GPDs from lattice QCD: new developments

Martha Constantinou

Temple University

Recent results

PHYSICAL REVIEW LETTERS **125,** 262001 (2020)

Unpolarized and Helicity Generalized Parton Distributions of the Proton within Lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1] Karl Jansen,[5] Aurora Scapellato,[3] and Fernanda Steffens[6]

★ **Twist-2 GPDs: "traditional" calculations**

PHYSICAL REVIEW D **105,** 034501 (2022)

Transversity GPDs of the proton from lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[1,2] Karl Jansen,[5] Aurora Scapellato,[4] and Fernanda Steffens[6]

PHYSICAL REVIEW D **108,** 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

★ **Twist-3 GPDs**

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

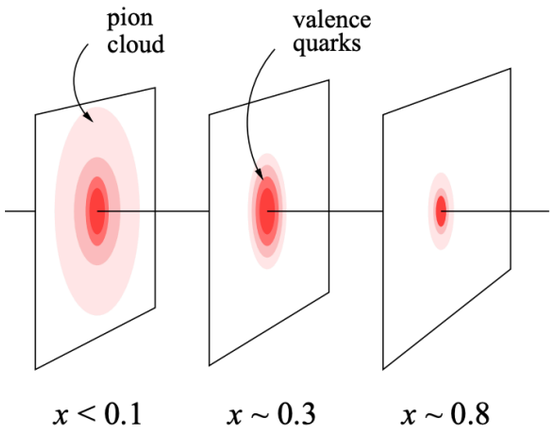
★ **Twist-2 GPDs: new approach**

arXiv:2310.13114v1

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Motivation



[H. Abramowicz et al., whitepaper for NSAC LRP, 2007]

$1_{mom} + 2_{coord}$ tomographic images of quark distribution in nucleon at fixed longitudinal momentum

3-D image from FT of the longitudinal mom. transfer

★ Also introduction from A. Metz and E. Voutier

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude: $\mathcal{H} = \int_{-1}^{+1} \frac{H(x,\xi,t)}{x-\xi+i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

Essential to complement the knowledge on GPD from lattice QCD

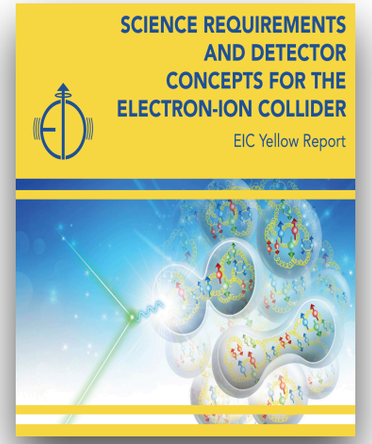
Hadron structure at core of nuclear physics

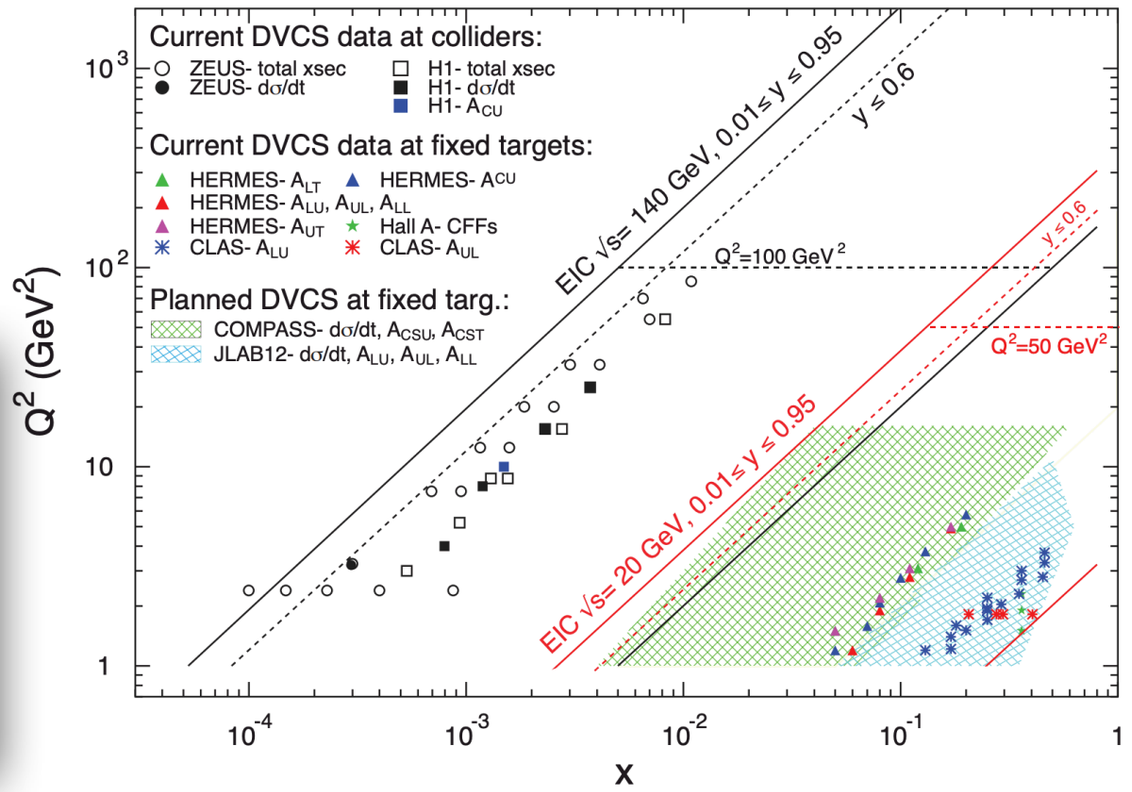
★ Tomographic imaging of proton has central role in the science program of EIC

GPDs, FFs, GFFs, TMDs, …

[R. Abdul Khalek et al.,

EIC Yellow Report 2021, arXiv:2103.05419]





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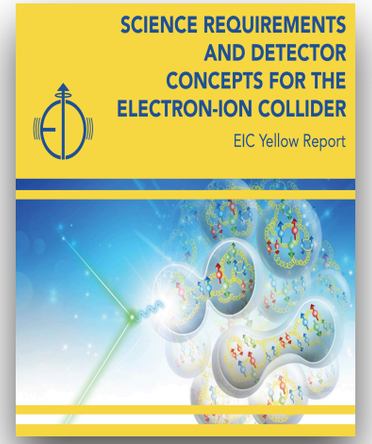

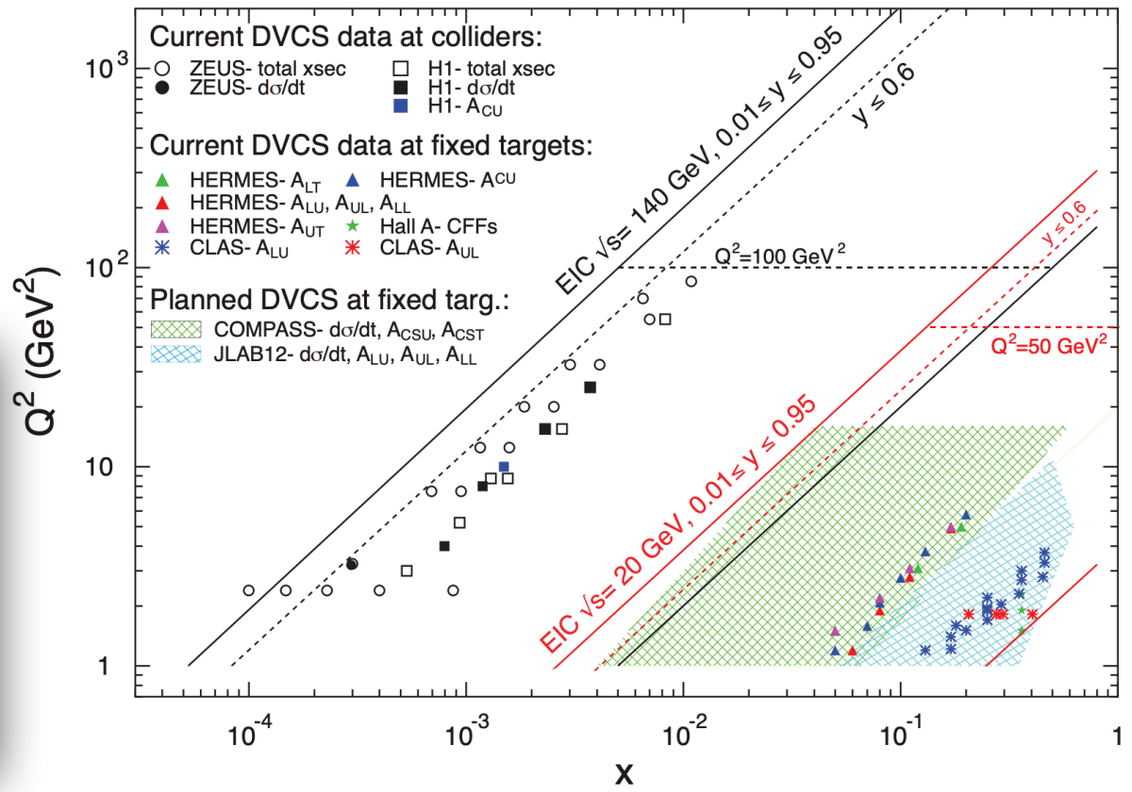

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QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number:
DE-SC0023646

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of t and ξ dependence

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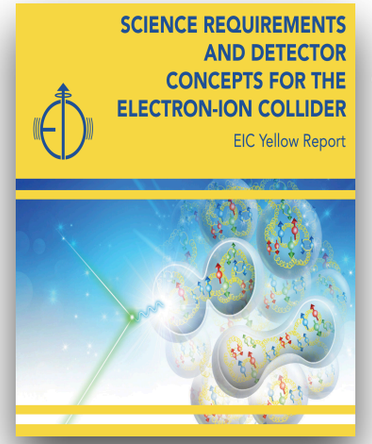

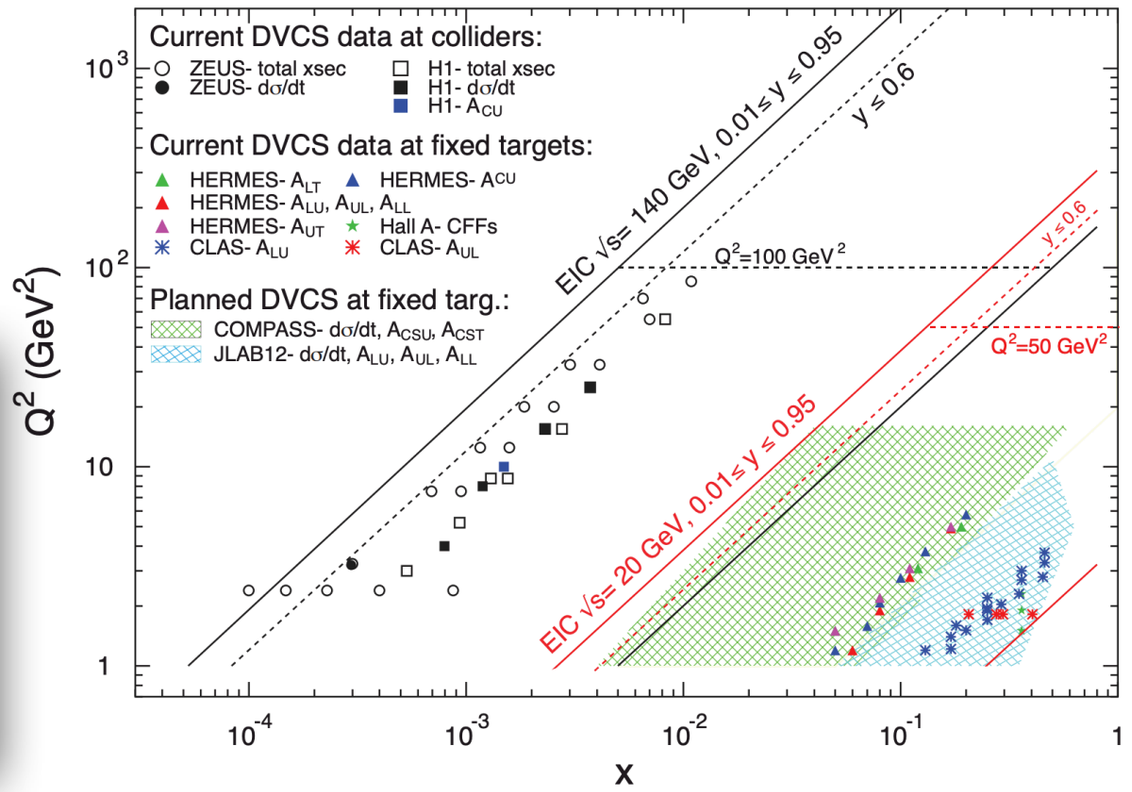

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Advances of lattice QCD are timely

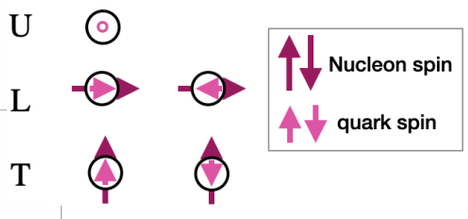
Twist-classification of PDFs, GPDs, TMDs

★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Nucleon \ Quark	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
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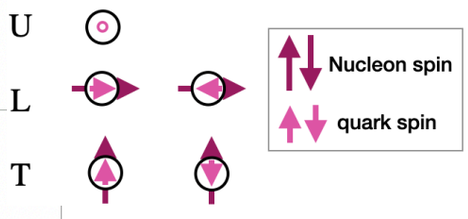

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- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)
- certain twist-3 PDFs are related to the TMDs
- physical interpretation (e.g. average force on partons inside hadron)

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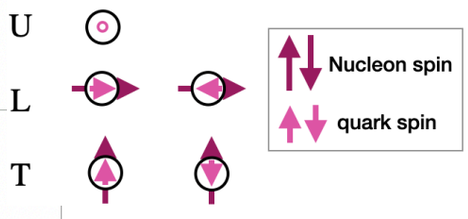

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While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

Accessing information on GPDs

★ **Mellin moments**
(local OPE expansion)

$$\bar{q}(-\tfrac{1}{2}z)\,\gamma^\sigma W[-\tfrac{1}{2}z,\tfrac{1}{2}z]\,q(\tfrac{1}{2}z) \;=\; \sum_{n=0}^{\infty}\frac{1}{n!}\,z_{\alpha_1}\ldots z_{\alpha_n}\Big[\bar{q}\gamma^\sigma \overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n} q\Big]$$

local operators

$$\langle N(P')|\mathcal{O}_V^{\mu\mu_1\cdots\mu_{n-1}}|N(P)\rangle \sim \sum_{\substack{i=0\\ \text{even}}}^{n-1}\left\{\gamma^{\{\mu}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}A_{n,i}(t) - i\frac{\Delta_\alpha\sigma^{\alpha\{\mu}}{2m_N}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\overline{P}^{\mu_{i+1}}\cdots\overline{P}^{\mu_{n-1}\}}B_{n,i}(t)\right\} + \frac{\Delta^\mu\Delta^{\mu_1}\cdots\Delta^{\mu_{n-1}}}{m_N}C_{n,0}(\Delta^2)\Big|_{n\ \text{even}}\Big)\Big\}$$

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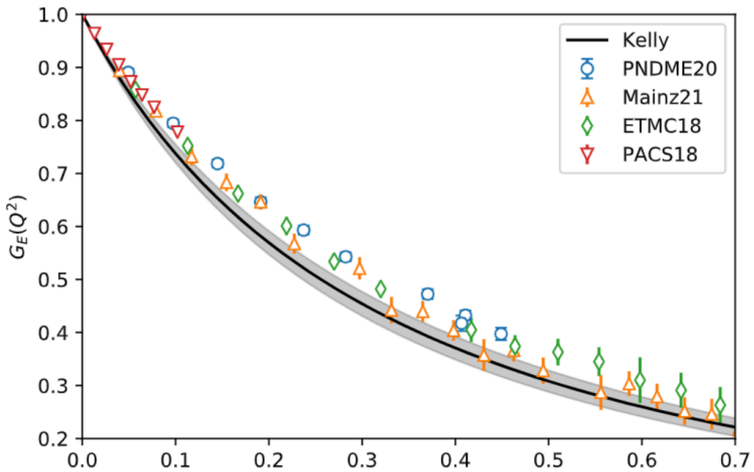

Wide -t range that comes at the cost of 1 (in the majority of cases)

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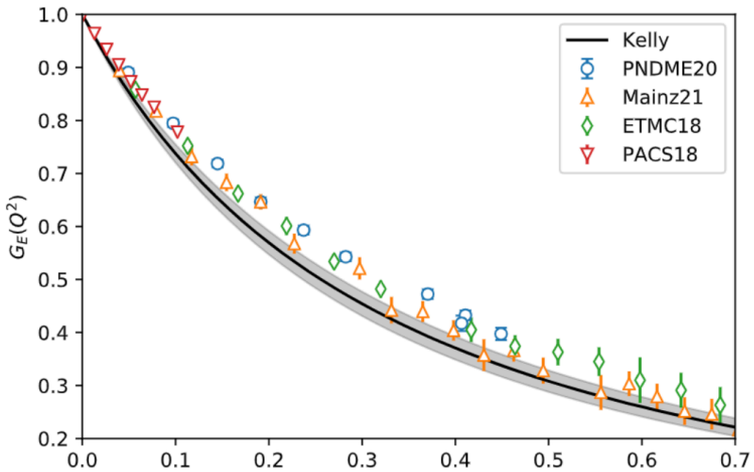

Wide -t range that comes at the cost of 1 (in the majority of cases)

★ Matrix elements of non-local operators (quasi-GPDs, pseudo-GPDs, …)

$$\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu$$

Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$

GPDs

Through non-local matrix elements

of fast-moving hadrons

Access of PDFs/GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

J. Karpie, plenary talk

$$\tilde{q}^{\mathrm{GPD}}_{\Gamma}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \quad \langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -\,Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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Accessing -t dependence: Computationally intensive

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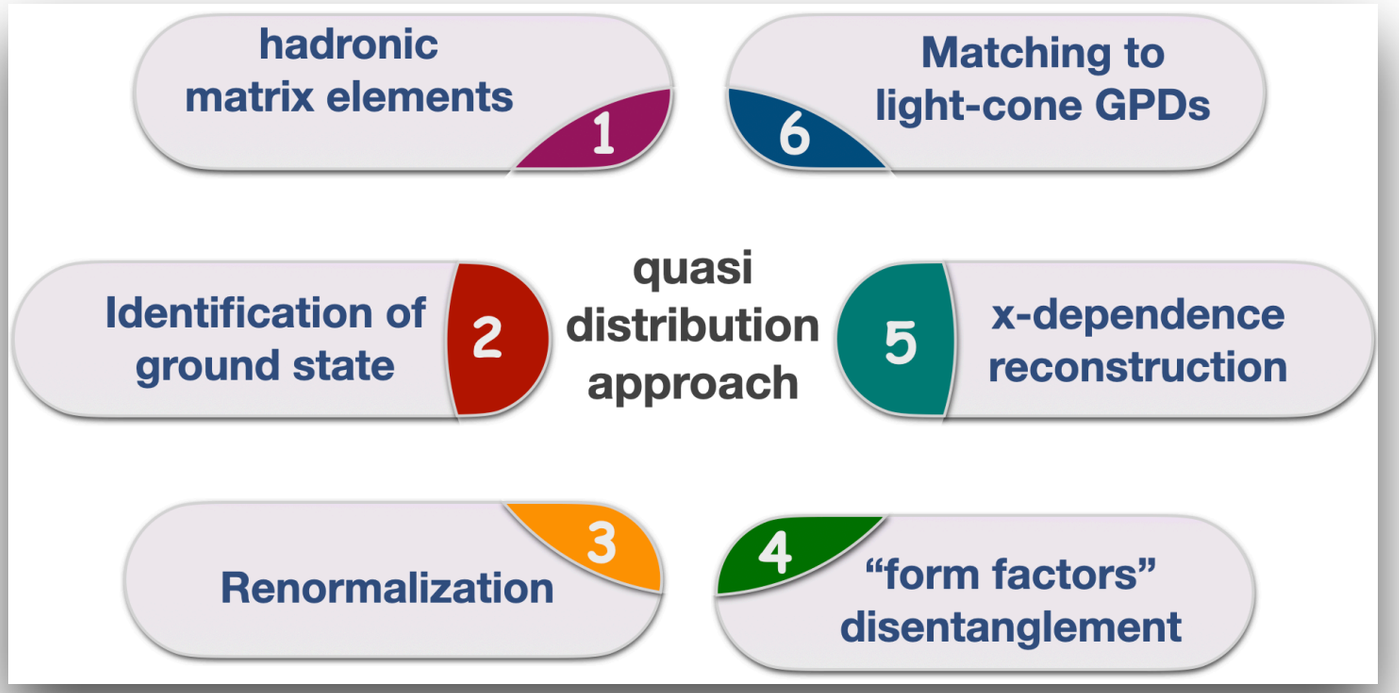

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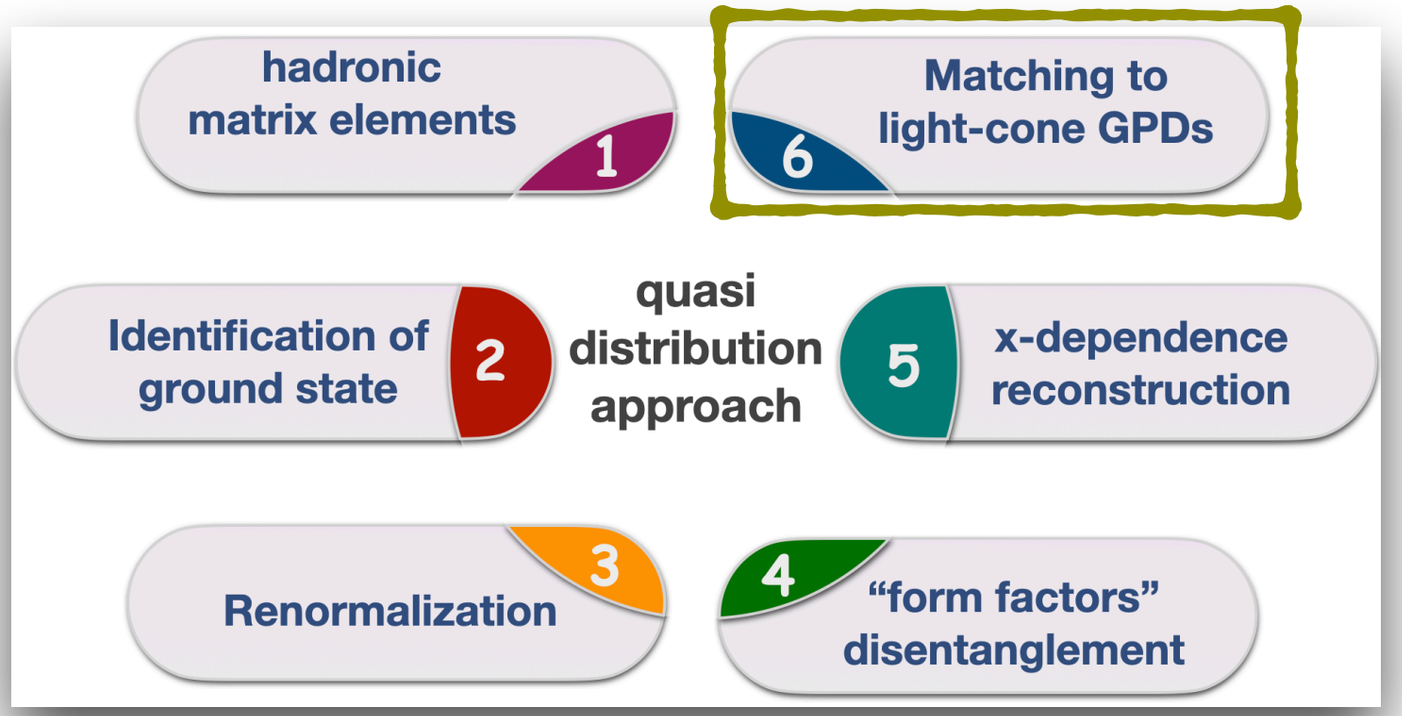

Twist-2 GPDs

★ γ^+ inspired parametrization (symmetric frame)

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3)=\frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3)+\frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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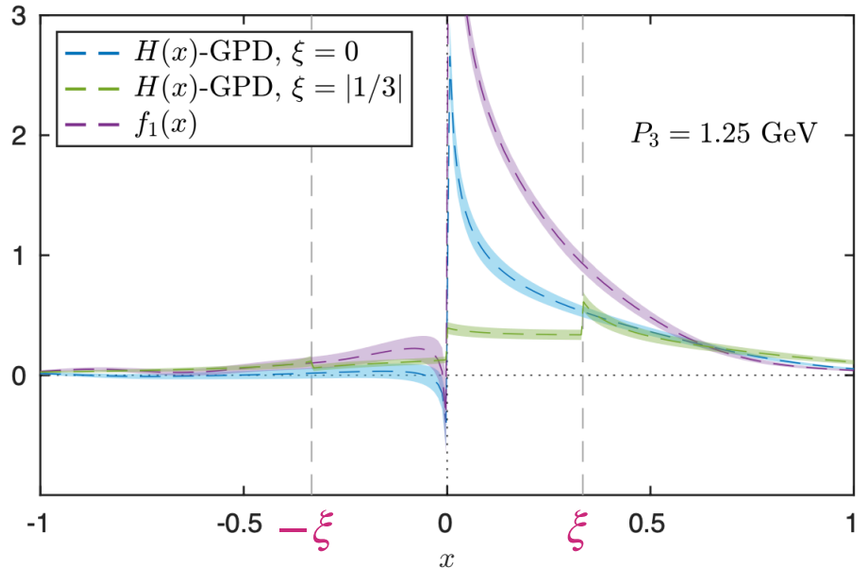
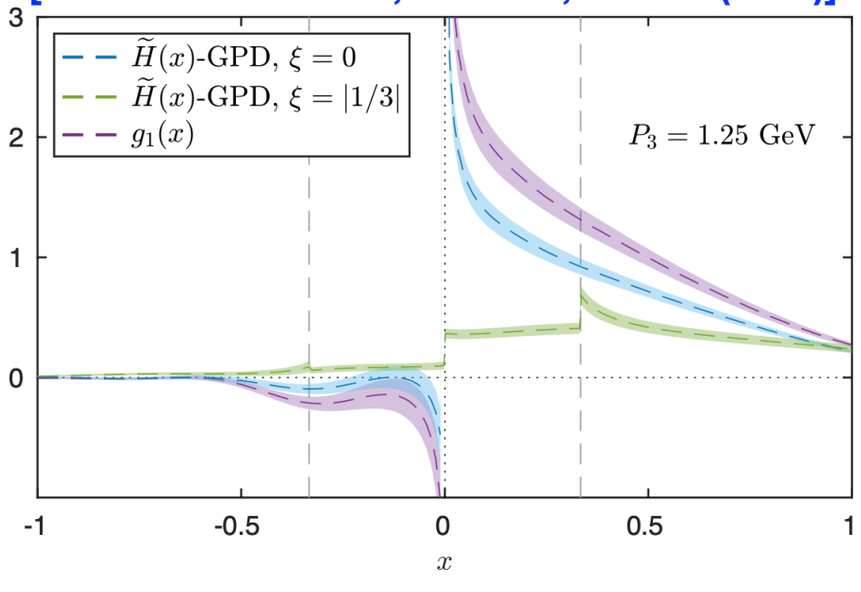
★ Nf=2+1+1 twisted mass fermions & clover term (pion mass 260 MeV)



[C. Alexandrou et al., PRL 125, 262001 (2020)]



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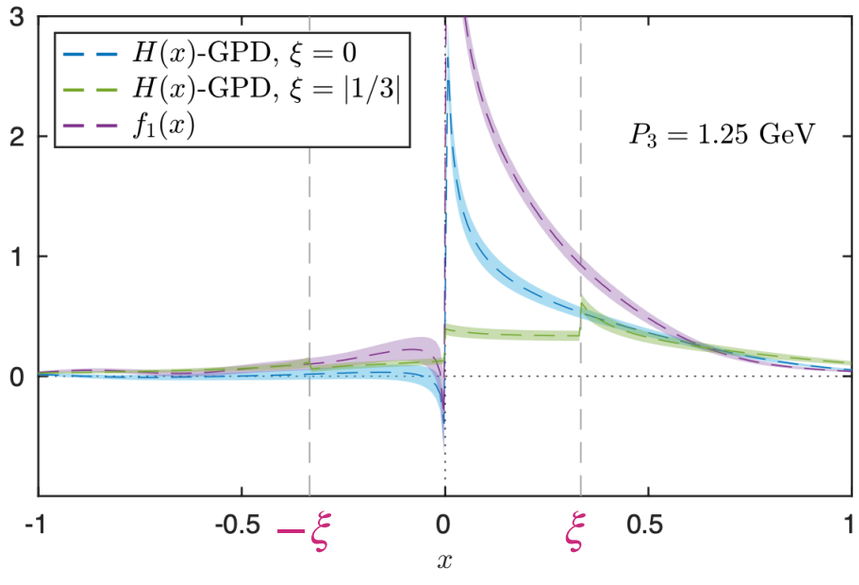

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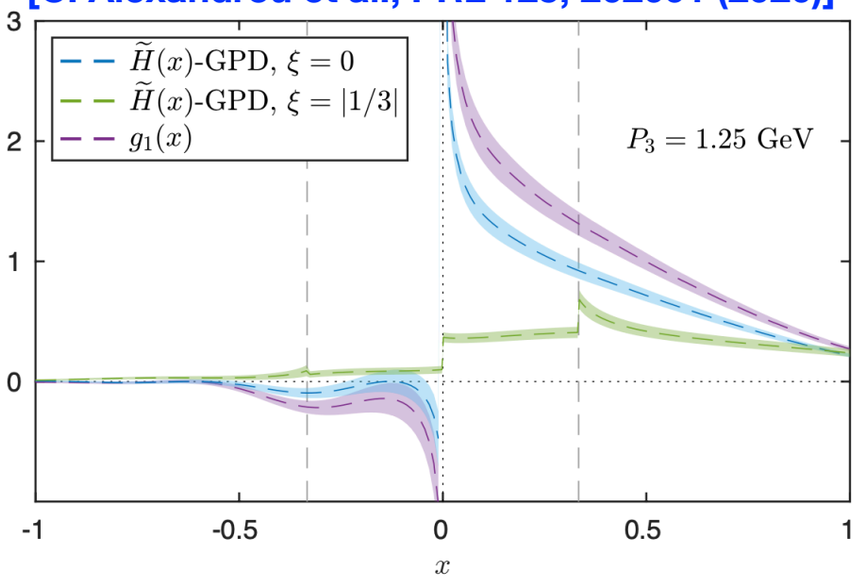

power counting analysis of GPDs ($x \to 1$)

[F. Yuan, Phys.Rev. D69 (2004) 051501, hep-ph/0311288]

- t-dependence vanishes at large-x
- $H(x,0)$ asymptotically equal to $f_1(x)$

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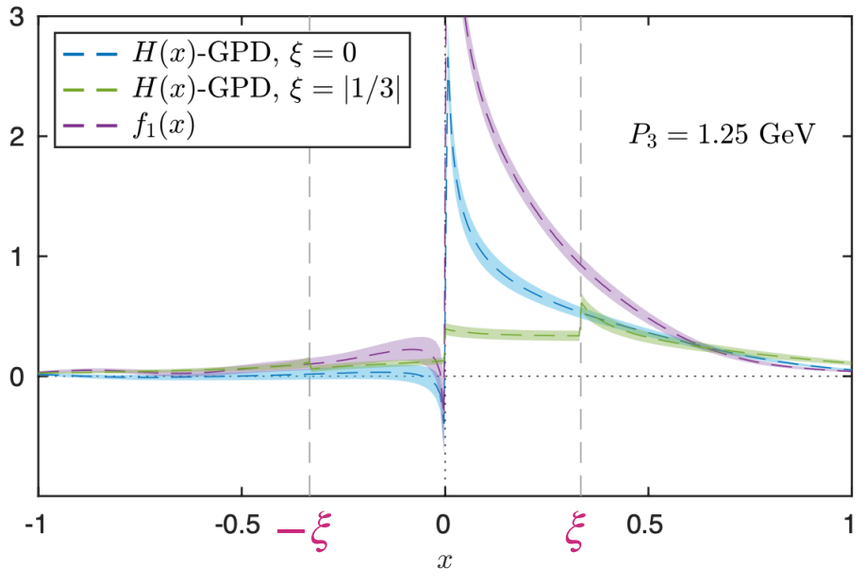
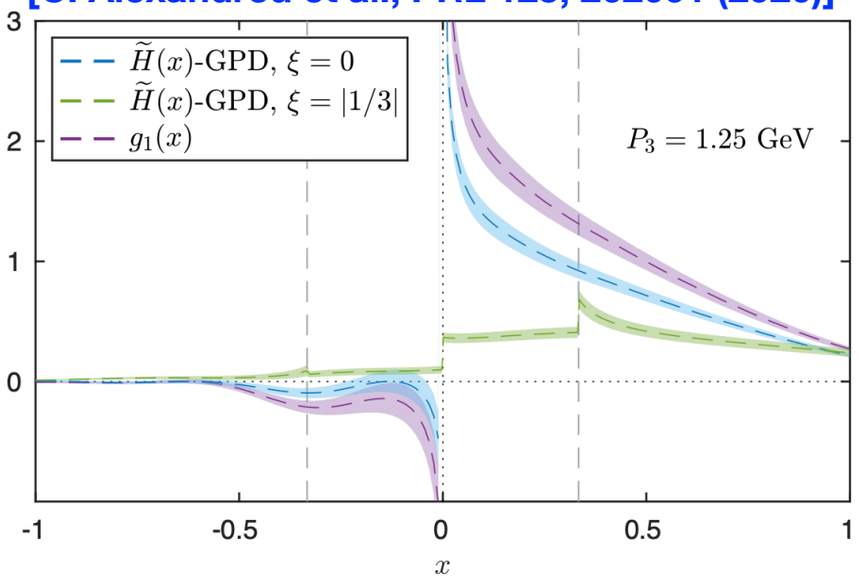


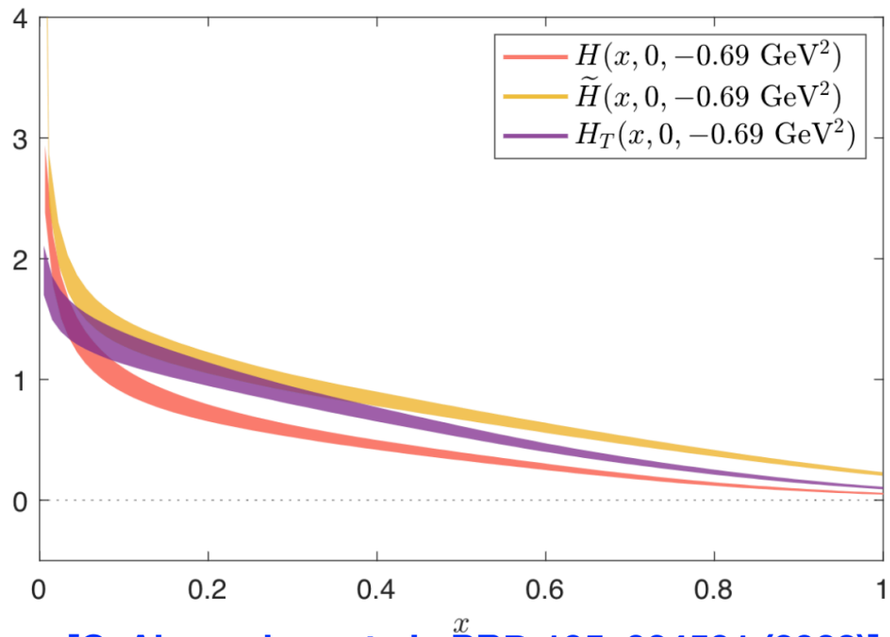
[C. Alexandrou et al., PRL 125, 262001 (2020)]



power counting analysis of GPDs ($x \to 1$)

[F. Yuan, Phys.Rev. D69 (2004) 051501, hep-ph/0311288]

- t-dependence vanishes at large-x
- $H(x,0)$ asymptotically equal to $f_1(x)$



[C. Alexandrou et al., PRD 105, 034501 (2022)]

Twist-3 GPDs

PHYSICAL REVIEW D **108,** 054501 (2023)

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[1] Jack Dodson,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[4]

Theoretical setup

★ Correlation functions in coordinate space

$$F^{[\Gamma]}(x,\Delta;P^3)=\frac{1}{2}\int\frac{dz^3}{2\pi}e^{ik\cdot z}\langle p_f,\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i,\lambda\rangle\Big|_{z^0=0,\vec{z}_\perp=\vec{0}_\perp}$$

★ Parametrization of coordinate-space correlation functions

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105] [F. Aslan et a., Phys. Rev. D 98, 014038 (2018)]

$$\begin{aligned}F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3)=\frac{1}{2P^3}\bar{u}(p_f,\lambda')\Bigg[&P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3)+P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)\\&+\Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3)+\gamma_\perp^\mu\gamma_5F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)\\&+\Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3)+i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Bigg]u(p_i,\lambda)\end{aligned}$$

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★ Twist-3 contributions to helicity GPDs: $\Gamma=\gamma^j\gamma_5,\ j=1,2$

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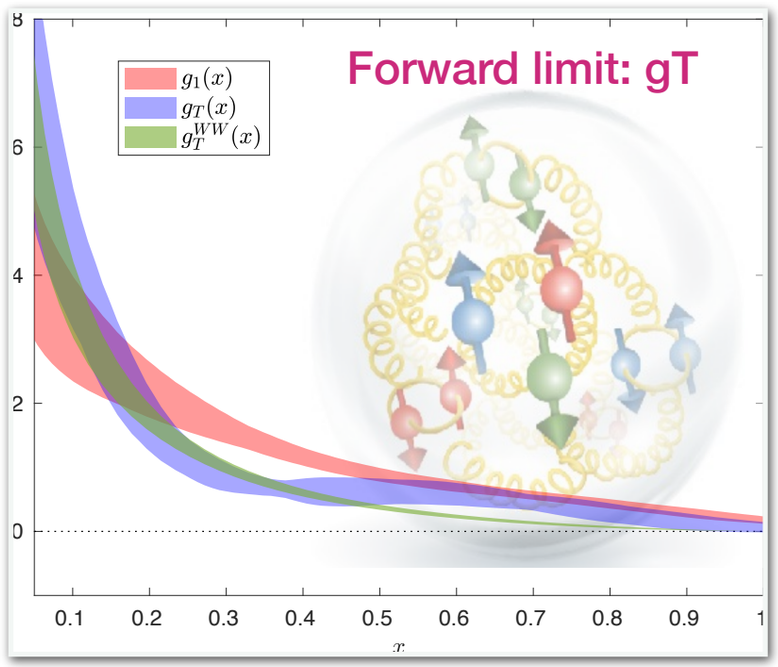


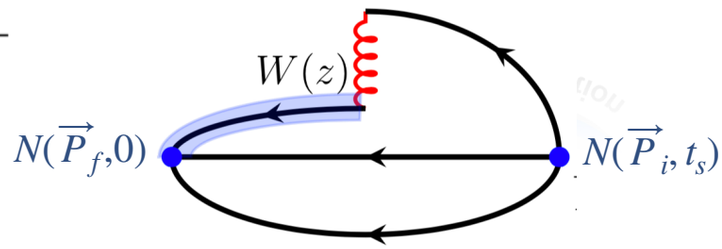
[S. Bhattacharya et al., PRD 102 (2020) 11] (Editors Highlight)

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4



★ Calculation of connected diagram

P_3 [GeV]	$\vec{q}$ [$\frac{2\pi}{L}$]	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm total}$
± 0.83	$(0,0,0)$	0	2	194	8	3104
± 1.25	$(0,0,0)$	0	2	731	16	23392
± 1.67	$(0,0,0)$	0	2	1644	64	210432
± 0.83	$(\pm 2,0,0)$	0.69	8	67	8	4288
± 1.25	$(\pm 2,0,0)$	0.69	8	249	8	15936
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± 1.25	$(\pm 2,\pm 2,0)$	1.38	16	224	8	28672
± 1.25	$(\pm 4,0,0)$	2.76	8	329	32	84224

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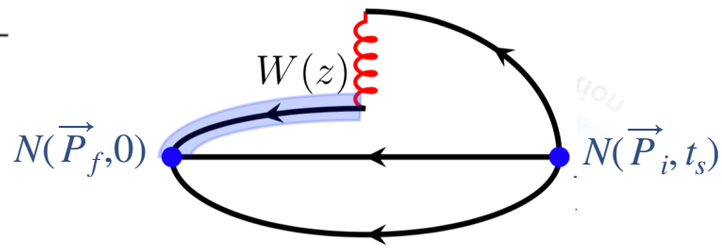

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Symmetric frame computationally expensive

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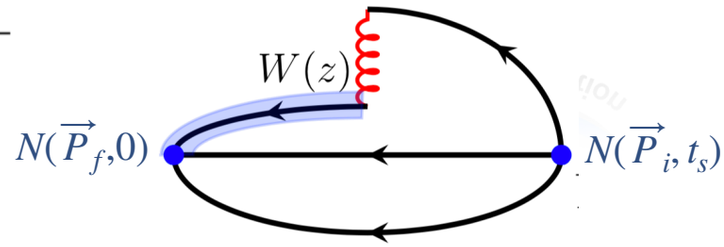

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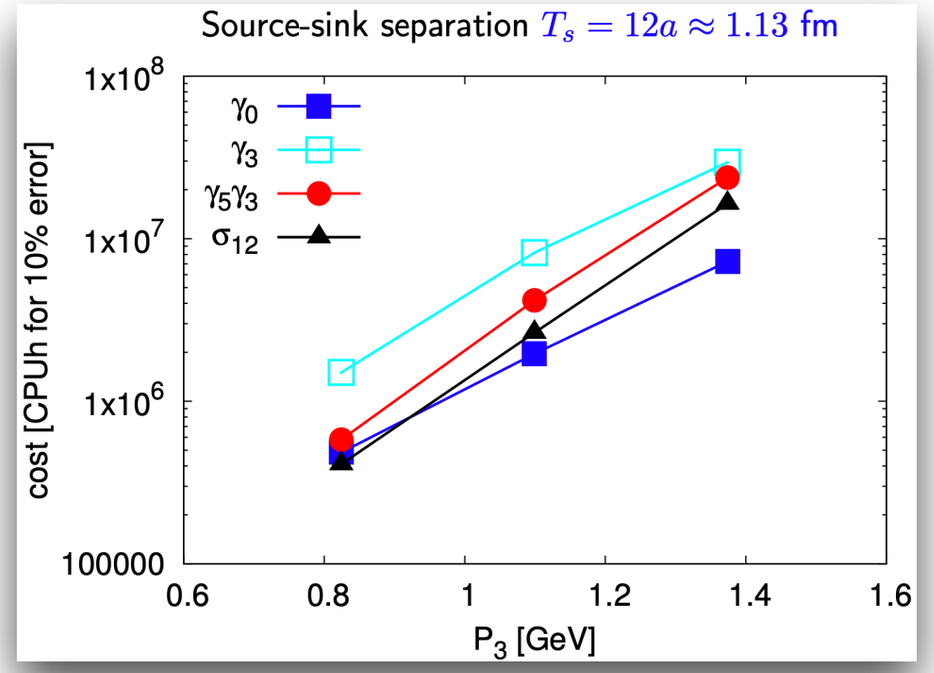

Symmetric frame computationally expensive

Suppressing gauge noise and reliably extracting the ground state comes at a significant computational cost

Decomposition

★ Requirement: four independent matrix elements

P_3 [GeV]	$\vec{q}\,[\frac{2\pi}{L}]$	$-t$ [GeV^2]
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± 1.25	$(\pm 2,\pm 2,0)$	1.38
± 1.25	$(\pm 4,0,0)$	2.76

★ Average kinematically equivalent matrix elements

$$\Pi^1(\Gamma_0) = C\left(- F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_y}{4m^2} - F_{\widetilde{G}_4}\frac{\text{sign}[P_3]\,\Delta_y(E+m)}{2m^2}\right),$$

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\frac{\text{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2P_3}\right)$$

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$$\Pi^1(\Gamma_3) = C\left(- F_{\widetilde{G}_3}\frac{E\Delta_x(E+m)}{2m^2P_3}\right),$$

$$\Pi^2(\Gamma_0) = C\left(F_{\widetilde{H}+\widetilde{G}_2}\frac{P_3\Delta_x}{4m^2} + F_{\widetilde{G}_4}\frac{\text{sign}[P_3]\,\Delta_x(E+m)}{2m^2}\right),$$

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$$\Pi^2(\Gamma_3) = C\left(- F_{\widetilde{G}_3}\frac{E\Delta_y(E+m)}{2m^2P_3}\right),$$

Consistency Checks

★ Sum Rules (generalization of Burkhardt-Cottingham)

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \quad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

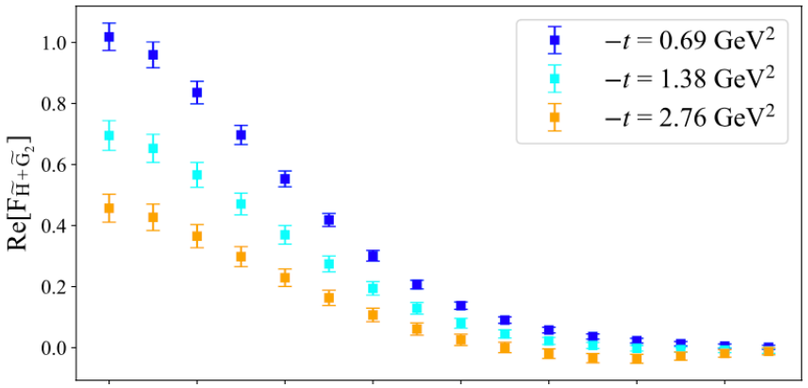
★ Sum Rules (generalization of Efremov-Leader-Teryaev)

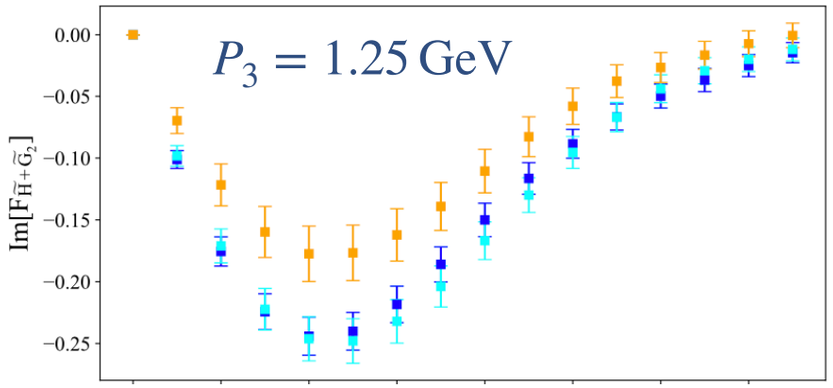
[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

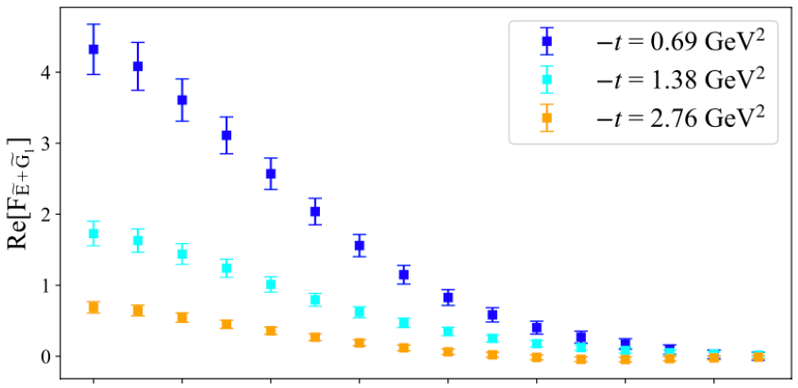
$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,0,t) = \frac{\xi}{4} G_E \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,0,t) = \frac{1}{4} G_E(t)$$

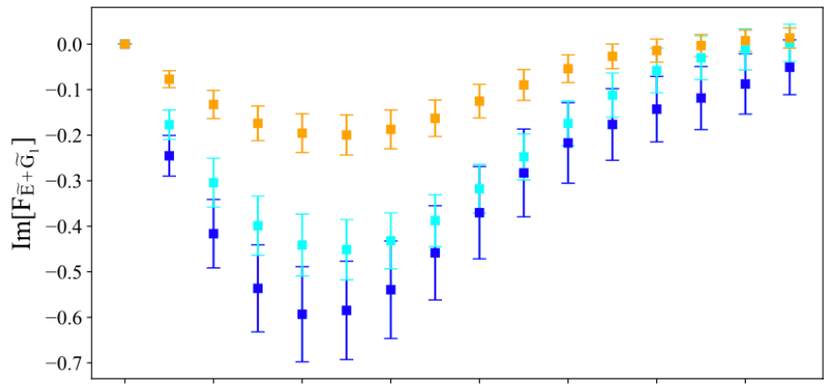
G_E : electric FF

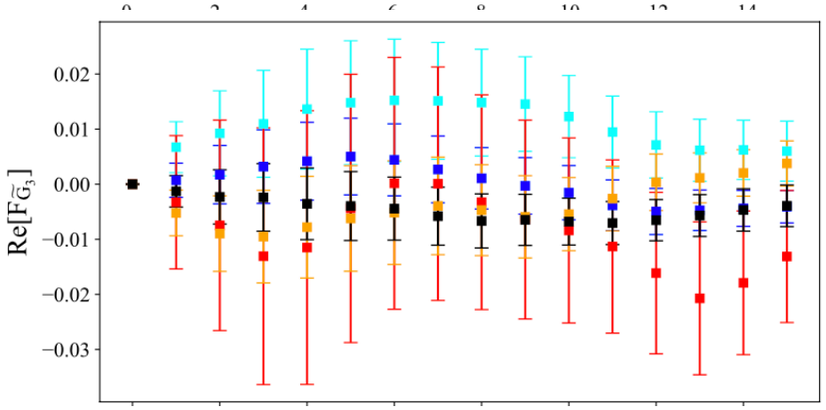
Lattice Results - quasi-GPDs

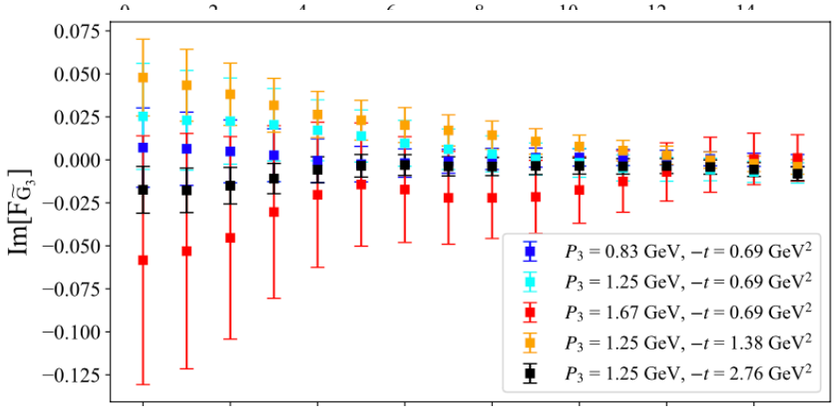


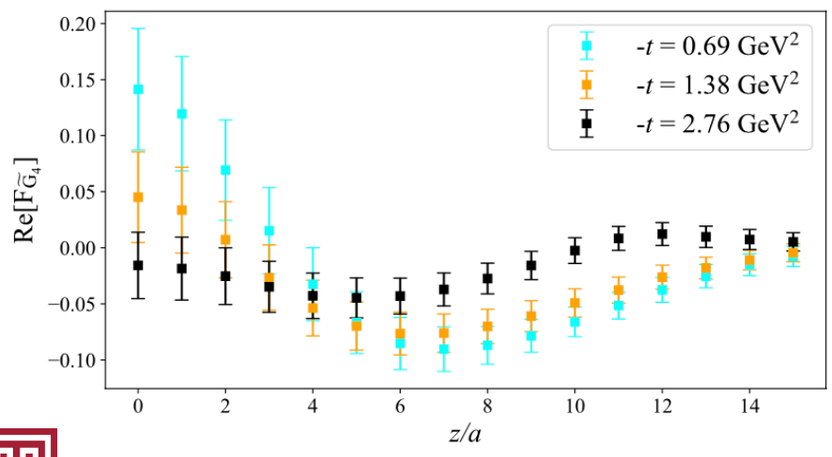


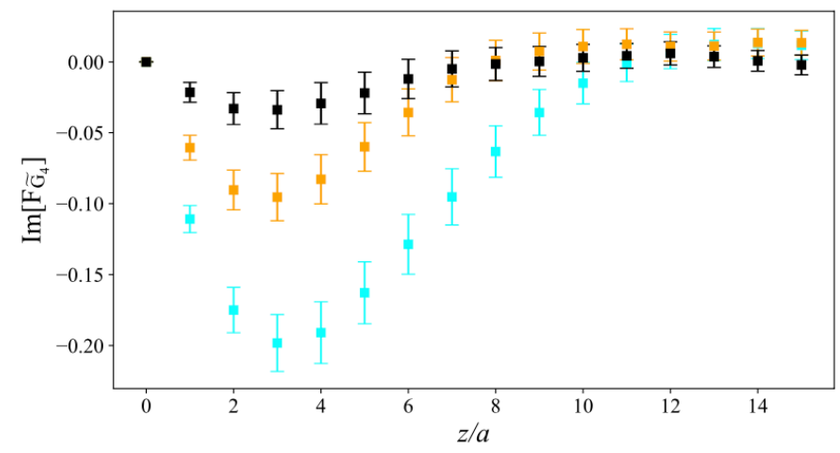




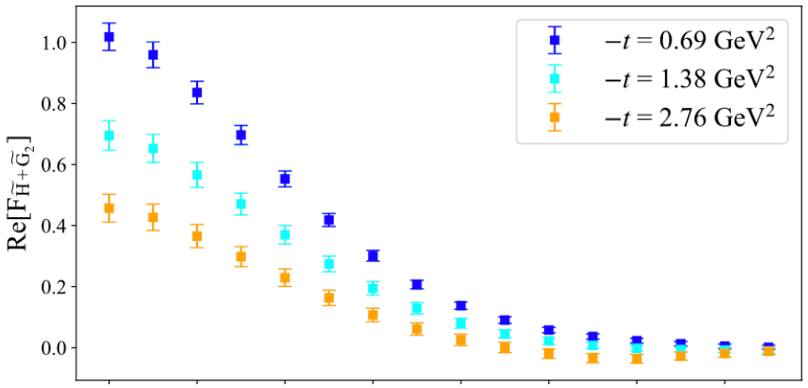


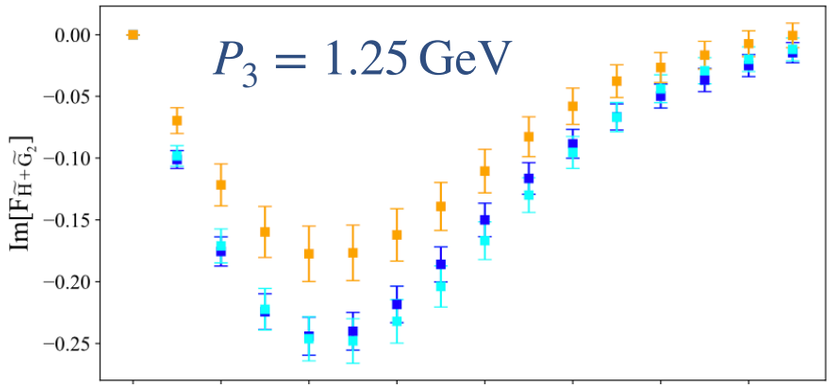


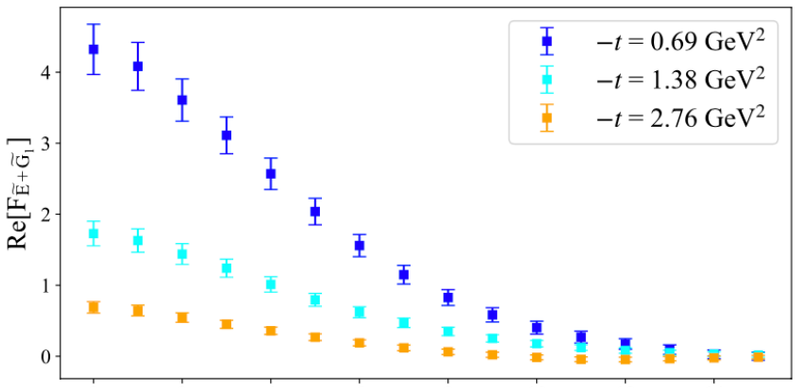


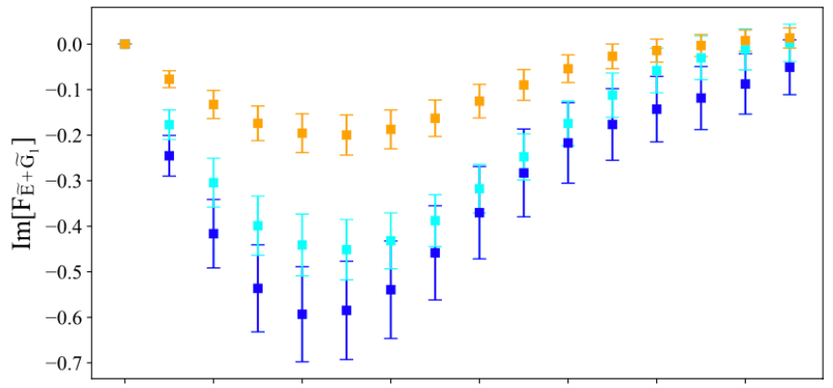


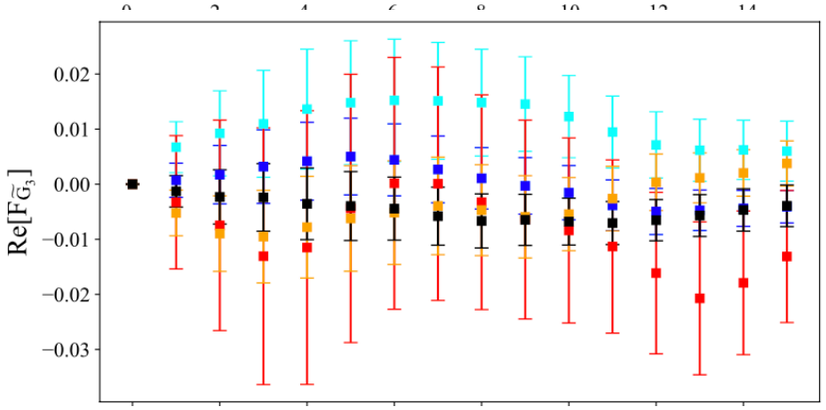
Lattice Results - quasi-GPDs

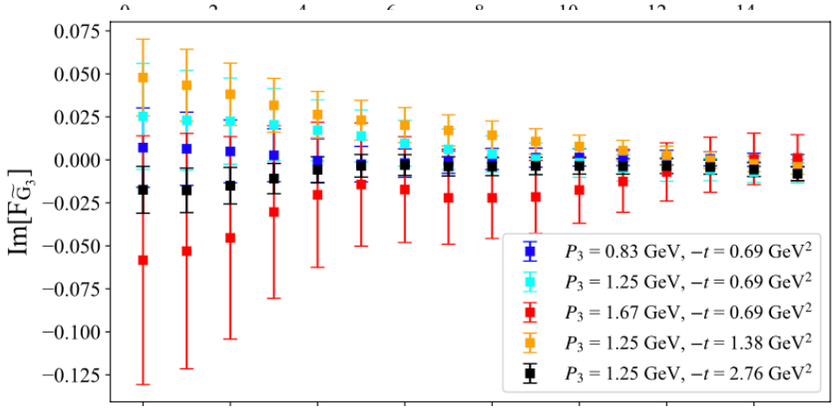


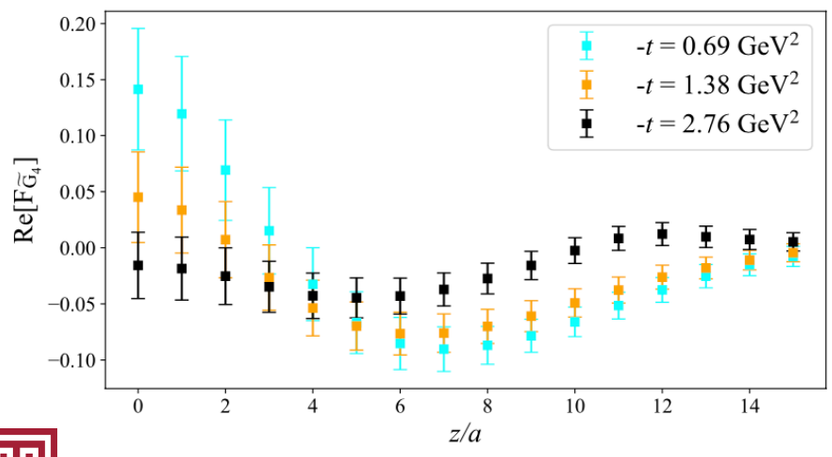


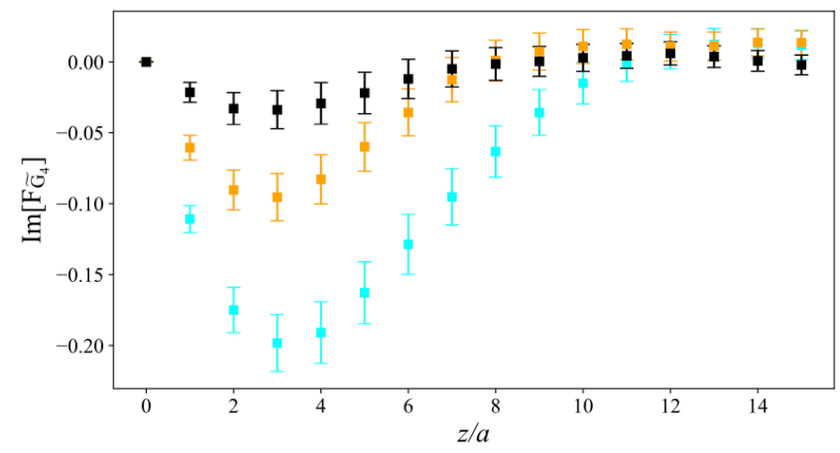












$$\int dx\, x\, \widetilde{G}_3 = \frac{\xi}{4} G_E(t)$$

Indeed, numerically found to be zero within uncertainties at ξ=0

Reconstruction of x-dependence & matching

★ quasi-GPDs transformed to momentum space using Backus Gilbert
[G. Backus and F. Gilbert, Geophysical Journal International 16, 169 (1968)]

★ Matching formalism to 1 loop accuracy level

$$F_X^{\mathrm{M\overline{MS}}}(x,t,P_3,\mu) = \int_{-1}^{1} \frac{dy}{|y|} C_{\gamma_j\gamma_5}^{\mathrm{M\overline{MS}},\overline{\mathrm{MS}}}\left(\frac{x}{y},\frac{\mu}{yP_3}\right) G_X^{\overline{\mathrm{MS}}}(y,t,\mu) + \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{t}{P_3^2},\frac{\Lambda_{\mathrm{QCD}}^2}{x^2P_3^2}\right)$$

★ Operator dependent kernel

PHYSICAL REVIEW D **102,** 034005 (2020)

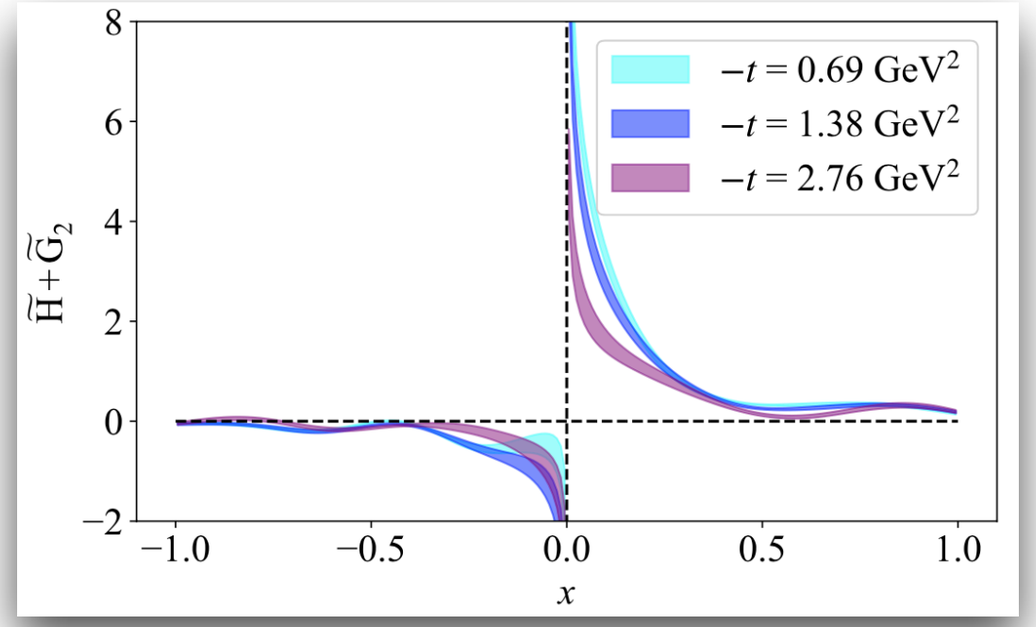
One-loop matching for the twist-3 parton distribution $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

$$C_{\mathrm{MM\overline{S}}}^{(1)}\left(\xi,\frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases}0\\ \delta(\xi)\\ 0\end{cases} + \frac{\alpha_s C_F}{2\pi}\begin{cases}\left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{\xi}{1-\xi}+\frac{3}{2\xi}\right]_+ & \xi>1\\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{4\xi(1-\xi)(xP_3)^2}{\mu^2}+\frac{\xi^2-\xi-1}{1-\xi}\right]_+ & 0<\xi<1\\ \left[\frac{-\xi^2+2\xi+1}{1-\xi}\ln\frac{\xi-1}{\xi}-\frac{\xi}{1-\xi}+\frac{3}{2(1-\xi)}\right]_+ & \xi<0\,,\end{cases}$$

★ Matching does not consider mixing with q-g-q correlators
[V. Braun et al., JHEP 05 (2021) 086]

Lattice Results - light-cone GPDs



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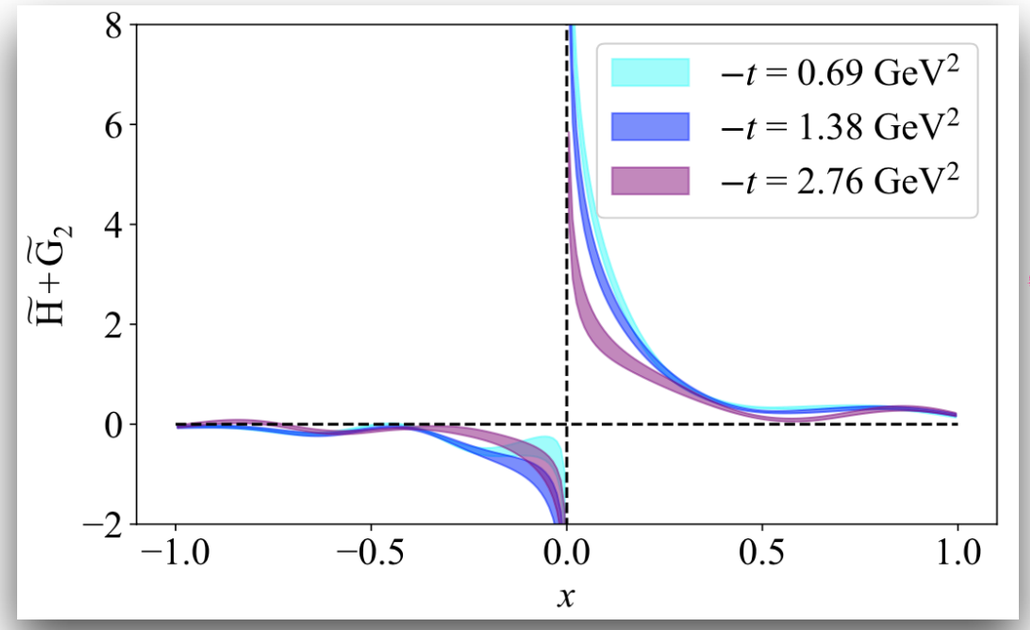
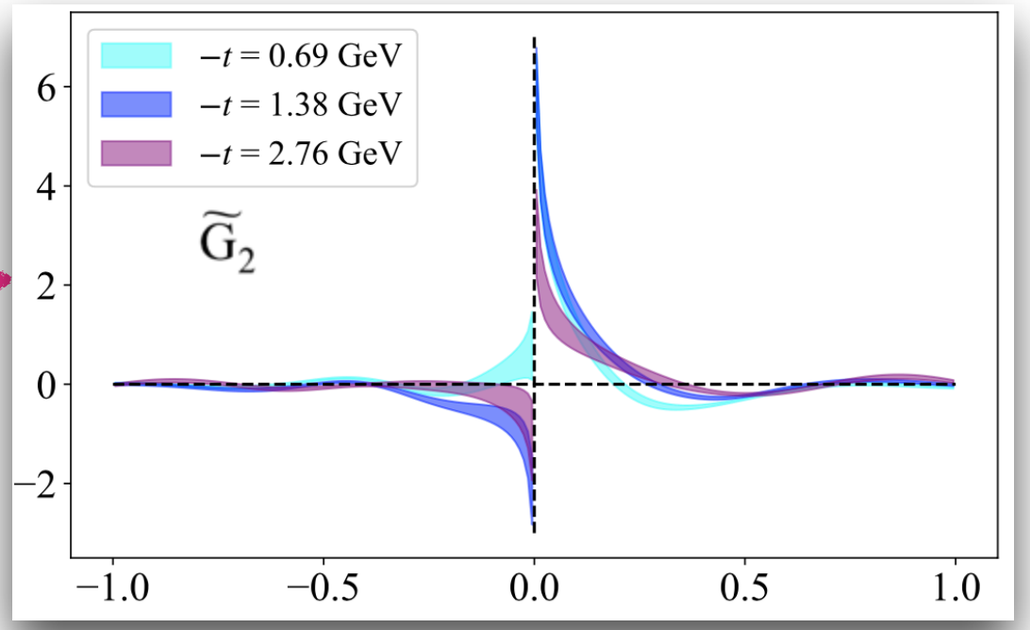


Isolating $\widetilde{G}_2$

using $\widetilde{H}$



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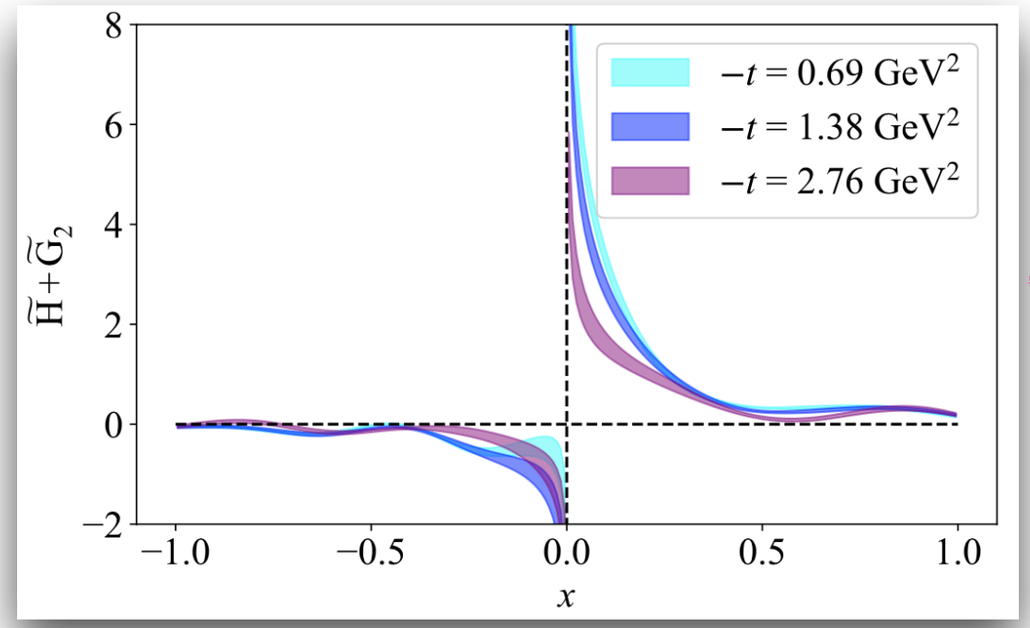

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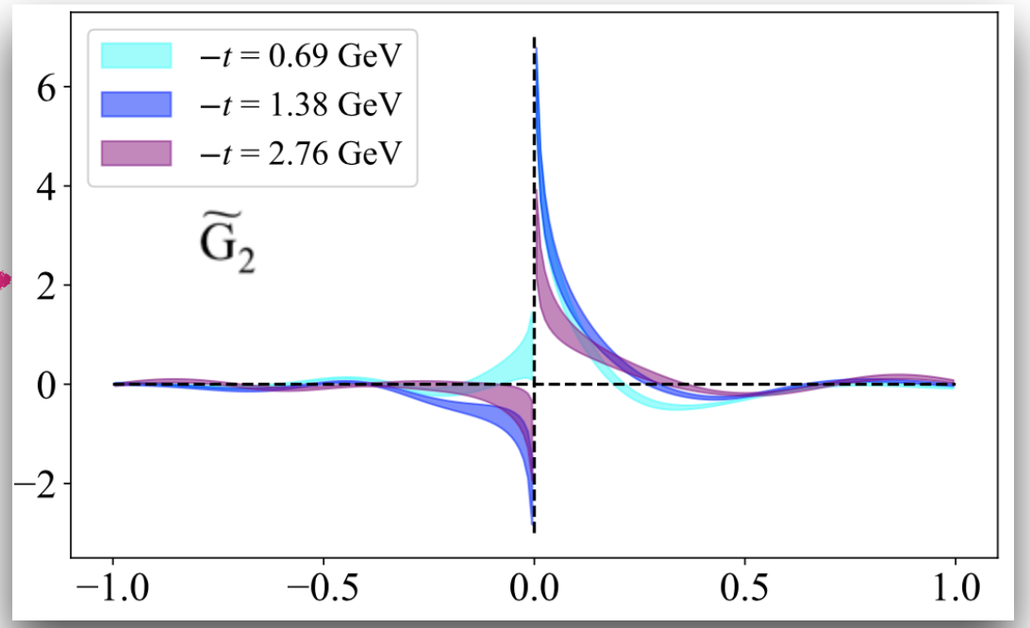

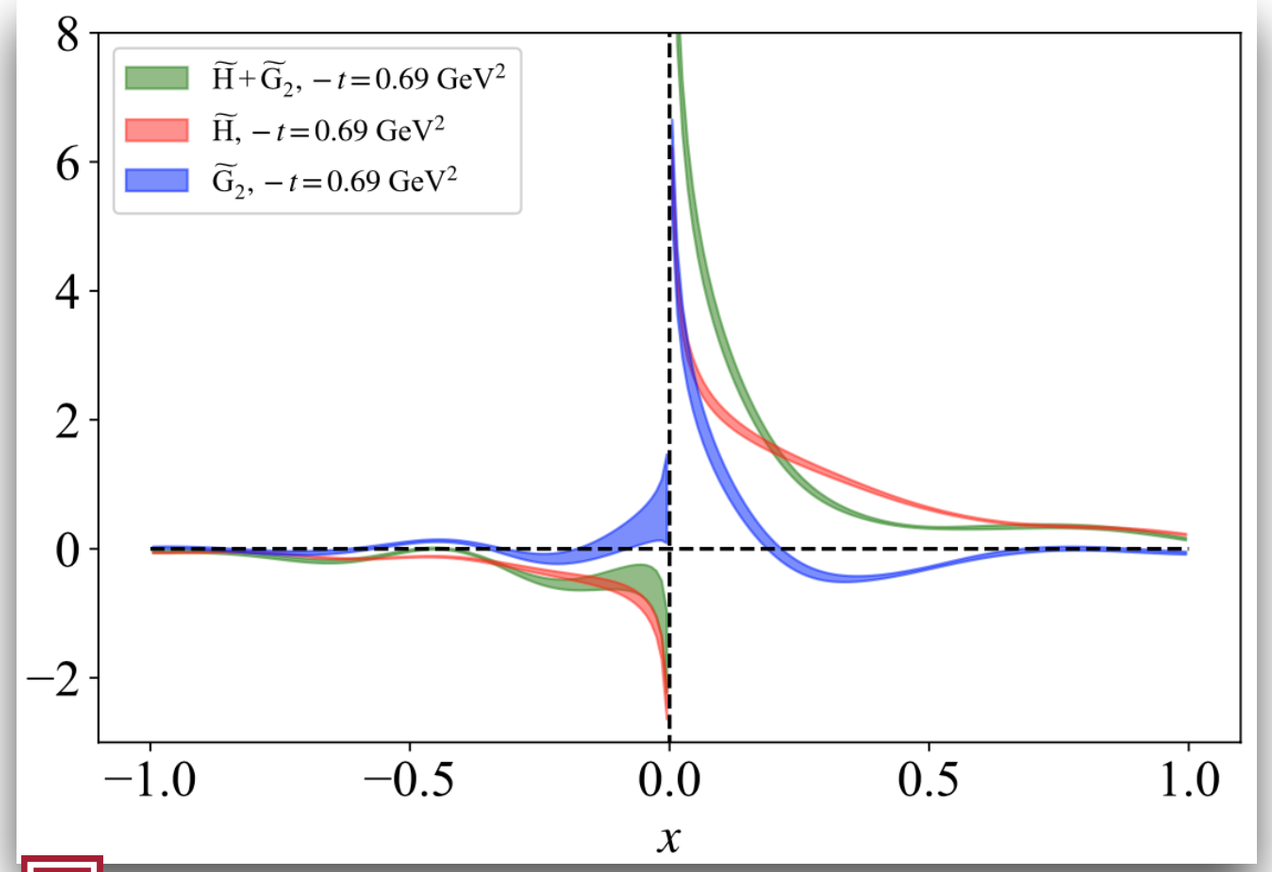

Negative areas in $\widetilde{G}_2$ theoretically anticipated:

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

- ★ Direct access to $\widetilde{E}$-GPD not possible for zero skewness
- ★ Glimpse into $\widetilde{E}$-GPD through twist-3 :

$$P^{\mu}\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$

Lattice Results - light-cone GPDs

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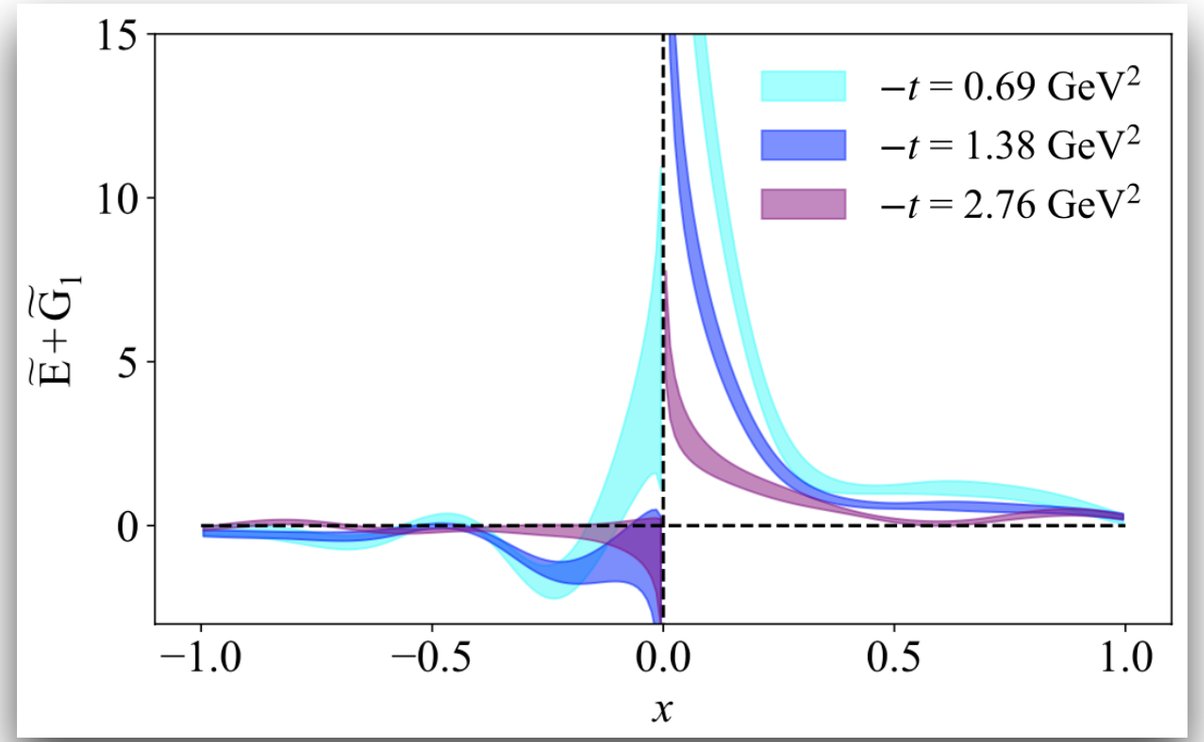

★ Sizable contributions as expected

$$\int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)$$

$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

Lattice Results - light-cone GPDs

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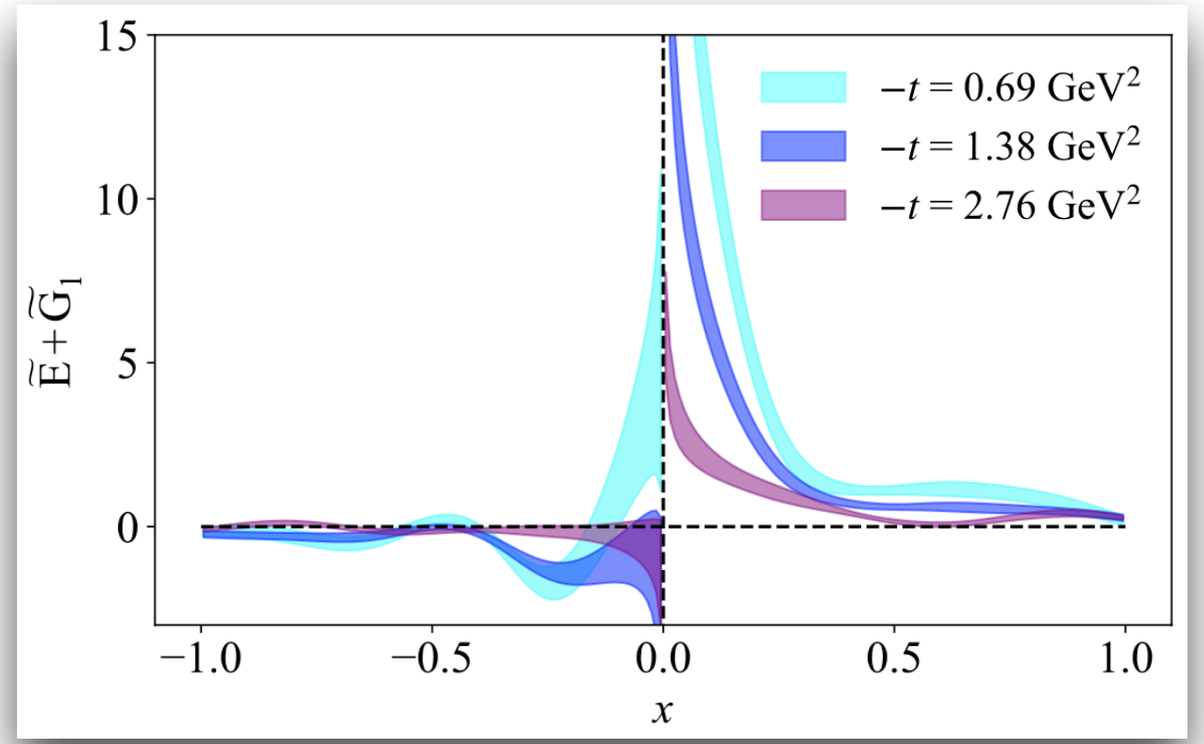

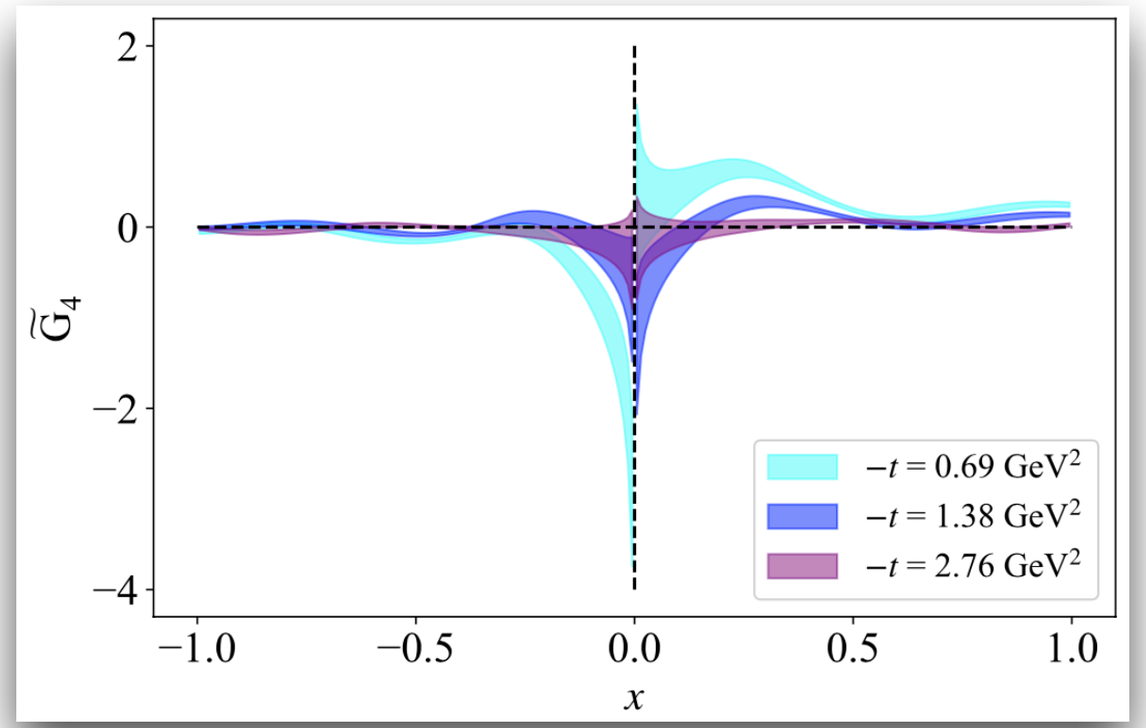

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$$\int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0\,, \quad i = 1,2,3,4$$

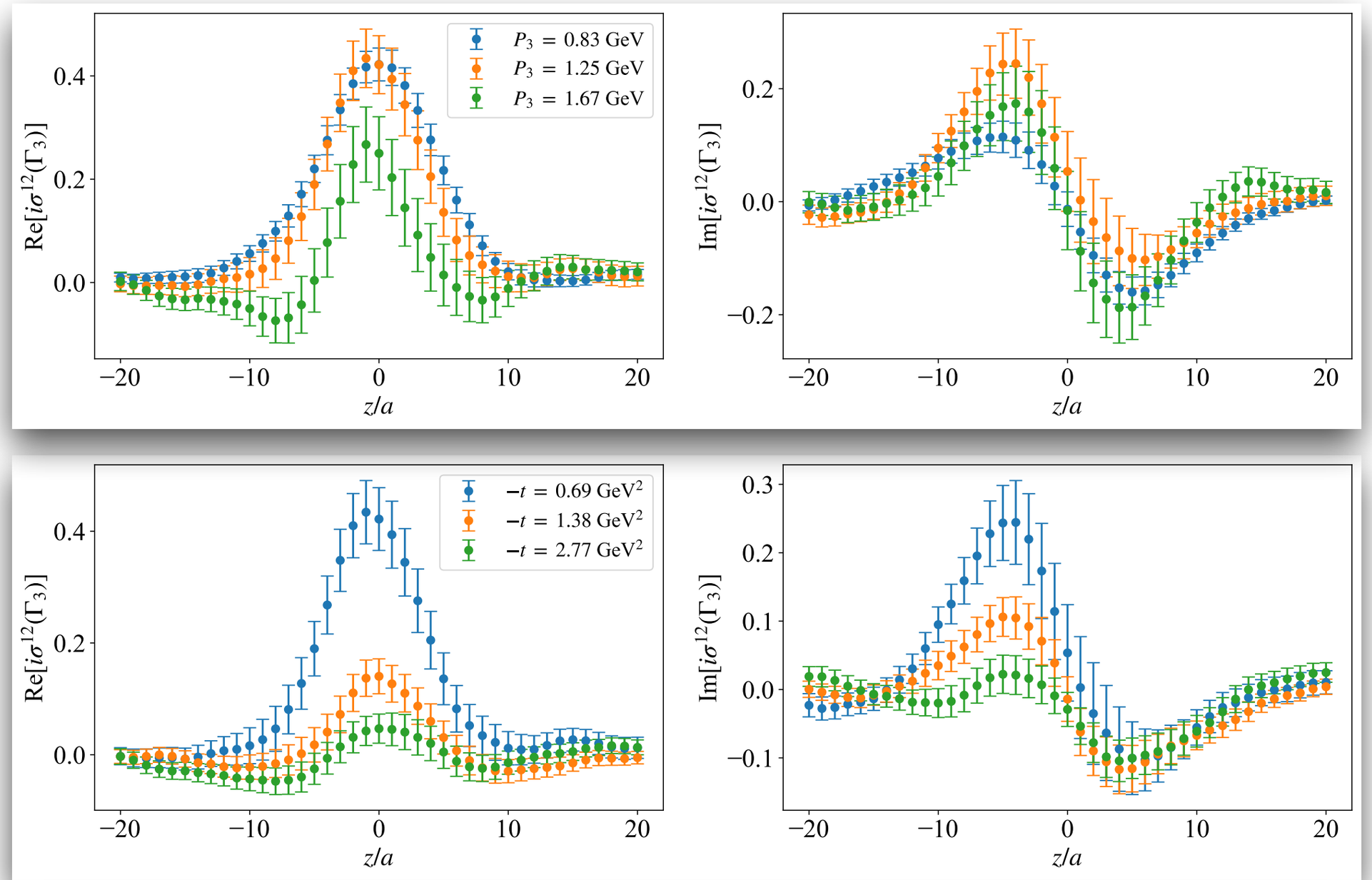
★ $\widetilde{G}_4$ very small; no theoretical argument to be zero

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E$$

Extension to twist-3 tensor GPDs

★ Parametrization [Meissner et al., *JHEP* 08 (2009) 056]

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p') \left(\gamma^+\gamma_5 \widetilde{H}'_2 + \frac{P^+\gamma_5}{M} \widetilde{E}'_2 \right) u(p)$$



New parametrization of GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	Δ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Theoretical setup

Theoretical setup

★ γ^+ inspired parametrization is prohibitively expensive

$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3)=\frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q(}}\right.$$ $$\left.+\frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q(0)}}(x,\xi,t;P^3)\right]u(p,\lambda)$$

Theoretical setup

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$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3)=\frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q(}}\right.$$ $$\left.+\frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q(0)}}(x,\xi,t;P^3)\right]u(p,\lambda)$$

$$F^{\mu}_{\lambda,\lambda'}=\bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1+z^\mu MA_2+\frac{\Delta^\mu}{M}A_3+i\sigma^{\mu z}MA_4+\frac{i\sigma^{\mu\Delta}}{M}A_5+\frac{P^\mu i\sigma^{z\Delta}}{M}A_6+\frac{z^\mu i\sigma^{z\Delta}}{M}A_7+\frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

★ Extraction of standard GPDs using A_i obtained from any frame

★ quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone

Theoretical setup

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$$F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3)=\frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q(}}\right.$$ $$\left.+\frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q(0)}}(x,\xi,t;P^3)\right]u(p,\lambda)$$

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Goals

★ Extraction of standard GPDs using A_i obtained from any frame

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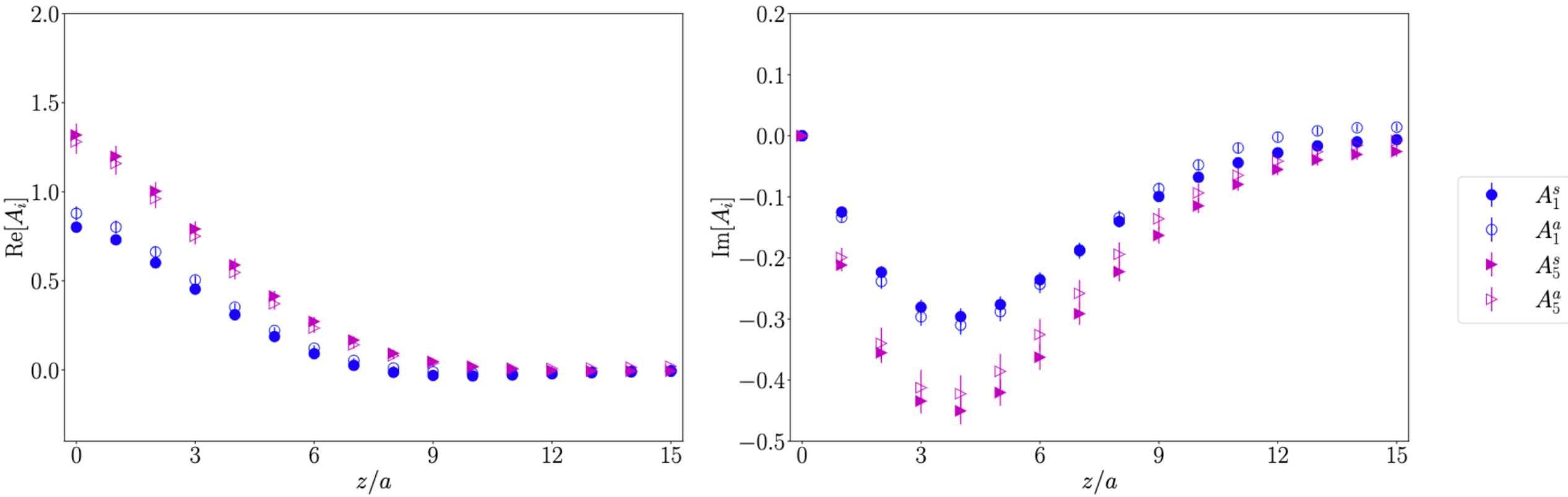
➡ Proof-of-concept calculation ($\xi=0$):

- symmetric frame: $\vec{p}_f^{\,s}=\overrightarrow{P}+\frac{\overrightarrow{Q}}{2}$, $\vec{p}_i^{\,s}=\overrightarrow{P}-\frac{\overrightarrow{Q}}{2}$ $\quad -t^s=\overrightarrow{Q}^2=0.69\,GeV^2$

- asymmetric frame: $\vec{p}_f^{\,a}=\overrightarrow{P}$, $\vec{p}_i^{\,a}=\overrightarrow{P}-\overrightarrow{Q}$ $\quad t^a=-\overrightarrow{Q}^2+(E_f-E_i)^2=0.65\,GeV^2$

Comparison of A_i in two frames

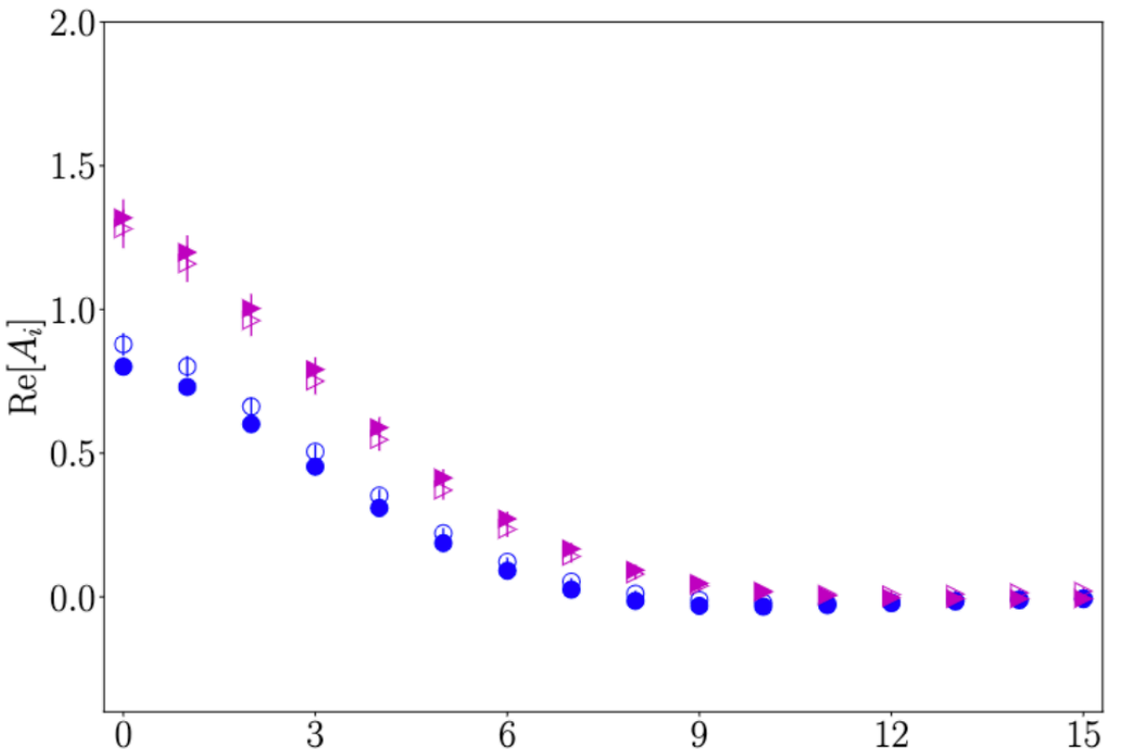
Unpolarized GPDs

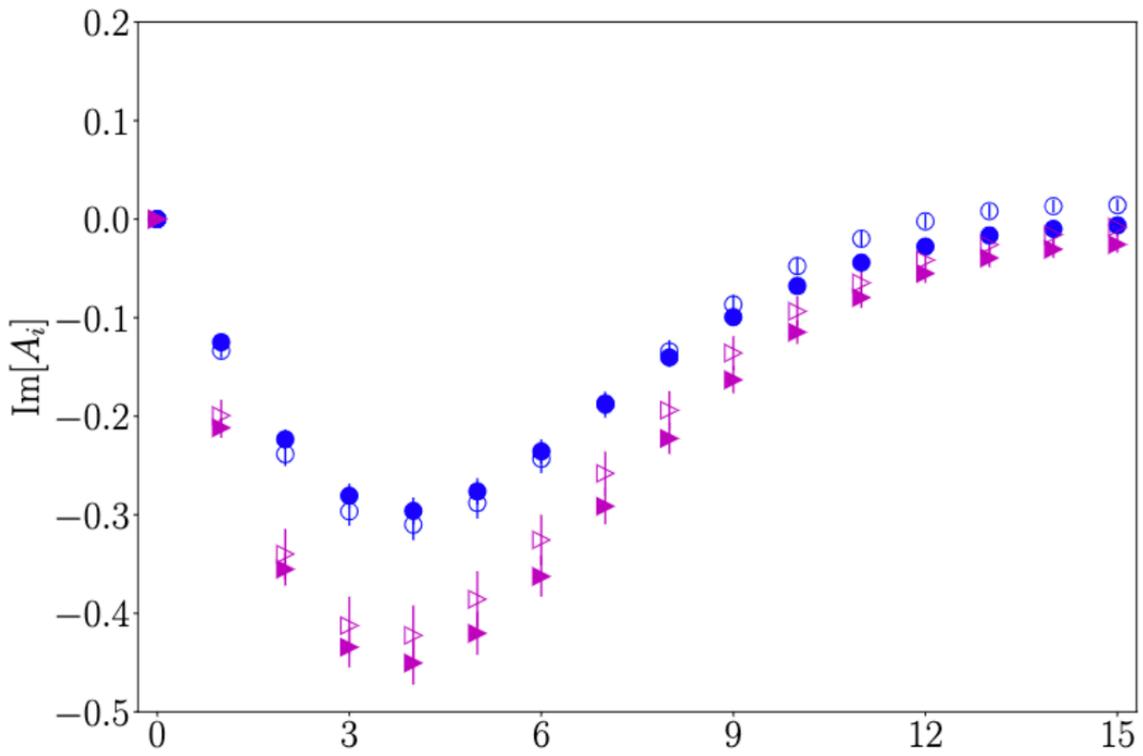


- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ A_3, A_4, A_8 zero at $\xi = 0$
- ★ A_2, A_6, A_7 suppressed (at least for this kinematic setup and $\xi = 0$)

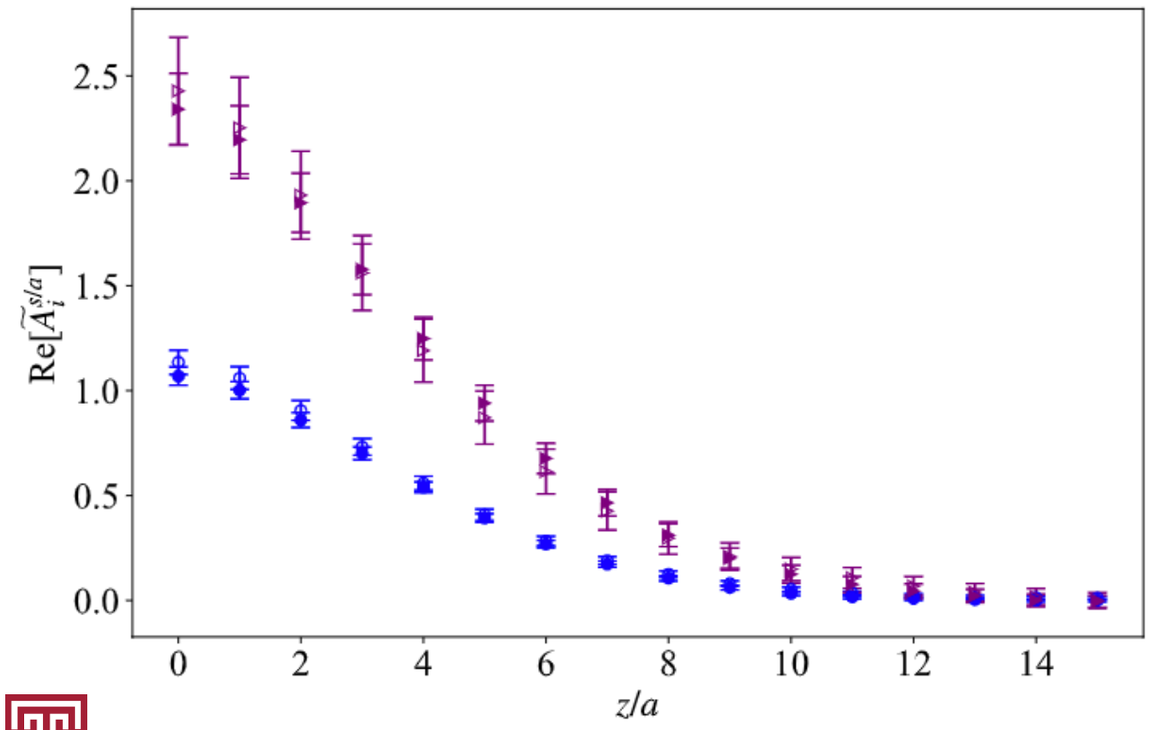
Comparison of A_i in two frames

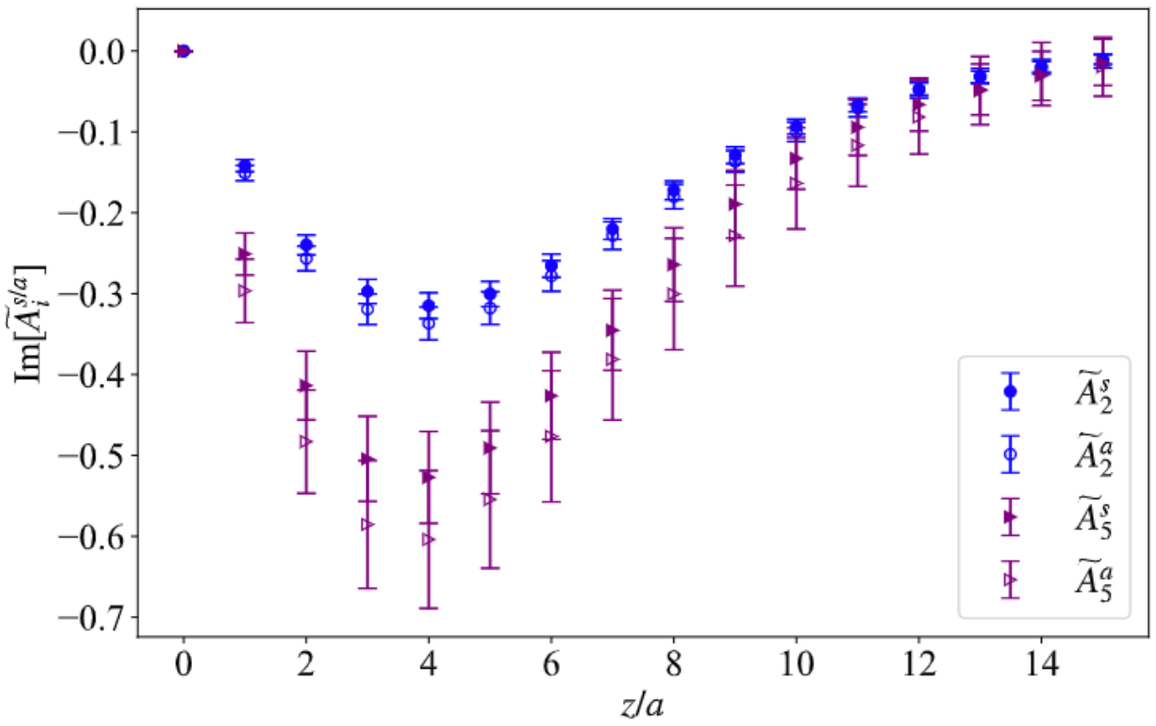
Unpolarized GPDs



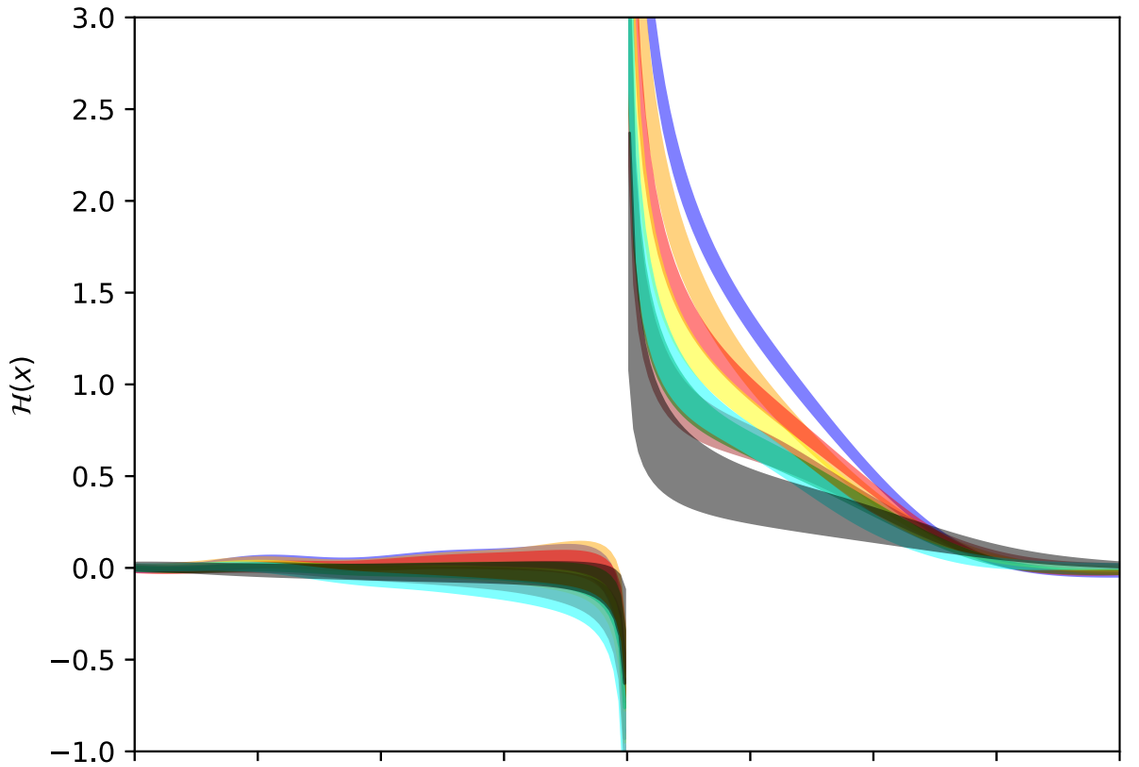


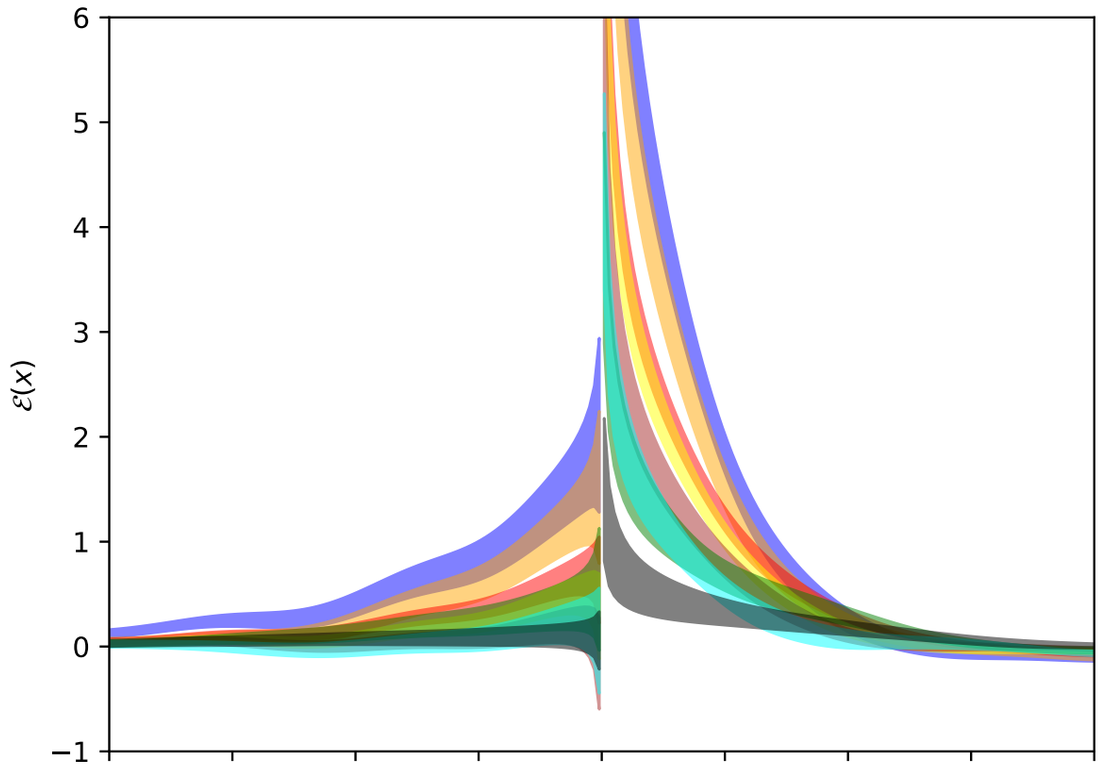
Helicity GPDs

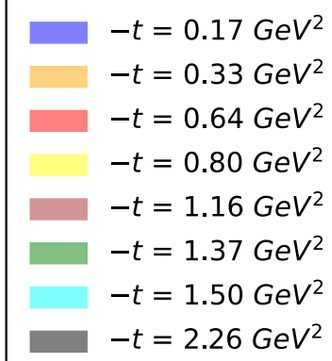


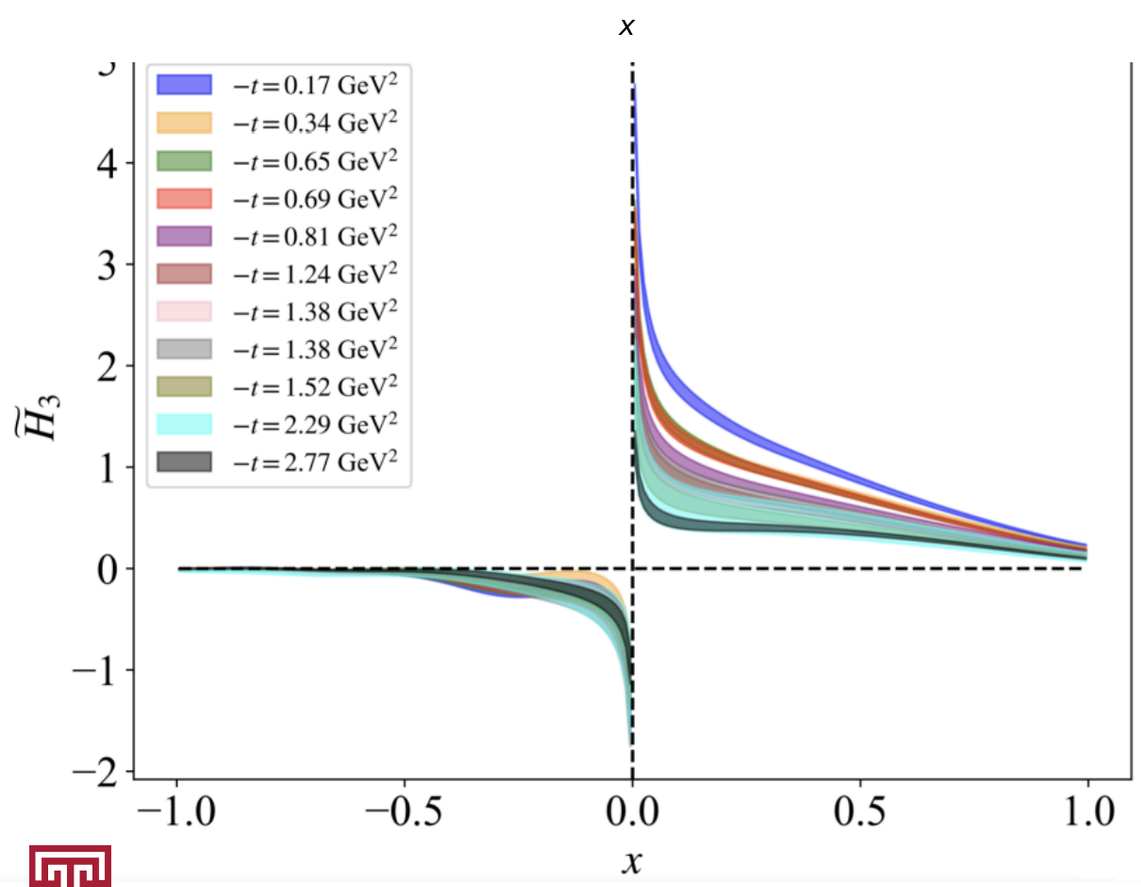


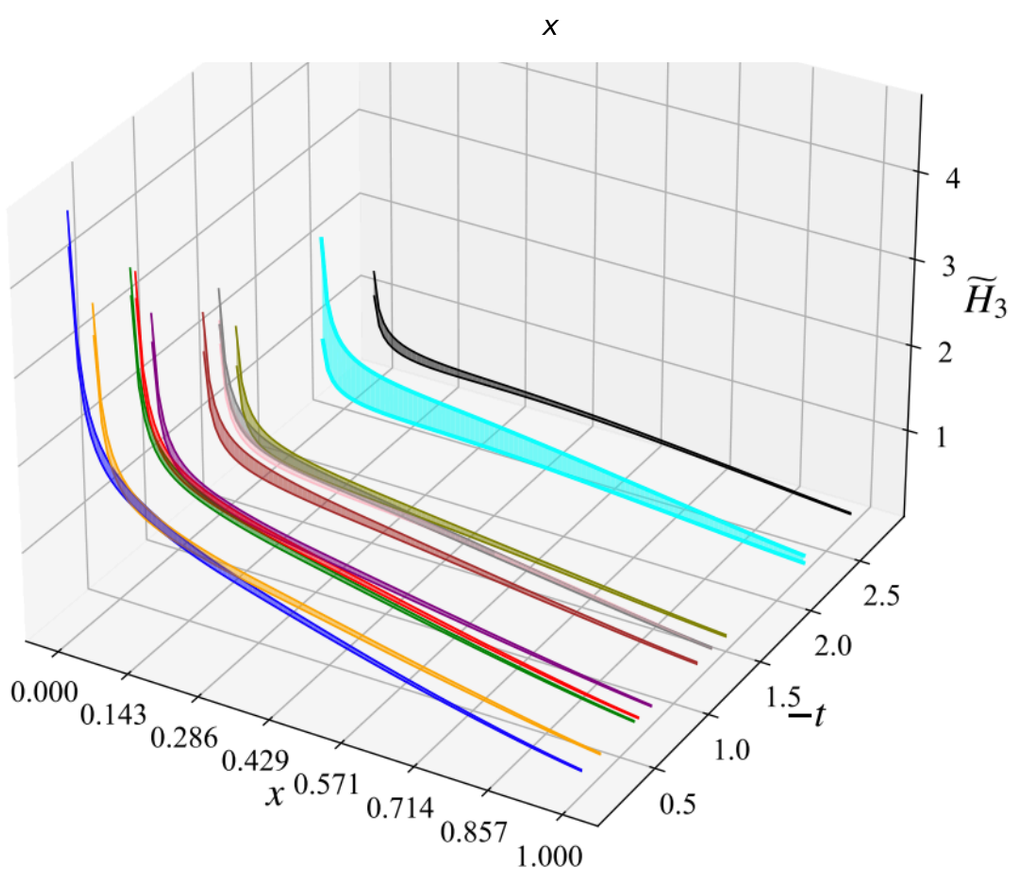
Light-cone GPDs

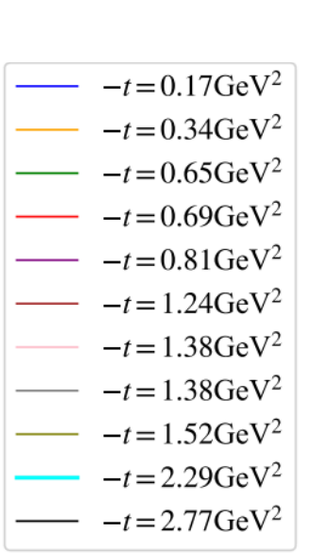






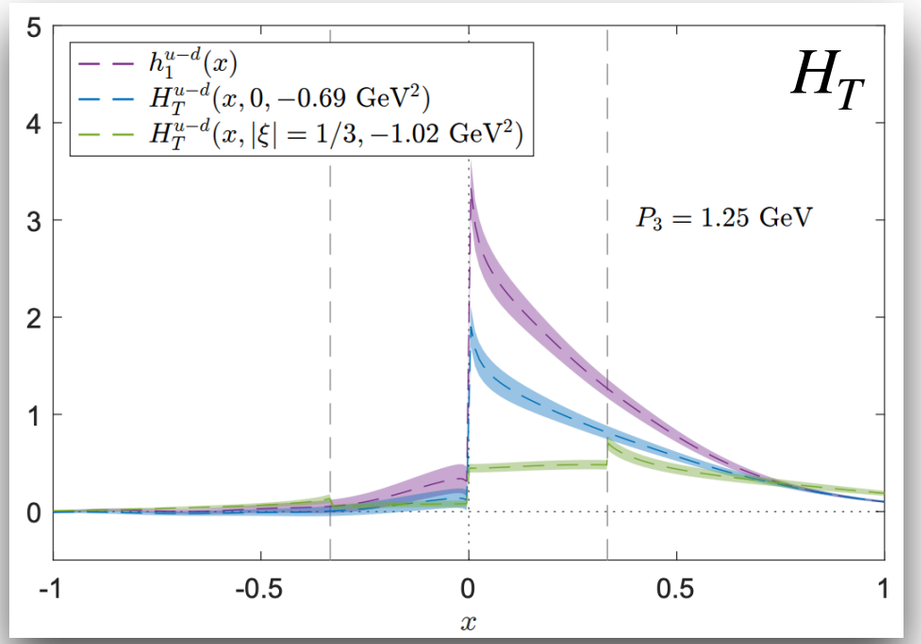






Transversity GPDs

Standard parametrization



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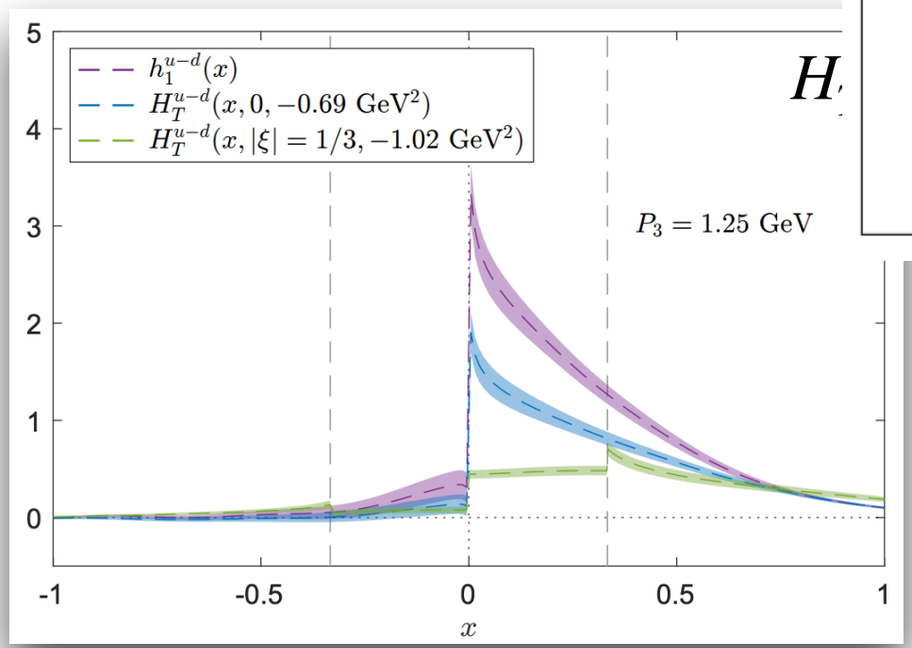


Lorentz covariant parametrization

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5$$
$$+ M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$
$$+ i\epsilon^{\mu\nu Pz}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

$$\Pi^s_{01}(\Gamma_0) = K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_2}{16m^2} + A_{T12}\frac{\left(P_3^2 - E(E+m)\right)z\Delta_2}{8m^2}\right)$$

$$\Pi^s_{01}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4(E+m)^2 + 4P_3^2 + \Delta_1^2 - \Delta_2^2\right)}{16m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right)$$

$$\Pi^s_{02}(\Gamma_0) = K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_1}{16m^2} + A_{T12}\frac{\left(E(E+m) - P_3^2\right)z\Delta_1}{8m^2}\right)$$

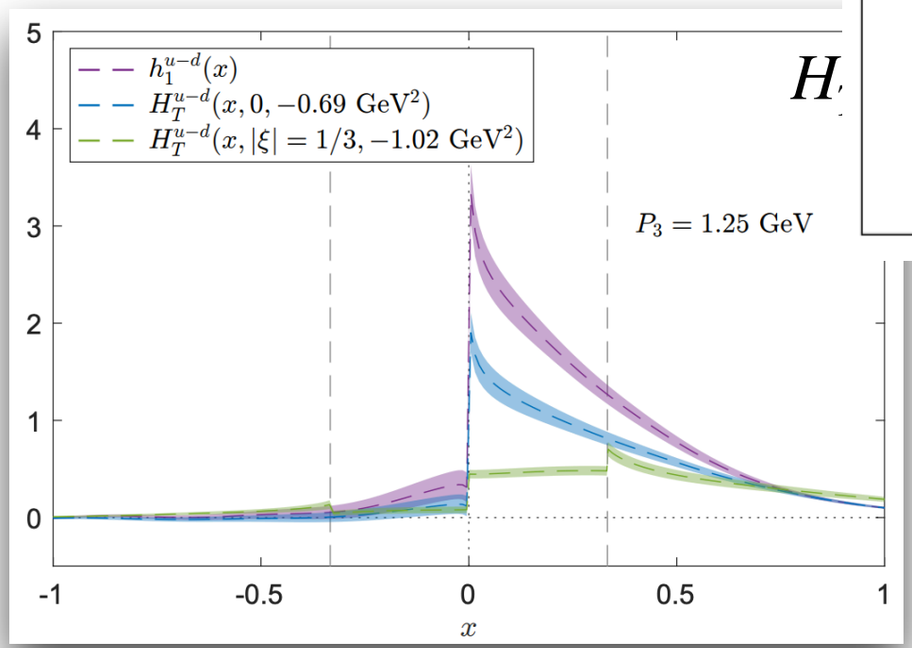
$$\Pi^s_{02}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4E(E+m) - \Delta_1^2\right)}{8m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)$$

Transversity GPDs

Standard parametrization



Lorentz covariant parametrization

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}\gamma_5 A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5$$
$$+ M\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$
$$+ i\epsilon^{\mu\nu Pz}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

$$\Pi^s_{01}(\Gamma_0) = K\left(-A_{T4}\frac{EP_3\Delta_2}{4m^3} + A_{T10}\frac{P_3\Delta_2}{4m^2} + A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_2}{16m^2} + A_{T12}\frac{\left(P_3^2 - E(E+m)\right)z\Delta_2}{8m^2}\right)$$

$$\Pi^s_{01}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1^2}{4m^4} + A_{T4}\frac{E\left(\Delta_2^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4(E+m)^2 + 4P_3^2 + \Delta_1^2 - \Delta_2^2\right)}{16m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_2^2\right)z}{32m^2} - A_{T12}\frac{P_3\Delta_2^2 z}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{01}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_1}{2m^3} - A_{T8}\frac{(E+m)\Delta_1 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_1 z}{8m}\right)$$

$$\Pi^s_{02}(\Gamma_0) = K\left(A_{T4}\frac{EP_3\Delta_1}{4m^3} - A_{T10}\frac{P_3\Delta_1}{4m^2} - A_{T11}\frac{\left(P_3^2 + E(E+m)\right)z\Delta_1}{16m^2} + A_{T12}\frac{\left(E(E+m) - P_3^2\right)z\Delta_1}{8m^2}\right)$$

$$\Pi^s_{02}(\Gamma_1) = iK\left(A_{T2}\frac{E(E+m)\Delta_1\Delta_2}{4m^4} - A_{T4}\frac{E\Delta_1\Delta_2}{8m^3} + A_{T10}\frac{\Delta_1\Delta_2}{8m^2} + A_{T11}\frac{P_3\Delta_1 z\Delta_2}{32m^2} + A_{T12}\frac{P_3\Delta_1 z\Delta_2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_2) = iK\left(A_{T2}\frac{E(E+m)\Delta_2^2}{4m^4} + A_{T4}\frac{E\left(\Delta_1^2 + 4m(E+m)\right)}{8m^3} + A_{T10}\frac{\left(4E(E+m) - \Delta_1^2\right)}{8m^2}\right.$$
$$\left. + A_{T11}\frac{P_3\left(8E(E+m) - \Delta_1^2\right)z}{32m^2} - A_{T12}\frac{P_3 z\Delta_1^2}{16m^2}\right)$$

$$\Pi^s_{02}(\Gamma_3) = iK\left(-A_{T6}\frac{(E+m)P_3\Delta_2}{2m^3} - A_{T8}\frac{(E+m)\Delta_2 zE^2}{2m^3} - A_{T12}\frac{(E+m)\Delta_2 z}{8m}\right)$$

On-going work

Summary

★ New proposal for Lorentz invariant decomposition has great advantages:
- significant reduction of computational cost
- access to a broad range of t and ξ

★ LaMET formalism is applicable beyond leading twist.
However, several improvements needed, e.g.,
mixing with quark-gluon-quark correlator

★ Future calculations have the potential to transform the field of GPDs

★ Synergy with phenomenology is an exciting prospect!
QGT Collaboration will be instrumental in such effort

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Thank you

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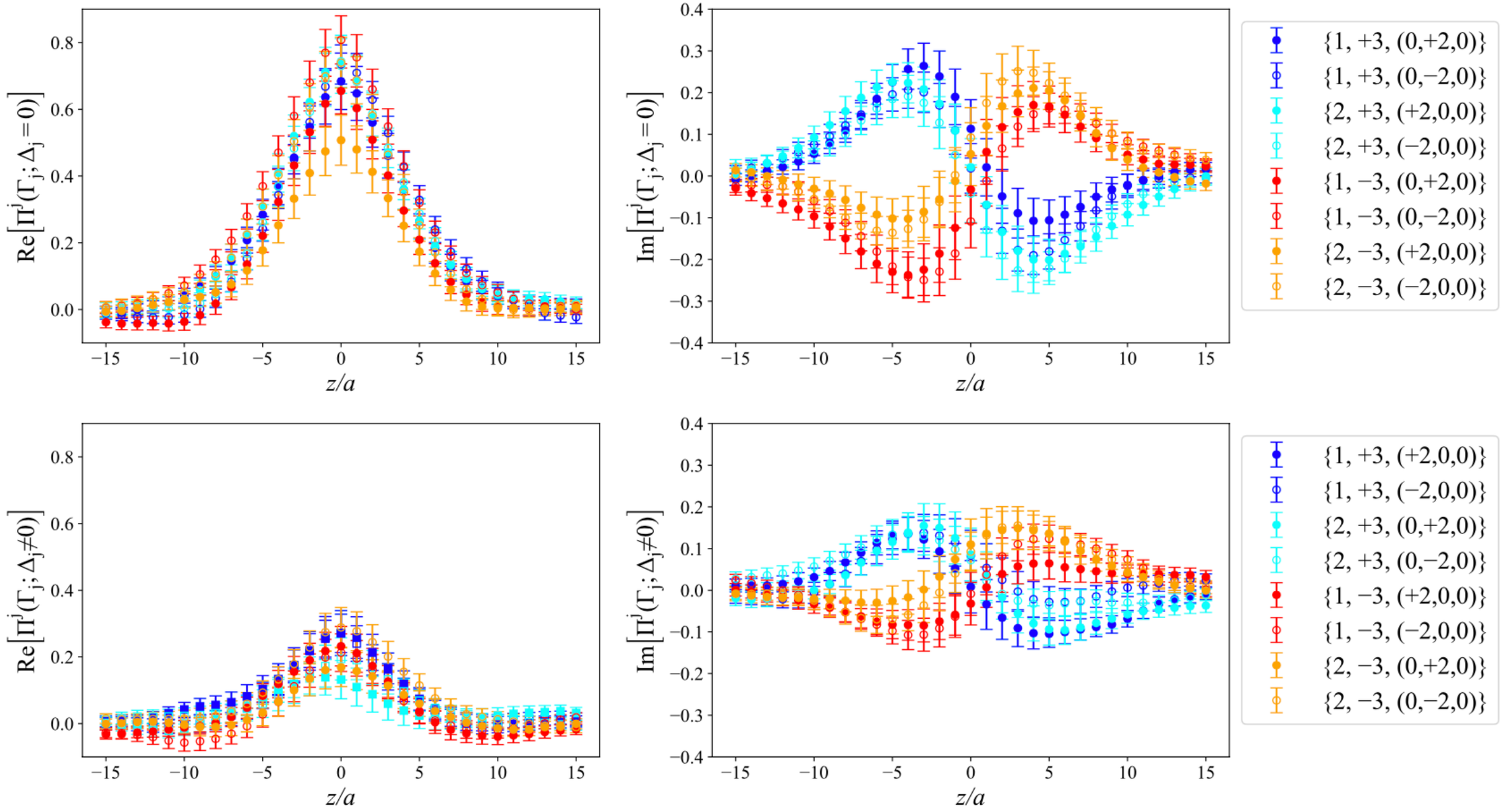
Miscellaneous

Lattice Results - Matrix Elements

★ Bare matrix elements

$$\Pi^1(\Gamma_1) = i\,C\left(F_{\widetilde{H}+\widetilde{G}_2}\,\frac{(4m(E+m)+\Delta_y^2)}{8m^2} - F_{\widetilde{E}+\widetilde{G}_1}\,\frac{\Delta_x^2(E+m)}{8m^3} + F_{\widetilde{G}_4}\,\frac{\mathrm{sign}[P_3]\,\Delta_y^2(E+m)}{4m^2 P_3}\right)$$



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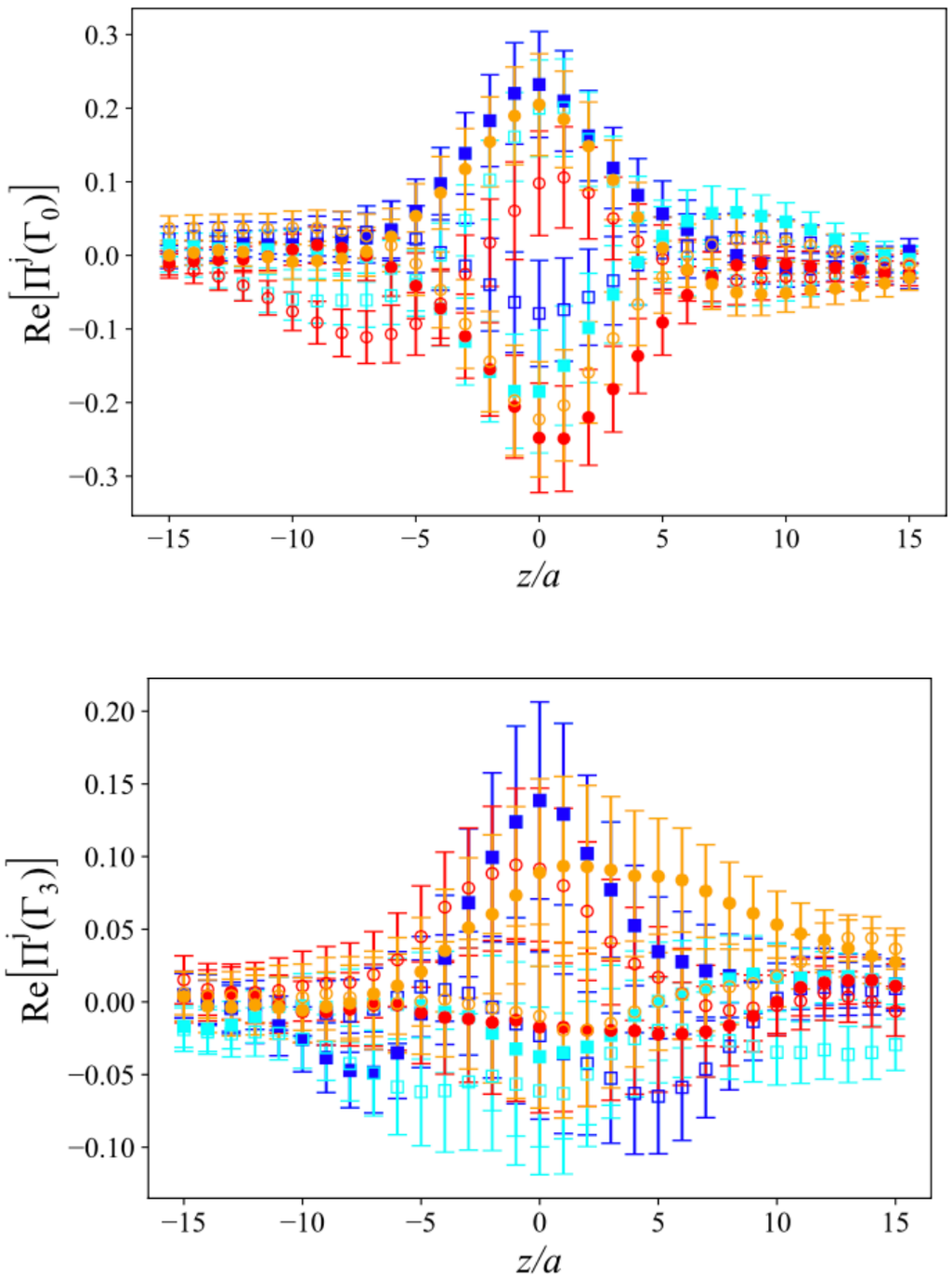

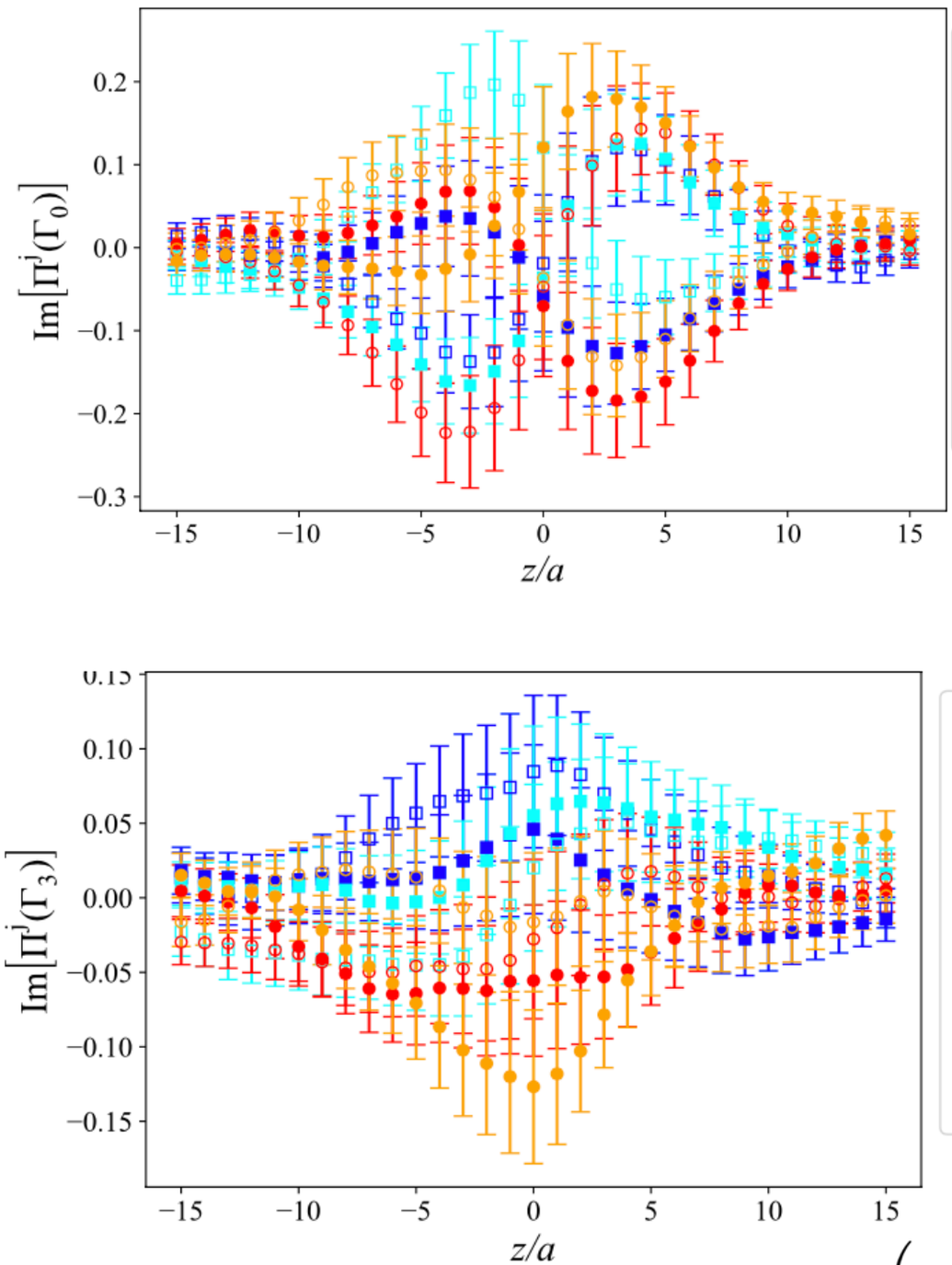

★ Suppressed signal compared to $\gamma_+ \gamma_5$ operators

$$\Pi^1(\Gamma_3) = C\left(- F_{\tilde{G}_3} \frac{E\Delta_x(E+m)}{2m^2 P_3} \right)$$

Consistency checks

★ Norms satisfied encouraging results

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

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★ Alternative kinematic setup can be utilized

[Fernanda Steffens]

$$F_{\widetilde{H}+\widetilde{G}_2} = \frac{1}{2m^2}\frac{z_3 P_0^2 (\Delta_\perp)^2}{P_3} + A_2 \qquad F_{\widetilde{G}_3} = \frac{1}{2m^2}\left(z_3 P_0^2 \Delta_3 - z_3 P_3 P_0 \Delta_0\right) A_1 - z_3 P_3 A_8$$

$$F_{\widetilde{E}+\widetilde{G}_1} = \frac{2 z_3 P_0^2}{P_3} + 2A_5 \qquad F_{\widetilde{G}_3} = \frac{1}{m^2}\left(z_3 P_0 P_3^2 - z_3 P_0^3\right) A_1$$

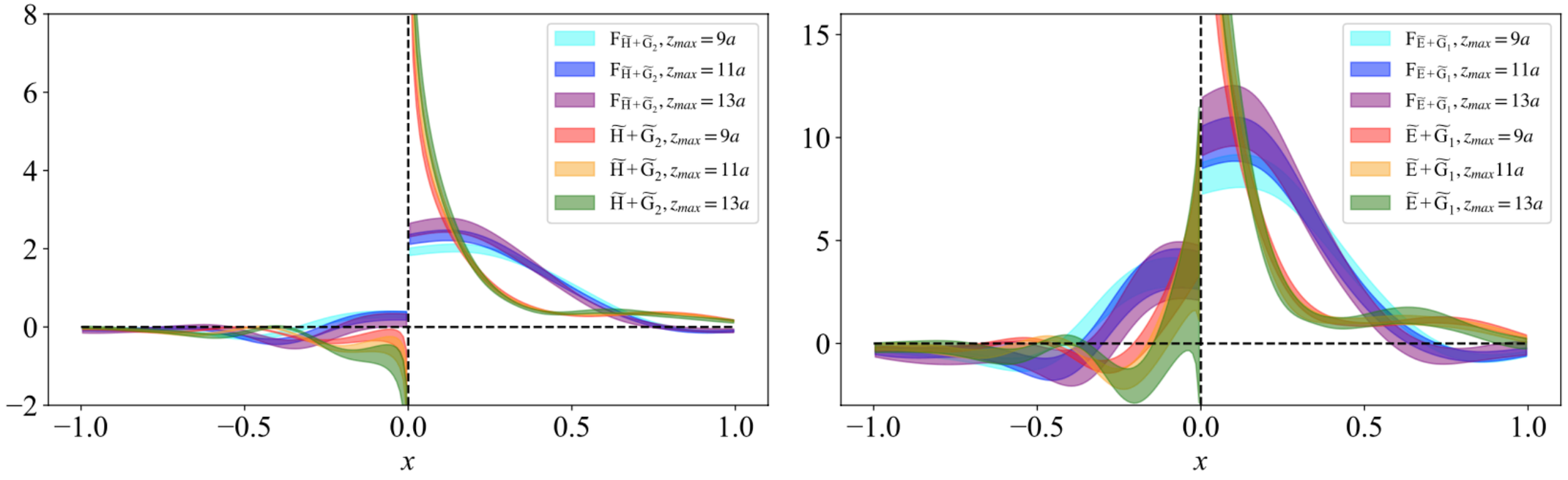

FIG. 10. $z_{\max}$ dependence of $F_{\widetilde{H}+\widetilde{G}_2}$ and $\widetilde{H}+\widetilde{G}_2$ (left), as well as $F_{\widetilde{E}+\widetilde{G}_1}$ and $\widetilde{E}+\widetilde{G}_1$ (right) at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in the $\overline{\text{MS}}$ scheme at a scale of 2 GeV.

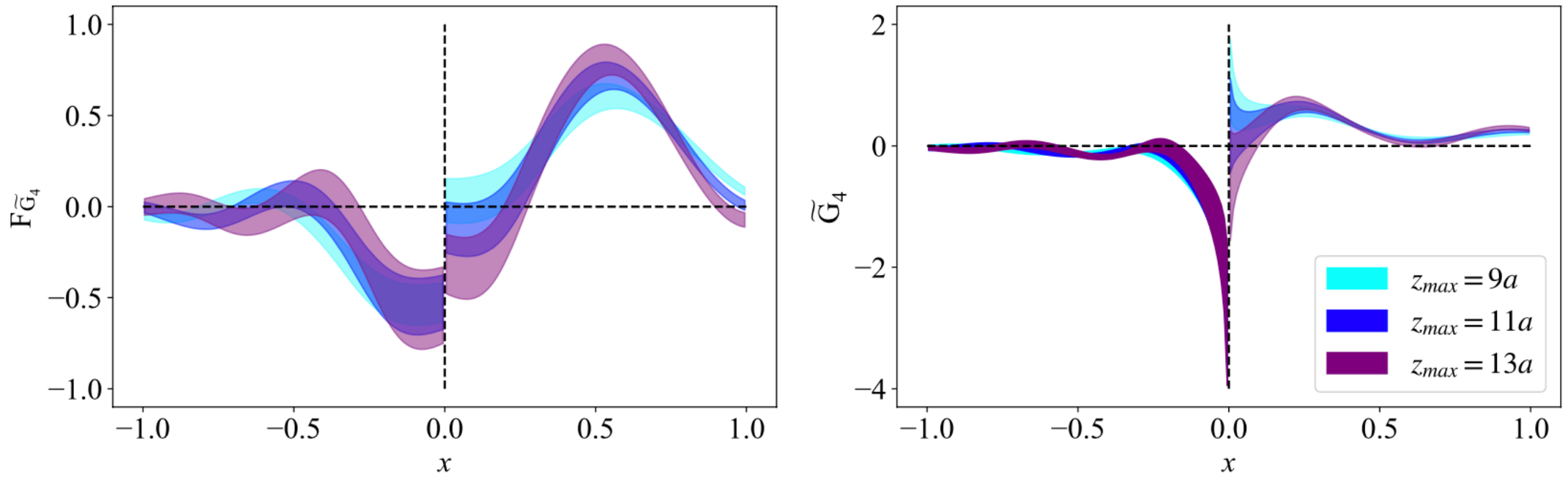

FIG. 11. $z_{\max}$ dependence of $F_{\widetilde{G}_4}$ and $\widetilde{G}_4$ at $-t = 0.69$ GeV^2 and $P_3 = 1.25$ GeV. Results are given in $\overline{\text{MS}}$ scheme at a scale of 2 GeV.