3D Nucleon Structure:
GPDs and TMDs

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Outline

- Introduction and motivation
- Transverse momentum dependent parton distributions (TMDs)
  - Unpolarized TMDs
  - Sivers function
  - Transversity and tensor charge
  - TMDs and lattice QCD
- Generalized parton distributions (GPDs)
  - Compton form factors
  - Deconvolution problem
  - Simultaneous fit of DVCS and DVMP data
  - GPDs and lattice QCD
- Further recent progress
- Conclusions

Disclaimer: subjective selection of topics / hardly any experimental data

$\rightarrow$ talks Diehl, Meziani, Joosten, Riedl, Niccolai, Surrow, Seidl, Voutier, ...

Nucleon Structure: the Nobel Prizes

- Protons' anomalous magnetic moment (Estermann, Frisch, Stern, 1933)

$$\mu_p \sim 3 \times \mu_{\text{Dirac}}$$

$\rightarrow$ proton cannot be pointlike

- Elastic electron-proton scattering (Hofstadter, McAllister, 1955)

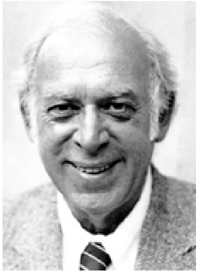
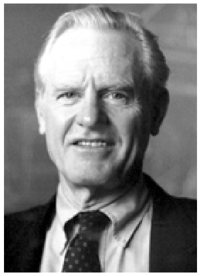
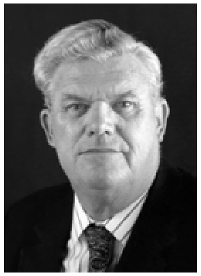
$$r_{p,\text{RMS}}^{\text{charge}} = (0.74 \pm 0.24)\,\text{fm}$$

$\rightarrow$ first rather precise idea about size of the proton

- Deep-inelastic electron-proton scattering (Friedman, Kendall, Taylor et al, 1968)

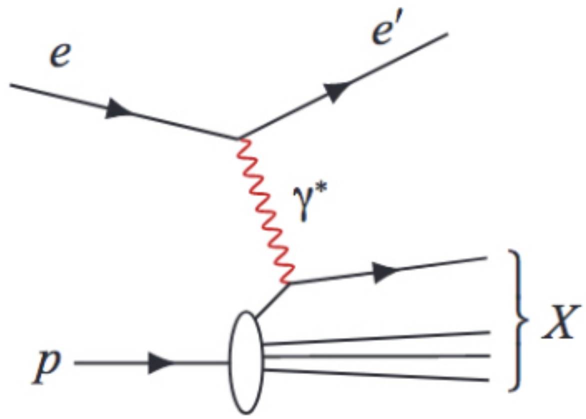
$\rightarrow$ proton has partonic substructure

$\rightarrow$ experiments paved ground for discovery of QCD

Inclusive Deep-Inelastic Scattering (DIS) and 1D Imaging

- Process: $e + p \to e + X$

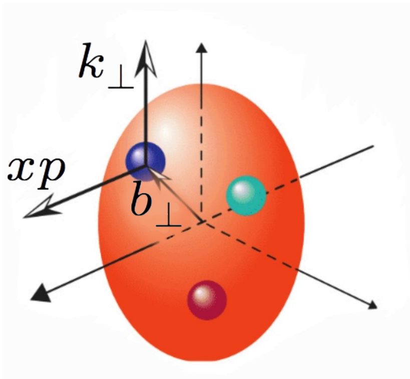


  - probing quarks and gluons with longitudinal momentum fraction x
  - modern PDF fits include data from other processes (gauge boson production in pp collisions, ...)
  - extractions of $f_1^{q/g}$ from global analysis (CTEQ, JAM, MSHT, NNPDF, ...)
  - extractions of $g_1^{q/g}$ from global analysis (DSSV, JAM, NNPDF, ...)

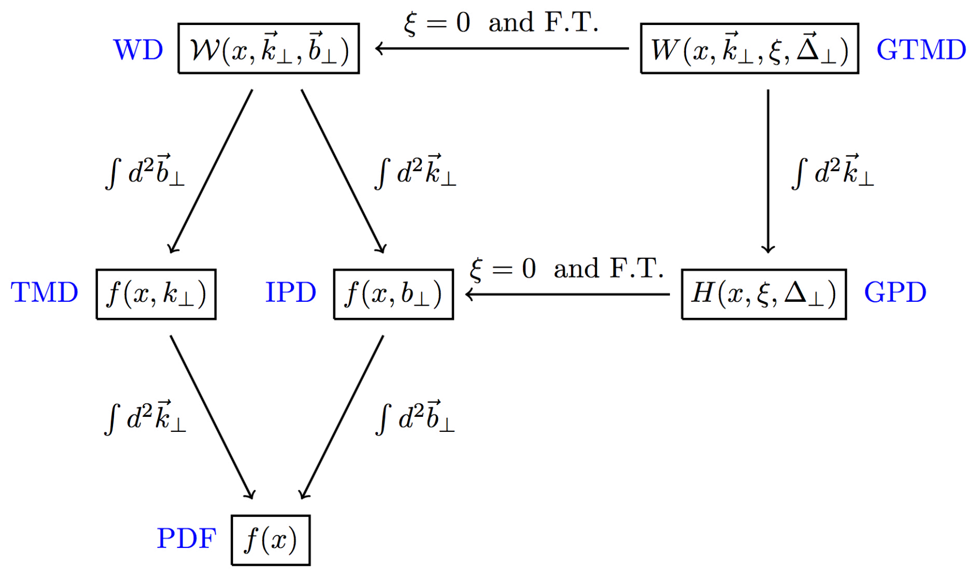
- 1D quark PDFs (including polarization of nucleon and quarks)
  - $f_1^q(x)$ unpolarized PDF ($\to$ talk Nadolsky, ...)
  - $g_1^q(x)$ helicity PDF ($\to$ talk Vogelsang, ...)
  - $h_1^q(x)$ transversity PDF

3D Imaging and Beyond

Adding $\vec{k}_\perp$ or/and $\vec{b}_\perp$



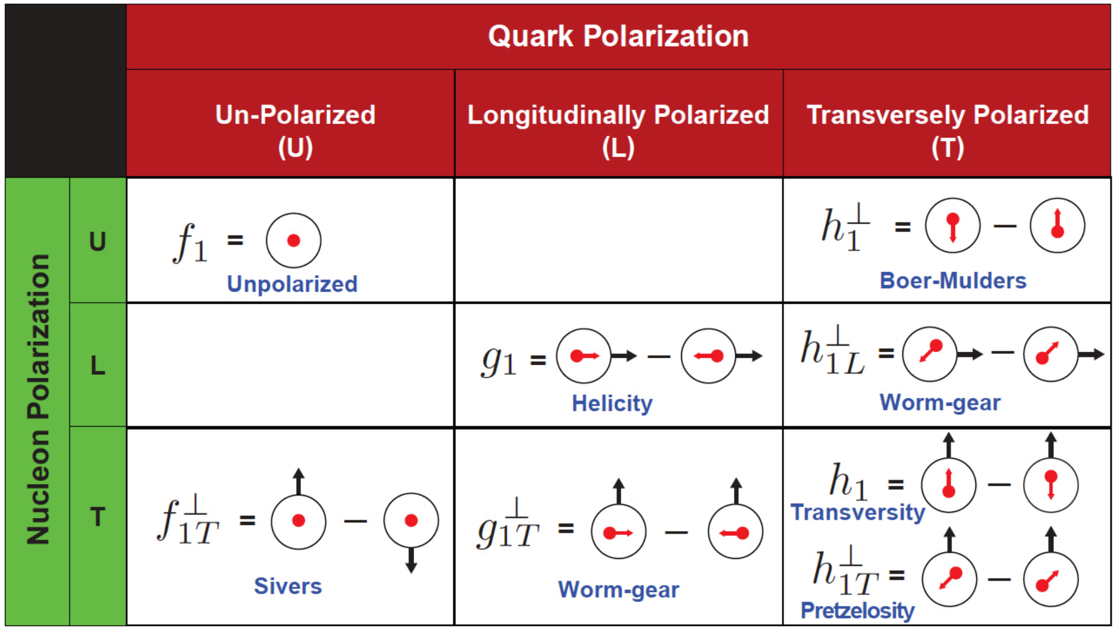
Overview of partonic functions



- Main objects of interest for multi-dimensional imaging
  1. $f(x, k_\perp)$ TMDs
  2. $f(x, b_\perp)$ Impact parameter distributions (Fourier transforms of GPDs)
  3. $\mathcal{W}(x, \vec{k}_\perp, \vec{b}_\perp)$ Wigner distributions (5-D quasi-probability distributions) (not covered in this talk)

Overview of TMDs

- Leading-power ("twist-2") quark TMDs for spin-$\frac{1}{2}$ hadron



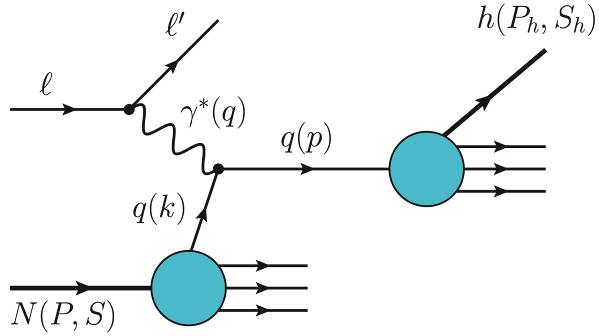
(TMD Handbook, 2304.03302)

  - $f_1(x, k_\perp)$ $\quad g_1(x, k_\perp)$ $\quad h_1(x, k_\perp)$ plus 5 additional functions
  - all TMDs contain unique physics
  - information on all quark TMDs available (experiment, LQCD, models)
  - two auxiliary scales: $f_1(x, k_\perp; \mu, \zeta)$

- Similarly, 8 leading-power gluon TMDs

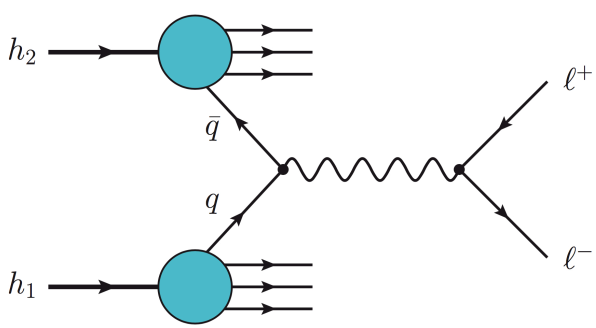
Key Processes for Measuring TMDs

- Semi-inclusive DIS: $\ell + N \to \ell + h + X$

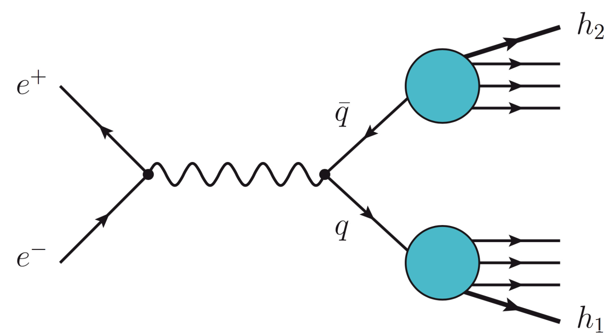


  - cross section for $P_{hT} \ll Q$
  - sensitive to TMD PDFs and TMD fragmentation functions (FFs) (→ talk Seidl, ...)

- Drell-Yan process and electron-positron annihilation



sensitive to TMD PDFs

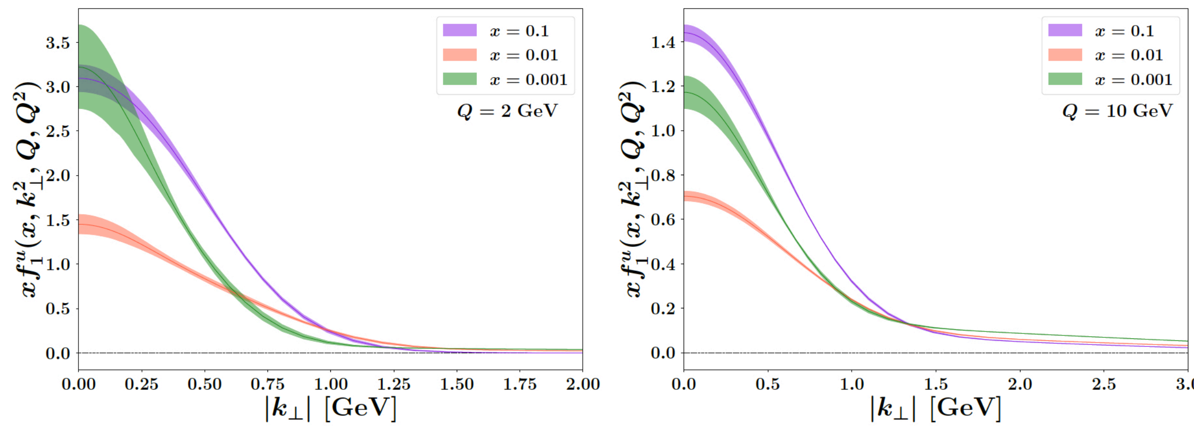


sensitive to TMD FFs

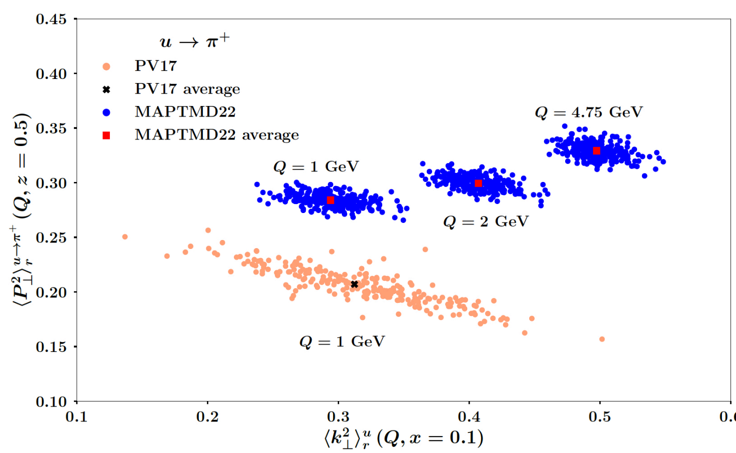
- Further processes
  - $\ell\, N \to \ell\,\text{jet X}$ $\quad \ell\, N \to \ell\,(h, \text{jet})\,\text{X}$ $\quad p\, p \to (h, \text{jet})\,\text{X}$ $\quad \ldots$
  - nuclear targets

Unpolarized TMDs

- Extraction: Example 1 (Bacchetta et al (MAP), 2206.07598)
  - based on SIDIS and DY data
  - $f_1^u(x, k_\perp)$ at different scales



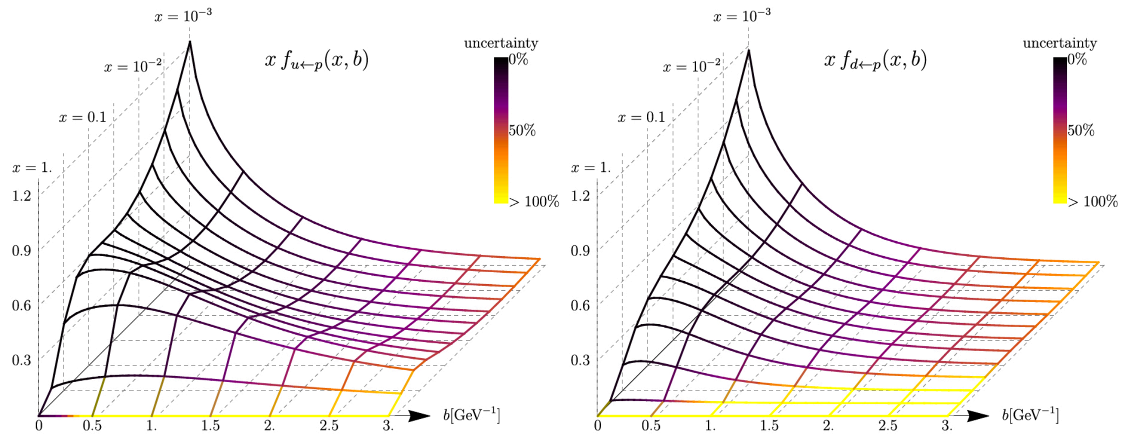
  - average transverse momenta of TMD PDF and TMD FF



- Extraction: Example 2 (Moos, Scimemi, Vladimirov, Zurita, 2305.07473) (→ talks Moos, Zurita)
  - based on DY data
  - TMDs in position space ($b_\perp$ is Fourier conjugate of $k_\perp$)

$$f_1^q(x, b_\perp) = \int \frac{db^-}{4\pi} \, e^{ixP^+b^-} \, \langle P| \, \psi_q(0) \, \gamma^+ \, \psi_q(b^-, 0, \vec{b}_\perp)|P\rangle$$

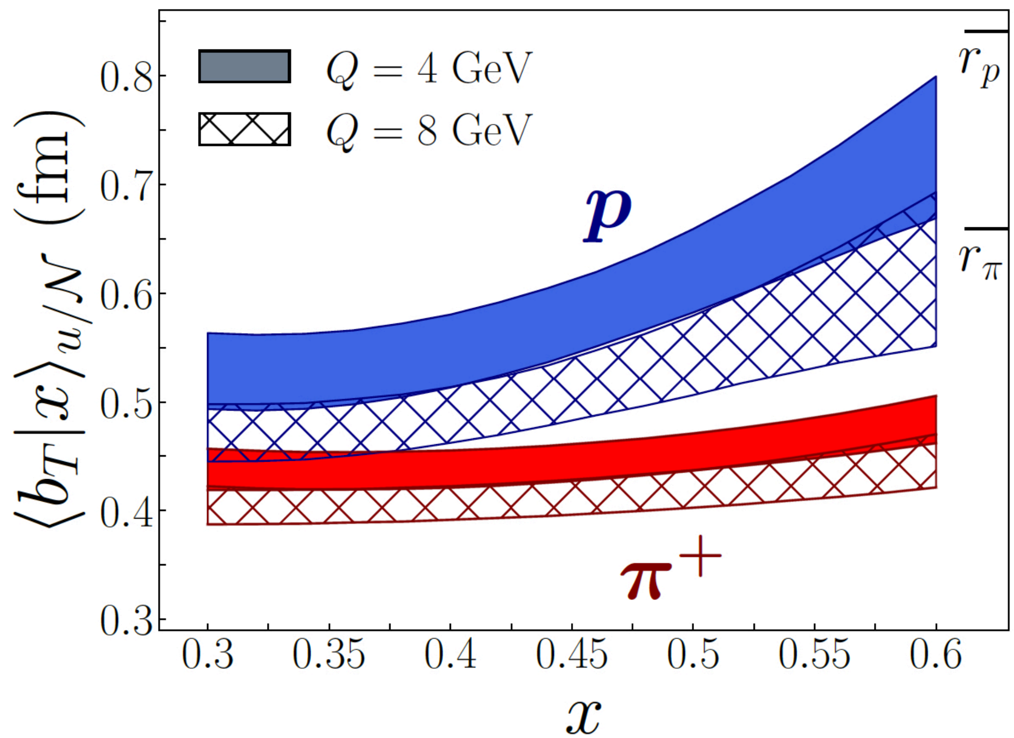
  - $f_1^u(x, b_\perp)$ and $f_1^d(x, b_\perp)$ at $\mu = 2\,\mathrm{GeV}$



  - uncertainty of TMD extractions due to PDF uncertainties is emphasized
    (see also Bury et al, 2201.07124)

$$\lim_{b_\perp \to 0} f_1^i(x, b_\perp) = \sum_j \int_x^1 \frac{dy}{y} \, C_{ij}(y, b_\perp) \, f_1^j(x/y)$$

- Extraction: Example 3 (Barry et al (JAM), 2302.01192)
  - mainly based on fixed-target proton-induced and pion-induced DY data
  - simultaneous extraction of pion collinear and TMD PDFs and proton TMD PDFs
  - average transverse separation of quark fields $b_\perp$: proton vs pion

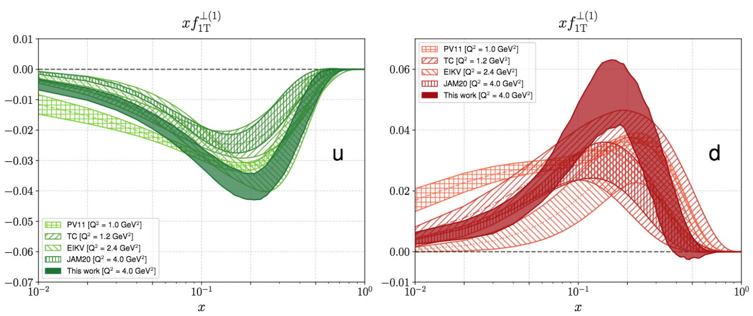


  - indication of TMD-dependent EMC effect (see also Alrashed et al, 2107.12401)

Sivers Function

(Bacchetta, Delcarro, Pisano, Radici, 2004.14278)

- (Moment of) Sivers function for proton (at $\mu = 2\,\mathrm{GeV}$)



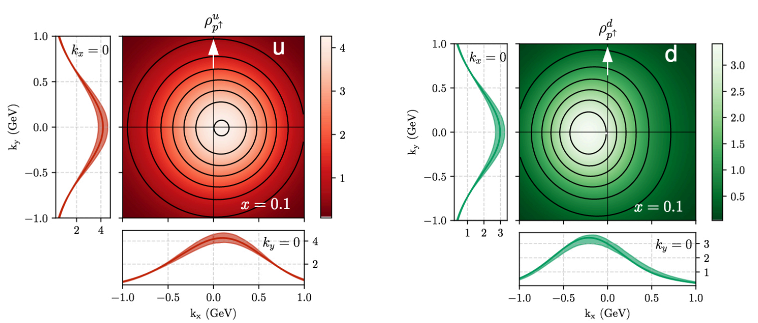
$$f_{1T}^{\perp(1)}(x) = \int d^2\vec{k}_\perp \, \frac{k_\perp^2}{2M^2} \, f_{1T}^\perp(x, k_\perp)$$

agreement with large-N_c prediction

$$f_{1T}^{\perp\, u} = -\, f_{1T}^{\perp\, d} + \mathcal{O}(1/N_c)$$

(Pobylitsa, hep-ph/0301236)

- Density $\rho_{\vec{S}}(x, \vec{k}_\perp)$ of unpolarized quarks in transversely polarized proton



$$\rho_{\vec{S}}(x, \vec{k}_\perp) = f_1(x, k_\perp) - \frac{(\vec{k}_\perp \times \vec{S}_\perp)_z}{M} f_{1T}^\perp(x, k_\perp)$$

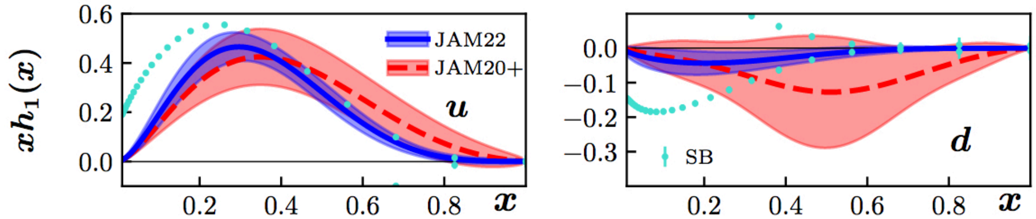
nonzero $f_{1T}^\perp$ "distorts" $\rho_{\vec{S}}(x, \vec{k}_\perp)$

- Results from other groups available as well

(Bury, Prokudin, Vladimirov, 2012.05135, 2103.03270 / Gamberg et al (JAM), 2205.00999 / ...)

Transvertsity and Tensor Charge

- Transversity from single-hadron production
  - extraction using data for Collins effect in SIDIS and e^+e^-, as well for A_N in pp
    (Gamberg et al (JAM), 2205.00999)



(results at $\mu = 2\,\mathrm{GeV}$)

  - related recent work (include TMD evolution, no pp data) (Zheng et al, 2310.15532)

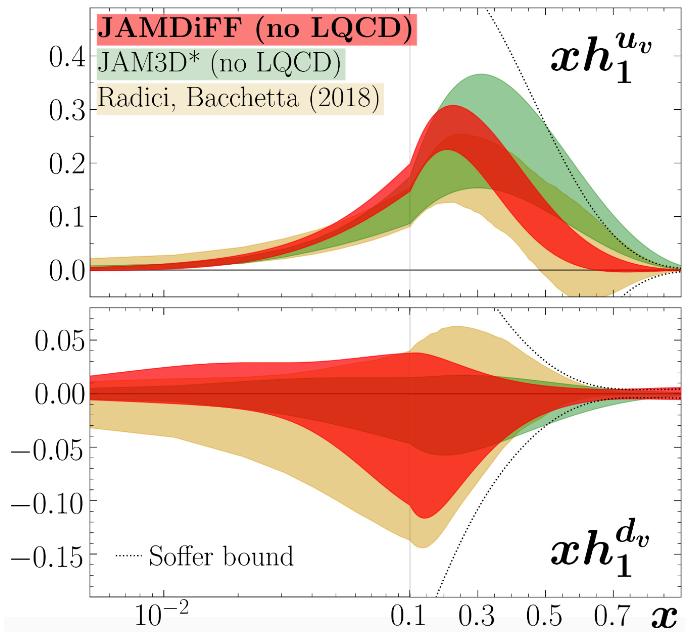
- Transversity from di-hadron production
  (Radici, Bacchetta, 1802.05212 / Benel, Courtoy, Ferro-Hernandez, 1912.03289 /
  Pitonyak et al, 2305.11995 / Cocuzza et al, 2306.12998, 2308.14857)

- Tensor charge

$$\delta q = \int_0^1 dx \big(h_1^q(x) - h_1^{\bar{q}}(x) \big) \qquad\qquad g_T = \delta u - \delta d$$

  - as fundamental as axial charge
  - can be computed well in LQCD

- Extracted transversity PDFs: di-hadron vs single-hadron production
  (from Cocuzza et al, 2306.12998, 2308.14857)



– fit of $h_1^{u_v}$, $h_1^{d_v}$, $h_1^{\bar{u}} = -h_1^{\bar{d}}$
  large-N_c constraint for antiquarks (Pobylitsa, 2003)

– Soffer bound (Soffer, hep-ph/9409254)

$$h_1^q(x) \leq \tfrac{1}{2}\left|f_1^q(x) + g_1^q(x)\right|$$

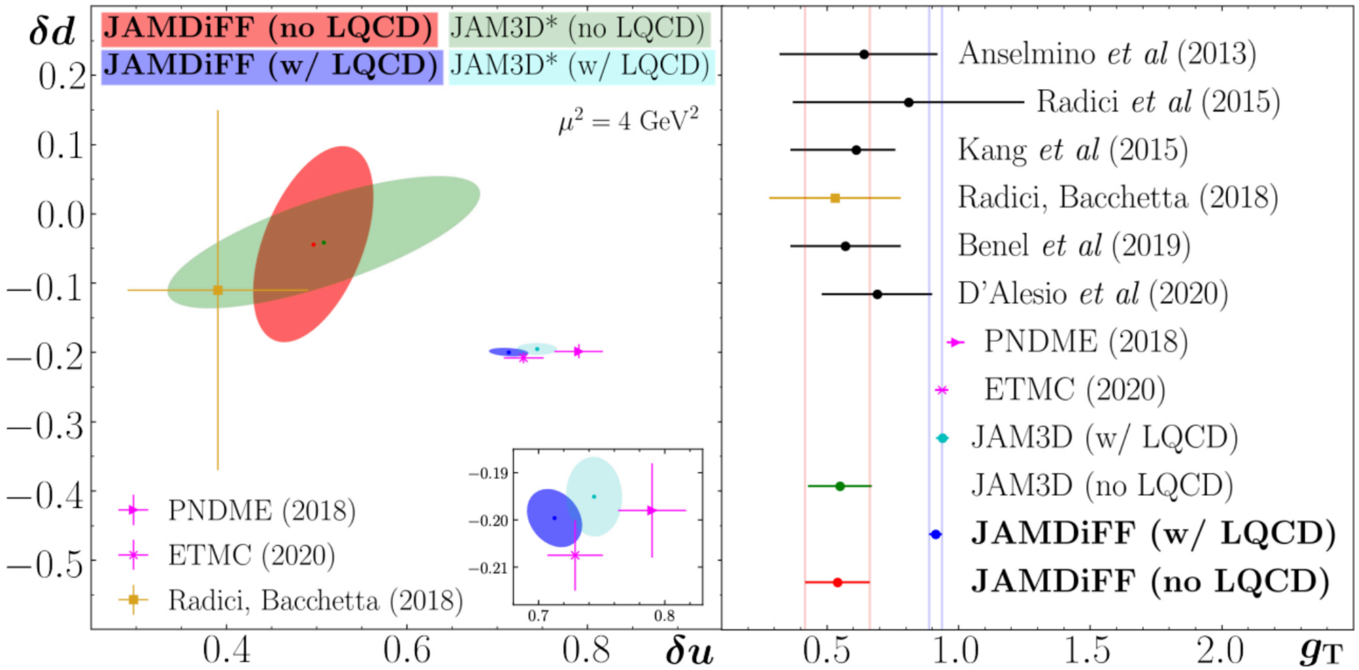
– small-x constraint (Kovchegov, Sievert, 1808.10354)

$$h_1^q \xrightarrow{x\to 0} x^{\alpha_q} \qquad \alpha_q \approx 0.17 \pm 0.085$$

– JAM3D* = JAM3D-22 (no LQCD)
  + antiquarks with $h_1^{\bar{u}} = -h_1^{\bar{d}}$
  + small-x constraint

– agreement between all three analyses within errors

- Tensor charge and input from LQCD (from Cocuzza et al, 2306.12998, 2308.14857)
  - results before and after inclusion of tensor charges from LQCD



LQCD results from PNDME (1808.07597) and ETMC (1909.00485)

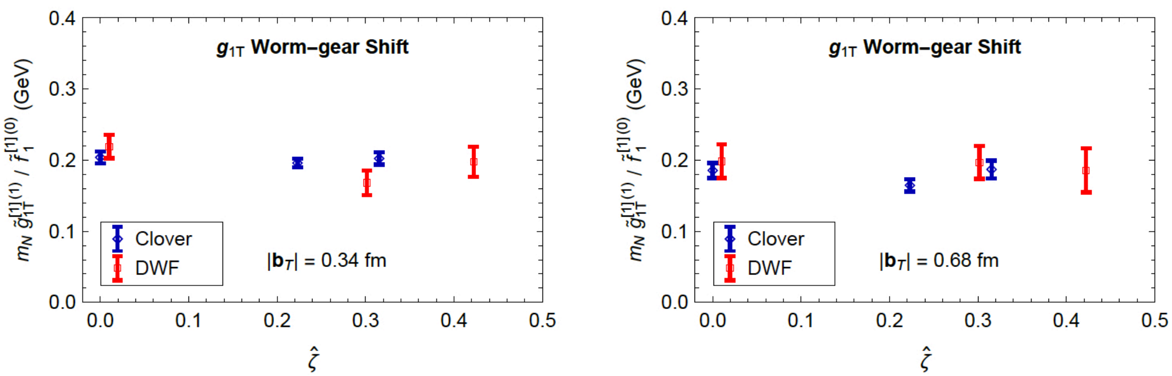
noticeable shift for δu after including LQCD results

overall finding: universal nature of all available information on h_1^q —
(1) data for di-hadron production, (2) data for single-hadron production,
(3) LQCD results for tensor charge, (4) Soffer bound, (5) small-x constraint

TMDs and Lattice QCD

- Calculation of generalized TMD shifts (Hägler et al, 0908.1283 / ...)
  - example: shift due to g_{1T} (Yoon et al, 1706.03406)

$$\langle k_\perp \rangle (b_\perp) = M \frac{\int_0^1 dx \left[g_{1T}^{(1)u}(x, b_\perp) - g_{1T}^{(1)d}(x, b_\perp) \right]}{\int_0^1 dx \left[f_1^u(x, b_\perp) - f^{(1)d}(x, b_\perp) \right]}$$



for relation to experiment, main interest in limits $b_\perp \to 0$ and $\hat{\zeta} \to \infty$

extraction of $\langle k_\perp \rangle$ from experimental data, agrees with LQCD result within errors (Bhattacharya et al, 2110.10253)

- Calculation of the Collins-Soper evolution kernel (Ebert, Stewart, Zhao, 1811.00026, ... / Schlemmer et al, 2103.16991, ... / Avkhadiev, Shanahan, Zhao, 2307.12359, ...)
  - overall: good agreement between LQCD and phenomenology

Overview of GPDs

- GPDs appear (at the amplitude level) in hard exclusive reactions
- Leading-twist (twist-2) quark GPDs for spin-$\frac{1}{2}$ hadron

N / q	U	L	T
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T \quad \tilde{H}_T$

  - GPDs depend on 3 variables and the renormalization scale: $\mathrm{GPD}(x, \xi, \Delta_T; \mu)$

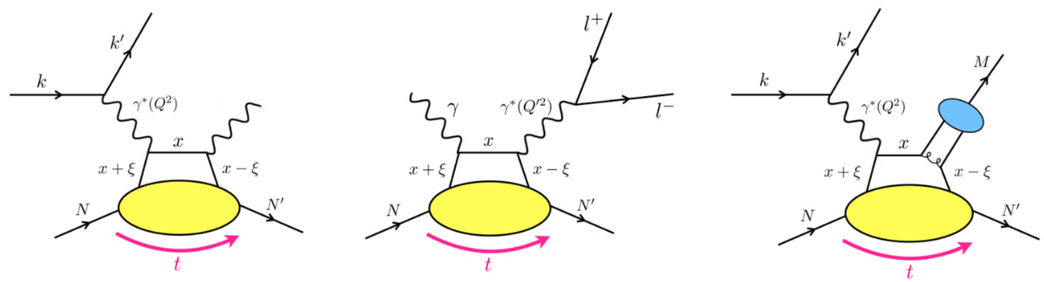
$$t = -\frac{1}{1-\xi^2}\left(4\,\xi^2 M^2 - \vec{\Delta}_T^2\right)$$

  - relation to PDFs: $H^q(x, 0, 0) = f_1^q(x)$ (similarly for $\tilde{H},\ H_T$)
  - relation to form factors: $\int_{-1}^{1} dx\, H^q(x, \xi, t) = F_1^q(t)$ etc.
  - GPDs for transverse quark polarization hard to measure (Collins, Diehl, hep-ph/9907498)

- Similarly, 8 leading-twist gluon GPDs

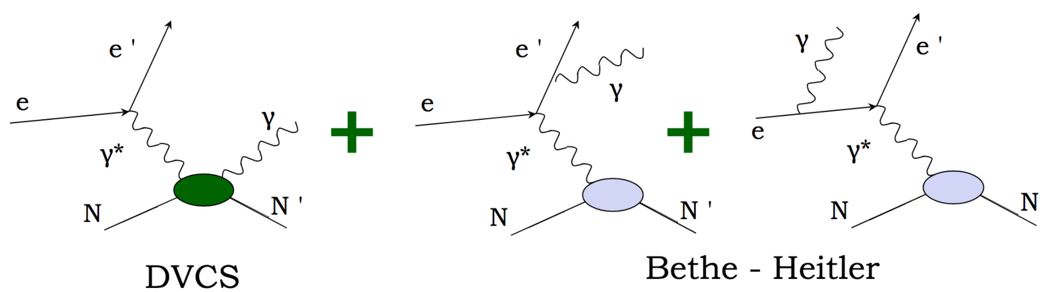
Key Processes for Measuring GPDs

- Deep virtual Compton scattering (DVCS), Time-like Compton scattering (TCS), Deep virtual meson production (DVMP)



  - data available for all those processes

- More details on important DVCS process



(Sokhan, HUGS Summer School, 2018)

  - interference between DVCS and Bethe-Heitler amplitude plays key role
  - $\sigma_{\rm int} \sim$ Compton form factor $\times$ electromagnetic form factor

Main Motivations for Studying GPDs

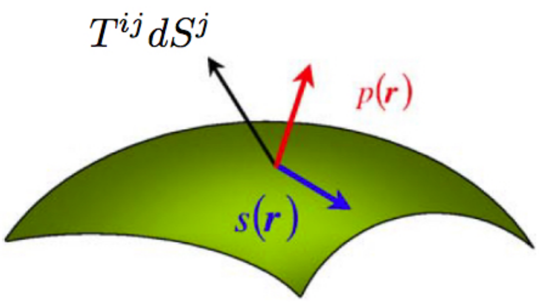
- Impact parameter distributions (Burkardt, hep-ph/0005108 / ...)

$$\mathrm{GPD}(x, \xi = 0, \Delta_T) \quad \overset{\mathcal{F.T.}}{\longleftrightarrow} \quad f(x, b_T)$$

- Spin sum rule and orbital angular momentum (Ji, hep-ph/9603249)

$$J_q = \int_{-1}^{1} dx\, x\, (H_q + E_q)\Big|_{t=0} \qquad\qquad J_g = \int_{0}^{1} dx\, (H_g + E_g)\Big|_{t=0}$$

- Mechanical properties (pressure, shear) inside nucleon
  (Polyakov, hep-ph/0210165 / Polyakov, Schweitzer, 1805.06596 / ...)



- Information in all three areas available (experiment, LQCD, models)

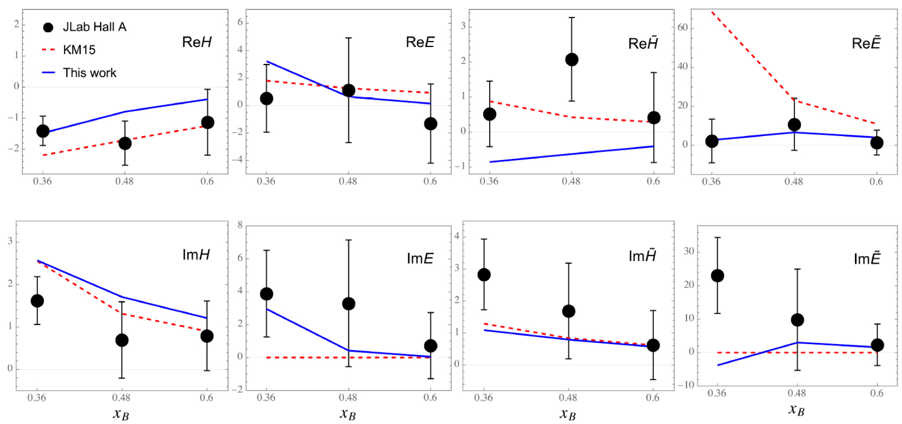
Compton Form Factors

- Example of Compton form factor (CFF)

$$\mathcal{H}(\xi, t; \mu) = \sum_q e_q^2 \int_{-1}^{1} dx\, H^q(x, \xi, t; \mu) \left(\frac{1}{\xi - x - i\varepsilon} - \frac{1}{\xi + x - i\varepsilon} \right) + \mathcal{O}(\alpha_s)$$

  – $\mathcal{H}$ has real and imaginary part $\rightarrow$ total of 8 CFFs

- Extracted CFFs (plot from Guo et al, 2302.07279; data points from JLab Hall A, 2201.03714)



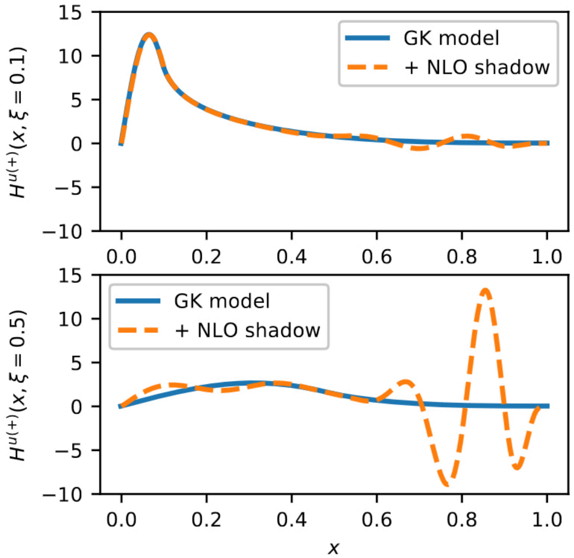
  – extraction of CFFs is difficult (multi-variable problem, complicated structure of cross section, power corrections, ...)

  – very little known about CFFs at small ξ (Moutarde, Sznajder, Wagner, 1905.02098)

  – how to get from CFFs to GPDs? $\rightarrow$ deconvolution problem

Deconvolution Problem

- Systematic study (Bertone et al, 2104.03836)



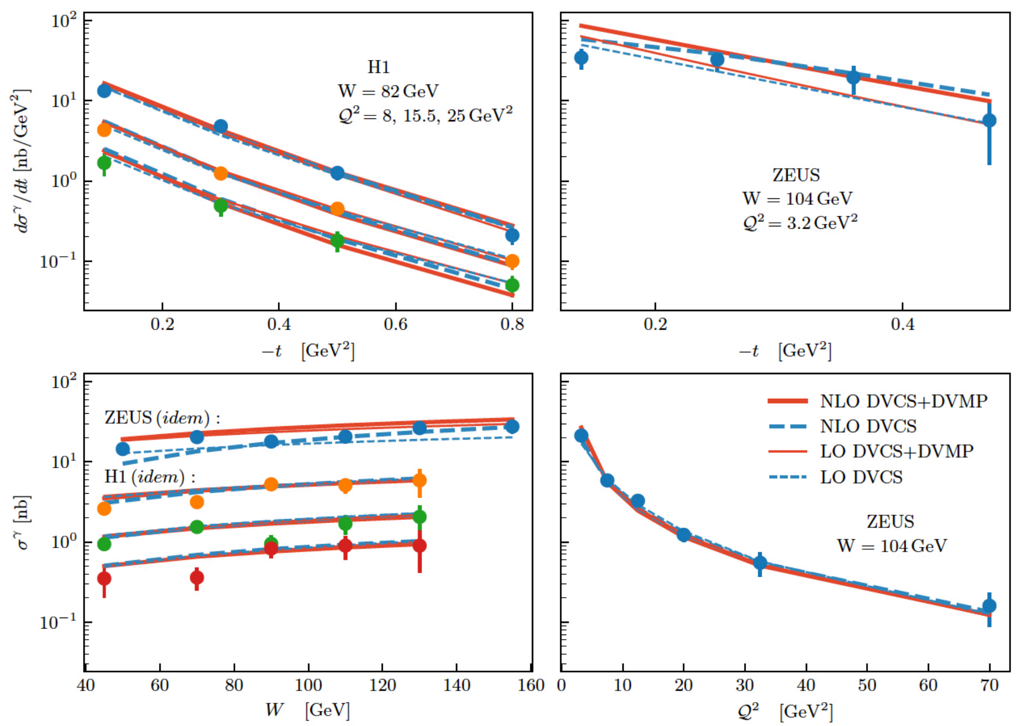
  - by construction, "shadow GPD" does not contribute to CFF at given scale μ_0
  - evolution hardly changes this picture
  - conceptual problem for model-independent extraction of GPDs

- Related recent work (Moffat et al, 2303.12006)
  - confirms qualitative finding of Bertone et al
  - but, with sufficient leverage in ξ and Q^2 situation may be more optimistic

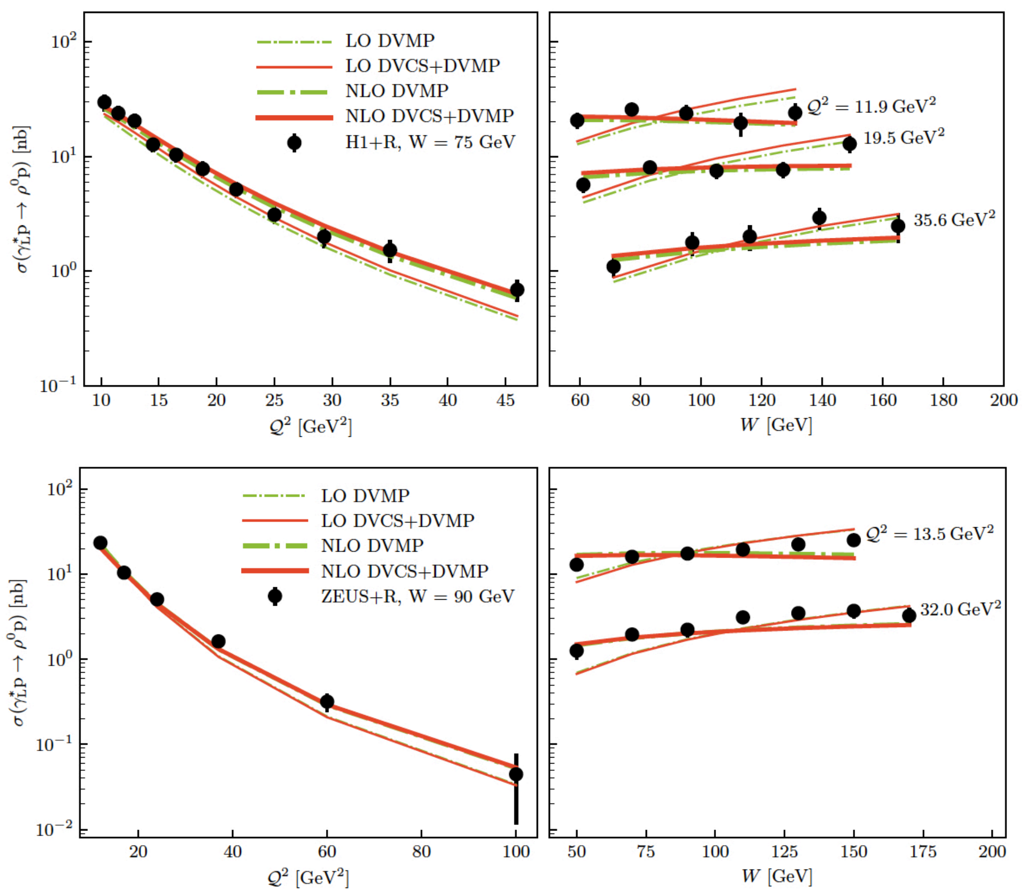
- Potential way out: other processes (with direct sensitivity to x-dependence of GPDs)
  - double DVCS (Guidal, Vanderhaeghen, hep-ph/0208275 / Deja et al, 2303.13668 / ...)
  - $\gamma\gamma$, γM production (Boussarie et al, 1609.03830 / Grocholski et al, 2204.00396 / Qiu, Yu, 2205.07846 / ...)

Simultaneous Fit of DVCS and DVMP Data

- Previous related work (Kroll, Moutarde, Sabatie, 1210.6975 / Lautenschlager, Müller, Schäfer, 1312.5493)
- Recent simultaneous fit (Čuić, Duplančić, Kumerički, Passek-K., 2310.13837)
  - data selection
    HERA data for DVCS $[Q^2 > 5\,\mathrm{GeV}^2]$
    HERA data for $\sigma_L(ep \to e\rho^0 p)$ $[Q^2 > 10\,\mathrm{GeV}^2]$
  - fit of DVCS data



– fit of DVMP data



– overall finding: successful combined fit at NLO but not at LO

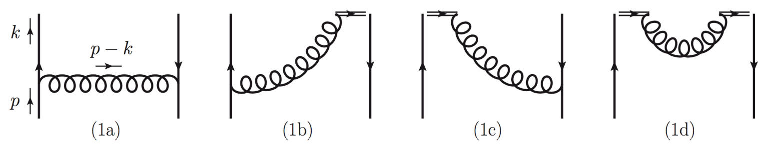
– important step forward in this field

– next step could be trying to include lower-energy DVCS data

GPDs and Lattice QCD

- Access to x-dependence of PDFs in LQCD via Euclidean parton correlators
  - quasi-PDFs (Ji, 1305.1539 / ...) ($\rightarrow$ talks Zhang, Mukherjee, Constantinou)
  - pseudo-PDFs (Radyushkin, 1705.01488 / ...) ($\rightarrow$ talk Karpie)
  - closely related previous works (Braun, Gornicki, Mankiewicz, hep-ph/9410318 / ...)

- Light-cone PDFs and quasi-PDFs have the same non-perturbative physics



sample result, with nonzero gluon mass m_g as IR regulator

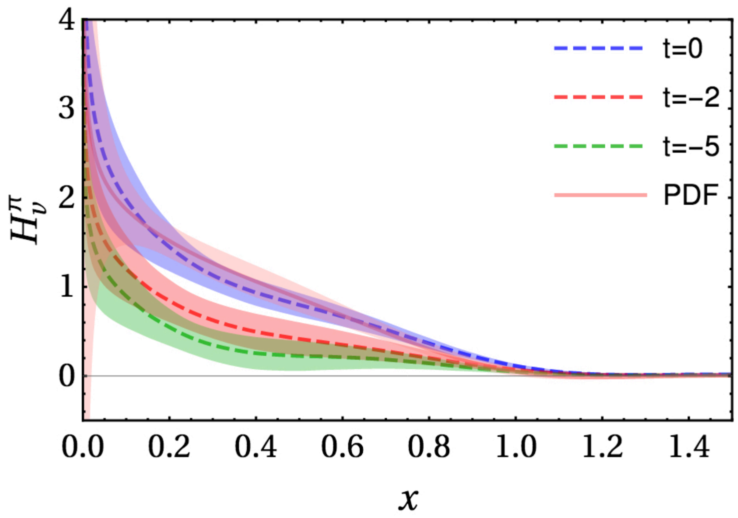
$$f_1^{(1a)} = \frac{\alpha_s C_F}{2\pi} (1-x) \left(\ln \frac{\mu^2}{x\, m_g^2} - 2 \right) \qquad 0 < x < 1$$

$$f_{1,\mathrm{Q}}^{(1a)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} (1-x)\ \ln \dfrac{x-1}{x} - 1 & x < 0 \\ (1-x)\ \ln \dfrac{4(1-x)p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x)\ \ln \dfrac{x}{x-1} + 1 & x > 1 \end{cases}$$

- Pioneering results for GPDs

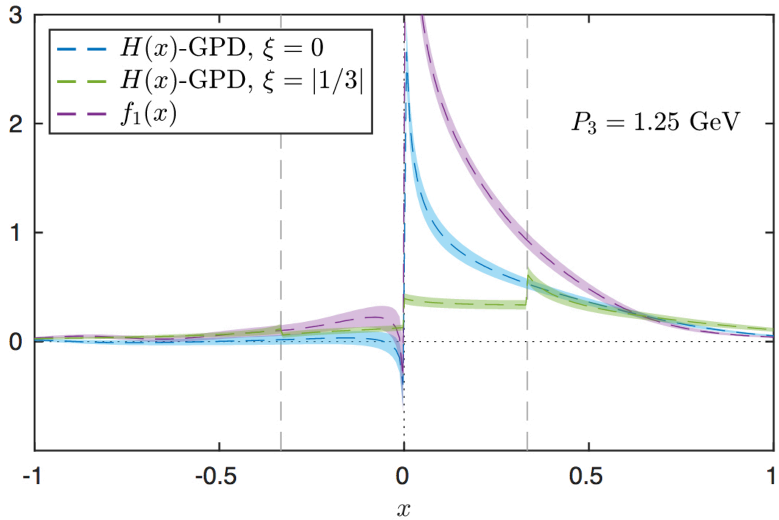
pion GPD ($\xi = 0$)

(Chen, Lin, Zhang, 1904.12376)



GPD H for proton ($t = -0.69\,\mathrm{GeV}^2$)

(ETM Collaboration, 2008.10573)



first LQCD results of GPDs quite encouraging

- Recent results
  - exploiting non-symmetric reference frame (Bhattacharya et al, 2209.05373, 2310.13114) (→ talk Constantinou)
  - first results of twist-3 GPDs (Bhattacharya et al, 2306.05533) (→ talk Constantinou)
  - moments of GPDs using short-distance factorization (Gao et al, 2305.11117) (→ talk Mukherjee)

Further Recent Progress (selected topics)

- TMD Handbook (2304.03322)
  - presently, most comprehensive discussion of TMD field (29 authors, 471 pages)

- TMDs at subleading ("twist-3") power (→ talk Gamberg)
  (Ebert, Gao, Stewart, 2112.07680 / Rodini, Vladimirov, 2204.03856 / Gamberg et al, 2211.13209 / ...)
  - main message: TMD factorization apparently works beyond leading power
  - tremendous new opportunities for reliable phenomenology

- First TMD extraction for nuclear targets (Alrashed et al, 2107.12401)
  - fit includes data with significant energy range (from HERMES to LHC)
  - broadening of TMD PDFs and TMD FFs (relative to vacuum) observed

- Addressing TMDs through jet measurements
  (Liu et al, 1812.08077, 2007.12866 / Kang et al, 2106.15624 / ...)
  - $\ell\, N \to \ell\, \mathrm{jet}\, \mathrm{X} \qquad \ell\, N \to \ell\, (h, \mathrm{jet})\, \mathrm{X} \qquad p\, p \to (h, \mathrm{jet})\, \mathrm{X} \quad \ldots$
  - particularly relevant for EIC and RHIC

- Higher-order pQCD corrections and power corrections for DVCS (→ talk Braun)
  (Braun et al, 2207.06818 / Ji, Schoenleber, 2310.05724 / Braun, Ji, Manashov, 2211.04902 / ...)
  - needed for precision studies
  - corrections can be sizeable

- Extraction of EMT form factor $D(t)$ from DVCS data
  (Burkert, Elouadrhiri, Girod, Nature 2018, 2310.11568 / ...)
  - pressure distribution inside proton
  - mechanical radius of the proton: $r_{p,\mathrm{RMS}}^{\mathrm{mech}} = (0.634 \pm 0.057)\,\mathrm{fm}$

- Threshold production of quarkonium and gluon EMT form factors
  (→ talks Diehl, Meziani, Joosten, Pefkou)
  - recent measurements of J/ψ production from JLab (Ali et al (GlueX), 1905.10811 / ...)
  - reliably information about gluon EMT form factors from those data ?
    (Boussarie, Hatta, 2004.12715 / Sun, Tong, Yuan, 2111.07034 / Guo, Ji, Yuan, 2308.13006 / ...)
  - phenomenology based on experimental results (Duran et al, Nature 2023 / ...)
  - LQCD does significantly contribute (Hackett, Pefkou, Shanahan, 2310.08484 / ...)

- Anomaly contributions in inclusive DIS and DVCS (→ talk Bhattacharya)
  (Tarasov, Venugopalan, 2008.08104, ... / Bhattacharya, Hatta, Vogelsang, 2210.13419, 2305.09431)

Conclusions

- Nucleon structure studies have a long and successful history
- 3D structure of hadronic systems (nucleons, nuclei, pions, ...), expressed through GPDs and TMDs, is an extremely rich and dynamic field (tremendous progress)
- Getting reliable information on the 3D parton structure from experiment remains complicated
- New theory developments may help: higher-order corrections, novel processes, modern data science tools, ...
- Models/approximations can help to further elucidate the underlying physics and guide experiments
- Future experimental data, especially from the EIC, can move the field to the next level
- LQCD already has made significant contributions to the field
- Combining information from experiment and LQCD, wherever appropriate, will further the field