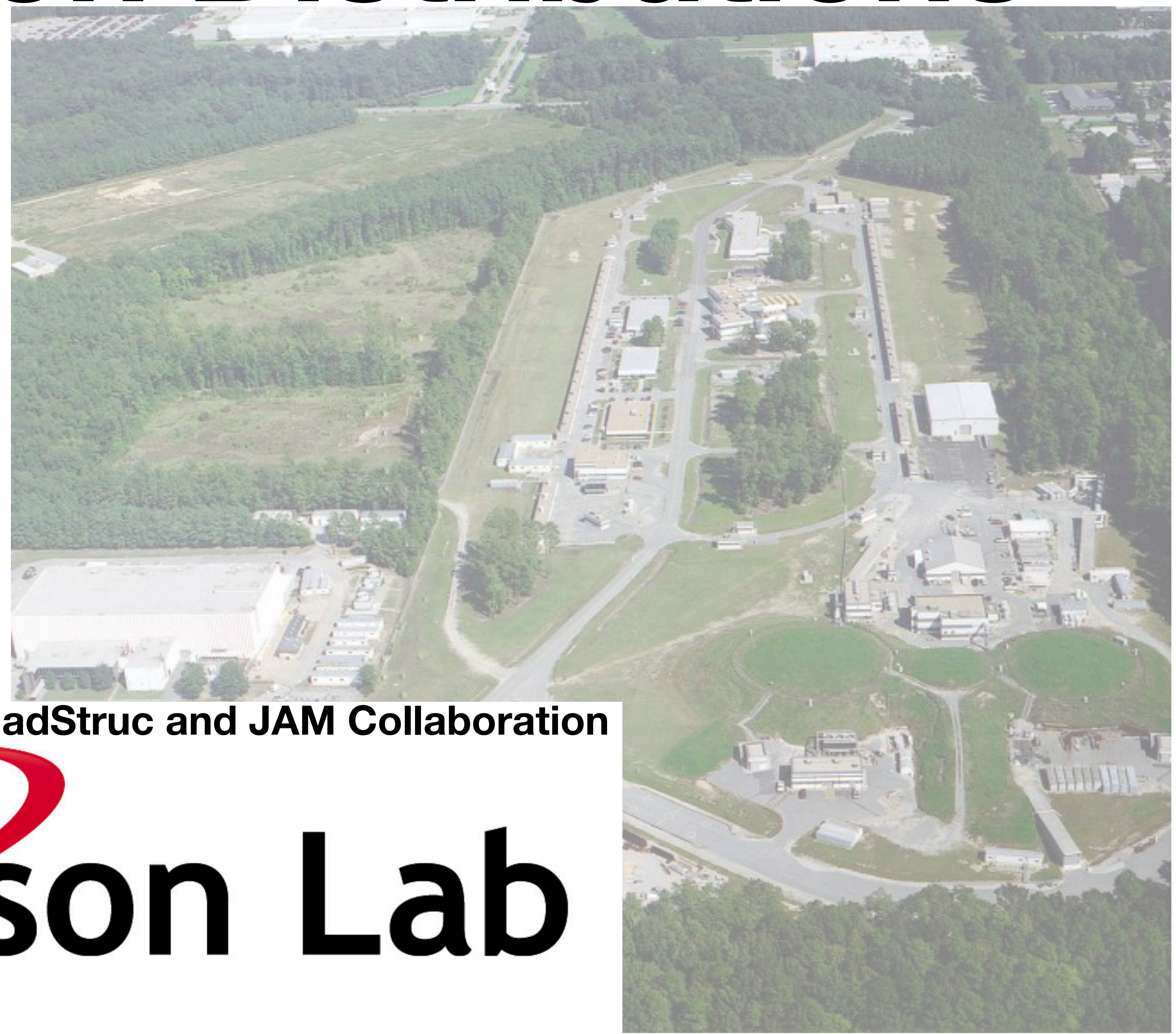
Lattice QCD Calculation Of Parton Distributions



Joe Karpie (JLab) part of the HadStruc and JAM Collaboration

Jefferson Lab

Parton Structure

For various flavors and spin combinations

Wigner Distribution/
Generalized Transverse Momentum
Distribution (GTMD)

$F(x, b_T, k_T)$

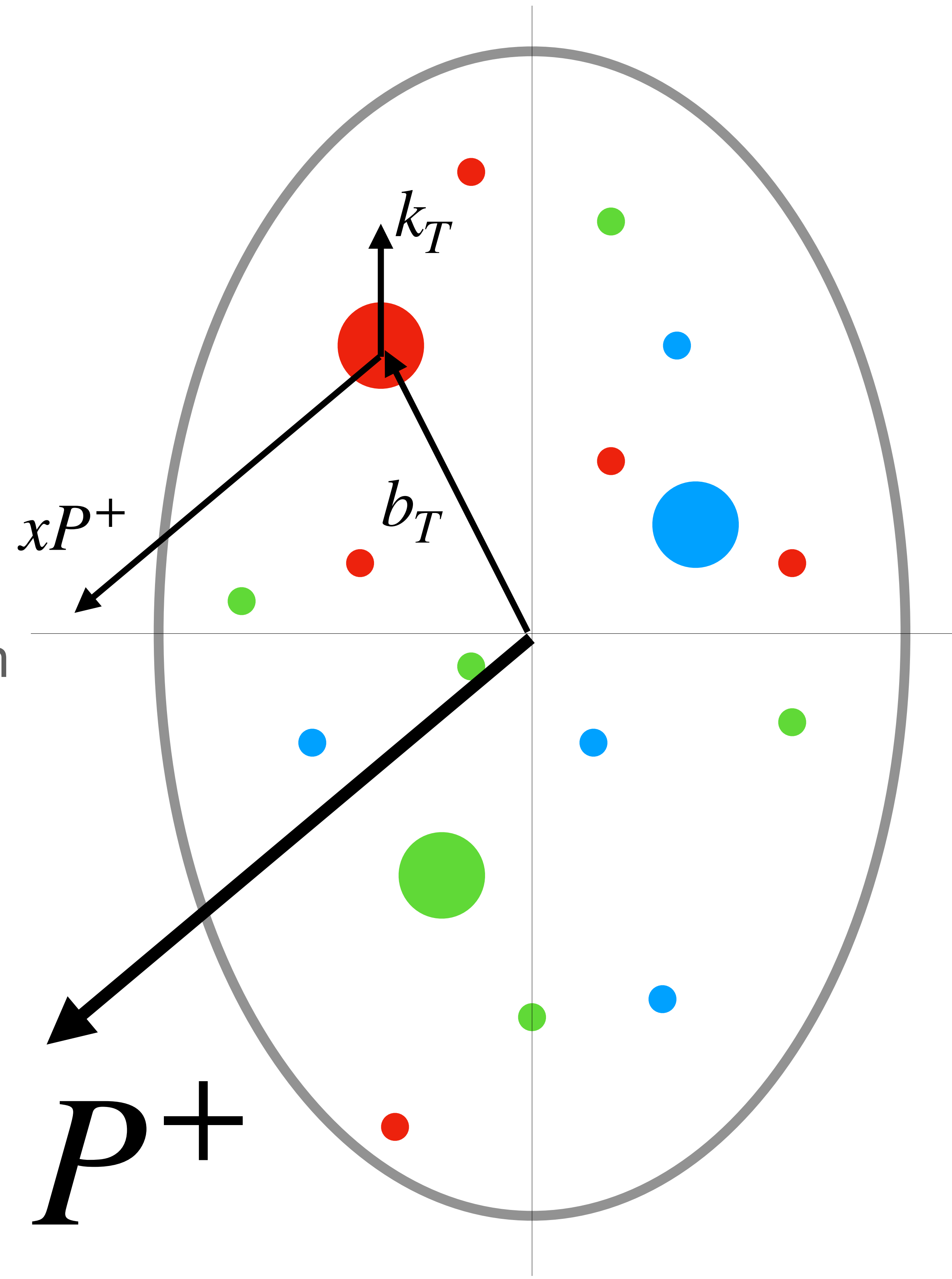
$\int d^2b_t$ $\int d^2k_t$

Transverse Momentum Distribution (TMD)
$f(x, k_T)$

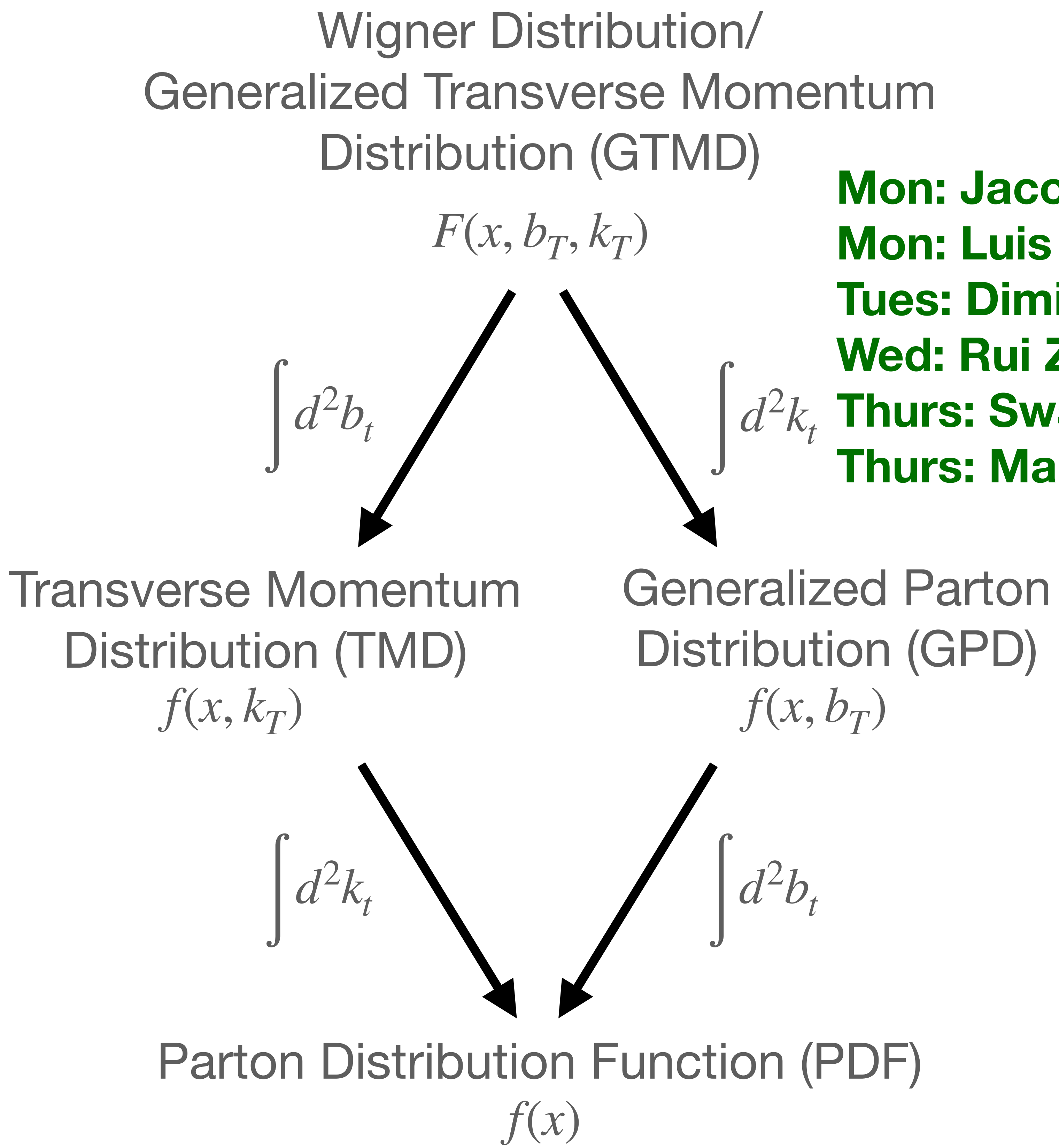
Generalized Parton Distribution (GPD)
$f(x, b_T)$

$\int d^2k_t$ $\int d^2b_t$

Parton Distribution Function (PDF)
$f(x)$



Parton Structure

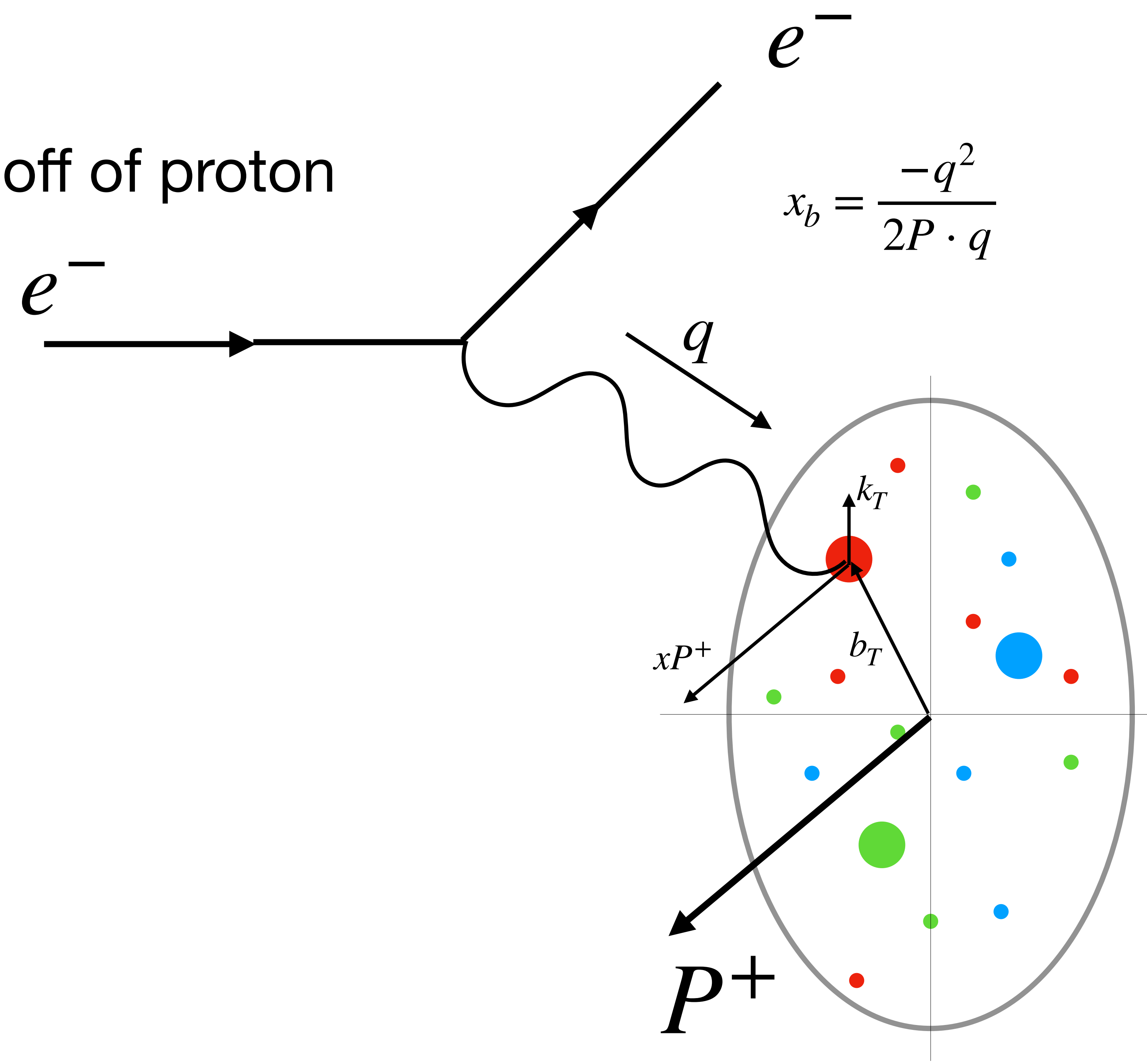


Mon: Jacopo Tarello (TMDs)
Mon: Luis Alberto Rodriguez Chacon (Moments)
Tues: Dimitra Pefkou (GFFs)
Wed: Rui Zhang (DAs)
Thurs: Swagato Mukherjee (Higher Order Evo)
Thurs: Martha Constantinou (GPDs)

Partons from Experiments

- Deep Inelastic Scattering
    - Hard Scattering of electron off of proton

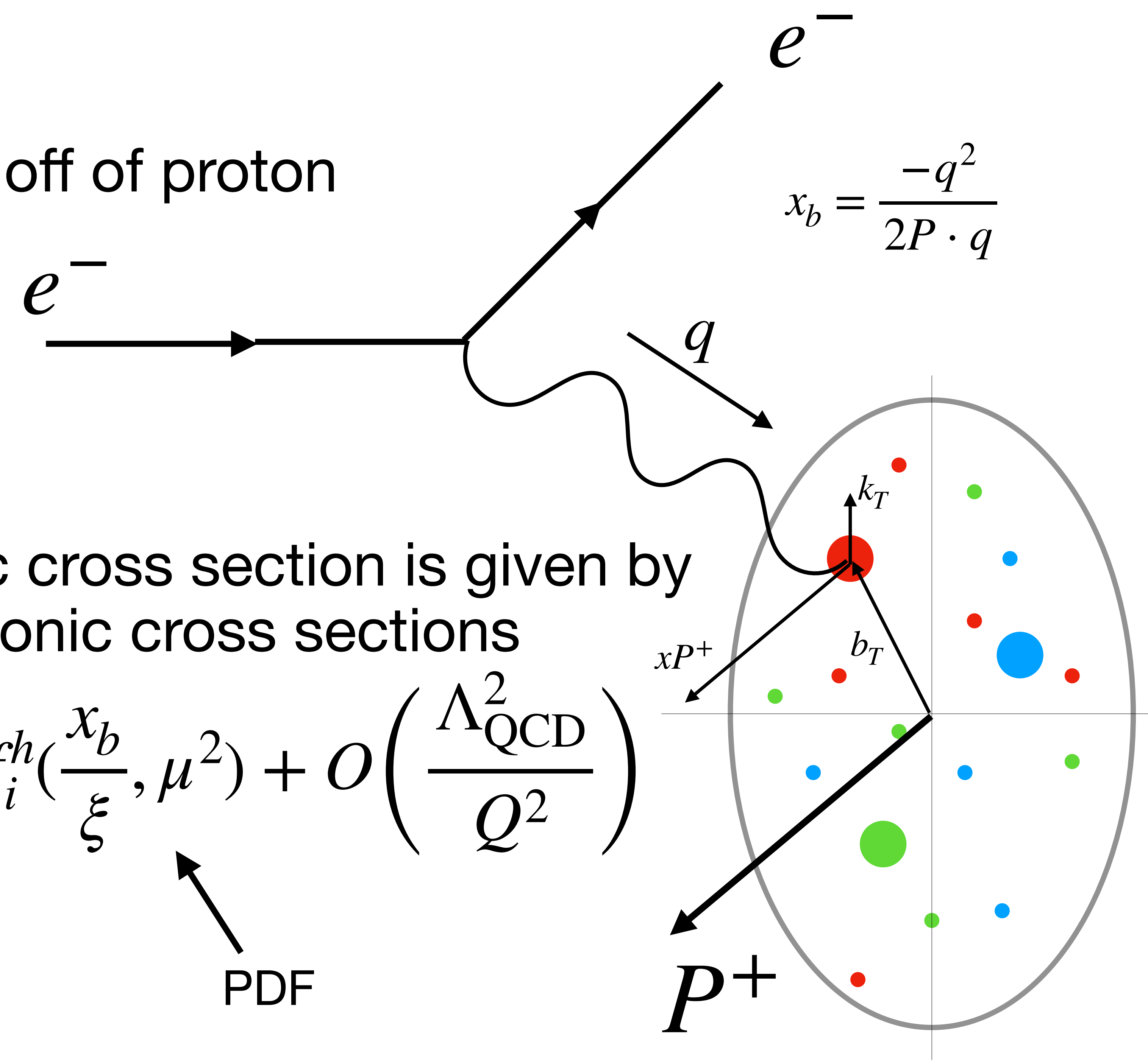
$$Q^2 = -q^2 \gg \Lambda_{QCD}^2$$



Partons from Experiments

- Deep Inelastic Scattering
  - Hard Scattering of electron off of proton

$$Q^2 = -q^2 \gg \Lambda^2_{\text{QCD}}$$



- **QCD Factorization:** Hadronic cross section is given by convolution of PDFs with partonic cross sections

$$F_2^h(x_b, Q^2) = \sum_i \int_{x_b}^1 d\xi \, F_2^i(\xi, \frac{\mu^2}{Q^2}) f_i^h(\frac{x_b}{\xi}, \mu^2) + O\left(\frac{\Lambda^2_{\text{QCD}}}{Q^2}\right)$$

Hadron Structure Function

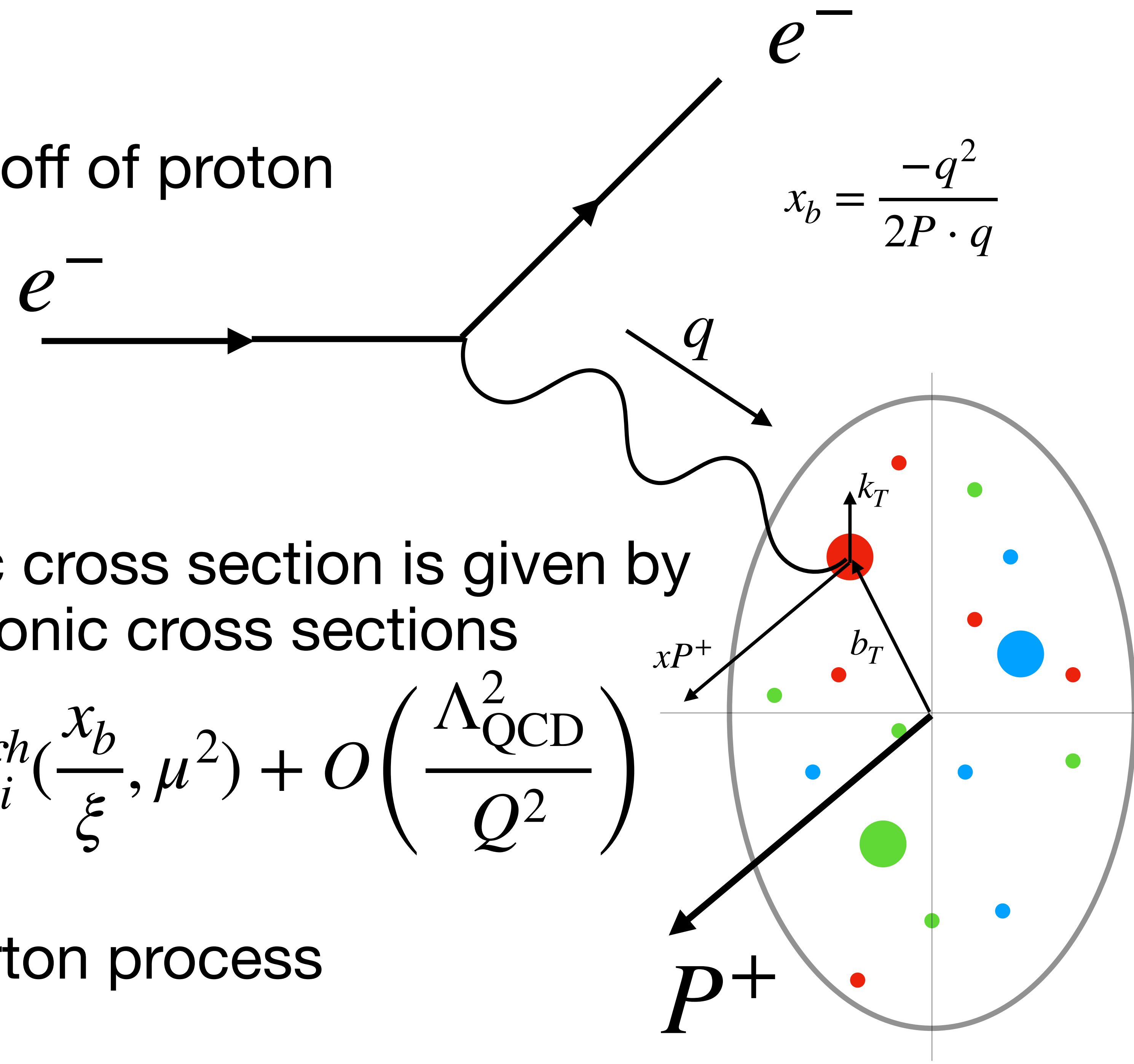
PDF

Parton Structure Function

Partons from Experiments

- Deep Inelastic Scattering
  - Hard Scattering of electron off of proton

$$Q^2 = -q^2 \gg \Lambda^2_{\rm QCD}$$



- **QCD Factorization:** Hadronic cross section is given by convolution of PDFs with partonic cross sections

$$F_2^h(x_b, Q^2) = \sum_i \int_{x_b}^1 d\xi \, F_2^i(\xi, \frac{\mu^2}{Q^2}) f_i^h(\frac{x_b}{\xi}, \mu^2) + O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

- PDFs interpolate between Parton process and Hadronic processes

- Global fits: Use global collider data to determine universal PDFs

Parton and Ioffe Time distributions

- Unpolarized Ioffe time distributions

Ioffe time: $\nu = p \cdot z$

V. Braun, P. Gornicki, L. Mankiewicz
Phys Rev D 51 (1995) 6036-6051

- $I_q(\nu, \mu^2) = \frac{1}{2p^+} \langle p \,|\, \bar{\psi}_q(z^-)\gamma^+ W(z^-;0)\psi_q(0) \,|\, p \rangle_{\mu^2}$

$z^2 = 0$

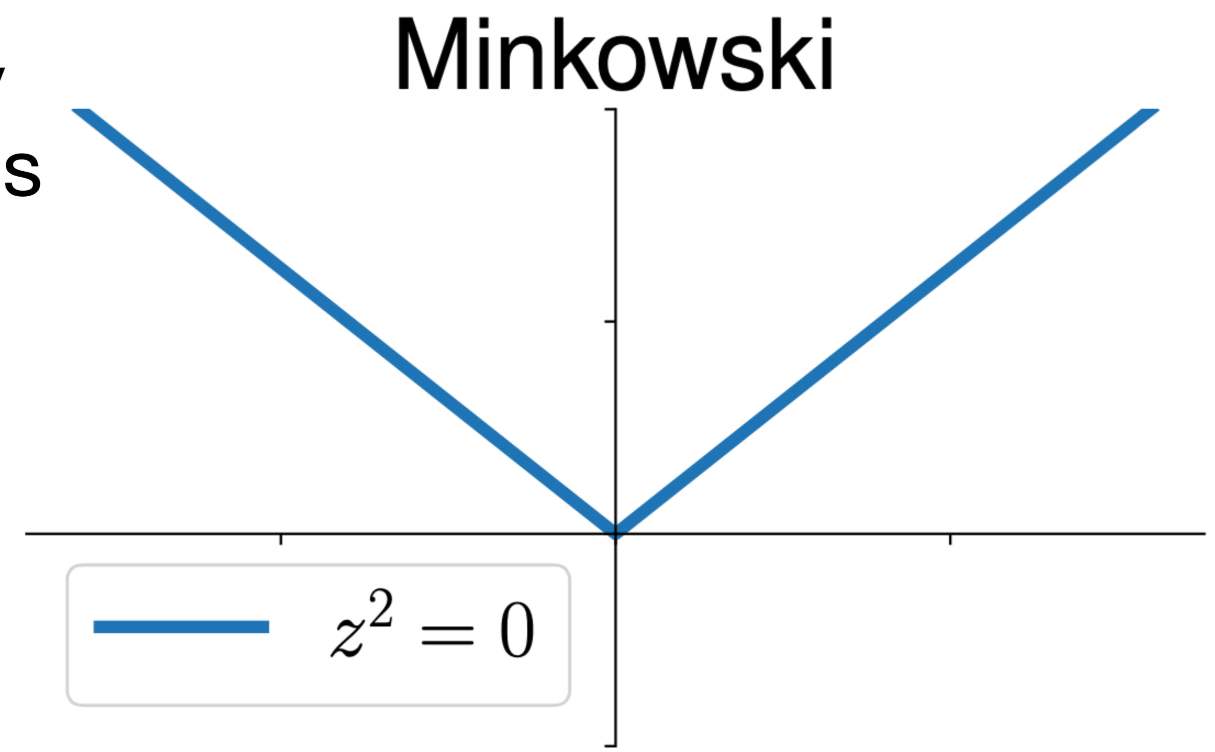
- $I_g(\nu, \mu^2) = \frac{1}{(2p^+)^2} \langle p \,|\, F_{+i}(z^-) W(z^-;0) F_+^i(0) \,|\, p \rangle_{\mu^2}$

$i = x, y$

- Parton Distribution Functions
  - $I_q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{ix\nu} f_q(x, \mu^2)$
  - $I_g(\nu, \mu^2) = \int_{0}^{1} dx\, \cos(x\nu)\, x f_g(x, \mu^2)$

Parton Distributions and the Lattice

- Parton Distributions are defined by operators with light-like separations



- Use space-like separations

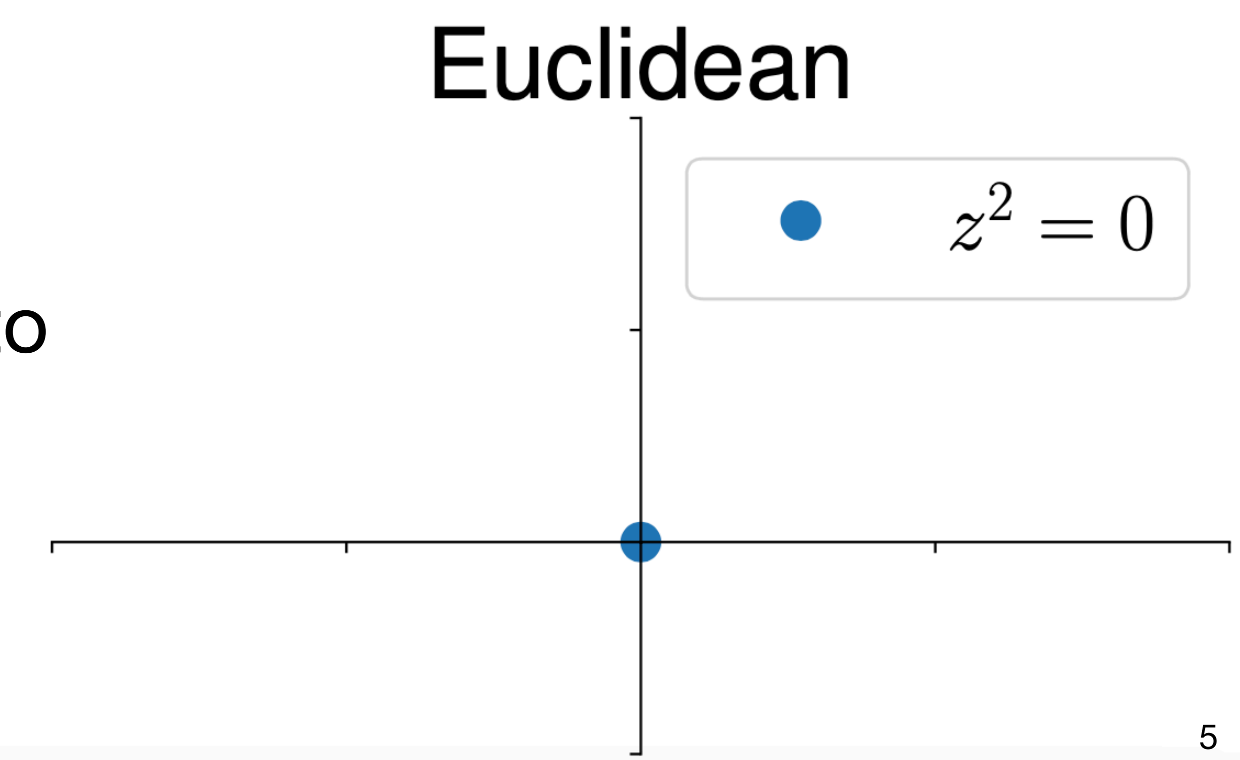
  X. Ji *Phys Rev Lett* 110 (2013) 262002

  - Wilson line operators

$$O_\Gamma^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0)$$

$$z^2 \neq 0$$

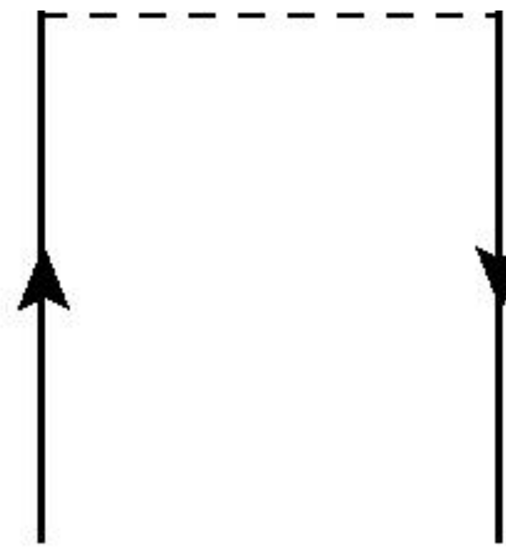
  - Factorizations exist analogous to cross sections



Many approaches

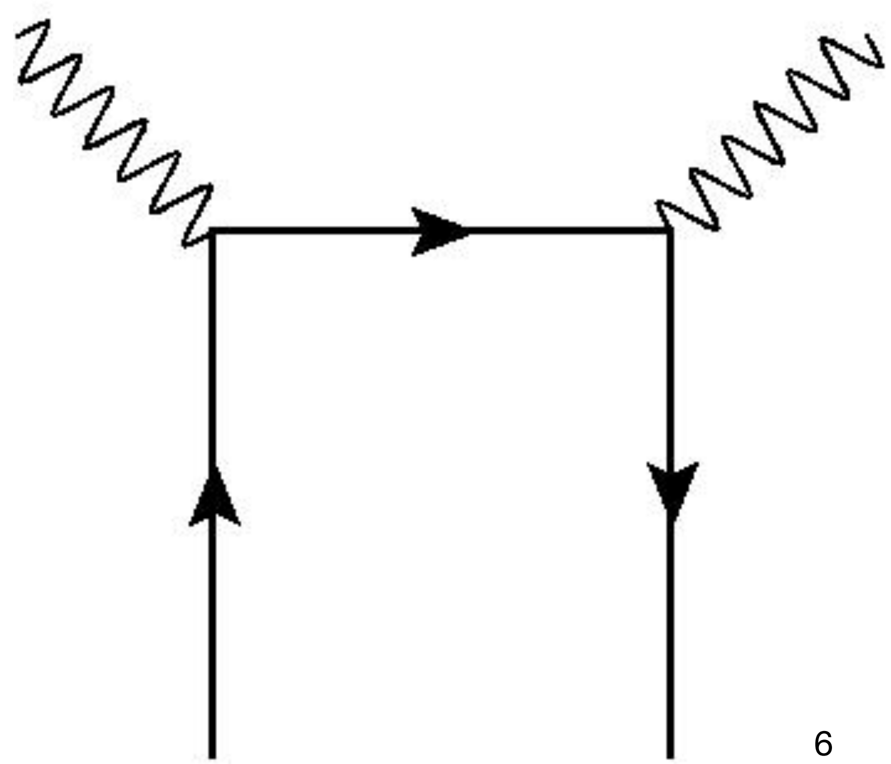
- **Wilson line operators**
  - LaMET X. Ji *Phys. Rev. Lett.* 110 (2013) 262002
  - Pseudo-PDF A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

$$O_{WL}(x;z) = \bar{\psi}(x+z)\Gamma W(x+z;x)\psi(x)$$



- Two current correlators
  - Hadronic Tensor
    K.-F. Liu et al *Phys. Rev. Lett.* 72 1790 (1994)
    Phys. Rev. D 62 (2000) 074501
  - HOPE
    W. Detmold and C.-J. D. Lin, *Phys. Rev. D* 73 (2006) 014501
  - Short distance OPE
    V. Braun and D. Muller *Eur. Phys. J. C* 55 (2008) 349
  - OPE-without-OPE
    A. Chambers et al, *Phys. Rev. Lett.* 118 (2017) 242001
  - Good Lattice Cross Sections
    Y.-Q. Ma and J.-W. Qiu *Phys. Rev. Lett.* 120 (2018) 2, 022003

$$O_{CC}(x,y) = J_{\Gamma}(x)J_{\Gamma'}(y)$$



Wilson Line Matrix Elements

- Matrix element $M(p, z) = \langle p | \bar{\psi}(z)\gamma^\alpha W(z; 0)\psi(0) | p \rangle$ $\quad z^2 \neq 0$

$$= 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

- Quasi-PDF: $\tilde{q}(y, p_z^2) = \frac{1}{2p_\alpha} \int dz e^{iyp_z z} M^\alpha(p_z, z)$

- Large Momentum Effective Theory: X. Ji *Phys. Rev. Lett.* 110 (2013) 262002

- $$\tilde{q}(y, p_z^2) = \int \frac{dx}{|x|} K\left(\frac{y}{x}, \frac{\mu^2}{(xp_z)^2}\right) q(x, \mu^2) + O\left(\frac{\Lambda_{\text{QCD}}^2}{(xp_z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x)p_z)^2}\right)$$

- Pseudo-PDF: A. Radyushkin *Phys. Rev. D* 96 (2017) 3, 034025

$$\mathcal{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda_{\text{QCD}}^2 z^2)$$

$$= \int du C'(u, \mu^2 z^2) I_q(u\nu, \mu^2) + O(\Lambda_{\text{QCD}}^2 z^2)$$

The Role of Separation and Momentum

- In quasi-PDF, pseudo-PDF, and Structure Functions, variables have common roles

Scale:

$p_z^2 \; / \; z^2 / Q^2$

- Scale for factorization to PDF
- Scale in power expansion
- Keep away from Λ^2_{QCD}
- Technically only requires single value

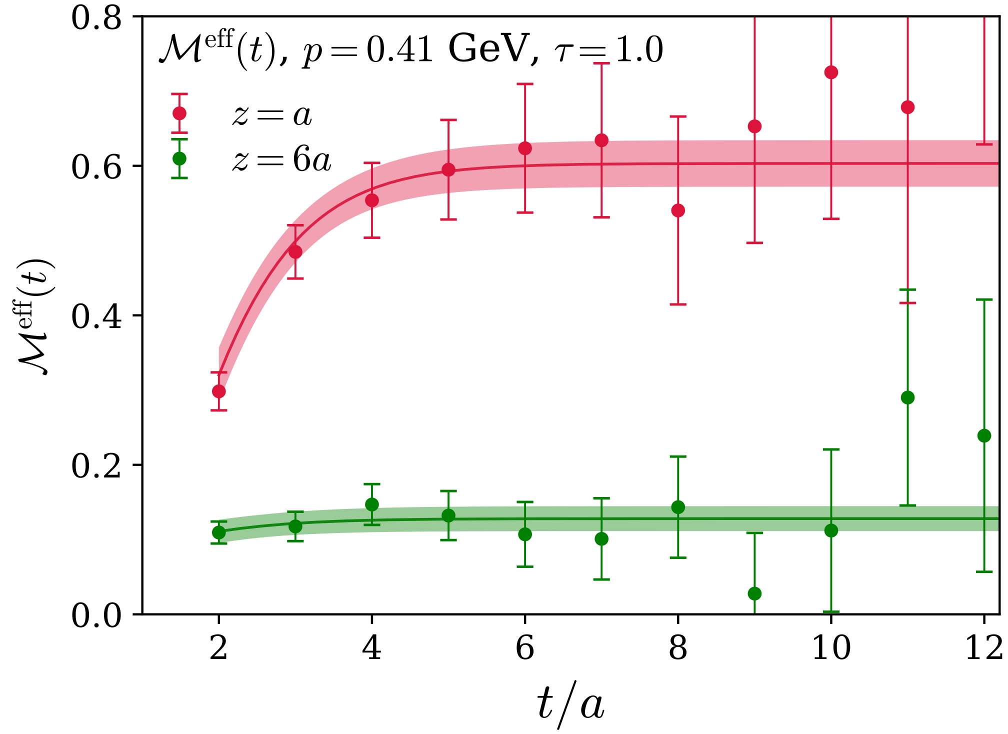
Dynamical variable:

$z \; / \; p_z$, or $\nu = p \cdot z$, x_B

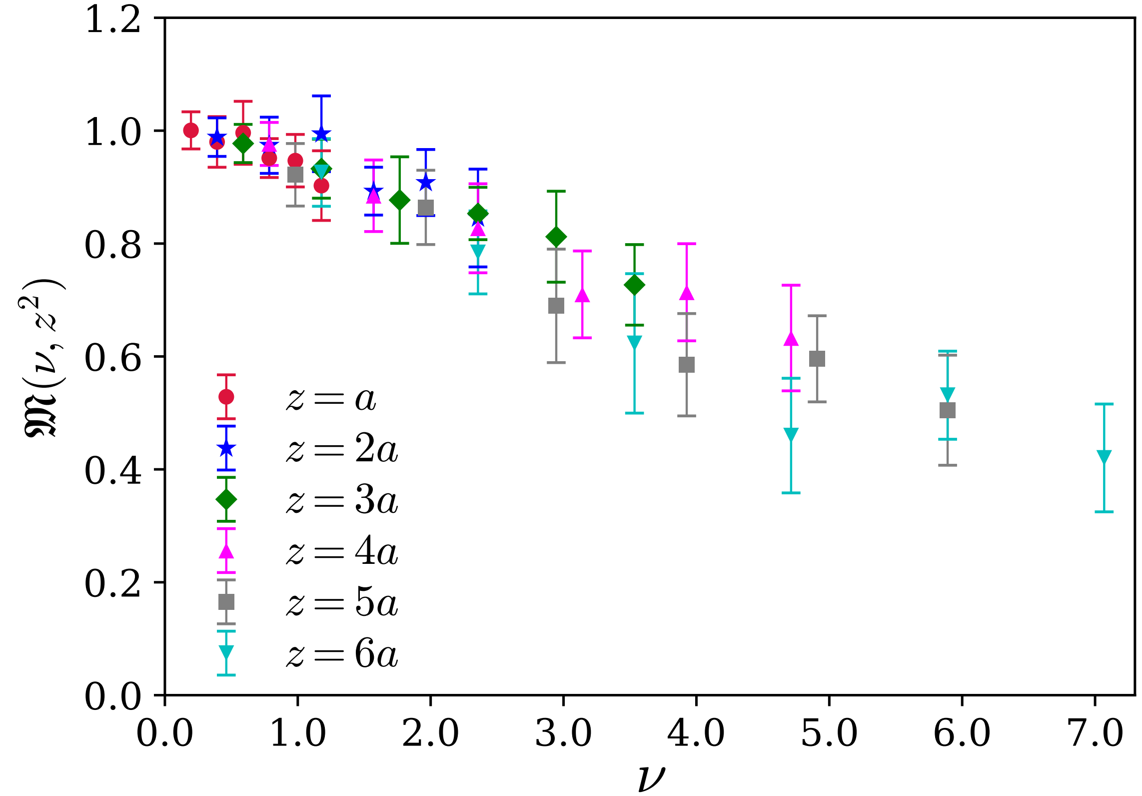
- Variable describes non-perturbative dynamics
- Can take large or small value
- Want as many as are available
- Wider range improves the inverse problem

From Lattice QCD to PDFs

Lattice Correlation Functions



Hadron Matrix Elements



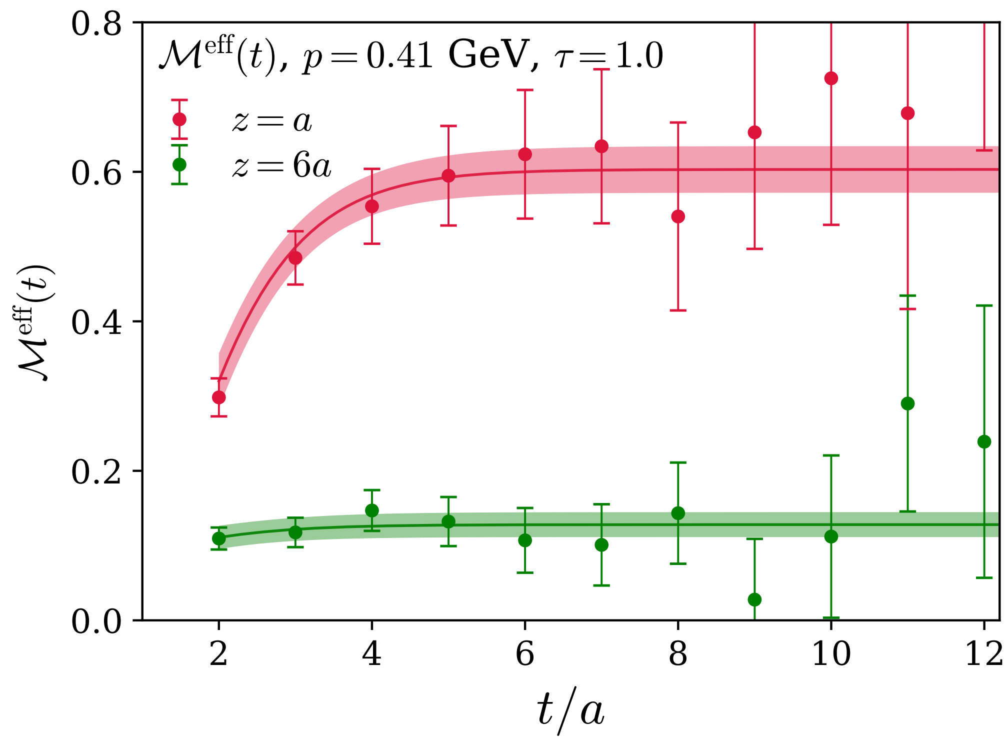
- Correlators are series of exponentials (Euclidean space)
- Model and remove subdominant at large time
- Common procedure in LQCD hadronic studies

Unpolarized Gluon PDF

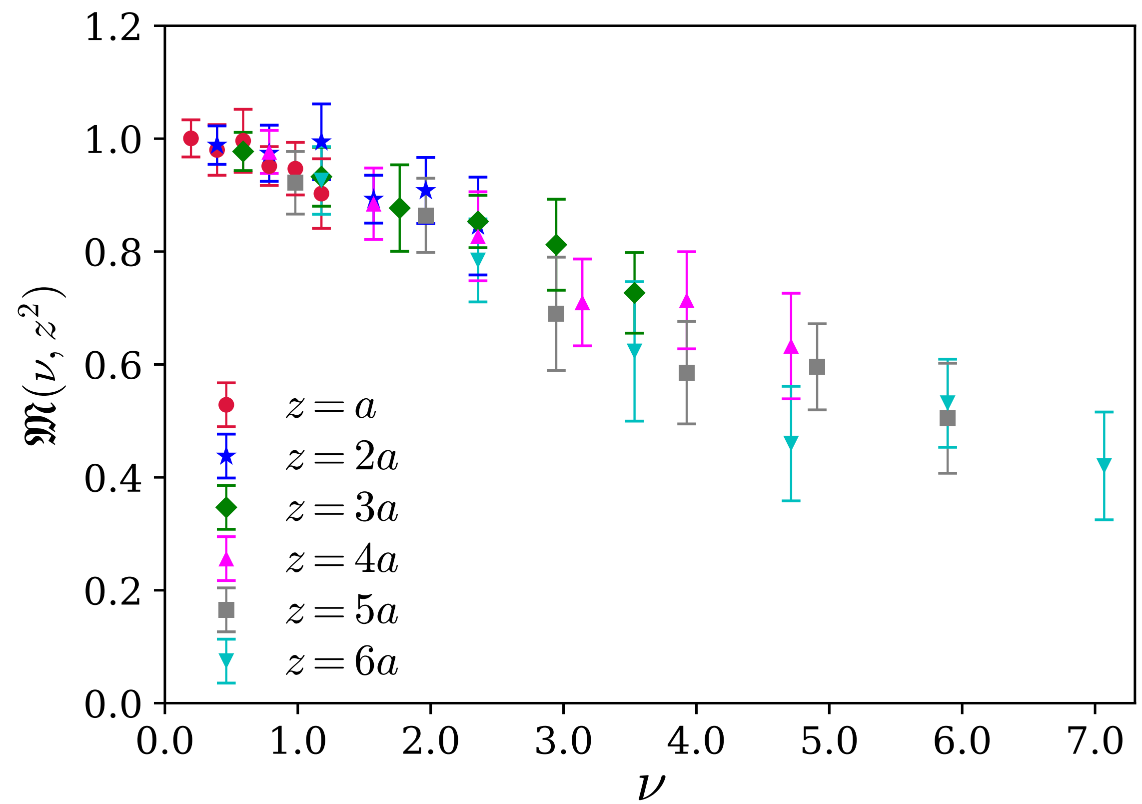
T. Khan, R. Sufian, JK, C. Monahan, C. Egerer, B. Joo, W. Morris, K. Orginos, A. Radyushkin, D. Richards, E. Romero, S. Zafeiropoulos
PRD 104 (2021) 9, 094516

From Lattice QCD to PDFs

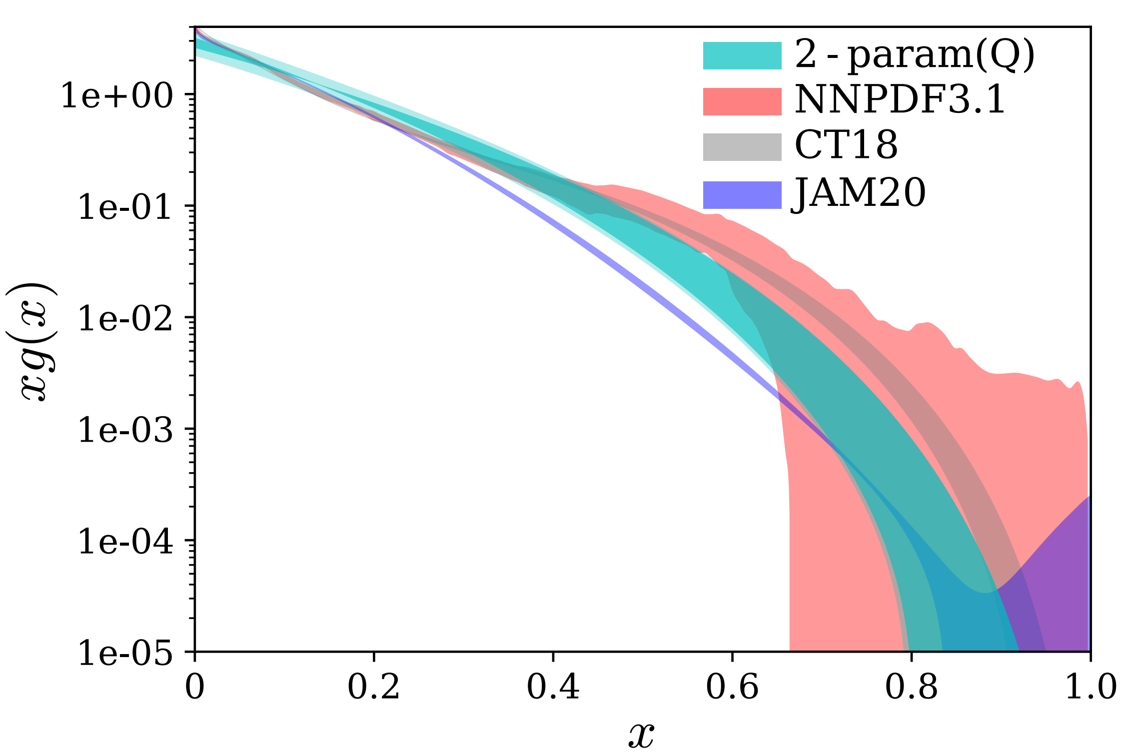
Lattice Correlation Functions



Hadron Matrix Elements



Parton Distributions



- Incomplete information gives integral inverse problem

$$xg(x) = x^a(1-x)^b / B(a+1, b+1)$$

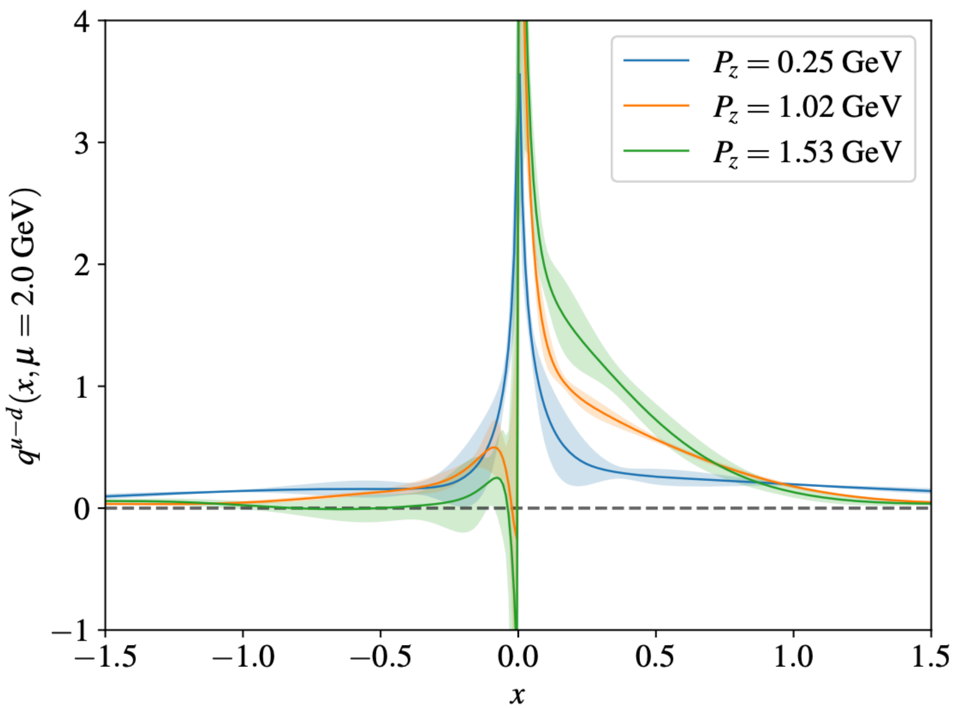
- To more accurately infer PDF, we need larger ν

Unpolarized Gluon PDF

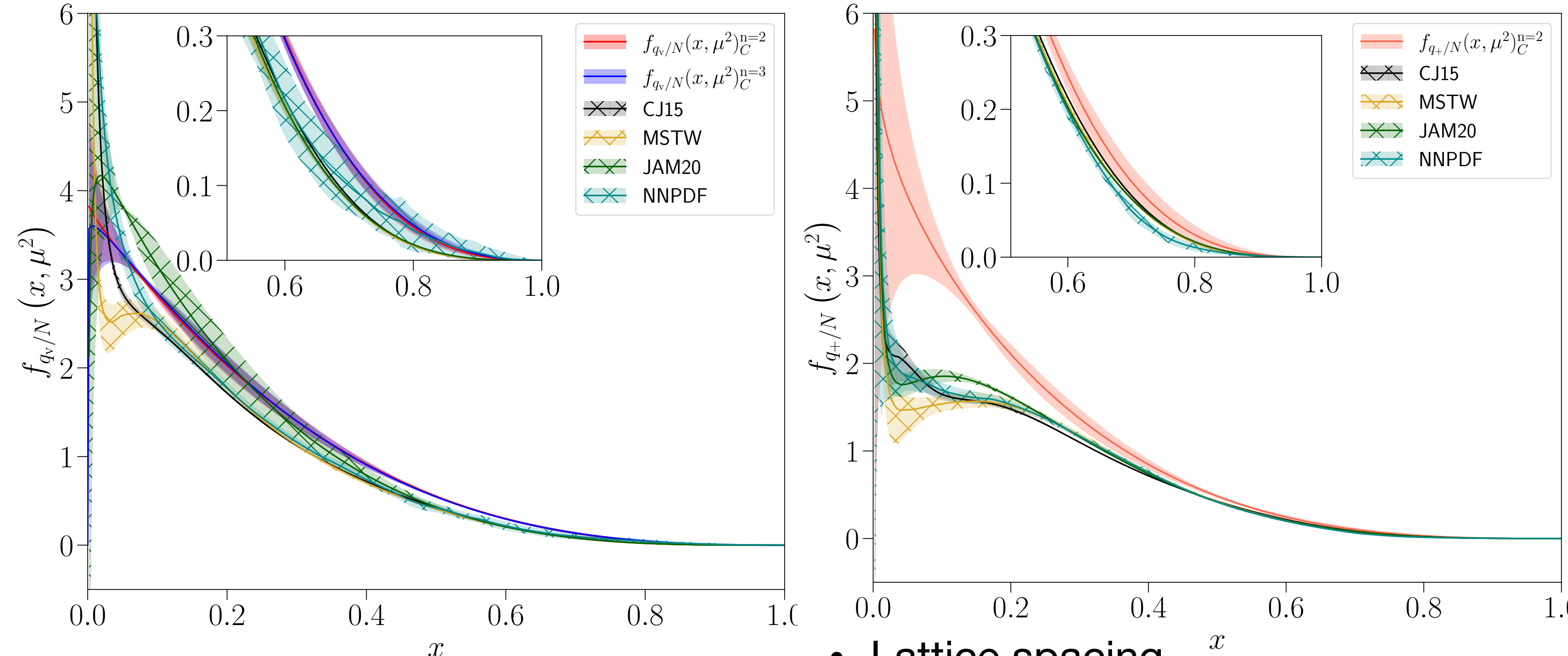
T. Khan, R. Sufian, JK, C. Monahan, C. Egerer, B. Joo, W. Morris, K. Orginos, A. Radyushkin, D. Richards, E. Romero, S. Zafeiropoulos
PRD 104 (2021) 9, 094516

Nucleon Unpolarized Quark PDF

X. Gao et al (ANL/BNL) 2212.12569



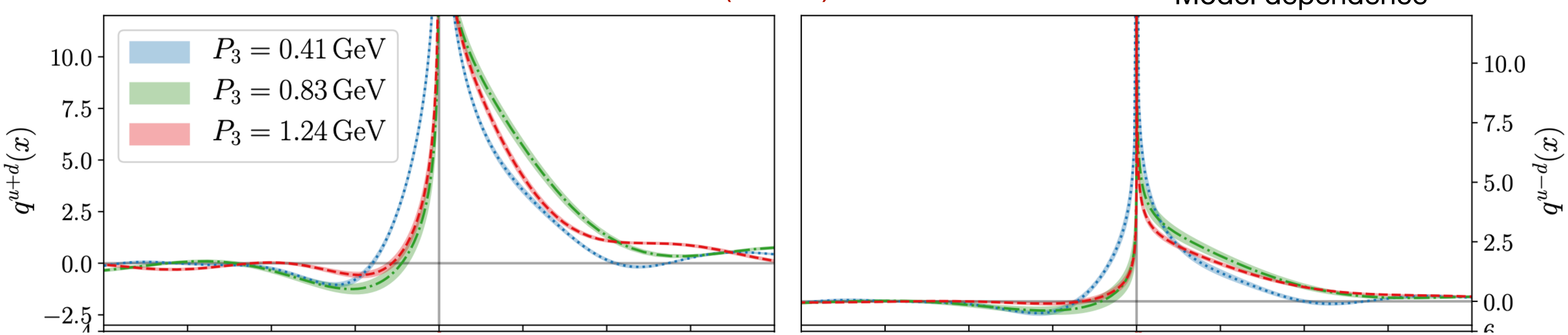
C. Egerer et al (HadStruc) 2107.05199



- Lattice spacing
- Pion mass
- Excited States
- Finite Volume
- Higher order matching
- Power Corrections
- Model dependence

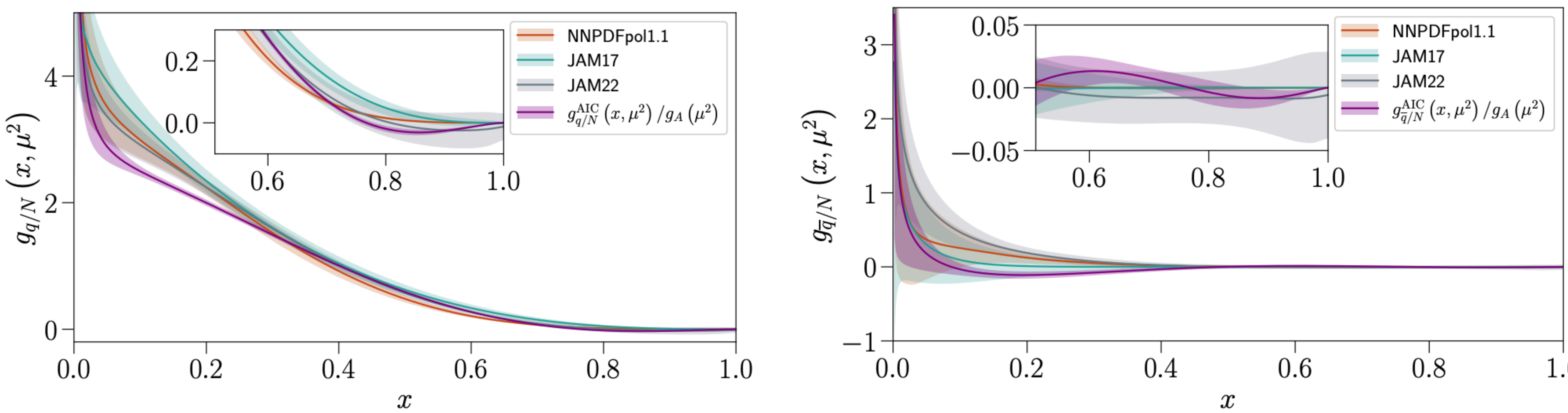
- Approaching a decade since first calculations
- Systematics have been continually improved

C. Alexandrou et al (ETMC) 2106.16065



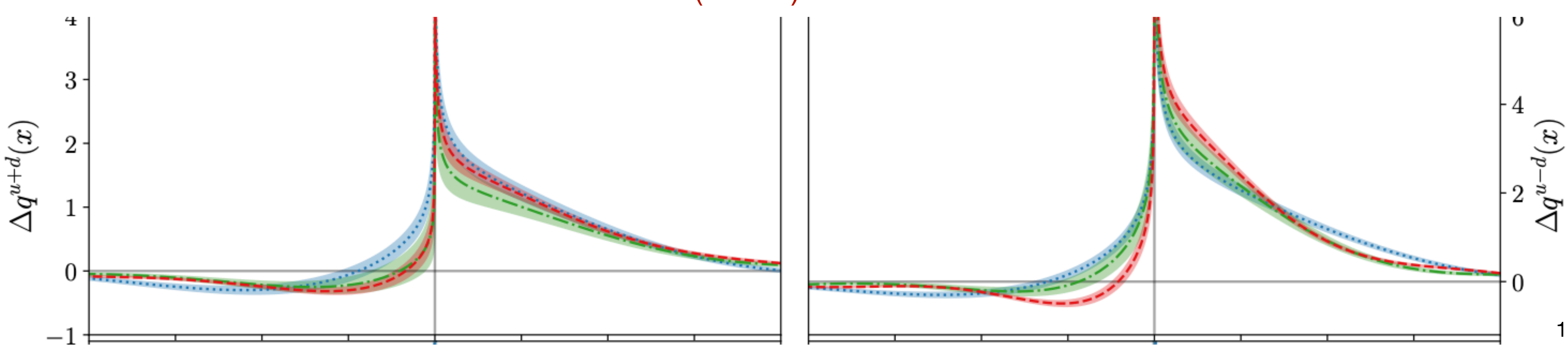
Nucleon Helicity Quark PDF

C. Egerer et al (HadStruc) 2211.04424

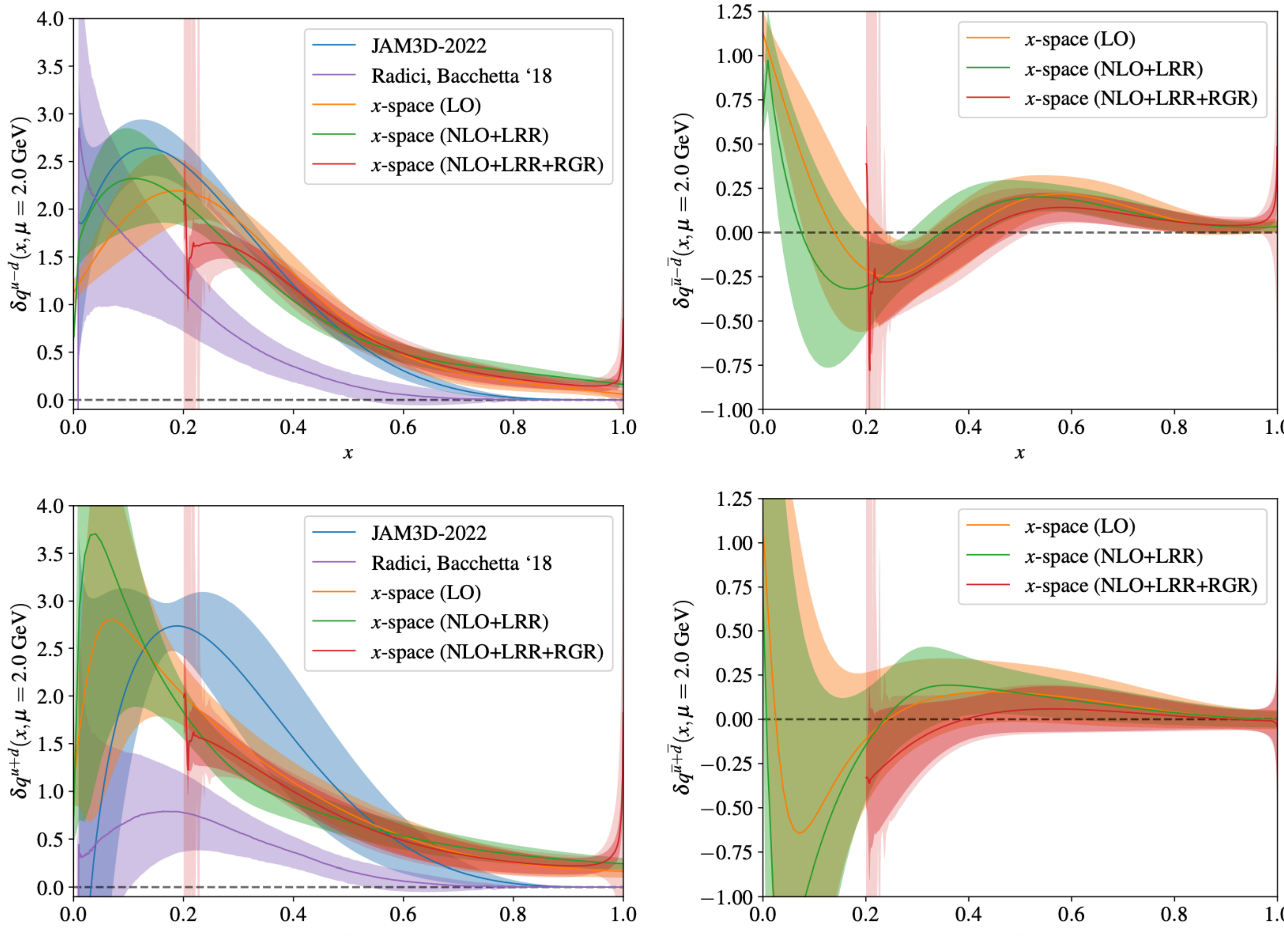


- Approaching a decade since first calculations
- Systematics have been continually improved

C. Alexandrou et al (ETMC) 2106.16065

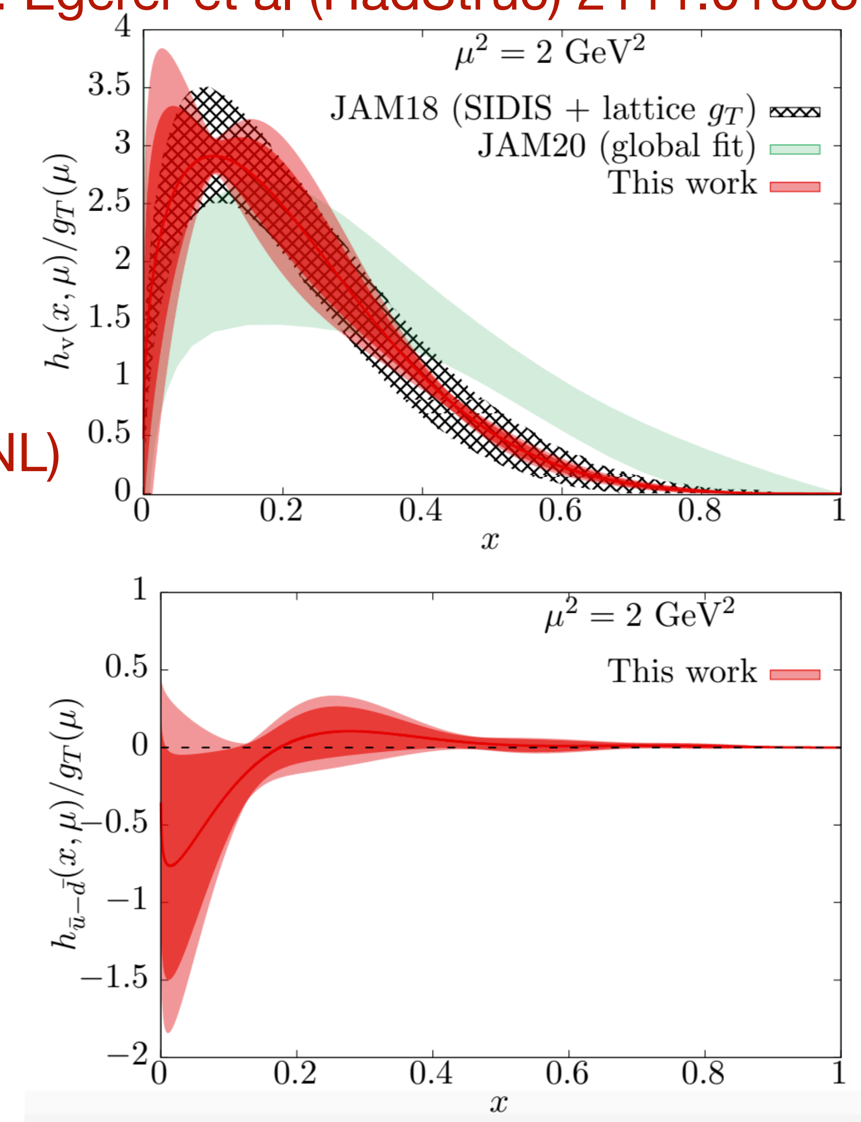


Nucleon Transversity Quark PDF



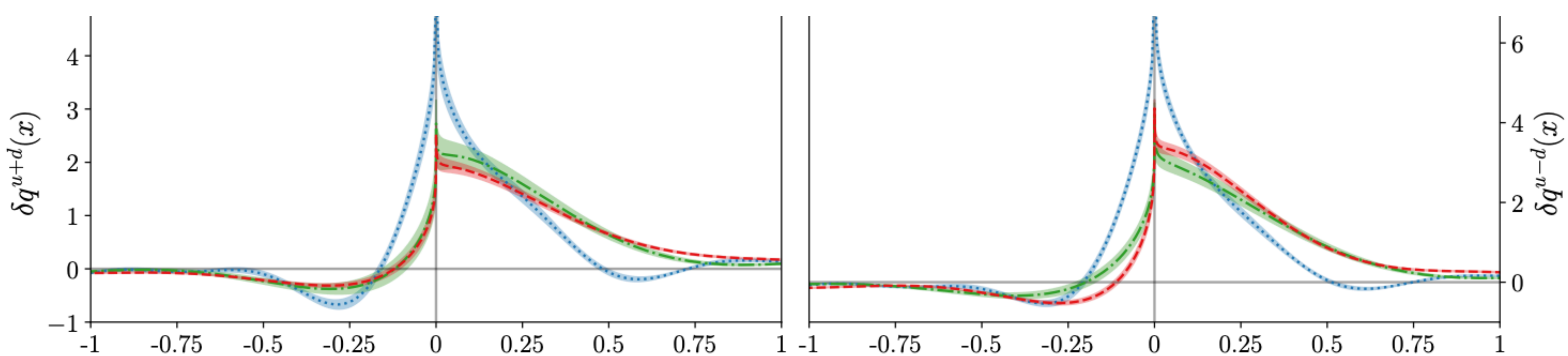
X. Gao et al (ANL/BNL) 2310.19047

C. Egerer et al (HadStruc) 2111.01808



- Approaching a decade since first calculations
- Systematics have been continually improved

C. Alexandrou et al (ETMC) 2106.16065



What can we do beyond looking at nice PDF fits?

Evolution of parton distributions

- Standard DGLAP evolution
  - Parton model: Splitting of partons into smaller x

$$\mu^2 \frac{d}{d\mu^2} q(x,\mu^2) = \int_x^1 dy\, P_{qq}(y) q(\frac{x}{y},\mu^2)$$

- MSbar Step Scaling function
  - Integrated or discretized version of evolution

$$q(x,\mu^2) = \int_x^1 dy\, \mathscr{E}(y,\mu^2,\mu_0^2) q(\frac{x}{y},\mu_0^2)$$

- pseudo-PDF evolution
  - $\mathfrak{M}(\nu,z^2) = \int_0^1 du\, C(u,\mu^2 z^2)\, I(u\nu,\mu^2)$
  - $\mu^2 \frac{d}{d\mu^2} \mathfrak{M}(\nu,z^2) = 0$

$$\mathfrak{M}(\nu,z^2) = \int_0^1 d\alpha\, \Sigma(\alpha,z^2,z_0^2) \mathfrak{M}(\alpha\nu,z_0^2)$$

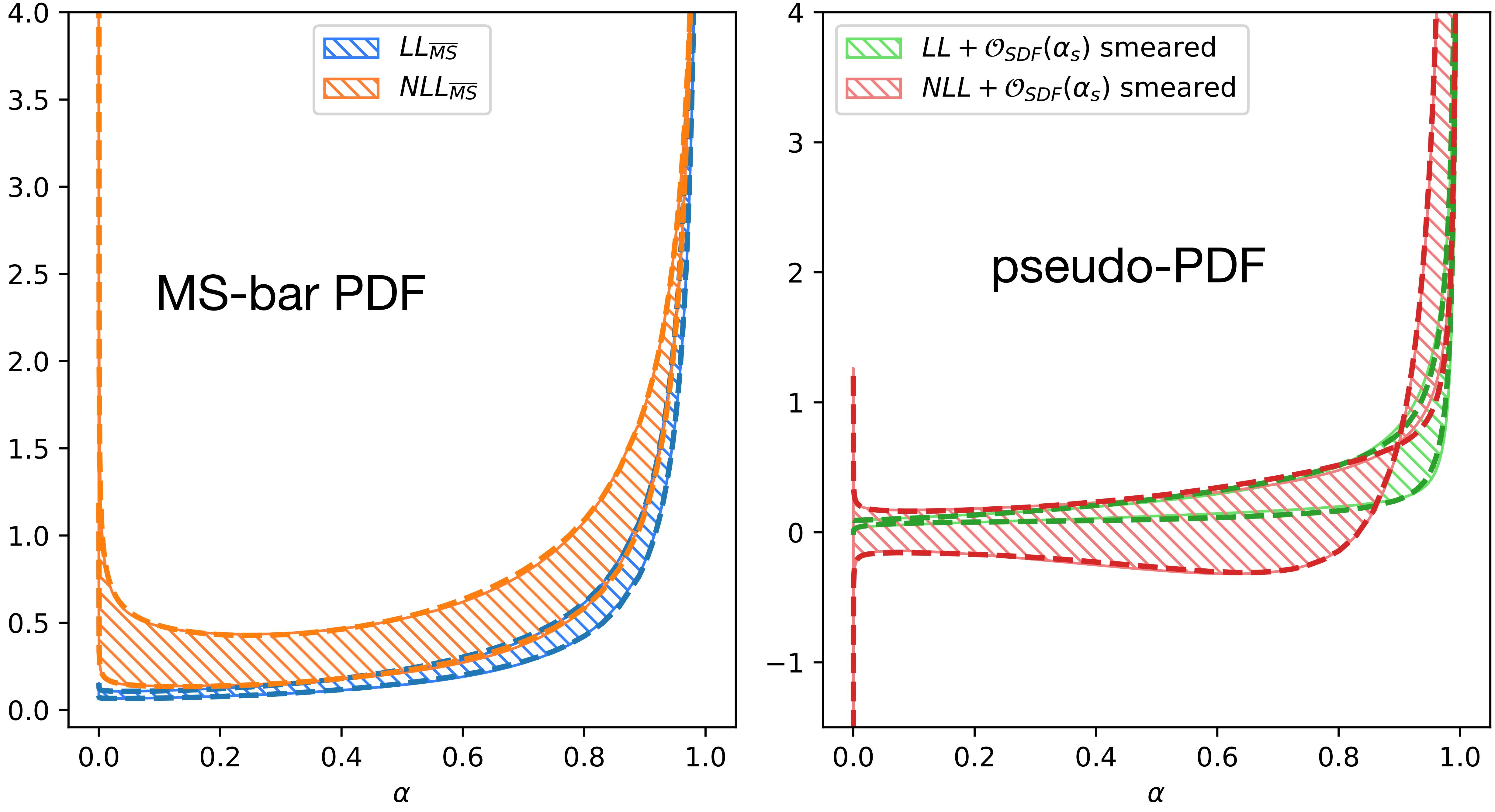
$$\mathscr{E}(\mu^2,\mu_0^2) = C^{-1}(\mu^2 z^2) \otimes \Sigma(z^2,z_0^2) \otimes C(\mu_0^2 z_0^2)$$

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

Evolution of parton distributions

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

- Perturbative evolution from ~700 MeV (0.282 fm) to ~1GeV (0.188 fm)
- Errors from varying scale by factor of 2

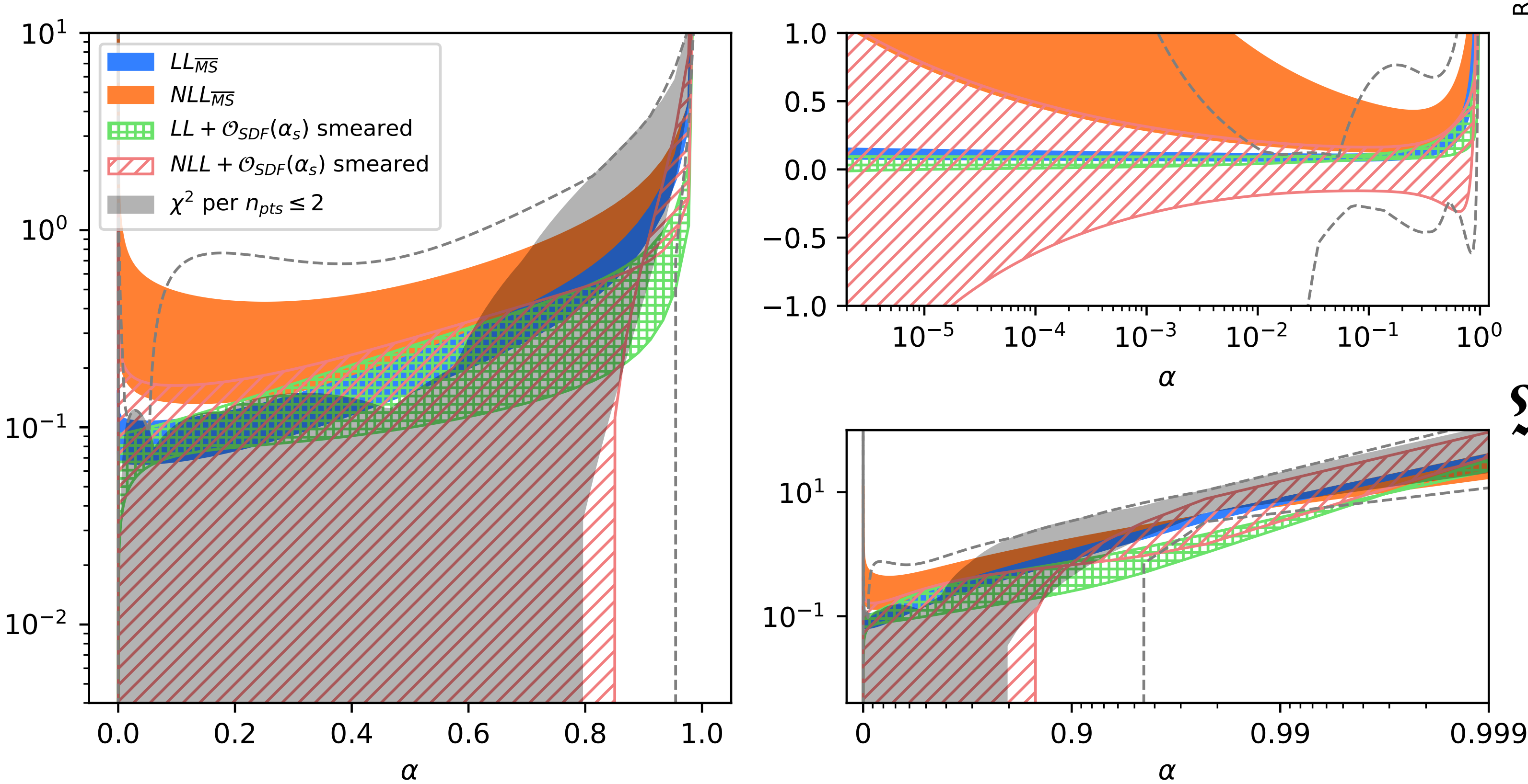


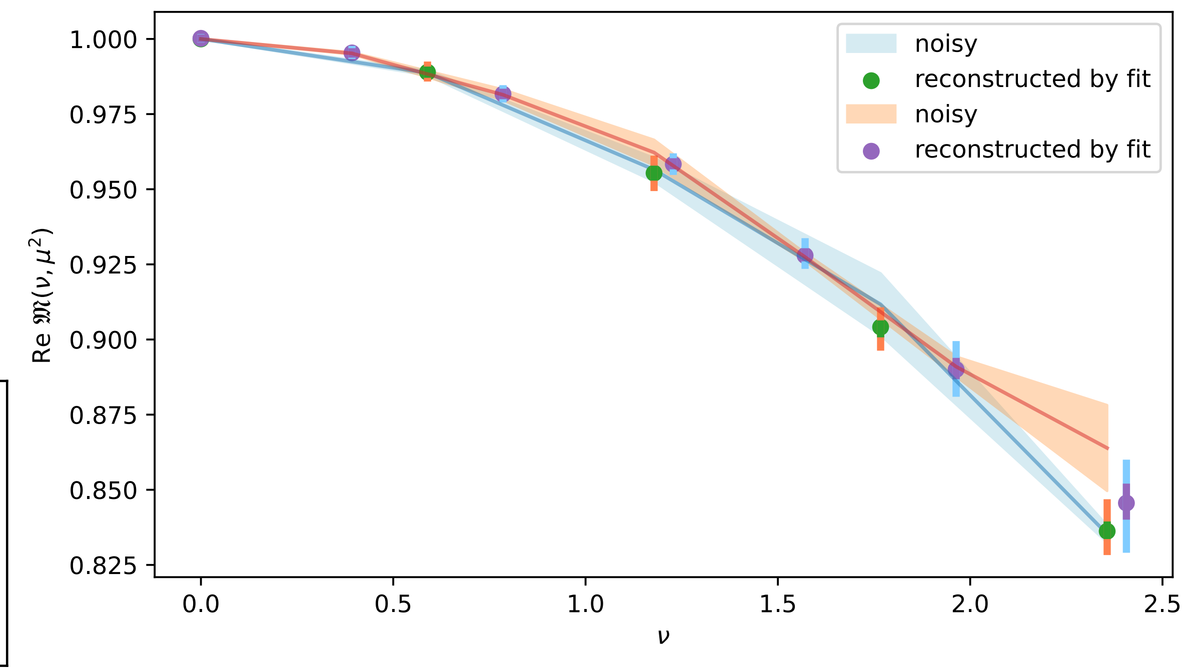
Step Scaling from the lattice

- Requires data in same range of ν and different z

- Model Function

$$\Sigma(\alpha) = A\alpha^{-\delta}(1 + r\alpha) + B(-\ln(\alpha))^{-\eta}\ln^2(1-\alpha) + \sigma\alpha(1-\alpha)$$





$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \Sigma(\alpha, z^2, z_0^2)\mathfrak{M}(\alpha\nu, z_0^2)$$

- Catch: Requires assumption of leading twist dominance and ranges of ν are limited
  - Need very fine lattices to do right.

H. Dutrieux, JK, C. Monahan, K. Orginos, S. Zafeiropoulos arXiv:2310.19926

If PDFs are universal….

*If the **same** PDFs are factorizable from lattice and experiment, and if power corrections can be controlled for both*

Why not analyze both simultaneously?

- Factorization of hadronic cross sections

$$d\sigma_h = d\sigma_q \otimes f_{h/q} + P.C.$$

- Factorization of Lattice observables

$$M_h = M_q \otimes f_{h/q} + P.C.$$

Consider Lattice as a theoretical prior to the experimental Global Fit

Complementarity in Lattice and Experiment

LATTICE

- Lattice limited to low ν, sensitive to $x \gtrsim 0.2$, but high sensitivity to large x
- Lattice matching relation is integral over all x
- Low p_z data can reach high signal-to-noise compared to experiment
- Lattice can evaluate independently each spin, flavor, and even hadron

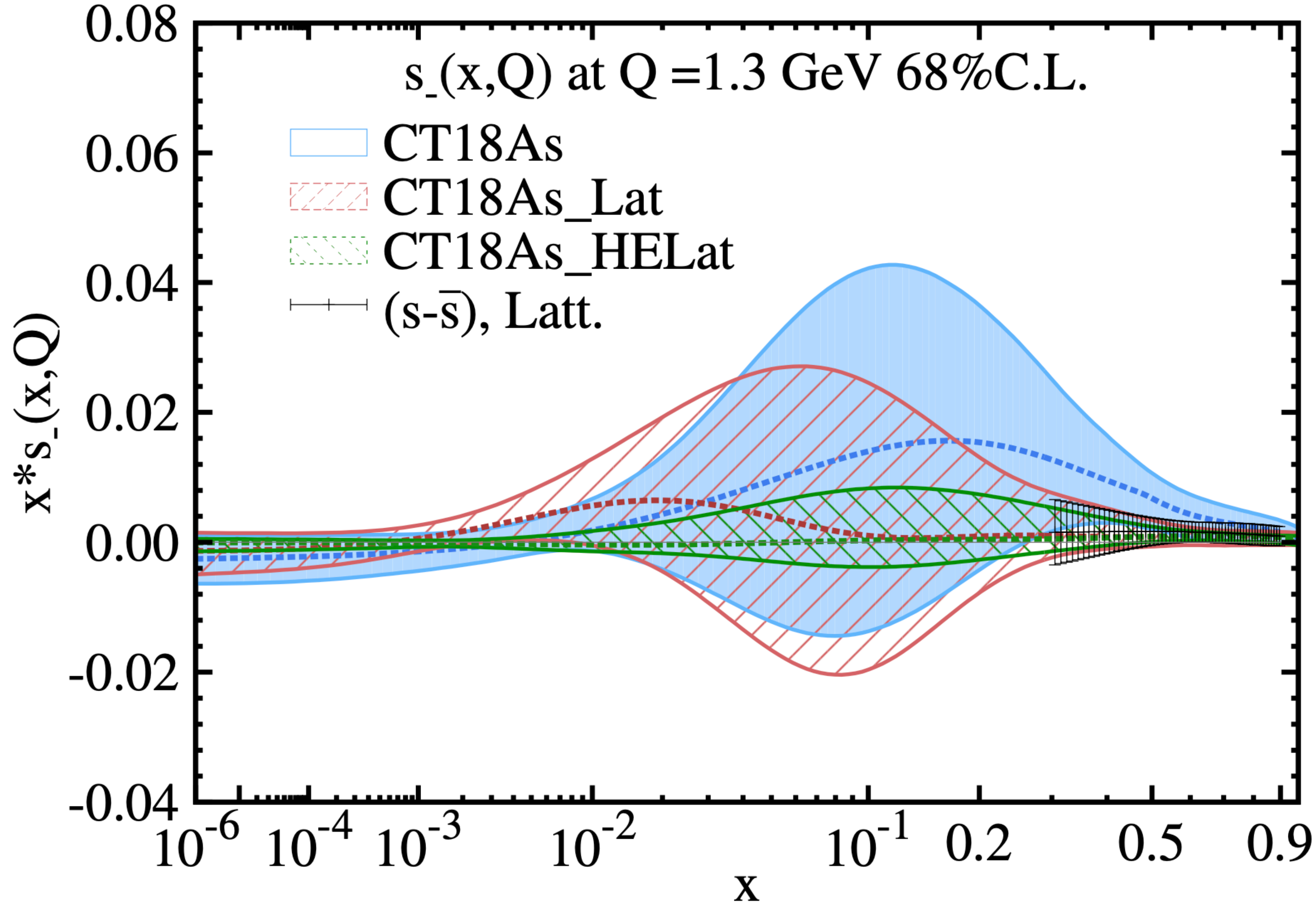
EXPERIMENT

- Cross Sections limited to specific max but can reach very low x_B
- Cross Section matching is integral from x_B to 1
    - Creates sensitively to hard kernel in large x region
- Wealth of decades of experimental data outweigh modern lattice

Strange quark distributions

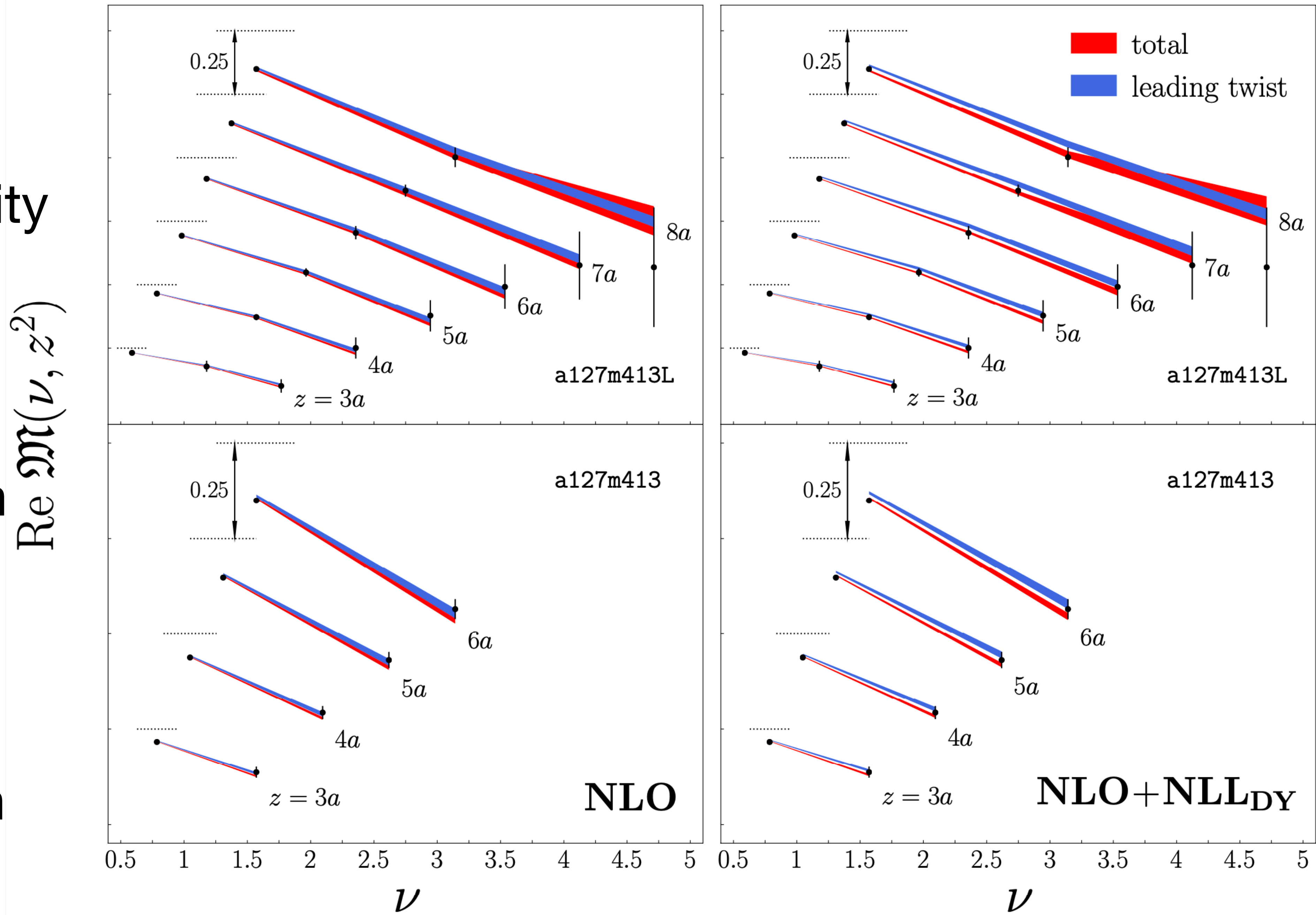
- Lattice can directly access individual quark flavors independently
- Flavor decomposed matrix elements have noisy “disconnected” contributions
- Studies of strange and charm PDFs have begun and give promising precision

Hou et al arXiv 2204.07944



Complementarity in pion PDF

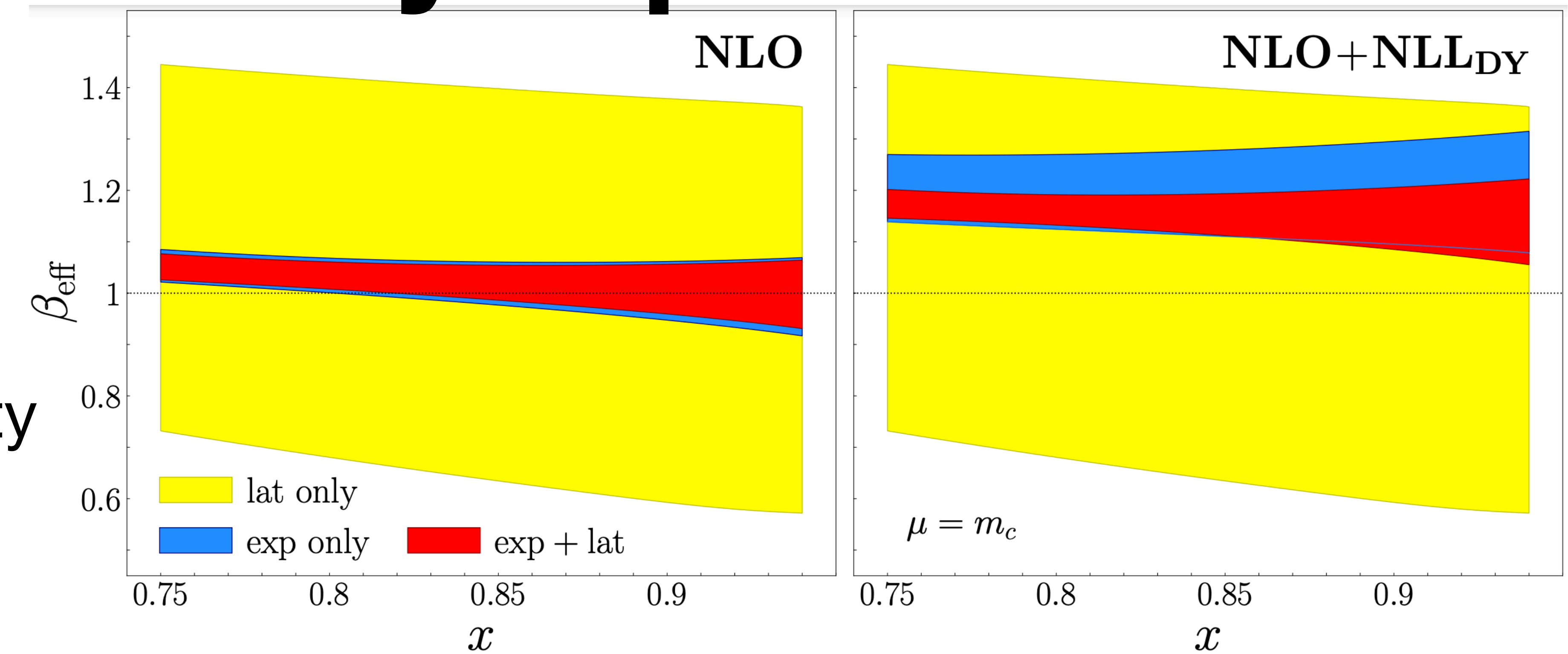
- Lattice can directly access different hadrons
- Lattice lacks sensitivity to threshold logs and can be used to test theoretical kernels
- Improves precision in large x where experimental data does not exist
- Low momentum pion data are extremely precise



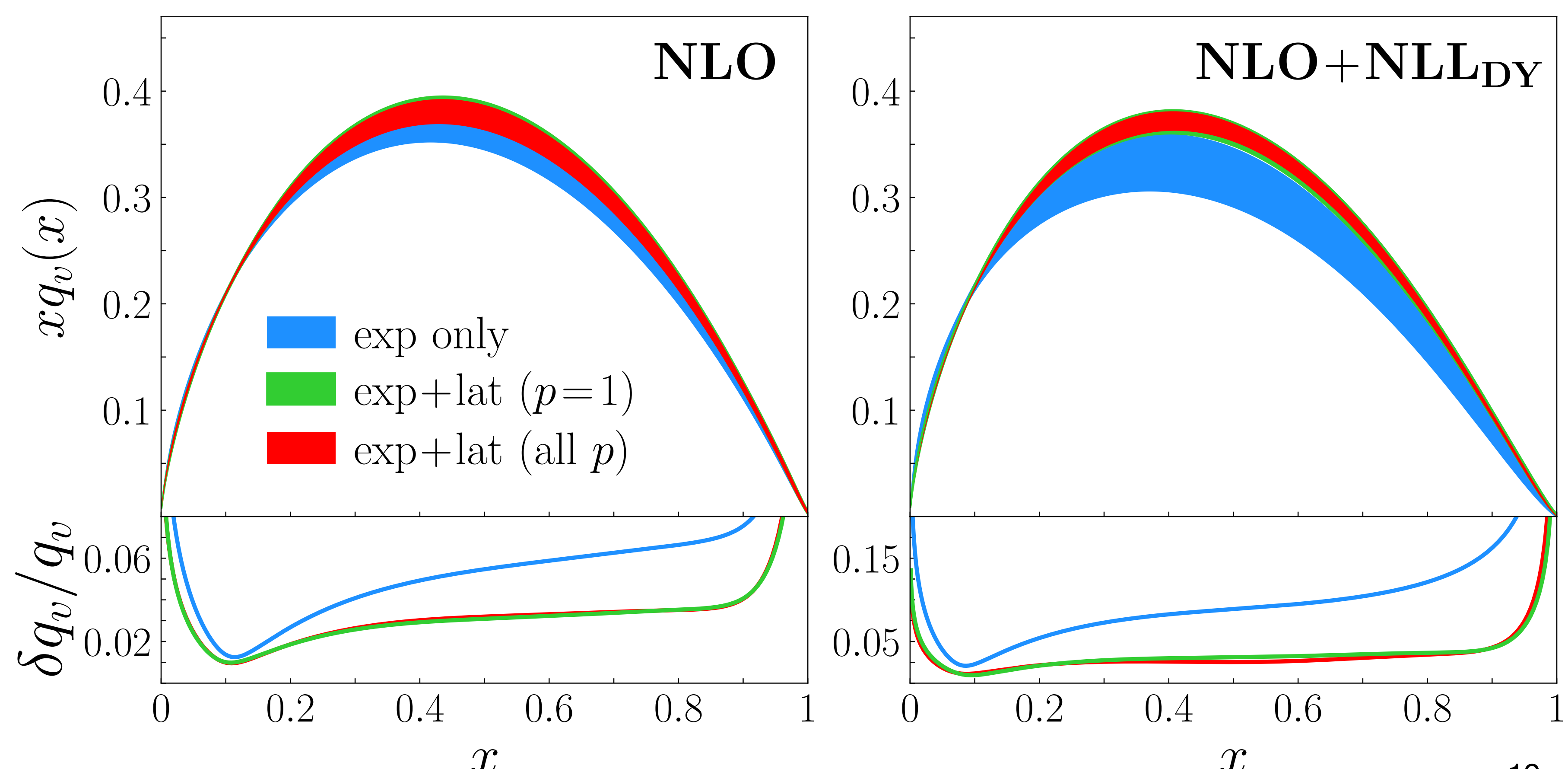
P. Barry et al, *Phys. Rev. D* 105 (2022) 11, 114051

Complementarity in pion PDF

- Lattice can directly access different hadrons
- Lattice lacks sensitivity to threshold logs and can be used to test theoretical kernels
- Improves precision in large x where experimental data does not exist
- Low momentum pion data are extremely precise

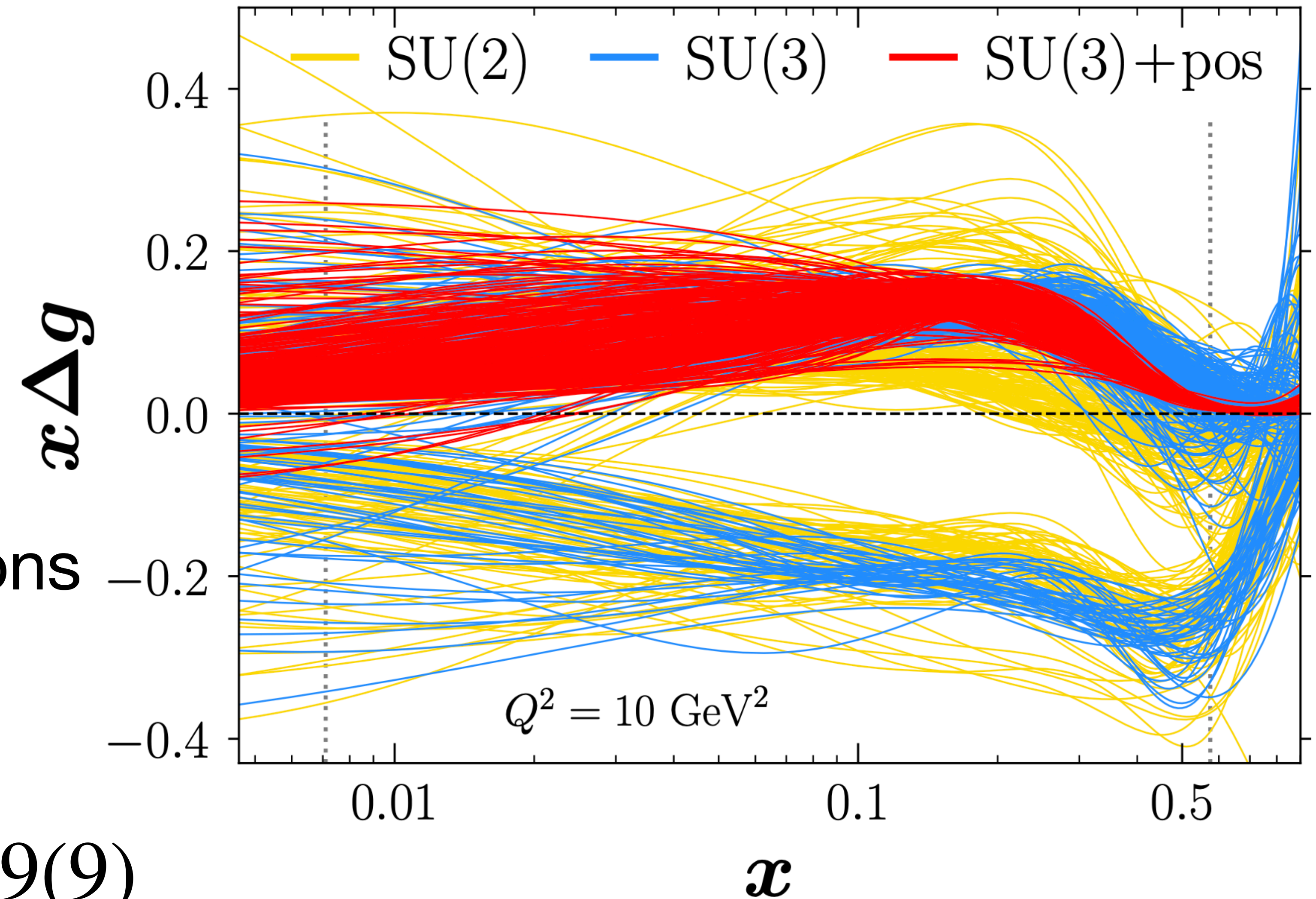


P. Barry et al, *Phys. Rev. D* 105 (2022) 11, 114051



Spinning gluons

Y. Zhou et al (JAM) Phys. Rev. D 105, 074022 (2022)



- Positivity removed from JAM helicity gluon PDF

$$|\Delta g| \leq g(x)$$

  - Reveals new band of solutions

- With constraint: $\Delta G = 0.39(9)$

- Without constraint: $\Delta G = 0.3(5)$

- Lattice: $\Delta G = 0.251(47)(16)$

Y-B. Yang et al (χ-QCD) Phys. Rev. Lett. 118, 102001 (2017)
K-F. Liu arXiv: 2112.08416

R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)

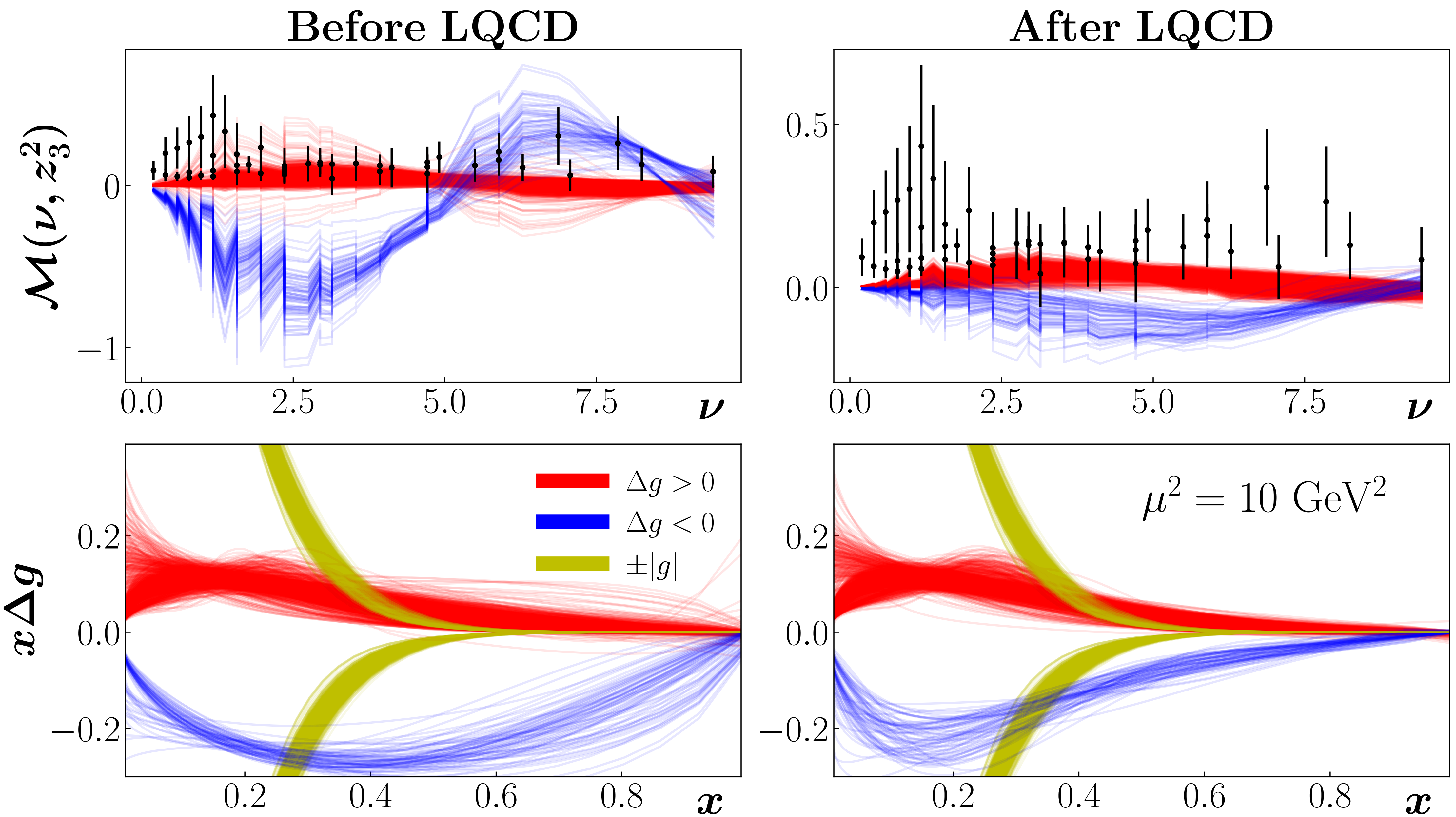
$$J = \frac{1}{2}\Delta\Sigma + L_q + L_G + \Delta G$$

$$\Delta G = \int dx\, \Delta g(x)$$

Spinning gluons

Y. Zhou et al Phys. Rev. D 105, 074022 (2022)
C. Egerer et al (HadStruc) arXiv:2207.08733

Can lattice data affect phenomenological polarized gluon analysis?



- The positive and negative solutions without positivity constraints
- Only positive band consistent with lattice data, but is too noisy to constrain.

$$\Delta G = \int d\nu\, I_g(\nu)$$

Conclusions

- Lattice matrix elements can be related to PDFs and their calculation have matured over the decade
- With control of systematic errors, lattice PDFs are approaching accuracy of global fits
- Non-perturbative PDF evolution can be determined from lattice data
- Adding Lattice data into global fits give better results than either could do alone
- All lessons can be extended to TMDs and GPDs

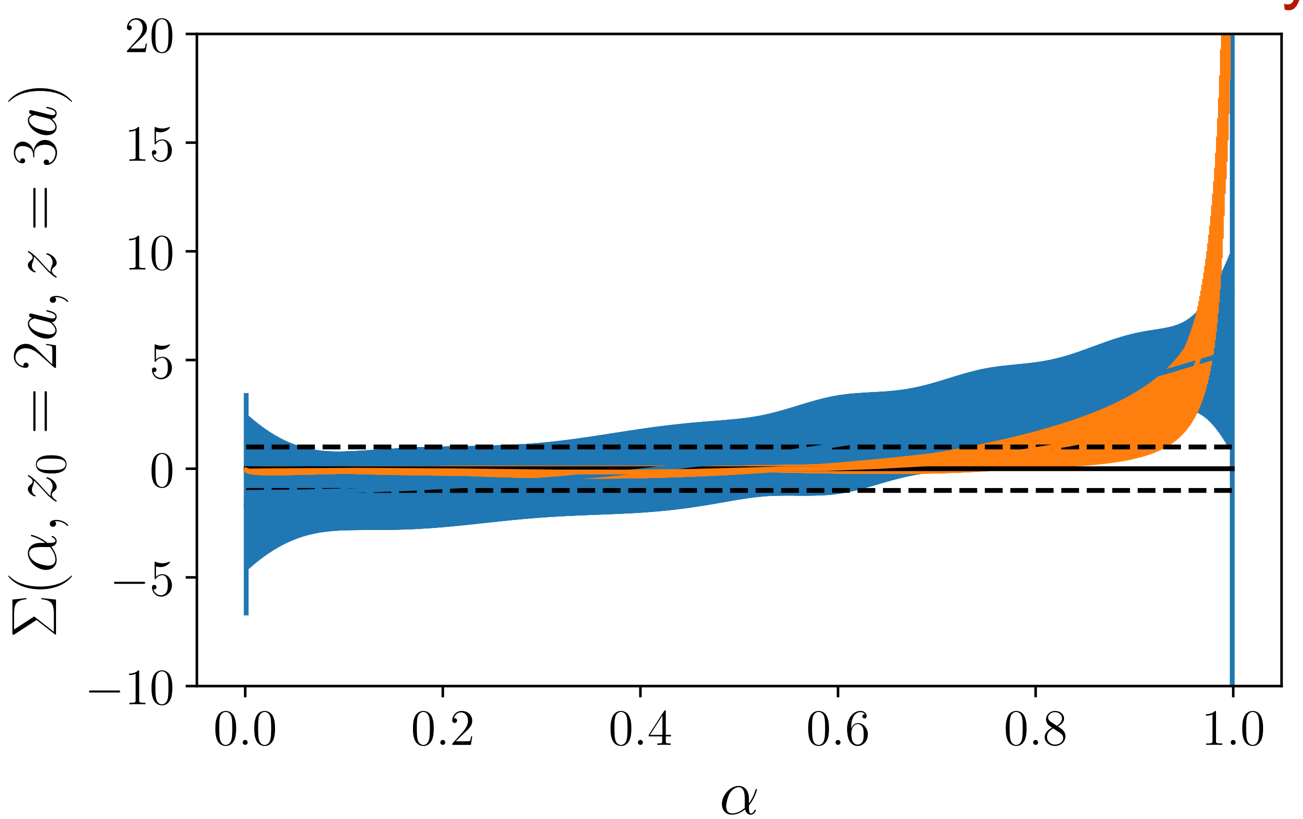
Thank you and the organizers!

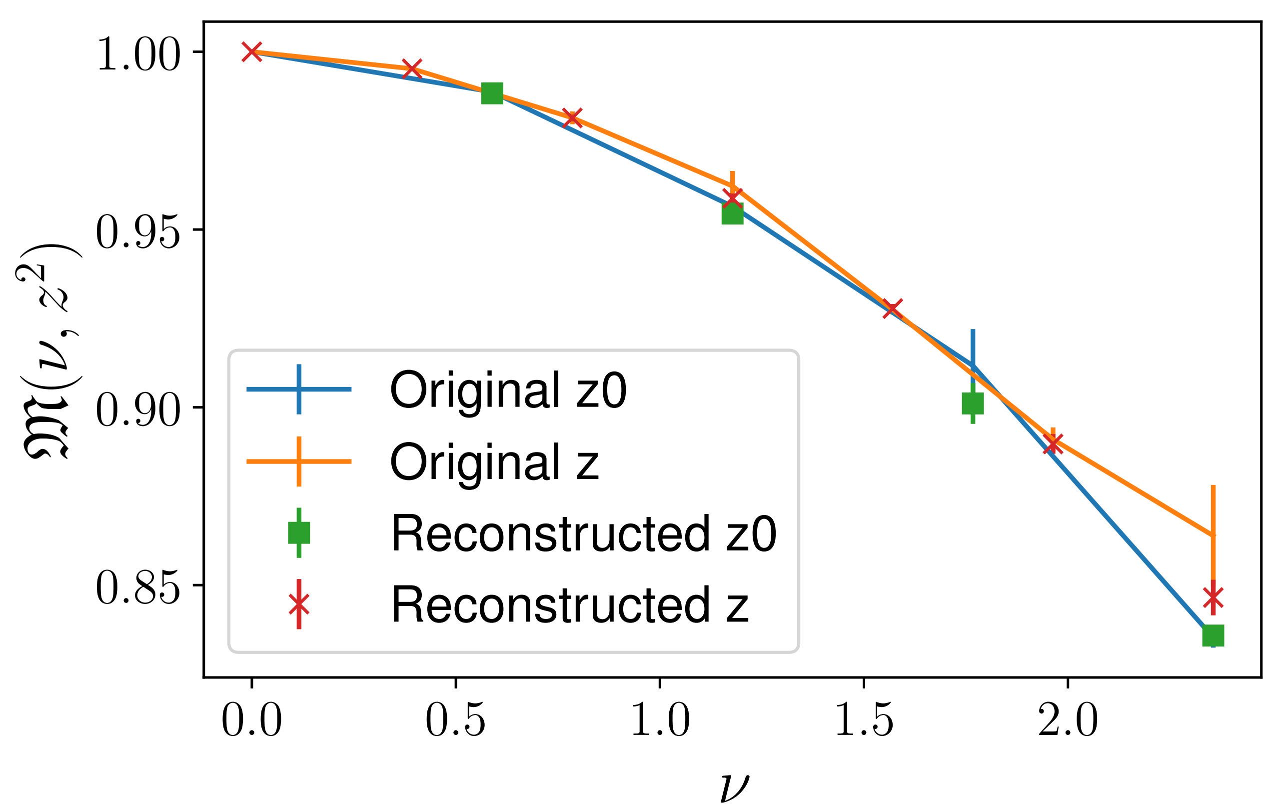
Back up slides

Bayesian Reconstruction

- Use different BR priors to study model dependencies
- First prior with easily understood biases
    - Quadratic Difference Ratio (QDR) $S(\Sigma) = u \int_0^1 d\alpha \, \frac{\left(\Sigma(\alpha) - h(\alpha)\right)^2}{\sigma(\alpha)^2}$

Preliminary!



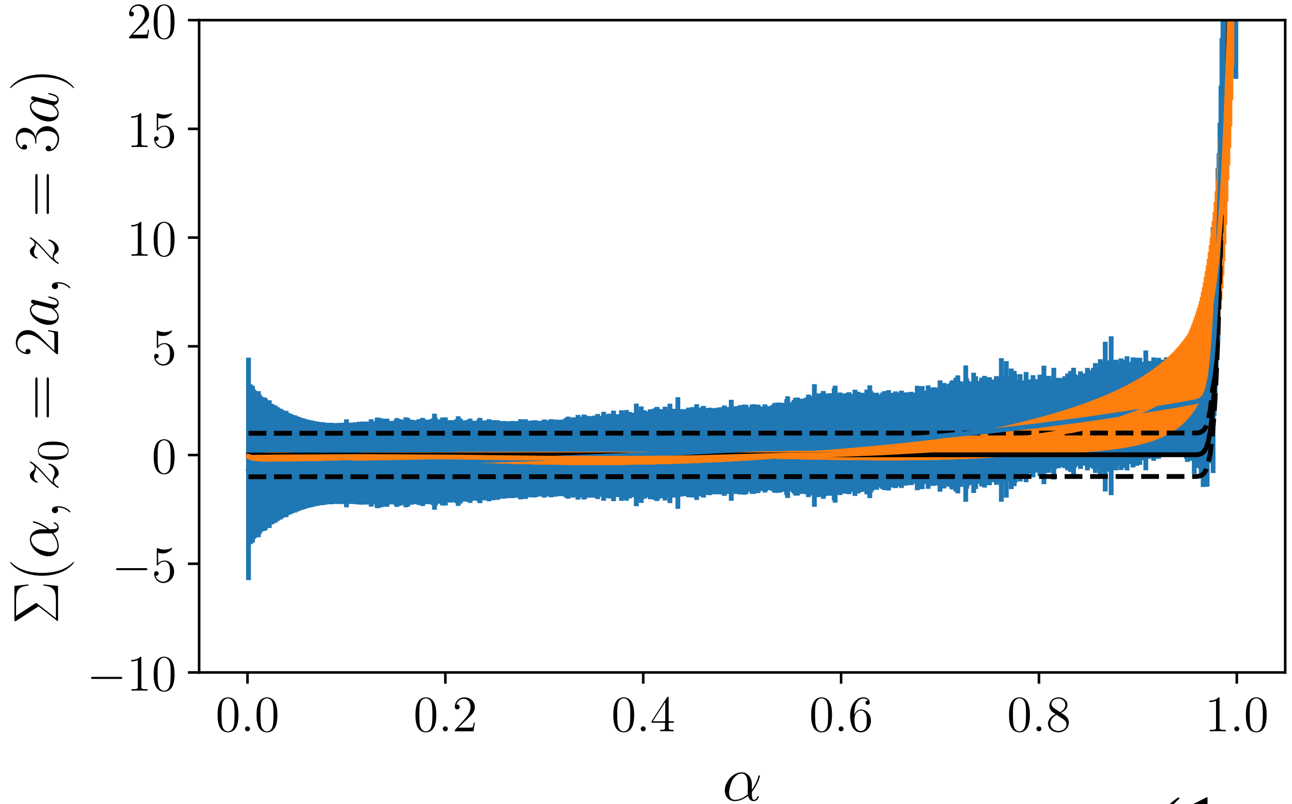


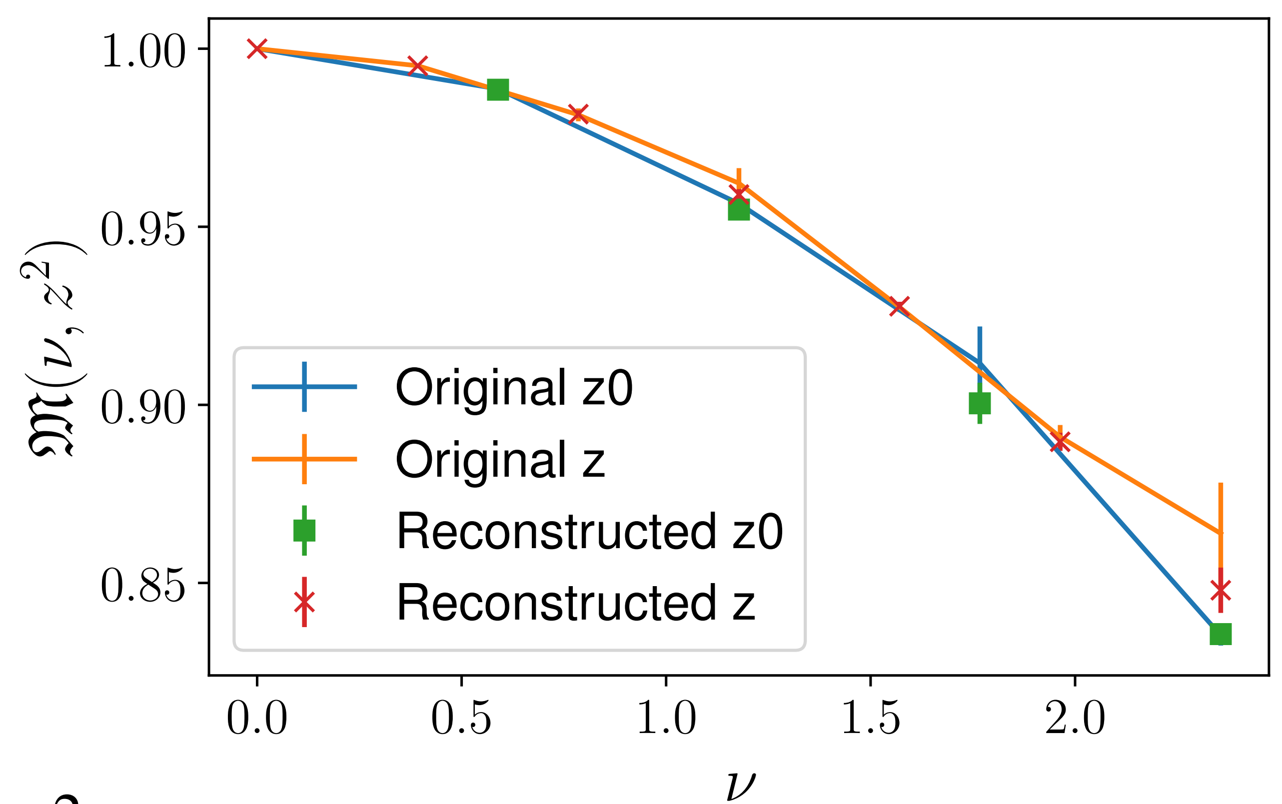
$$u = 1 \quad h(\alpha) = 0 \quad \sigma(\alpha) = 1$$

Bayesian Reconstruction

- Use different BR priors to study model dependencies
- First prior with easily understood biases
  - Quadratic Difference Ratio (QDR) $S(\Sigma) = u \int_0^1 d\alpha \, \frac{\left(\Sigma(\alpha) - h(\alpha)\right)^2}{\sigma(\alpha)^2}$

Preliminary!





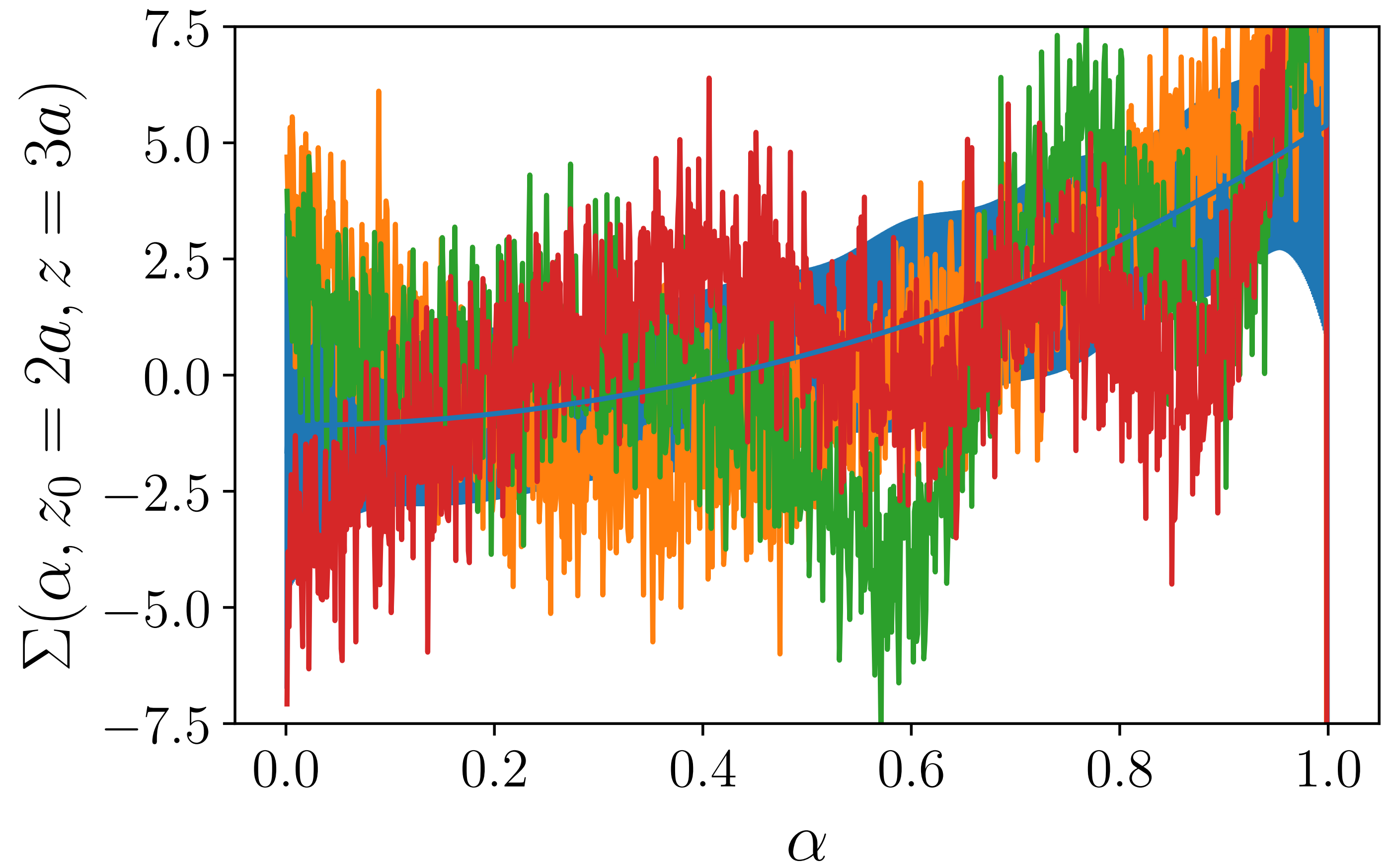
$$u = 1 \quad h(\alpha) = \exp(-\frac{(1-\alpha)^2}{w^2})/(w\sqrt{2\pi}) \quad \sigma(\alpha) = 1$$

$$w = 0.01$$

“I’m sorry, Nature hates Wiggles”

-A. Radyushkin

- Characteristic curves from fit
- QDR has no correlations between neighbors
- Need better priors!



“I’m sorry, Nature hates Wiggles”

-A. Radyushkin

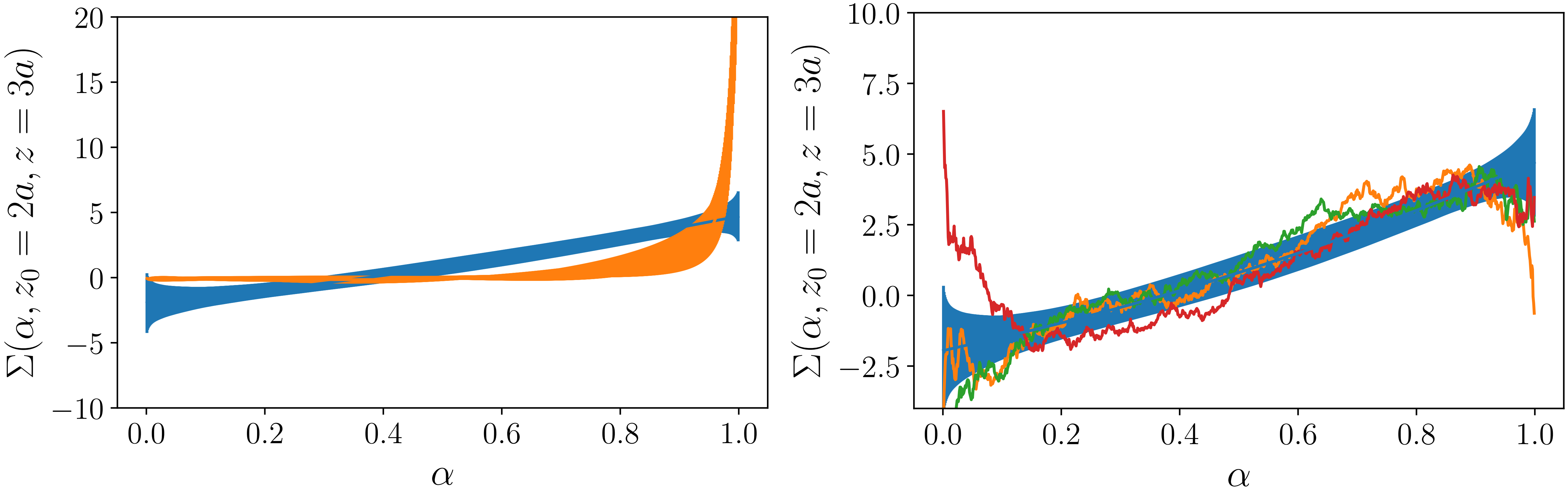
- Use different BR priors to study model dependencies
- Can we remove the wiggles?
  - A smoothing prior

$$S(\Sigma) = u \int_0^1 d\alpha\, \alpha(1-\alpha)\left(\frac{\partial \Sigma}{\partial \alpha}\right)^2$$

- Set u too large and it forces a flat result.

Preliminary!

$u = 1$



Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\Big[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\Big]\,|p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
    - Gives **two** amplitudes, one has no leading twist contribution

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\right]|\,p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution
- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\left[\widetilde{\mathscr{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathscr{M}(0,z^2)/m^2}$$

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\Big[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\Big]\,|p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \Big[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\Big]$
  - Gives **two** amplitudes, one has no leading twist contribution
- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\Big[\widetilde{\mathscr{M}}(z,p)/p_z p_0\Big]/Z_L(z/a)}{\mathscr{M}(0,z^2)/m^2}$$

- Relation to gluon and quark singlet ITD

$$\langle x\rangle_g \widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)$$

Helicity Gluon matrix element

I. Balitsky, W. Morris, A. Radyushkin JHEP 02 (2022) 193
C. Egerer et al (HadStruc) arXiv:2207.08733

- Helicity Gluon Matrix Element:

$$\widetilde{M}_{\mu\alpha;\nu\beta}(z,p,s) = \frac{1}{2}\epsilon_{\nu\beta\rho\sigma}M_{\mu\alpha;\rho\sigma} = \langle p,s\,|\,\mathrm{Tr}\left[F^{\mu\alpha}(z)W(z;0)\,\widetilde{F}^{\nu\beta}(0)\right]|\,p,s\rangle$$

- Useful Combination $\widetilde{\mathscr{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$
  - Gives **two** amplitudes, one has no leading twist contribution
- Use ratio with finite continuum limit

$$\widetilde{\mathfrak{M}}(\nu,z^2) = i\frac{\left[\widetilde{\mathscr{M}}(z,p)/p_z p_0\right]/Z_L(z/a)}{\mathscr{M}(0,z^2)/m^2}$$

- Relation to gluon and quark singlet ITD

$$\langle x\rangle_g\,\widetilde{\mathfrak{M}}(\nu,z^2) = \int_0^1 \widetilde{C}^{gg}(u,\mu^2z^2)\widetilde{I}_g(u\nu,\mu^2) + \widetilde{C}^{qg}(u,\mu^2z^2)\widetilde{I}_s(u\nu,\mu^2)$$

Pol Gluon Lorentz decomposition

I. Balitsky, W. Morris, A. Radyushkin
JHEP 02 (2022) 193

$$\widetilde{M}^{(2)}_{\mu\alpha;\lambda\beta}(z,p) = (sz)\big(g_{\mu\lambda}p_\alpha p_\beta - g_{\mu\beta}p_\alpha p_\lambda - g_{\alpha\lambda}p_\mu p_\beta + g_{\alpha\beta}p_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{pp}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha z_\beta - g_{\mu\beta}z_\alpha z_\lambda - g_{\alpha\lambda}z_\mu z_\beta + g_{\alpha\beta}z_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{zz}$$
$$+(sz)\big(g_{\mu\lambda}z_\alpha p_\beta - g_{\mu\beta}z_\alpha p_\lambda - g_{\alpha\lambda}z_\mu p_\beta + g_{\alpha\beta}z_\mu p_\lambda\big)\widetilde{\mathcal{M}}_{zp}$$
$$+(sz)\big(g_{\mu\lambda}p_\alpha z_\beta - g_{\mu\beta}p_\alpha z_\lambda - g_{\alpha\lambda}p_\mu z_\beta + g_{\alpha\beta}p_\mu z_\lambda\big)\widetilde{\mathcal{M}}_{pz}$$
$$+(sz)\big(p_\mu z_\alpha - p_\alpha z_\mu\big)\big(p_\lambda z_\beta - p_\beta z_\lambda\big)\widetilde{\mathcal{M}}_{ppzz}$$
$$+(sz)\big(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\big)\widetilde{\mathcal{M}}_{gg}$$

$$\widetilde{M}^{(1)}_{\mu\alpha;\lambda\beta}(z,p) = \Big(g_{\mu\lambda}s_\alpha p_\beta - g_{\mu\beta}s_\alpha p_\lambda - g_{\alpha\lambda}s_\mu p_\beta + g_{\alpha\beta}s_\mu p_\lambda\Big)\widetilde{\mathcal{M}}_{sp}$$
$$+\Big(g_{\mu\lambda}p_\alpha s_\beta - g_{\mu\beta}p_\alpha s_\lambda - g_{\alpha\lambda}p_\mu s_\beta + g_{\alpha\beta}p_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{ps}$$
$$+\Big(g_{\mu\lambda}s_\alpha z_\beta - g_{\mu\beta}s_\alpha z_\lambda - g_{\alpha\lambda}s_\mu z_\beta + g_{\alpha\beta}s_\mu z_\lambda\Big)\widetilde{\mathcal{M}}_{sz}$$
$$+\Big(g_{\mu\lambda}z_\alpha s_\beta - g_{\mu\beta}z_\alpha s_\lambda - g_{\alpha\lambda}z_\mu s_\beta + g_{\alpha\beta}z_\mu s_\lambda\Big)\widetilde{\mathcal{M}}_{zs}$$
$$+(p_\mu s_\alpha - p_\alpha s_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{pspz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(p_\lambda s_\beta - p_\beta s_\lambda)\widetilde{\mathcal{M}}_{pzps}$$
$$+(s_\mu z_\alpha - s_\alpha z_\mu)(p_\lambda z_\beta - p_\beta z_\lambda)\widetilde{\mathcal{M}}_{szpz}$$
$$+(p_\mu z_\alpha - p_\alpha z_\mu)(s_\lambda z_\beta - s_\beta z_\lambda)\widetilde{\mathcal{M}}_{pzsz}$$

Want: $M_{\Delta g}(\nu, z^2) = \left[\widetilde{\mathcal{M}}^{(+)}_{sp} - \nu\,\widetilde{\mathcal{M}}_{pp}\right]$

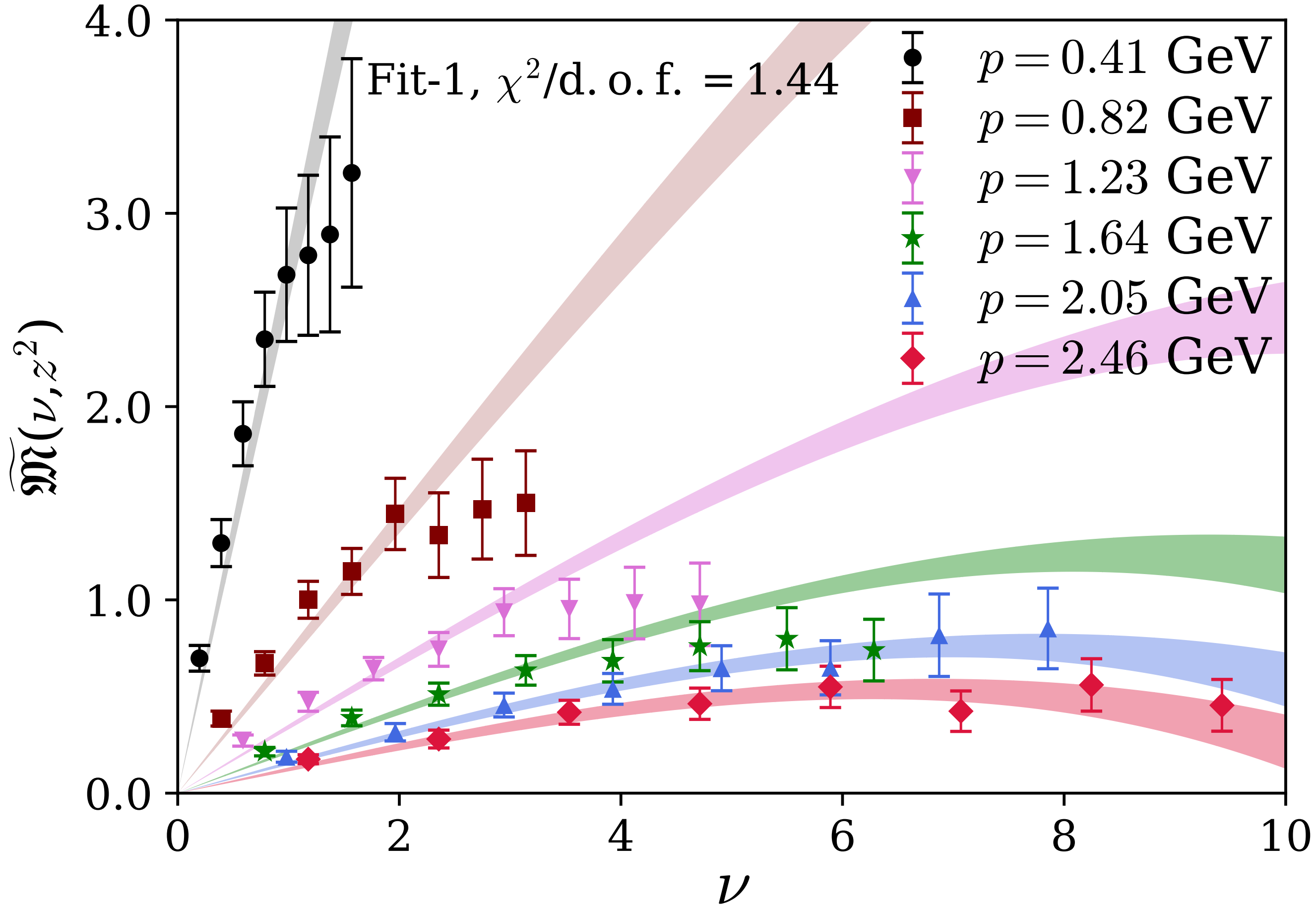
Can get: $\widetilde{\mathcal{M}}(z,p) = \left[\widetilde{M}_{ti;it} + \widetilde{M}_{ij;ij}\right]$

$$= M_{\Delta g} - \frac{m^2 z^2}{\nu}\widetilde{\mathcal{M}}_{pp}$$

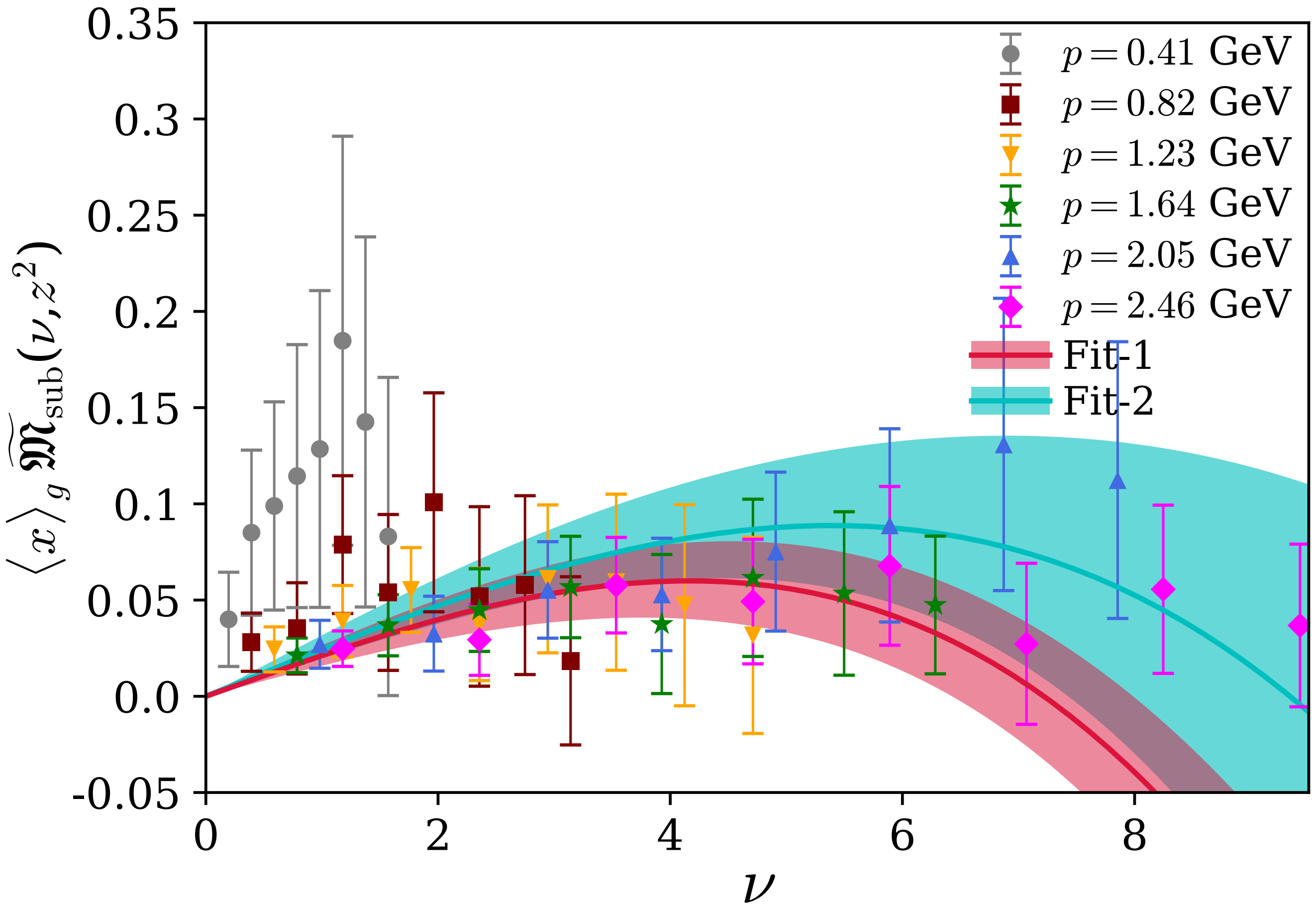
$$= M_{\Delta g} - \frac{m^2}{p_z^2}\nu\,\widetilde{\mathcal{M}}_{pp}$$

Helicity Gluon PDF

- Model both terms



- Subtract rest frame

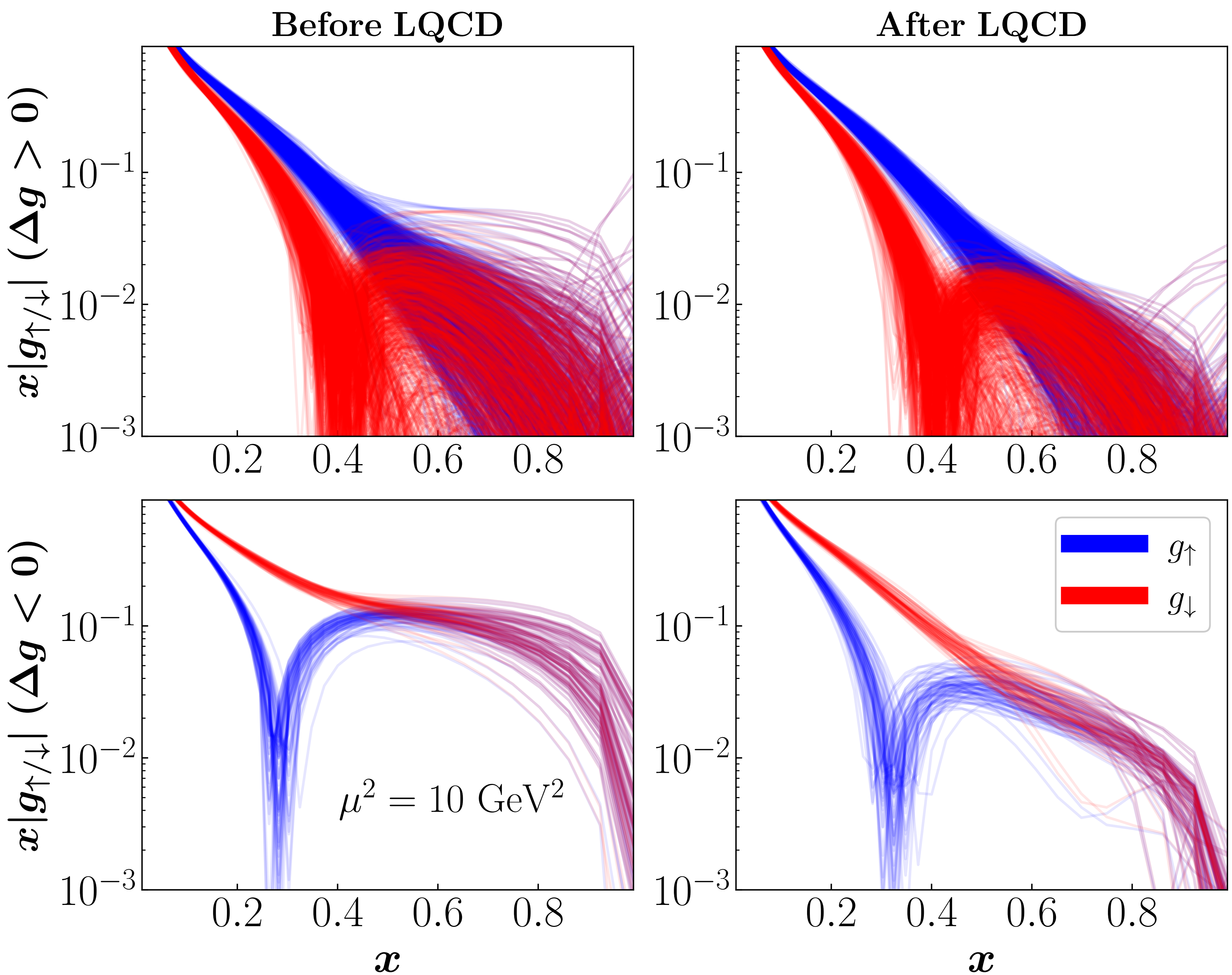


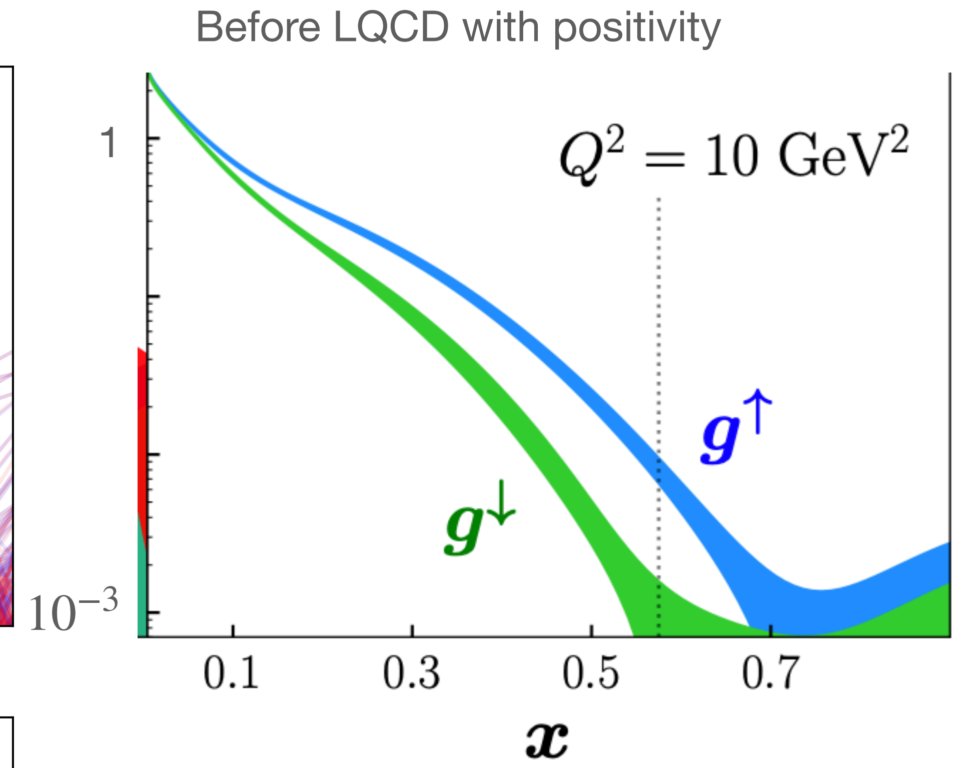
$a = 0.094$ fm

$m_\pi = 358$ MeV

C. Egerer et al (HadStruc) arXiv: 2207.08733

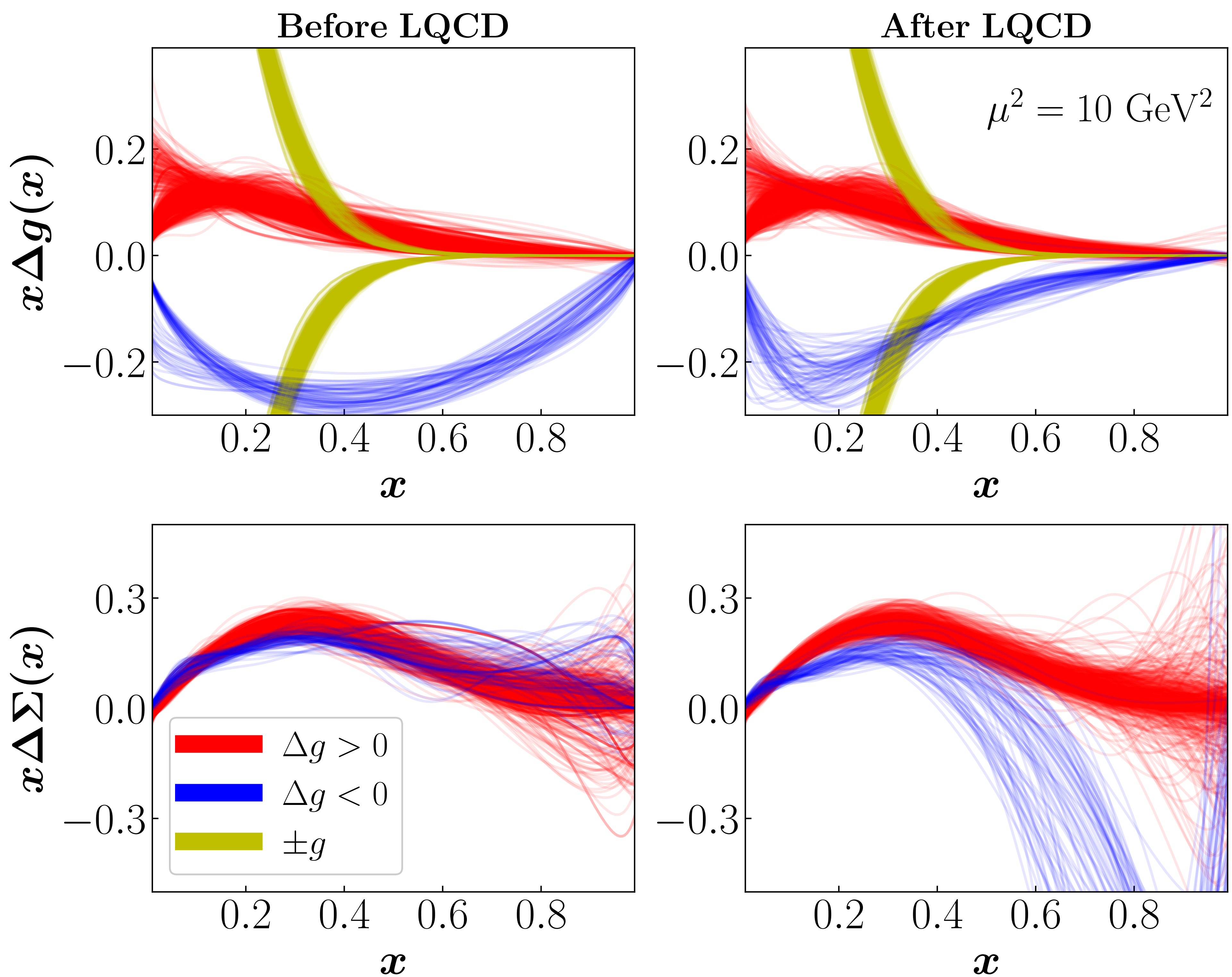
Which ways do gluons spin?





Lattice gluon data impacts quarks

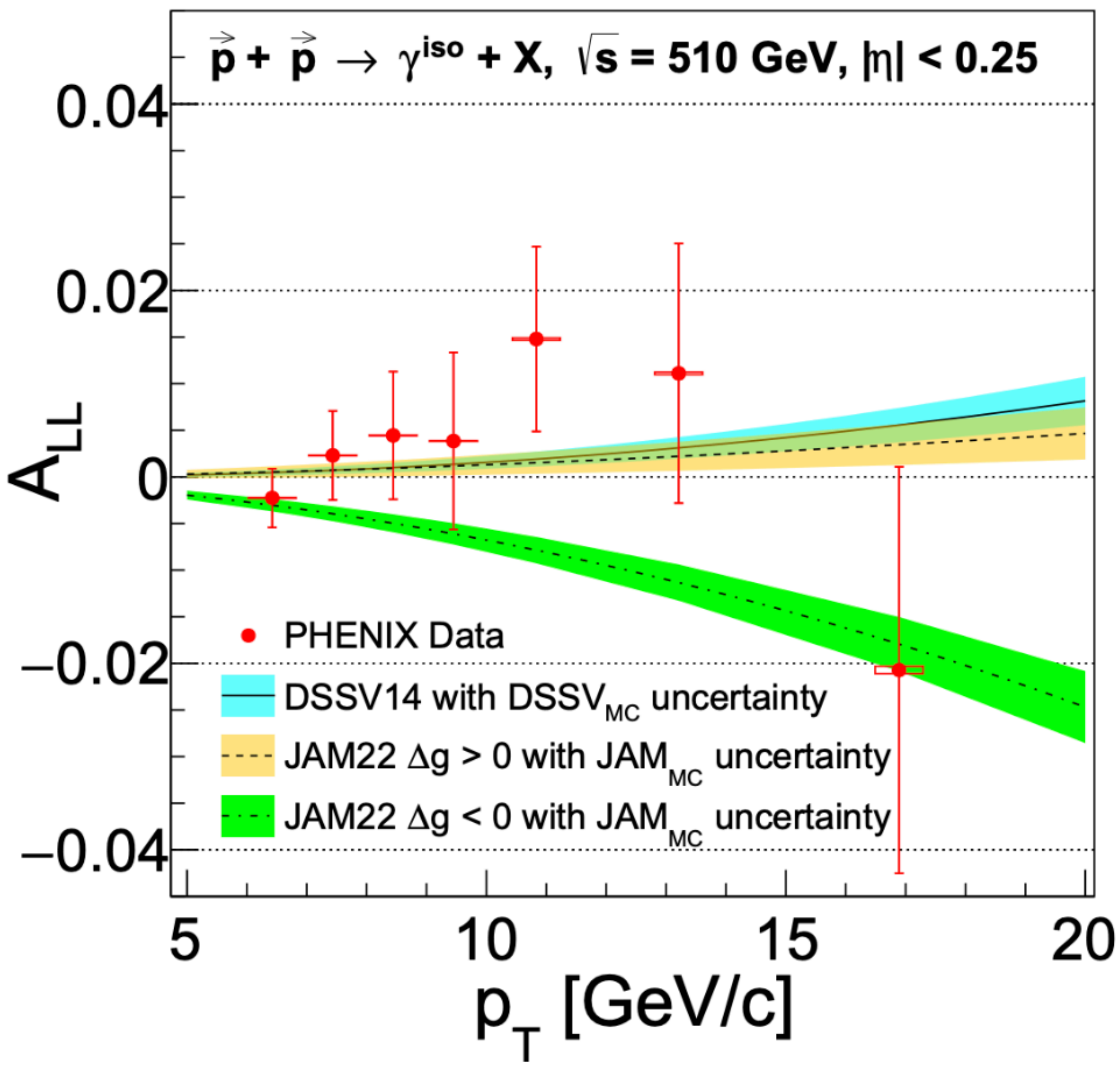
- Quark gluon mixing leads to impact on singlet
- Unexpected change in extrapolation region
- Compensates reduced magnitude of Δg in relation to cross sections

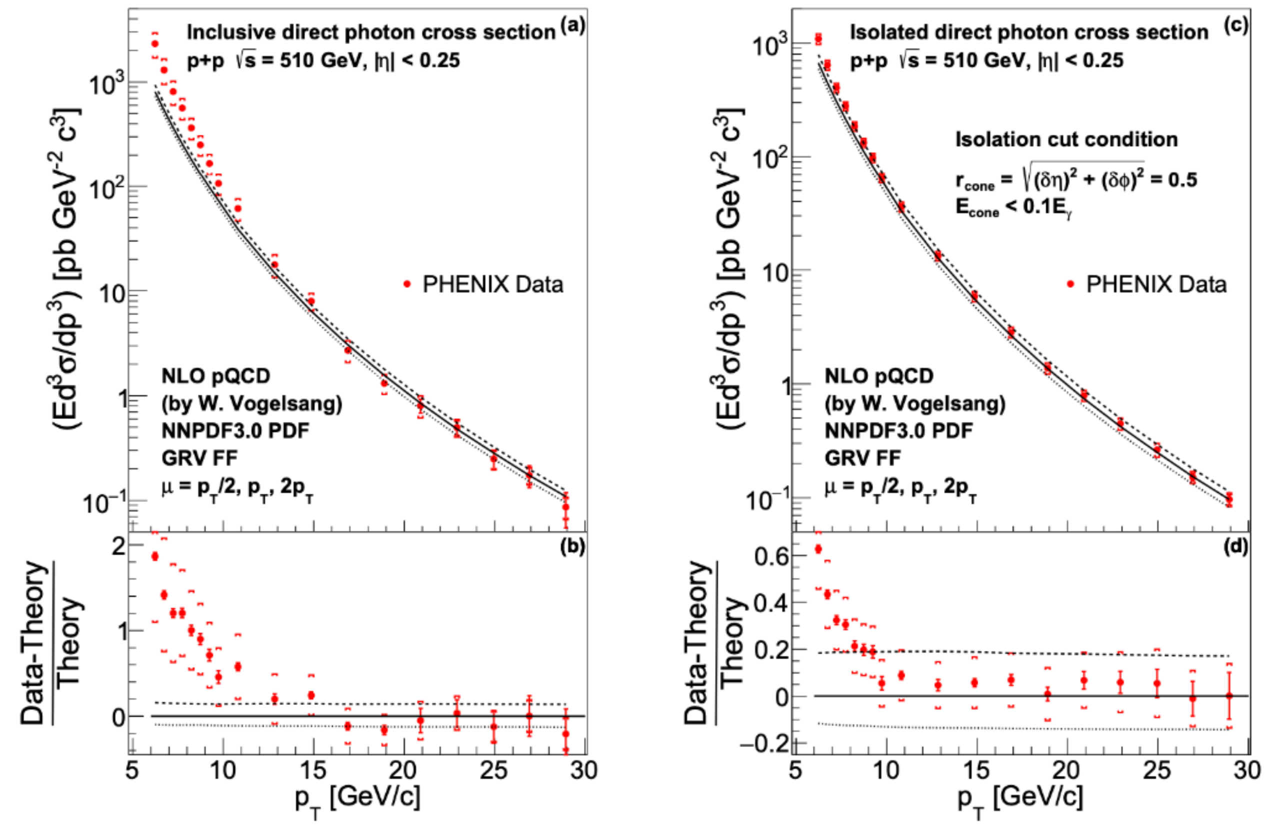


Measurement of Direct-Photon Cross Section and Double-Helicity Asymmetry at $\sqrt{s} = 510$ GeV in $\vec{p} + \vec{p}$ Collisions

PHENIX Collaboration • U. Acharya (Georgia State U., Atlanta) et al. (Feb 16, 2022)

e-Print: 2202.08158 [hep-ex]





The two dashed curves in Fig. 2 come from the global analysis of the JAM Collaboration [15, 16]. They found there are two distinct sets of solutions for the polarized gluon PDF, Δg, which differ in sign. Even though the solutions with $\Delta g < 0$ violate the positivity assumption, $|\Delta g| < g$, all previous data cannot exclude those solutions due to the mixed contributions from quark-gluon and gluon-gluon interactions. However, the direct-photon A_{LL} comes mainly from the quark-gluon interactions and has $\chi^2 = 4.7$ and 12.6 for 7 data points for the $\Delta g > 0$ and $\Delta g < 0$ solutions, respectively, with the difference of 7.9 between χ^2 values implying that the negative solution is disfavored at more than 2.8σ level.

Nobuo Sato's slide from SPIN 2023