The Δ resonance at different physical parameters

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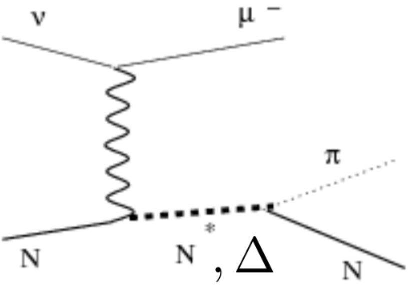
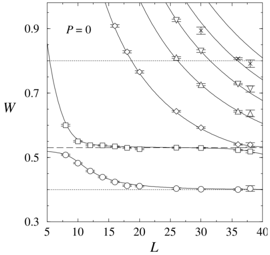
Delta resonance

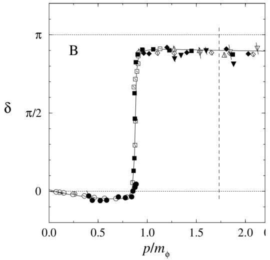
- $\Delta(1232)$
- Lightest baryonic resonance
- Dominant in the *p* wave $N\pi$ scattering
- Simplest resonance: 3 quark and $N\pi$ contribution
- Resonances are not eigenstates of the QCD Hamiltonian



- They decay via strong interactions
- Finite volume influences the two-hadron spectrum

Lüscher Method (Nucl.Phys.B 1991)





- Turn the finite volume to an advantage
- $\det(F^{-1}(E_{\text{cm}}) - 8\pi i T(E_{\text{cm}})) = 0$
- F is the box function (L), T is scattering amplitude
- E_{cm} can be determined
- We have two Different L-s at two different pion masses
- Physical point: threshold very low

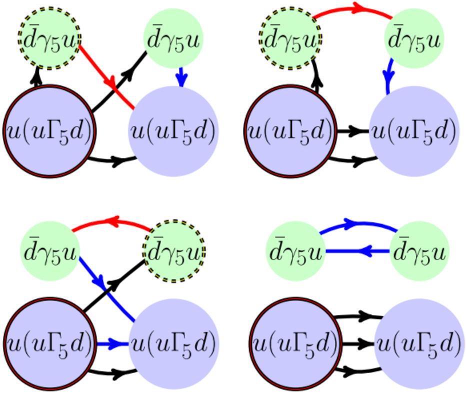
Rummukainen, Gottlieb Nucl.Phys.B(1995)

Diagrams: $N-N, N-\pi N, \pi N-\pi N$

2pt correlator

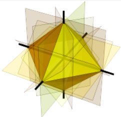
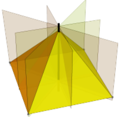
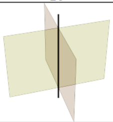
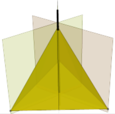
- $C(t) = \langle 0|O_{\mathscr{X}}(t)\bar{O}_{\mathscr{X}'}(0)|0\rangle \; \mathscr{X}, \mathscr{X}' \in \{N, N\pi\}$

- Simplest case *NN* is already expensive
- Three point-to-all propagators have to be multiplied. Each has dimensions $\sim \mathscr{O}(10^9)$
- Per lattice site:
- $\varepsilon_{a,b,c}\varepsilon_{l,m,n} S1^{c,l}_{\alpha\alpha_0} \Gamma_{i\,\alpha_0,\alpha_1} S2^{b,m}_{\beta_0,\alpha_1} \Gamma_{f\,\beta_0,\beta_1} S3^{a,n}_{\beta_1,\beta}$
- Ideal task for a GPU-kernel https://github.com/cylqcd/PLEGMA



Finite volume lattice: projections

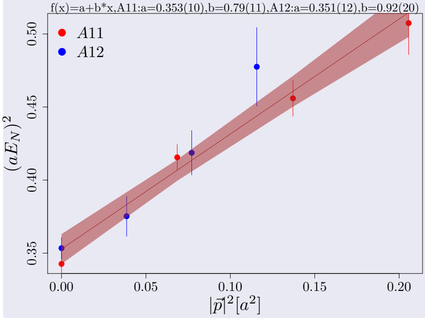
- We use single and two hadron interpolating operators with $I = 3/2, I_3 = 3/2$
- Finite volume we no longer have continuous rotational symmetry
- Symmetry group in the centre-off-mass frame is the double cover of the octahedral group $2O_h$
- Finite number of irreducible representations

$\frac{L}{2\pi}\vec{P}$	$(0,0,0)$	$(0,0,1)$	$(0,1,1)$	$(1,1,1)$
Group LG	$O_h^{(D)}$	$C_{4v}^{(D)}$	$C_{2v}^{(D)}$	$C_{3v}^{(D)}$
Axis and planes of symmetry				
g_{LG}	96	16	8	12
$\Lambda(J^P) : \pi(\ 0^-\ )$	$A_{1u}(\ 0^-\ , 4^-, ...)$	$A_2(\ 0\ , 1, ...)$	$A_2(\ 0\ , 1, ...)$	$A_2(\ 0\ , 1, ...)$
$\Lambda(J^P) : N(\ \frac{1}{2}^+\ )$	$G_{1g}(\ \frac{1}{2}^+\ , \frac{7}{2}^+, ...)$	$G_1(\ \frac{1}{2}\ , \frac{3}{2}, ...)$	$G(\ \frac{1}{2}\ , \frac{3}{2}, ...)$	$G(\ \frac{1}{2}\ , \frac{3}{2}, ...)$
$\Lambda(J^P) : \Delta(\frac{3}{2}^+)$	$H_g(\frac{3}{2}^+, \frac{5}{2}^+, ...)$	$G_1(\frac{1}{2}, \frac{3}{2}, ...) \oplus G_2(\frac{3}{2}, \frac{5}{2}, ...)$	$(2)G(\frac{1}{2}, \frac{3}{2}, ...)$	$G(\frac{1}{2}, \frac{3}{2}, ...) \oplus F_1(\frac{3}{2}, \frac{5}{2}, ...) \oplus F_2(\frac{3}{2}, \frac{5}{2}, ...)$

Simulation details

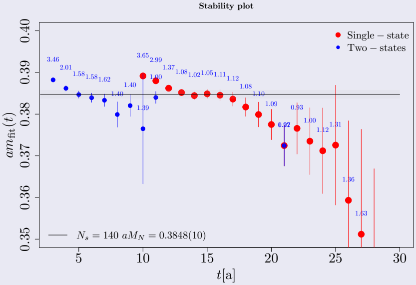
$N_f = 2+1$ Clover, $a = 0.1163\text{fm}$

- A11,A12: $M_\pi = 200\text{MeV}, L = 2.8-3.7\text{fm}$
- A7,A8: $M_\pi = 250\text{MeV}, L = 2.8-3.7\text{fm}$
- A15: $M_\pi = 137\text{MeV}, L = 5.5\text{fm}$



$N_f = 2+1+1$ Twisted-Clover $a = 0.08\text{fm}$

- $M_\pi = 139\text{MeV}, L = 5.12\text{fm}$

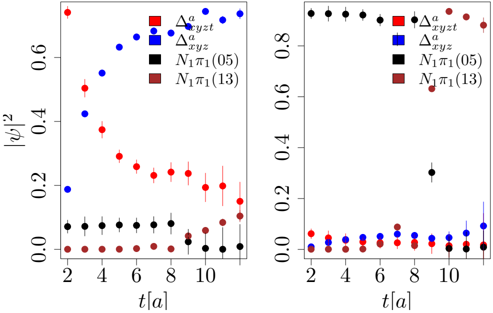


Generalized eigenvalue problem (GEVP)

$$C_{ik}^{\Lambda,\vec{P}}(t)u_k^n(t,t_0) = \lambda^n(t,t_0)C_{ij}(t_0)u_j^n(t)$$

$$\Lambda^n(t,t_0) \propto e^{-E_n^{\Lambda,\vec{P}}(t-t_0)}$$

- Key point: Selecting a basis
- Aim: Robostness, "good" signal quality, eigenvectors



$$\langle \mathrm{GS}|\mathrm{GS}\rangle = |\psi|^2_{\Delta xyzt} + |\psi|^2_{\Delta xyz} + |\psi|^2_{N\pi(05)} + |\psi|^2_{N\pi(13)} \equiv 1$$

Four different methods

Single state fits

- For each principal correlators of the GEVP $\Lambda^n(t,t_0) \propto e^{-E_n^{\Lambda,\vec{P}}(t-t_0)}$

Hankel (Fischer et.al. Eur.Phys.J.A(2020))

- For each principal correlators of the GEVP
- $H^0_{ij}(t) = C^0(t+i\Delta+j\Delta)$
- $\sum_{k=0}^{n-1} e^{-E_k t} e^{-E_k i\Delta} e^{-E_k j\Delta} c_k$

AMIAS (Finkenrath et.al. PoS LATTICE2016)

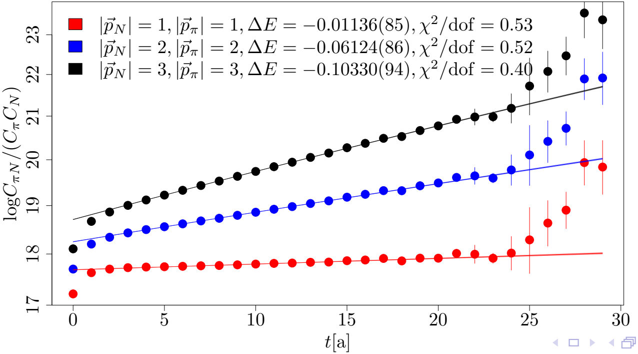
- Statistically sampling the space of fit parameters according to the χ^2 value of the fit function

Ratio method

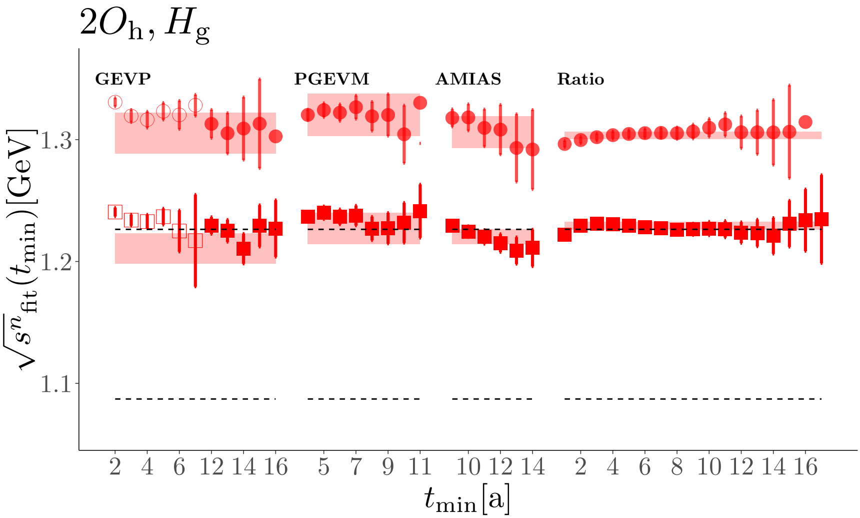
- We fit the energy shift directly

Ratio method

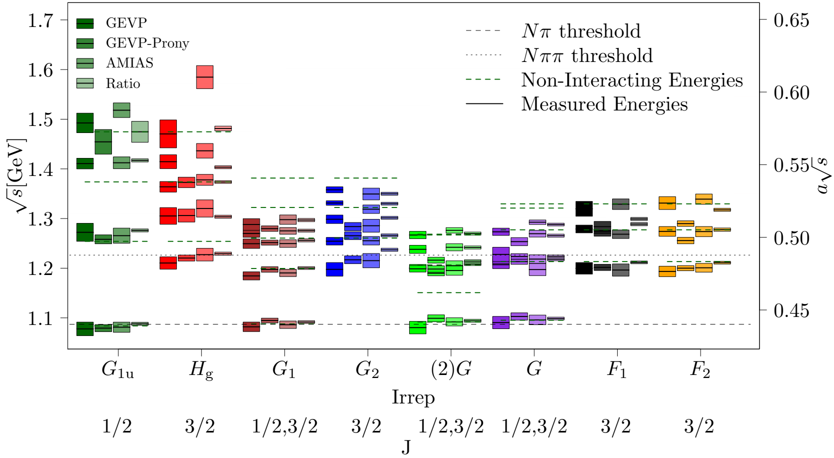
- Single hadron 2pt and two hadron 2pt functions are correlated
- Take the log of the ratio of $C_{\pi N}(t)/(C_N(t)C_\pi(t))$
- We can measure the shift relative to different non-interacting levels



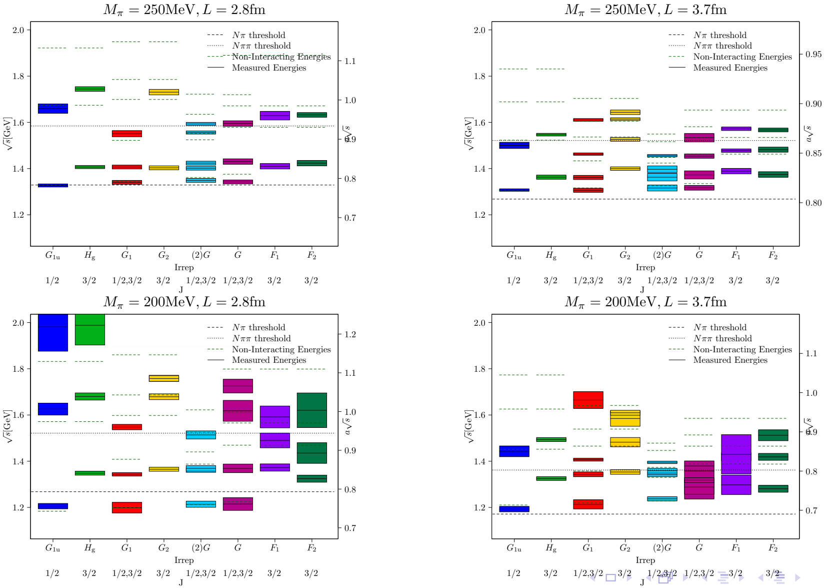
Comparison of different methods



Spectrum summary



Spectrum summary Clover ensembles



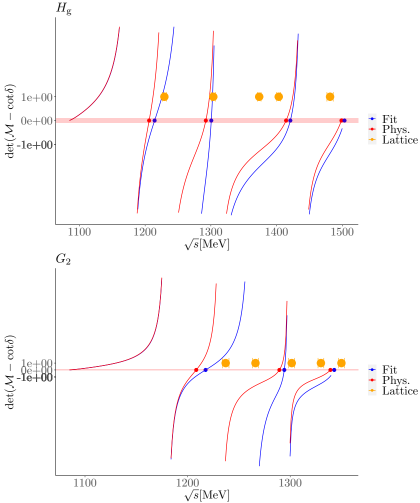
Getting the phase shift

Lüscher-method

- Two particle energy levels in a finite box with size L
- Volume dependence of the energy shift related to scattering observables at $L = \infty$

$$\det\left(\mathcal{M}^{\vec{P}}_{J\ell\mu,J'\ell'\mu'} - \delta_{JJ'}\delta_{\ell\ell'}\delta_{\mu\mu'}\cot\delta_{J\ell}\right) = 0$$

- Determinant is taken in angular momentum space
- Important: For $\ell = 1$ dominant irreps there is a one-to-one correspondence between phase-shift and finite volume energy levels (ignoring contributions from higher partial waves)



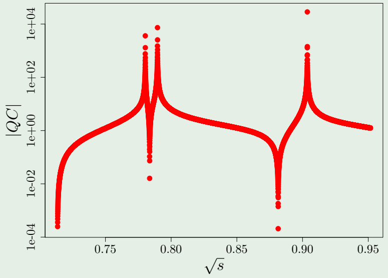
Parametrization of the resonance

Possible mixing of partial waves

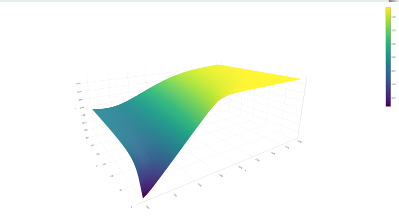
Quantization conditions (QC) Göckeler et. al PRD 2012

- Phase shift parametrization:
  - $\ell = 0 \rightarrow \cot\delta_{\ell=0} = a_0 q_{\text{cmf}}, \qquad \ell = 1 \rightarrow \tan\delta_{\ell=1} = \frac{\sqrt{s}\Gamma(\Gamma_R, s)}{{M_R}^2 - s}$
- We restrict ourselves to $\ell = 0, 1$ and check for $\ell \geq 2$

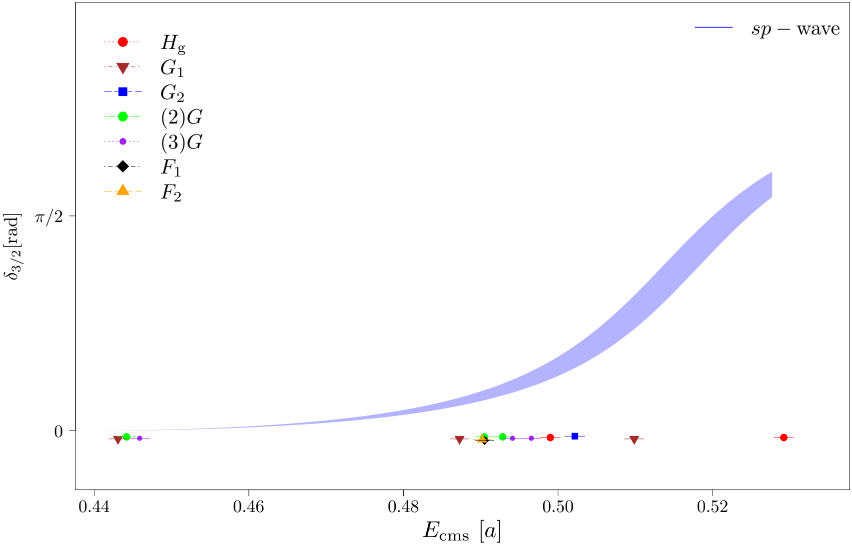
Example



2*d* Spline interpolation



LQC fit


H_g
G_1
G_2
$(2)G$
$(3)G$
F_1
F_2
sp − wave
$\pi/2$
0
$\delta_{3/2}$[rad]
0.44
0.46
0.48
0.50
0.52
E_{cms} [a]

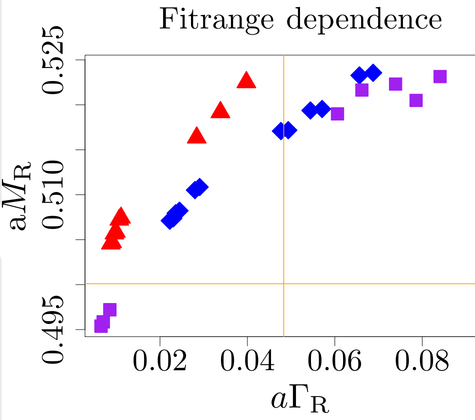
Luescher fits on the physical point ensemble

Performing several fits:

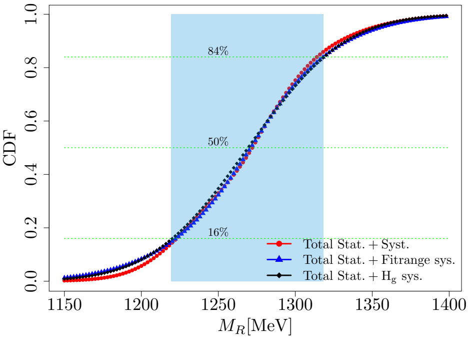
- $p - waveonly$
- $sp - wave$
- below-$N\pi\pi$
- Different fitranges
- Different extraction

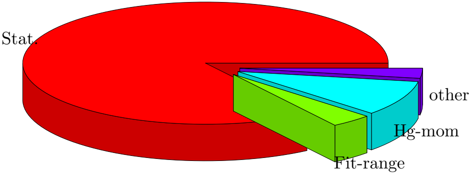
Stat.+Syst. error

- Model averaging
- Each fit is weighted
- $\sim e^{-0.5\left(\chi^2 - 2N_{\mathrm{data}} + 2N_{\mathrm{param}}\right)}$



Result at the physical point twisted clover





Conclusion, outlook

This work (Details [arXiv:2307.12846])

Ensemble	m_π[MeV]	L	m_Δ[MeV]	$g_{\Delta-\pi N}$
Twisted-Clover	139 MeV	5.12 fm	1267(46) MeV	
Nf2+1 Clover	200 MeV	3.7 fm	1320(10) MeV	17.6(2.7)
Nf2+1 Clover	250 MeV	2.8 fm	1380(7) MeV	13.6(5)
Nf2+1 Clover	250 MeV	3.7 fm	1373(6) MeV	10.3(1.6)

Collaboration	m_π[MeV]	Methodology	m_Δ[MeV]	$g_{\Delta-\pi N}$
Verduci(2014)	266	Distillation, Lüscher	1396(19)	19.9(8)
Alexandrou et.al. (2013)	360	LO pert., Michael & McNeile	1535(25)	26.7(1.5)
Alexandrou et.al. (2015)	180	LO pert., Michael & McNeile	1350(50)	23.7(1.3)
Andersen et.al. (2017)	280	Stoch. distillation, Lüscher	1344(20)	37.1(9.2)
Morningstar et.al.(2022)	200	Stoch. distillation, Lüscher	1290(7)	$14.41(53)_{\mathrm{BW}}$
Silvi et.al. (2021)	255	Smeared sources, Lüscher	$1380(7)(9)_{\mathrm{BW}}$	$13.6(5)_{\mathrm{BW}}$

Summary

- Perform analysis on all the ensembles
- Perform chiral extrapolations

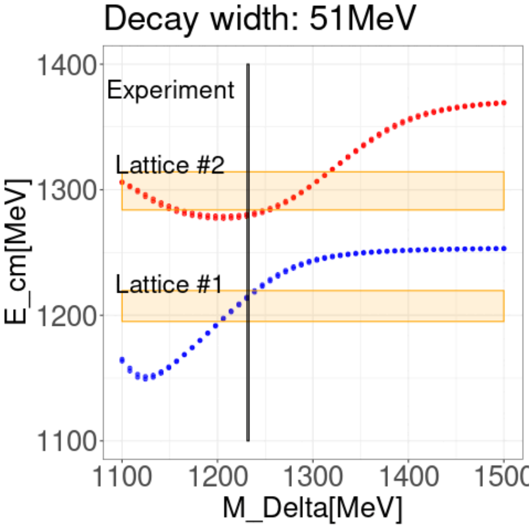
Acknowledgement

Thank you very much for your attention

The project acknowledge support

- Nice Quarks project
- Pizdaint supercomputer
- Juwelsbooster supercomputer
- NERSC supercomputer

Backup: The decay width

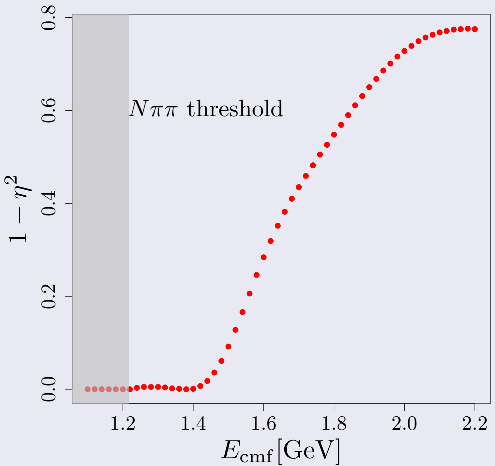


Backup: Scattering at the physical point

Data from http://gwdac.phys.gwu.edu

Challange: $N\pi\pi$ threshold is very low

At the physical point $m_N + m_\pi < m_\Delta \longrightarrow \Delta$ is unstable



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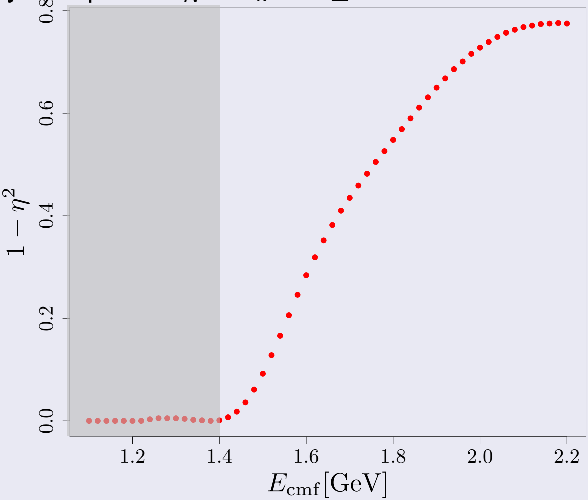
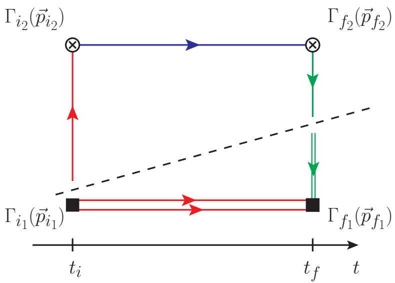
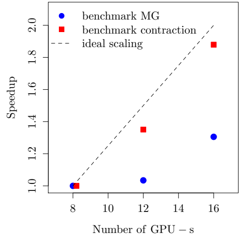


Backup Code: Reductions on the GPU

B diagram: $U(x_{f1},x_{i1})(\Gamma_{f1}D(x_{f1},x_{f2})\Gamma_{f2}U(x_{f2},x_{i2})\Gamma_{i2}D(x_{i2},x_{i1})\Gamma_{i1})^t U(x_{f1},x_{i1})$





- Two correlated spatial sum (pion(f_2), nucleon(f_1))
- The problematic is the green line (sink-to-sink)
- Estimate it stochastically $D(x_{f1},x_{f2}) = \sum_r \xi_r(f_1)\phi_r^\dagger(f_2)$
- Cut the diagram into factors
- Factors be combined to diagrams
- Many different diagrams share the same factors

Backup: Gramm-Schmidt irreducible representations

- irrep, irrep row(μ), # occurances,# combinations of momenta
- As an example we have a 12×12 correlation matrix for the delta
- In the process of projection this matrix will be block diagonalized GS transformations

- Pion nucleon correlation matrix

$\vec{p}_{\text{tot}}$, irrep name	N_{dim}
$\vec{p} = (0,0,0), \mathrm{G1_u}$	8x8
$\vec{p} = (0,0,0), \mathrm{Hg}$	9x9
$\vec{p} = (0,0,1), \mathrm{G1}$	24x24
$\vec{p} = (0,0,1), \mathrm{G2}$	18x18
$\vec{p} = (1,1,0), (2)\mathrm{G}$	30x30
$\vec{p} = (1,1,1), (3)\mathrm{G}$	16x16
$\vec{p} = (1,1,1), \mathrm{F1}$	6x6
$\vec{p} = (1,1,1), \mathrm{F2}$	6x6