Nucleon Axial, Scalar, and Tensor Charges From Lattice QCD Simulations at the Physical Point

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RESEARCH·TECHNOLOGY·INNOVATION

EINN2023

Nucleon Structure with Physical Point Ensembles

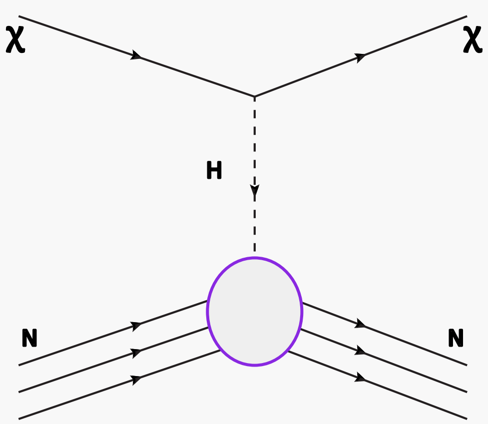
Outline

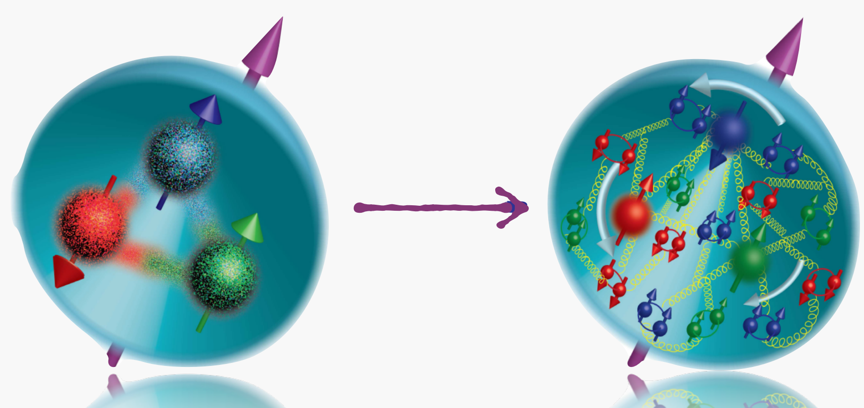
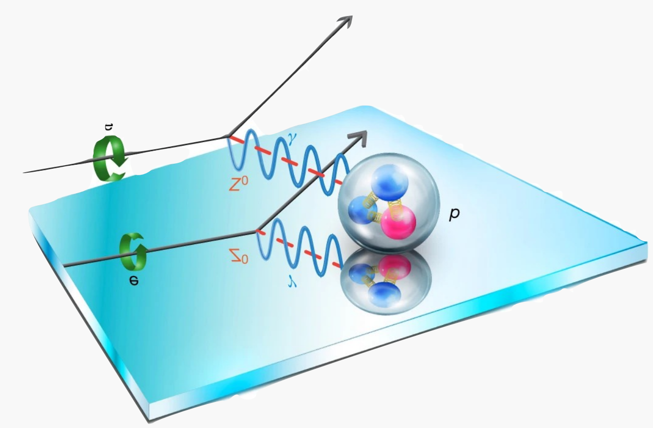
- Matrix elements on the lattice
- Lattice methods
- $N_f=2+1+1$ ensembles at physical point (twisted mass + clover)
- Statistics and uncertainties
- Results

Extended Twisted Mass Collaboration

Nucleon Matrix Elements

- Scalar and tensor charges ➜ novel interactions/dark matter searches
- Axial matrix elements ➜ origin of nucleon spin
- σ-terms ➜ mass decomposition of nucleon
- Electromagnetic form factors ➜ radii and moments well known experimentally
- Axial form factors ➜ PCAC and pion pole dominance relations
- Strange form factors ➜ connect to weak charges and constraints on new physics
- Momentum fraction, moments of PDFs and GPDs, …



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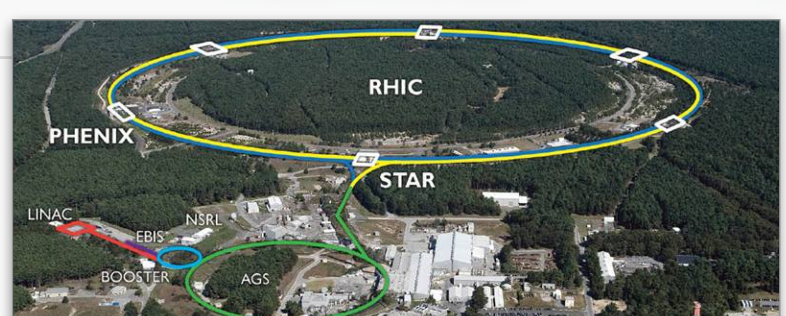

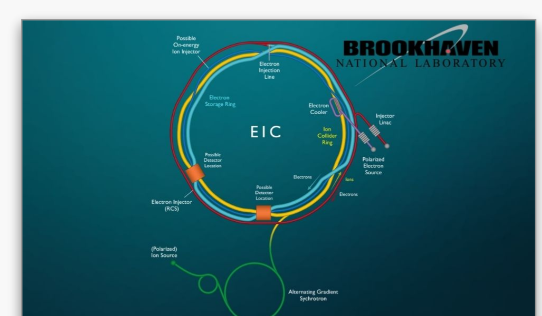

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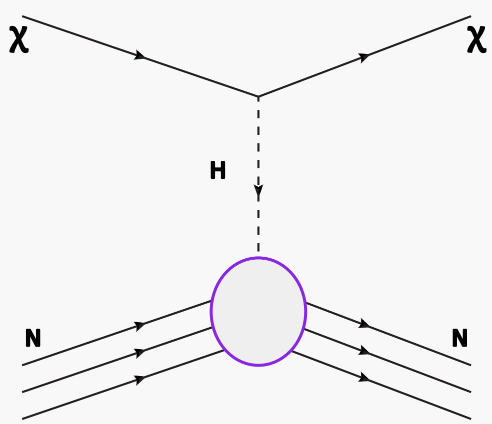

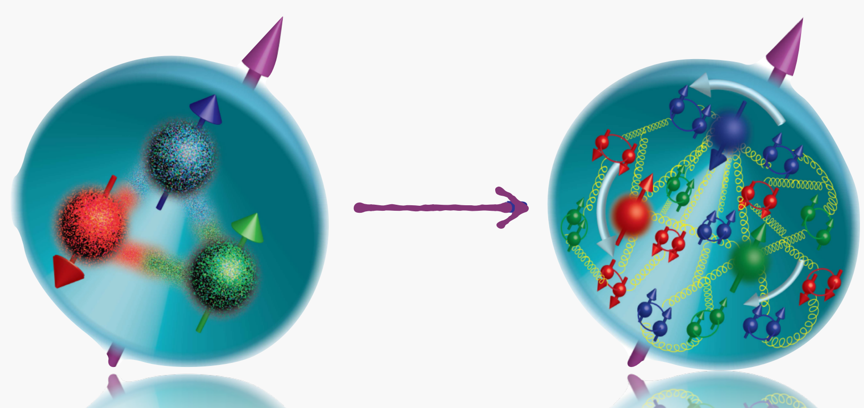

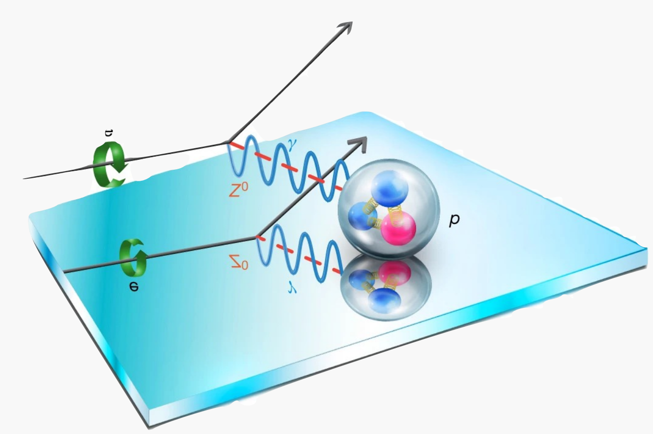

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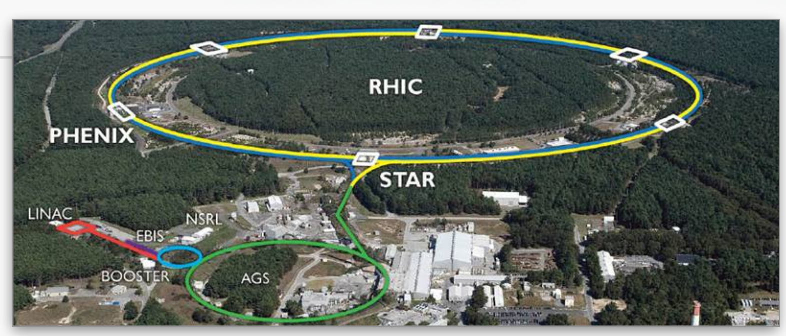
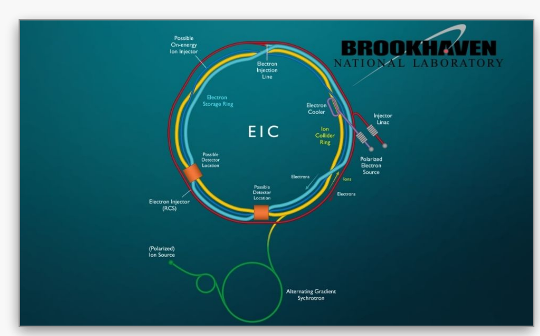




Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{QCD}[U]}$$

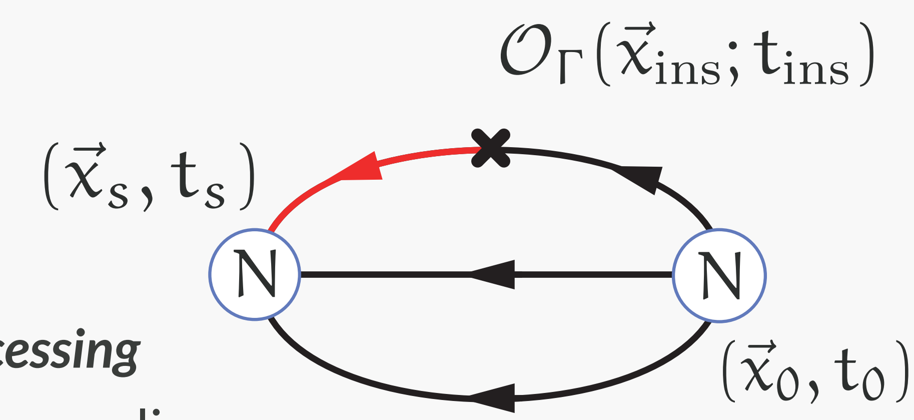
Simulation

- Markov chain Monte Carlo to generate ensembles of gluon-field configurations $\{U\}$

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{QCD}[U]}$$

Analysis

- Construction of hadron correlation functions on background field configurations

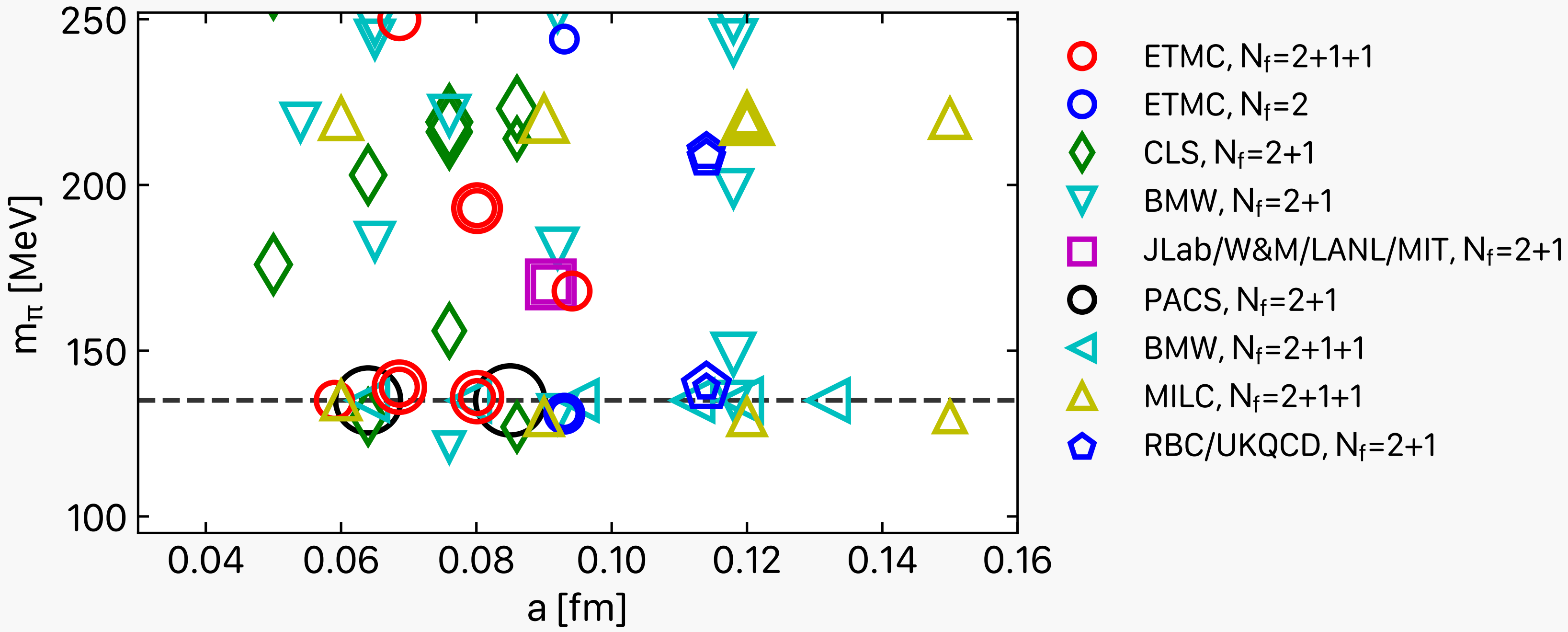


Data analysis – post-processing

- Statistical analysis, resampling
- Statistical and stochastic errors
- Continuum and infinte volume extrapolation

Ensembles

Landscape of ensembles used for nucleon structure



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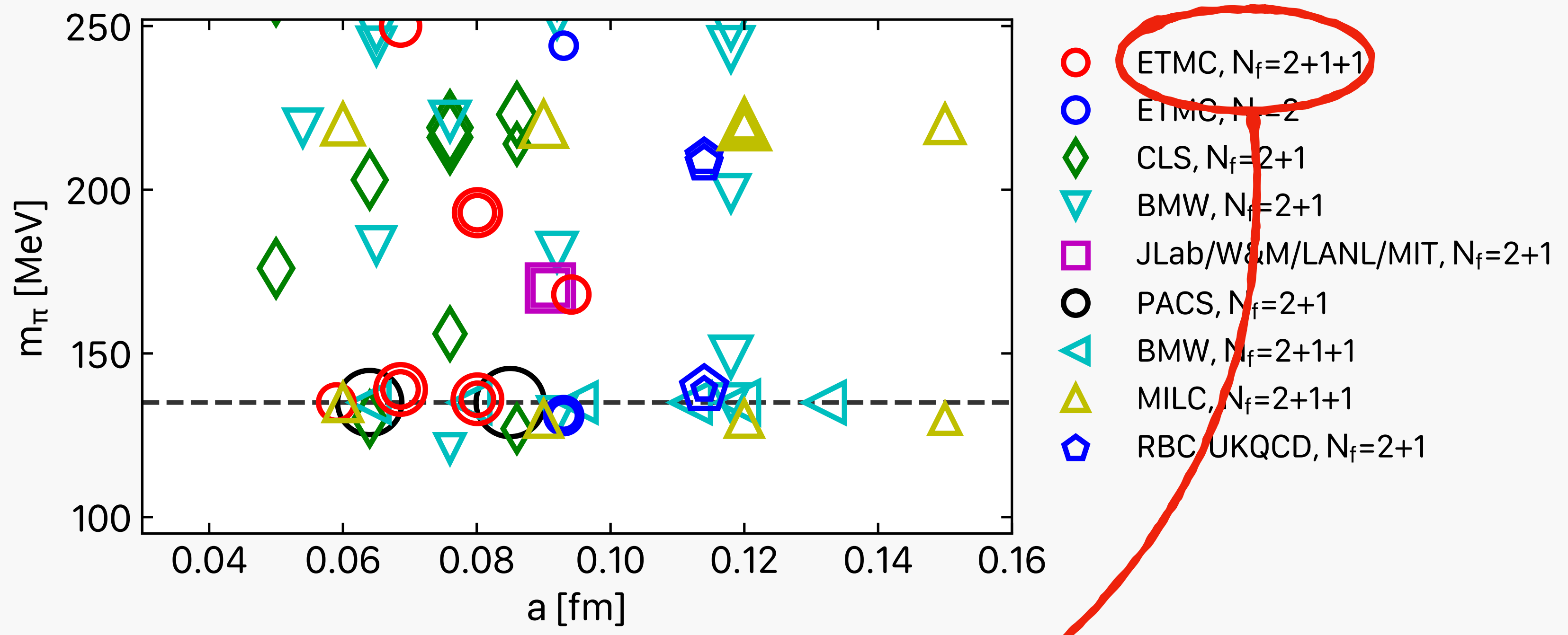


ETMC: three $N_f=2+1+1$ ensembles at physical pion mass

Ens. ID (abbrv.)	Vol.	a [fm]
cB211.072.64 (cB64)	64×128	0.080
cC211.060.80 (cC80)	80×160	0.068
cD211.054.96 (cD96)	96×192	0.057

- Three lattice spacings at physical point
- Ongoing generation of finer ensembles and larger volumes
- ***This talk:*** 3 ensembles with:

$$a = 0.057 - 0.068 \text{ fm}$$

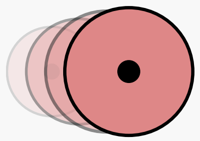
Local matrix elements

$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}_N^\beta(\vec{x}_s;t_s)|\mathcal{O}^\Gamma(\vec{x}_{ins};t_{ins})|\chi_N^\alpha(\vec{0};0)\rangle$$

Unpolarised

$$\mathcal{O}_V^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

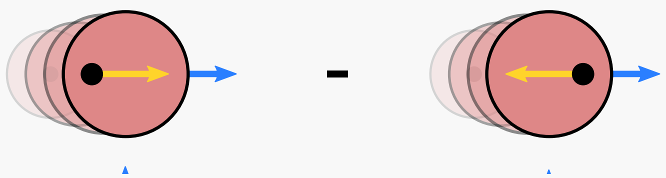
$$\langle 1\rangle_{u-d} = g_V\,,\quad \langle x\rangle_{u-d},\quad \ldots$$



Helicity

$$\mathcal{O}_A^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

$$\langle 1\rangle_{\Delta u-\Delta d} = g_A,\quad \langle x\rangle_{\Delta u-\Delta d},\quad \ldots$$



Transverse

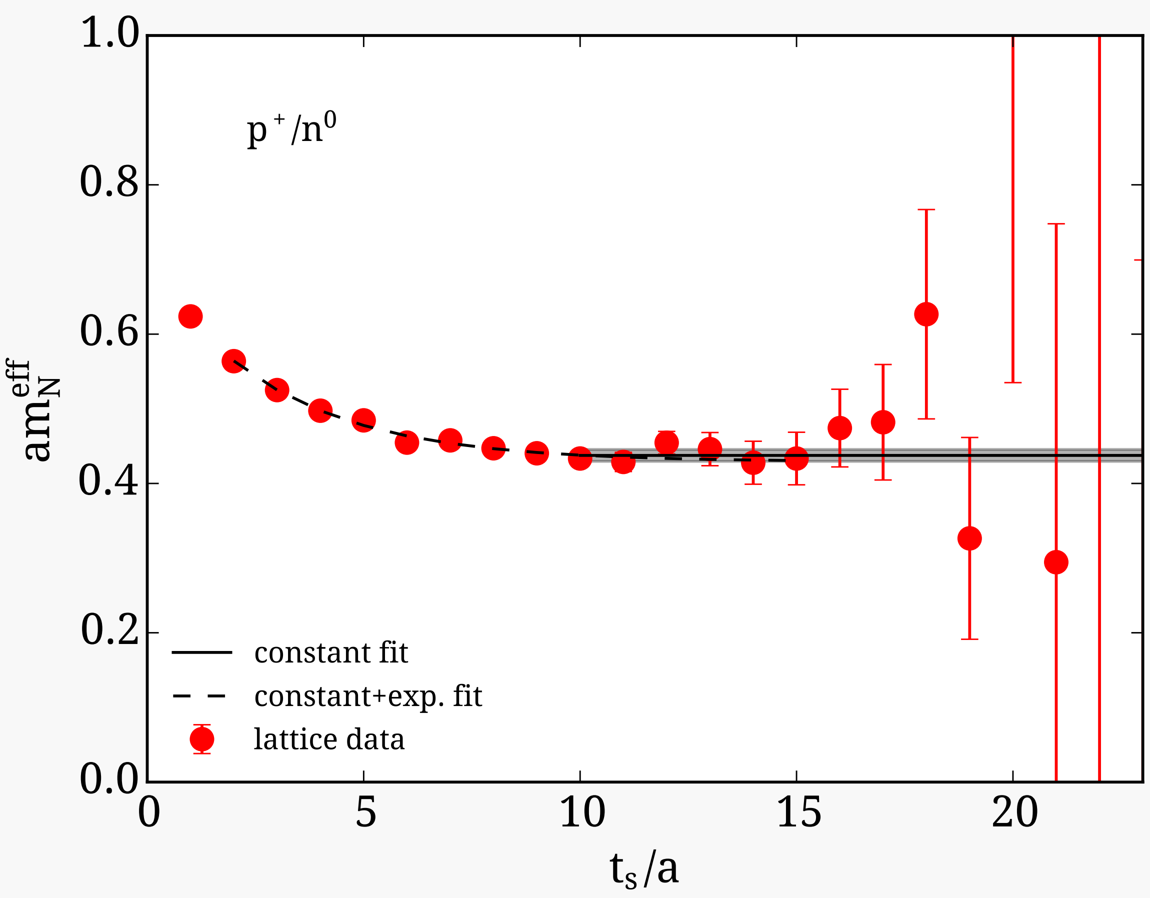
$$\mathcal{O}_T^{\nu\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

$$\langle 1\rangle_{\delta u-\delta d} = g_T,\quad \langle x\rangle_{\delta u-\delta d},\quad \ldots$$

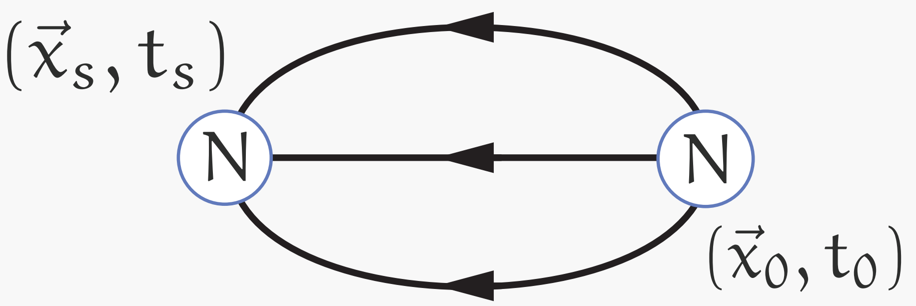
Nucleon structure on the lattice

Two-point correlation functions

- **Statistical error:** N^−½ with Monte Carlo samples
- Correlation functions exponentially decay with time-separation
- Contamination from higher energy states



$$\sum_{\vec{x}_s} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(x_s) | \chi_N^{\alpha}(0) \rangle = c_0 e^{-E_0 t_s} + c_1 e^{-E_1 t_s} + \ldots$$



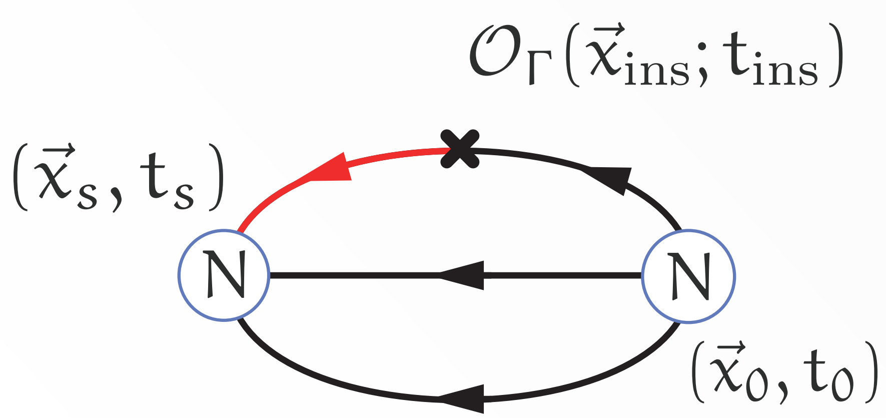
Matrix elements on the Lattice

General three-point function:

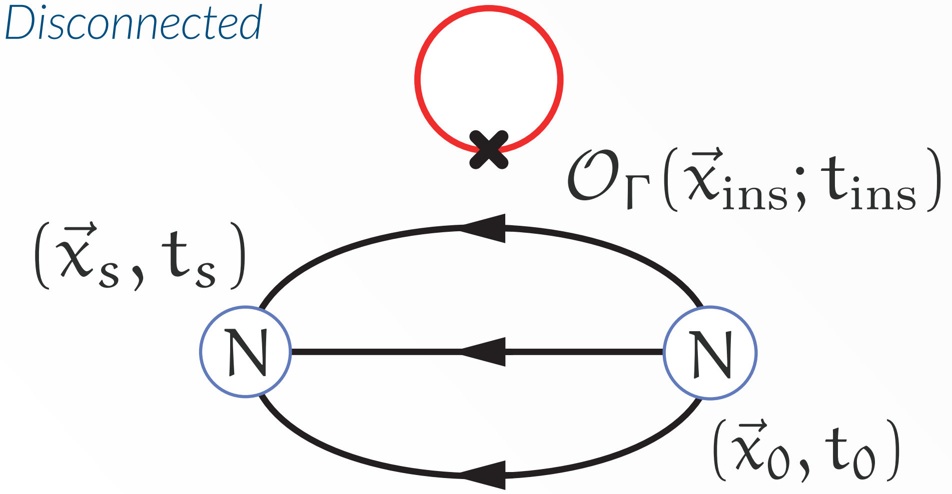
$$G_\Gamma(P;\vec{q};t_s,t_{ins}) = \sum_{\vec{x}_s\,\vec{x}_{ins}} e^{-i\vec{q}.\vec{x}_{ins}} P^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(\vec{x}_s;t_s)|\mathcal{O}^{\Gamma}(\vec{x}_{ins};t_{ins})|\chi_N^{\alpha}(\vec{0};0)\rangle$$

At quark level gives rise to both so-called connected and disconnected contributions

Connected



Disconnected



Matrix elements on the Lattice

General three-point function:

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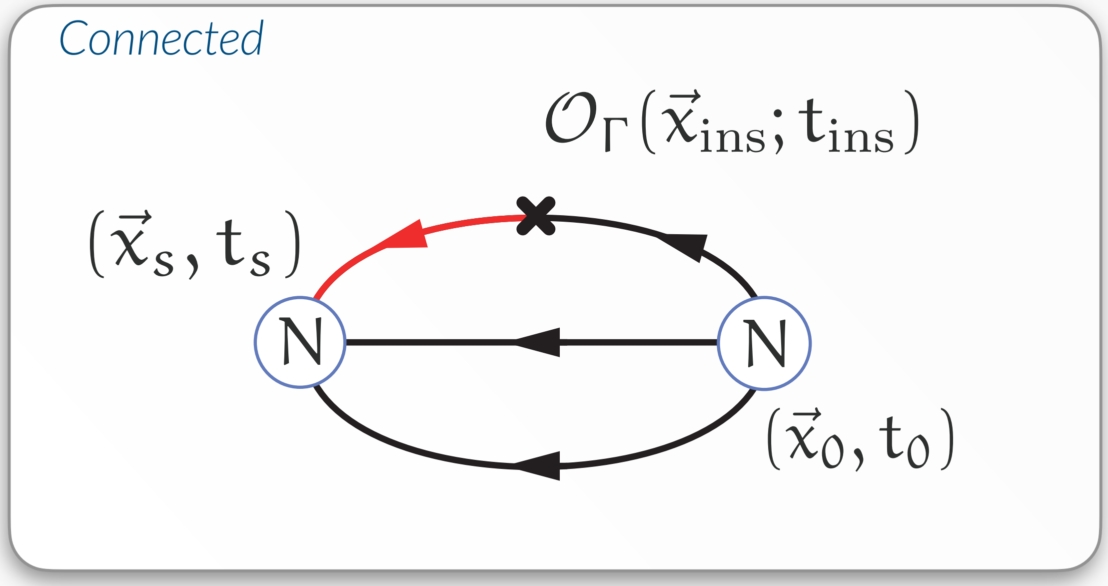
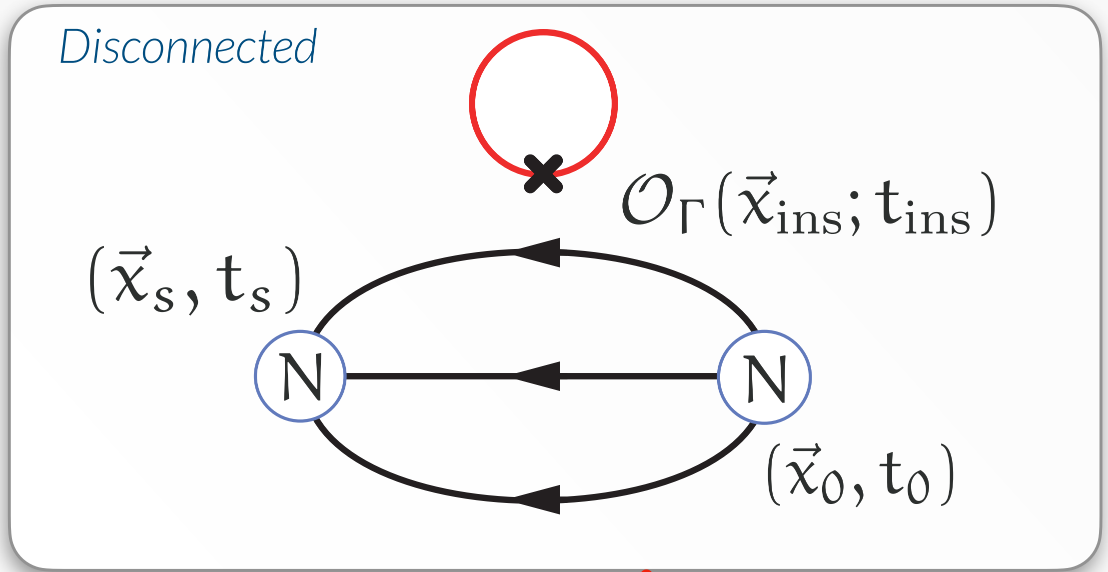
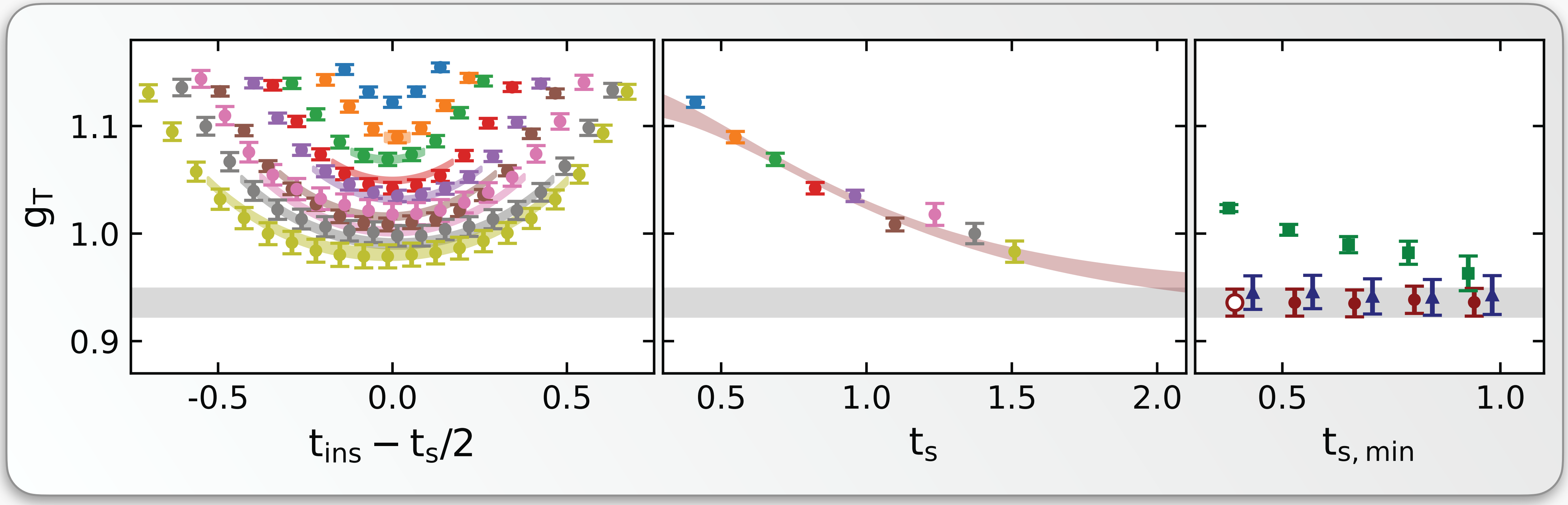




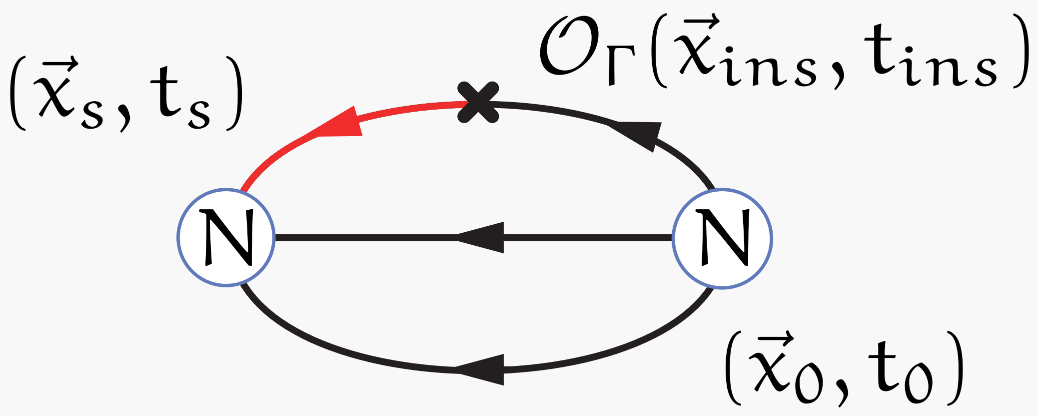
Stochastic evaluation of loop – stochastic error in addition to statistical

Treatment of excited states

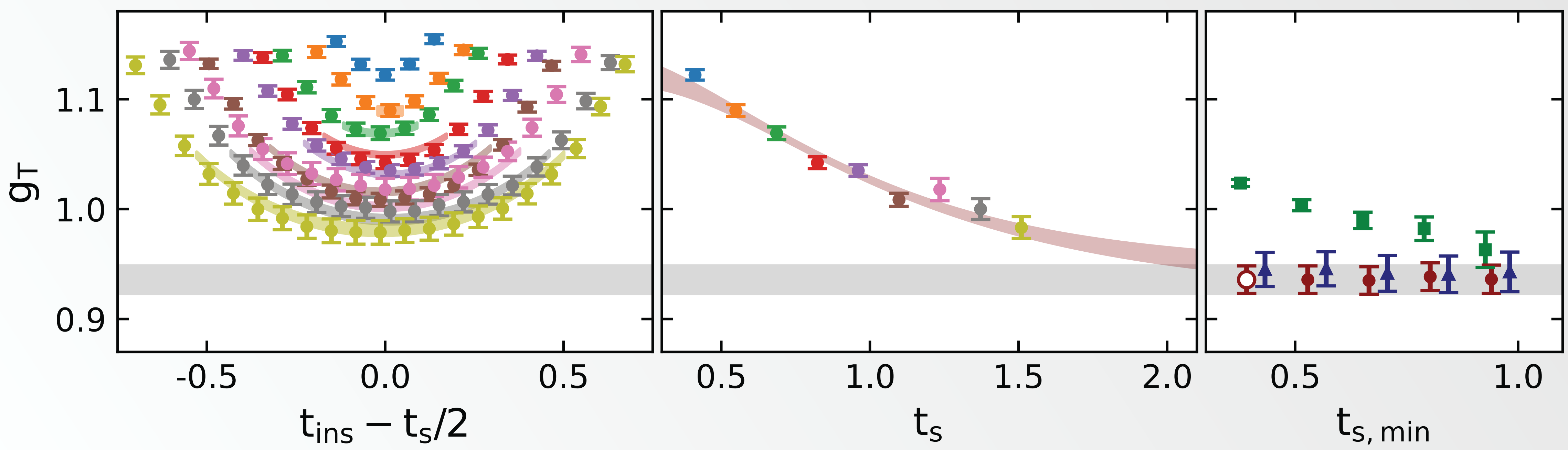


Example from intermediate a

- Isovector tensor charge (only connected)
- Increasing statistics with separation t_s
- Summation, two- and three-state fits



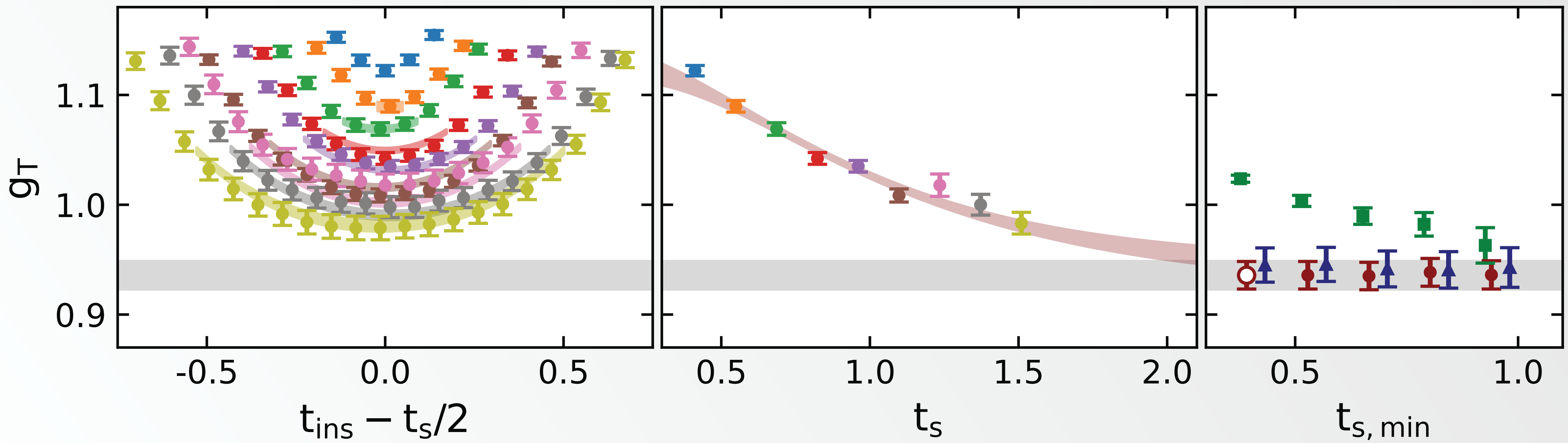
Treatment of excited states



Summation method

$$S_\Gamma(\vec{q}; t_s) = \sum_{t_{ins}=\tau}^{t_s-\tau} R_\Gamma(\vec{q}; t_s; t_{ins}) \rightarrow \mathcal{M} t_s + C$$

Treatment of excited states



Two-state fit

$$G_\Gamma(\vec{q}; t_s, t_{ins}) = \sum_{i=0}^{1} \sum_{j=0}^{1} c_{ij} e^{-E_i(0)(t_s - t_{ins})} e^{-E_j(\vec{q}) t_{ins}}$$

$$\mathcal{M} = \frac{c_{00}}{\sqrt{a_0(\vec{0}) a_0(\vec{q})}}$$

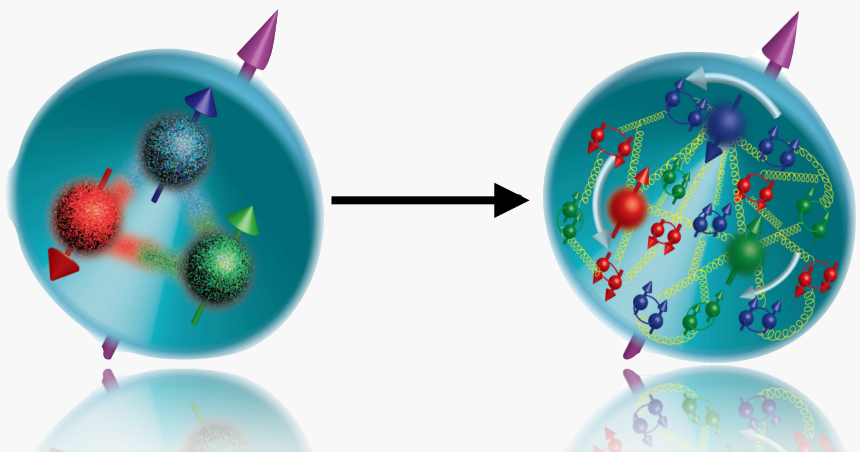
$$G(\vec{q}; t_s) = a_0(\vec{q}) e^{-\varepsilon_0(\vec{q}) t_s} + a_1(\vec{q}) e^{-\varepsilon_1(\vec{q}) t_s}$$

Nucleon axial charge

Matrix element of the axial current

Isovector case well known from β-decay: $\langle p|\bar{u}\gamma_5\gamma_k d|n\rangle$

Flavor–separated contributions to axial charge relate to quark intrinsic spin contributions to nucleon spin

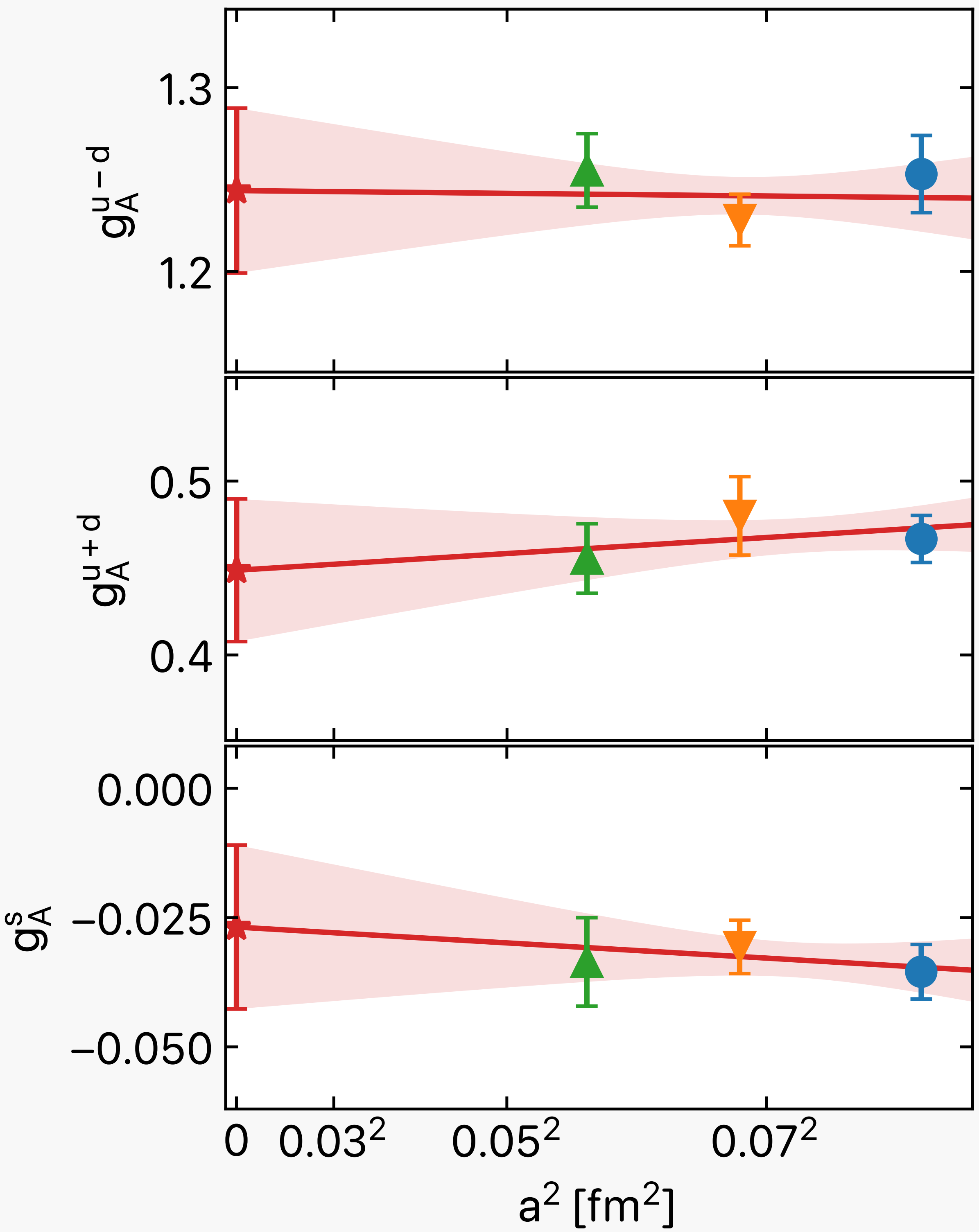


$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{q=u,d,s,\ldots} g_A^q$$

Quark intrinsic spin contributions to nucleon spin

- Need linear combination of isovector (u-d) and isoscalar (u+d) contributions for individual up- and down-quarks
- Strange quark contribution is sea-quark contribution only (disconnected diagrams)

Nucleon axial charge



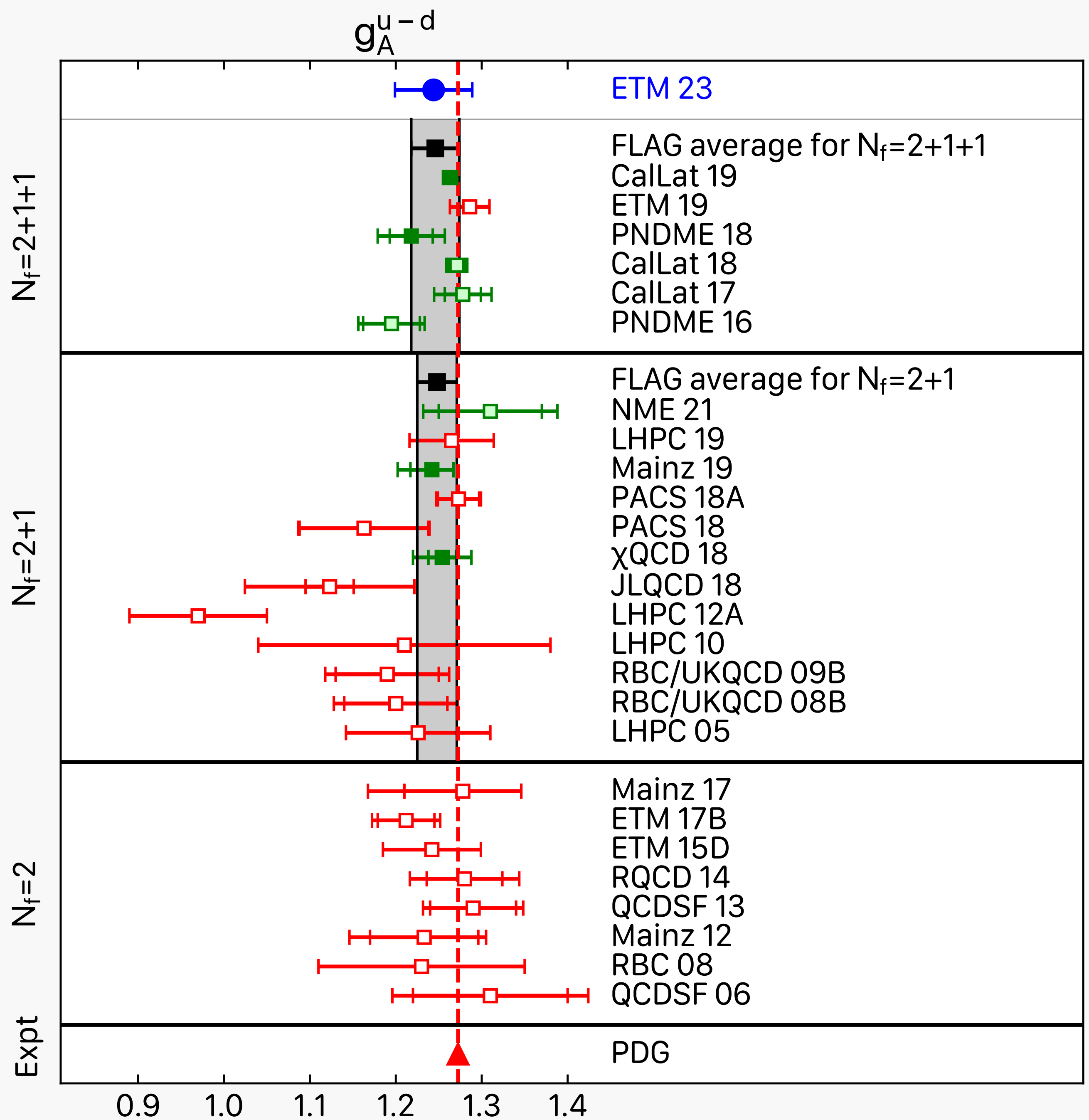
See `arXiv:2309.05774 [hep-lat]`; Includes isovector axial form-factors

- Errors for each ensemble include *statistical* and *systematic* due to excited state contamination
- Model averaged based on AIC (see e.g. `arXiv:2208.14983`)

Preliminary!

Complete analysis for flavour separated charges ongoing

Nucleon axial charge



Latest FLAG21 values

- ETM23 consistent with FLAG average
- Only result with three physical point ensembles
- Agreement for g_A means confidence for more challenging quantities
- E.g.
  - Scalar ME, σ-terms

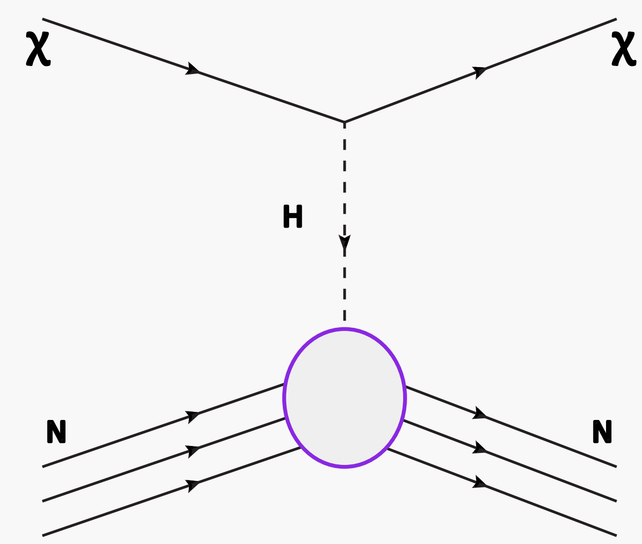
$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

  - Tensor ME

$$g_T = \langle 1\rangle_{\delta u - \delta d} \leftarrow \langle N|\bar{u}\sigma_{\mu\nu}u + \bar{d}\sigma_{\mu\nu}d|N\rangle$$

Scalar charge – σ-terms

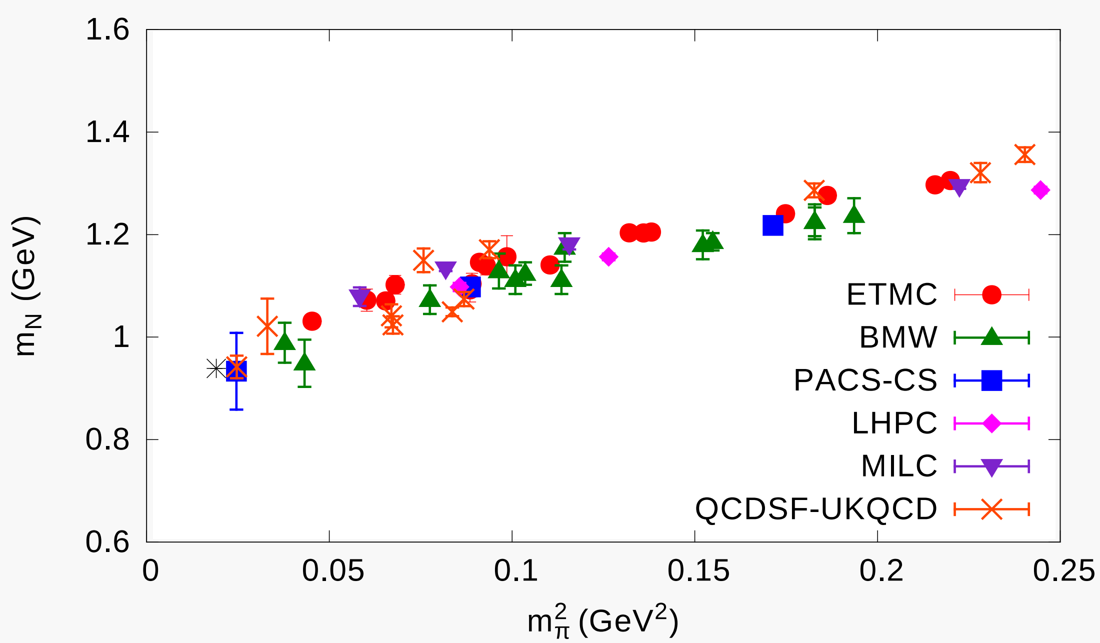
- Pion nucleon σ-term: $\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$
- Strange σ-term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
- Enter super-symmetric candidate particle scattering cross sections with nucleon (e.g. neutralino through Higgs)



1. Direct calculation of matrix elements

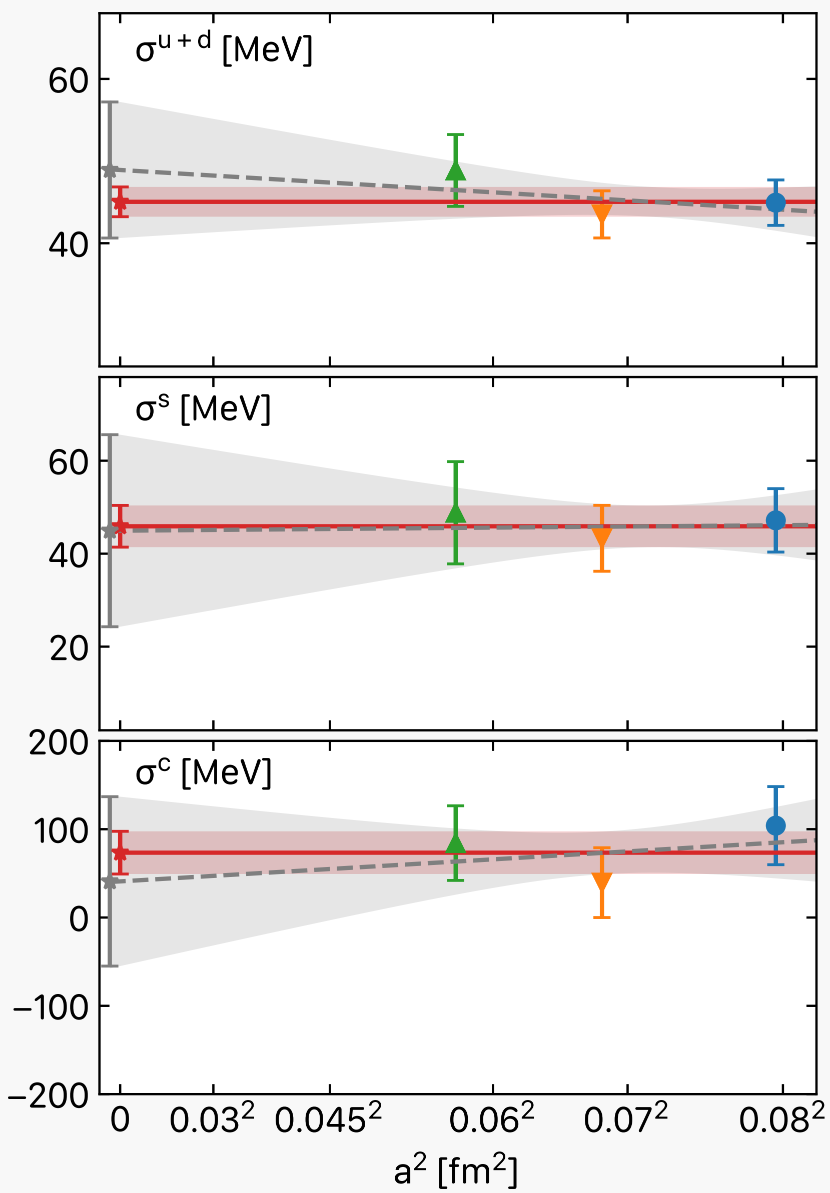
Involves disconnected contributions

2. Through Feynman - Hellmann theorem: $\sigma_{\pi N} = m_{ud}\frac{\partial m_N}{\partial m_{ud}}$ $\sigma_s = m_s\frac{\partial m_N}{\partial m_s}$

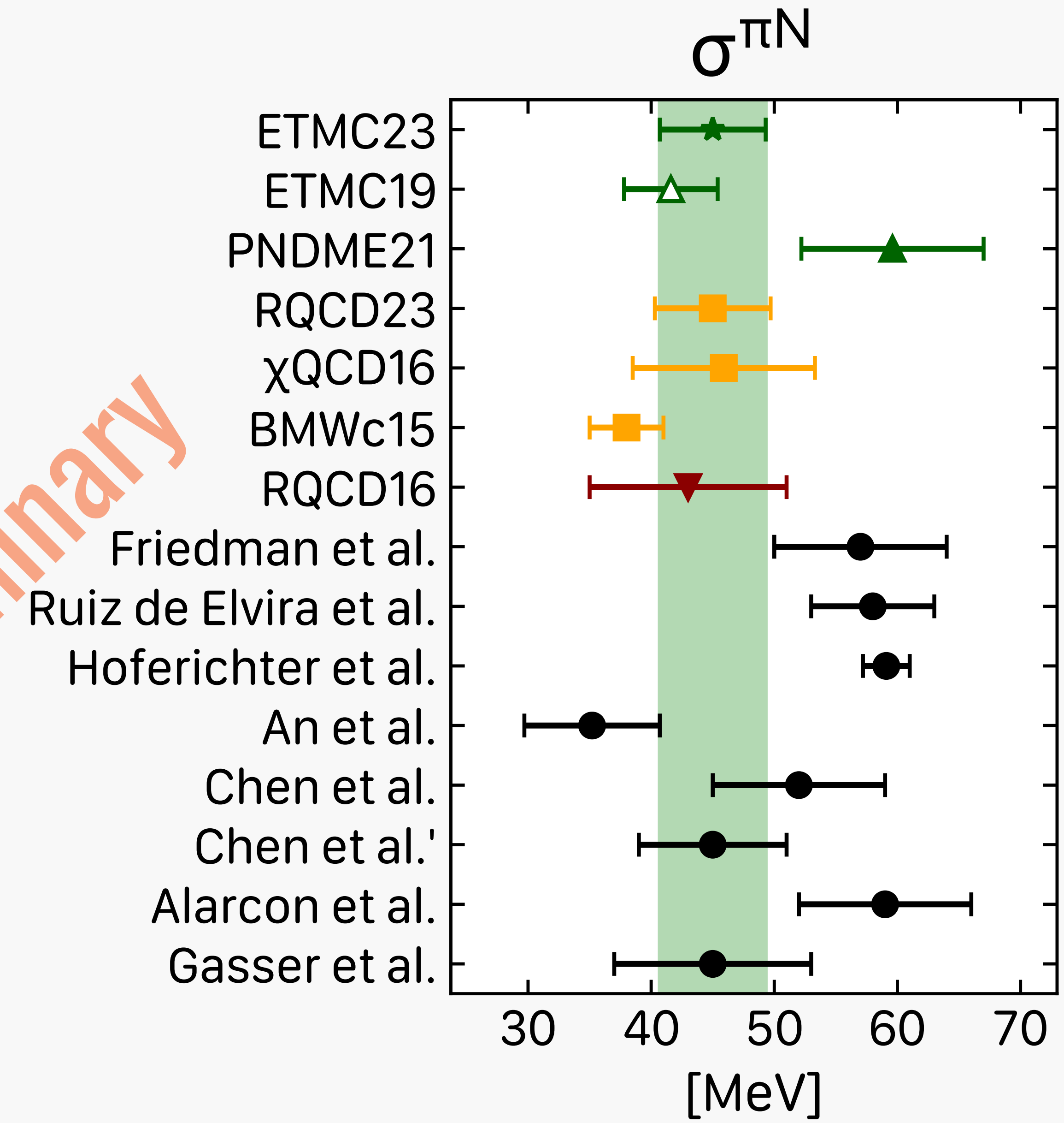


- Reliance on effective theories for dependence on m_π
- Weak dependence on m_s

Scalar charge – σ-terms

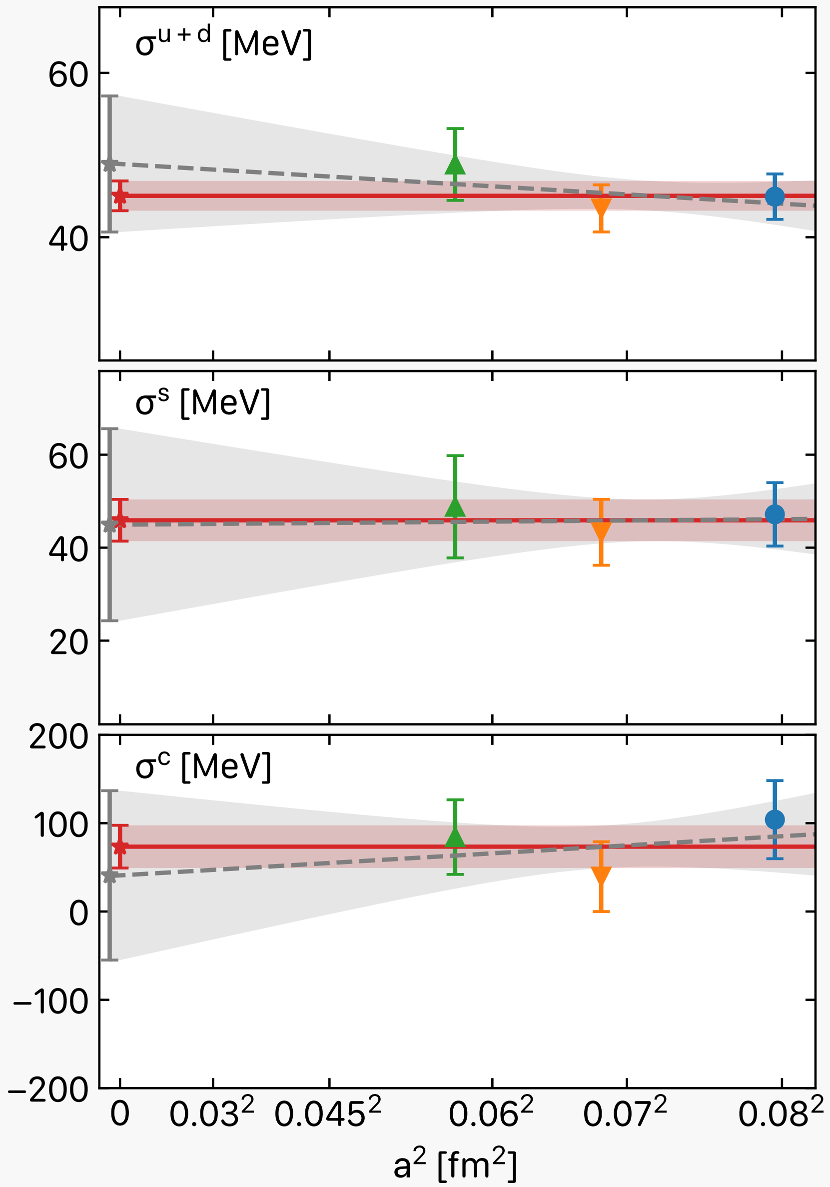


Preliminary



- One result using FH method (BMW)
- Los Alamos group: $59.6(7.4)\ MeV$, when explicitly including πN energy as prior

Scalar charge – σ-terms



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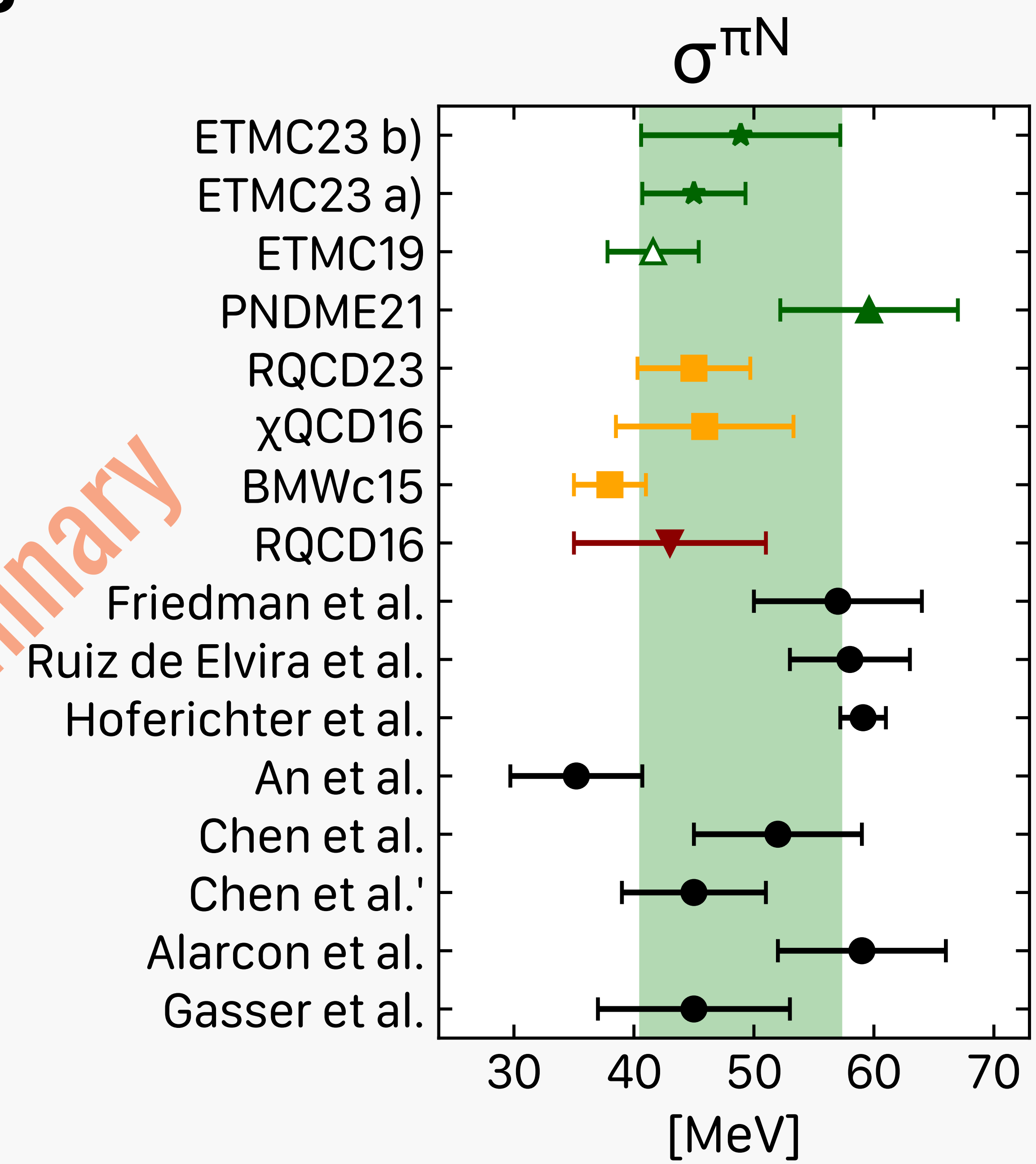
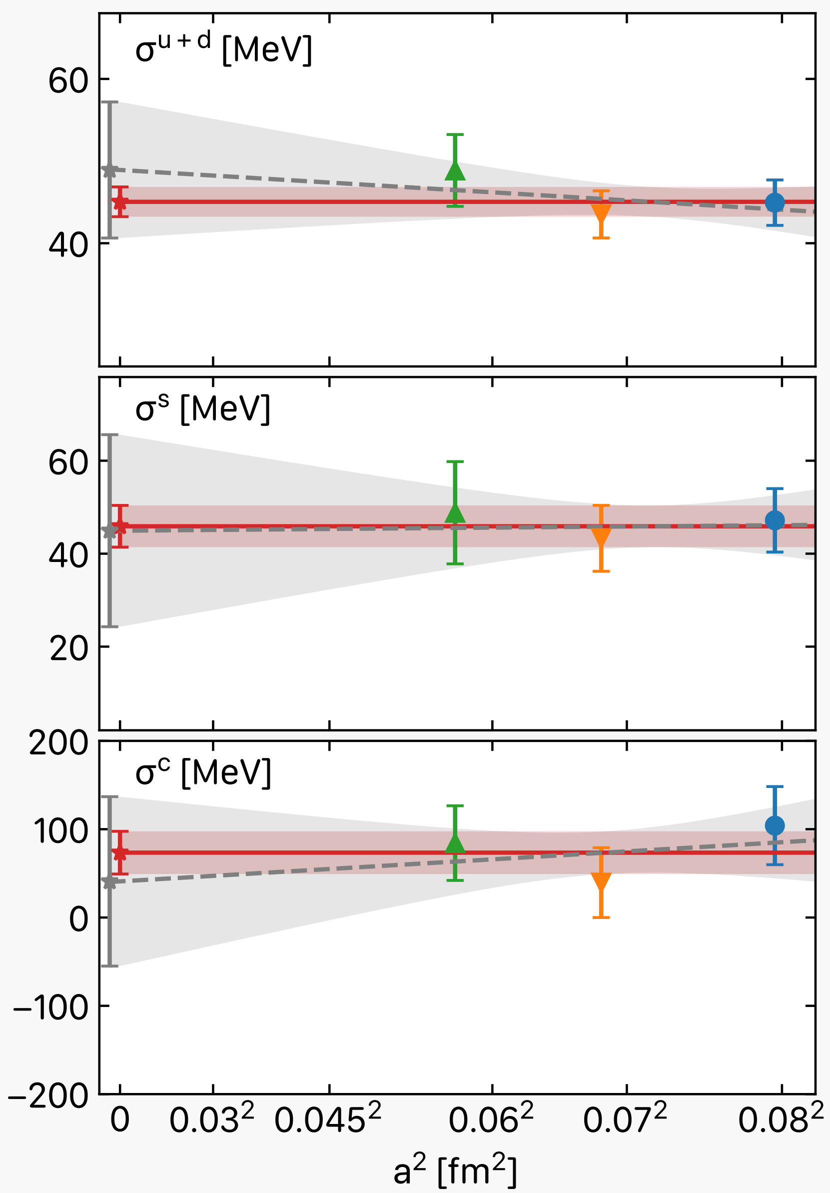
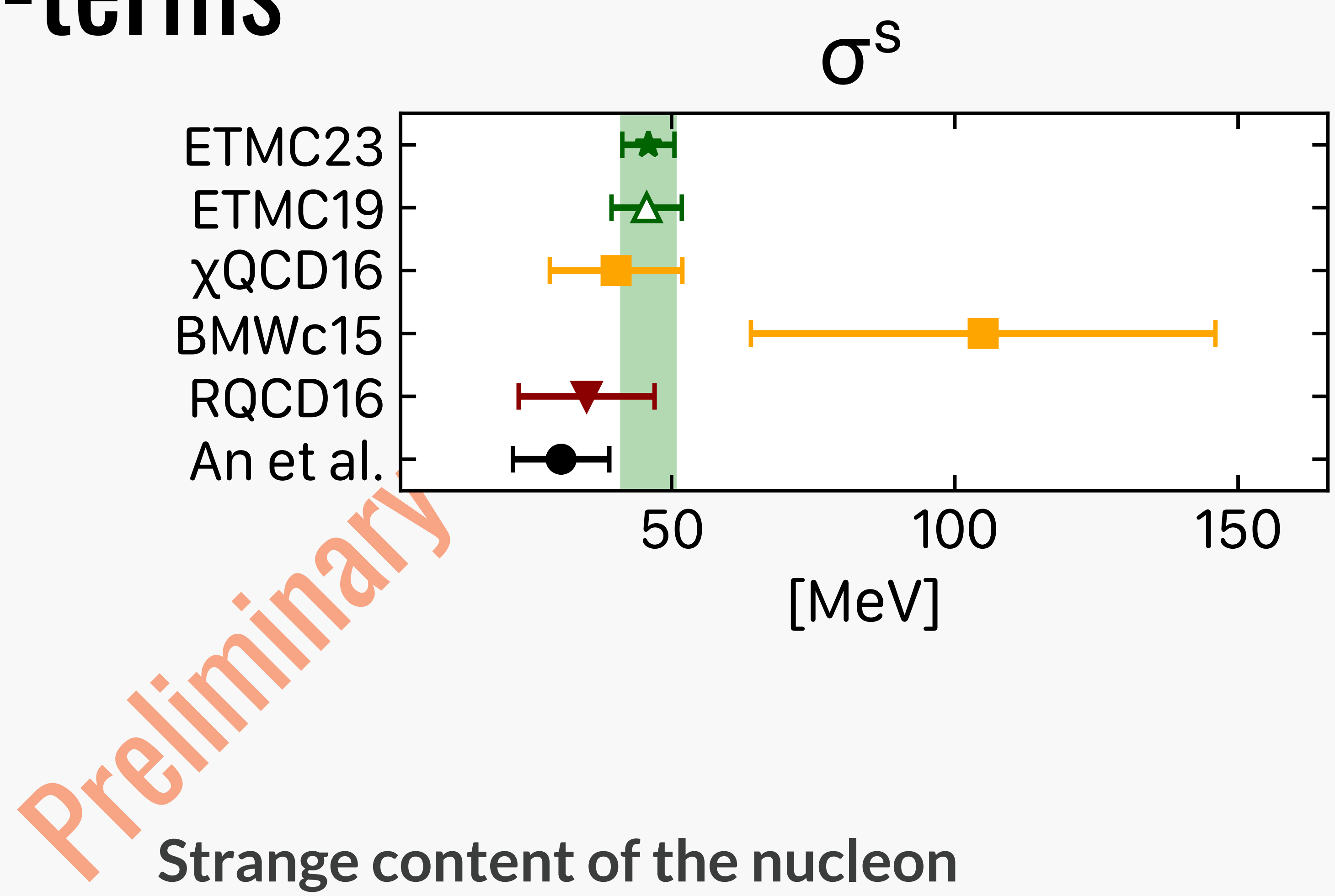


- One result using FH method (BMW)
- Los Alamos group: $59.6(7.4)\ MeV$, when explicitly including πN energy as prior
- Our result with linear continuum extrapolation (Grey band in left plot)

Scalar charge – σ-terms





Strange content of the nucleon

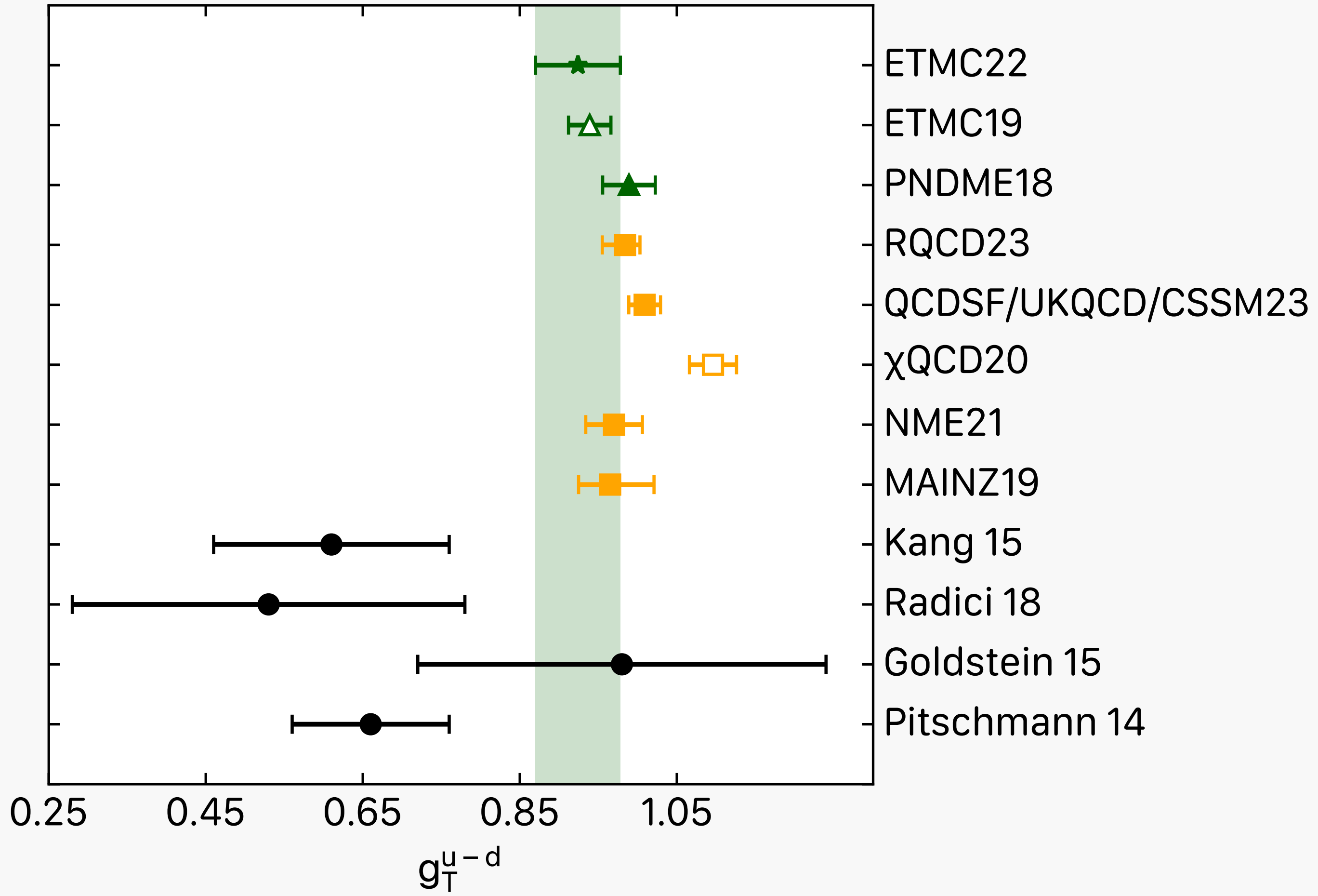
- Weaker dependence on lattice spacing
- Overall general agreement between lattice formulations

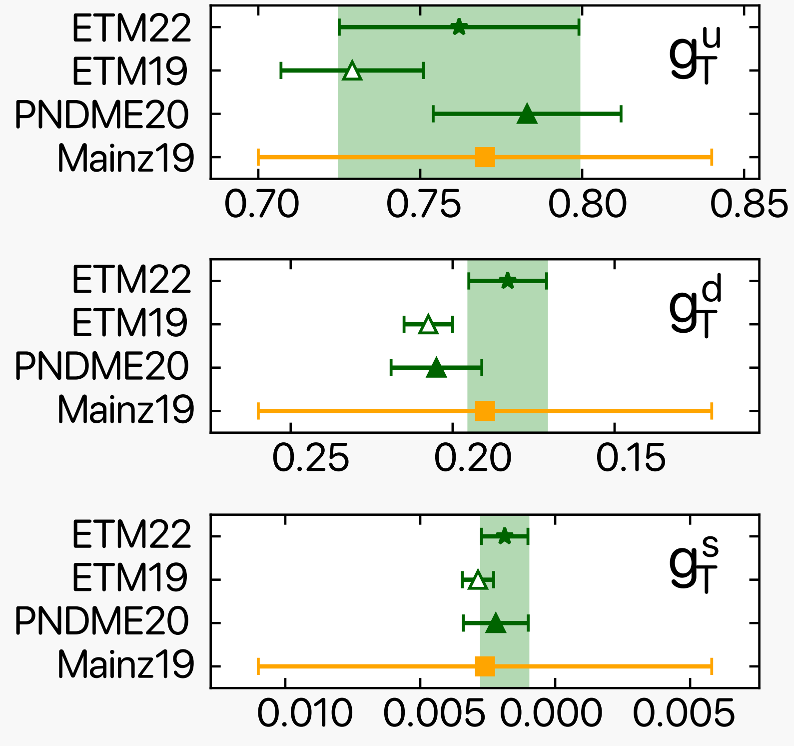
Tensor charge

- Tensor matrix element

$$g_T = \langle 1 \rangle_{\delta u - \delta d} \leftarrow \langle N | \bar{u}\sigma_{\mu\nu}u + \bar{d}\sigma_{\mu\nu}d | N \rangle$$

- Moment of transversity PDF; Can be used to constrain experimental analyses, e.g. JAM in `arXiv:2205.00999`





Preliminary

See: "First moments of the nucleon transverse quark spin densities using lattice QCD" `arXiv:2202.09871`

Summary & Outlook

- Lattice QCD with physical point ensembles at multiple lattice spacings
  - Here, three lattice spacings ➜ Continuum limit directly at physical point
- Reproduction of well-known nucleon structure quantities, e.g. axial charge
- Requires thorough study of systematic uncertainties
  - Main systematic is excited state effects
- Agreement for axial charge leads to confidence in other less well known quantities
  - Flavor-separated axial charges
  - σ-terms
  - Tensor charges
- Analysis ongoing
  - For full systematic due to excited state effects in all charges and form factors
  - Additional lattice spacing at smaller values of a e.g. $a \simeq 0.05$ fm
  - Moments of PDFs, e.g. $\langle x \rangle_{u-d}$, $\langle x \rangle_{\Delta u - \Delta d}$, $\langle x \rangle_{\delta u - \delta d}$

Acknowledgements

Με τη συγχρηματοδότηση της Ευρωπαϊκής Ένωσης

ΚΥΠΡΙΑΚΗ ΔΗΜΟΚΡΑΤΙΑ

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