Electroweak structure of light nuclei and its implication for neutrino scattering



Electromagnetic interactions with nucleons and nuclei EINN 2023 Paphos, Cyprus

1 November 2023 Saori Pastore

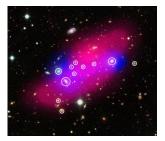
https://physics.wustl.edu/quantum-monte-carlo-group

Quantum Monte Carlo Group @ WashU Lorenzo Andreoli (PD) Jason Bub (GS) Graham Chambers-Wall (GS) Garrett King (GS) Anna McCoy (FRIB TA Fellow) Maria Piarulli and Saori Pastore

Computational Resources awarded by the DOE ALCC, INCITE and SciDAC programs

Understand Nuclei to Understand the Cosmos





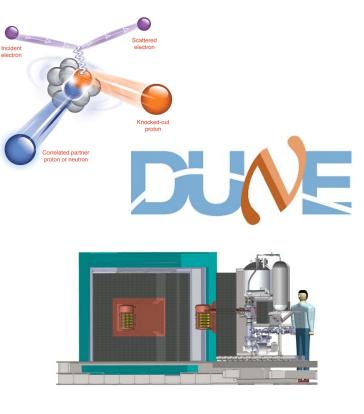
ESA, XMM-Newton, Gastaldello, CFHTL

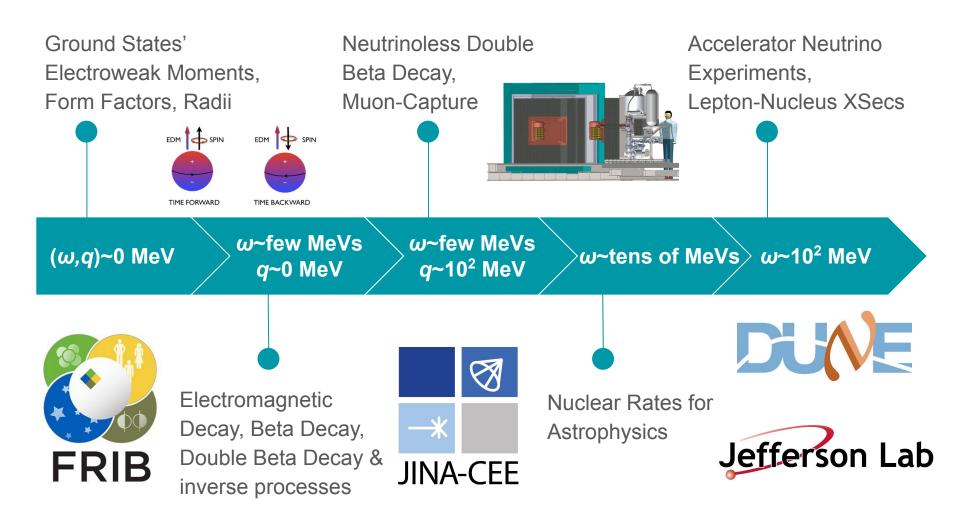


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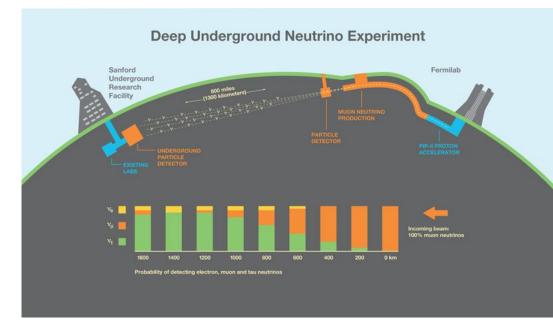
ar distances repeating up to 10,000 rates the trackar distances. If the electron cloud were shown to acits, this clust would corest a small town.



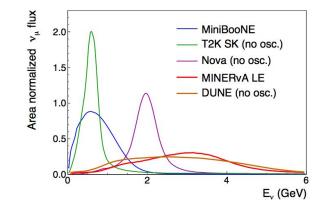


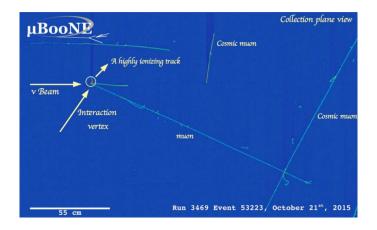


Accelerator Neutrinos' Experiments

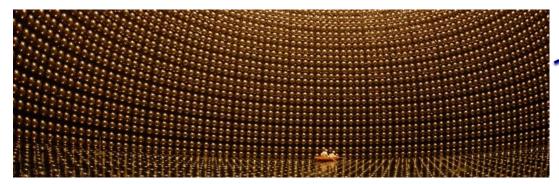


DUNE - Fermilab

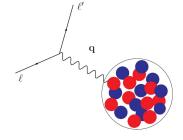




Nuclei for Neutrino Oscillations' Experiments

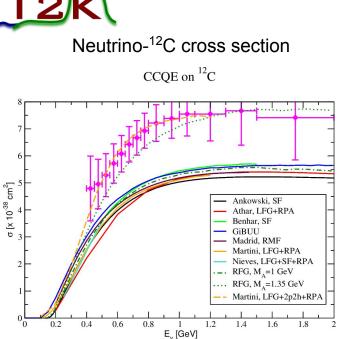


$$P(\mathbf{v}_{\mu} \to \mathbf{v}_{e}) = \sin^{2}2\theta \sin^{2}\left(\frac{\Delta m_{21}^{2}L}{2\mathbf{E}_{v}}\right)$$



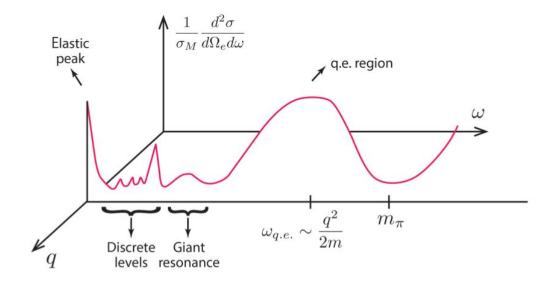
Nuclei are the active material in

the detector. The energy of the incident neutrino is reconstructed from the observed final states using **neutrino event generators** that require **theoretical cross-sections**.



Alvarez-Ruso arXiv:1012.3871

Electron-Nucleus Scattering Cross Section



Energy and momentum transferred (ω ,q)

Current and planned experimental programs rely on theoretical calculations at different kinematics

Strategy

Validate the Nuclear Model against available data for strong and electroweak observables

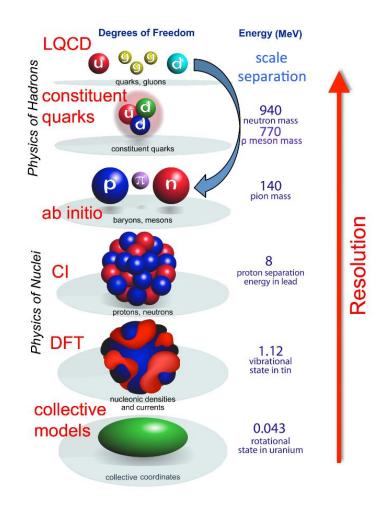
- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- Electron-Nucleus Scattering Cross Sections, ...

Use attained information to make (accurate) predictions for BSM searches and precision tests

- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- Neutrino-Nucleus Scattering Cross Sections, ...
- ...

From Quarks to Nuclei

- Nuclei are complex systems made of interacting protons and neutrons, which in turns are composite objects made of interacting constituent quarks.
- All fundamental forces are at play in nuclei.
- **EFTs** low-energy approximations of QCD whose d.o.f. are bound states of QCD (e.g., protons, neutrons, pions, ...)
- **EFTs** are used to construct many-nucleon interactions and currents



Microscopic (or ab initio) Description of Nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

- Accurate understanding of the interactions/correlations between nucleons in **paris**, **triplets**, ... (two- and three-nucleon forces)
- Accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated nucleon-pairs, ... (one- and two-body electroweak currents)
- **Computational methods** to solve the many-body nuclear problem of strongly interacting particles



Erwin Schrödinger

 $H\Psi = E\Psi$

Many-body Nuclear Problem

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

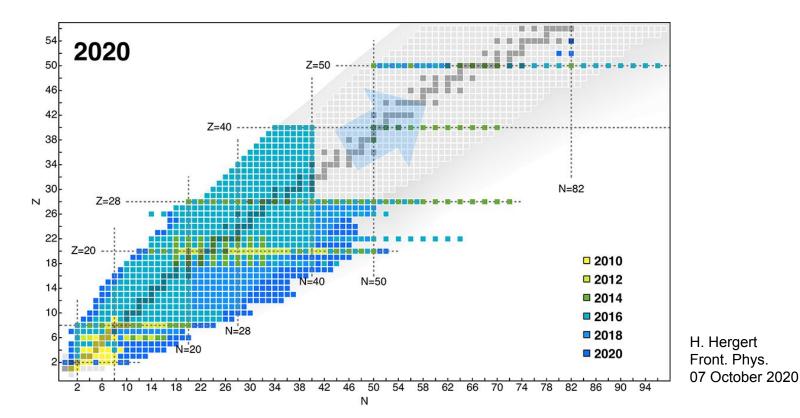
$$\Psi(\mathbf{r}_1,\mathbf{r}_2,...,\mathbf{r}_A,\mathbf{s}_1,\mathbf{s}_2,...,\mathbf{s}_A,\mathbf{t}_1,\mathbf{t}_2,...,\mathbf{t}_A)$$



$$\Psi$$
 are spin-isospin vectors in 3A dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components
⁴He : 96
⁶Li : 1280
⁸Li : 14336
¹²C : 540572

(numerically) exactly or within approximations that are under control the many-body nuclear problem

Current Status

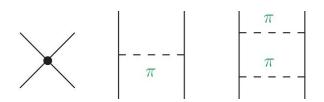


Many-body Nuclear Interactions

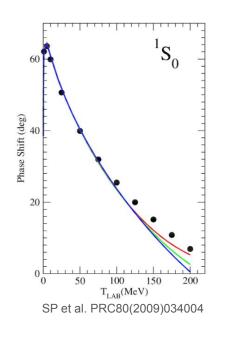
Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

 v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



Contact term: short-range Two-pion range: intermediate-range $r\propto (2\,m_\pi)^{-1}$ One-pion range: long-range $r\propto m_\pi^{-1}$



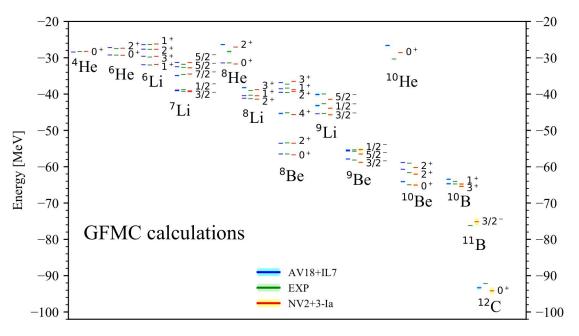


Hideki Yukawa

AV18+UIX; AV18+IL7 Wiringa, Schiavilla, Pieper *et al.*

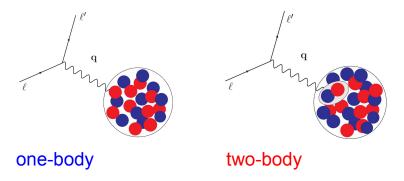
chiral πNΔ N3LO+N2LO Piarulli *et al.* Norfolk Models

Energies



Piarulli et al. PRL120(2018)052503

Many-body Nuclear Electroweak Currents



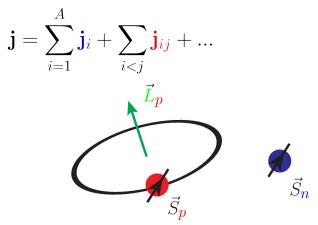
- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i < j} \rho_{ij} + \dots$$

Nuclear (Vector) Current Operator



Magnetic Moment: Single Particle Picture

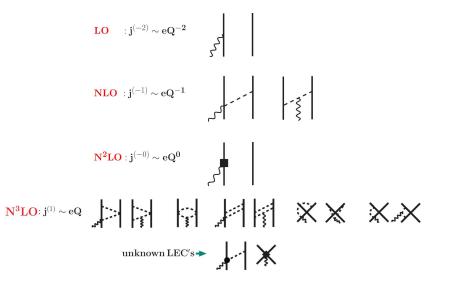
Many-body Currents

• Meson Exchange Currents (MEC)

Constrain the MEC current operators by imposing that the current conservation relation is satisfied with the given two-body potential

Chiral Effective Field Theory Currents

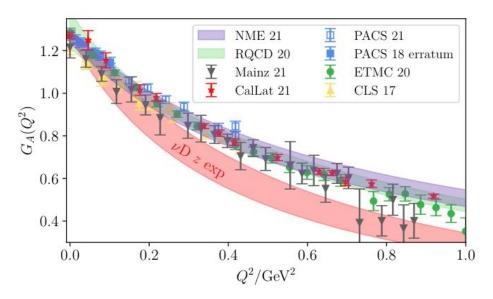
Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (LECs), need to be determined by either fits to experimental data or by Lattice QCD calculations



Electromagnetic Current Operator

SP *et al.* PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001, PRC87(2013)014006 Park *et al.* NPA596(1996)515, Phillips (2005) Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

LCQD inputs for neutrino-nucleus scattering



Snowmass WP: Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators; arXiv:2203.09030

Building blocks of ab initio nuclear approaches:

Nucleonic form factors Transition form factors Pion production amplitudes Two-nucleon couplings (strong and EW)

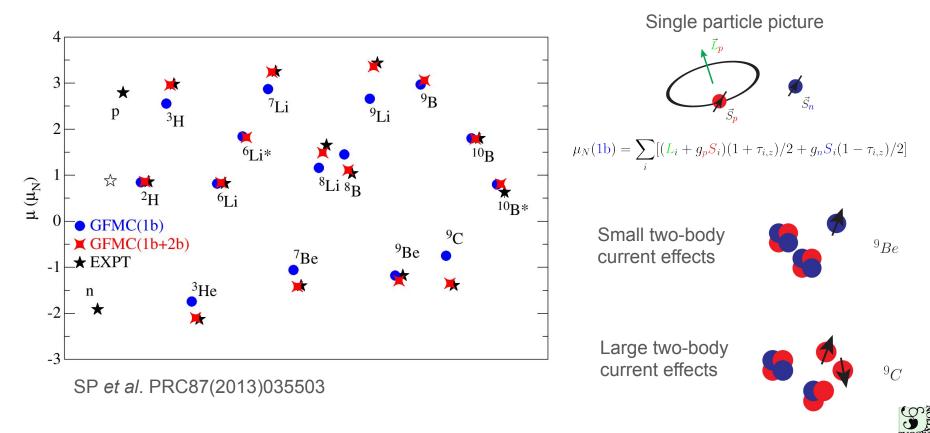
. . .

Taken from data where available, or from theory

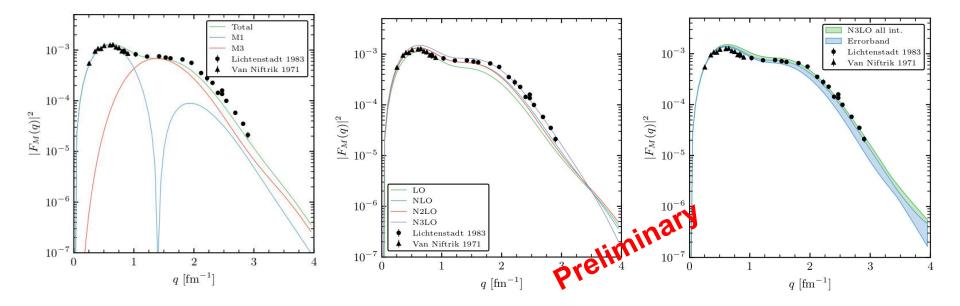
See talks by Bacchio, Koutsou, Tomalak, ...

Meyer, Walker-Loud, Wilkinson (2022)

Magnetic Moments of Light Nuclei

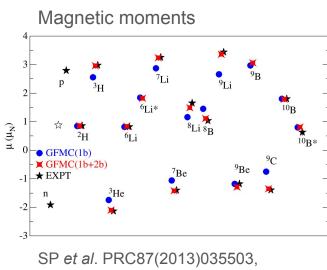


Electromagnetic form factors from chiEFT

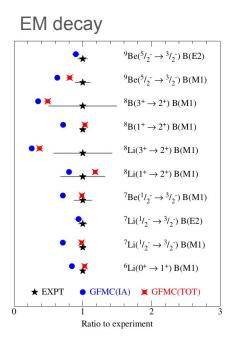


⁷Li magnetic form factor - A. Gnech, G. Chambers-Wall, G. King *et al.* (in preparation)

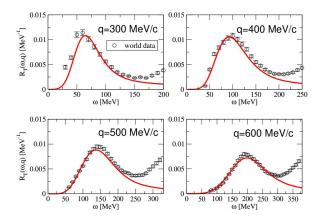
Electromagnetic Observables



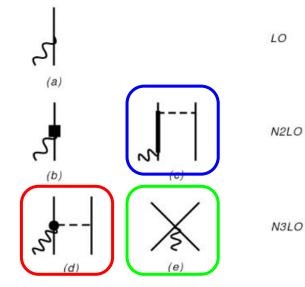
PRC101(2020)044612



e-⁴He particle scattering

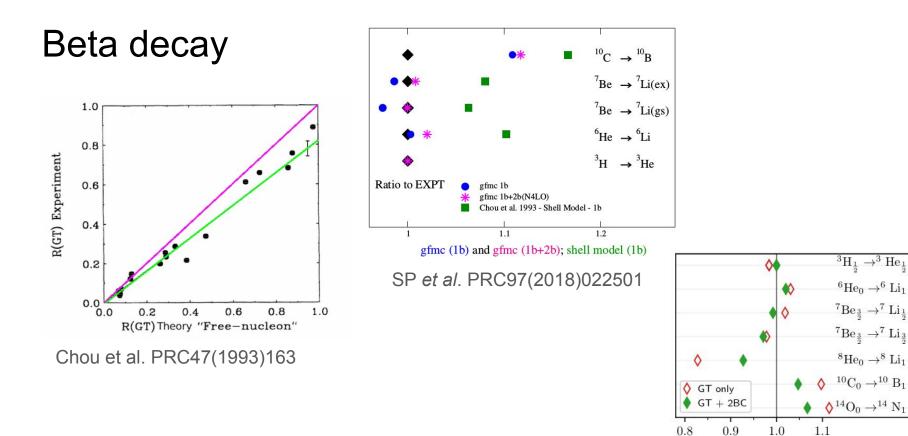


Axial currents with Δ at tree-level



Two body currents of one pion range (red and blue) with $c_3 c_4$ from Krebs *et al.* Eur.Phys.J.(2007)A32

Contact current involves the LEC c_p



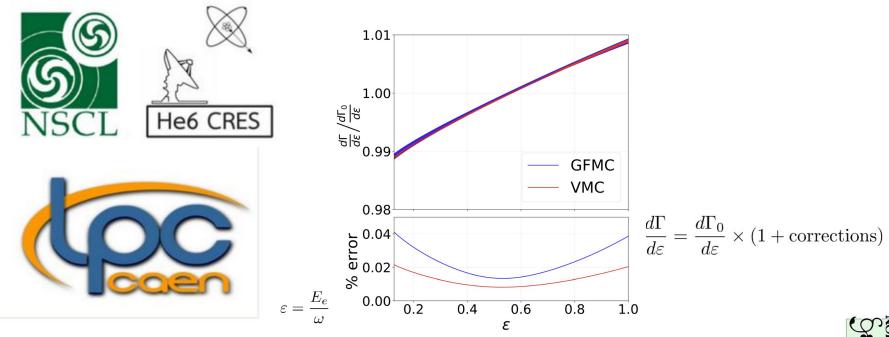
P. Gysbers Nature Phys. 15 (2019)

 $|M_{\rm GT}|$ ratio to experiment

 ${}^{8}\mathrm{He}_{0} \rightarrow {}^{8}\mathrm{Li}_{1}$

Beta decay spectrum

⁶He Beta decay spectrum for BSM searches with NCSL, He6-CRES, LPC-Caen



Standard Model spectrum for ⁶He

Garrett King et al. PRC (2023)

Partial muon capture rates: VMC calculations

$$\Gamma_{VMC}(avg.) = 1495 \text{ s}^{-1} \pm 19 \text{ s}^{-1}$$

 $\Gamma_{expt} = 1496.0 \text{ s}^{-1} \pm 4.0 \text{ s}^{-1}$

Ackerbauer et al. PLB417, 224(1998)

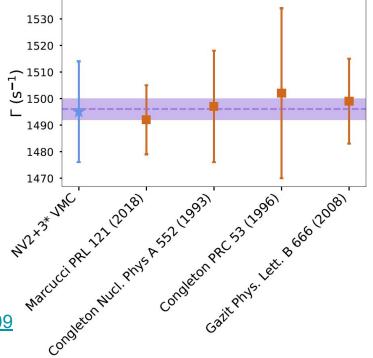
Momentum transfer q~ 100 MeV

Two-body correction is \sim 8% of total rate on average for A=3

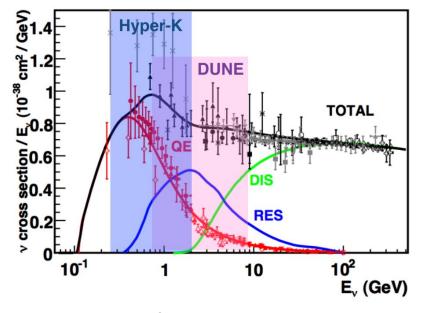
Garrett King et al. PRC2022

Review by Measday Physics Reports 354 (2001) 243-409

 ${}^{3}\text{He}(1/2^{+};1/2) \rightarrow {}^{3}\text{H}(1/2^{+};1/2)$



Neutrino cross section anatomy



Formaggio & Zeller

Quasi-elastic: dominated by single-nucleon knockout

Resonance: excitation to nucleonic resonant states which decay into mesons

Deep-inelastic scattering: where the neutrino resolves the nucleonic quark content

Each of these regimes requires knowledge of both the **nuclear ground state** and the **electroweak coupling** and **propagation of the struck nucleons, hadrons, or partons**

A challenge for achieving precise neutrino-nucleus cross-section is **reliably bridging the transition regions which use different degrees of freedom**

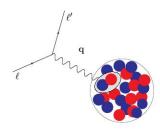
Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) |\langle f|O_{\alpha}(\mathbf{q})|0\rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$ Transverse response induced by the current operator $O_T = j$ 5 Responses in neutrino-nucleus scattering

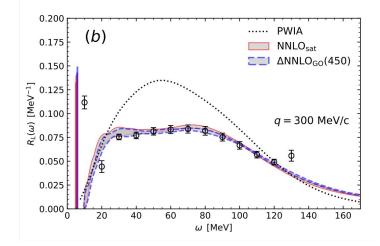
$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$



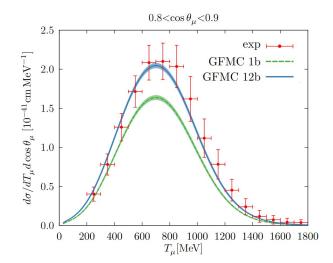
For a recent review on QMC, SF methods see Rocco Front. In Phys.8 (2020)116

Inclusive Cross Sections with Integral Transforms

Exploit integral properties of the response functions and closure to avoid explicit calculation of the final states (Lorentz Integral Transform **LIT**, **Euclidean**, ...)



Sobczyk et al, PRL127 (2021)



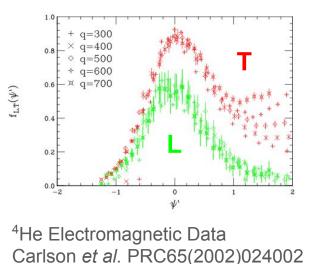
Lovato et al. PRX10 (2020)

Lepton-Nucleus scattering: Data

5

Transverse Sum Rule

 $S_T(q) \propto \langle 0 | \mathbf{j}^{\dagger} \mathbf{j} | 0 \rangle \propto \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{1b} | 0 \rangle + \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{2b} | 0 \rangle + \dots$



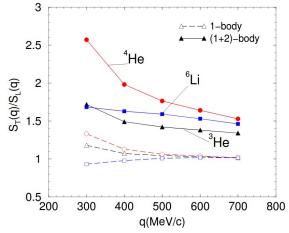
Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term

$$\langle \mathbf{j}_{1b}^{\dagger} \ \mathbf{j}_{1b} \rangle > 0$$

Leading one-body term

$$\langle \mathbf{j}_{1b}^{\dagger} \; \mathbf{j}_{2b} \; v_{\pi} \rangle \propto \langle v_{\pi}^2 \rangle > 0$$

Interference term

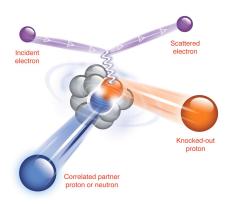


Transverse/Longitudinal Sum Rule Carlson *et al.* PRC65(2002)024002

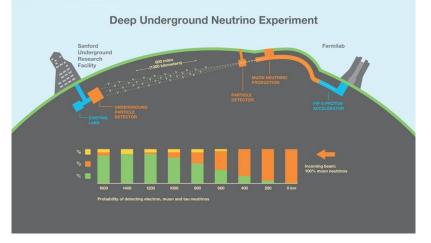
Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A
 > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



Subedi et al. Science320(2008)1475



Stanford Lab article



Short-Time-Approximation

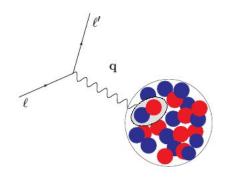
Short-Time-Approximation:

- Based on Factorization (also SF by Noemi Rocco)
- Retains two-body physics
- Correctly accounts for interference

$$R(q,\boldsymbol{\omega}) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\boldsymbol{\omega}+E_0)t} \langle 0|O^{\dagger} e^{-iHt} O|0\rangle$$

$$O_i^{\dagger} e^{-iHt} O_i + O_i^{\dagger} e^{-iHt} O_j + O_i^{\dagger} e^{-iHt} O_{ij} + O_{ij}^{\dagger} e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$



STA: regime of validity

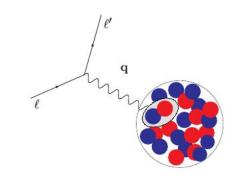
The typical (conservative estimate) energy (time) scale in a nucleus with A correlated nucleons in pairs is

$$\epsilon_{pair} \sim 20 \text{ MeV}$$
 (t ~ 1/ ϵ_{pair})

This sets a natural expansion parameter in the QE region characterized by ω_{QE}

 ϵ_{pair} / ω_{QE}

The STA neglects terms of order $O((\epsilon_{pair} / \omega_{QE})^2)$



Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization
- Retains two-body physics
- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions ∞ Cross Sections

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) \left|\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle\right|^2$$

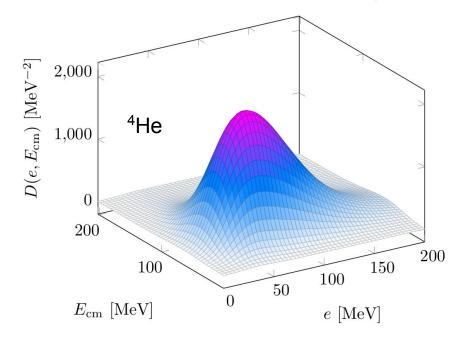
Response *Densities*

$$R(q,\omega) \sim \int \delta \left(\omega + E_0 - E_f\right) dP' dp' \mathcal{D}(p',P';q)$$

P' and *p*' are the CM and relative momenta of the struck nucleon pair

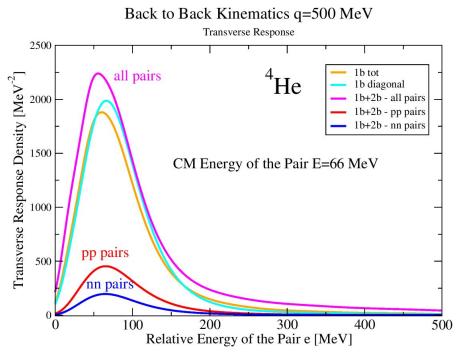
Transverse Response Density: *e*-⁴He scattering

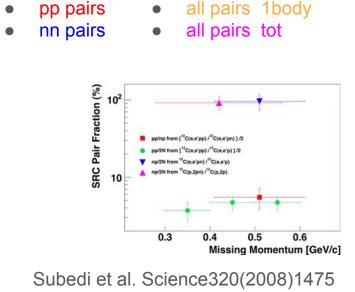
Transverse Density q = 500 MeV/c



SP et al. PRC101(2020)044612

e-⁴He scattering in the back-to-back kinematic

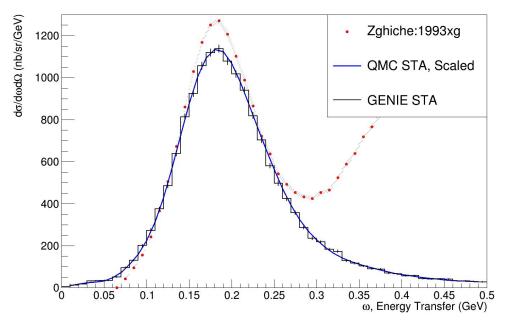




SP et al. PRC101(2020)044612

GENIE validation using e-scattering

Z = 2, A = 4, Beam Energy = 0.64 GeV, Angle = 60° ± 0.25°

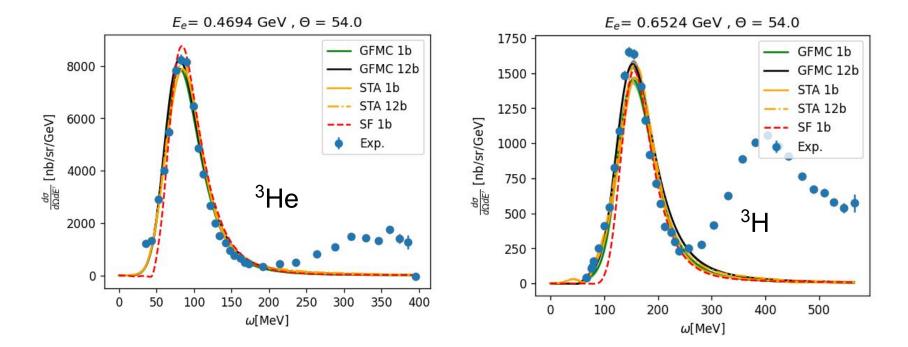


- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE (a Monte Carlo neutrino event generator)
- Here, we use electromagnetic processes (for which data are available) to validate the generator

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Barrow, Gardiner, SP et al. PRD 103 (2021) 5, 052001

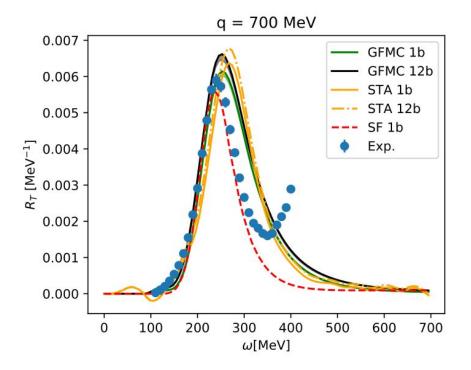
GFMC SF STA: Benchmark & error estimate



Lorenzo Andreoli, et al. PRC 2021



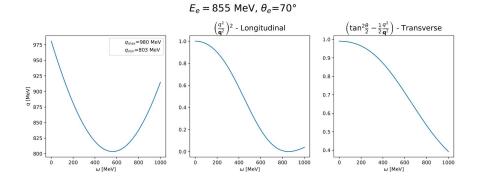
Importance of relativistic corrections

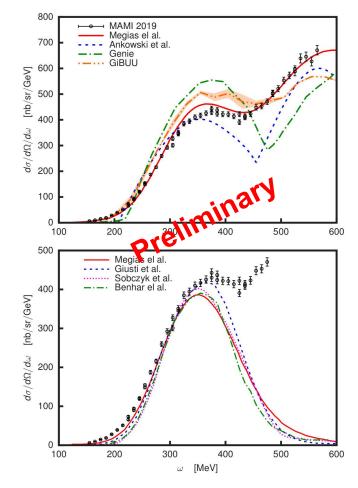


Lorenzo Andreoli, et al. PRC 2021

(Available) Mainz kinematics

$$E_e = 855 \text{ MeV}$$
 $\theta_e = 70$







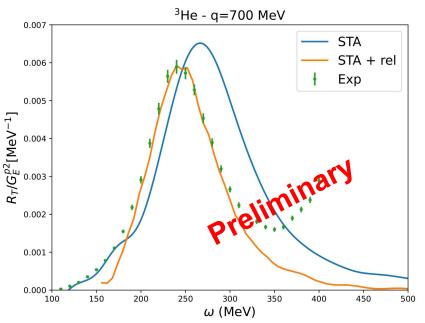
Relativistic corrections

So far we used non relativistic reduction of the single nucleon covarian current (low-momentum expansion both p and p')

With Ronen Weiss

Relativistic corrections obtained expanding the covariant one-nucleon current for high values of momentum transfer **q**

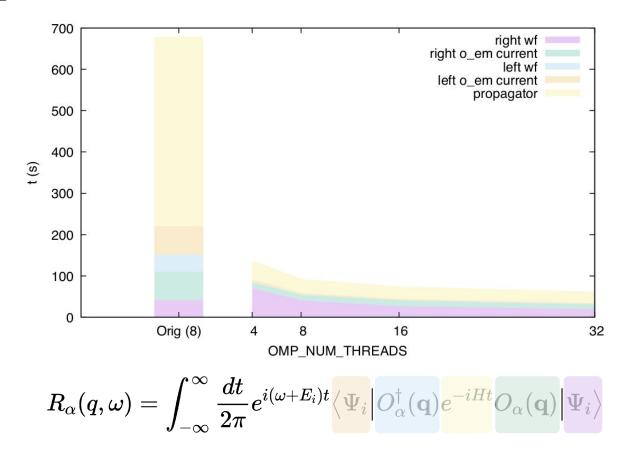
$$j^{\mu}=ear{u}ig(oldsymbol{p}'s'ig)ig(e_N\gamma^{\mu}+rac{i\kappa_N}{2m_N}\sigma^{\mu
u}q_{
u}ig)u(oldsymbol{p}s)$$



R. Weiss & L. Andreoli *et al.* (in preparation)

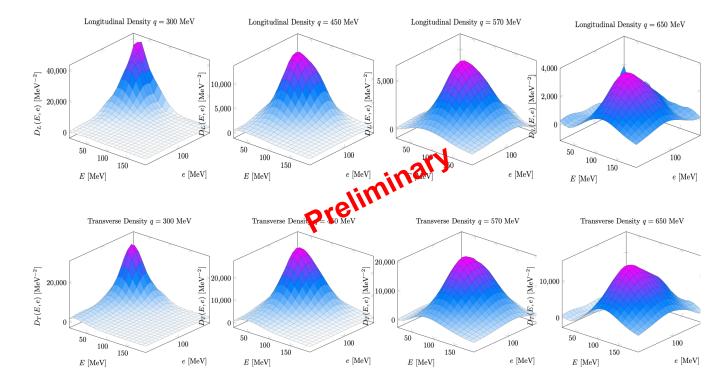
 $oldsymbol{p}' = oldsymbol{p} + oldsymbol{q}$

Towards A=12



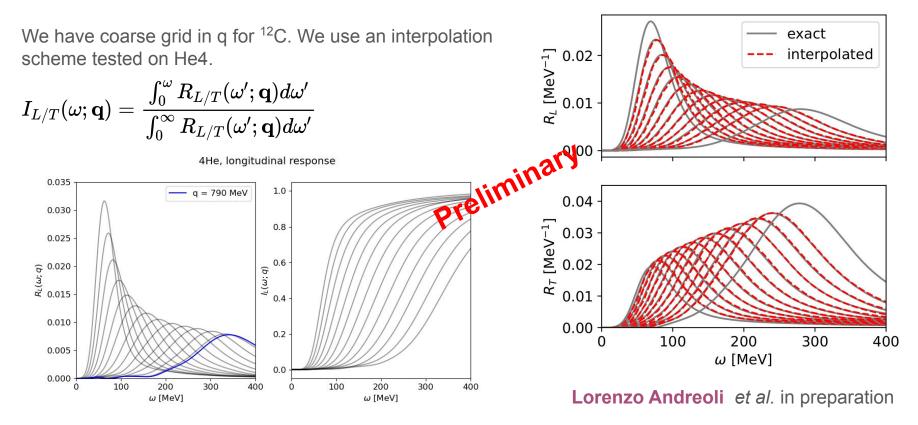
Lorenzo Andreoli et al. in preparation

¹²C Response Densities

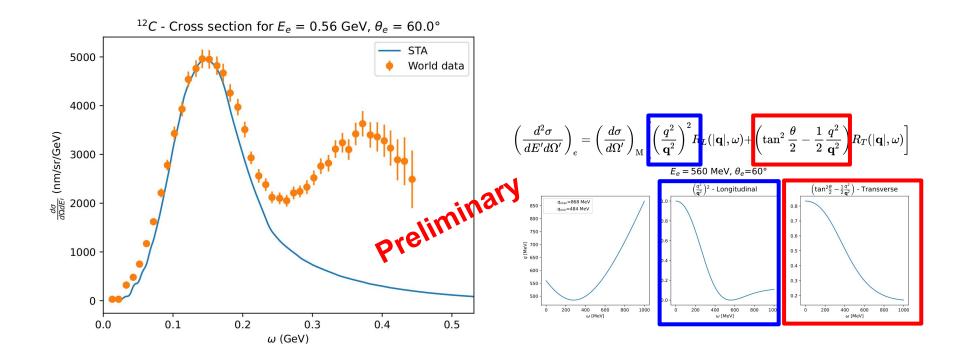


Lorenzo Andreoli et al. in preparation

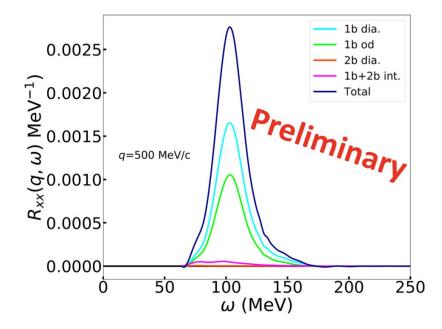
¹²C cross sections: interpolation scheme



¹²C cross sections

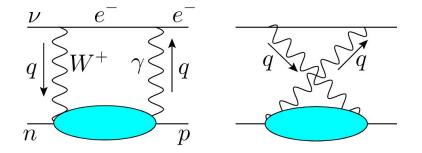


NC processes on deuteron with STA



Garrett King et al. in preparation

Ties to fundamental symmetry: CKM unitarity



Superallowed beta decay used to test CKM unitarity

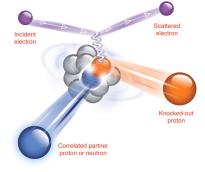
resonance

Momentum distributions



Nuclear properties are strongly affected by two-body correlations and currents in a wide range of energy and momentum transfer

Two-body momentum distribution Wiringa et al. PRC89(2014)024305 12**C** ⁸B^r 61 ⁴He $\rho_{pN}(q,Q=0) \ (fm^3)$ 10^{5} 10^{3} 10^{1} 10-3 $q (fm^{-1})$



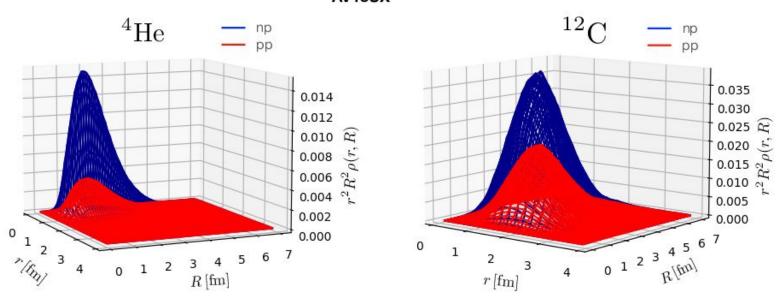
New two-body momentum distributions from chiEFT potentials

Piarulli, SP, Wiringa PRC (2023)



pp-pairs; np-pairs

Two-body densities



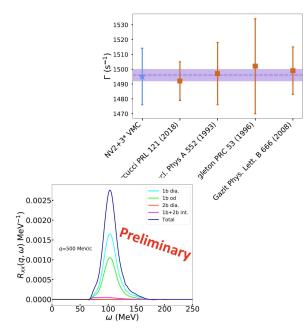
AV18UX

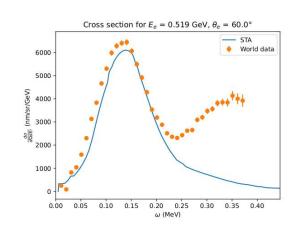
Piarulli, SP, Wiringa PRC (2023)

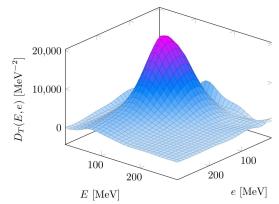
<u>data</u>

Summary

Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where many-body effects play an essential role to explain available data.







Transverse Density q = 570 MeV

Comp, Expt, ... are required to progress e.g., NP is represented in the Snowmass process and LRP

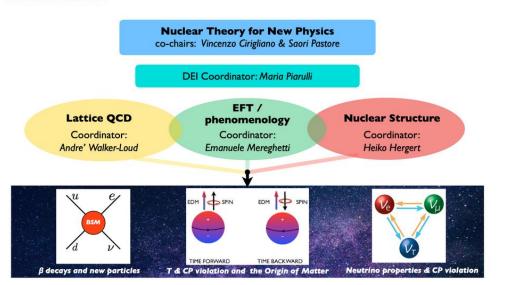
It's a very exciting time!

Nuclear Theory for New Physics NP&HEP TC

Nuclear Theory for New Physics

About Us

- Commitment to Diversity
- Funding Acknowledgement



Snowmass:

Topical groups and Frontier Reports, Whitepapers, ...

LRP: White papers, 2301.03975, FSNN,

. . .

Funding Acknowledgement



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Collaborators

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Theory Alliance facility for rare isotope beams























Quantum Monte Carlo Group for Nuclear Physics

https://physics.wustl.edu/quantum-monte-carlo-group





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Quantum Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: $H = T + v_{ii} + V_{iik}$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \ge E_V$$

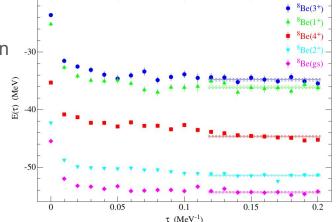
using the trial wave function:

$$|\Psi_V\rangle = \left[\mathcal{S}\prod_{i< j} (1 + U_{ij} + \sum_{k\neq i,j} U_{ijk})\right] \left[\prod_{i< j} f_c(r_{ij})\right] |\Phi_A(JMTT_3)\rangle$$

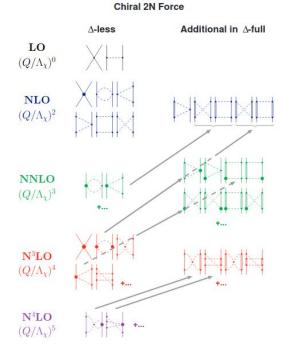
Further improve the trial wave function by eliminating spurious contaminations via a Green's Function Monte Carlo propagation in imaginary time

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n\psi_n$$
$$\Psi(\tau \to \infty) = a_0\psi_0$$

Carlson, Wiringa, Pieper et al.



Norfolk Two- and Three-body Potentials

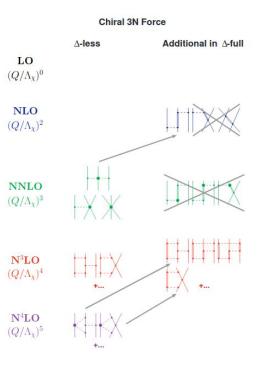


Norfolk Chiral Potentials

NV2+3

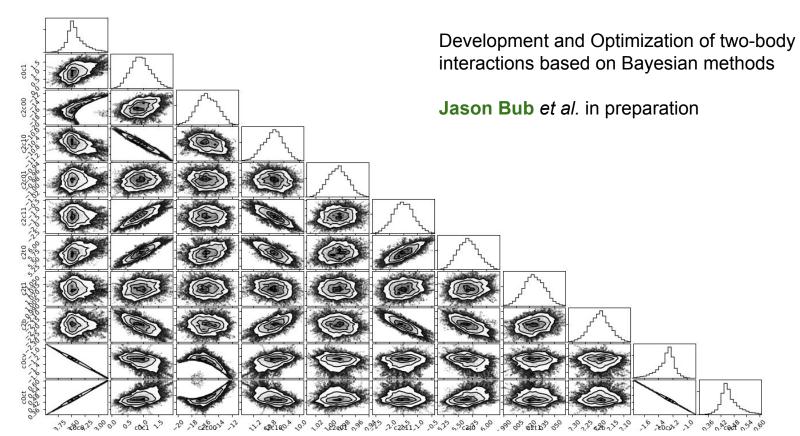
developed in Piarulli *et al.* PRC91(2015)024003 PRC94(2016)054007

26 LECs fitted to np and pp Granada database (2700-3700 data points; 125-200 MeV) with a chi-square/datum ~1

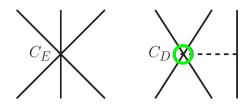


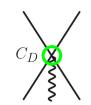
Figs. credit Entem and Machleidt Phys.Rept.503(2011)1

Optimization of Nuclear Two-body Interactions



Three-body Force and the Axial Contact Current





Three-body force

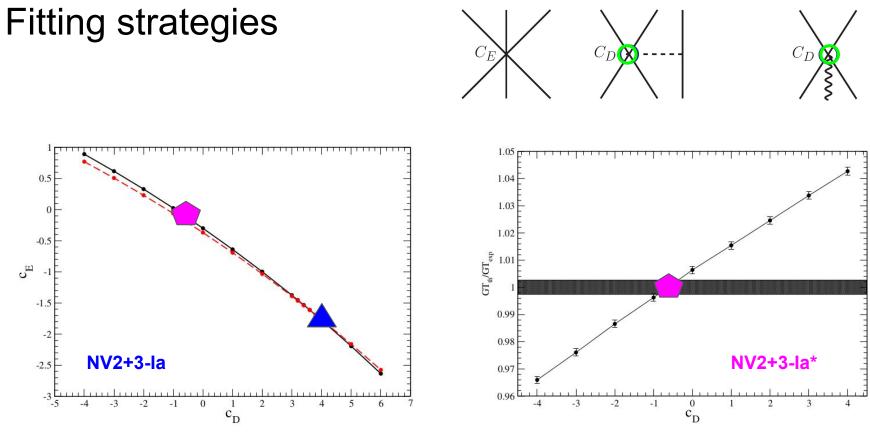
Axial two-body contact current

LECs c_{D} and c_{E} are fitted to:

- trinucleon B.E. and nd doublet scattering length in NV2+3-la
- trinucleon B.E. and Gamow-Teller matrix element of tritium NV2+3-la*

Baroni *et al.* PRC98(2018)044003

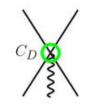
Energies A=8-10 slightly better with non-starred models



Baroni et al. PRC98(2018)044003

Contact Current

	NV2+3-Ia	NV2+3-Ia*
c_D	3.666	-0.635
c_E	-1.638	-0.090
z_0	0.090	1.035



$$\mathbf{j}_{5,a}^{\text{N3LO}}(\mathbf{q};\text{CT}) = \mathbf{z_0} e^{i\mathbf{q}\cdot\mathbf{R}_{ij}} \frac{e^{-\tilde{r}_{ij}^2}}{\pi^{3/2}} \left(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j\right)_a \left(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j\right)$$

$$z_{0} = \frac{g_{A}}{2} \frac{m_{\pi}^{2}}{f_{\pi}^{2}} \frac{1}{(m_{\pi}R_{S})^{3}} \left[-\frac{m_{\pi}}{4g_{A}\Lambda_{\chi}} c_{D} + \frac{m_{\pi}}{3} (c_{3} + 2c_{4}) + \frac{m_{\pi}}{6m} \right]$$

Beta Decay and Electron Capture in Light Nuclei

0.96 1 1.04	0.96 1 1.04	0.96 1 1.04	0.96 1 1.04
3 H β -decay	⁶ He β-decay	⁷ Be ε -cap(gs)	⁷ Be ε -cap(ex)
0	()	0•	0
⁸ Li β-decay	⁸ B β-decay	⁸ He β-decay	¹⁰ C β-decay
0 •	0•	0	••
■ NV2+3-Ia NV2+3-Ia* NV2+3-Ia* NV18+IL7			
0.4 0.6 0.8 1	0.4 0.6 0.8 1	0.4 0.6 0.8 1	1 1.1

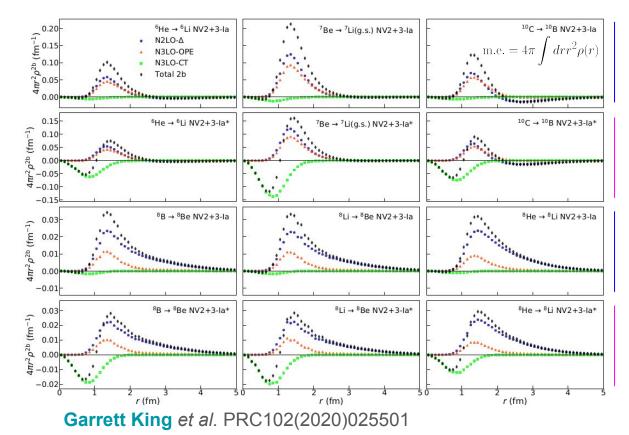
Garrett King et al. PRC102(2020)025501

Calculations based on

- chiral interactions and currents NV2+3-la Norfolk unstarred NV2+3-la* Norfolk* starred Piarulli *et al.* PRL120(2018)052503 Baroni *et al.* PRC98(2018)044003
- phenomenological AV18+IL7 potential and chiral axial currents (hybrid calculation)

Two-body currents are small/negligible; Results for A=6-7 are within 2% of data; Results for A=8 are off by a 30-40%; Results for A=10 are affected by the second J^{π} =(1⁺) state in ¹⁰B

Axial Two-body Transition Density



NV2+3-la; NV2+3-la*

enhanced contribution from contact current in the starred model gives rise to nodes in the two-body transition density

Two-body axial currents

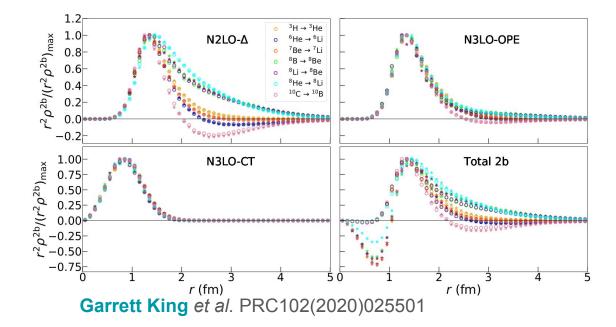


long-range at N2LO and N3LO

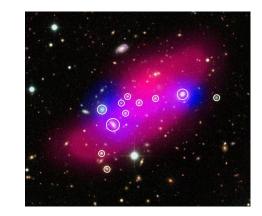


contact current at N3LO

Scaling & Universality of Short-Range Dynamics



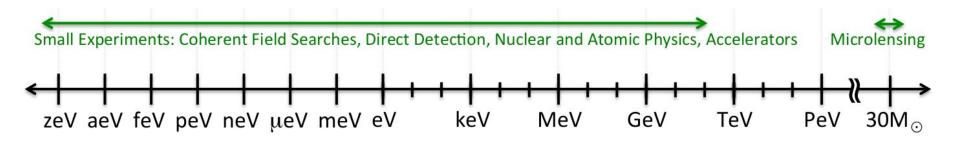
NV2+3-Ia empty circles; NV2+3-Ia* stars Different colors refer to different transitions



Candidates

Dark Matter

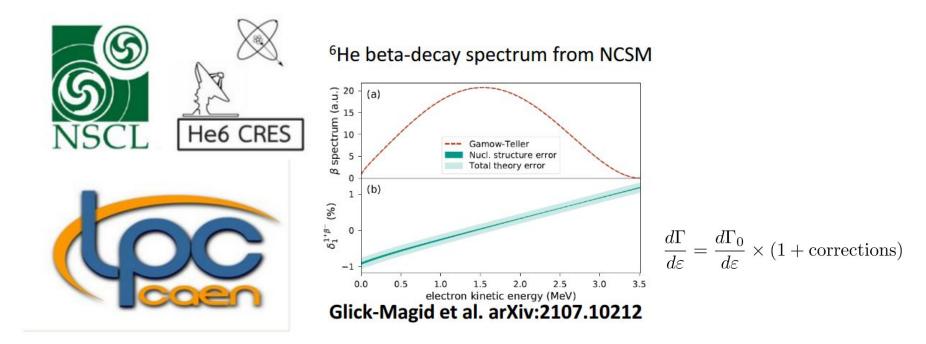
ESA, XMM-Newton, Gastaldello, CFHTL



US cosmic vision 2017

Beta decay spectrum

⁶He Beta decay spectrum for BSM searches with NCSL, He6-CRES, LPC-Caen



⁶He Beta Decay Spectrum

$$d\Gamma = \frac{2\pi}{2J_i + 1} \sum_{s_e, s_\nu} \sum_{M_i, M_f} |\langle f | H_W | i \rangle|^2 \delta(\Delta E) \frac{d^3 k_e}{(2\pi)^3} \frac{d^3 k_\nu}{(2\pi)^3}$$

Multipoles

$$C_{1}(q; A) = \frac{i}{\sqrt{4\pi}} \langle {}^{6}\mathrm{Li}, 10|\rho_{+}^{\dagger}(q\hat{\mathbf{z}}; A)|{}^{6}\mathrm{He}, 00 \rangle$$

$$L_{1}(q; A) = \frac{i}{\sqrt{4\pi}} \langle {}^{6}\mathrm{Li}, 10|\hat{\mathbf{z}} \cdot \mathbf{j}_{+}^{\dagger}(q\hat{\mathbf{z}}; A)|{}^{6}\mathrm{He}, 00 \rangle$$

$$E_{1}(q; A) = -\frac{i}{\sqrt{2\pi}} \langle {}^{6}\mathrm{Li}, 10|\hat{\mathbf{z}} \cdot \mathbf{j}_{+}^{\dagger}(q\hat{\mathbf{x}}; A)|{}^{6}\mathrm{He}, 00 \rangle$$

$$M_{1}(q; V) = -\frac{1}{\sqrt{2\pi}} \langle {}^{6}\mathrm{Li}, 10|\hat{\mathbf{y}} \cdot \mathbf{j}_{+}^{\dagger}(q\hat{\mathbf{x}}; V)|{}^{6}\mathrm{He}, 00 \rangle$$

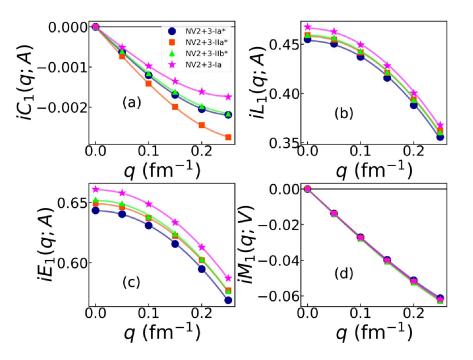
$$C_{1}(q;A) = -i\frac{qr_{\pi}}{3} \left(C_{1}^{(1)}(A) - \frac{(qr_{\pi})^{2}}{10} C_{1}^{(3)}(A) + \mathcal{O}\left((qr_{\pi})^{4}\right) \right)$$

$$L_{1}(q;A) = -\frac{i}{3} \left(L_{1}^{(0)}(A) - \frac{(qr_{\pi})^{2}}{10} L_{1}^{(2)}(A) + \mathcal{O}\left((qr_{\pi})^{4}\right) \right)$$

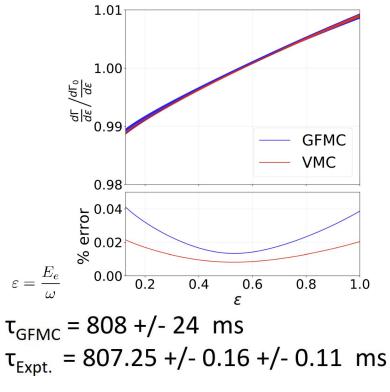
$$M_{1}(q;V) = -i\frac{qr_{\pi}}{3} \left(M_{1}^{(1)}(V) - \frac{(qr_{\pi})^{2}}{10} M_{1}^{(3)}(V) + \mathcal{O}\left((qr_{\pi})^{4}\right) \right)$$

$$E_{1}(q;A) = -\frac{i}{3} \left(E_{1}^{(0)}(A) - \frac{(qr_{\pi})^{2}}{10} E_{1}^{(2)}(A) + \mathcal{O}\left((qr_{\pi})^{4}\right) \right)$$

Beta Decay Spectrum



Dominant terms $L_{1^{(0)}}$ and $E_{1^{(0)}}$ have model dependence of ~1% to ~2%



Garrett King et al. PRC (2023)

