# Nucleon axial and pseudoscalar form factors from Lattice QCD simulations at the physical point

Extended Twisted Mass Collaboration

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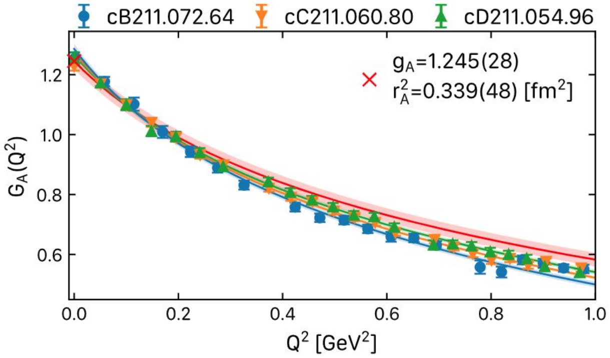
# Overview

Results on nucleon isovector axial, induced pseudoscalar, and pseudoscalar form factors (FF)

- Three physical point ensemble
- Thorough excited state analysis
- Combined fit of $Q^2$-dependence and continuum limit




https://arxiv.org/abs/2309.05774

**Nucleon axial and pseudoscalar form factors using twisted-mass fermion ensembles at the physical point**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(Extended Twisted Mass Collaboration)

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(Dated: October 28, 2023)


We compute the nucleon axial and pseudoscalar form factors using three $N_f = 2+1+1$ twisted mass fermion ensembles with all quark masses tuned to approximately their physical values. The values of the lattice spacings of these three physical point ensembles are 0.080 fm, 0.068 fm and 0.057 fm, and spatial sizes 5.1 fm, 5.44 fm, and 5.47 fm, respectively, yielding $m_\pi L > 3.6$. Convergence to the ground state matrix elements is assessed using multi-state fits. We study the momentum dependence of the three form factors and check the partially conserved axial-vector current (PCAC) hypothesis and the pion pole dominance (PPD). We show that in the continuum limit, the PCAC and PPD relations are satisfied. We also show that the Goldberger-Treimann relation is approximately fulfilled and determine the Goldberger-Treiman discrepancy. We find for the nucleon axial charge $g_A = 1.245(28)(14)$, for the axial radius $\langle r_A^2\rangle = 0.339(48)(06)$ fm$^2$, for the pion-nucleon coupling constant $g_{\pi NN} \equiv \lim_{Q^2\to -m_\pi^2} G_{\pi NN}(Q^2) = 13.25(67)(69)$ and for $G_P(0.88m_\mu^2) \equiv g_P^* = 8.99(39)(49)$.

# The weak axial-vector matrix element

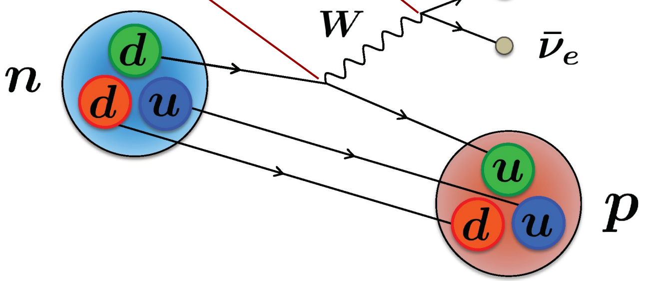
The transition matrix element of the neutron β-decay is

$$\mathcal{M}(n \to p\, e^-\, \bar{\nu}_e) = \frac{G_F}{\sqrt{2}}\, V_{ud}\, \sum_\mu \underbrace{\langle p(p')|W_\mu|n(p)\rangle}\, L_\mu$$

with
$$W_\mu = V_\mu - A_\mu$$
$$V_\mu = \bar{u}\gamma_\mu d$$
$$A_\mu = \bar{u}\gamma_\mu\gamma_5 d$$

*Axial-vector matrix element*

$$\langle p(p')|A_\mu|n(p)\rangle$$




Neutrino-nucleon scattering processes are related to matrix elements at finite momentum transfer.

# The axial and induced pseudoscalar FF

Neglecting isospin-breaking effects, transition FFs are equivalent to isovector FFs

$$\langle p(p')|A_\mu|n(p)\rangle \quad \xrightarrow[u=d]{} \quad \langle N(p')|A_\mu^{\text{isov}}|N(p)\rangle$$

$$A_\mu = \bar{u}\gamma_\mu\gamma_5 d \qquad\qquad A_\mu^{\text{isov}} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$$

Matrix elements are decomposed into Lorentz-invariant form factors (FF)

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N}G_P(Q^2)\right]\gamma_5 u_N(p,s),$$

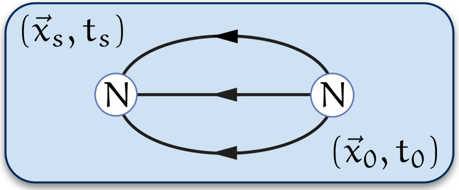
*Axial FF* *Induced pseudoscalar FF*

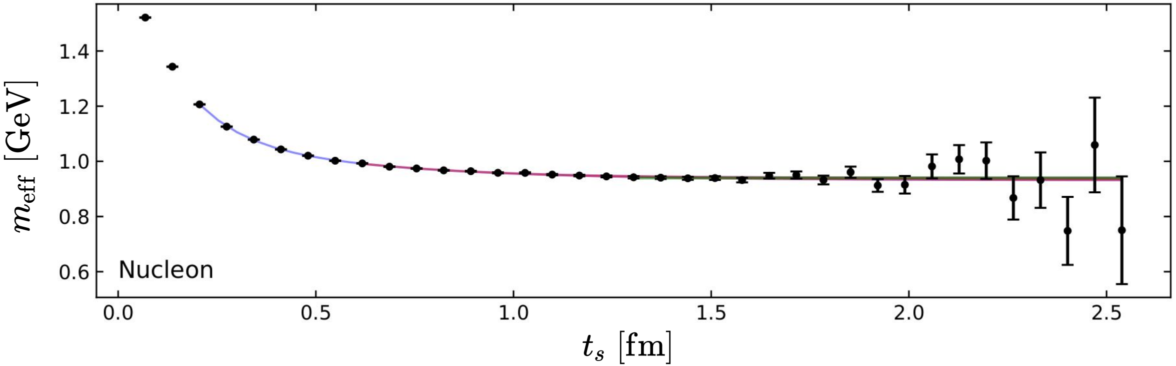
# Nucleon two-point functions

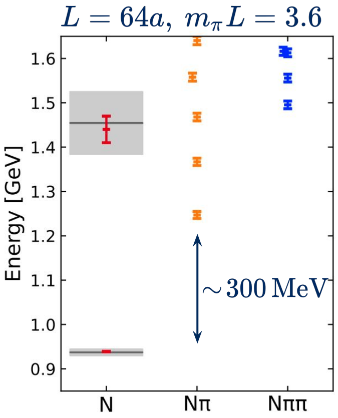
$$G(t_s) = \sum_{\vec{x}} P_0^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(\vec{x}_s, t_s) | \chi_N^{\alpha}(\vec{0}, 0) \rangle = \sum_k c_k e^{-t_s E_k}$$

*with* $\chi_N^{\alpha}(x) = \epsilon^{abc} u_{\alpha}^{a}(x)[u^b(x) C\gamma_5 d^c(x)]$

- Two-point functions
  - Ground state dominance at large-time limit $G(t_s) = c_0^{-t_s m_N}\Big|_{t_s \to \infty}$
  - Error increases exponentially with $t$
  - Density of excited states increases with volume



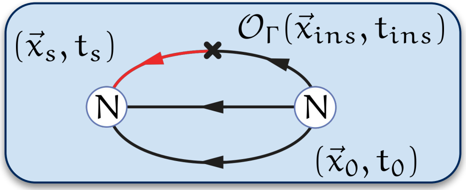
# Nucleon three-point functions

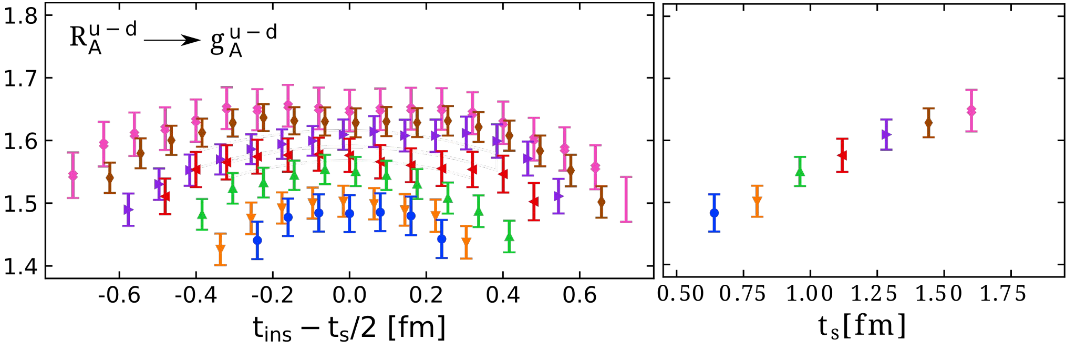
$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s,\vec{x}_{ins}} e^{-i\vec{q}\cdot\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}^\beta_N(\vec{x}_s,t_s) | \mathcal{O}_\Gamma(\vec{x}_{ins},t_{ins}) | \chi^\alpha_N(\vec{0},0)\rangle$$

suitable projector

e.g. $\mathcal{O}_A(x) = \bar{\psi}(x)\gamma_5\gamma^\mu\psi(x)$

- Three-point functions
  - Ground state at $t_s \to \infty$, $(t_s - t_{ins}) \to \infty$
  - Error increases exponentially with $t_s$
  - Statistics increased to keep errors constant




×750 configurations

| $t_s/a$ | $t_s$[fm] | $n_{src}$ |
|---|---|---|
| 8 | 0.64 | 1 |
| 10 | 0.80 | 2 |
| 12 | 0.96 | 5 |
| 14 | 1.12 | 10 |
| 16 | 1.28 | 32 |
| 18 | 1.44 | 112 |
| 20 | 1.60 | 128 |
| Nucleon 2pt | | 477 |

~30M inversions!

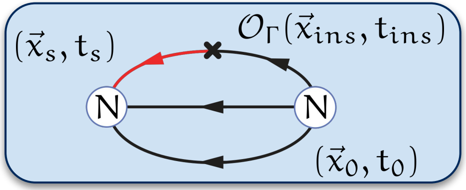
# Nucleon three-point functions

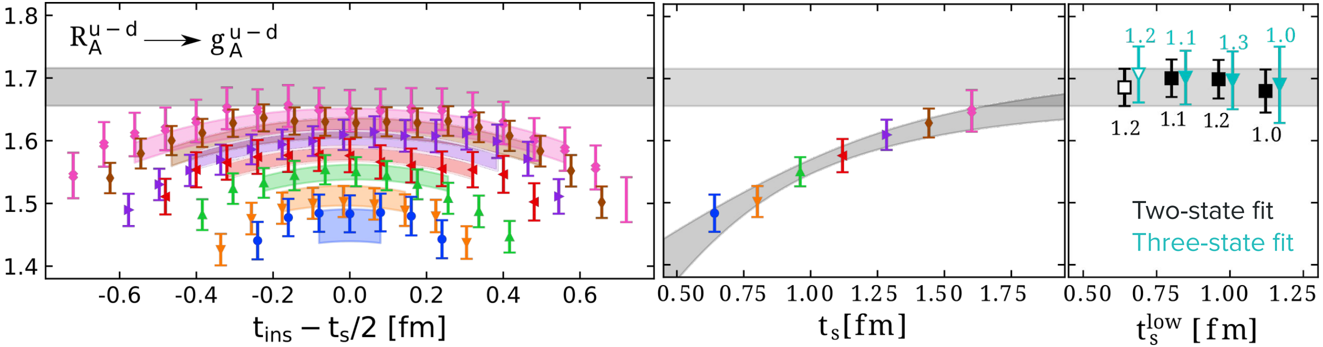
$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s,\vec{x}_{ins}} e^{-i\vec{q}\cdot\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}^\beta_N(\vec{x}_s,t_s) | \mathcal{O}_\Gamma(\vec{x}_{ins},t_{ins}) | \chi^\alpha_N(\vec{0},0)\rangle$$

$$G_\Gamma(t_s,t_{ins}) \simeq A_{00} e^{-m_N t_s} + A_{01}\left(e^{-E_1 t_{ins}} + e^{-E_1 t_s + (E_1 - m_N) t_{ins}}\right) + A_{11} e^{-E_1 t_s}$$

$$G(t) \simeq c_0 e^{-m_N t_s} + c_1 e^{-E_1 t_s}$$

*Desired matrix element:* $\mathcal{M} = \dfrac{A_{00}}{c_0}$




$\times 750$ configurations

| $t_s/a$ | $t_s$[fm] | $n_{src}$ |
|---|---|---|
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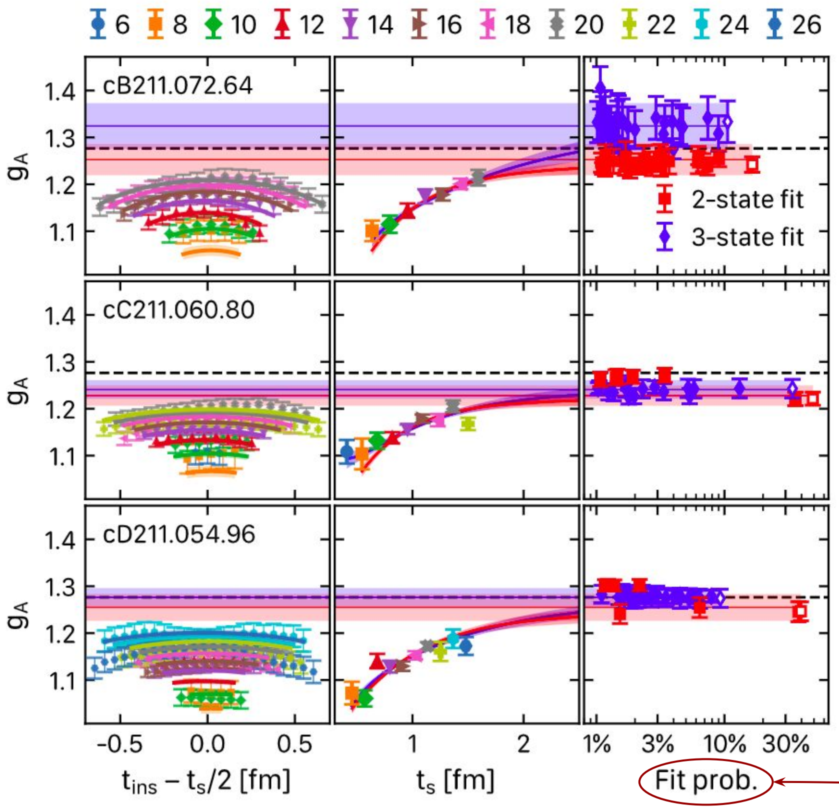
*~30M inversions!*

[C. Alexandrou, S. B., et al. "Nucleon axial, tensor, and scalar charges and σ-terms in lattice QCD". Phys. Rev., D102(5):054517, 2020]

# The three ensembles and model averaging




| Ensemble | $V/a^4$ | $\beta$ | $a$ [fm] | $m_\pi$ [MeV] | $m_\pi L$ |
|---|---|---|---|---|---|
| cB211.072.64 | $64^3 \times 128$ | 1.778 | 0.07957(13) | 140.2(2) | 3.62 |
| cC211.060.80 | $80^3 \times 160$ | 1.836 | 0.06821(13) | 136.7(2) | 3.78 |
| cD211.054.96 | $96^3 \times 192$ | 1.900 | 0.05692(12) | 140.8(2) | 3.90 |

| cB211.072.64 | | |
|---|---|---|
| 750 configurations | | |
| $t_s/a$ | $t_s$[fm] | $n_{src}$ |
| 8 | 0.64 | 1 |
| 10 | 0.80 | 2 |
| 12 | 0.96 | 5 |
| 14 | 1.12 | 10 |
| 16 | 1.28 | 32 |
| 18 | 1.44 | 112 |
| 20 | 1.60 | 128 |
| Nucleon 2pt | | 477 |

| cC211.060.80 | | |
|---|---|---|
| 400 configurations | | |
| $t_s/a$ | $t_s$[fm] | $n_{src}$ |
| 6 | 0.41 | 1 |
| 8 | 0.55 | 2 |
| 10 | 0.69 | 4 |
| 12 | 0.82 | 10 |
| 14 | 0.96 | 22 |
| 16 | 1.10 | 48 |
| 18 | 1.24 | 45 |
| 20 | 1.37 | 116 |
| 22 | 1.51 | 246 |
| Nucleon 2pt | | 650 |

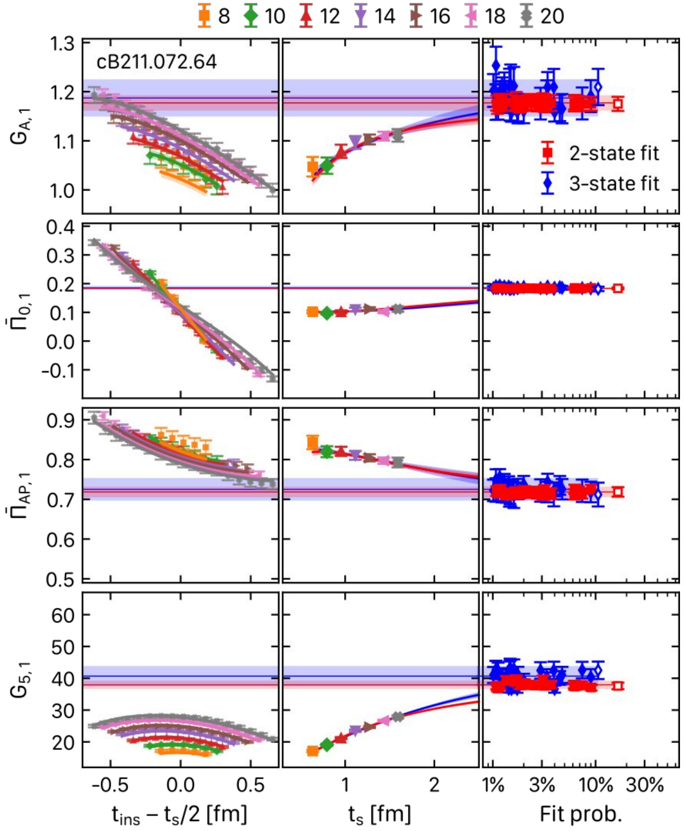
| cD211.054.96 | | |
|---|---|---|
| 500 configurations | | |
| $t_s/a$ | $t_s$[fm] | $n_{src}$ |
| 8 | 0.46 | 1 |
| 10 | 0.57 | 2 |
| 12 | 0.68 | 4 |
| 14 | 0.80 | 8 |
| 16 | 0.91 | 16 |
| 18 | 1.03 | 32 |
| 20 | 1.14 | 64 |
| 22 | 1.25 | 16 |
| 24 | 1.37 | 32 |
| 26 | 1.48 | 64 |
| Nucleon 2pt | | 480 |

*Up to 1.5fm for all ensembles*

Model average over thousands of fits: $\log(w_i) = -\frac{\chi_i^2}{2} + N_{\mathrm{dof},i}$

$p_i = \frac{w_i}{Z}$ with $Z = \sum_i w_i$. [E. T. Neil, J. W. Sitison, arXiv:2208.14983]

# … and at finite momentum transfer




$$\Pi_\mu(\Gamma_k;\vec{q}) = \frac{\mathcal{A}^{0,0}_\mu(\Gamma_k,\vec{q})}{\sqrt{c_0(\vec{0})c_0(\vec{q})}}$$

Three-point ground state

Two-point ground state

Combined fit of all three-point functions at the same $Q^2$

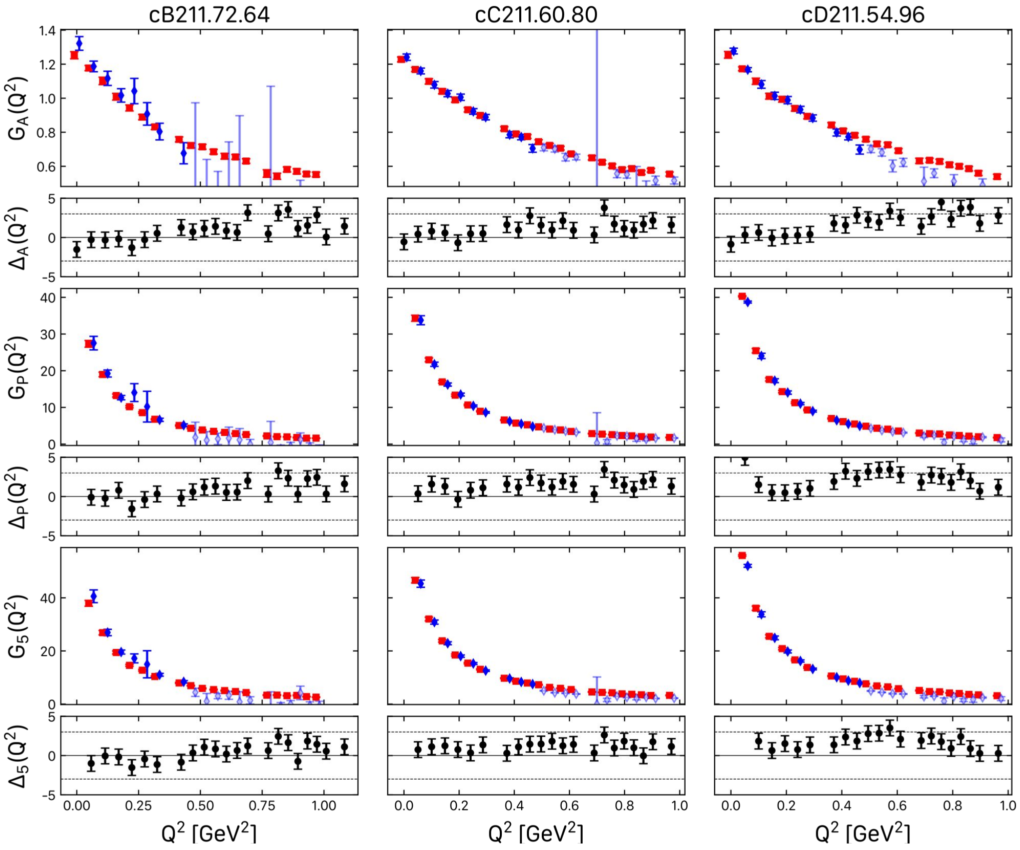
$$\Pi_i(\Gamma_k,\vec{q}) = \frac{i\mathcal{K}}{4m_N}\left[\frac{q_k q_i}{2m_N}G_P(Q^2) - \delta_{i,k}(m_N+E_N)G_A(Q^2)\right]$$

$$\Pi_0(\Gamma_k,\vec{q}) = -\frac{q_k\mathcal{K}}{2m_N}\left[G_A(Q^2) + \frac{(m_N-E_N)}{2m_N}G_P(Q^2)\right]$$

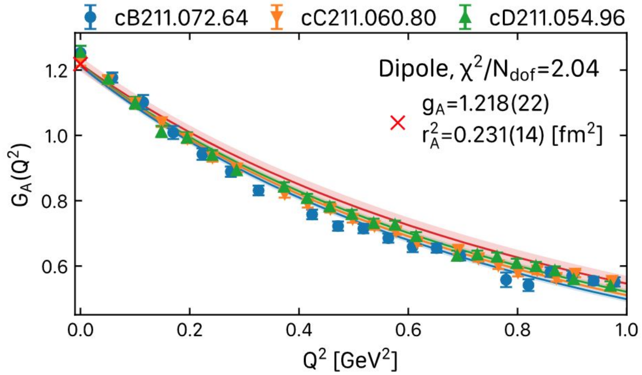
$$\Pi_5(\Gamma_k,\vec{q}) = -\frac{iq_k\mathcal{K}}{2m_N}G_5(Q^2)$$

Pseudoscalar FF

# Comparing two- and three-state fit FFs




- Most values compatible within 3σ
- Combined fit of $Q^2$ and $a^2$ dependence

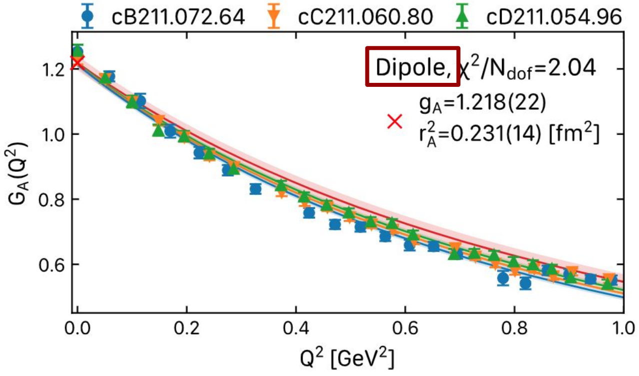


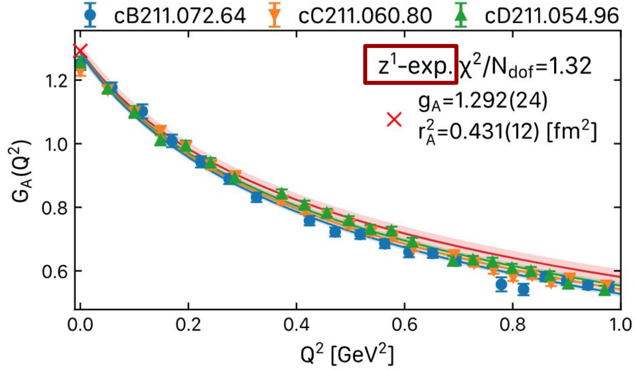

$$G(Q^2, a^2) = \frac{g(a^2)}{\left(1+\frac{Q^2}{12}r^2(a^2)\right)^2}$$

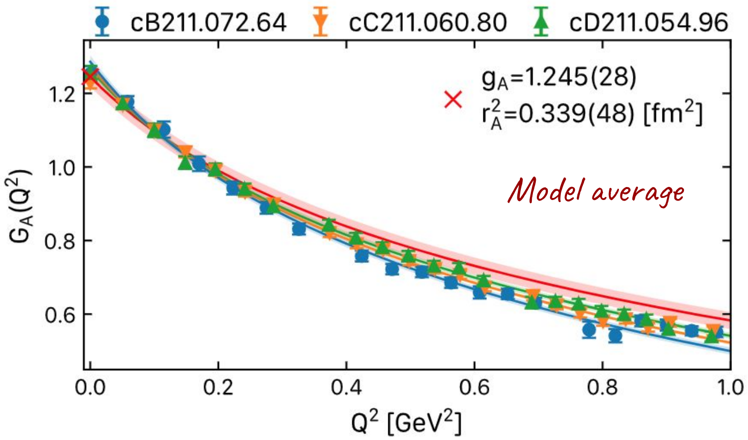
$$g(a^2) = g_0 + a^2 g_2$$

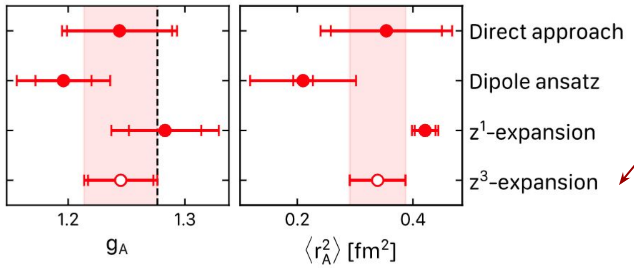
$$r^2(a^2) = r_0^2 + a^2 r_2^2$$

# Dipole vs z-expansion




*Compatible with the direct approach but smaller error because all information used*

# Pseudoscalar FF and operator relations

Axial FFs are commonly studied together with the pseudoscalar FF

$$\langle N(p',s')|P|N(p,s)\rangle = \bar{u}_N(p',s')\ G_5(Q^2)\,\gamma_5 u_N(p,s),$$

$$P^{\rm isov} = \bar{u}\gamma_5 u - \bar{d}\gamma_5 d$$

*Pseudoscalar FF*

Two important operator relations are

*i)* $\partial^\mu A_\mu = 2m_q P$ *ii)* $\partial^\mu A_\mu = F_\pi m_\pi^2 \psi_\pi$ $\Longrightarrow$ *iii)* $\psi_\pi = \frac{2m_q}{F_\pi m_\pi^2} P$

*i)* The axial Ward-Takahashi identity leads to the partial conservation of the axial-vector current (PCAC)

*ii)* The spontaneous breaking of chiral symmetry relates the axial-vector current to the pion field $\psi_\pi$

# The PCAC and PPD relations

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N}G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$\partial^\mu$

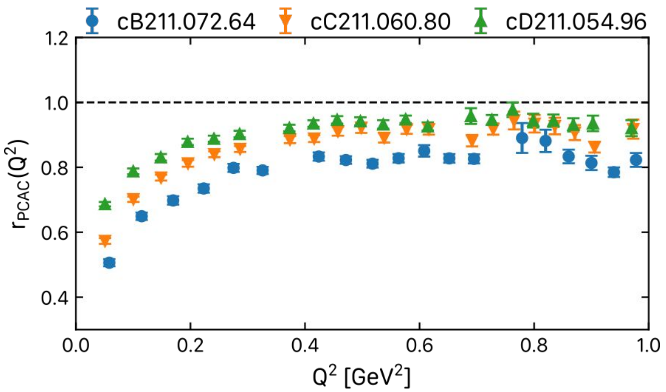
$$\langle N(p',s')|\partial^\mu A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[2m_N G_A(Q^2) - \frac{Q^2}{2m_N}G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$\langle N(p',s')|P|N(p,s)\rangle = \bar{u}_N(p',s')\;G_5(Q^2)\,\gamma_5 u_N(p,s)$$

i) $\partial^\mu A_\mu = 2m_q P$

**PCAC**

$$\boxed{G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)}$$




$$r_{\mathrm{PCAC}}(Q^2) = \frac{\frac{m_q}{m_N}G_5(Q^2) + \frac{Q^2}{4m_N^2}G_P(Q^2)}{G_A(Q^2)}$$

# The PCAC and PPD relations

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N} G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$\partial^\mu$

$$\langle N(p',s')|\partial^\mu A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[2m_N G_A(Q^2) - \frac{Q^2}{2m_N} G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$\langle N(p',s')|P|N(p,s)\rangle = \bar{u}_N(p',s')\; G_5(Q^2)\,\gamma_5 u_N(p,s)$$

i) $\partial^\mu A_\mu = 2m_q P$

**PCAC**

$$\boxed{G_A(Q^2) - \frac{Q^2}{4m_N^2} G_P(Q^2) = \frac{m_q}{m_N} G_5(Q^2)}$$

iii) $\psi_\pi = \frac{2m_q}{F_\pi m_\pi^2} P$

$$\langle N(p',s')|\psi_\pi|N(p,s)\rangle = \bar{u}_N(p',s')\; \underbrace{(m_\pi^2 + Q^2)}_{\text{Pole at } Q^2 = -m_\pi^2}{}^{-1} G_{\pi NN}(Q^2)\,\gamma_5 u_N(p,s)$$

**PPD**

$$\boxed{m_q G_5(Q^2) = \frac{F_\pi m_\pi^2}{m_\pi^2 + Q^2} G_{\pi NN}(Q^2)}$$

PPD = Pion-pole dominance

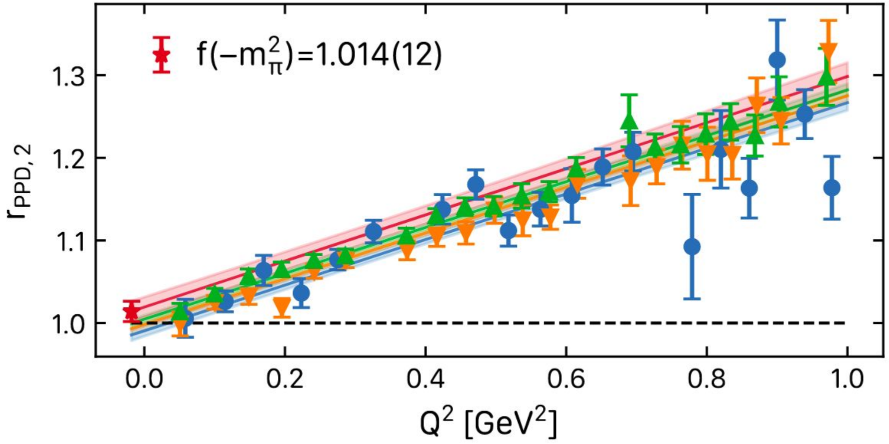
# The Goldberger-Treiman relation

$$m_q G_5(Q^2) = \frac{F_\pi m_\pi^2}{m_\pi^2+Q^2} G_{\pi NN}(Q^2)$$

$$G_A(Q^2) - \frac{Q^2}{4m_N^2} G_P(Q^2) = \frac{F_\pi m_\pi^2}{m_N(m_\pi^2+Q^2)} G_{\pi NN}(Q^2)$$

$$\lim_{Q^2\to -m_\pi^2} (Q^2+m_\pi^2) m_q G_5(Q^2) = F_\pi m_\pi^2 g_{\pi NN}$$

$$\lim_{Q^2\to -m_\pi^2} (Q^2+m_\pi^2) G_P(Q^2) = 4 m_N F_\pi g_{\pi NN}$$



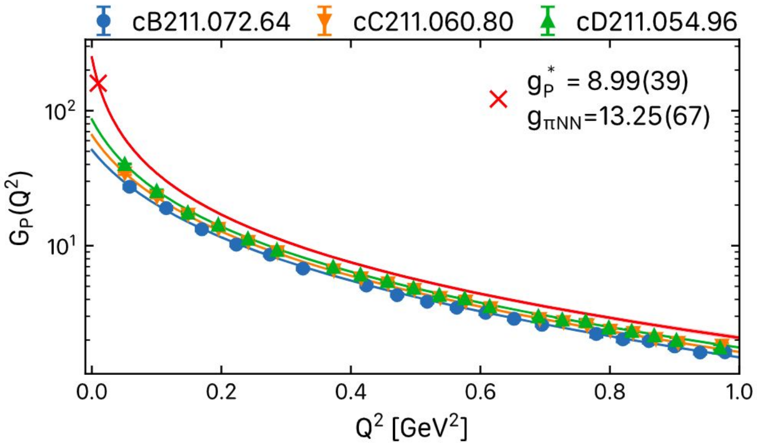

with $g_{\pi NN} = G_{\pi NN}(-m_\pi^2)$

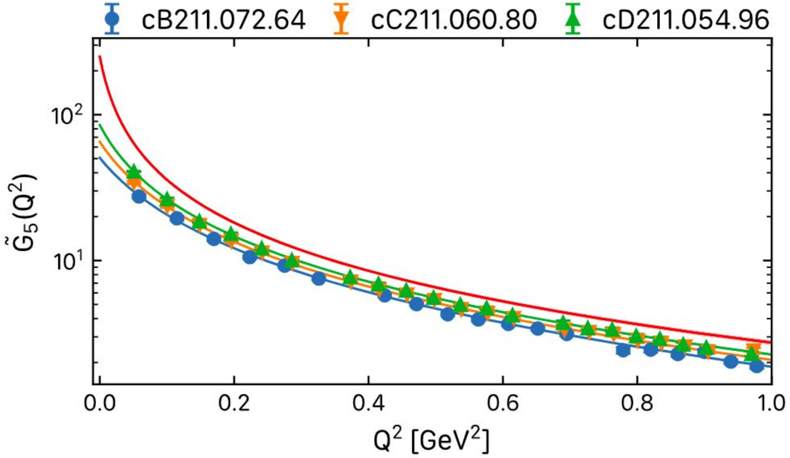
$$r_{\mathrm{PPD},2}(Q^2) = \frac{4m_N}{m_\pi^2}\frac{m_q G_5(Q^2)}{G_P(Q^2)}$$

The slope is connected to the GT discrepancy

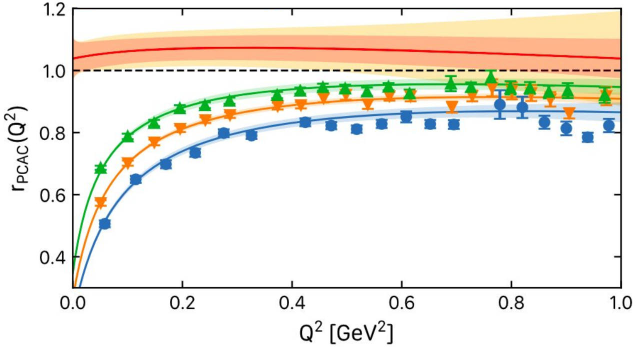
$$\Delta_{\mathrm{GT}} = 1 - \frac{g_A m_N}{g_{\pi NN} F_\pi} = 2.13(38)\% \approx 2\%$$

# Continuum limit of (induced) pseudoscalar FFs

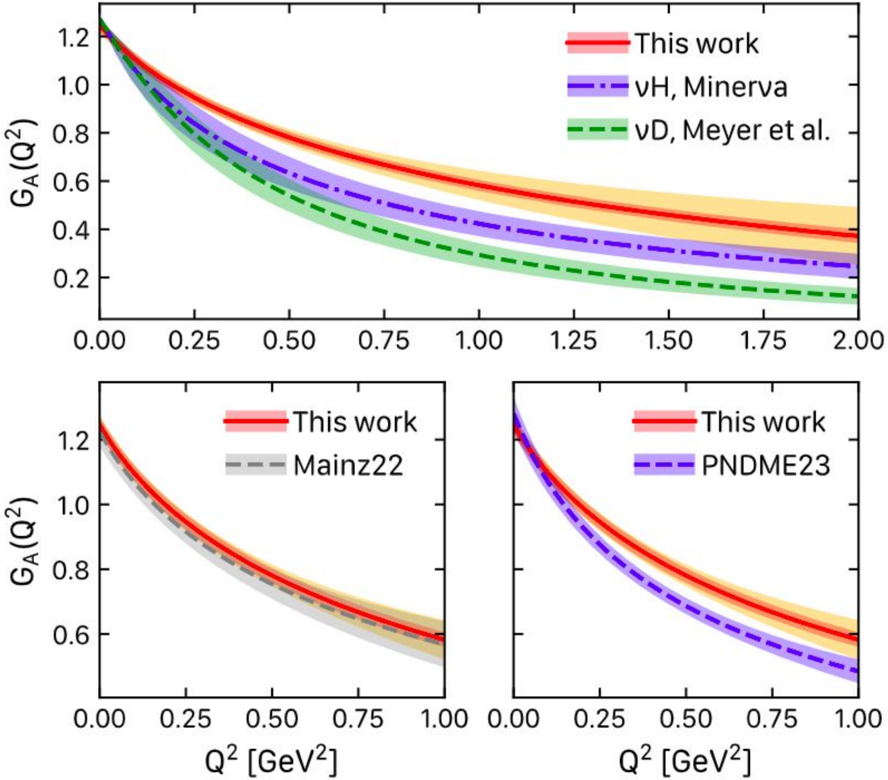


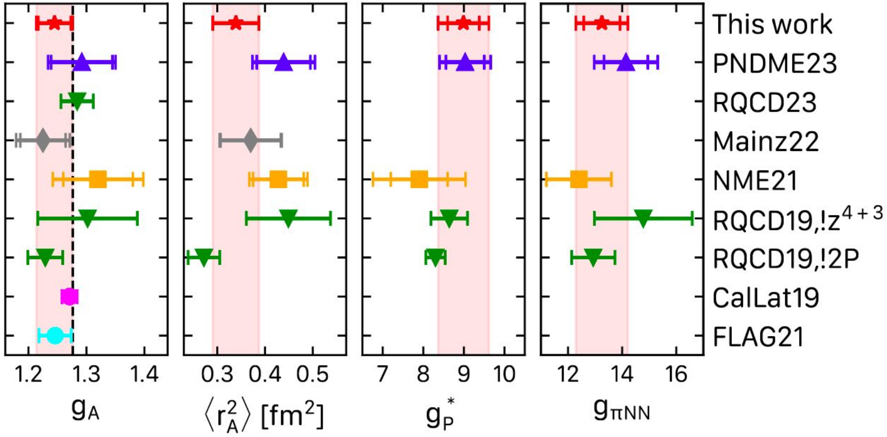

- We observe significant cut-off effects on both form factors
- Combined fit of both form factor with common pole
- The PCAC relation is satisfied at the continuum limit



# Conclusions




- Overall good agreement between recent lattice results and better agreement with the very recent results from Minerva



# Thank you for you attention!

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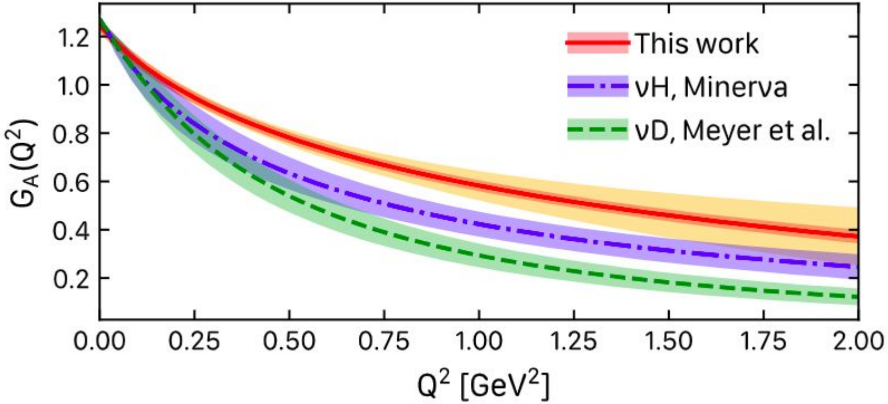


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We compute the nucleon axial and pseudoscalar form factors using three $N_f = 2+1+1$ twisted mass fermion ensembles with all quark masses tuned to approximately their physical values. The values of the lattice spacings of these three physical point ensembles are 0.080 fm, 0.068 fm and 0.057 fm, and spatial sizes 5.1 fm, 5.44 fm, and 5.47 fm, respectively, yielding $m_\pi L > 3.6$. Convergence to the ground state matrix elements is assessed using multi-state fits. We study the momentum dependence of the three form factors and check the partially conserved axial-vector current (PCAC) hypothesis and the pion pole dominance (PPD). We show that in the continuum limit, the PCAC and PPD relations are satisfied. We also show that the Goldberger-Treimann relation is approximately fulfilled and determine the Goldberger-Treiman discrepancy. We find for the nucleon axial charge $g_A = 1.245(28)(14)$, for the axial radius $\langle r_A^2 \rangle = 0.339(48)(06)$ fm$^2$, for the pion-nucleon coupling constant $g_{\pi NN} \equiv \lim_{Q^2 \to -m_\pi^2} G_{\pi NN}(Q^2) = 13.25(67)(69)$ and for $G_P(0.88 m_\mu^2) \equiv g_P^* = 8.99(39)(49)$.

# Backup slide - OS pion pole

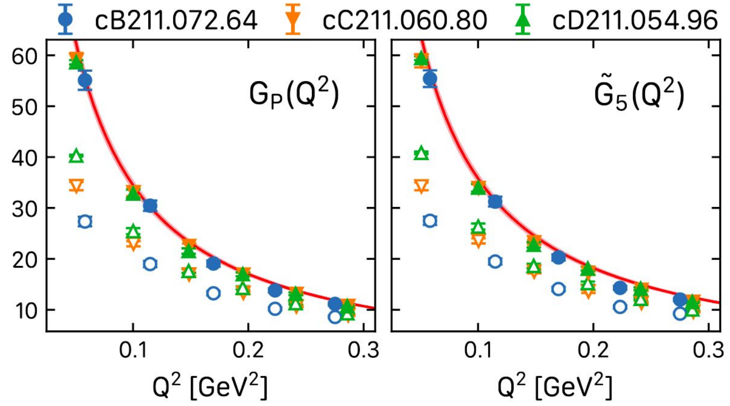
$$\langle N(p',s')|P|N(p,s)\rangle \qquad P^{\text{isov}} = \bar{u}\gamma_5 u - \bar{d}\gamma_5 d$$

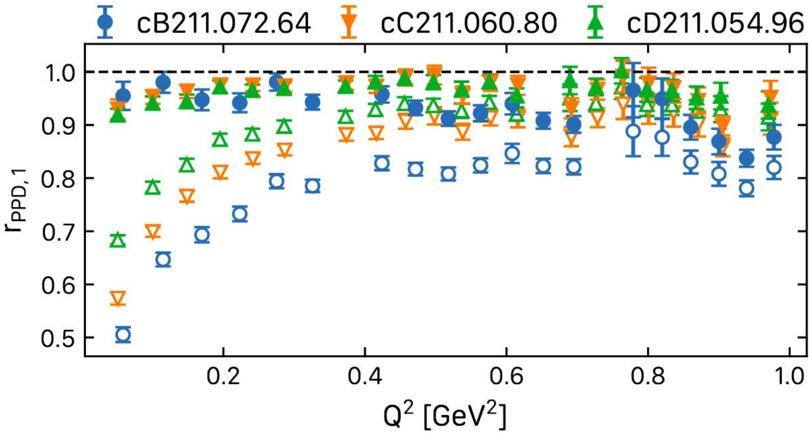
Connected neutral pion

Matrix elements couples to the Osterwalder-Seiler (OS) pion

Charged pion

| Ensemble | $m_\pi^{\text{pole}}$ [MeV] | $m_\pi^{\text{TM}}$ [MeV] | $m_\pi^{\text{OS}}$ [MeV] |
|---|---|---|---|
| cB211.72.64 | 299.3(4.5) | 140.2(2) | 297.5(7) |
| cC211.60.80 | 266.7(3.2) | 136.6(2) | 248.9(5) |
| cD211.54.96 | 235.8(4.8) | 140.8(3) | 210.0(4) |




$$G_{\text{improved}}(Q^2,a^2) = \frac{Q^2 + m^2_{\pi,\text{OS}}}{Q^2 + m^2_{\pi,\text{TM}}} G_{\text{w pole}}(Q^2,a^2)$$