



Parton Fragmentation Functions

EINN2023, Paphos, Cyprus

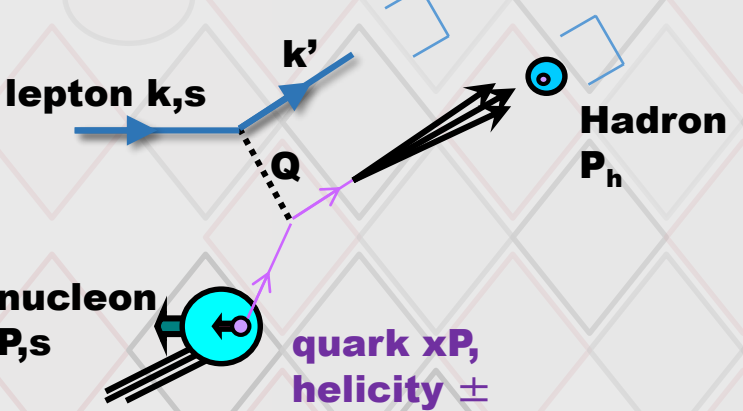
November 2nd

Ralf Seidl (RIKEN)

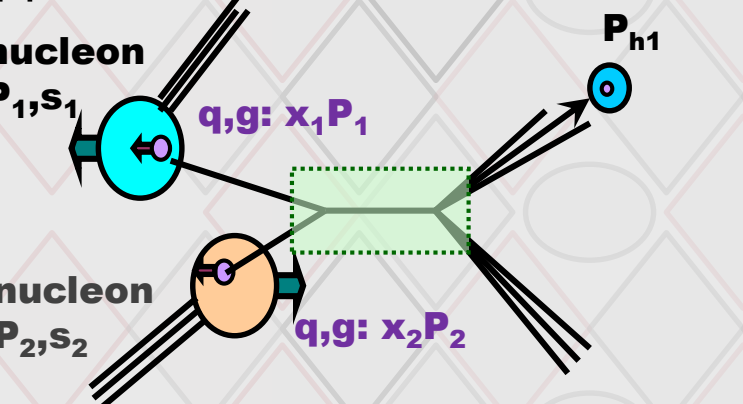
Help of Fragmentation Functions (FF) for nucleon (spin, transverse) structure

- Correlate a hadron in the final state with the struck parton:
 - Parton flavor (HF, favored vs disfavored, strange fragmentation, etc; ie: u quark into a π^+ is **favored**, d quark **disfavored**, s, ubar quarks **favored** in K^+ fragmentation)
 - Spin of the struck quark (Collins and Interference Fragmentation, other polarized FFs)
 - Transverse momentum of struck parton (though convolution with fragmentation transverse momentum)
- Closely related to confinement mechanism

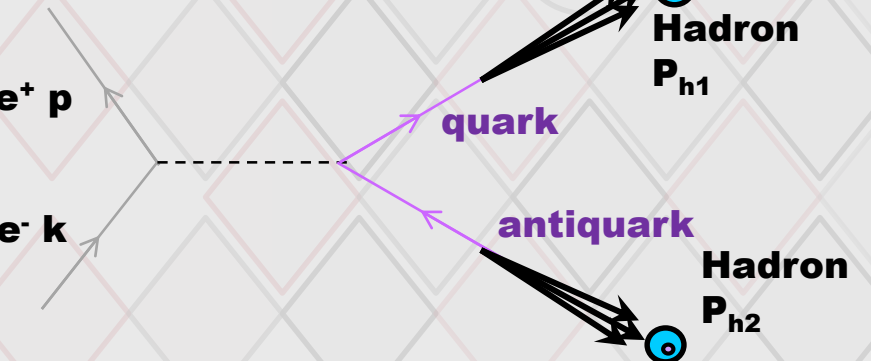
SIDIS



pp collisions



e+e- Annihilation



Access to Fragmentation functions

- SIDIS:

$$\sigma^h(x, z, Q^2, P_{h\perp}) \propto \sum_q e_q^2 q(x, p_t, Q^2) D_{1,q}^h(z, k_t, Q^2)$$
 - Relies on unpol PDFs
 - Parton momentum known at LO
 - Flavor structure directly accessible
 - Transverse momenta convoluted between FF and PDF
- pp:

$$\sigma^h(P_T) \propto \int_{x_1, x_2, z} \sum_{a, a' \in q, g} f_a(x_1) \otimes f_{a'}(x_2) \otimes \sigma_{aa'} \otimes D_{1,q}^h(z)$$
 - Relies on unpol PDFs
 - leading access to gluon FF
 - Parton momenta not directly known
- e+e-:

$$\sigma^h(z, Q^2, k_t) \propto \sum_q e_q^2 (D_{1,q}^h(z, k_t, Q^2) + D_{1,\bar{q}}^h(z, k_t, Q^2))$$
 - No PDFs necessary
 - Clean initial state, parton momentum known at LO
 - Flavor structure not directly accessible*

Single hadron FF


Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Single hadron cross sections: $e^+e^- \rightarrow hX$ $D_{1,q}^h(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRL111 (2013) 062002 PRD101(2020) 092004 </div>	Azimuthal asymmetries: $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2)$ $H_{1,q}^{\perp(1)h}(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRL 96 (2006) 232002 PRD 78 (2008) 032011 </div>	Unpol SIDIS, pp: $\frac{d\sigma}{dz}$ $e^+e^- \rightarrow (h)(h)X$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRD92 (2015) 092007 PRD101(2020) 092004 </div> and scale dependence
Transverse momentum dependent FFs: $e^+e^- \rightarrow (h)X$ $D_{1,q}^h(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRD 99 (2019) 112006 </div>	Transverse momentum dependent asymmetries $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2), Q_t$ $H_{1,q}^{\perp h}(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRD100 (2019) 92008 </div>	Polarizing Λ fragmentation <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRL 122 (2019), 042001 </div> $D_{1,q}^{\perp h}(z, k_T, Q^2)$



Dihadron FF (IFF)

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Dihadron cross sections $e^+e^- \rightarrow (hh)X$ $D_{1,q}^{h_1 h_2}(z, m, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRD96 (2017) 032005 </div>	Azimuthal asymmetries: $e^+e^- \rightarrow (hh)(hh)X,$ $\cos(\phi_1 + \phi_2),$ $H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> PRL107 (2011) 072004 </div>	Unpol SIDIS, pp: $\frac{d^2\sigma}{dzdm}$

Single hadron measurements

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
<p>Single hadron cross sections:</p> $e^+e^- \rightarrow hX$ $D_{1,q}^h(z, Q^2)$ <p>PRD 88 (2013) 032011 (Babar)</p>	<p>Azimuthal asymmetries:</p> $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2)$ $H_{1,q}^{\perp(1)h}(z, Q^2)$ <p>PRD 92 (2015) 111101 (Babar K) PRL 116 (2016) 042001 (BESIII)</p>	<p>Unpol SIDIS, pp: $\frac{d\sigma}{dz}$</p> $e^+e^- \rightarrow (h)(h)X$ <p>and scale dependence</p>
<p>Transverse momentum dependent FFs:</p> $e^+e^- \rightarrow (h)X$ $D_{1,q}^h(z, k_T, Q^2)$	<p>Transverse momentum dependent asymmetries</p> $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2), Q_t$ $H_{1,q}^{\perp h}(z, k_T, Q^2)$ <p>PRD 90 (2014) 052003 (Babar)</p>	

Dihadron measurements

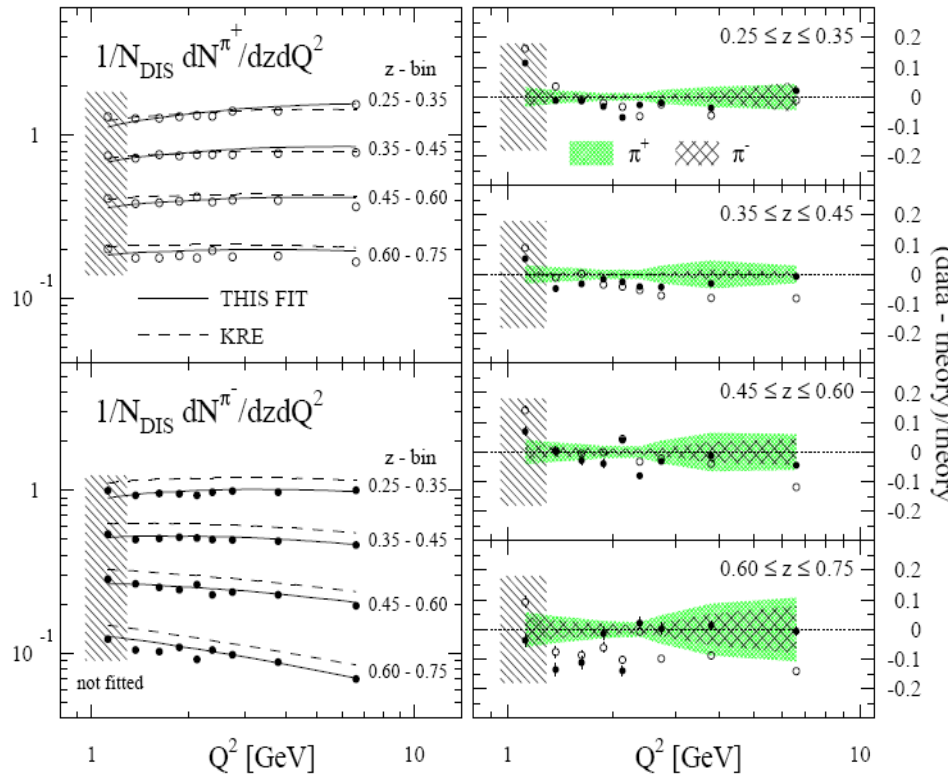
Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
<p>Dihadron cross sections</p> $e^+e^- \rightarrow (hh)X$ $D_{1,q}^{h_1 h_2}(z, m, Q^2)$	<p>Azimuthal asymmetries:</p> $e^+e^- \rightarrow (hh)(hh)X,$ $\cos(\phi_1 + \phi_2),$ $H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$ <p>Ralf Seidl: Fragmentation</p>	<p>Unpol SIDIS, pp:</p> $\frac{d^2\sigma}{dzdm}$

Light hadron Fragmentation functions

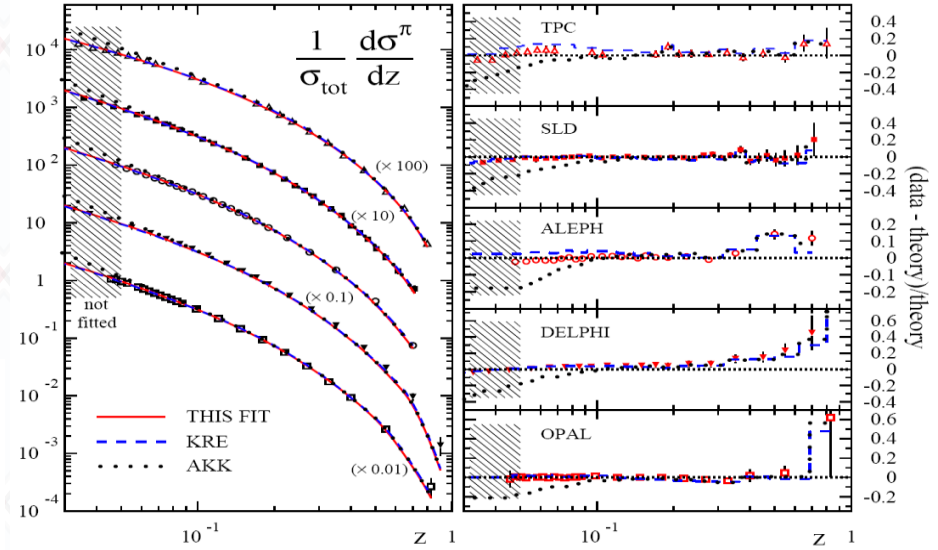
de Florian, Sassot, Stratmann

- results for π^\pm, K^\pm , chg. hadrons
- full flavor separation for $D_i^H(z)$ and D_g^H
- fits to all LEP, HERMES, SMC, RHIC, ... data
- supersede old fits based only on e^+e^- data

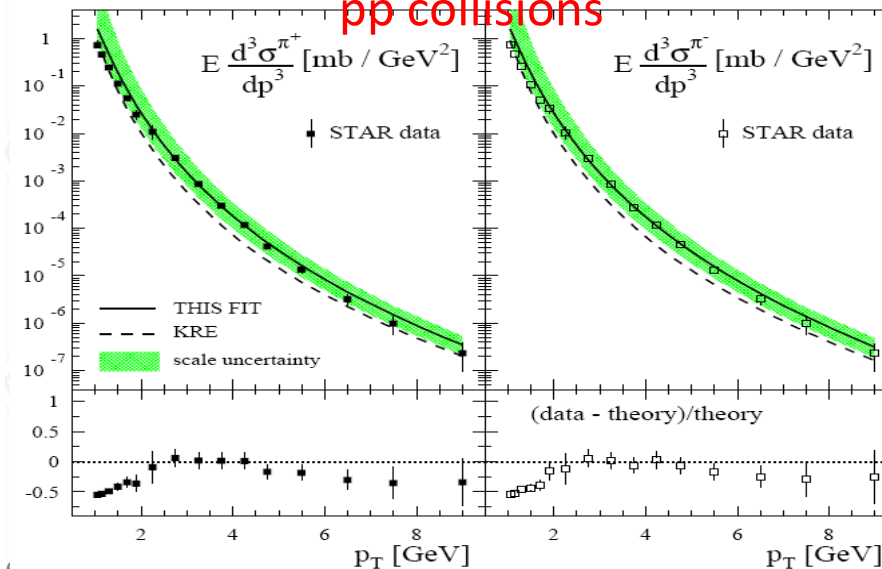
SIDIS



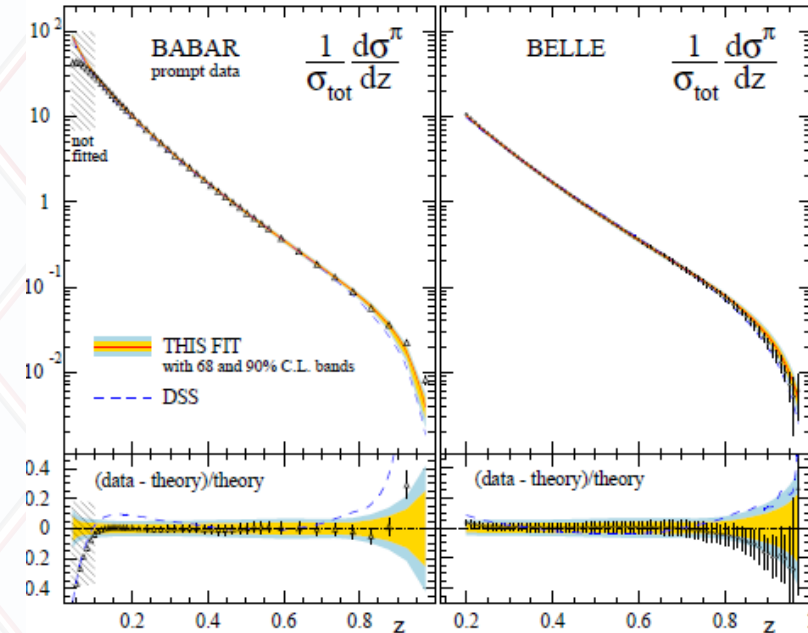
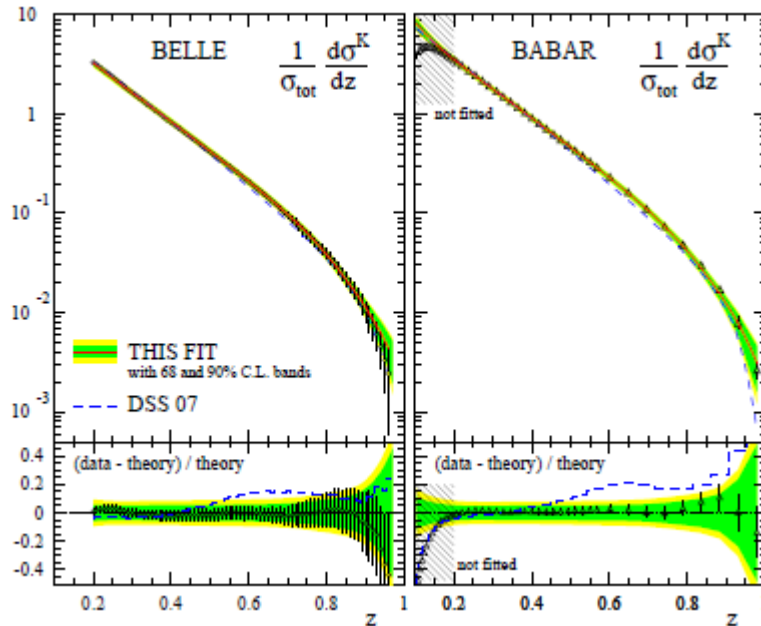
e^+e^-



pp collisions



B factory data ($Q \sim 10\text{GeV}$)

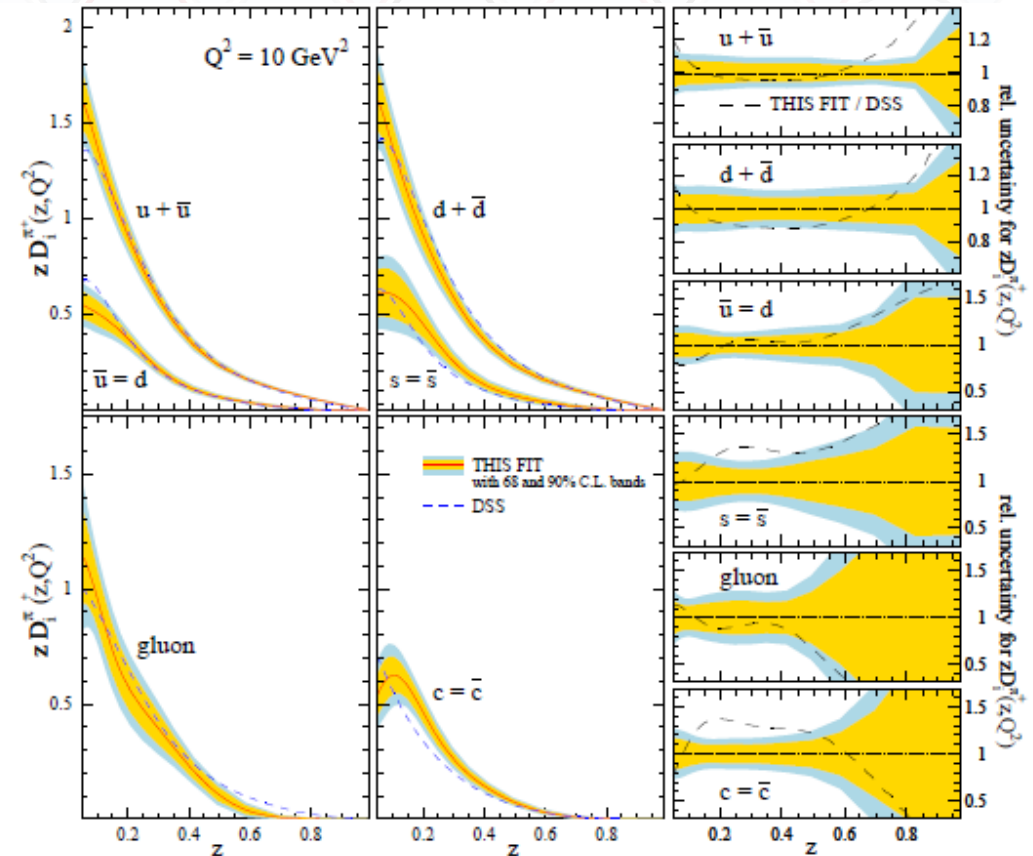


- High precision pion and kaon data from both B factory experiments
- Precision up to very high z
- Lever arm to much higher energy ($Q \sim 20 - 200\text{GeV}$) data allows for determination of gluon fragmentation over evolution

Pion fragmentation

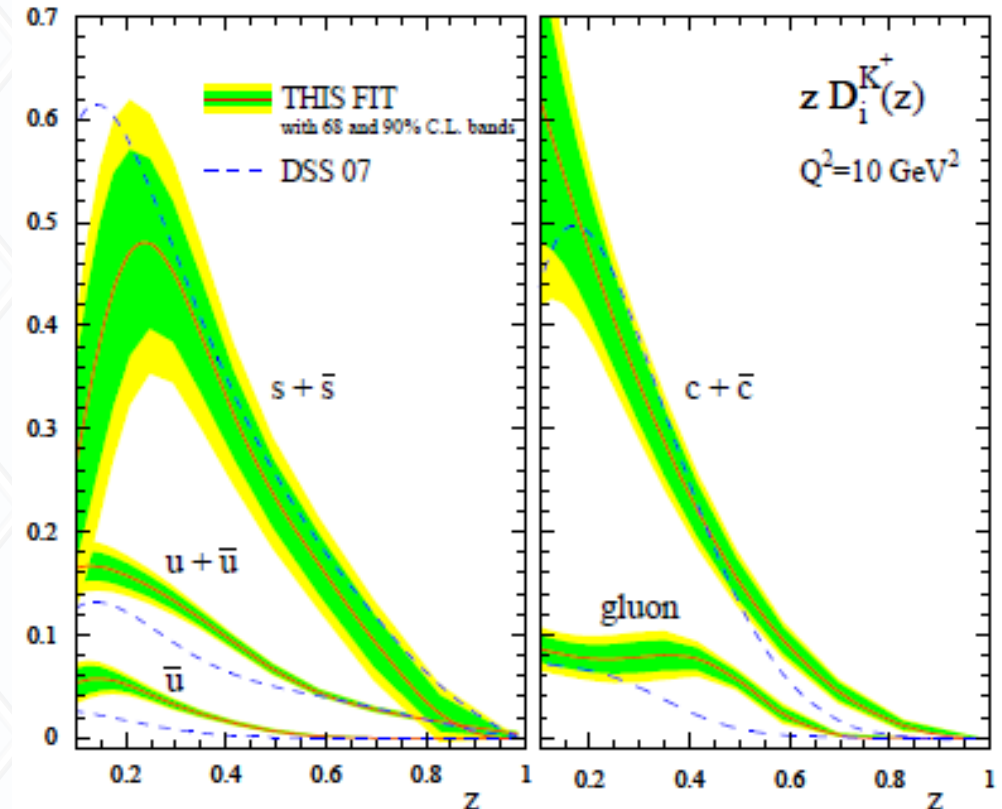
DSS15: [deFlorian et.al., Phys.Rev. D91 \(2015\) 014035](#)

- Light quarks symmetric
- Dominated by favored fragmentation especially at high z
- Gluon substantial but falling off faster than quarks



Kaon fragmentation

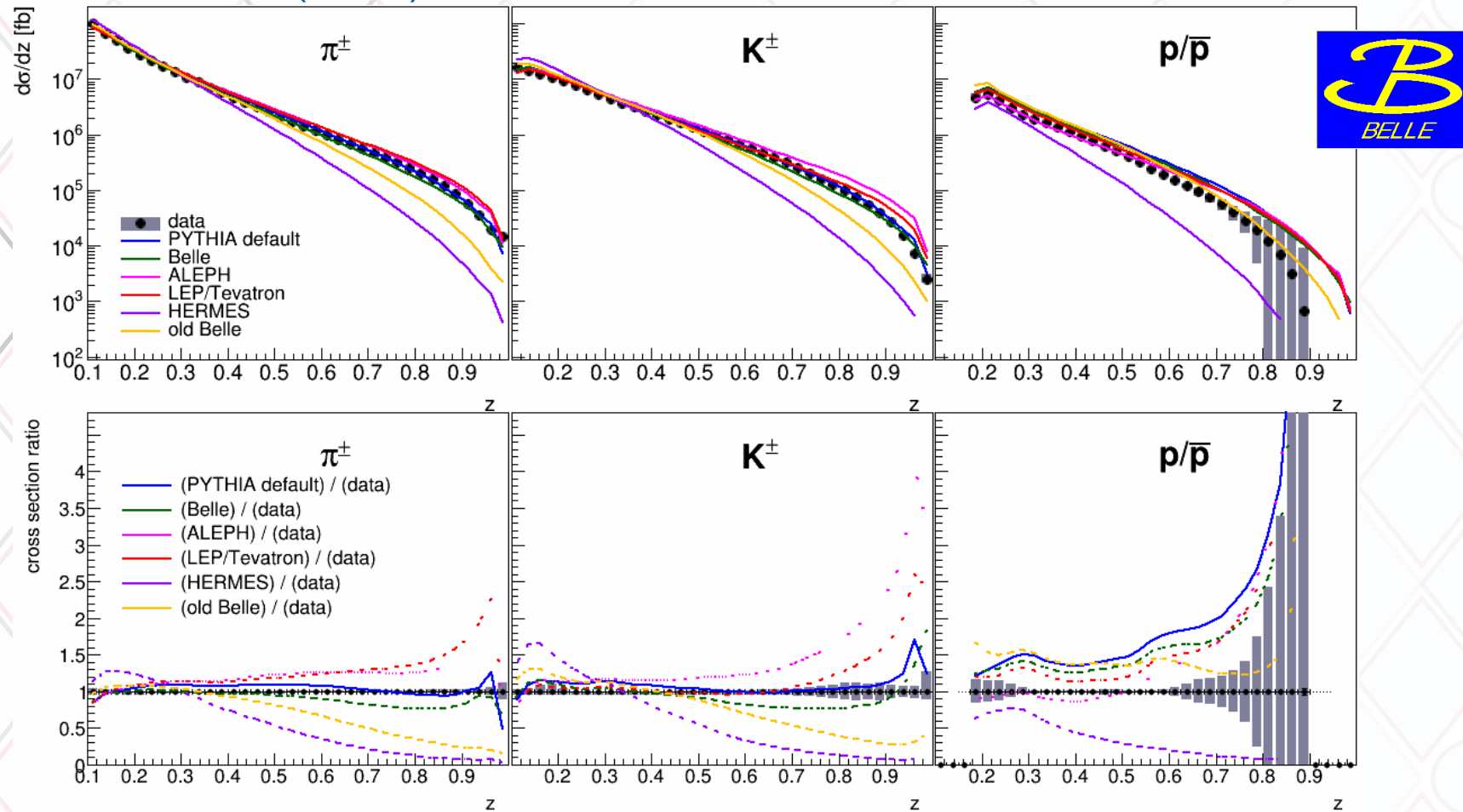
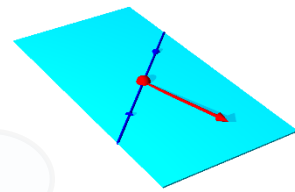
- Strange quarks are dominating kaon fragmentation
- Also dominated by favored u quark fragmentation at high-z
- At lower z penalty for producing $s\bar{s}$ pair in fragmentation ($u+\bar{u} < s+\bar{s}$)
- Charm fragmentation comparable (what about weak decays?)



[DEHSS *Phys.Rev.D* 95 \(2017\) 9, 094019](#)

Unpolarized single hadrons

PRD 101 (2020) 092004

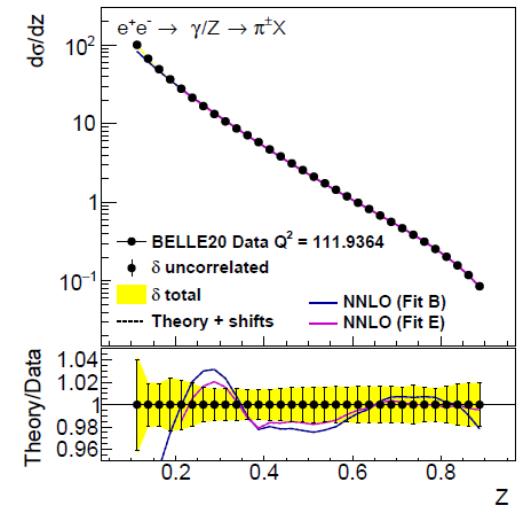
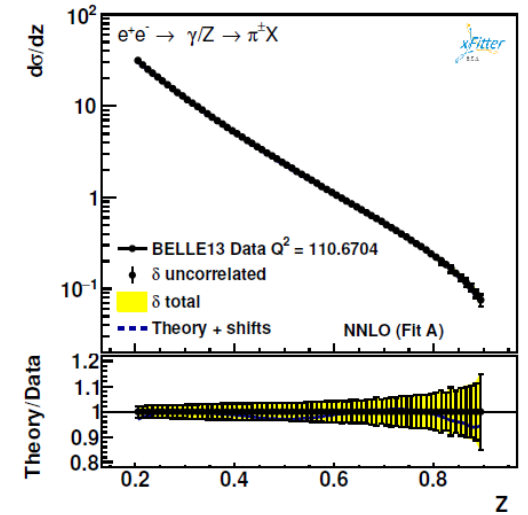
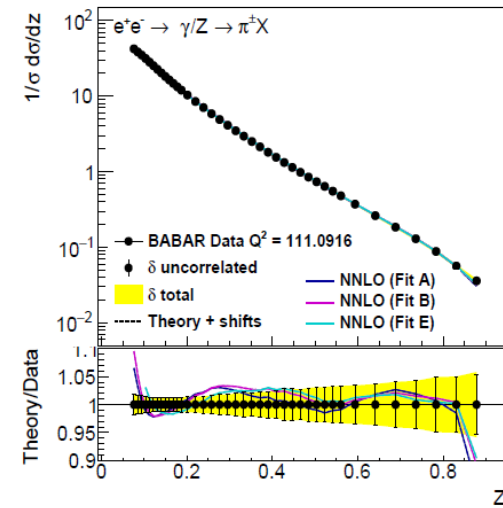


- Update with better ISR correction
- Correlated and uncorrelated uncertainties separated \rightarrow improve global unpolarized FF fits

From “your errors are too conservative” to “your errors are too precise”

One group: “However we do not consider it because of a poor control of the degree of correlation of systematic uncertainties”

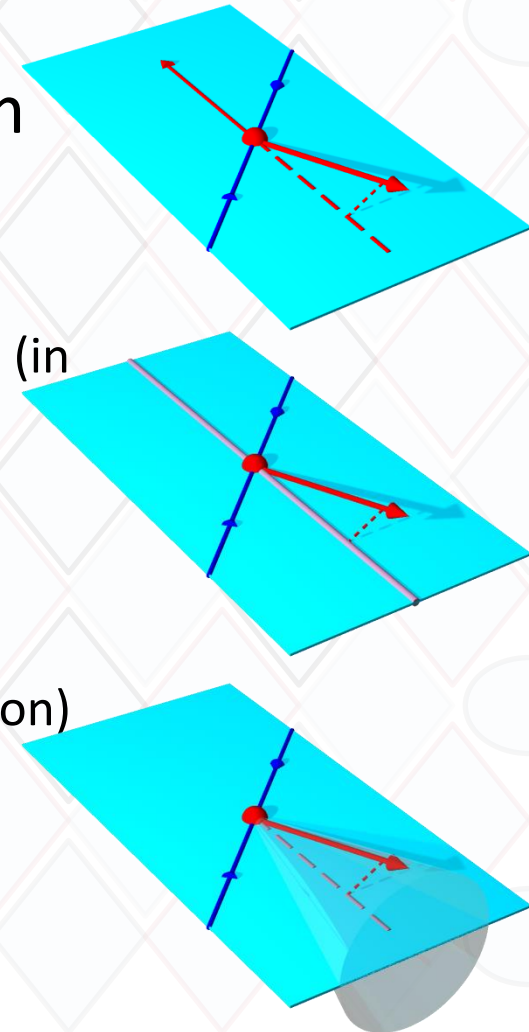
- Initial single hadron cross section measurement in very fine z binning, thus large bin-to-bin migration. Unfolding performed but assigned very conservative uncertainties \rightarrow global fit's χ^2 generally too low for our data set
- Recent update ('20) with more realistic binning, much better understanding of all systematic uncertainty sources, correlated and uncorrelated uncertainties provided separately
- However, fitters would prefer:
 - all systematics separately (will be done in the future),
 - all systematics symmetric (INCORRECT!)



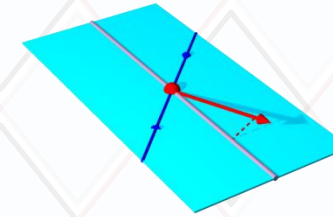
xFitter: [Phys.Rev.D 104 \(2021\) 056019](#)

K_T Dependence of FFs in e^+e^-

- Gain also sensitivity into transverse momentum generated in fragmentation
- Two ways to obtain transverse momentum dependence
 - Traditional 2-hadron FF
 - use transverse momentum between two hadrons (in opposite hemispheres)
 - Usual convolution of two transverse momenta
 - Single-hadron FF wrt to **Thrust** or jet axis
 - No convolution
 - Need correction for $q\bar{q}$ axis (similar to a Jet function)



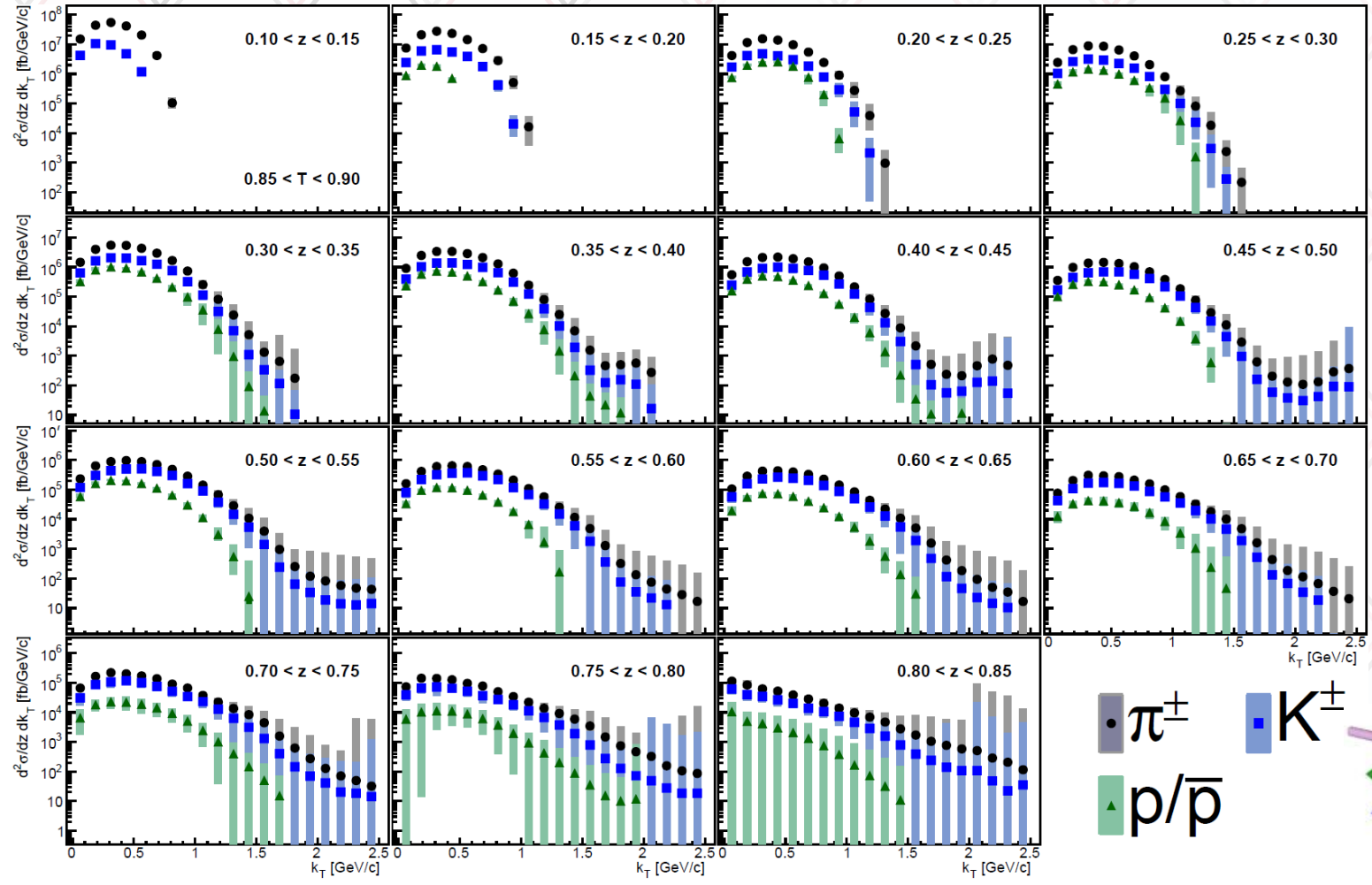
Transverse momentum dependent cross sections for pions, kaons and protons



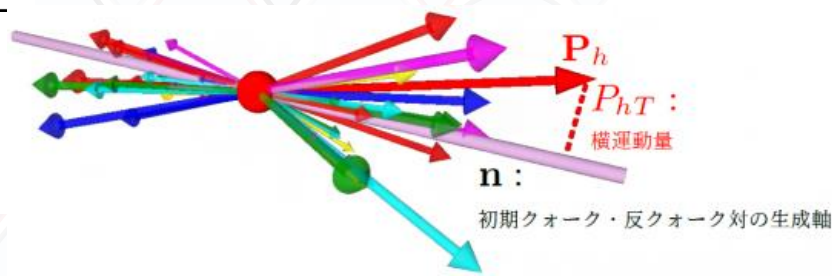
Important baseline for most transverse momentum/spin dependent measurements at RHIC and EIC

RIKEN Press release:
https://www.riken.jp/press/2019/20190615_1/

$$\frac{d^2\sigma}{dzdP_{hT}}$$



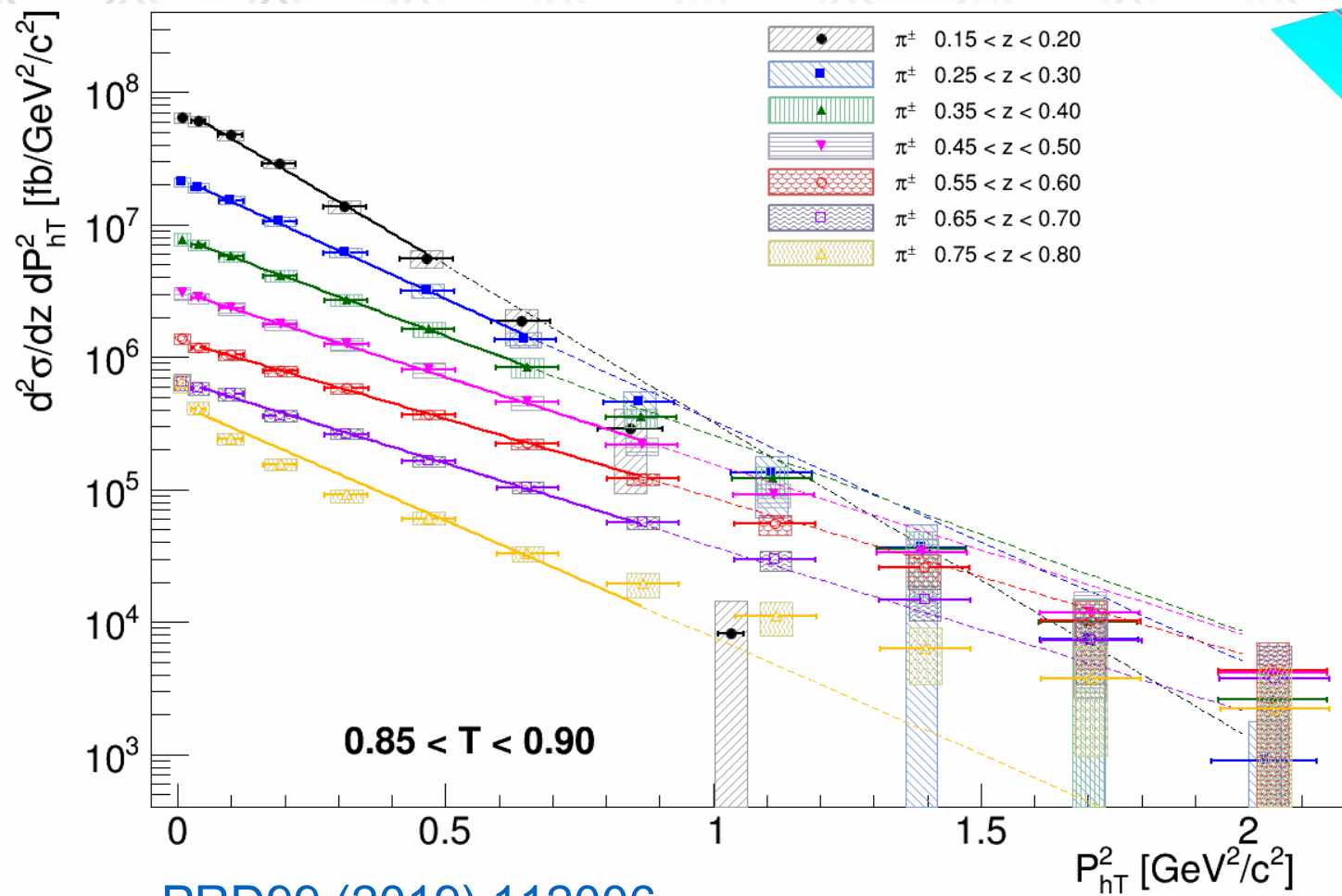
π^\pm
 K^\pm
 p/\bar{p}



RS. et. al. [PRD99 \(2019\) 112006](#)

Fits vs P_{hT}^2

Fit exponential to smaller transverse momenta for
Gaussian P_{hT} dependence and power law at higher P_{hT}



[PRD99 \(2019\) 112006](#)

Gaussian widths comparison to MC

first direct (no convolutions) measurement of z dependence of Gaussian widths

MSTP(21):

0.28

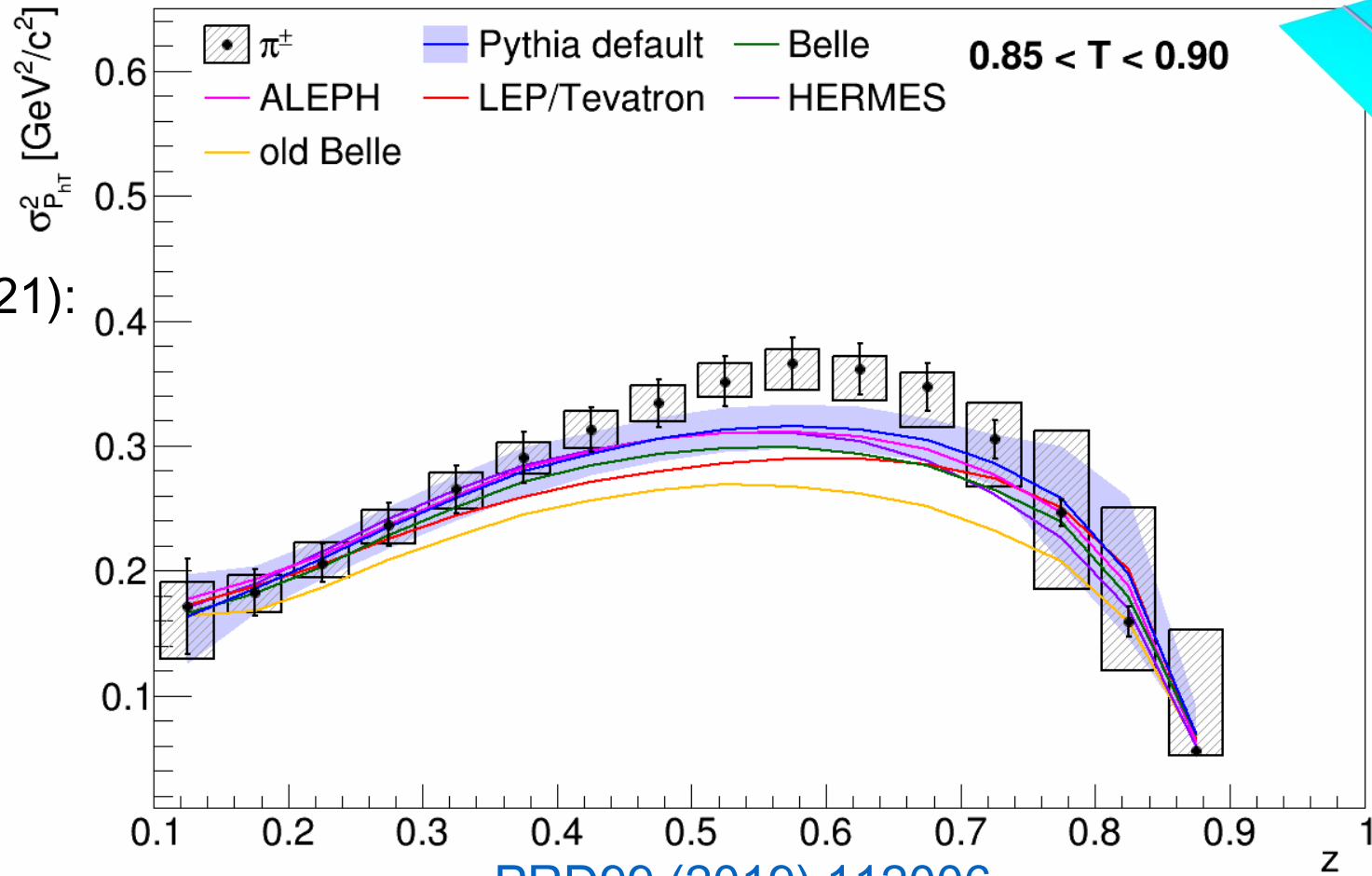
0.325

0.36

0.36

0.37

0.40



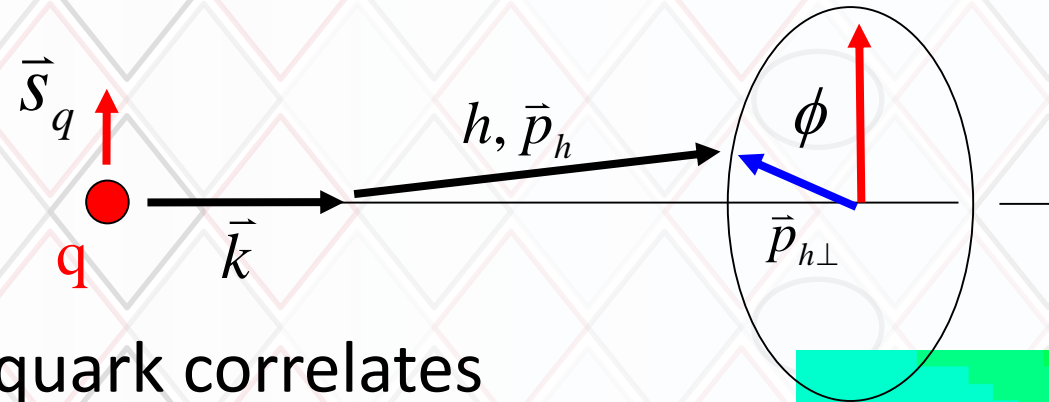
$0.85 < T < 0.90$

[PRD99 \(2019\) 112006](#)

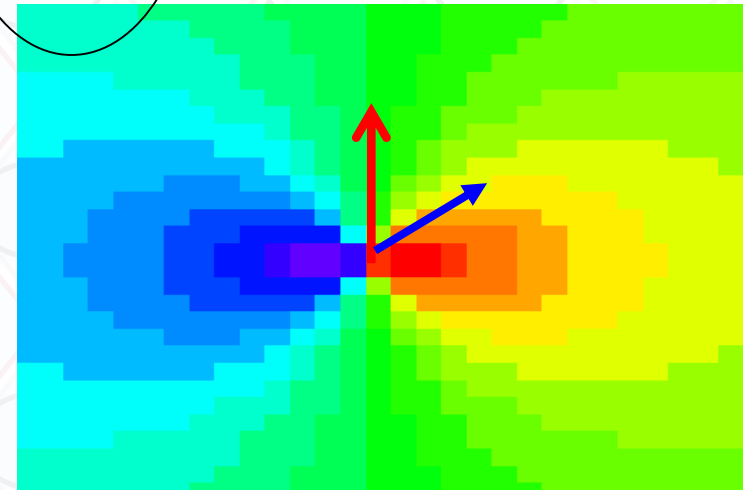
Collins fragmentation function

J. Collins, Nucl. Phys. B396, (1993) 161

$$D_{q\uparrow}^h(z, P_{h\perp}) = D_{1,q}^h(z, P_{h\perp}^2) + H_{1,q}^{\perp h}(z, P_{h\perp}^2) \frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{zM_h}$$



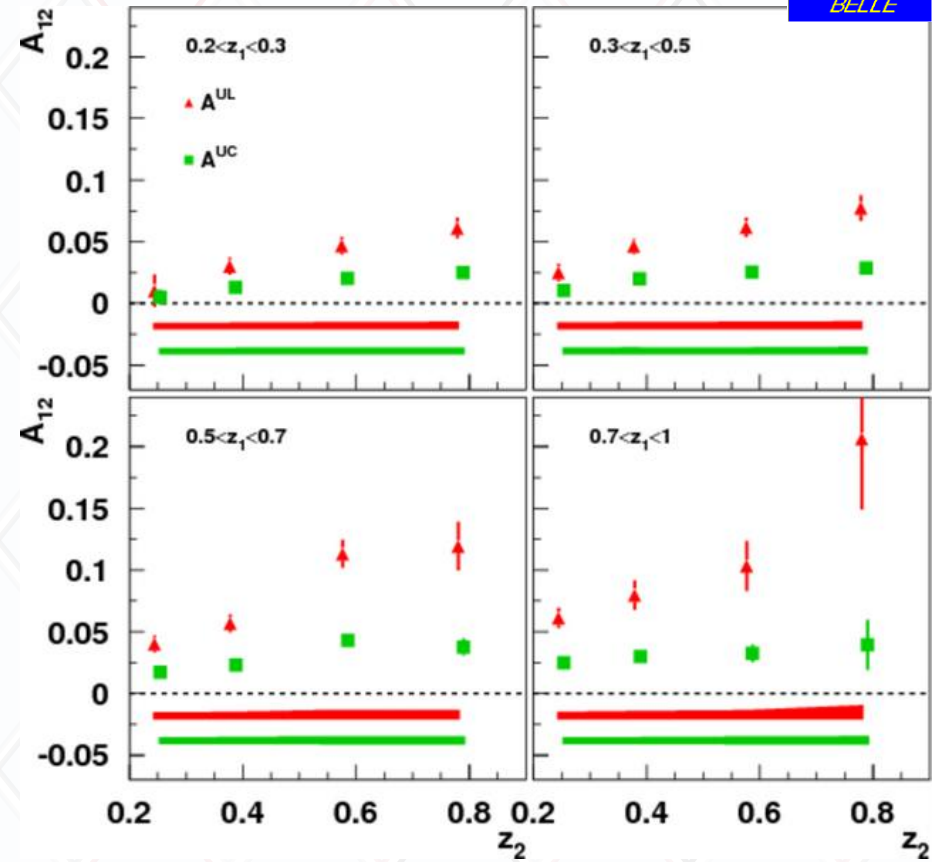
- Spin of quark correlates with hadron transverse momentum
- translates into azimuthal anisotropy of final state hadrons



Belle Collins asymmetries



- **Red points** : $\cos(\phi_1 + \phi_2)$ moment of **Unlike** sign pion pairs over **like** sign pion pair ratio : A^{UL}
- **Green points** : $\cos(\phi_1 + \phi_2)$ moment of **Unlike** sign pion pairs over **any charged** pion pair ratio : A^{UC}
- Collins fragmentation is large effect
- Consistent with SIDIS indication of sign change between favored and disfavored Collins FF

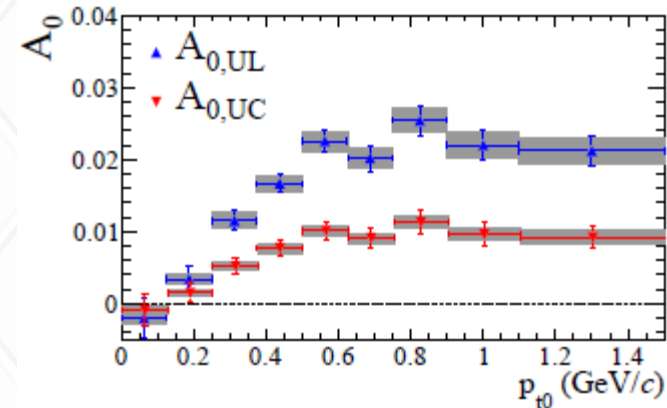
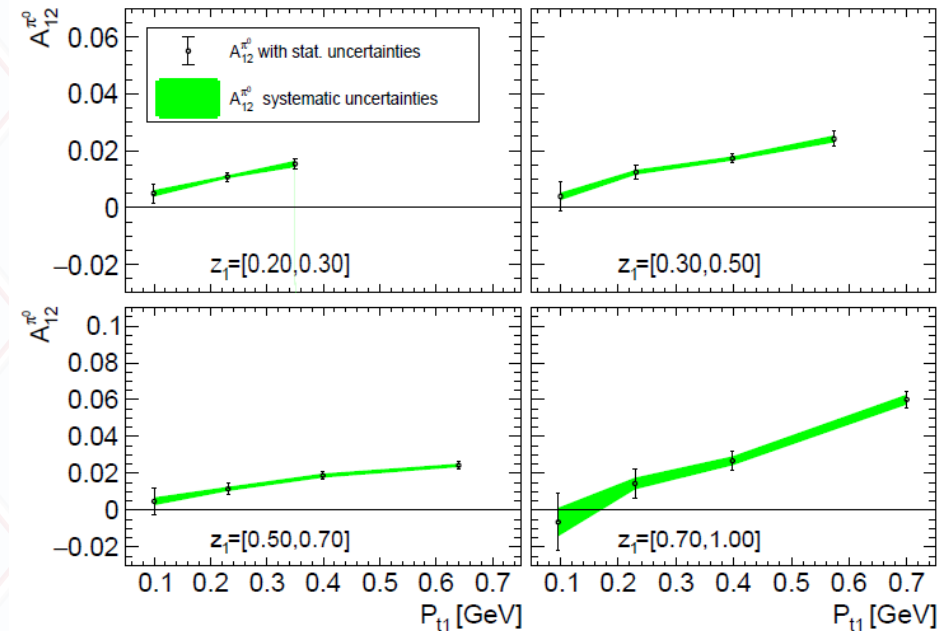


RS et al (Belle), PRL96: 232002
PRD 78:032011, Erratum D86:039905

Transverse momentum

- Add transverse momentum to Collins asymmetries' z dependence
- Currently only 1 or 2-dimensional extractions available ($q_t, z_1 \times z_2, p_{t1} \times p_{t2}, z_1 \times p_{t1}$)
- Increasing asymmetries with both z and p_t , but p_t reach limited
- Multidimensional extractions needed

PRD100 (2019) 92008

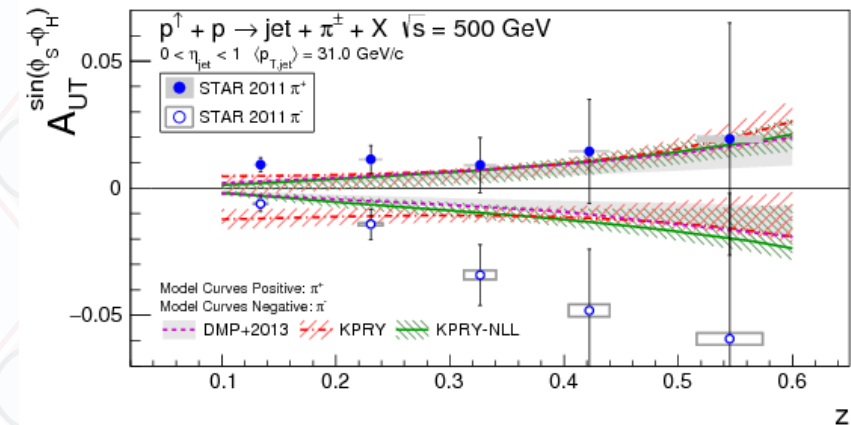
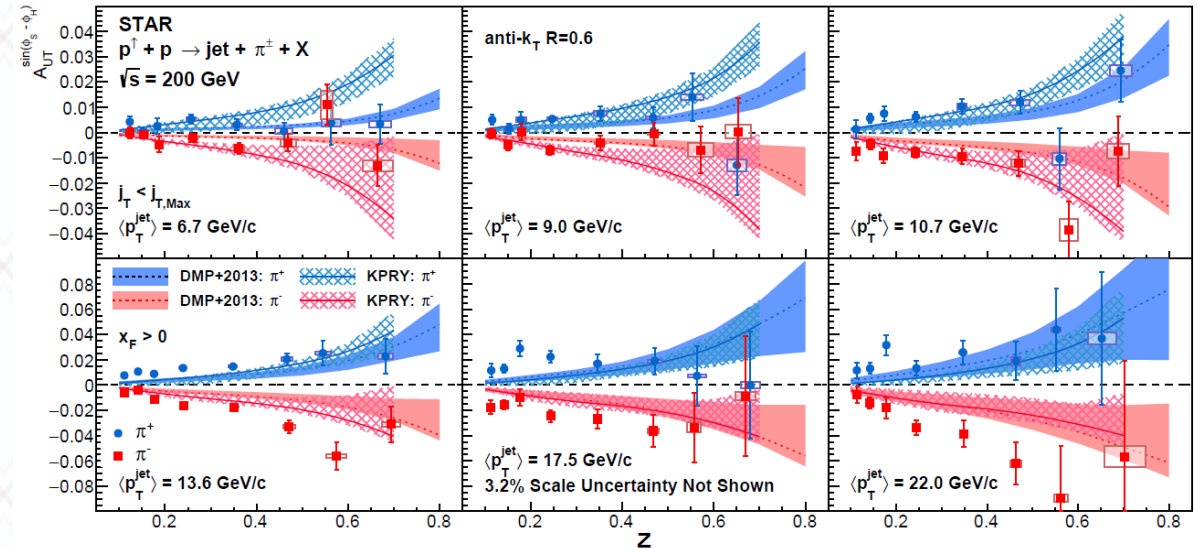


PRD 90 (2014) 052003 (Babar)

Transversity in proton collisions

- Nonzero Collins asymmetries (hadron in jets) at central rapidities at 200 and 500 GeV
- Substantial theoretical progress for hadron in jet measurements
 - unpolarized: Kaufmann et al.
 - polarized Kang et al.
- For roughly same x and k_t similar size \rightarrow evolution effects moderate?
- But generally slightly larger than global fits from SIDIS/e+e-
- More to come from sPHENIX in near future

STAR: [Phys.Rev.D 106 \(2022\) 072010, 2022](#)

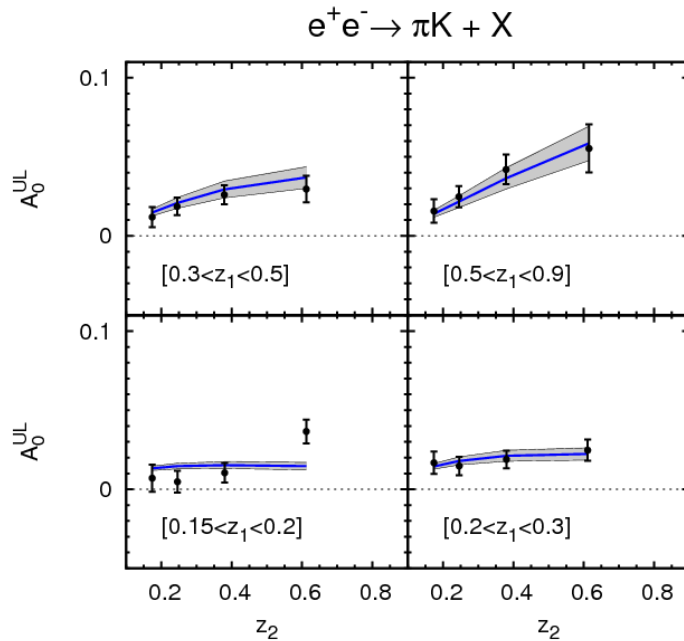


STAR: [Phys.Rev.D 97 \(2018\) 032004](#)

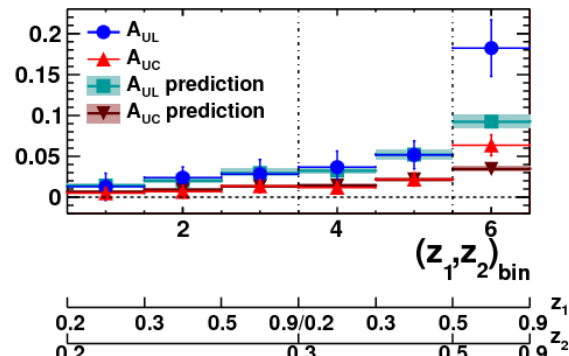
Quark transversity via Collins: Kaons

BABAR: [PRD 92 \(2015\) 111101](#)

Anselmino et al: [PRD 93 \(2016\) 034025](#)



BESIII: [PRL 116 \(2016\) 042001](#)

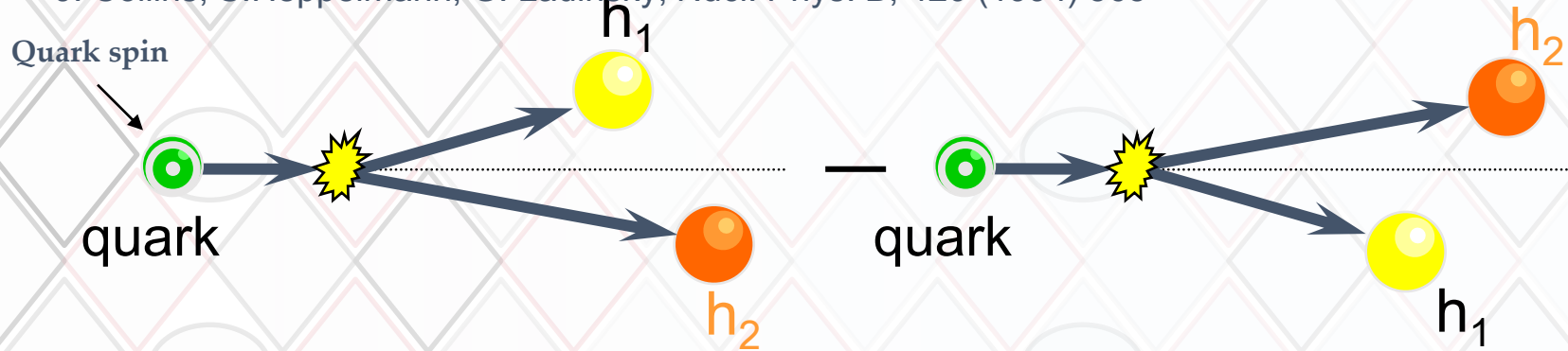


- Addition of kaon Collins fragmentation strongly needed for flavor decomposition of quark transversity
- Large amount of potentially participating FFs well described by light and “heavy” favored and disfavored FFs
- Allows inclusion of HERMES and COMPASS kaon asymmetries (+eventually EIC) in fits
- Also: pion Collins at lower scale (BESIII) consistent with TMD evolution

Interference fragmentation as alternative Transversity channel

Interference fragmentation function H_1^{\triangleleft}

J. Collins, S. Heppelmann, G. Ladinsky, Nucl. Phys. B, 420 (1994) 565

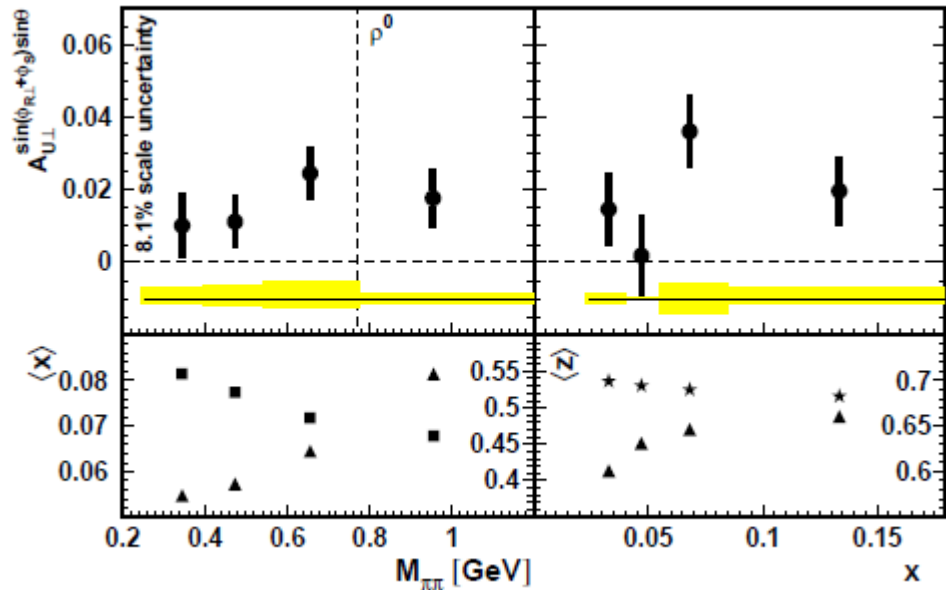


- Di-hadron vs single hadron

- Collinear factorization is shown to be valid \rightarrow TMD factorization is less certain in $p+p$ (Rogers, Mulders, arXiv:1010.2977)
- No model uncertainties from transverse momentum dependence of FF and PDF
- No need to separate Sivers/Collins effects as in single hadron measurement
- Completely independent measurement
- Doesn't need jet reconstruction
- Evolution is known
- But unpolarized and polarized di-hadron fragmentation functions needed

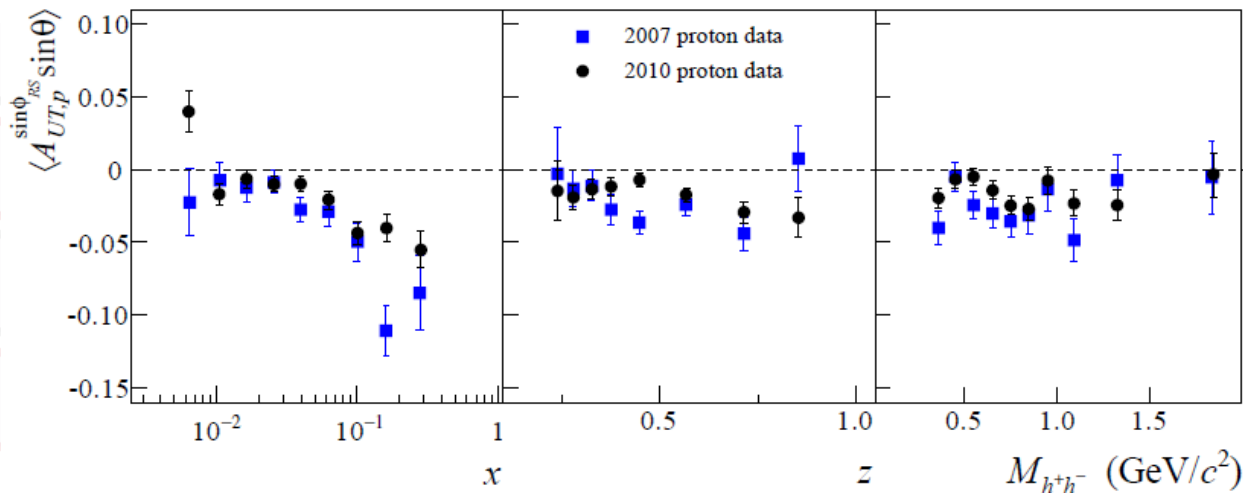
SIDIS IFF measurements

HERMES: [JHEP 0806, 017 \(2008\)](#)

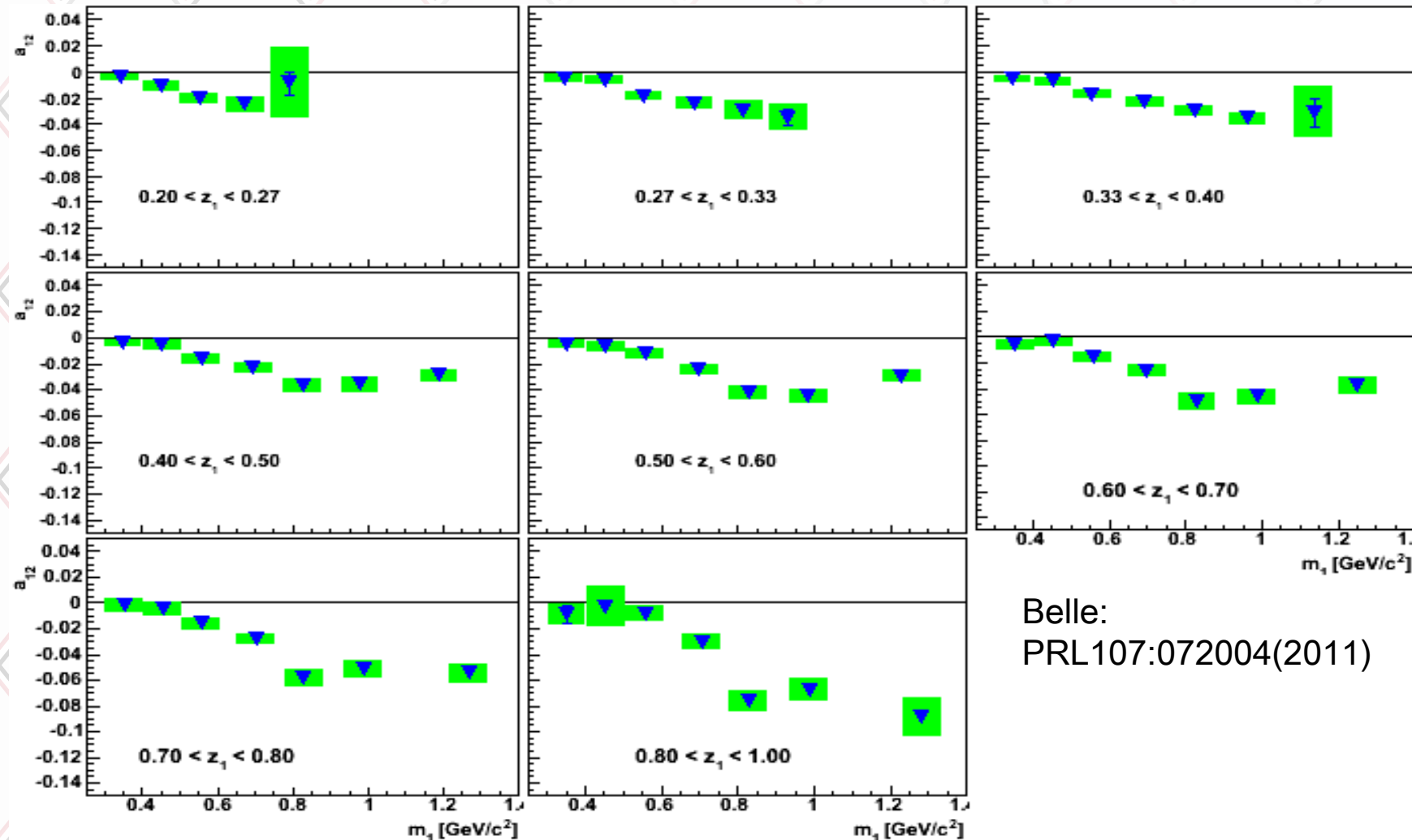


- Again sizeable asymmetries in SIDIS seen from HERMES and COMPASS
- Consistent with each other

COMPASS: [Phys.Lett. B736 \(2014\)](#)



Also Belle fragmentation measurements available

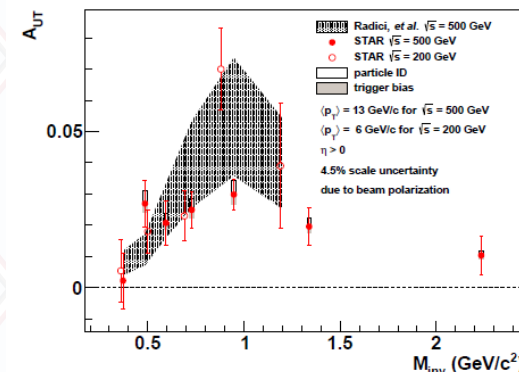
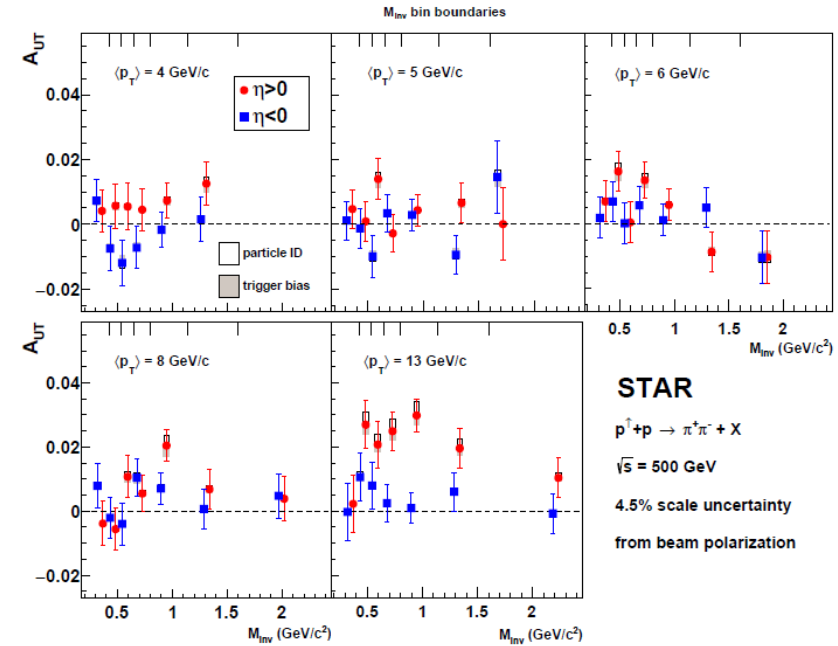


Belle:
PRL107:072004(2011)

STAR IFF results

- Now both 200 and 510 GeV results finalized
- Both with substantial nonzero effects at:
 - Forward rapidities
 - Higher Pt
 - Masses around 1 GeV
- First theory predictions from SIDIS+Belle consistent with magnitude
 - will help improve transversity uncertainties
 - but gluon DIFFs not well known

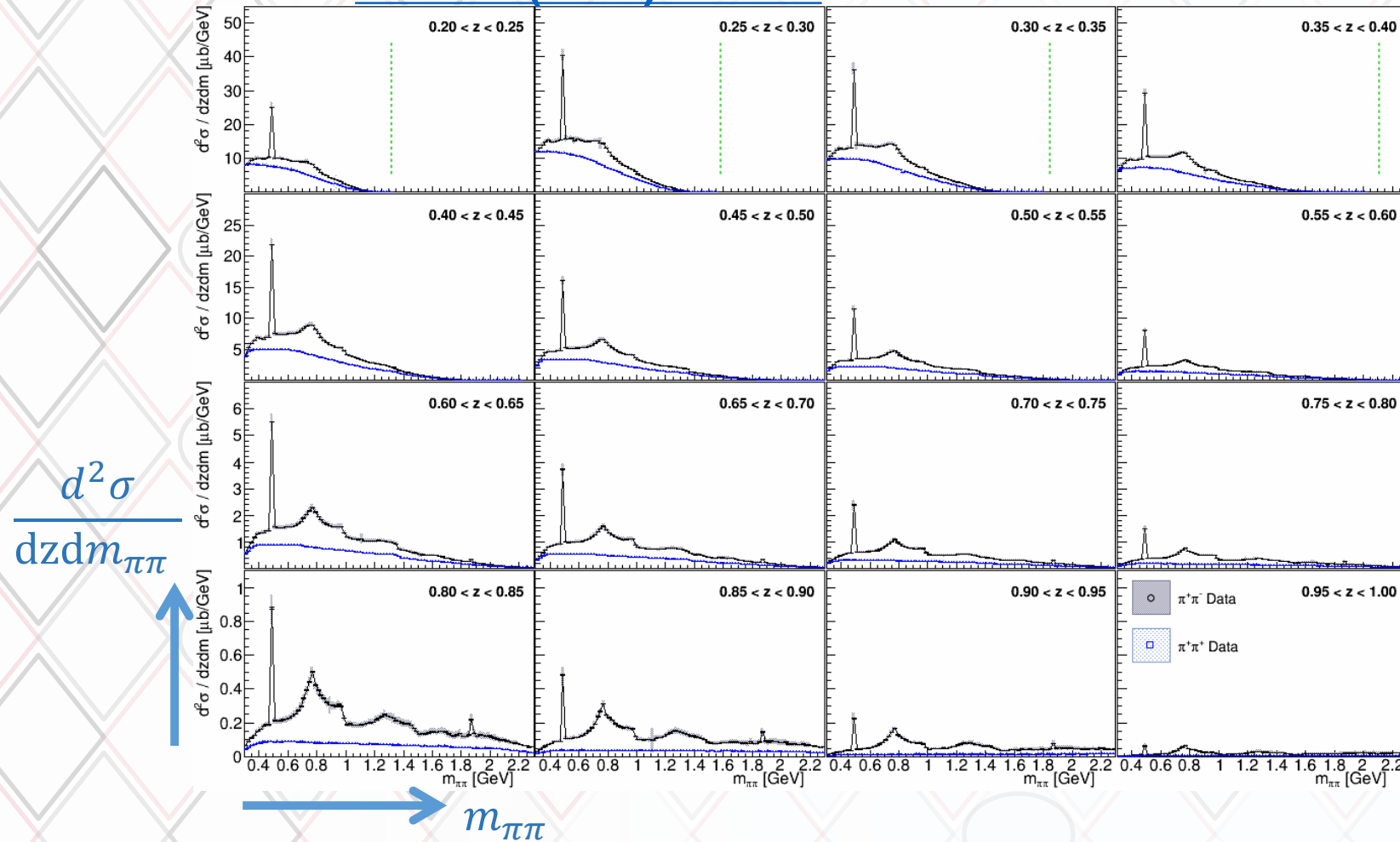
STAR 510 GeV: [PLB780 \(2018\) 332](#)
 200GeV: [PRL 115 \(2015\) 242501](#)



Radici et al: [PRD97 \(2018\) 074019](#)

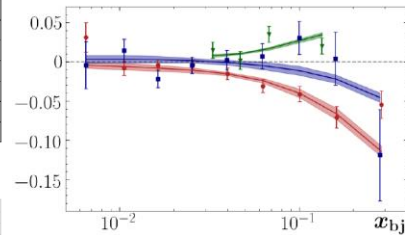
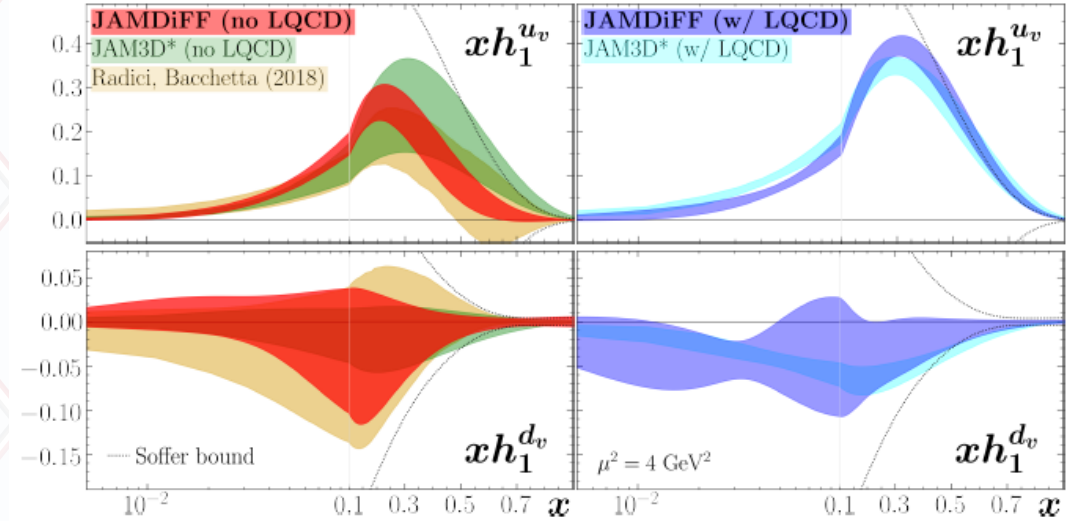
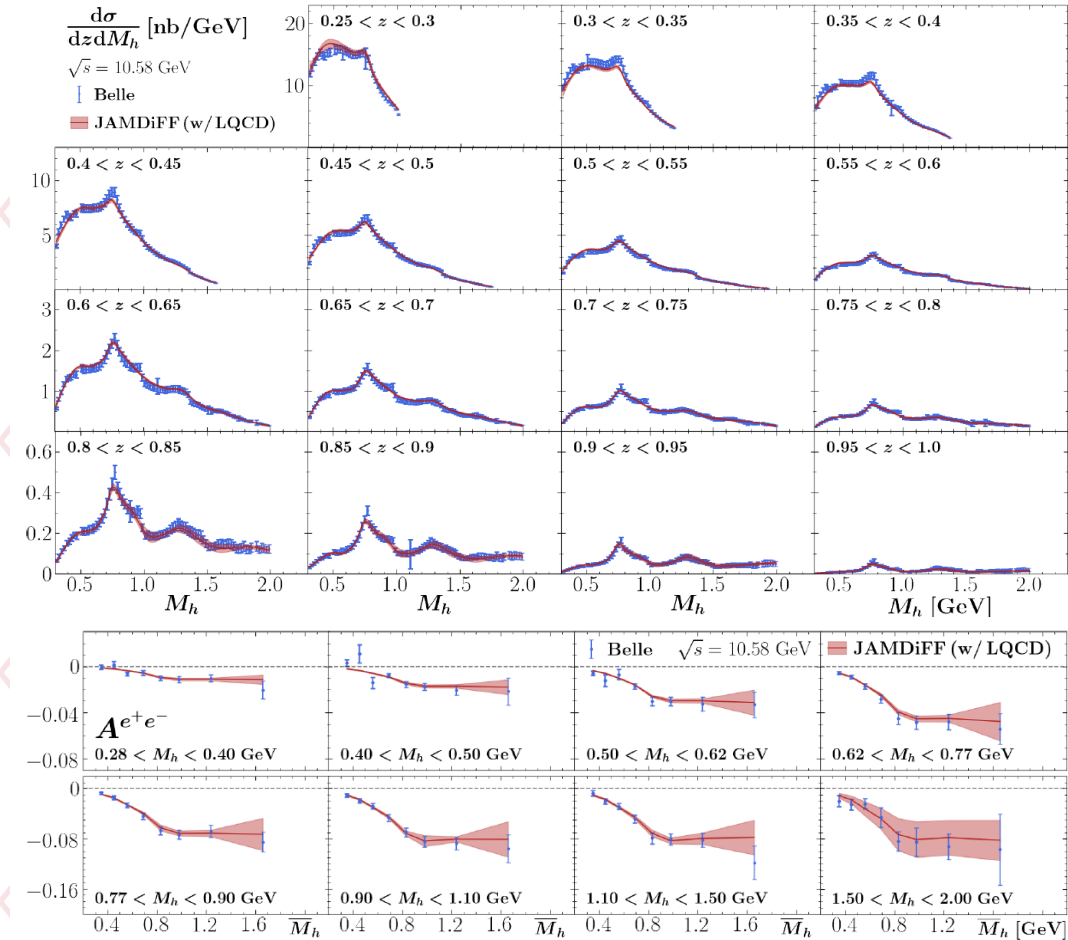
Di-hadron FF mass dependence (same hemisphere)

Belle: RS et.al. [PRD96 \(2017\) 032005](#)



- Important input for IFF based transversity global analysis
- Individual resonances, etc quite visible; interesting for FF in itself (→soon)

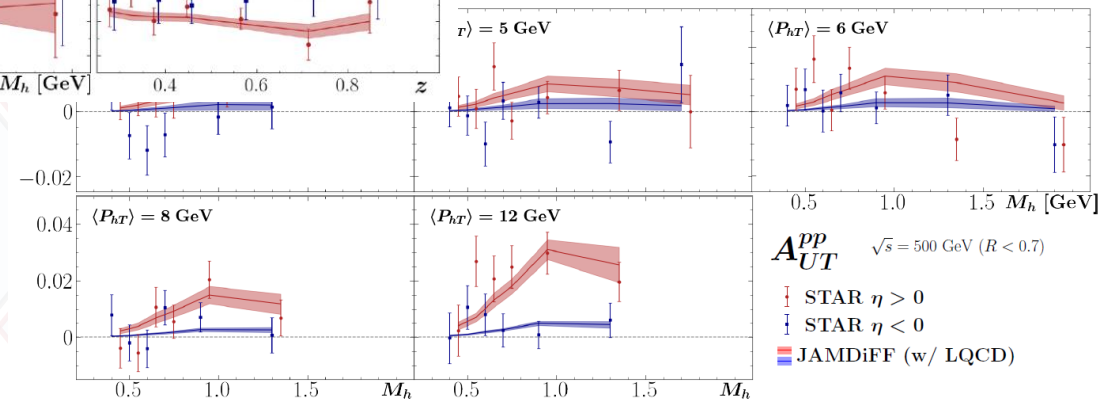
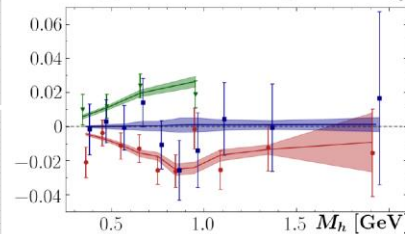
Global fits of Dihadron FFs



Cocuzza et al.

<https://arxiv.org/abs/2306.12998>

Also Radici et al. *PRL* 120 (2018) 19, 192001



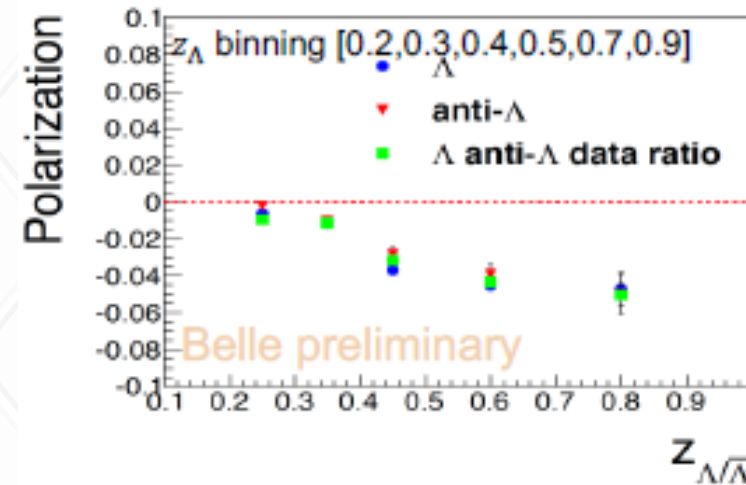
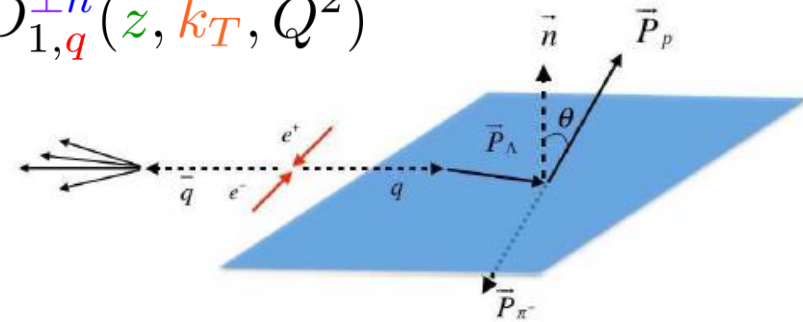
- Global fits taking SIDIS data, RHIC data, Belle data (polarized and unpolarized) into account

Single Λ polarization measurements

- Related to open question about Λ polarization in hadron collisions from 40 years ago!
- Fragmentation counterpart to the Sivers Function:
 - unpolarized parton fragments into transversely polarized baryon with transverse momentum wrt to parton direction
- Reconstruct Λ , its transverse momentum and polarization

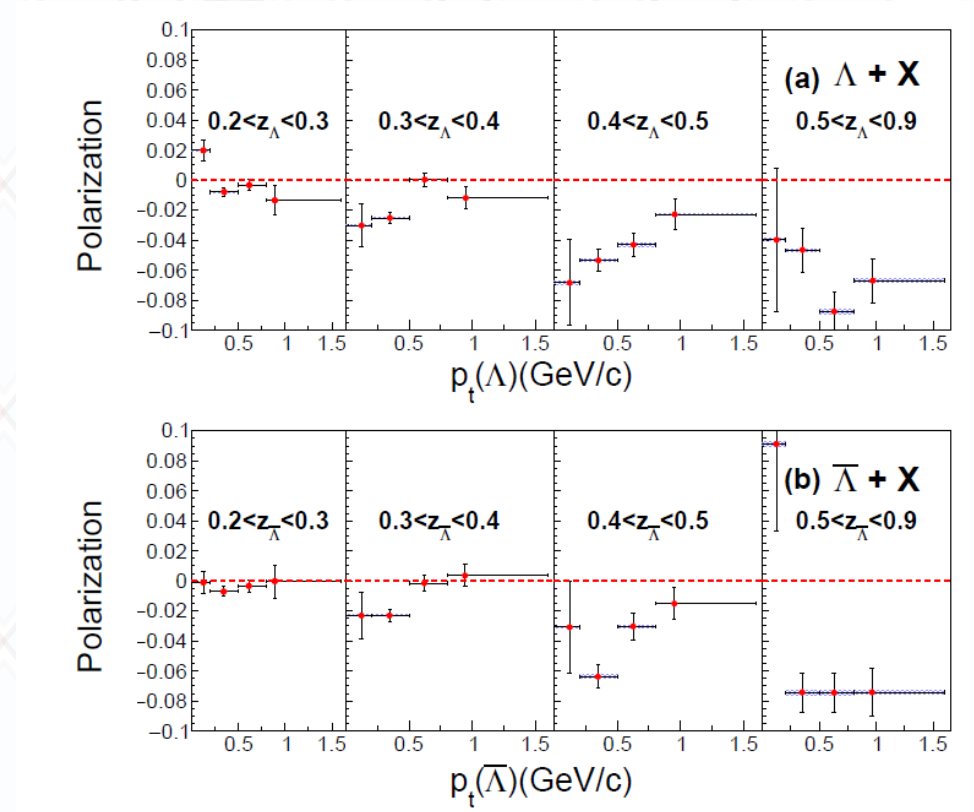
YingHui Guan (Indiana/KEK):
[PRL 122 \(2019\), 042001](#)

$$D_{1,q}^{\perp h}(z, k_T, Q^2)$$



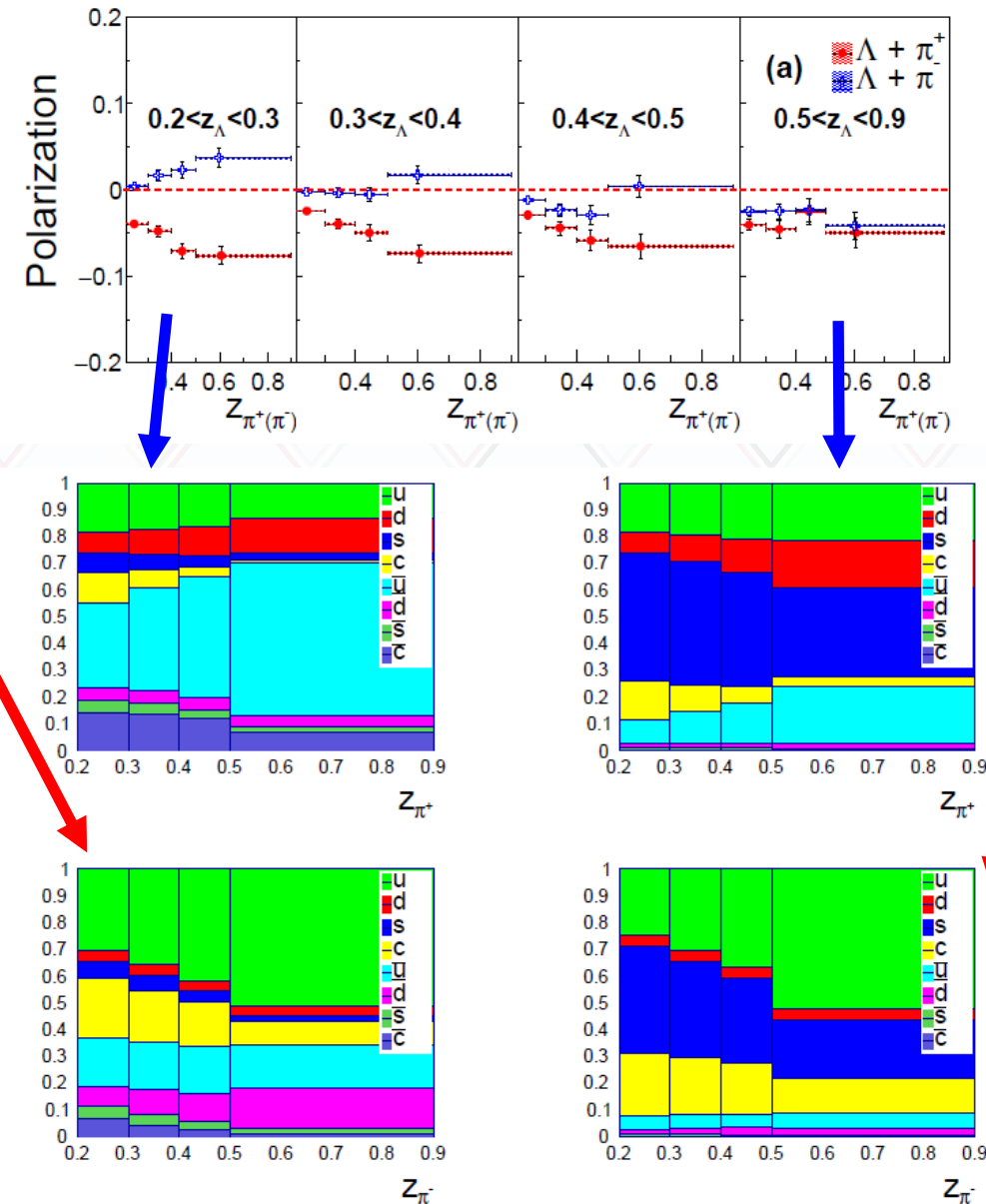
Transverse momentum dependence

- Different behavior for low and high- z :
- At low z small
- At intermediate z falling Polarization with P_t
- At high z increasing polarization with P_t



Opposite hemisphere pion correlation

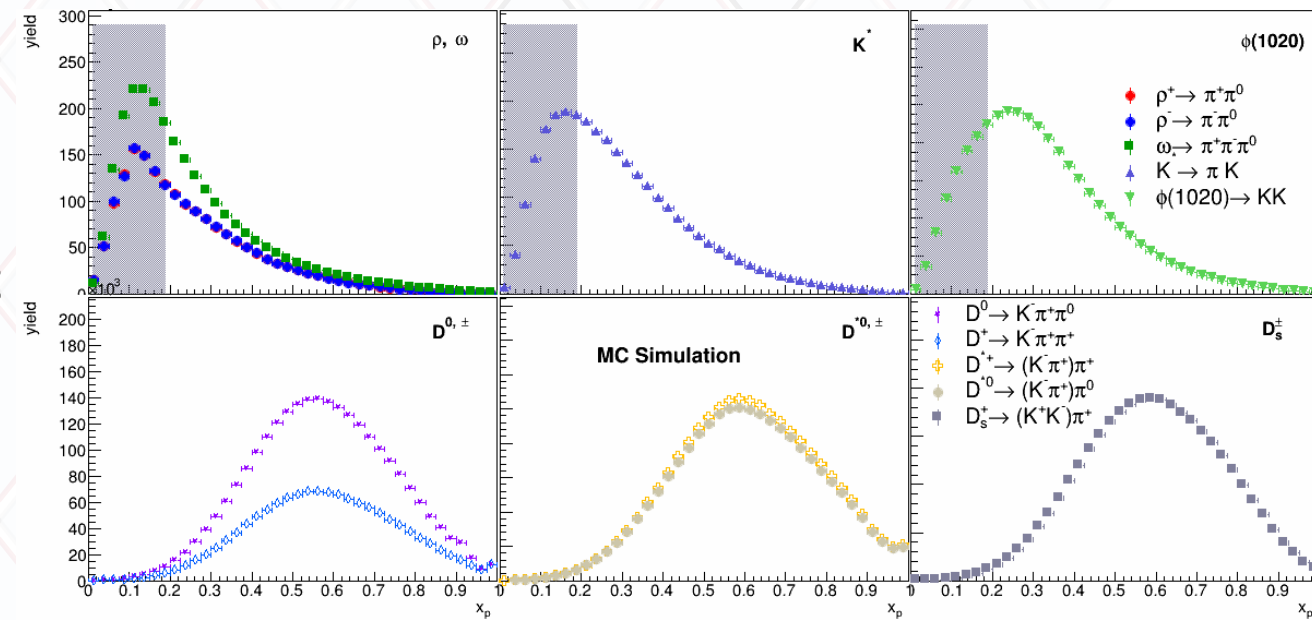
- Interesting z_π and z_Λ dependence :
- At low z_Λ light quark fragmentation dominant, some charm in $\pi^- \rightarrow$ different signs
- At high z_Λ strange + charm fragmentation more relevant \rightarrow same signs
- Several fits to data with slightly different results



Ongoing: Decaying particle FFs

- Study the explicit differential cross sections for VMs, D mesons as a function of x_p
- Mostly mass distributions and fits well-behaved, except for ρ – ω (interference) and more exotic resonances
- Also of interest for ultra high-energetic cosmic ray air shower research (muon problem)

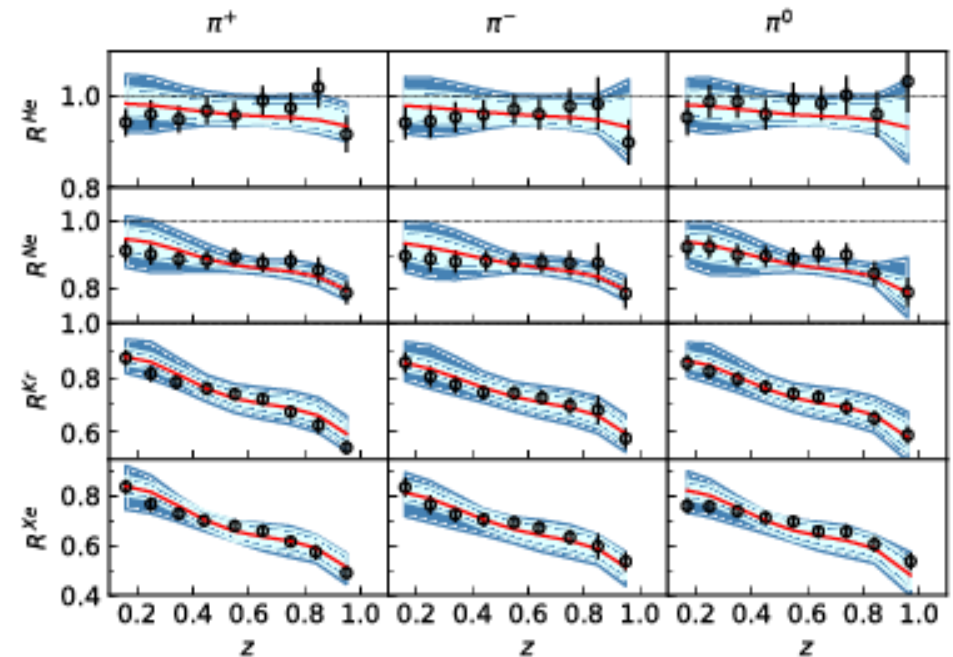
- Example from MC at Belle energies (for 4π acceptance):



Important resource for EIC, RHIC and HI physics

Nuclear Fragmentation functions

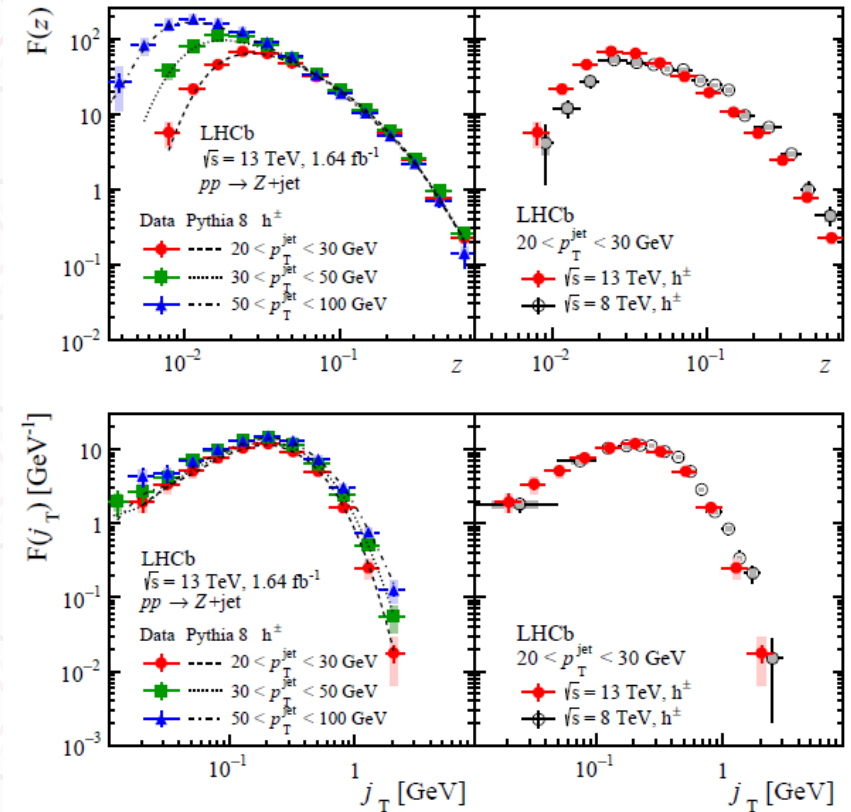
- Revisiting global nFF analysis using xFitter at NLO
- Good description of HERMES data
- predictions differ from the low-scale preliminary CLAS data



[Zurita: arxiv2101.01088](https://arxiv.org/abs/2101.01088)

Other venues to access fragmentation: h in jet

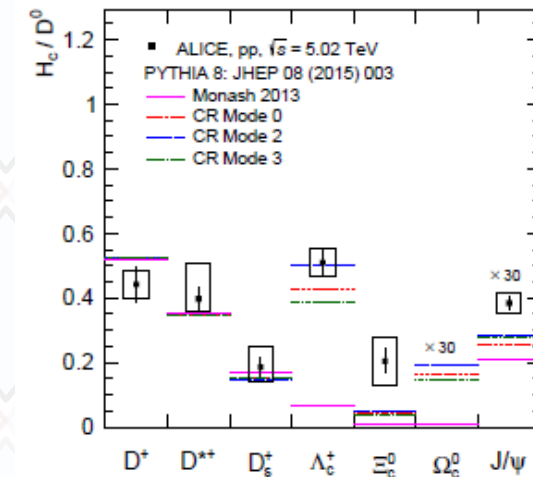
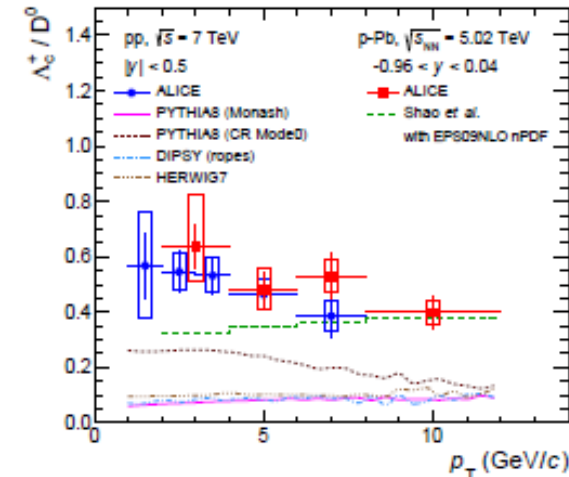
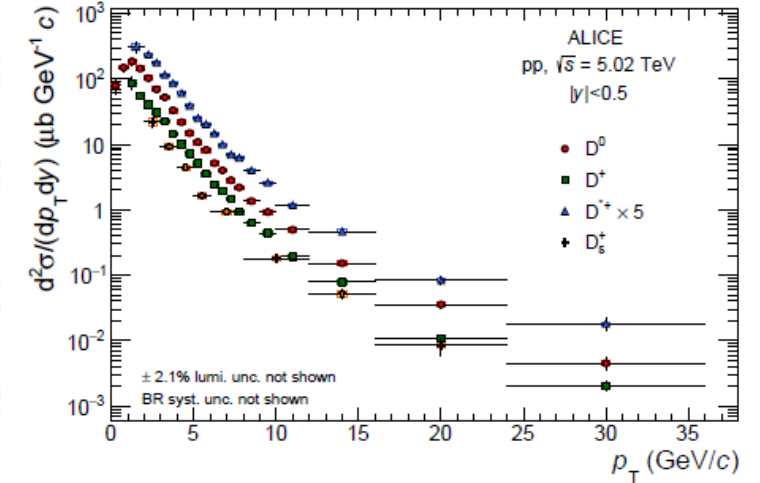
- Study hadron fragmentation within a jet
- Momentum fraction and transverse momentum defined relative to jet momentum and direction: $z = \frac{p_h \cdot P_{jet}}{P_{jet}^2}$
- Can be related to conventional fragmentation functions via jet Functions



[LHCb: arxiv2208.11691](https://arxiv.org/abs/2208.11691)

Universality of Fragmentation!?

- Generally light hadron production can be well-described by universal FFs in different processes
- Charmed mesons appear to be also ok
- However, there appear to be substantially more charmed baryons produced in hadronic collisions
- Suggestions that color-recombination with remnants can cause increased rates



ALICE: *JHEP*
04 (2018) 108

ALICE: *PRD* 105
(2022) L011103

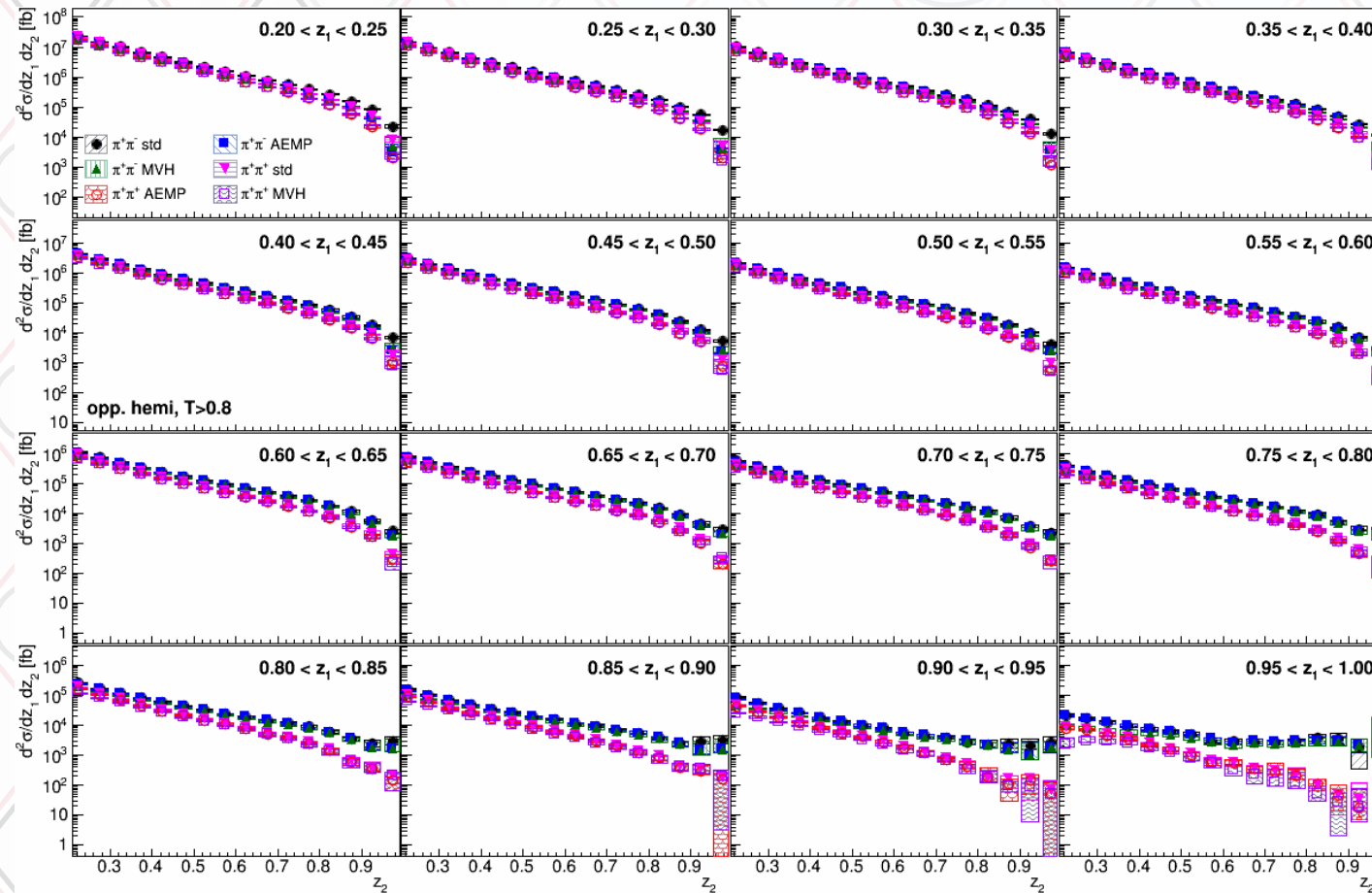
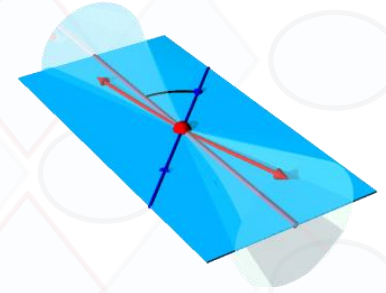


Summary

- e^+e^- is an excellent source to study fragmentation
- Crucial information for any SIDIS/pp measurements to access PDFs
- Sensitivity not only to collinear FFs but also TMD FFs:
 - Light hadron fragmentation
 - Explicitly k_t dependent unpol FFs
 - Collins asymmetries
 - Polarizing Λ fragmentation
 - Dihadron fragmentation (pol and unpol)
 - Nearly back-to-back di-hadron measurements
 - Role of D and Vector Meson fragmentation and its impact on light hadrons
- New additions such as hadron in jet measurements, hint at non-universality for charmed baryons in hadronic collisions
- More to come

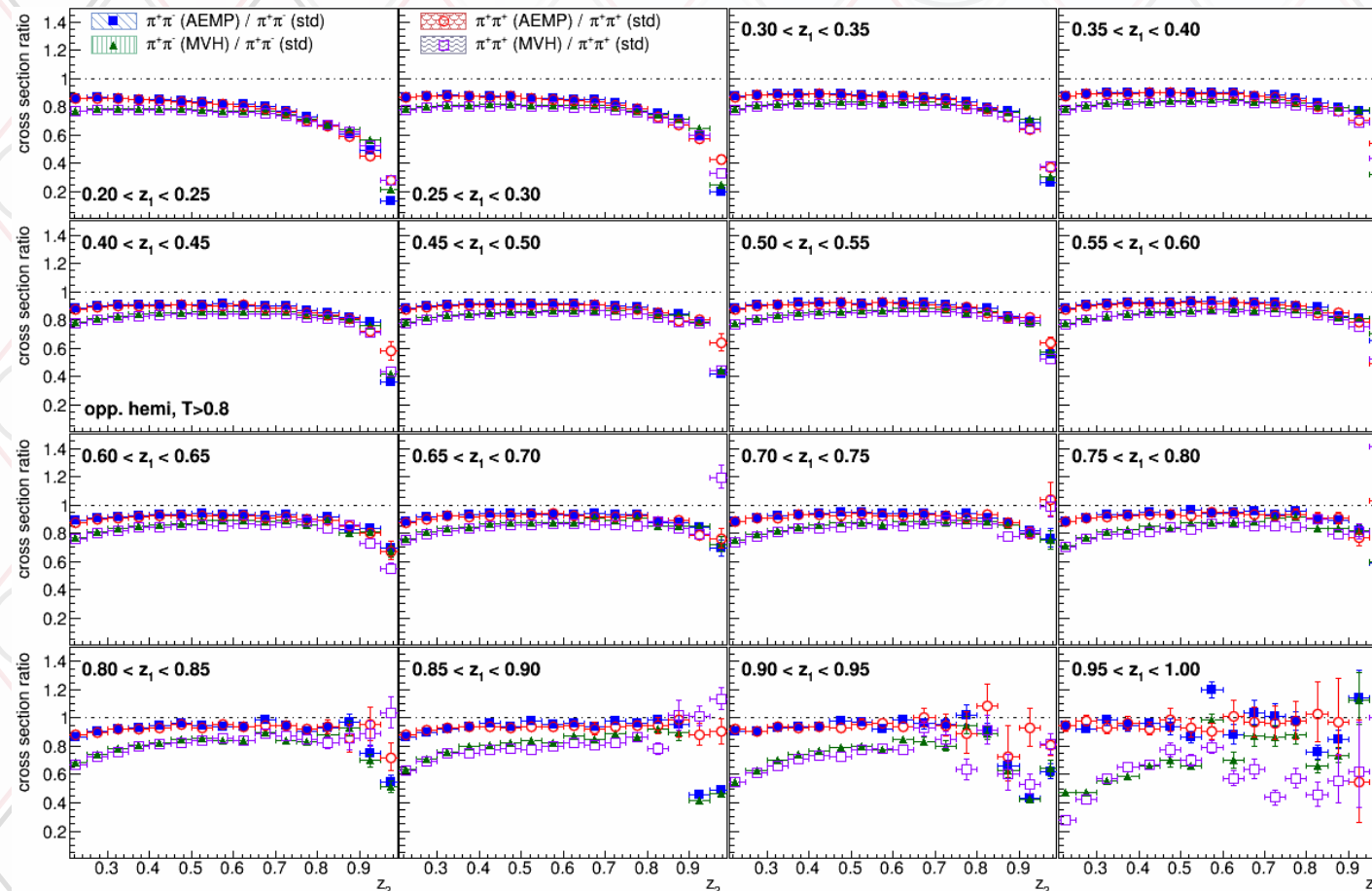
Comparison of definitions: pion pairs in opposite hemispheres

[RS et al, PRD 101 \(2020\) 092004](#)



Ratios to standard definitions: pion pairs in opposite hemisphere

[RS et al, PRD 101 \(2020\) 092004](#)



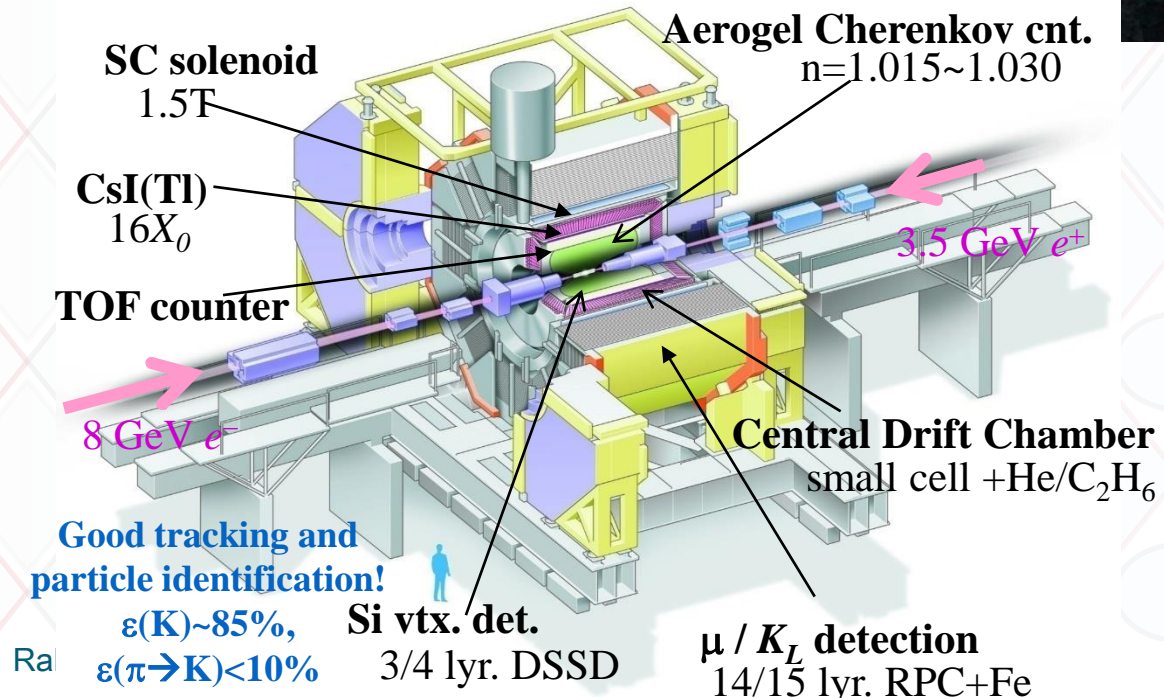
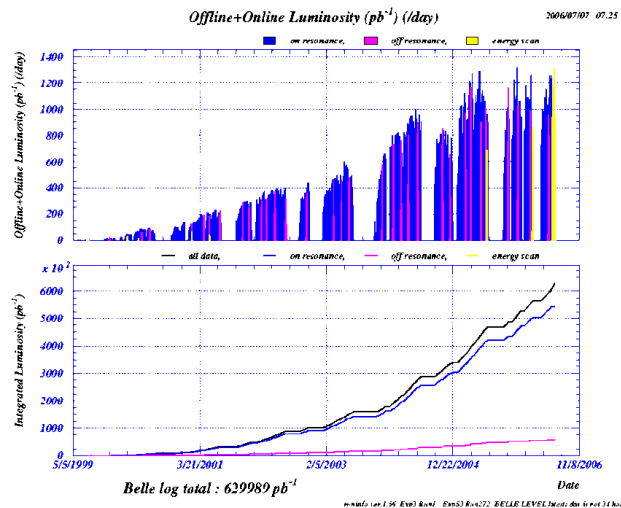
AEMP : suppressed at small fractional energies, at high z close to unity
 MvH : further suppressed due to transverse momentum

Ongoing work: Collins multidimensional analysis and Kaon combinations

- Currently revisiting kaon combinations of the Collins asymmetries
- While doing so, try to perform a full multi-dimensional analysis:
 - Consider :
 - $6(z_1) \times 6(z_2) \times 5(k_{t1}) \times 5(k_{t2}) \times 1(\text{costheta}) \times 8(\text{phi})$ for A_{12} method
 - $6(z_1) \times 6(z_2) \times 10(q_t) \times 1(\text{costheta}) \times 8(\text{phi})$ for A_0 method
- Perform most correction steps similar to recent analyses (PID, smearing)
 - Possibly simplified smearing unfolding as each z_1 - z_2 bin separately (z smearing almost nonexistent in such a binning)
 - non-qqbar removal, charm removal, ISR correction and acceptance might require introduction of nonzero MC asymmetries

Belle Detector and KEKB

- Asymmetric collider
- $8\text{GeV } e^- + 3.5\text{GeV } e^+$
- $\sqrt{s} = 10.58\text{GeV } (Y(4S))$
- $e^+e^- \rightarrow Y(4S) \rightarrow B \bar{B}$
- Continuum production: 10.52 GeV
- $e^+e^- \rightarrow q \bar{q} \text{ (u,d,s,c)}$
- Integrated Luminosity: $>1000 \text{ fb}^{-1}$
- $>70\text{fb}^{-1} \Rightarrow \text{continuum}$



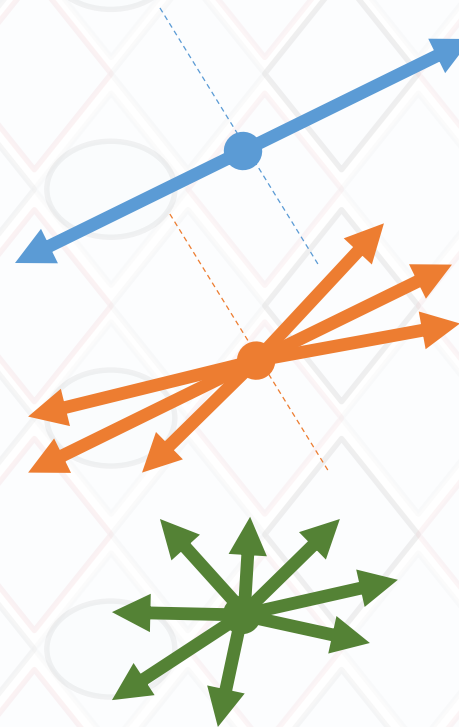
Thrust definition

- Event shape variable thrust is defined as:

$$T \stackrel{max}{=} \frac{\sum_h |\mathbf{P}_h \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h|}$$

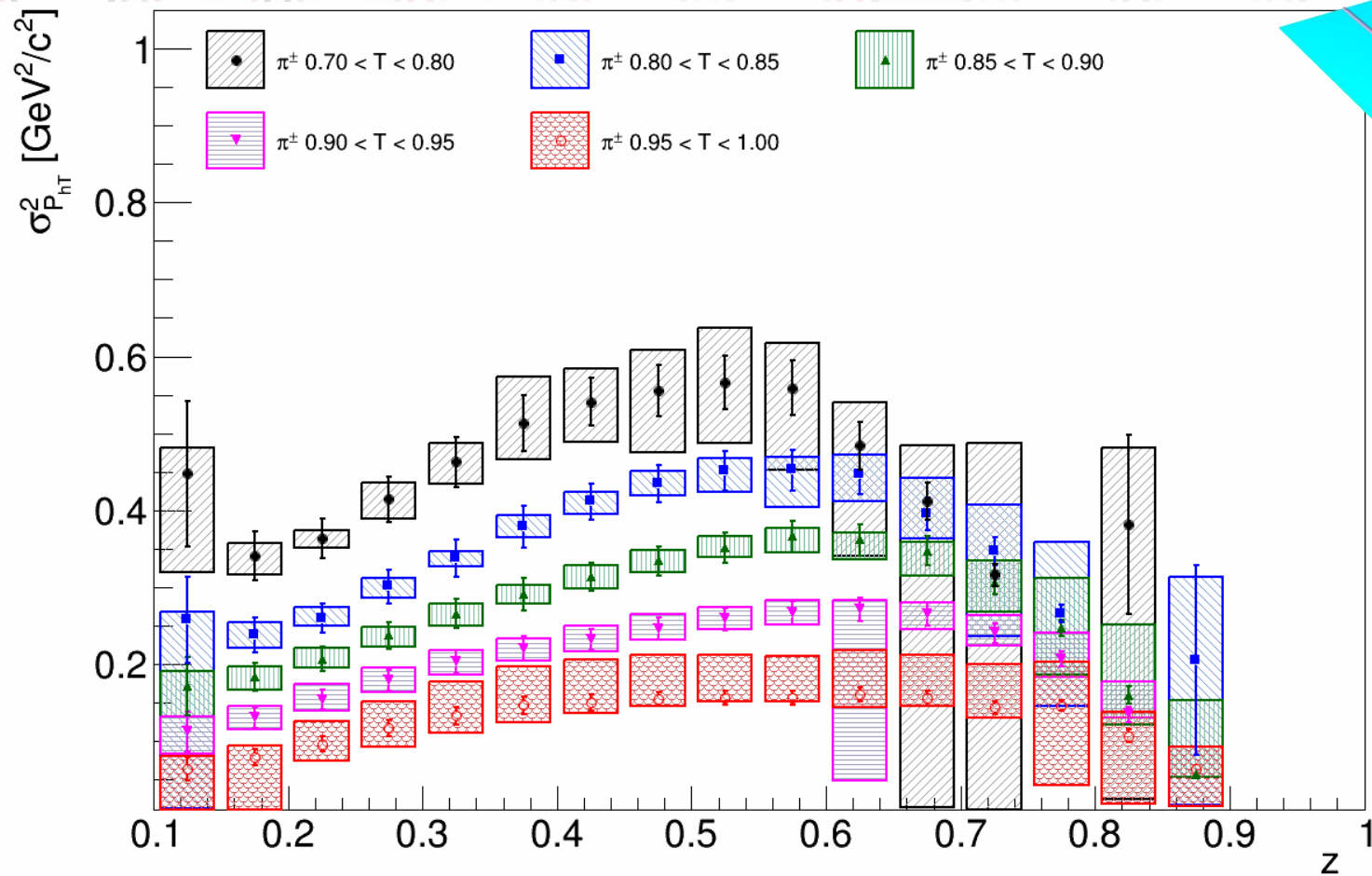
- All final-state particles are included in the sum
- A **two-jet-like** event has a high thrust value
- A completely **spherical** event has a thrust value of 0.5

- Thrust axis \mathbf{n} also defines the hemispheres



Gaussian widths, thrust dependence

Gaussian widths get narrower with higher Thrust

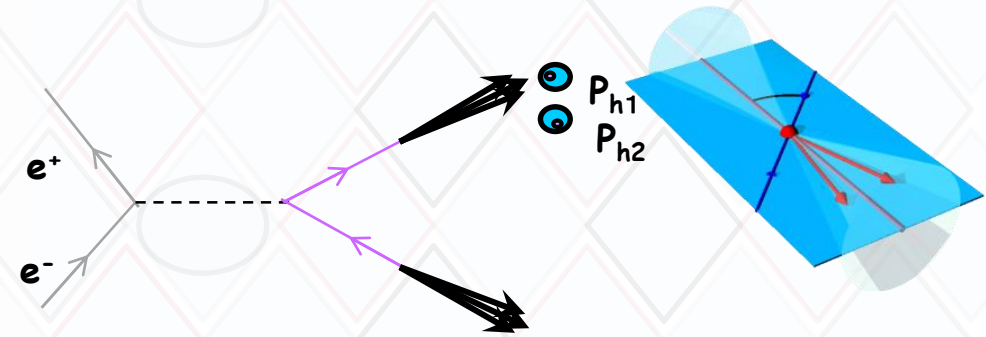


[PRD99 \(2019\) 112006](#)

Ralf Seidl: Fragmentation

Di-hadron fragmentation functions

$$D_{1,q}^{h_1 h_2}(z, m, Q^2)$$



$$D_{1,q}^h(z_1, Q^2) D_{1,q}^h(z_2, Q^2)$$

