# Axial & trace anomalies in DVCS


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In Collaboration with:


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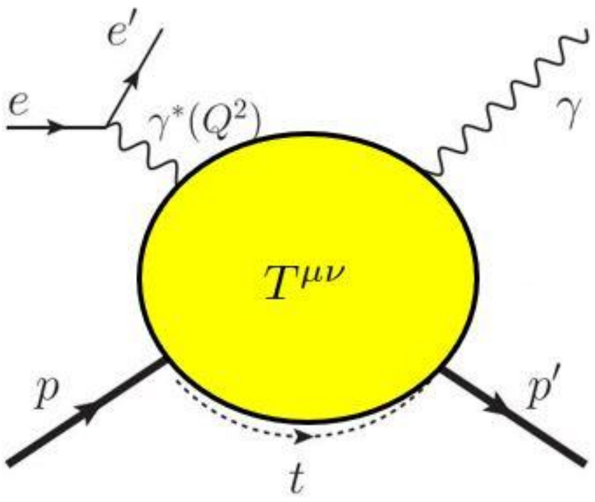
**Werner Vogelsang (Tubingen U.)**


Based on:
arXiv:2210.13419, 2305.09431

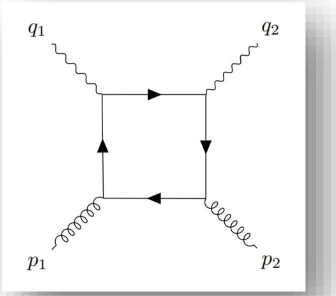

**Parallel Workshop 2**

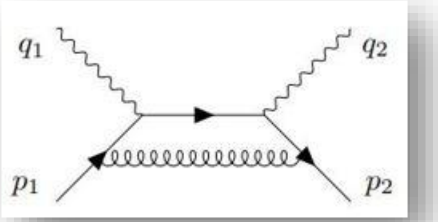
# Outline

- Recap on chiral & trace anomalies in QCD
- Connection between GPDs & anomalies:




Calculation of box diagrams relevant for Compton scattering:




- Polarized case
- Unpolarized case

# Outline

- **Recap on chiral & trace anomalies in QCD**

- **Connection between GPDs & anomalies:**




**Calculation of box diagrams relevant for Compton scattering:**







- Polarized case
- Unpolarized case

# Axial / Chiral anomaly

**Recap on chiral anomaly in QCD:**

- **Lagrangian invariant under global chiral rotation** $\psi \to e^{i\alpha\gamma_5}\psi$
- **Axial-vector current:** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

# Axial / Chiral anomaly

**Recap on chiral anomaly in QCD:**

- **Lagrangian invariant under global chiral rotation** $\psi \to e^{i\alpha\gamma_5}\psi$
- **Axial-vector current:** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$
- **But measure of the path integral is not invariant, which breaks the conservation of the axial current**

**K. Fujikawa, PRL 1979**

# Axial / Chiral anomaly

**Anomaly equation:**

$$\boxed{\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}} \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

**A fundamental property of axial-vector current is the anomaly equation**

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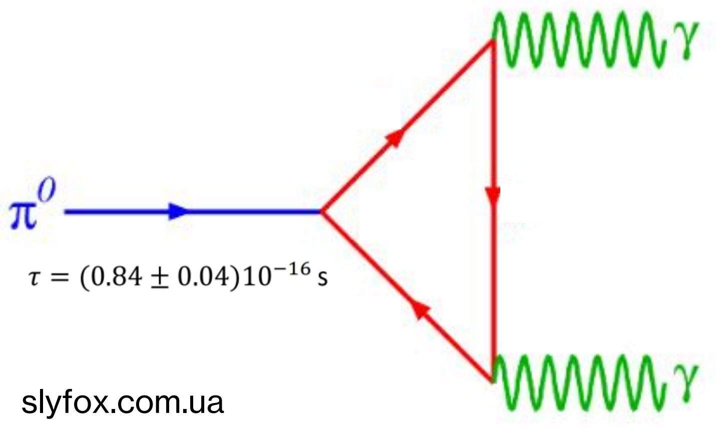
**A fundamental property of axial-vector current is the anomaly equation**

## Adler – Bell - Jackiw chiral anomaly

**Famous example: ABJ anomaly contribution to** $\pi^0 \to 2\gamma$




**In the chiral limit, without the anomaly,**

$\pi^0$ **does not decay!**

# Axial / Chiral anomaly

**Anomaly equation:**

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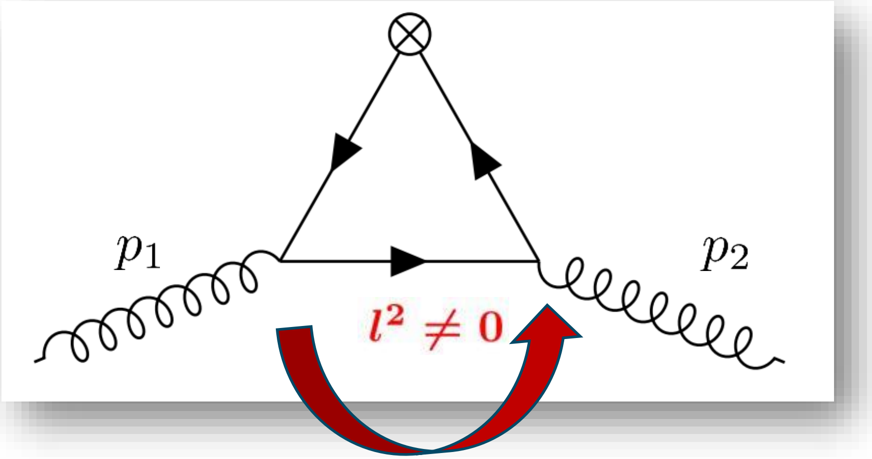
**A fundamental property of axial-vector current is the anomaly equation**

**A perturbative solution to anomaly equation:**




**Calculation in off-forward kinematics** $(l = p_2 - p_1)$**:**

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}^a_{\alpha\beta} | p_1 \rangle$$

**Triangle diagram is dominated by infra-red pole**

# Axial / Chiral anomaly

## Axial Form Factors:

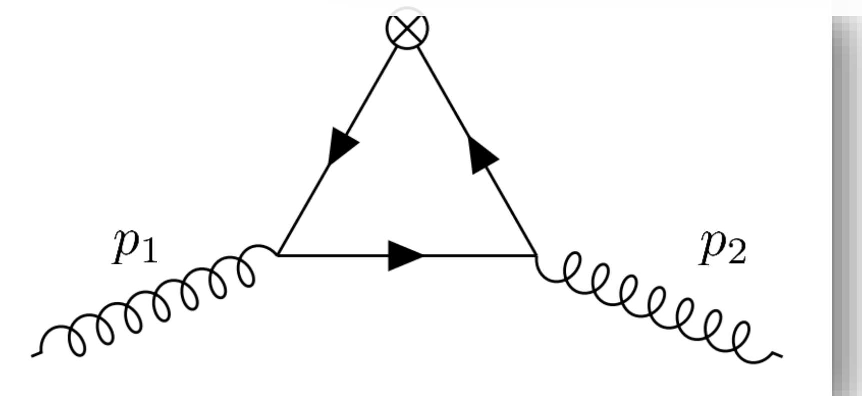
$$\boxed{\langle P_2|J_5^\mu|P_1\rangle = \bar{u}(P_2)\left[\gamma^\mu\gamma_5 g_A(l^2) + \frac{l^\mu\gamma_5}{2M} g_P(l^2)\right]u(P_1)}$$

A fundamental property of axial-vector current is the anomaly equation

**Massless pole in pseudo scalar Form Factor?** $g_P(l^2) \sim \dfrac{1}{l^2}$

A perturbative solutio

$g_A(0) = \Delta\Sigma$ **: Fraction of proton spin carried by quarks**

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**Massless pole in pseudo scalar Form Factor?** $g_P(l^2) \sim \cancel{\frac{1}{l^2}}$

**In QCD, we expect:** $g_P(l^2) \sim \dfrac{1}{l^2 - m_{\eta'}^2}$

Calculation in off-forward kinematics $(l = p_2 - p_1)$:

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**eta meson mass generation**

$$\frac{1}{l^2 - m_{\eta'}^2} \quad \eta'$$

Triangle diagram is dominated by infra-red pole

**Deeply tied to the UA(1) problem: Why is the $\eta'$ so massive (957 MeV!)?**

# Axial / Chiral anomaly

Axial Form Factors:

Twist-2 GPDs

| Pol. \ $\Gamma$ | $\gamma^+$ | $\gamma^+\gamma_5$ | $\sigma^{+j}\gamma_5$ |
|---|---|---|---|
| U | $H$ | | $E_T$ |
| L | | $\widetilde{H}$ | $\widetilde{E}_T$ |
| T | $E$ | $\widetilde{E}$ | $H_T\ \widetilde{H}_T$ |

**Any implications for the corresponding GPD?**

$$g_P(l^2) = \int_{-1}^{1} dx\, \boldsymbol{\tilde{E}(x,\xi,l^2)}$$

Deeply tied to the UA(1) problem: Why is the $\eta'$ so massive (957 MeV!)?

# Trace anomaly

**Recap on trace anomaly in QCD:**

- **Lagrangian invariant under scale transformation** $x^\mu \to e^\sigma x^\mu \quad \phi \to e^{-D\sigma}\phi$

- **Dilatation current:** $D^\mu = \Theta^{\mu\nu} x_\nu$ $\qquad \Theta^{\mu\nu}$ **: Energy Momentum Tensor (EMT)**

- **Conformal symmetry explicitly broken by quantum effects**

$$\partial_\mu D^\mu = \Theta^\mu_\mu \neq 0$$

# Trace anomaly

**Recap on trace anomaly in QCD:**

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance

**Trace anomaly:**

$$\boxed{\Theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}}$$

$\Theta^{\mu\nu}$ : Energy Momentum Tensor (EMT)

**Fundamentally important in QCD: Trace anomaly is the origin of hadron masses**

$$\langle P | \Theta^{\mu}_{\mu} | P \rangle = 2M^2$$

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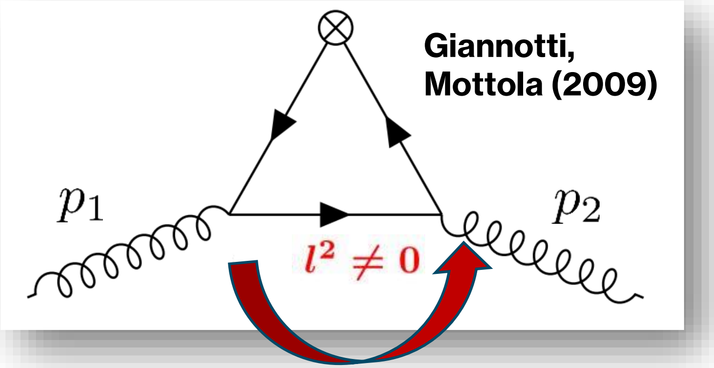



**Calculation in off-forward kinematics** $(l = p_2 - p_1)$**:**

$$\langle p_2 | \Theta^{\mu\nu} \quad | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left( p^{\mu} p^{\nu} + \frac{l^{\mu} l^{\nu} - l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{\alpha\beta} F_{\alpha\beta} | p_1 \rangle$$

**Triangle diagram is dominated by infra-red pole**

# Trace anomaly

Rec

## Gravitational Form Factors:

$$\boxed{\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)}$$

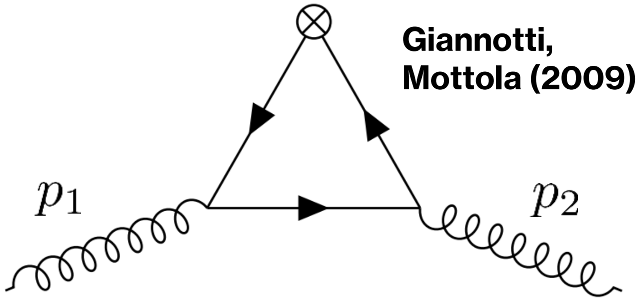
omaly)

**Massless poles in Gravitational Form Factors?**

$$A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2}$$

Tensor (EMT)

## A perturbative solution to anomaly equation:




Giannotti, Mottola (2009)

**Calculation in off-forward kinematics** $(l = p_2 - p_1)$**:**

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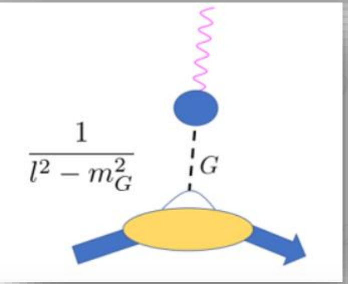
**In QCD, we expect:**

$$\frac{1}{l^2} \to \frac{1}{l^2 - m_G^2}$$

**glueball mass generations**




Fujita, Hatta, Sugimoto, Ueda (2022)
Mamo, Zahed (2019)

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# Trace anomaly

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$$A_f(l^2), B_f(l^2), D_f(l^2) \sim \cancel{\frac{1}{l^2}}$$

**Any implications for the corresponding GPD?**

$$A(l^2) + \xi^2 D(l^2) = \int_{-1}^{1} dx\, x\, \boldsymbol{H(x,\xi,l^2)}$$

$$B(l^2) - \xi^2 D(l^2) = \int_{-1}^{1} dx\, x\, \boldsymbol{E(x,\xi,l^2)}$$

**In QCD, we expect:**

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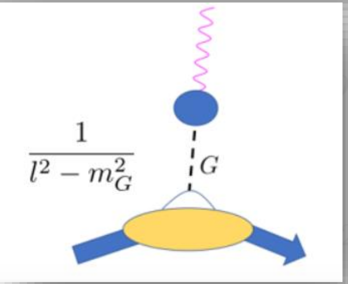


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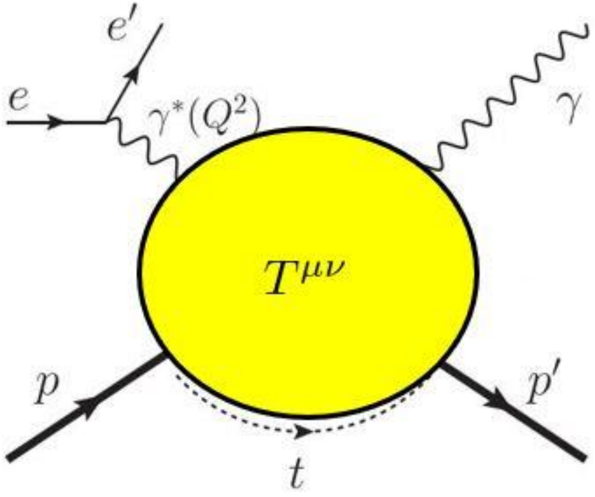


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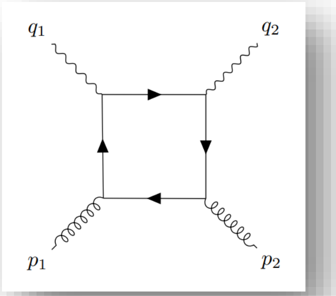
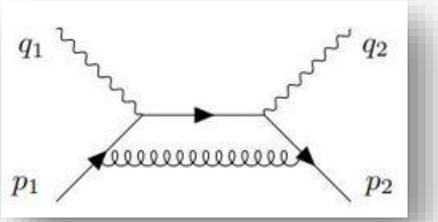
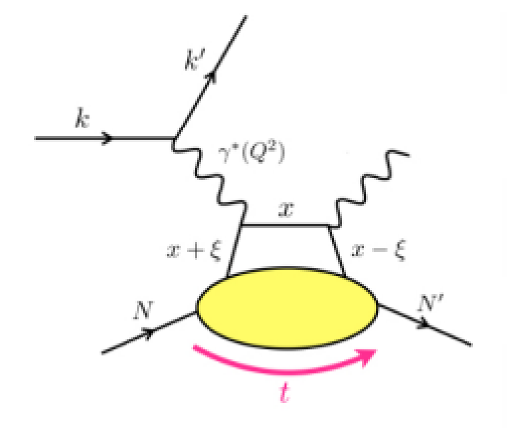



- **Polarized case**
- **Unpolarized case**

# Imprint of Anomalies in QCD Compton scattering

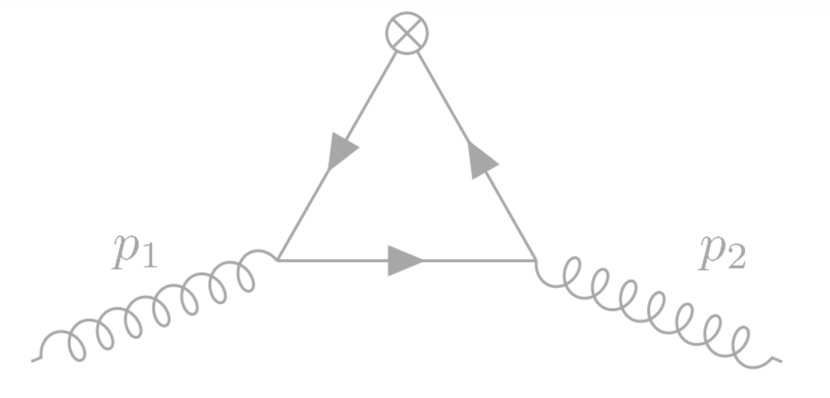
## QCD Compton Scattering




In QCD Compton scattering, box diagrams appear in perturbation theory at one-loop

Box diagram can be viewed as a non-local generalization of triangle diagram

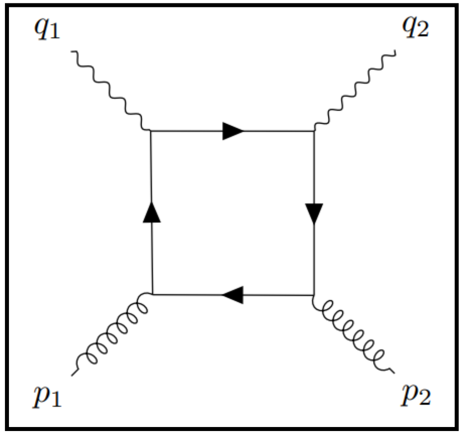
If triangle is dominated by anomaly pole, trace of that should be visible in box diagram




Calculation in off- $= p_2 - p_1$):

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f C}{4\pi}$$

Triangle diagram ... ed pole




Box diagram

# Imprint of Anomalies in DIS

**First calculation of box diagram with $l^2 \neq 0$:**

Anomaly equation:

**The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics**

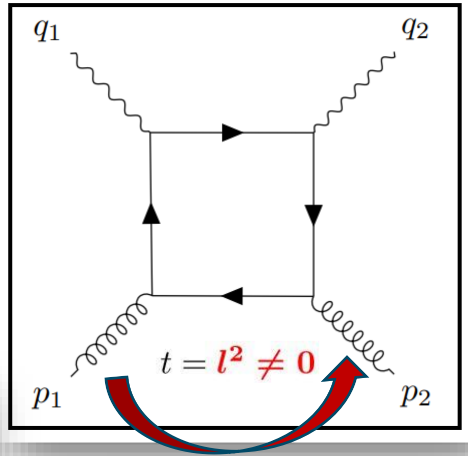
Andrey Tarasov[1,2] and Raju Venugopalan[3]

A fundamental prop

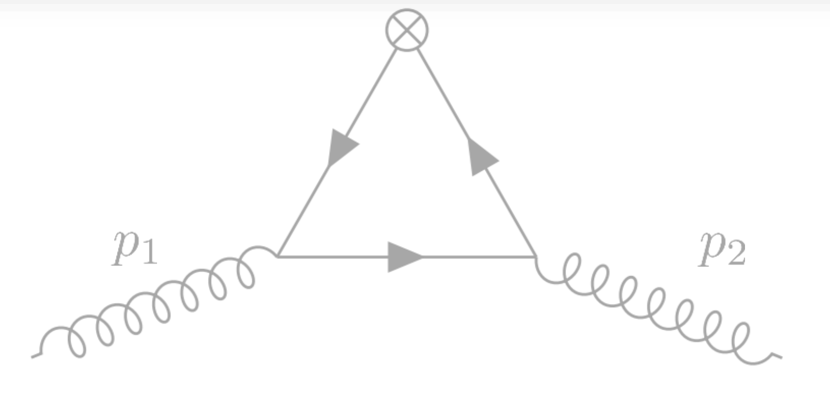
**The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics**

Andrey Tarasov[1,2] and Raju Venugopalan[3]

**Andrey & Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics for polarized DIS**




**Box diagram**

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**Triangle diagram is dominated by infra-red pole**

# Imprint of Anomalies in QCD Compton scattering

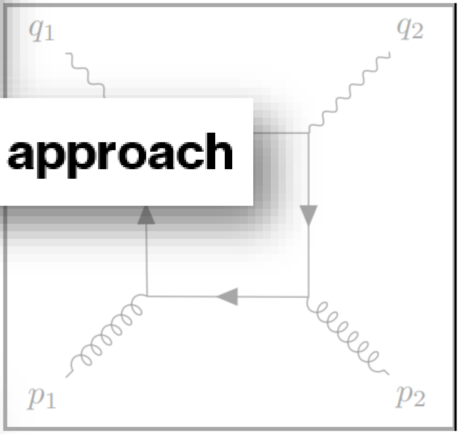
First calculation of b

The role of the
triangle gr

Chiral and trace anomalies in Deeply Virtual Compton Scattering : QCD factorization and beyond

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

**We explored the physics of anomaly in DVCS using Feynman-diagram approach**



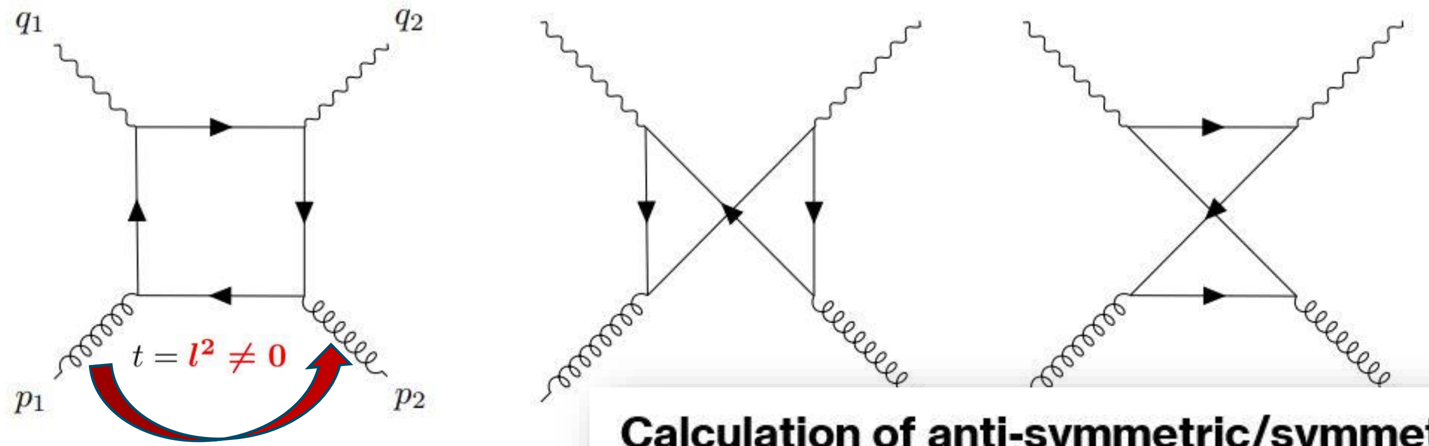




FIG. 1: Diagrams for the su

**Calculation of anti-symmetric/symmetric $(\mu, \nu)$ of Compton amplitude with $t = l^2 \neq 0$** $(\Lambda_{\rm QCD}^2 \ll t \ll Q^2)$

**Different than $t \sim \Lambda_{\rm QCD}^2$ regime**

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by infra-red pole

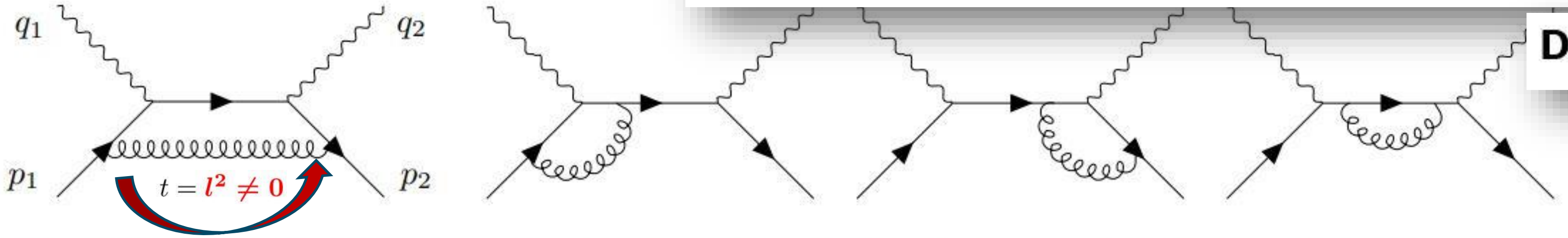
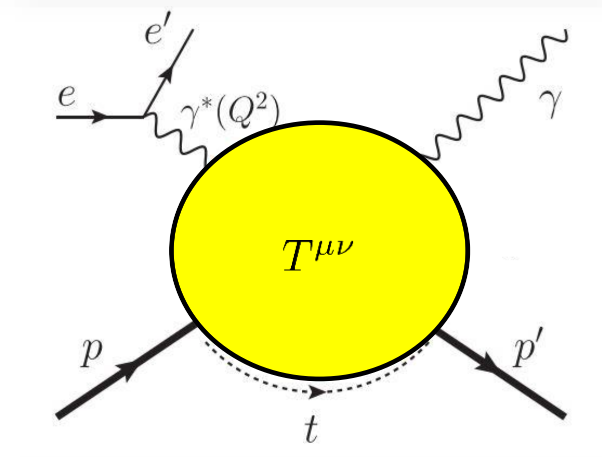


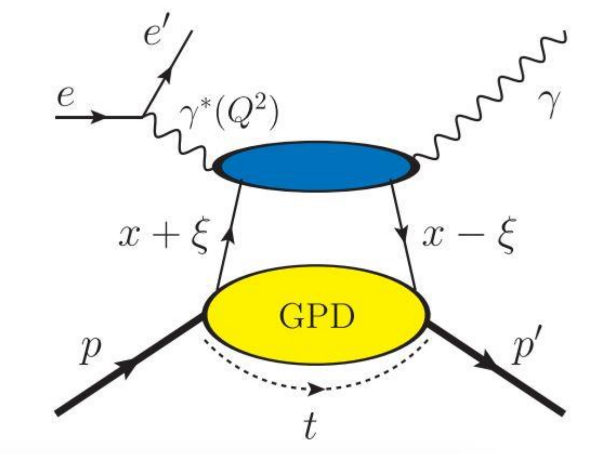

FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon lines crossed are not shown.

# Remarks on DVCS:




**Bjorken limit** $\longrightarrow$

$t, \Lambda^2_{\mathrm{QCD}} \ll Q^2$




**The QCD factorization theorem:** Collins, Freund; Ji, Osborne (1998)

$$T^{\mu\nu} = \sum_{a=q,g} \int \frac{dx}{x} C_a^{\mu\nu}\left(\frac{x_B}{x}, \frac{\xi}{x}\right) f_a(x,\xi,t) + \mathcal{O}(1/Q^2)$$

**Original proof of factorization is for** $t \sim \Lambda^2_{\mathrm{QCD}}$ **regime**

**Different than** $t \sim \Lambda^2_{\mathrm{QCD}}$ **regime**

**We extended factorization for the first time within** $\Lambda^2_{\mathrm{QCD}} \ll t \ll Q^2$ **regime**

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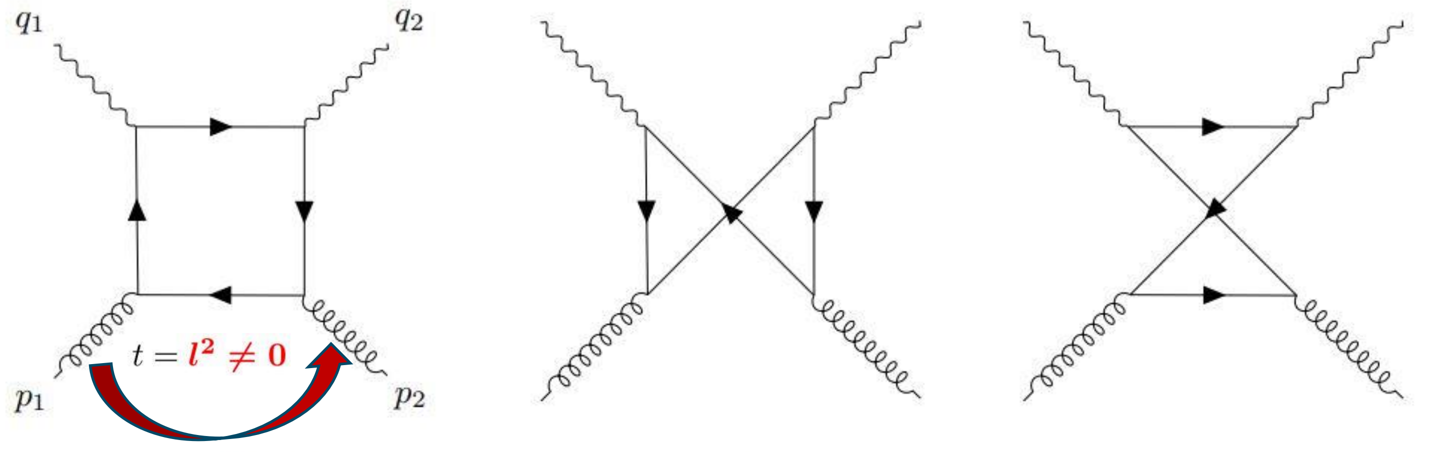
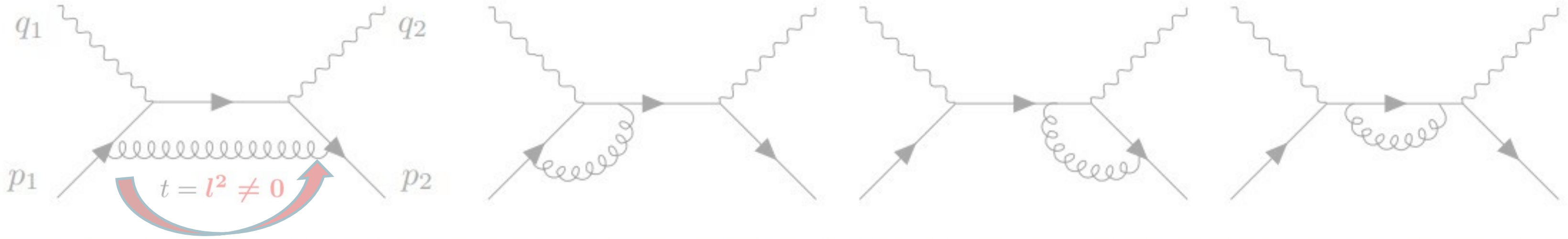



FIG. 1: Diagrams for the subprocess $\gamma^* g \to \gamma^* g$ in Compton scattering.

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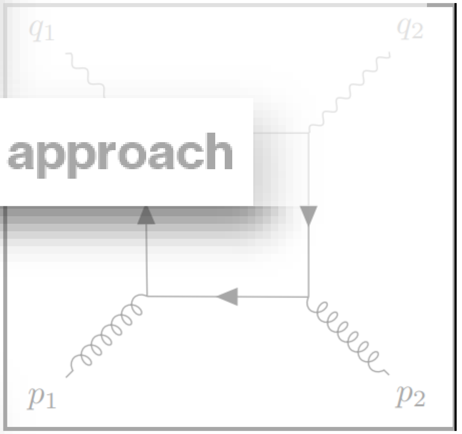


Box diagram

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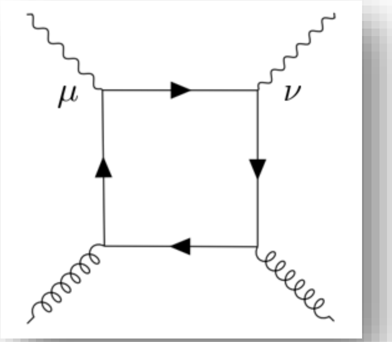
$\tilde{F}^a_{\alpha\beta}|p_1\rangle$

d by infra-red pole

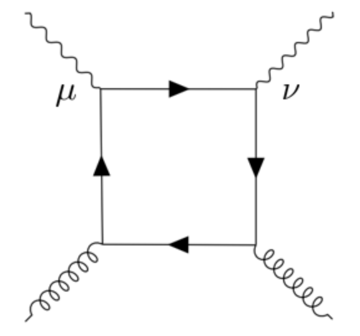
# Imprint of Anomalies in QCD Compton scattering




**Antisymmetric part of Compton amplitude**

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\mathrm{asym}}_{\mu\nu}$$

# Imprint of Anomalies in QCD Compton scattering




**Antisymmetric part of Compton amplitude**

**Collinear singularity regularized by $l^2$**

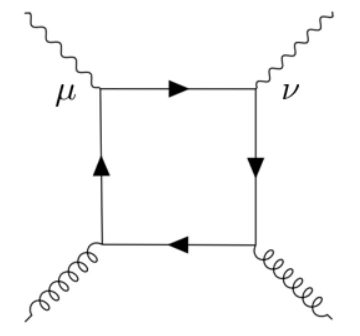
$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \times T^{\mathrm{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right) \bar{u}(P_2)\left[\left(\tilde{\kappa}_{qg} \ln \frac{Q^2}{-l^2} + \delta\tilde{C}_g^{\mathrm{off}}\right) \otimes \langle F^{+\mu}\tilde{F}^{+}_{\mu}\rangle \gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\tilde{A}_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B)\gamma_5\right] u(P_1)$$

**Reproduced the known logarithms from literature**

$$\tilde{\kappa}_{qg}(\hat{x},\hat{\xi}) = \frac{2\hat{x} - 1 - \hat{\xi}^2}{2(1-\hat{\xi}^2)^2} \ln \frac{\hat{x}-1}{\hat{x}} - \frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x})$$

**Ji, Osborne; Belitsky, Mueller**

# Imprint of Anomalies in QCD Compton scattering




**Antisymmetric part of Compton amplitude**

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\rm asym}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\tilde{\kappa}_{qg}\ln\frac{Q^2}{-l^2}+\delta\tilde{C}_g^{\rm off}\right)\otimes\langle F^{+\mu}\tilde{F}^{+}_{\mu}\rangle\gamma^\alpha\gamma_5+\frac{l^\alpha}{l^2}\tilde{A}_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

**Imaginary part of pole term ($\xi=0$) in agreement with Andrey & Raju**

**Anomalous coefficient function:**

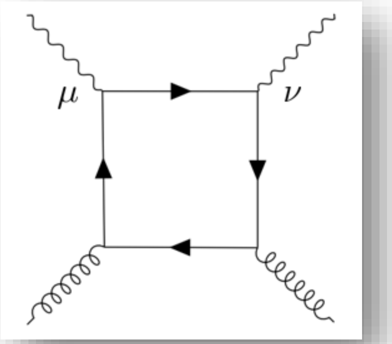
$$\tilde{A}_g^{\rm anom}=\frac{8T_R}{x}\frac{(1-\hat{x})\ln\frac{\hat{x}-1}{x}+(\hat{x}-\hat{\xi})\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}-(\hat{x}\to-\hat{x})}{1-\hat{\xi}^2}$$

## Twist-4 GPD:

$$\tilde{\mathcal{F}}(x,l^2)=\frac{iP^+}{\bar{u}(P_2)\gamma_5 u(P_1)}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle P_2|F_a^{\mu\nu}(-z^-/2)\tilde{F}^a_{\mu\nu}(z^-/2)|P_1\rangle$$

**(Non-local) chiral anomaly manifests itself in high energy scattering amplitude**

# Imprint of Anomalies in QCD Compton scattering




**Antisymmetric part of Compton amplitude**

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\rm asym}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\tilde{\kappa}_{qg}\ln\frac{Q^2}{-l^2}+\delta\tilde{C}_g^{\rm off}\right)\otimes\langle F^{+\mu}\tilde{F}^{+}_{\mu}\rangle\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\tilde{A}_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

**Twist-4 GPD**

**But no suppression in $1/Q^2$!**

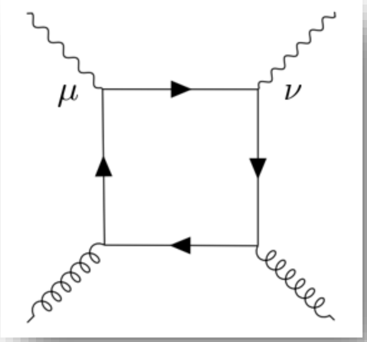
**The QCD factorization theorem:** **Collins, Freund; Ji, Osborne (1998)**

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\rm asym}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\left[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2)) + \frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\rm bare}(x_B,\xi,l^2)+\tilde{E}_f^{\rm bare}(-x_B,\xi,l^2))\right]u(P_1)$$
$$+\mathcal{O}(\alpha_s)+\mathcal{O}(1/Q^2)\,,$$

**Twist-2 GPDs to all orders**

**(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization**

# Imprint of Anomalies in QCD Compton scattering




## Antisymmetric part of Compton amplitude

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**Anomalous contribution to GPD $\tilde{E}$ at one loop**

## The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\text{asym}}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\left[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2))+\frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\text{bare}}(x_B,\xi,l^2)+\tilde{E}_f^{\text{bare}}(-x_B,\xi,l^2))\right]u(P_1)$$
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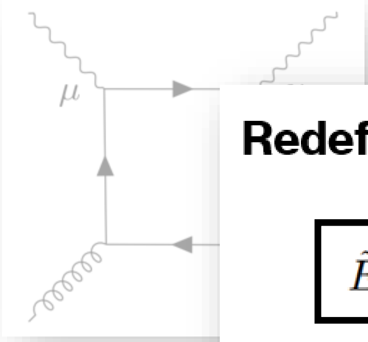



Antisymmetric p

## Justifying factorization

**Redefine**

$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\,\tilde{A}_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

$\otimes \tilde{\mathcal{F}}(x_B)\gamma_5\Big] u(P_1)$

one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\rm asym}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\left[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2)) + \frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\rm bare}(x_B,\xi,l^2)+\tilde{E}_f^{\rm bare}(-x_B,\xi,l^2))\right]u(P_1)$$
$$+\mathcal{O}(\alpha_s)+\mathcal{O}(1/Q^2)\,,$$

**Perhaps not an ad hoc argument?**

to all orders

chiral anomaly manifests itself in high energy scattering amplitude &

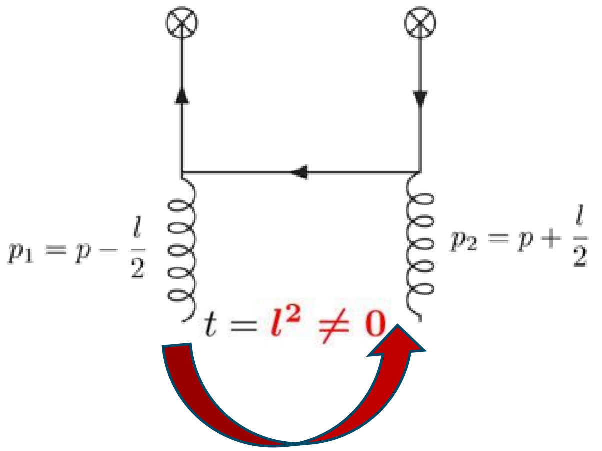
apparently breaks QCD factorization

# Imprint of Anomalies in QCD Compton scattering

Antisymmetric p

## Justifying factorization

Redefine




**Perturbative pole in GPD**

$$\int \frac{dz^-}{4\pi} e^{i\hat{x}p^+z^-} \langle p_2|\bar{\psi}(-z^-/2)\gamma^+\gamma_5\psi(z^-/2)|p_1\rangle\Big|_{\text{pole}} \sim \frac{\alpha_s}{2\pi} T_R \frac{2il^+}{l^2}(1-\hat{x}) \otimes \delta(1-\hat{x})\epsilon^{\epsilon_1\epsilon_2^* lp}$$

**Same pole in one-loop calculation!**

**Perhaps not an ad hoc argument !**

to all orders

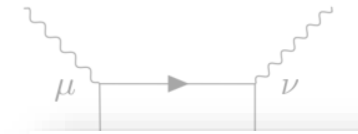
chiral anomaly manife

**The pole "belongs" to GPD**

nplitude &

apparently breaks QCD factorization

# Imprint of Anomalies in QCD Compton scattering




Antisymmetric part

**Elusive pole**

$$\sum_f e_f^2 \Big) \, \bar{u}(P_2) \left[ \left( \tilde{\kappa}_{qg} \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_g^{\rm off} \right) \otimes \langle F^{+\mu} \tilde{F}_\mu^{\ +} \rangle \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \tilde{A}_g^{\rm anom} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \right] u(P_1)$$

Anomalous contribution to GPD $\tilde{E}$ at one loop

ONE-LOOP QCD CORRECTIONS TO DEEPLY-VIRTUAL COMPTON SCATTERING: THE PARTON HELICITY-INDEPENDENT CASE

Xiangdong Ji and Jonathan Osborne

, Osborne (1998)

Predictions from conformal algebra for the deeply virtual Compton scattering.

A.V. Belitsky D. Müller

$$\tilde{H}_f(x_B, \xi, l^2) + \tilde{H}_f(-x_B, \xi, l^2)) + \frac{l^\alpha \gamma_5}{2M} (\tilde{E}_f^{\rm bare}(x_B, \xi, l^2) + \tilde{E}_f^{\rm bare}(-x_B, \xi, l^2)) \Big] u(P_1)$$

NLO Corrections to Deeply-Virtual Compton Scattering *

L. Mankiewicz[†a], G. Piller[a], E. Stein[b], M. Vänttinen[a] and T. Weigl[a]

Twist-2 GPDs

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude &

...ation

NLO corrections to timelike, spacelike and double deeply virtual Compton scattering.

B. Pire[1] and L. Szymanowski[2] and J. Wagner[2]

# Imprint of Anomalies in QCD Compton scattering

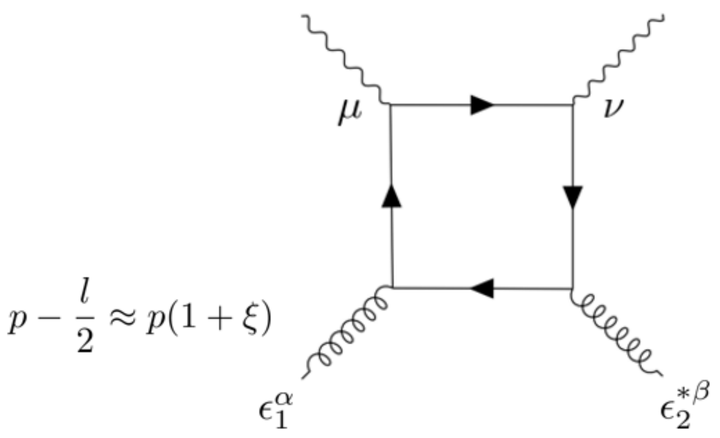
## Elusive pole

**Pole was unnoticed in the GPD literature because one typically assumes**

$$l^\mu = -2\xi p^\mu \;\rightarrow\; t = l^2 = 0$$

**before loop integration**

**Usual rationale: Corrections supposedly higher twist** $\dfrac{t}{Q^2}$




Twist-2 GPDs
to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude &
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# Imprint of Anomalies in QCD Compton scattering

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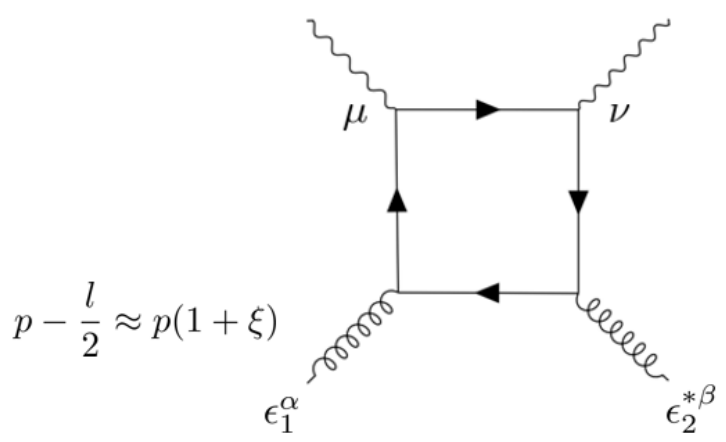



**However, box diagram is power-divergent in the IR!**

$$\frac{l^\alpha}{l^2}\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B)\gamma_5 \;\longrightarrow$$

**Still, pole was never seen before because:**

$$\langle p_2|F^{\mu\nu}\tilde{F}_{\mu\nu}|p_1\rangle \propto \epsilon^{\mu\nu\alpha\beta} l_\mu p_\nu \epsilon_{1\alpha}\epsilon^*_{2\beta}$$

$$\rightarrow 0 \quad \textbf{when} \quad l^\mu \propto p^\mu$$

# Imprint of Anomalies in QCD Compton scattering

## Beyond factorization: Fate of anomaly pole

Redefine

$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\,\tilde{A}_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

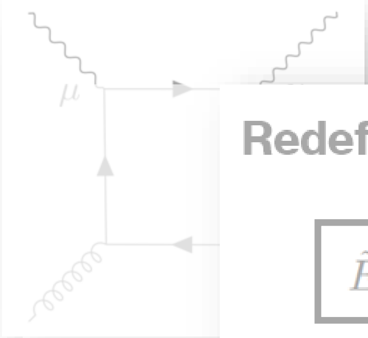
Perturbative pole (one loop)

**Perturbative calculations suggest that massless poles are induced in GPD $\tilde{E}$**

**However, we know there are no massless poles in axial form factor (moment of GPD $\tilde{E}$ )**

$$g_P(l^2) = \int dx\,\tilde{E}(x) \sim \cancel{\frac{1}{l^2}}$$

# Imprint of Anomalies in QCD Compton scattering



## Beyond factorization: Fate of anomaly pole

**Redefine**

$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\,\tilde{A}_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

**Postulate that the perturbative pole cancels the pre-existing pole in "bare" GPD:**

$$\boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} \approx -\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\,\tilde{A}_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2=0)$$

**Postulate that the "renormalized" GPD integrates to $g_P(l^2)$ :**

$$g_P(l^2)=\sum_f\int_{-1}^{1}dx\tilde{E}_f(x,\xi,l^2)=\sum_f\int_0^1\boxed{dx(\tilde{E}_f(x,\xi,l^2)+\tilde{E}_f(-x,\xi,l^2))}$$

# Imp… malies in QCD Compton scattering

**Connection between twist 2 & twist 4 GPDs due to anomaly**

## Beyond factorization: Fate of anomaly pole

**Redefine**

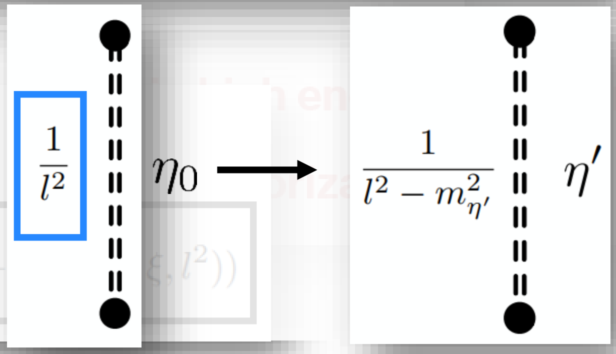
$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\tilde{A}_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

**Pole cancellation at $\int dx$**

**We find:** $$\frac{g_P(l^2)}{2M} = -\left[\boxed{\frac{2M\Delta\Sigma}{l^2}} - \frac{2M\Delta\Sigma}{l^2} + \frac{2M\Delta\Sigma}{l^2-m_{\eta'}^2}\right] \sim \frac{1}{l^2-m_{\eta'}^2}$$

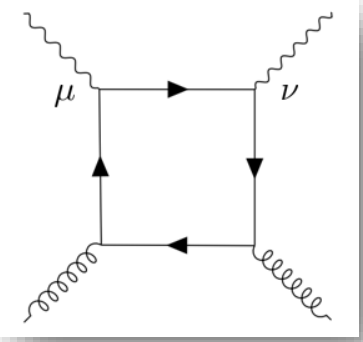



"We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the $\eta'$ and the cancellation of the anomaly pole"

**- Tarasov, Venugopalan**

**See also Jaffe Manohar, 1990**

# Equivalence with $\overline{\mathrm{MS}}$ scheme




**Antisymmetric part of Compton amplitude**

**Coefficient function**

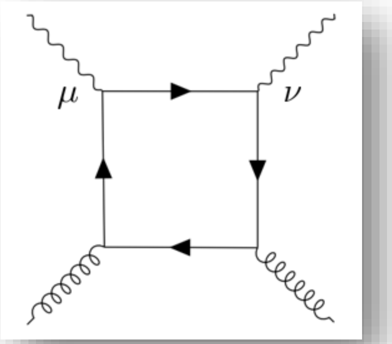
$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \times T^{\text{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\tilde{\kappa}_{qg}\ln\frac{Q^2}{-l^2} + \delta\tilde{C}_g^{\text{off}}\right)\otimes\langle F^{+\mu}\tilde{F}^{+}_{\mu}\rangle\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\tilde{A}_g^{\text{anom}}\otimes\tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

$$\delta\tilde{C}_1^g(\hat{x},\hat{\xi}) = -\frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-1}{\hat{x}} + 2\frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}$$
$$+\frac{2\hat{x}-1-\hat{\xi}^2}{2(1-\hat{\xi}^2)^2}\ln^2\frac{\hat{x}-1}{\hat{x}} + \frac{\hat{\xi}}{1-\hat{\xi}^2}\ln^2\frac{\hat{x}-\hat{\xi}}{\hat{x}} - \frac{\hat{x}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}}$$
$$+\frac{2\hat{\xi}}{(1-\hat{\xi}^2)^2}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}} + \mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right) - (\hat{x}\to-\hat{x})$$

**After subtracting IR singularities and finite terms, $t \neq 0$ regularization is equivalent to $\overline{\mathrm{MS}}$ scheme**

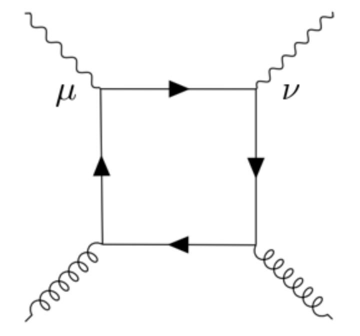
**This means that the result can be smoothly connected to the regime $t \sim \Lambda^2_{\mathrm{QCD}}$ as considered in the works by Collins, Freund; Ji, Obsborne**

# Imprint of Anomalies in QCD Compton scattering




**Symmetric part of Compton amplitude**

# Imprint of Anomalies in QCD Compton scattering




**Symmetric part of Compton amplitude**

**Pole! (New result)**

$$\boxed{(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2))} = \boxed{(H_f^{\mathrm{bare}}(x_B,\xi,l^2) - H_f^{\mathrm{bare}}(-x_B,\xi,l^2))} + \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\mathrm{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

$$\boxed{(E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2))} = \boxed{(E_f^{\mathrm{bare}}(x_B,\xi,l^2) - E_f^{\mathrm{bare}}(-x_B,\xi,l^2))} - \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\mathrm{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

Hatta, Zhao (2020);
Radyushkin, Zhao (2021)

**Twist-4 GPD:**

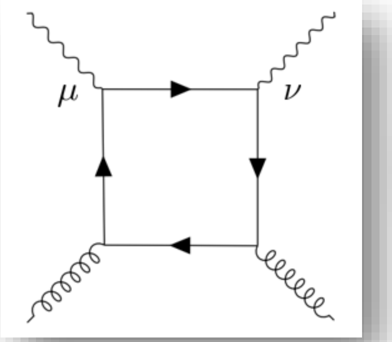
$$\mathcal{F}(x,\xi,l^2) = -4xP^+M\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)F_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$

**"Bare GPD" (tree level)**

**Perturbative pole (one loop)**

**(Non-local) trace anomaly manifests itself in high energy scattering amplitude**

# Imprint of Anomalies in QCD Compton scattering




**Symmetric part of Compton amplitude**

**Pole! (New result)**

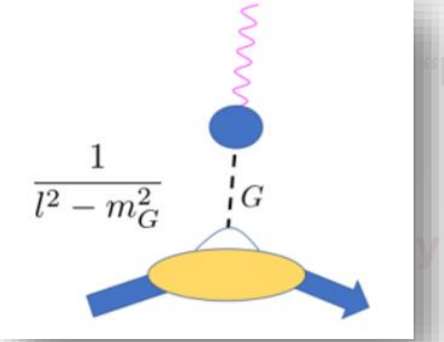
**Glueball pole**

**Pole cancellation**

$$\boxed{(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2))} = \boxed{(H_f^{\rm bare}(x_B,\xi,l^2) - H_f^{\rm bare}(-x_B,\xi,l^2))}$$
$$+ \frac{\alpha_s}{2\pi}\frac{1}{l^2} C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

**We proposed a possible scenario of pole cancellation**

**glueball mass generations**



# Imprint of Anomalies in QCD Compton scattering

First calculation of b

Chiral and trace anomalies in Deeply Virtual Compton Scattering : QCD factorization and beyond

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

The role of the
triangle gr

We explored the physics of anomaly in DVCS using Feynman-diagram approach

The role of the chiral anomaly in polarized deeply inelastic scattering II:

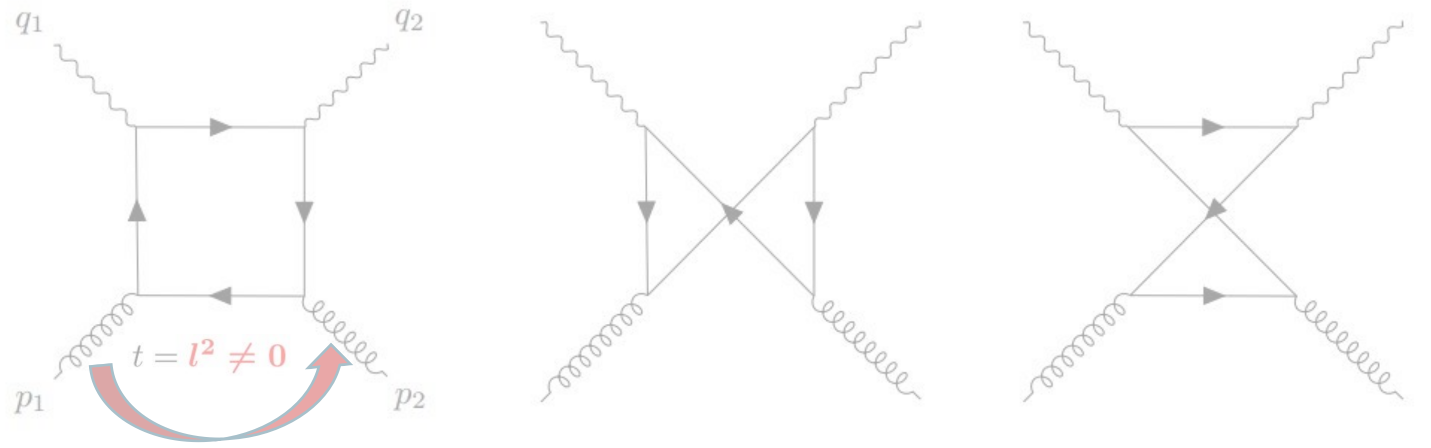



FIG. 1: Diagrams for the subprocess $\gamma^* g \to \gamma^* g$ in Compton scattering.

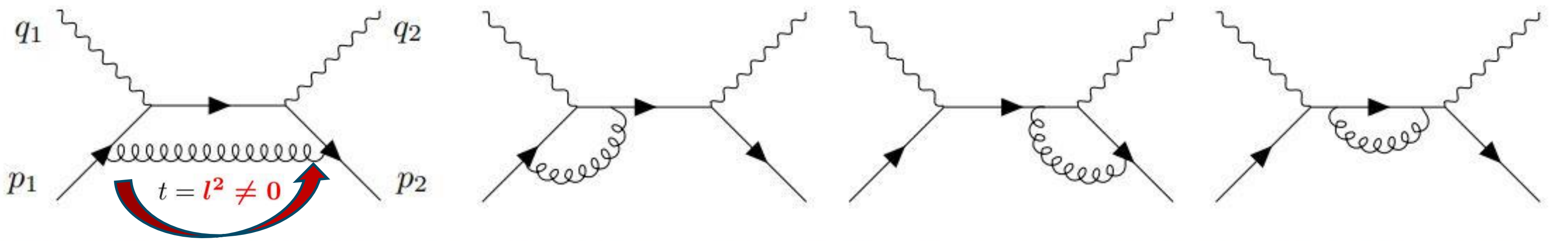
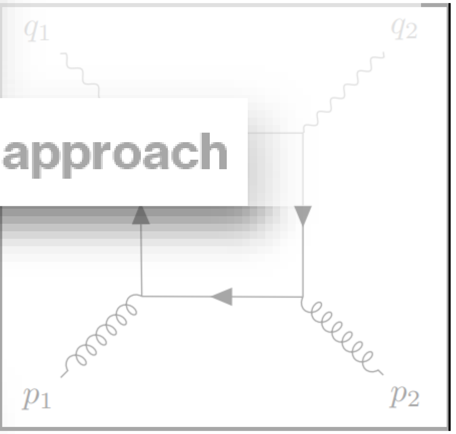



FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon lines crossed are not shown.




Box diagram

maly

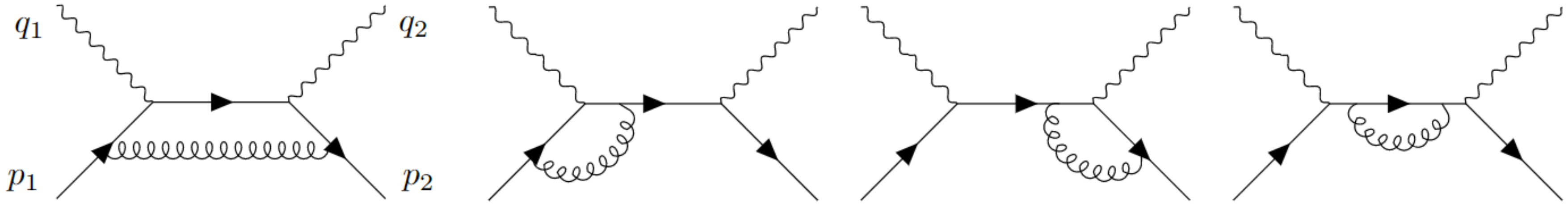
ematics $(l = p_2 - p_1)$:

$\beta \tilde{F}^a_{\alpha\beta}|p_1\rangle$

d by infra-red pole

# Imprint of Anomalies in QCD Compton scattering

## Quark-channel diagrams in DVCS



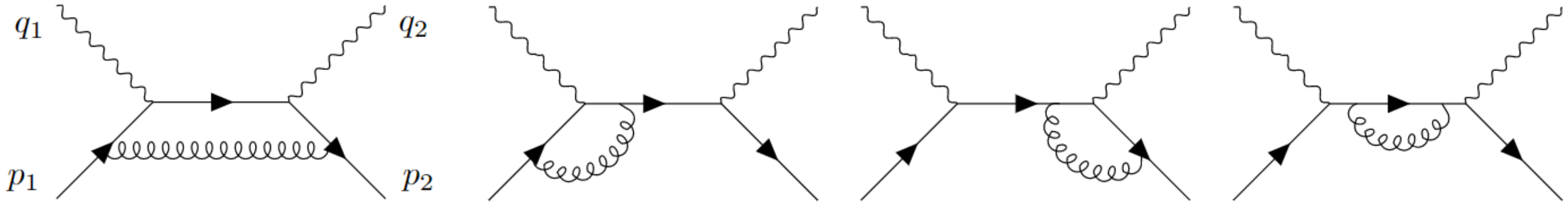

### Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} \quad + \tilde{\kappa}_{qq}(\hat{x}, \hat{\xi}) \ln \frac{Q^2}{-l^2} \quad + \delta \tilde{C}_1^q(\hat{x}, \hat{\xi})$$

**No pole!**

# Imprint of Anomalies in QCD Compton scattering

## Quark-channel diagrams in DVCS




**Example: Antisymmetric case**

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_1^q(\hat{x},\hat{\xi})$$

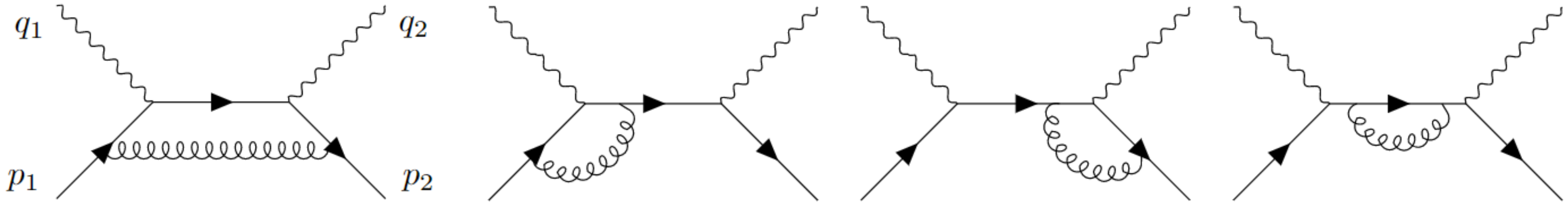
**Reproduced the known logarithms from literature**

$$\tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) = \frac{3}{2(1-\hat{x})} + \frac{\hat{x}^2+1-2\hat{\xi}^2}{(1-\hat{\xi}^2)(1-\hat{x})} \ln \frac{\hat{x}-1}{\hat{x}} - \frac{(\hat{x}-\hat{\xi})(1+\hat{x}^2+2\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x})$$

Ji, Osborne; Belitsky, Mueller

# Imprint of Anomalies in QCD Compton scattering

## Quark-channel diagrams in DVCS

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$$\sim \cancel{\frac{1}{l^2}} + \tilde\kappa_{qq}(\hat x,\hat\xi)\ln\frac{Q^2}{-l^2} + \delta\tilde C_1^q(\hat x,\hat\xi)$$

**Coefficient function**

$$\delta\tilde C_1^q(\hat x,\hat\xi) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat x)} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat x)} + \frac{-1+2\hat x-4\hat x^2+3\hat\xi^2}{2(1-\hat x)(1-\hat\xi^2)}\ln\frac{\hat x-1}{\hat x} + \frac{(\hat x-\hat\xi)(1+2\hat x^2+3\hat x\hat\xi)}{(1-\hat x^2)(1-\hat\xi^2)}\ln\frac{\hat x-\hat\xi}{\hat x}$$

$$\ldots \ln^2\frac{\hat x-1}{\hat x} \ldots + \hat x^2 - 2\hat\xi^2)\ldots \ln\frac{\hat x-\hat\xi}{\hat x}\ln\frac{\hat x+\hat\xi}{\hat x} + \frac{\pi^2-54}{12(1-\hat x)}$$

$$+\frac{\hat\xi}{1-\hat\xi^2}\mathrm{Li}_2\ldots + \frac{1+\hat x^2}{(1-\hat x)}\ldots$$

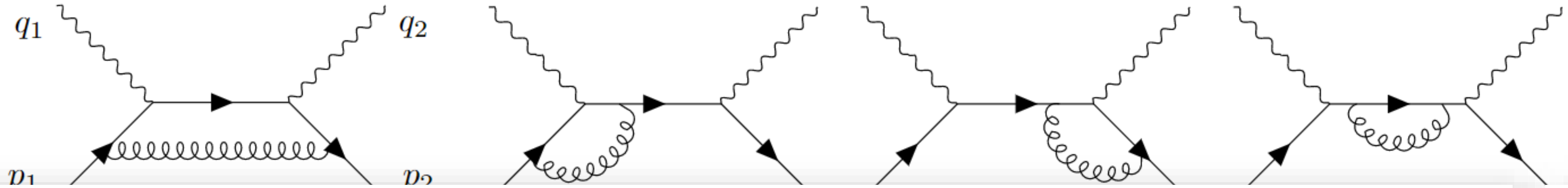
**Unexpected double IR pole**

**Unexpected single IR pole**

**Sudakov logs!** $\ln\left(\frac{Q^2}{-l^2}\right), \ln^2\left(\frac{Q^2}{-l^2}\right)$

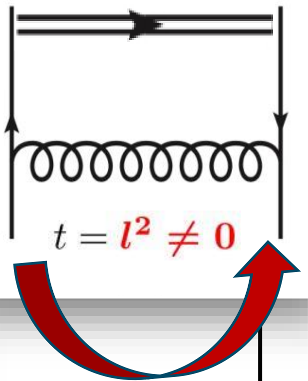
# Imprint of Anomalies in QCD Compton scattering

## Quark-channel diagrams in DVCS




**It looks like factorization is broken due to the unexpected double, single IR poles**

**But, when you compute GPD itself, you find the same double, single IR poles!**
**These poles can be systematically absorbed into GPD**




**Factorization restored**

$$\delta \tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)}\ln\frac{\hat{x}-1}{\hat{x}} + \frac{\cdots}{(1-\hat{x}^2)(1-\hat{\xi}^2)}\ln\frac{\cdots}{\hat{x}}$$

$$\cdots + \hat{x}^2 - 2\hat{\xi}^2}{\cdots\hat{x})(1-\hat{\xi}^2)}\ln^2\frac{\hat{x}-1}{\hat{x}} \cdots + \hat{x}^2 - 2\hat{\xi}^2)}{2(1-\hat{x}^2)(1-\hat{\xi}^2)}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

$$+\frac{\hat{\xi}}{1-\hat{\xi}^2}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2-2\hat{\xi}^2}{(1-\hat{x})(1-\hat{\xi}^2)}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}} + \mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right) - (\hat{x}\to-\hat{x})$$

**Unexpected double IR pole**

**Unexpected single IR pole**

# Summary

**Factorization**

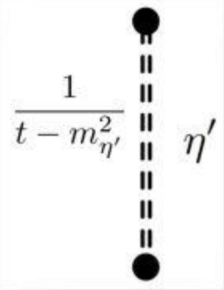
- **Off-forwardness is an alternative factorization scheme that clarifies the physics of anomaly (More physical than other schemes.)**

- **Clarified QCD factorization for the first time within $\Lambda_{\mathrm{QCD}}^2 \ll t \ll Q^2$ regime:**
  **Crucial topic for ongoing & future experiments including at EIC**

**Fate of anomaly poles**

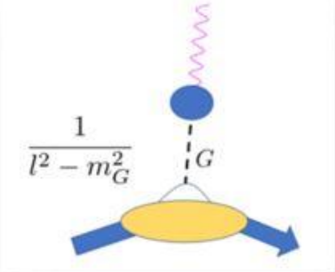
- **Novel connection between twist 2 & twist 4 sectors at the density level due to anomaly**

**Imprints of chiral & trace anomalies in GPDs:**




**Eta meson mass generation:**

$$\boxed{\tilde{E}(x) \sim \frac{1}{t - m_{\eta'}^2}}$$




**Glueball mass generation:**

$$\boxed{H(x), E(x) \sim \frac{1}{t - m_G^2}}$$

**Novel avenue of GPD research**

**Profound physical implication of anomaly poles:**
**Touches questions on mass generations, Chiral symmetry breaking, ...**