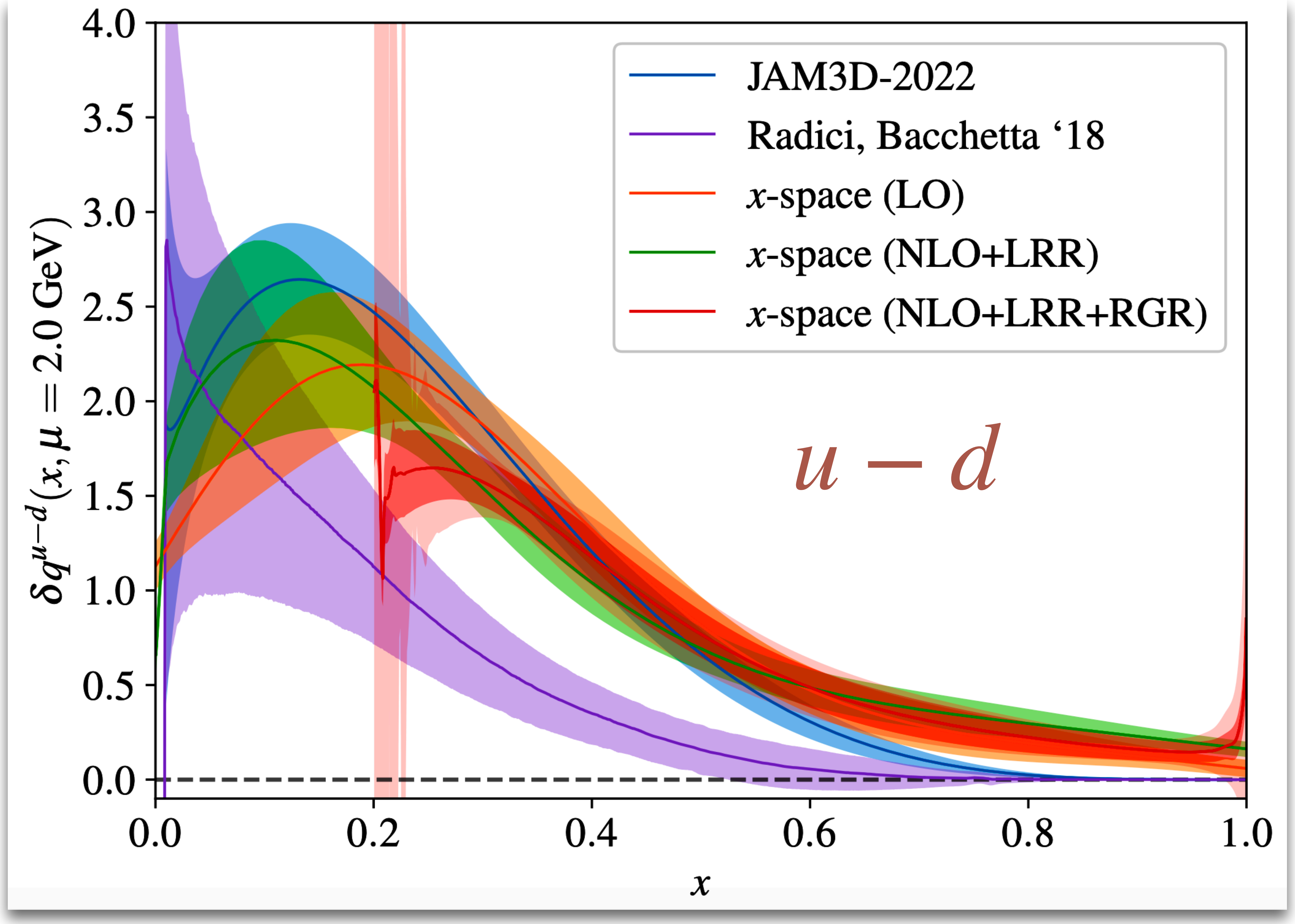
resummation of large logs in matching

- $\alpha_s(\mu')$ becomes too large for small μ'; large z^2 and small $2xP_z$
- breakdown of pQCD matching
- estimate trustworthy x-region for a give P_z

proton transversity PDF

Andrew Hanlon *et al.*, arXiv:2310.19047



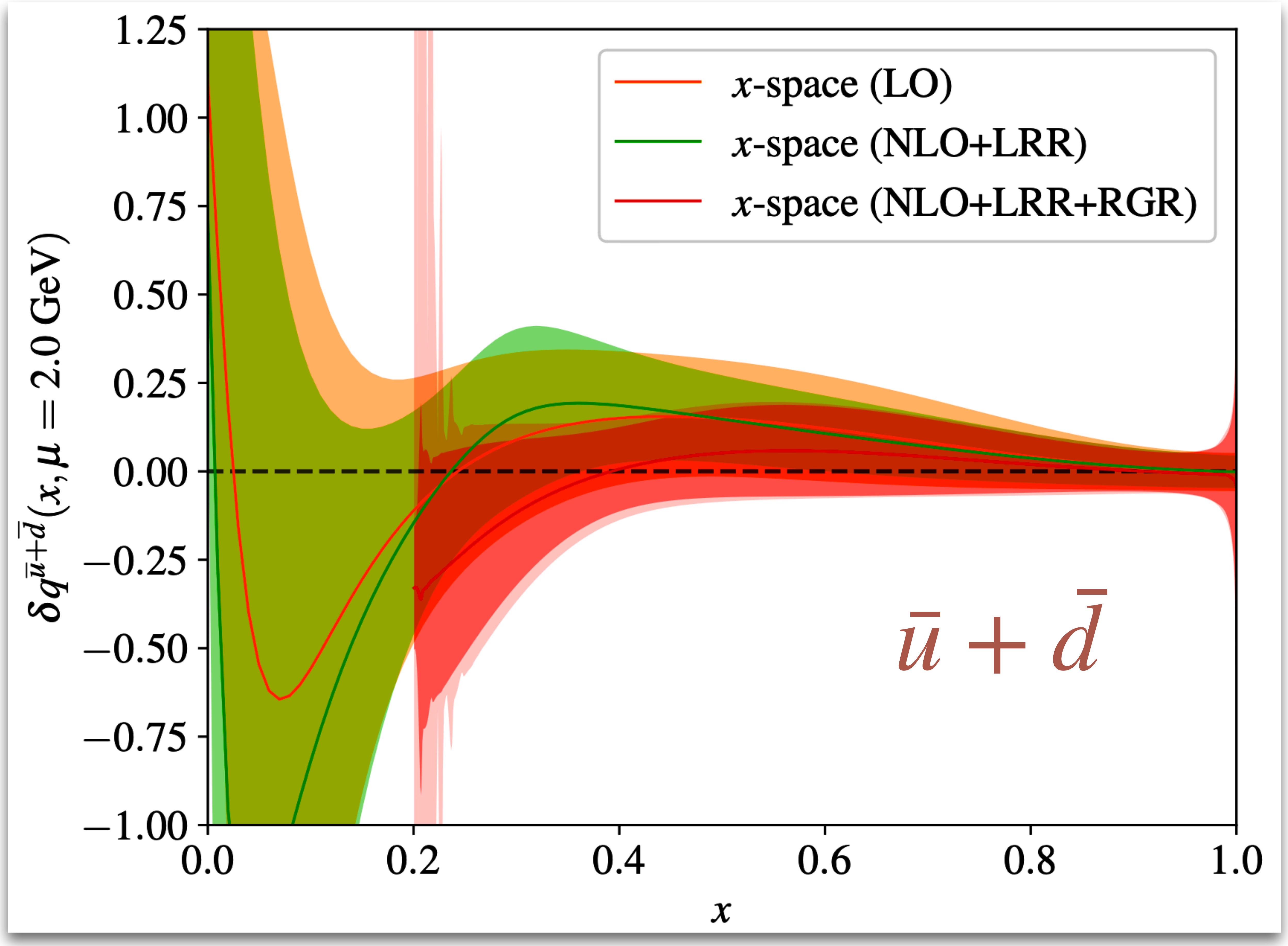


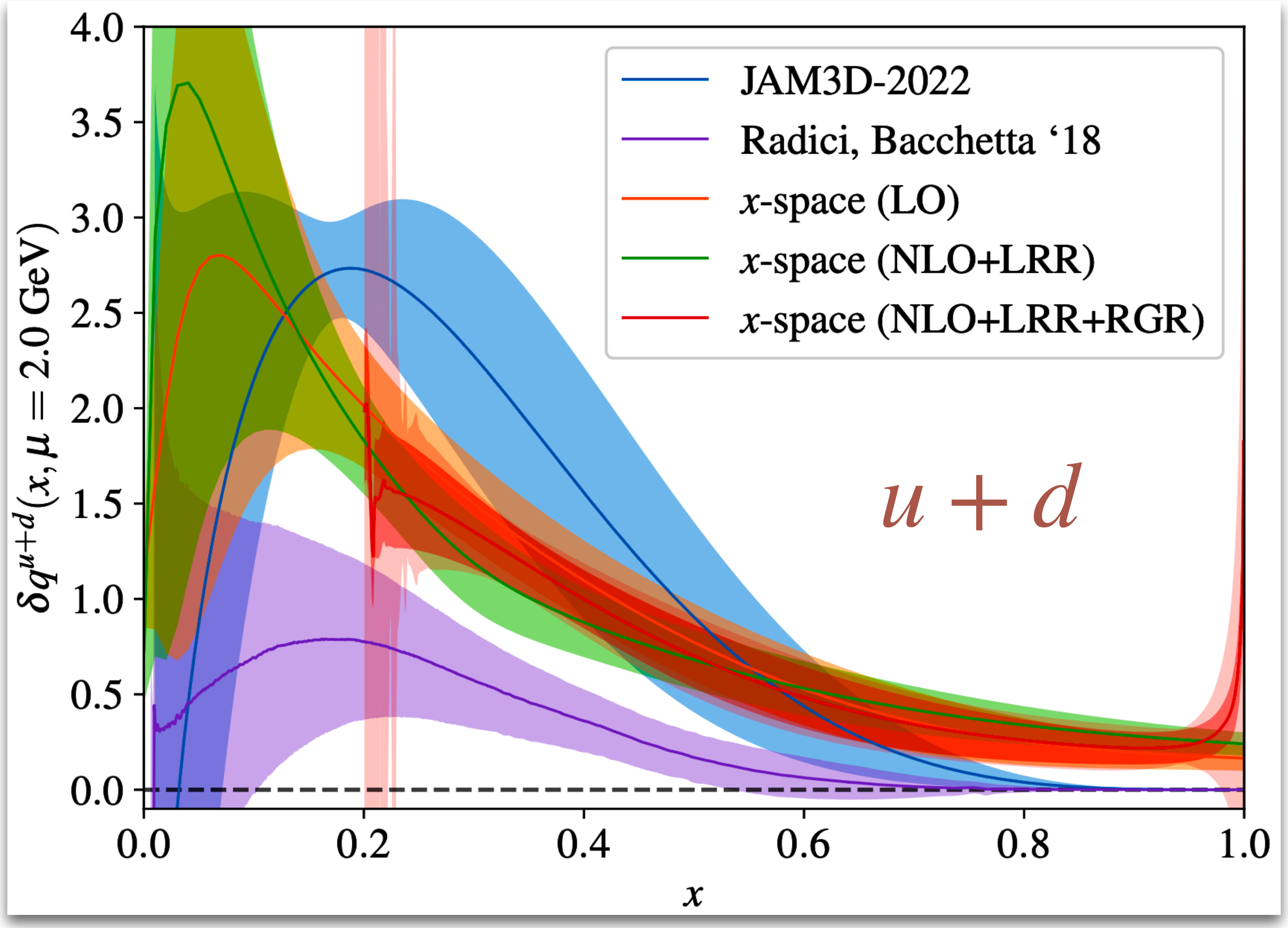
NLO+LRR+NLL(RGR) momentum matching

lattice spacing 0.075 fm, proton momenta 1.53 GeV, physical pions

proton transversity PDF

Andrew Hanlon *et al.*, arXiv:2310.19047





NLO+LRR+NLL(RGR) momentum matching

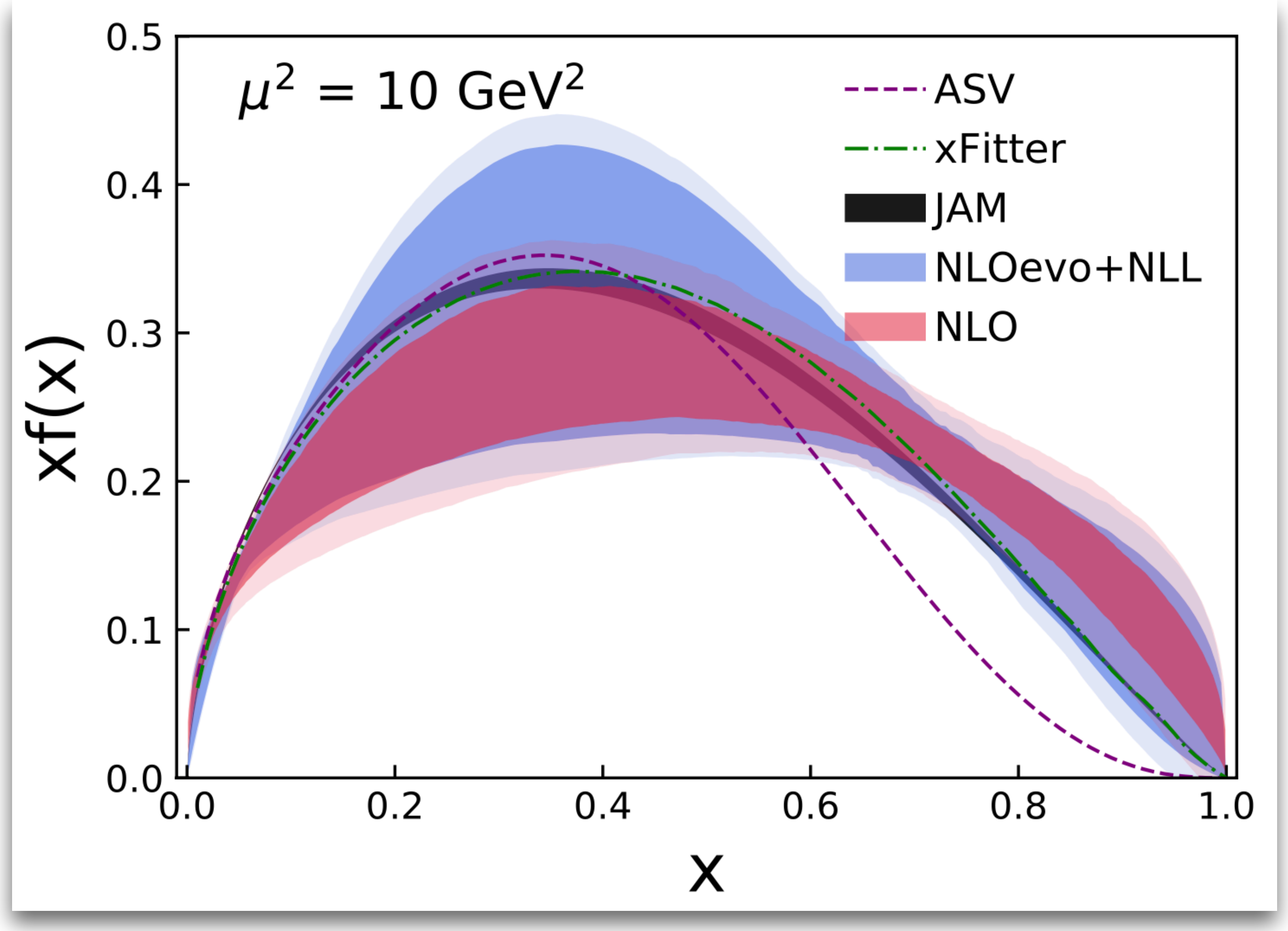
resummation of threshold logs

sorry, no time left to discuss

pion valance PDF

first LQCD calculations of PDF including threshold resummation

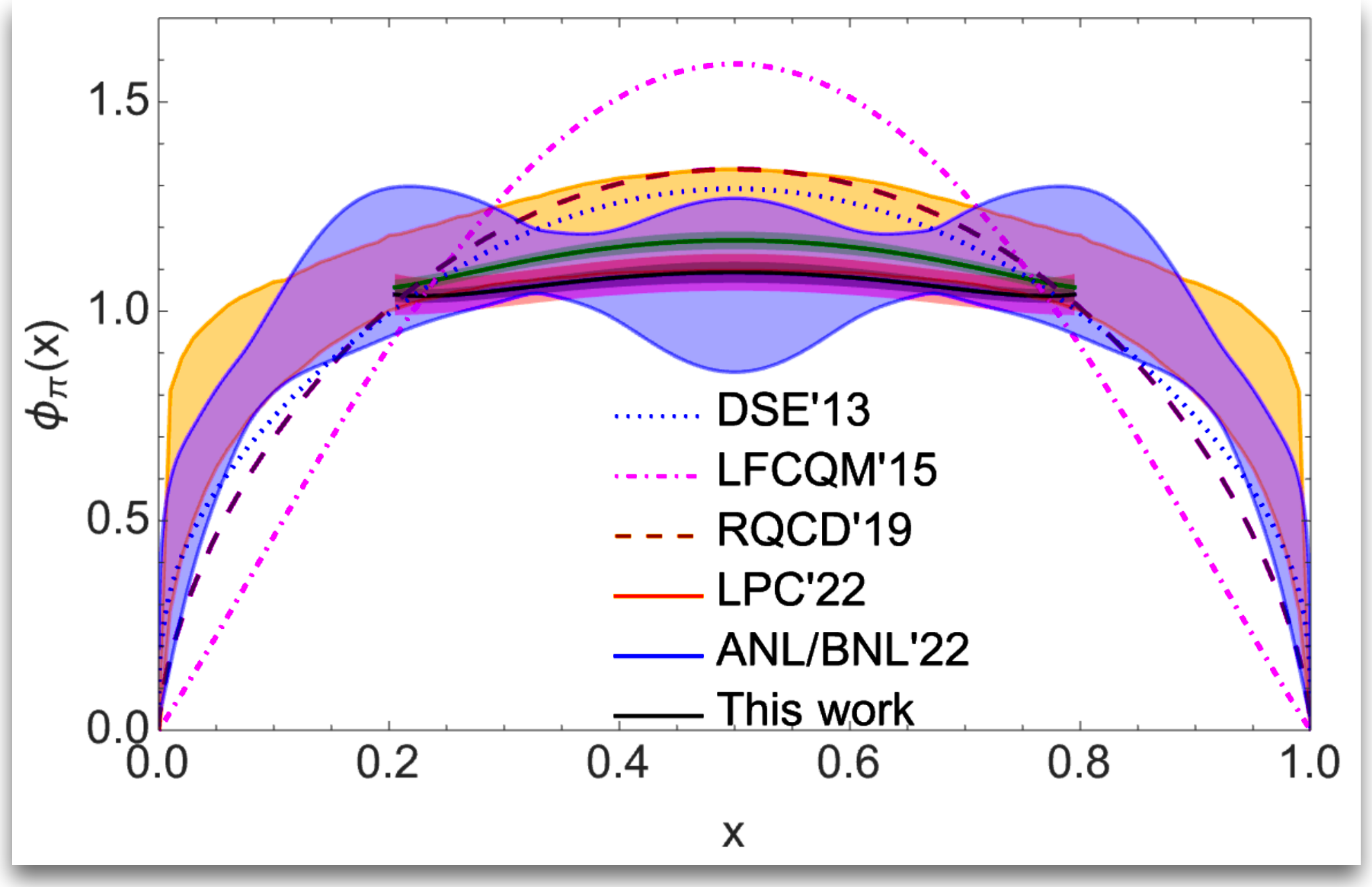
Yong Zhao *et al.*, Phys.Rev.D 103 (2021) 9, 094504



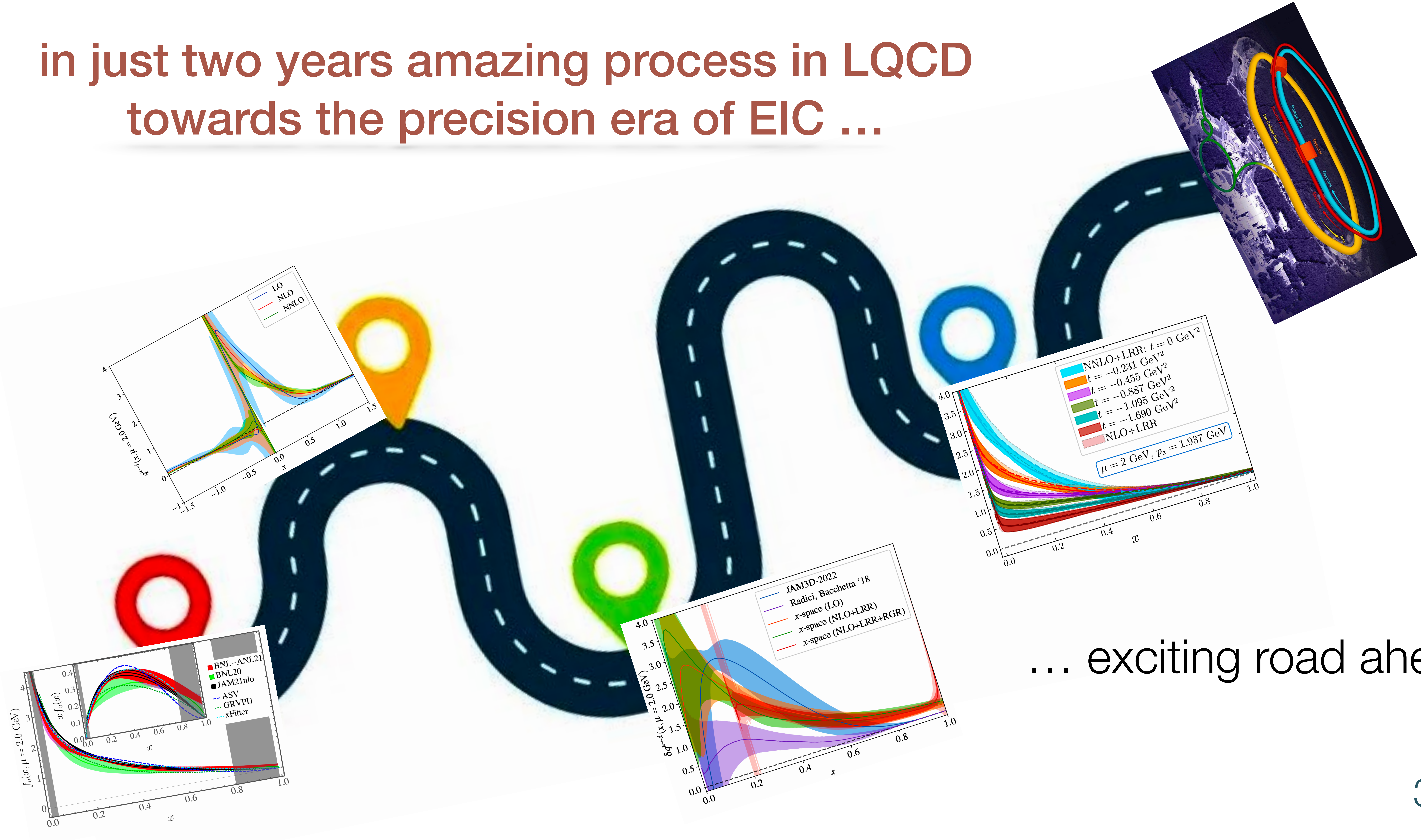
pion distribution amplitude

Talk: Rui Zhang

first LQCD calculations of x-dependent partonic structure with chiral fermions



in just two years amazing process in LQCD towards the precision era of EIC ...



… exciting road ahead