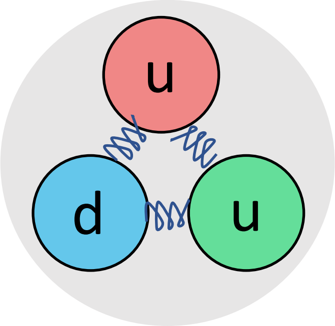
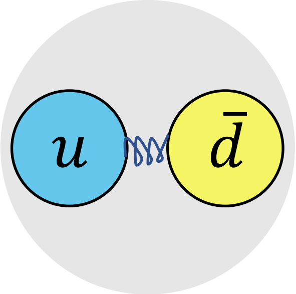
The gravitational form factors of the nucleon and the pion from lattice QCD

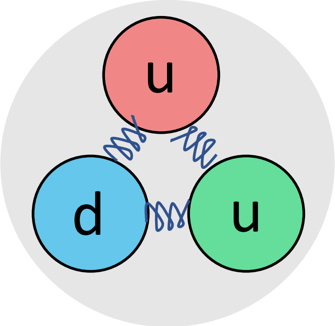


Dimitra Anastasia Pefkou

Berkeley
UNIVERSITY OF CALIFORNIA



The gravitational form factors of the nucleon and the pion from lattice QCD



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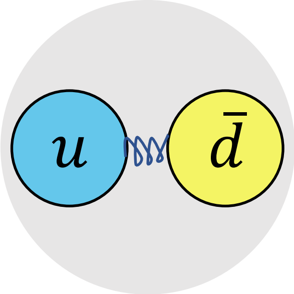


Dan Hackett
MIT → FNAL

Patrick Oare
MIT

Phiala Shanahan
MIT

Energy-momentum tensor (EMT)

Classical field theory: EMT is the conserved current due to symmetry under space-time translations

Langrangian $\xrightarrow[\text{theorem}]{\text{Noether's}}$ EMT

coordinate space:

$$T^{\mu\nu} = \begin{bmatrix} T^{tt} & \\ & T^{ij} \end{bmatrix} = \begin{bmatrix} \varepsilon(r) & \\ & \left(\frac{r^i r^j}{r^2} - \frac{\delta^{ij}}{d}\right) s(r) + \delta^{ij} s(r) \end{bmatrix}$$

QCD : $\mathcal{L} = -\frac{1}{4} F^{a,\mu\nu} F^{a}_{\mu\nu} + \sum_f [\bar{\psi}_f i\, \gamma^\mu\, D_\mu \psi_f + m_f \bar{\psi}_f \psi_f]$

$$T^{\mu\nu} = \underbrace{-F_a^{\mu\alpha} F^{\nu}_{a,\alpha} + \frac{1}{4} g^{\mu\nu} F_a^{\alpha\beta} F_{a,\alpha\beta}}_{T_g^{\mu\nu}} + \underbrace{\sum_f i\bar{\psi}_f \gamma^{\{\mu} D^{\nu\}} \psi_f}_{T_q^{\mu\nu}}$$

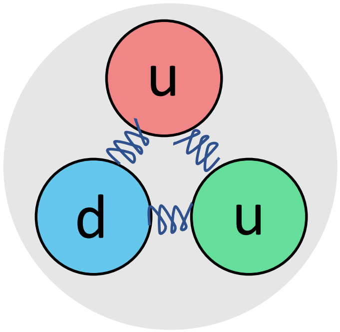
this talk: symmetric EMT

$\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu T_i^{\mu\nu} \neq 0$

→ quark and gluon EMTs scale μ and scheme dependent

→ traceless $T_i^{\mu\nu}$: terms in OPE for GPDs

Gravitational form factors - proton



$$\langle \mathrm{N}(p',s')|T_{\mu\nu}|\mathrm{N}(p,s)\rangle = \frac{1}{m_N}\bar{u}(\boldsymbol{p}',s')\begin{bmatrix} P_\mu P_\nu A^N(t) \\ iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho J^N(t) \\ \frac{1}{4}\left(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2\right)D^N(t) \end{bmatrix} u(\boldsymbol{p},s)$$

P_μ: $(p'_\mu + p_\mu)/2$

Δ_μ: $p'_\mu - p_\mu$

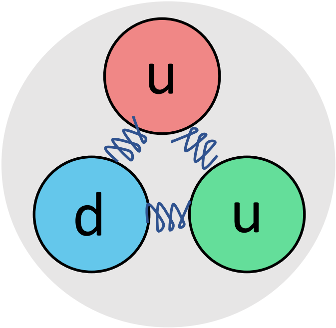
$t = \Delta^2$

$$A^N(t) = A_g^N(t) + A_q^N(t)\ ,\ A^N(0) = 1$$

$$J^N(t) = J_g^N(t) + J_q^N(t)\ ,\ J^N(0) = 1/2$$

$$D^N(t) = D_g^N(t) + D_q^N(t)\ ,\ D^N(0) = ?$$

Gravitational form factors - proton



$$\langle \mathrm{N}(p', s') | T_{\mu\nu} | \mathrm{N}(p, s) \rangle = \frac{1}{m_N} \bar{u}(\boldsymbol{p}', s') \begin{bmatrix} P_\mu P_\nu A^N(t) \\ i P_{\{\mu} \sigma_{\nu\}\rho} \Delta^\rho J^N(t) \\ \frac{1}{4}(\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2) D^N(t) \end{bmatrix} u(\boldsymbol{p}, s)$$

$(p'_\mu + p_\mu)/2$

$p'_\mu - p_\mu$

$t = \Delta^2$

$$A^N(t) = A_g^N(t) + A_q^N(t), \; A^N(0) = 1$$
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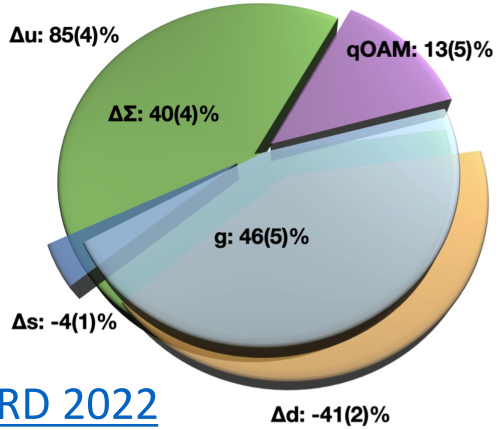
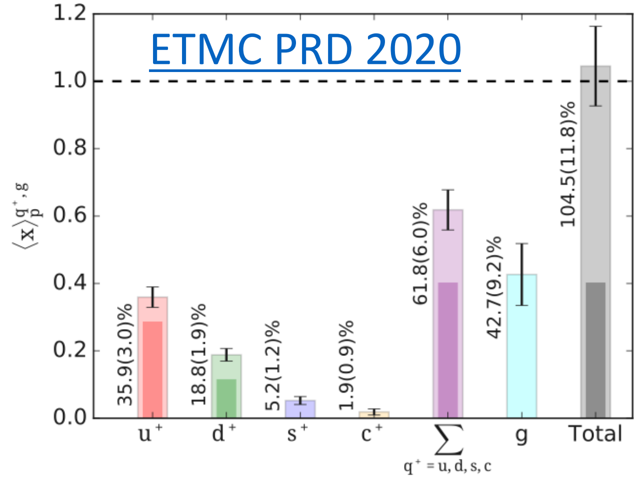
$$D^N(t) = D_g^N(t) + D_q^N(t), \; D^N(0) = ?$$

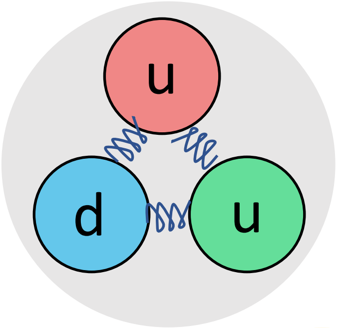
momentum and spin decomposition: long history
see, e.g, recent lattice QCD results



χQCD PRD 2022



Gravitational form factors - proton



$$\langle \mathrm{N}(p',s')|T_{\mu\nu}|\mathrm{N}(p,s)\rangle = \frac{1}{m_N}\bar{u}(\boldsymbol{p}',s')\begin{bmatrix} P_\mu P_\nu A^N(t) \\ iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho J^N(t) \\ \frac{1}{4}(\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2)D^N(t)\end{bmatrix}u(\boldsymbol{p},s)$$

P_μ: $(p'_\mu + p_\mu)/2$; Δ_μ: $p'_\mu - p_\mu$; $t = \Delta^2$

t –dependence: much less is known

$$A^N(t) = A_g^N(t) + A_q^N(t)\,,\ A^N(0) = 1$$
$$J^N(t) = J_g^N(t) + J_q^N(t)\,,\ J^N(0) = 1/2$$
$$D^N(t) = D_g^N(t) + D_q^N(t)\,,\ D^N(0) = ?$$

this talk:
Lattice A+J+D for q + g : Hackett **DAP** Shanahan 2310.08484

Experiment (A+D):
Duran Meziani et al Nature 2023

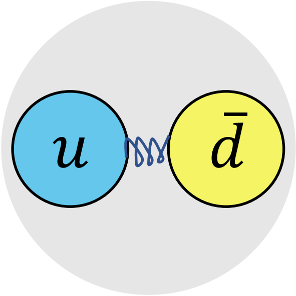
Lattice (A+J+D):
Shanahan Detmold PRL 2018,
DAP Hackett Shanahan PRD 2022

Lattice (A+J+D):
ETMC PRD 2020,
LHPC PRD 2008

Experiment (D):
Burkert Elouadrhiri Girod Nature 2018
Dispersive (D):
Pasquini et al 2014

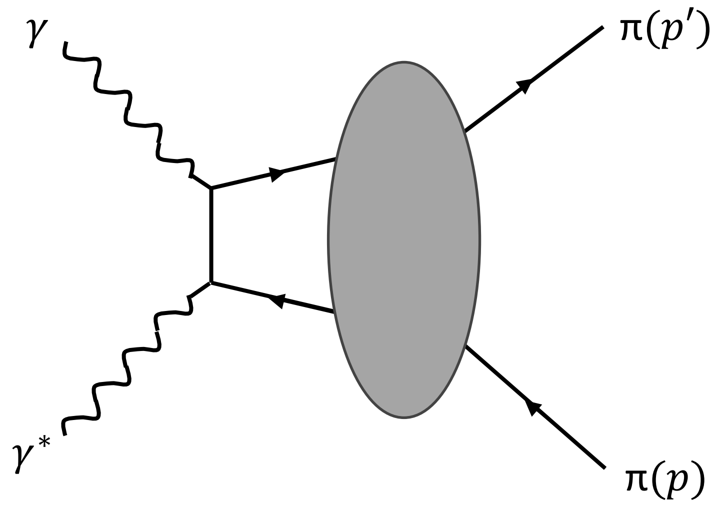
+ more from theory and models
(see 2023 Colloquium Burkert et al
for review)

Gravitational form factors - pion



$$\langle \pi(p') | T_{\mu\nu} | \pi(p) \rangle = \begin{bmatrix} 2P_\mu P_\nu A^\pi(t) \\ \frac{1}{4}(\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2) D^\pi(t) \end{bmatrix}$$

quark:



from GDAs in timelike region

gluon? other hadrons?

$$A^\pi(t) = A_g^\pi(t) + A_q^\pi(t),\ A^\pi(0) = 1$$
$$D^\pi(t) = D_g^\pi(t) + D_q^\pi(t),\ D^\pi(0) \approx -1$$

→ chiral perturbation theory

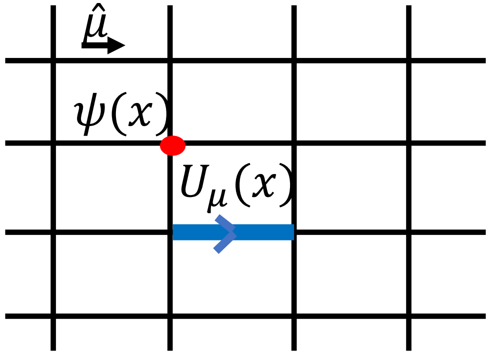
Lattice (A+D): Shanahan Detmold PRD 2018, **DAP** Hackett Shanahan PRD 2022

Experiment (A+D): Kumano Song Teryaev PRD 2018

Lattice (A+D): Brommel PhD thesis 2007 Joe Delmar Lattice 2023

Lattice A+D for q + g : Hackett Oare **DAP** Shanahan 2307.11707 (accepted at PRD)

→ this talk



GFFs from lattice QCD

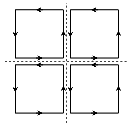
Pion A+D for q + g : Hackett Oare **DAP** Shanahan 2307.11707 (accepted at PRD)

Proton A+J+D for q + g : Hackett **DAP** Shanahan 2310.08484

$$T^{\mu\nu} = -F_a^{\mu\alpha} F_{a,\alpha}^{\nu} + \frac{1}{4} g^{\mu\nu} F_a^{\alpha\beta} F_{a,\alpha\beta} + \sum_f i\bar{\psi}_f \gamma^{\{\mu} D^{\nu\}} \psi_f$$
$$= \sum_{i \in \{q,g\}} T_i^{\mu\nu}$$

→ $T_i^{\mu\nu}$: write in terms of Euclidean lattice fields

$G_{\mu\nu} \sim (Q_{\mu\nu} - Q_{\mu\nu}^\dagger)$

$Q_{\mu\nu} \sim$ 

$\overleftrightarrow{D} = (\vec{D} - \overleftarrow{D})/2$

$\vec{D}_\mu \psi(x) = \frac{1}{2}(U_\mu(x)\psi(x+\mu) - U_\mu^\dagger(x-\mu)\psi(x-\mu))$

→ $T_i^{\mu\nu}$: isotropic hypercubic lattice

symmetric traceless components transform in two irreps of $H(4)$ group → $\tau_1^{(3)}, \tau_3^{(6)}$

→ $T_i^{\mu\nu}$: flavor singlet $q = u + d + s + \cdots$ mixes with g

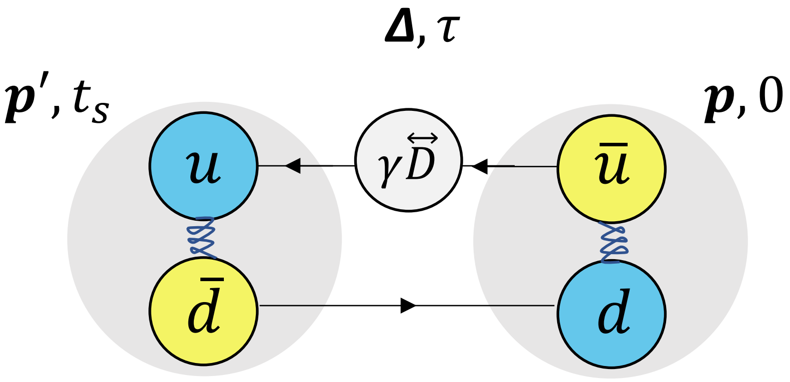
non-singlet $u - d, u + d - 2s$ renormalize ~ multiplicatively

Lattice simulation

m_π (MeV)	a (fm)	$L^3\times T$	N_f
169(1)	0.091(1)	$48^3\times 96$	$2+1$

Clover-improved Wilson quarks, Lüscher-Weisz gauge action
generated by JLab/LANL/MIT/WM groups

similarly for nucleon



Connected contribution

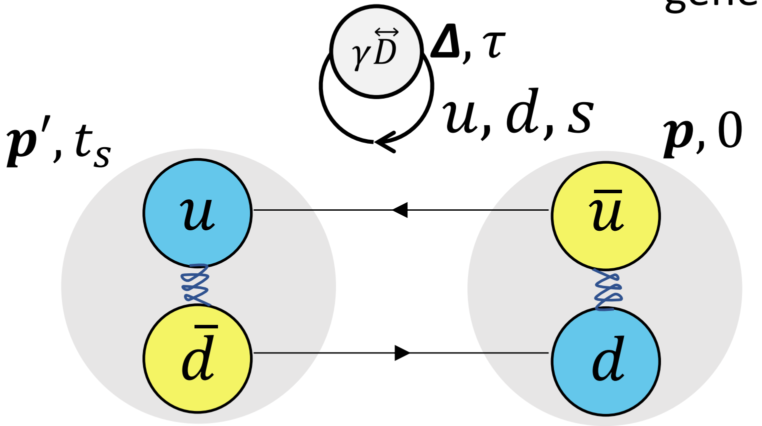
→ 1381 configurations

→ sequential sources

→ $t_s \in \{6-18\}$

→ $|\mathbf{\Delta}|^2 \le 25(\frac{2\pi}{L})^2$

→ $\mathbf{p}' \in \{(1,-1,0), (-2,-1,0), (-1,-1,-1)\}2\pi/L$



Disconnected contribution

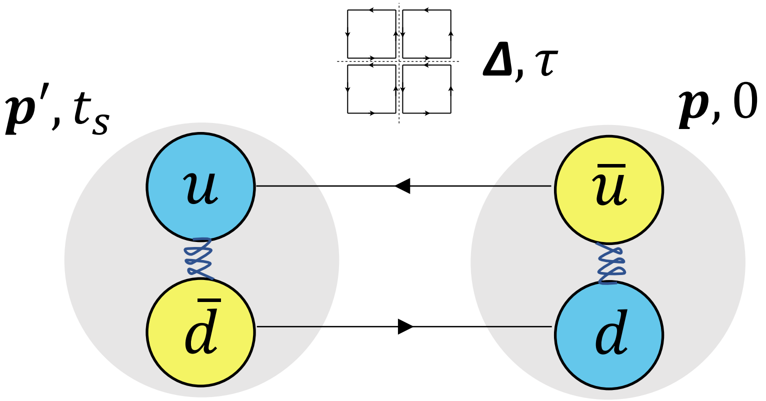
→ 1381 configurations

→ Z_4 noise, hierarchical probing, 512 Hadamard vectors

→ 1024 sources

→ $|\mathbf{\Delta}|^2 \le 25(\frac{2\pi}{L})^2$

→ $|\mathbf{p}'|^2 \le 10(\frac{2\pi}{L})^2$



Gluon contribution

→ 2511 configurations

→ Gradient flow to reduce UV fluctuations

→ 1024 sources

→ $|\mathbf{\Delta}|^2 \le 25(\frac{2\pi}{L})^2$

→ $|\mathbf{p}'|^2 \le 10(\frac{2\pi}{L})^2$

Bare matrix elements

$\mathcal{R} \in \{\tau_1^{(3)}, \tau_3^{(6)}\}$

- Form ratios $R_{\mathcal{R}}(\boldsymbol{p}', \boldsymbol{\Delta}, t_s, \tau) \sim \#\langle p'|T_{\mathcal{R}}|p\rangle + \mathcal{O}(e^{-\Delta\mathrm{E}\,\tau - \Delta\mathrm{E}'(t_s-\tau)})$

- Bin together ratios with same # (same linear combination of GFFs): $\bar{R}_c(t_s, \tau)$
  π: $\sim 10^5\ \{\mu, \nu, \boldsymbol{p}', \boldsymbol{\Delta}\} \rightarrow 1379\ \text{connected} + 3364\ \text{disco/glue}$
  N: $\sim 10^6\ \{\mu, \nu, \boldsymbol{p}', \boldsymbol{\Delta}, s\} \rightarrow 3081\ \text{connected} + 11454\ \text{disco/glue}$

- Summation method [Capitani et al PRD 2012 etc.] :

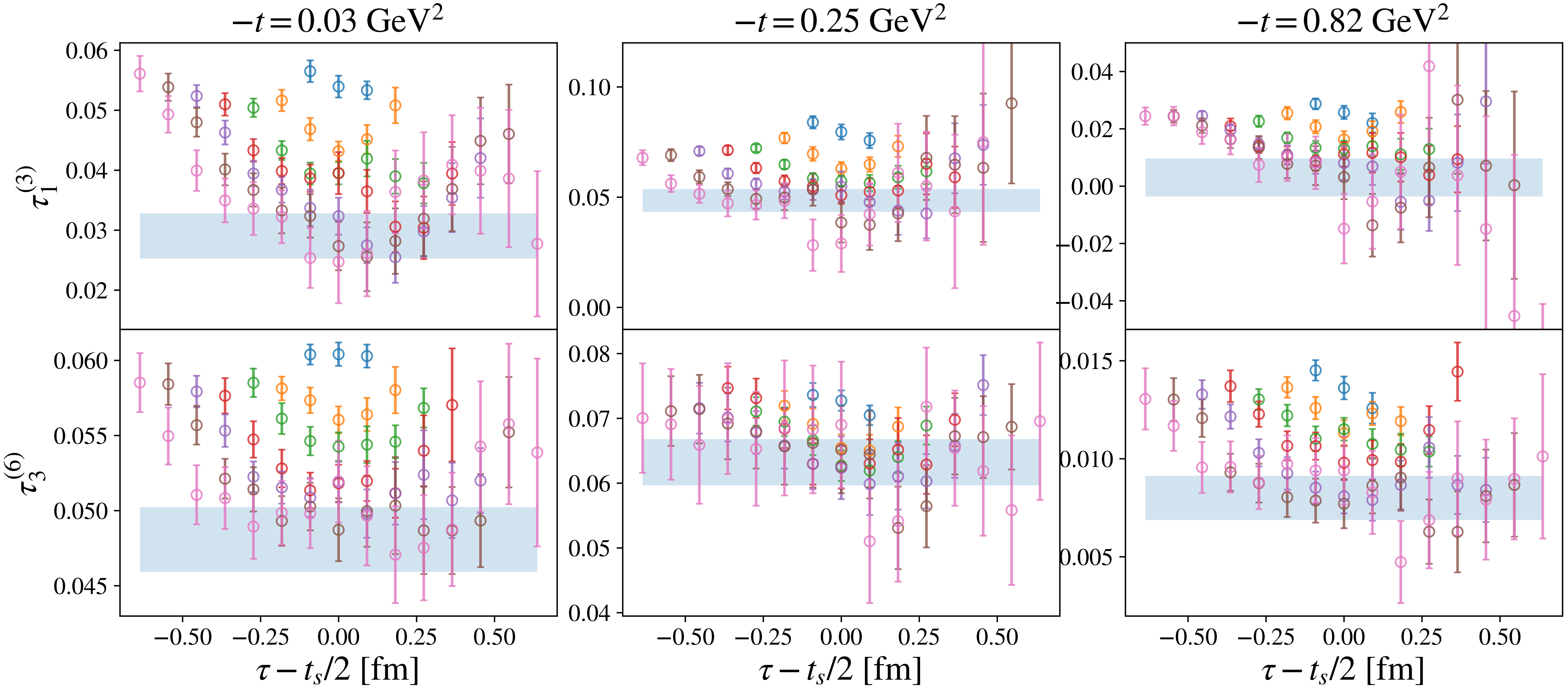
$$\bar{\Sigma}_c(t_s) = \sum_{\tau=\tau_{cut}}^{t_s-\tau_{cut}} \bar{R}_c(t_s, \tau) \sim (\text{const}) + \#\, t_s\langle p'|T_{\mathcal{R}}|p\rangle + + \mathcal{O}(e^{-\delta\mathrm{E}\,t_s})$$

Fit with Bayesian model averaging over time ranges (Akaike Information Criterion weights)

Jay Neil PRD 2021

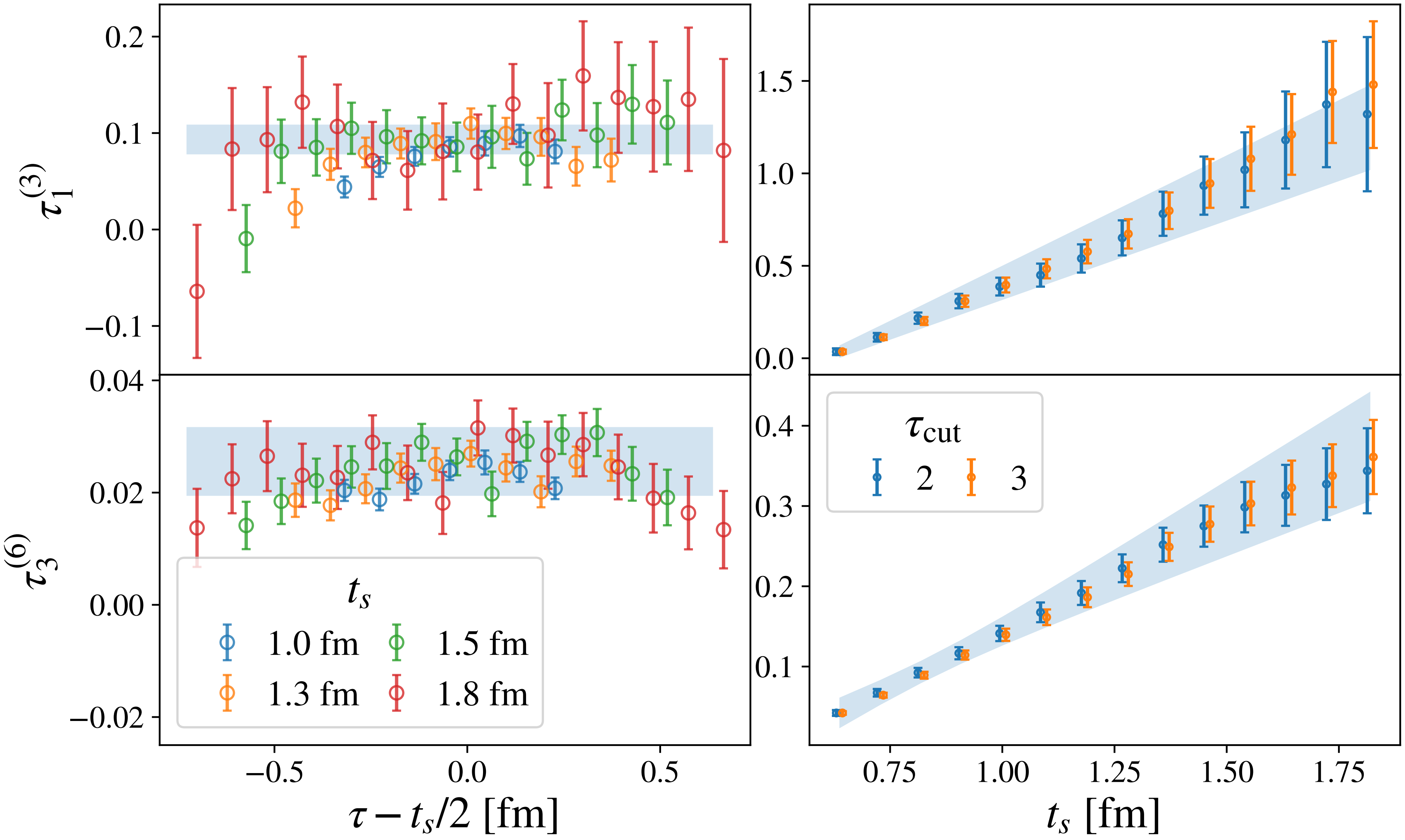
Pion connected quark contribution

$\tau_1^{(3)}$: diagonal elements irrep
$\tau_3^{(6)}$: off-diagonal elements irrep



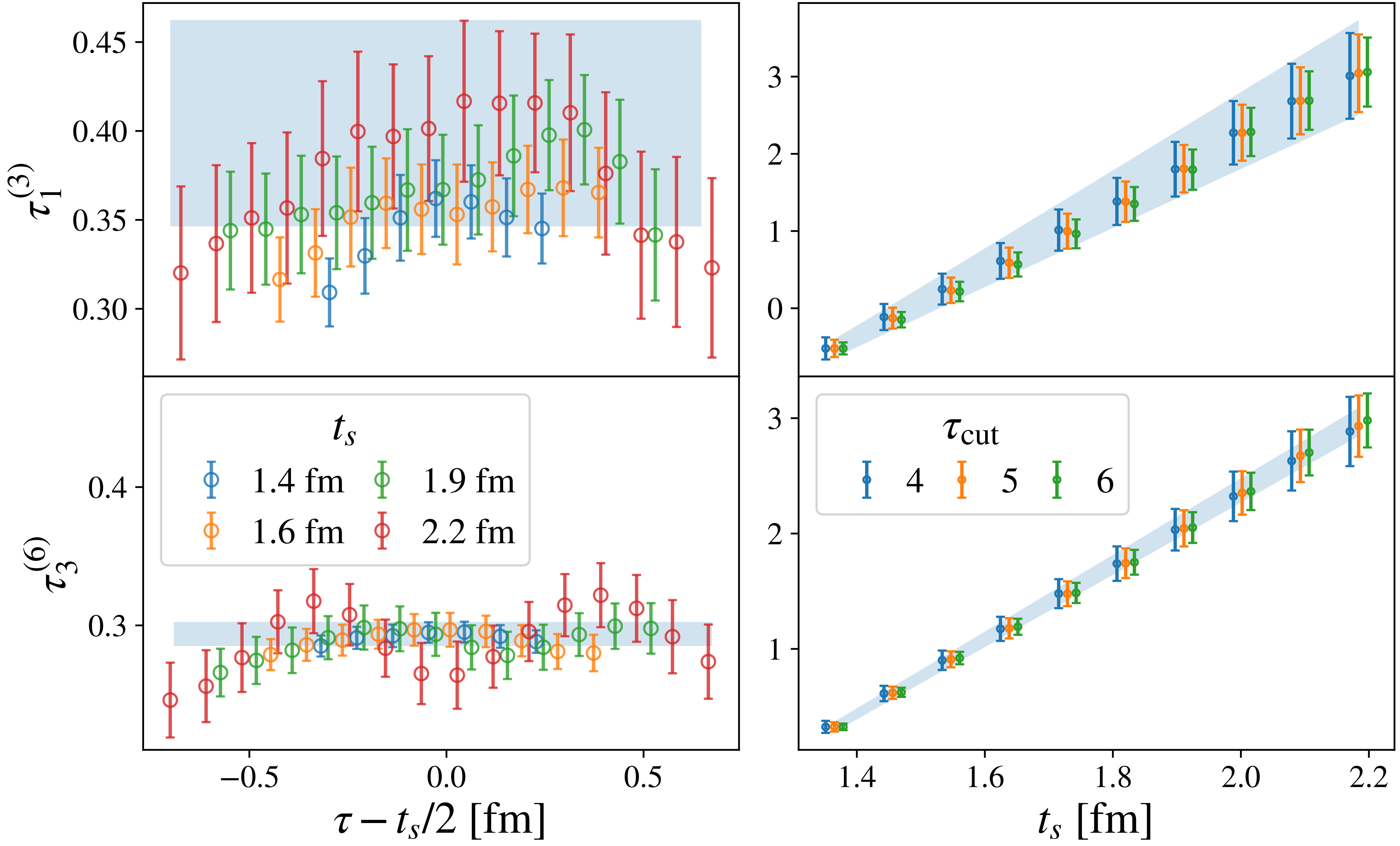
Pion disconnected quark contribution

$-t = 0.08 \text{ GeV}^2$



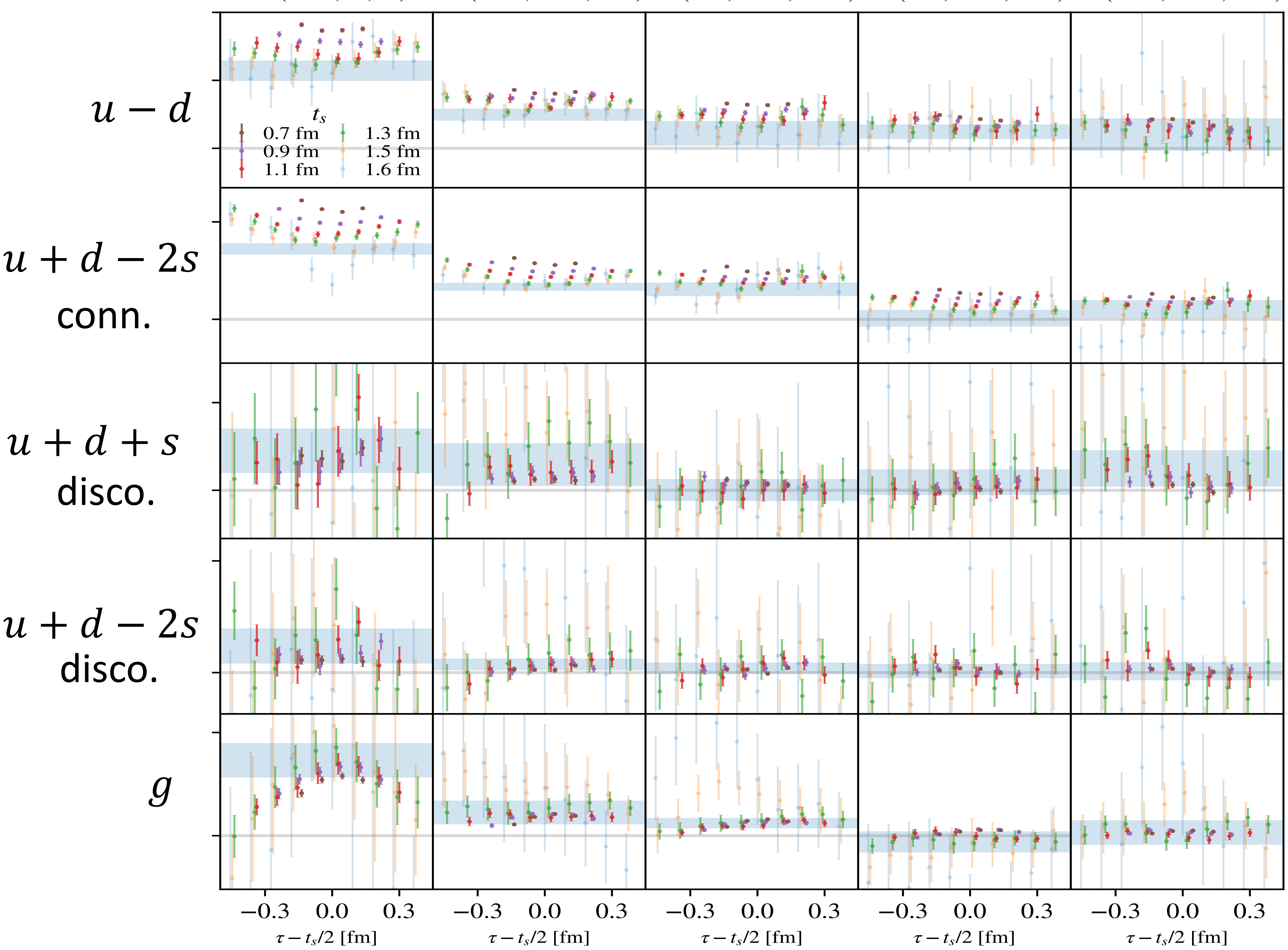
Pion gluon contribution

$-t = 0.13\ \text{GeV}^2$



Nucleon

$\tau_1^{(3)}$



Renormalization

$\mathcal{R} \in \{\tau_1^{(3)}, \tau_3^{(6)}\}$

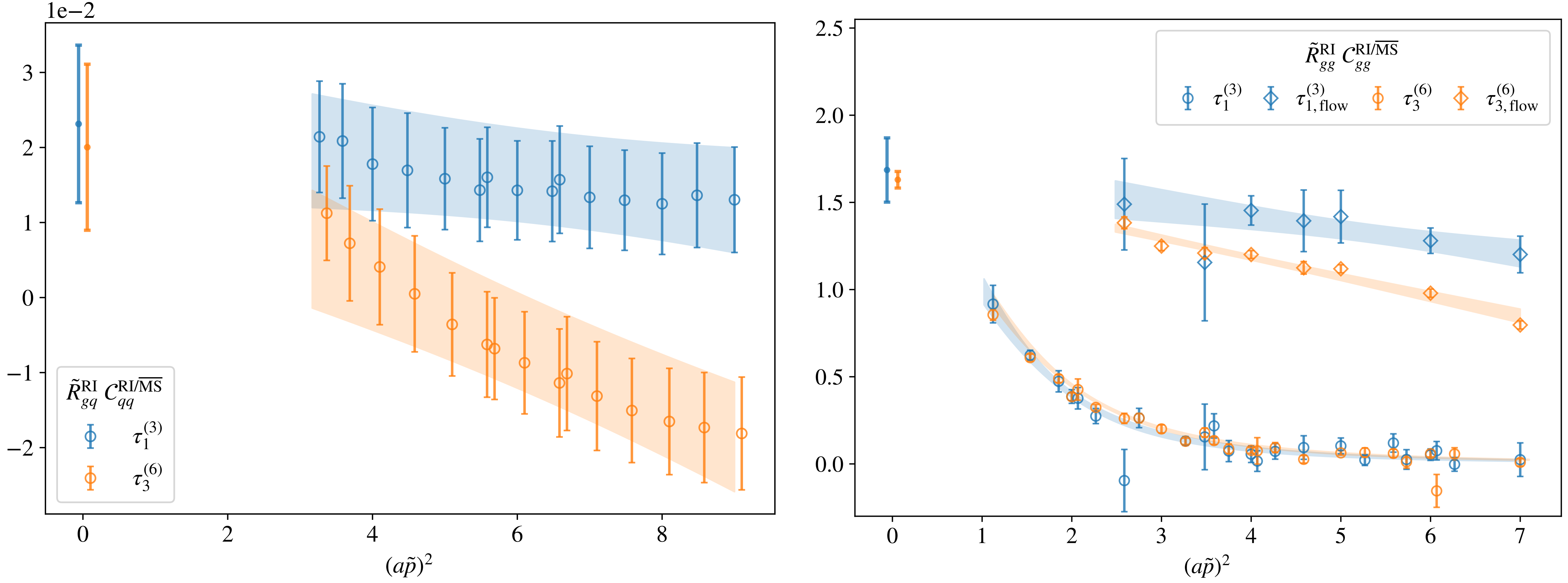
m_π (MeV)	a (fm)	$L^3 \times T$	N_f
450(5)	0.117(2)	$12^3 \times 24$	$2+1$

- $\begin{pmatrix} T_q^{\overline{\text{MS}}} \\ T_g^{\overline{\text{MS}}} \end{pmatrix} = \begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix} \begin{pmatrix} T_{q\mathcal{R}}^{\text{bare}} \\ T_{g\mathcal{R}}^{\text{bare}} \end{pmatrix}$: quark isosinglet and gluon mix under renormalization
- $T_v^{\overline{\text{MS}}} = Z_{v\mathcal{R}}^{\overline{\text{MS}}} T_{v\mathcal{R}}^{\text{bare}}$: non-singlet do not mix in the chiral limit
- Compute non-perturbatively via the RI-MOM scheme, convert to $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV
- each $R_{ij}^{\text{RI}} C_{jk}^{\text{RI}/\overline{\text{MS}}}$ has residual dependence on $(a\tilde{p})^2$ due to lattice artifacts, non-perturbative effects, etc..
  →model and fit to extract the renormalization coefficients

$$\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix}^{-1} (\mu^2) = \begin{pmatrix} R_{qq\mathcal{R}}^{\text{RI}} & R_{qg\mathcal{R}}^{\text{RI}} \\ R_{gq\mathcal{R}}^{\text{RI}} & R_{gg\mathcal{R}}^{\text{RI}} \end{pmatrix} (\mu_R^2) \times \begin{pmatrix} C_{qq}^{\text{RI}/\overline{\text{MS}}} & C_{qg}^{\text{RI}/\overline{\text{MS}}} \\ C_{gq}^{\text{RI}/\overline{\text{MS}}} & C_{gg}^{\text{RI}/\overline{\text{MS}}} \end{pmatrix} (\mu^2, \mu_R^2)$$

Panagopoulos et al PRD 2021

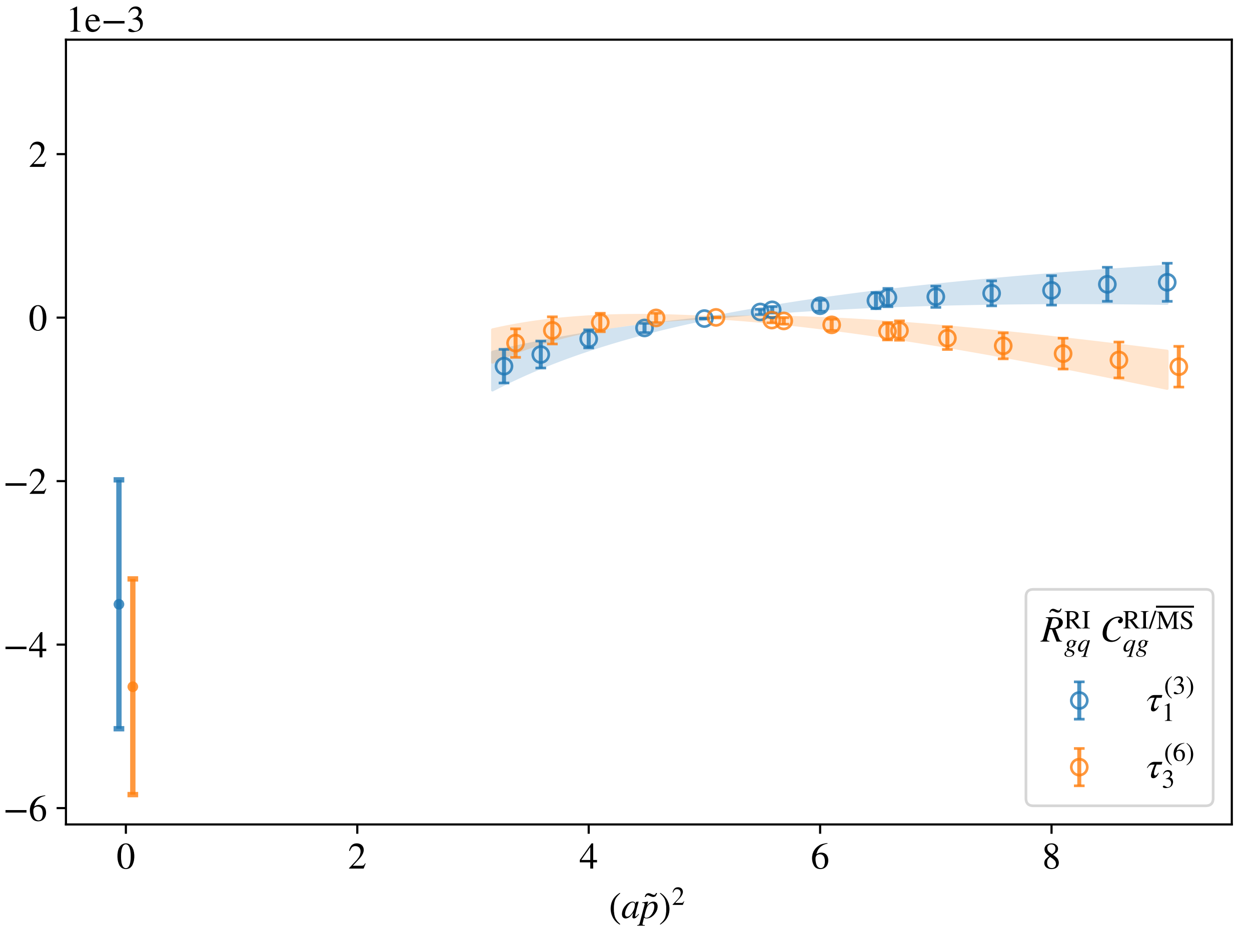
Extraction of renormalization coefficients

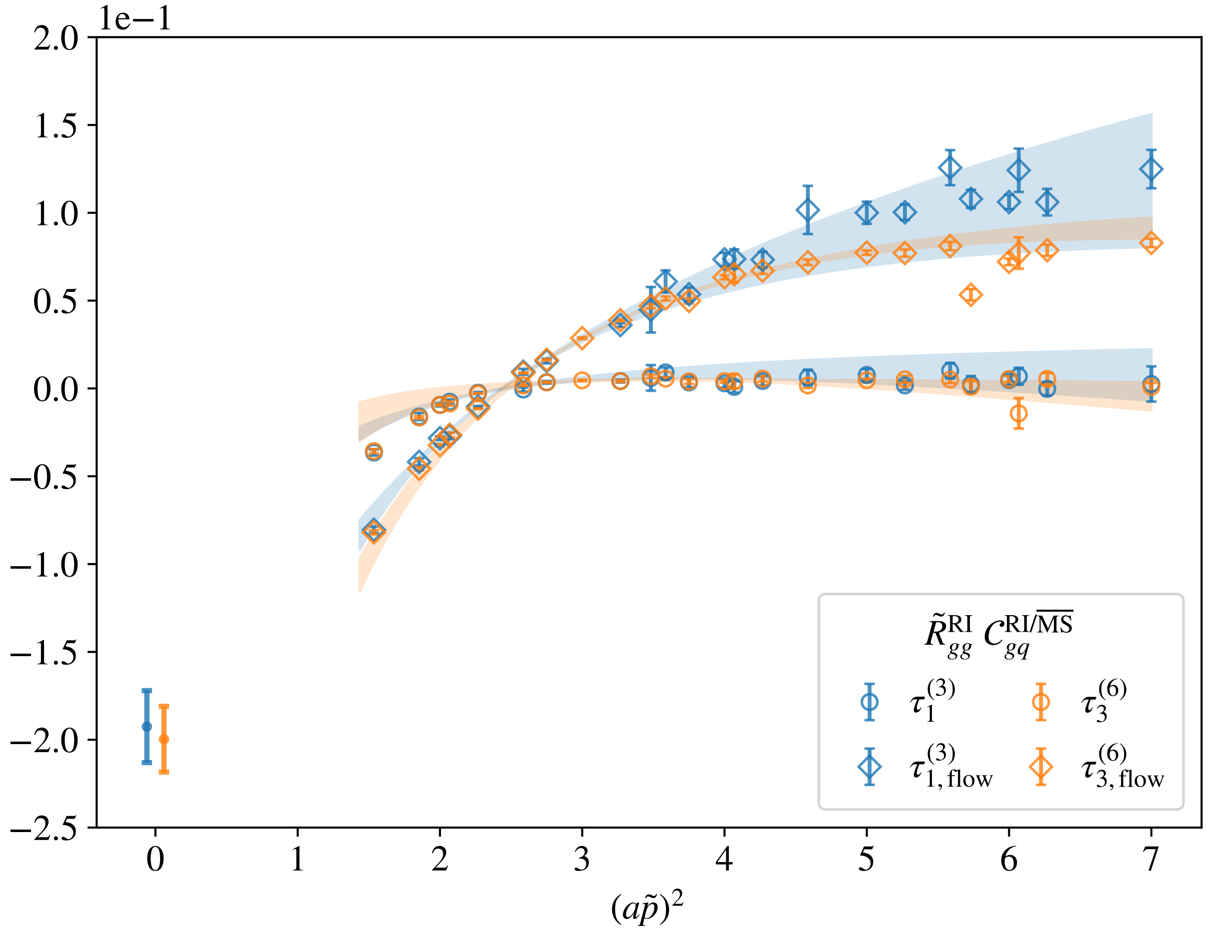


Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

(inverse) polynomial

Extraction of renormalization coefficients





Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

logarithmic

Obtain renormalized GFFs

We have: 1) bare matrix elements $\langle h | T_i^{\mu\nu} | h \rangle, i \epsilon \{g, q, v\}$ for each irrep $\mathcal{R}$

2) mixing matrix renormalization $\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix}^{-1}$, non-singlet ${Z_{v\mathcal{R}}^{\overline{\text{MS}}}}^{-1}$ for each $\mathcal{R}$

→ recast into a single system of equations including both irreps

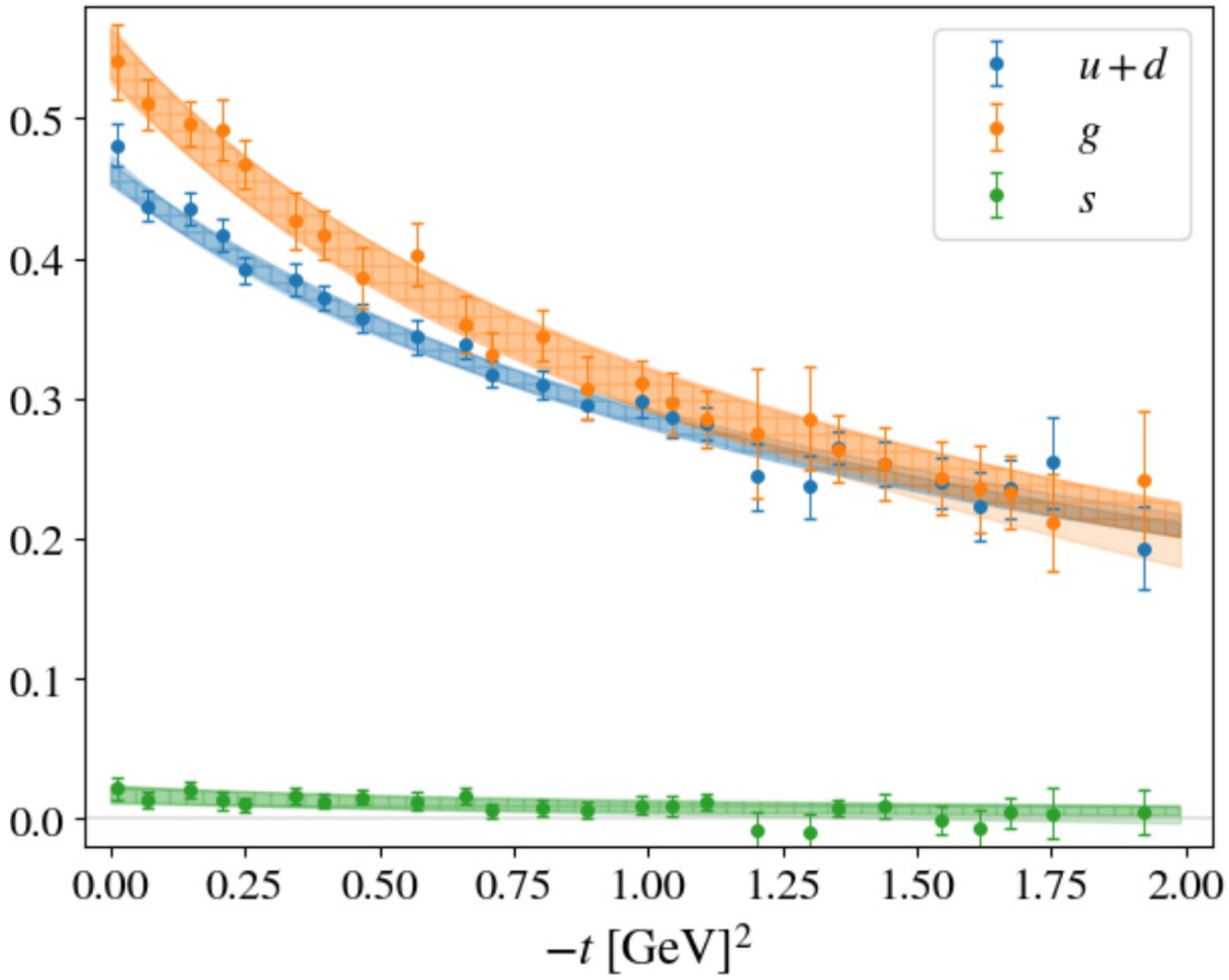
→ solve to get GFFs for discrete values of t

Fit with 1) multipole : $F_n = \frac{\alpha}{(1+\frac{t}{\Lambda^2})^n}$,

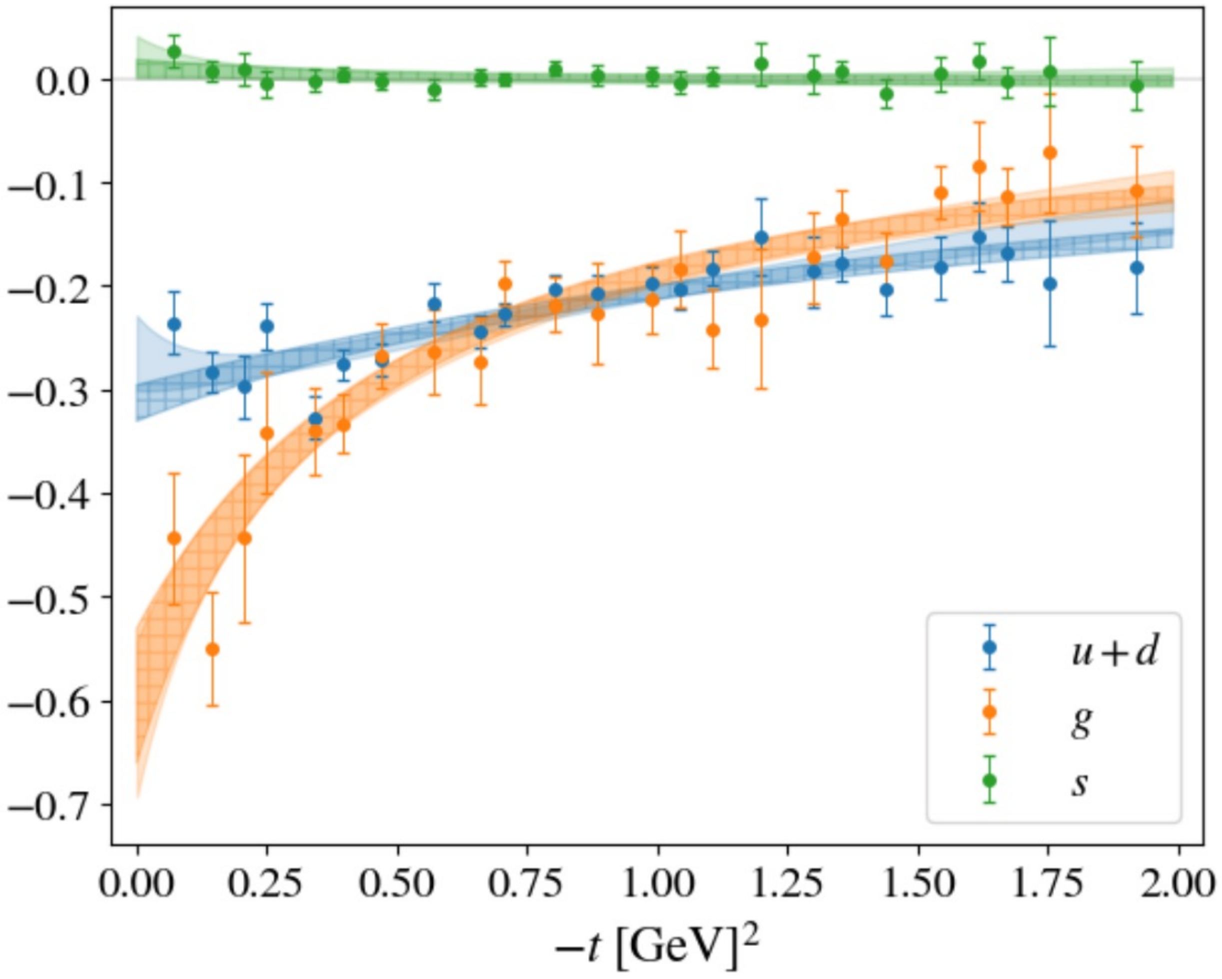
2) z-expansion : $F = \sum_k \alpha_k [z(t)]^k$

Renormalized pion GFFs

$A_i^{\pi}(t)$

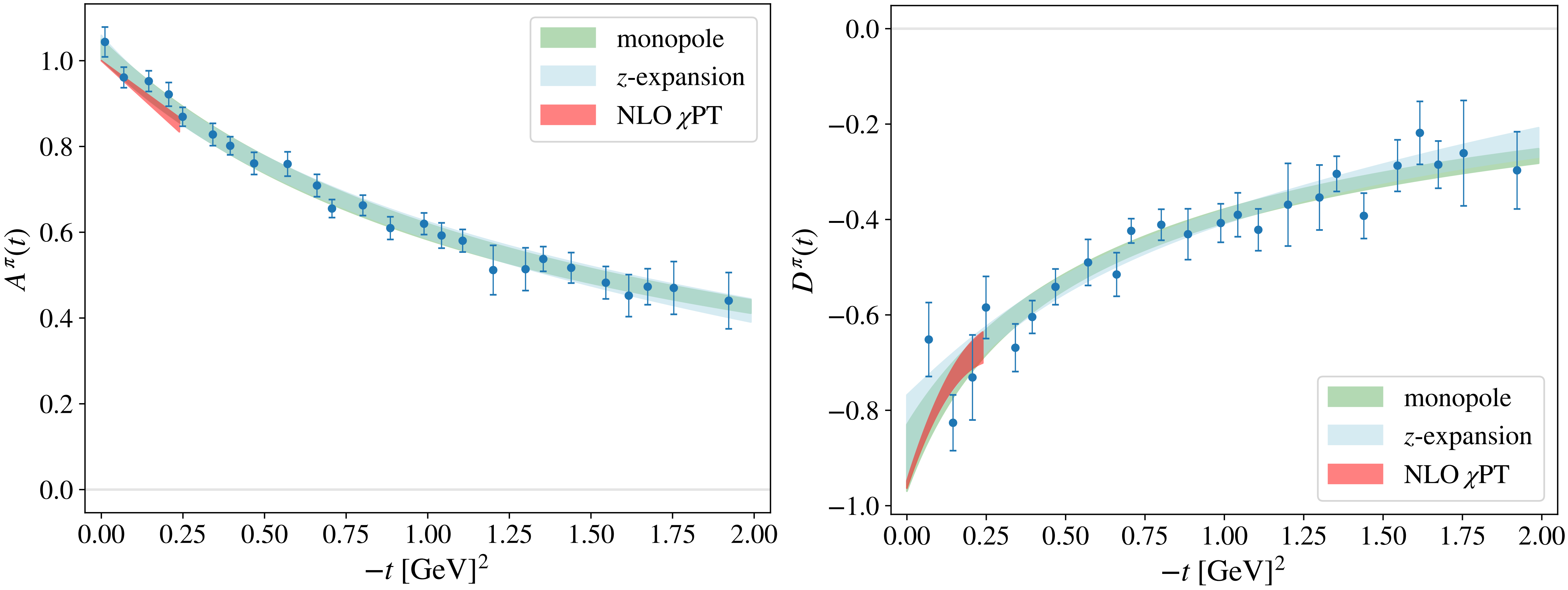


$D_i^{\pi}(t)$



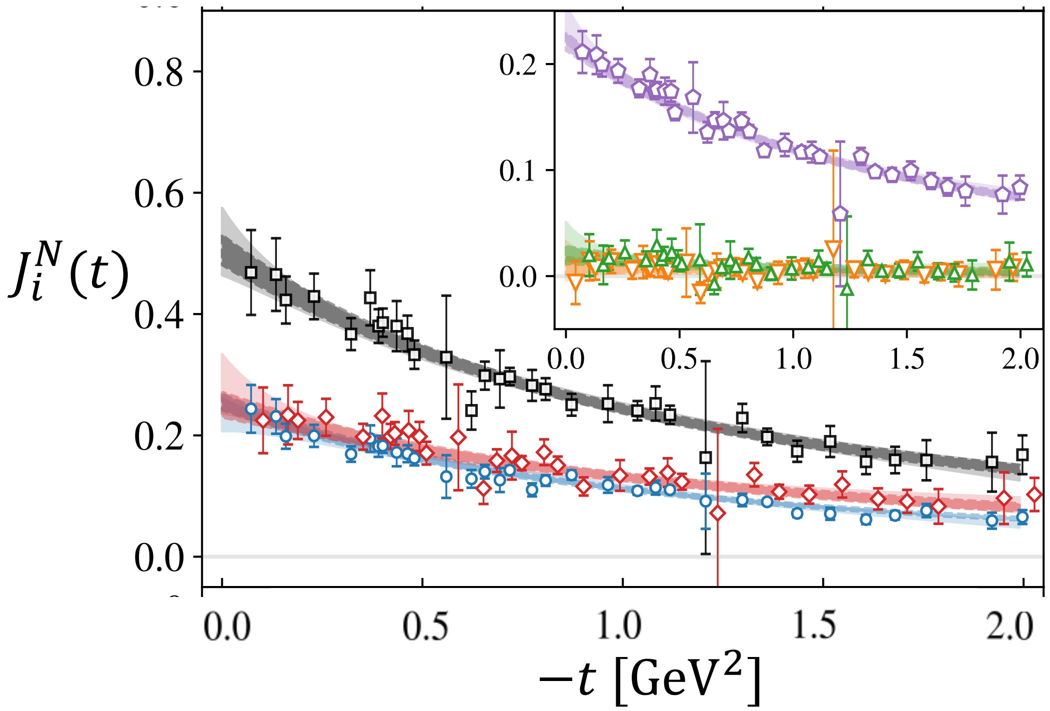
hatched bands : monopole, opaque bands : z-expansion with $k_{\max} = 2$

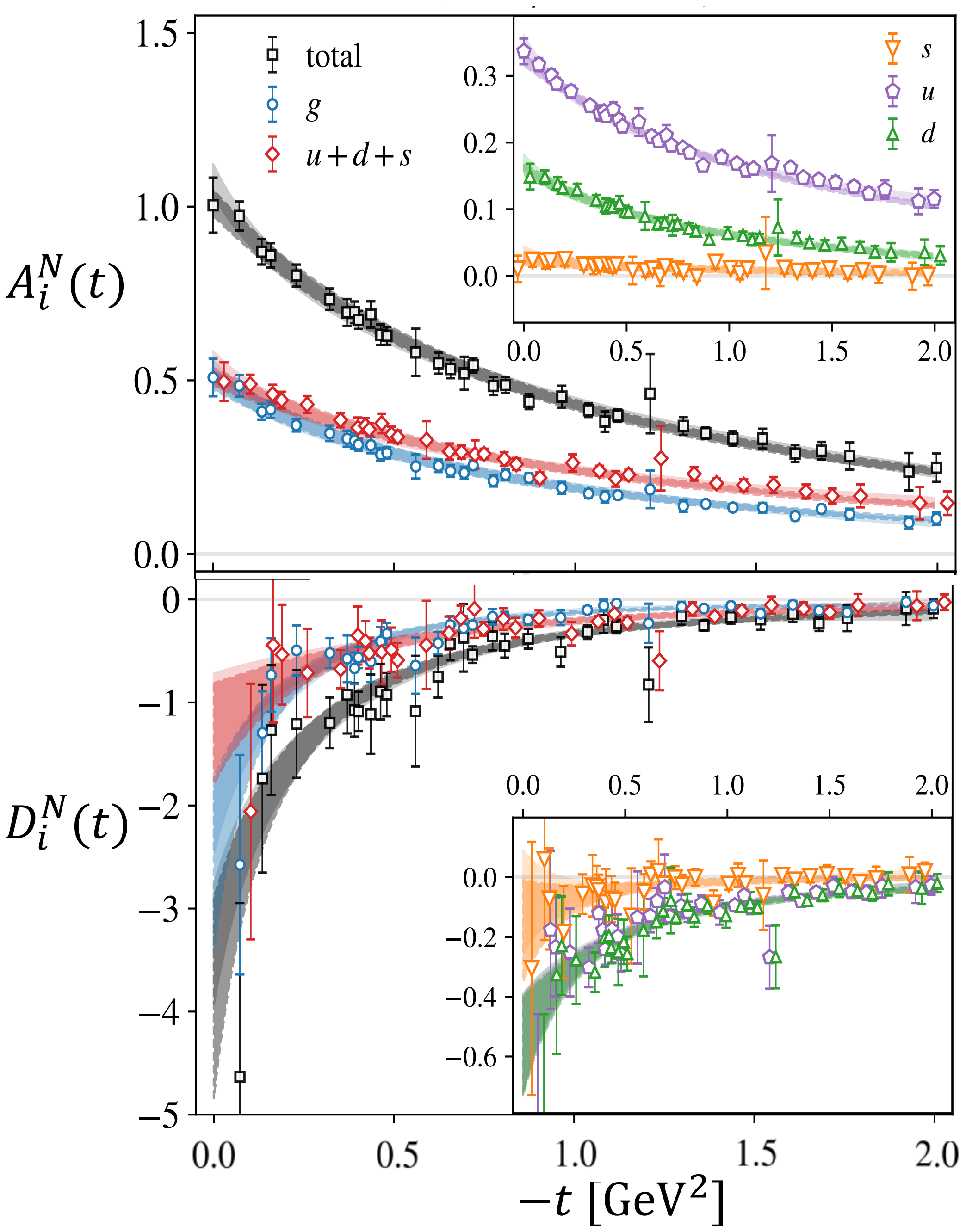
Pion : total GFFs



Red band spread due to different estimates for low energy constants [Donoghue Leutwyler Z.Phys.C 1991]

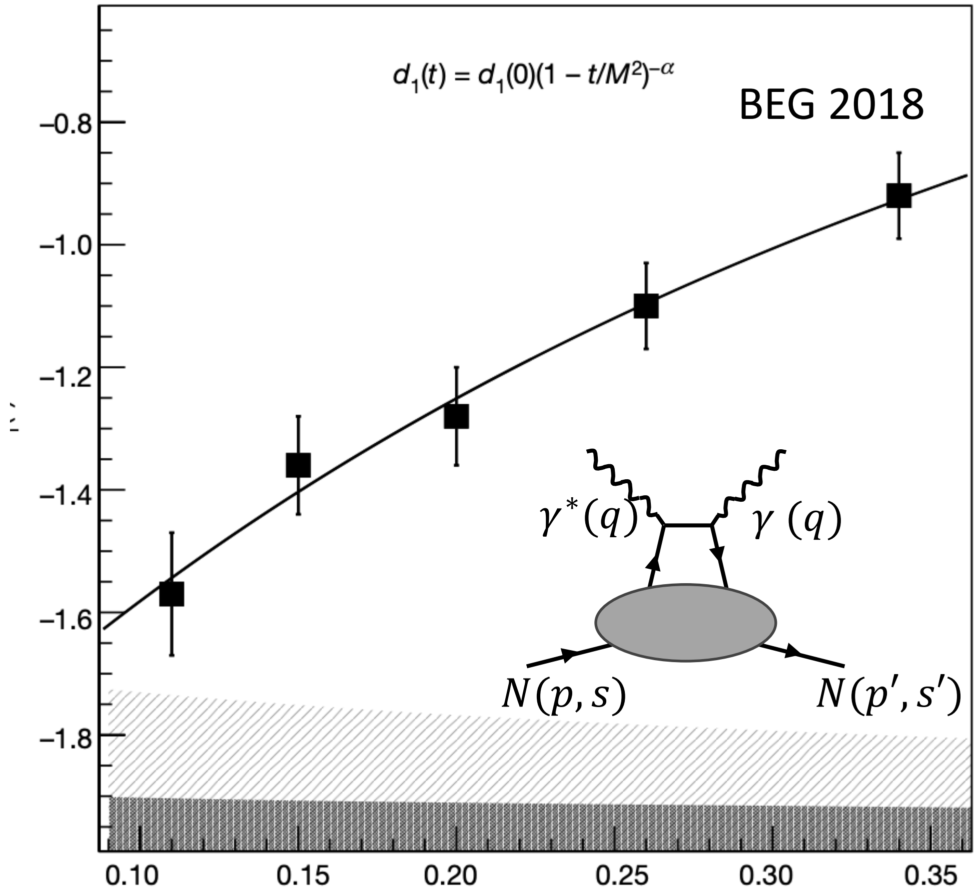
Renormalized nucleon GFFs





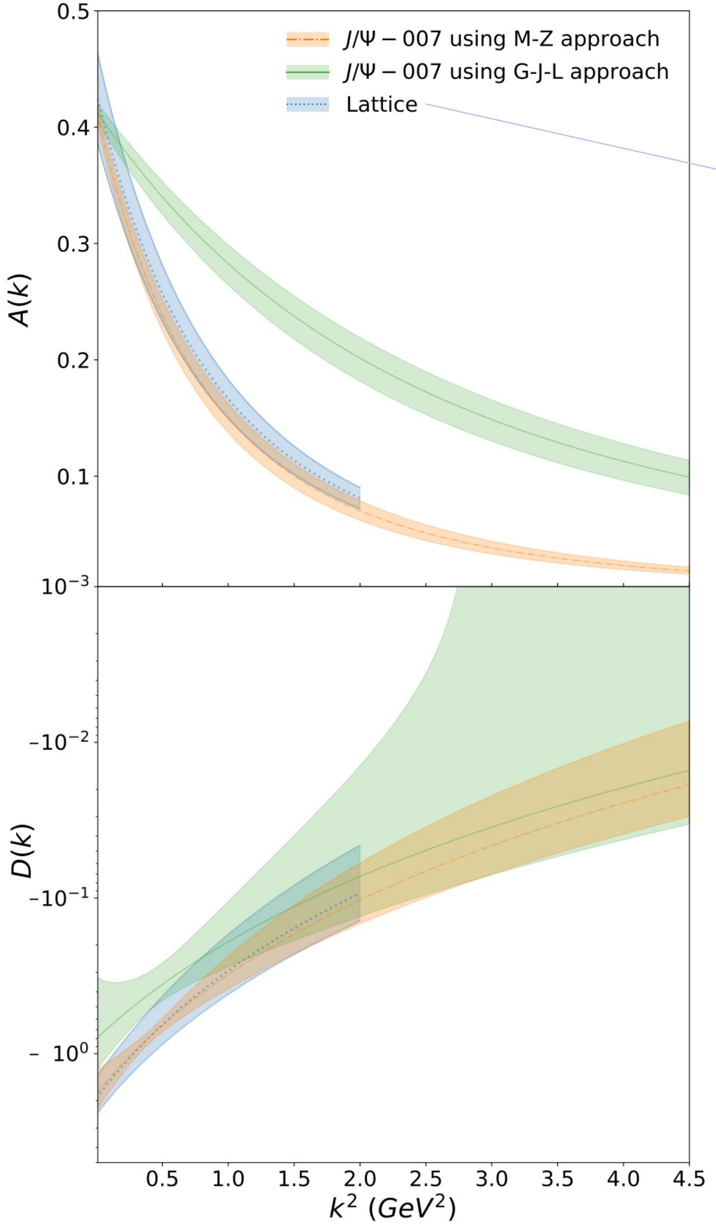
Nucleon: first experimental results

Quark D^N_{u+d} from DVCS



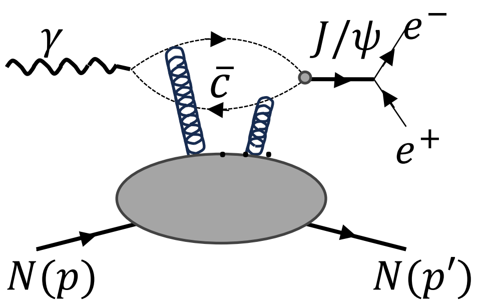
Burkert Elouadrhiri Girod Nature 2018

Gluon A^N_g and D^N_g from J/ψ photoproduction

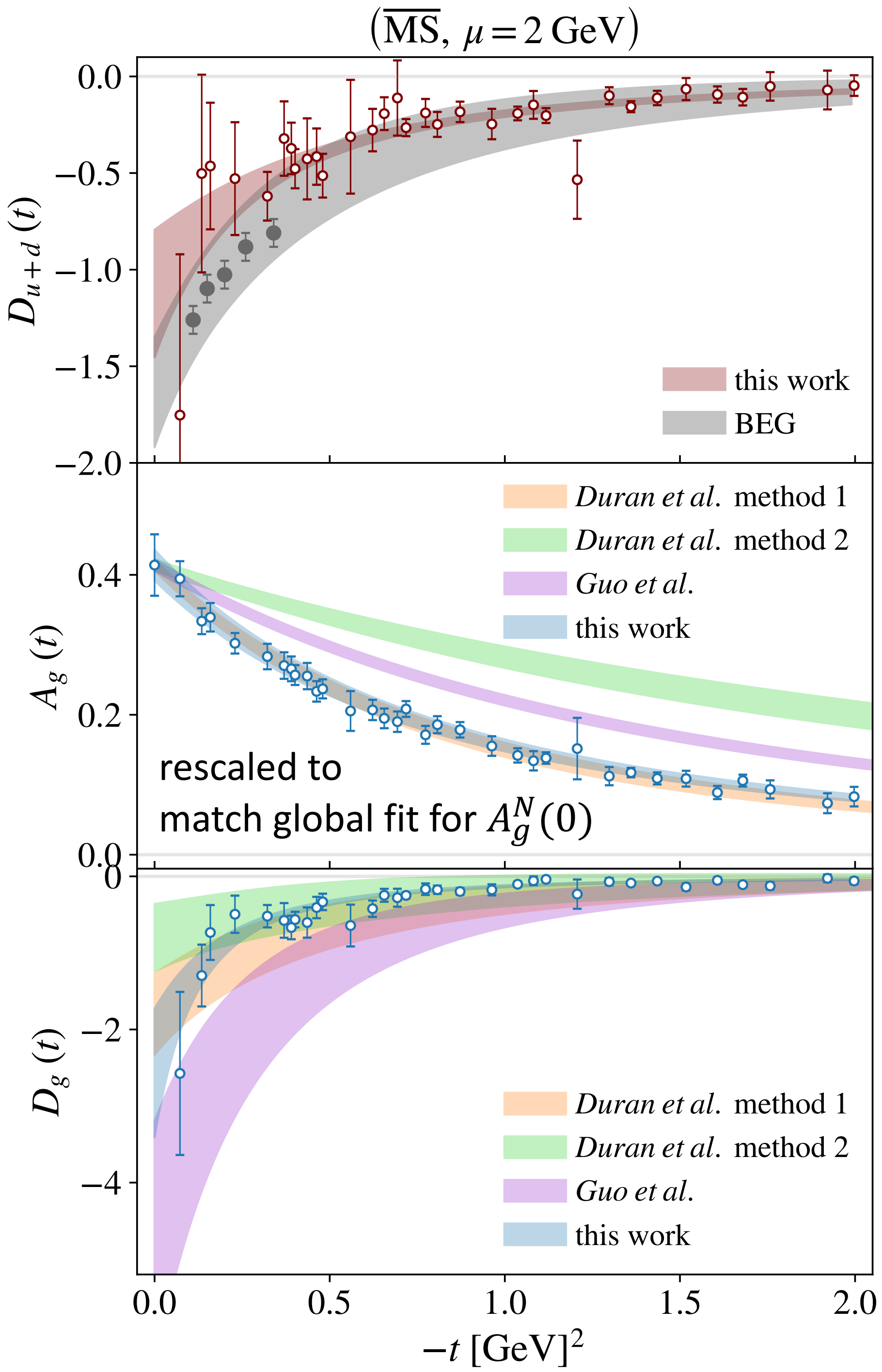


Duran Meziani et al Nature 2023

Lattice: **DAP Hackett** Shanahan *PRD* 2022
heavier pion mass + neglecting mixing with quark



Renormalized nucleon GFFs – comparison to experiments



Burkert Elouardhiri Girod Nature 2018

Duran et al Nature 2023 (J/ψ)

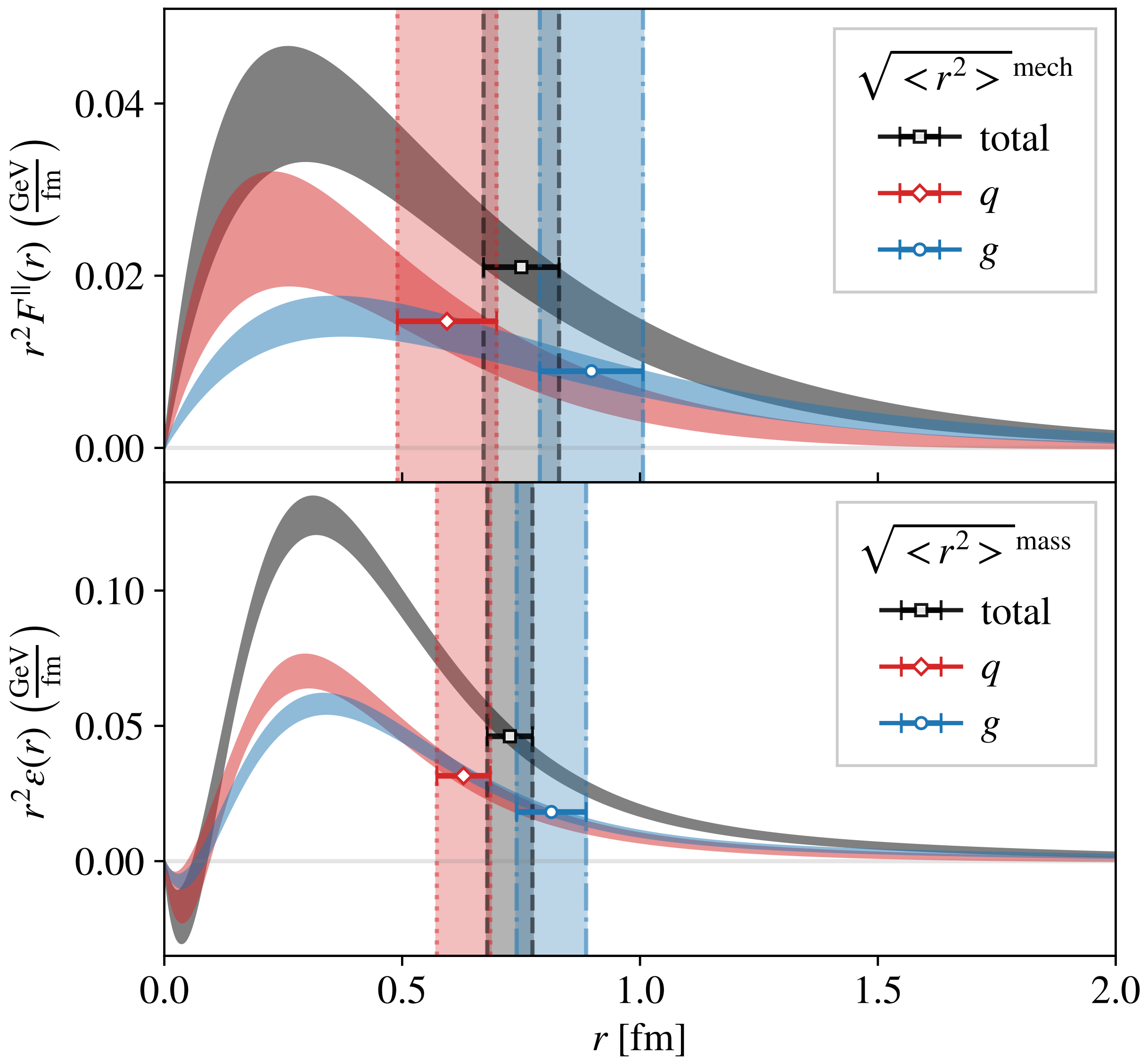
method 1: holographic QCD (Mamo Jahed PRD 2021+2022)

method 2: GPDs (Guo Ji Liu PRD 2021)

Guo et al PRD 2023 (+ GlueX data)

method 2 updated

EMT densities



$$\varepsilon_i(r) = m\left[A_i(t) - \frac{t(D_i(t)+A_i(t)-2J_i(t))}{4m^2}\right]_{\text{FT}}$$

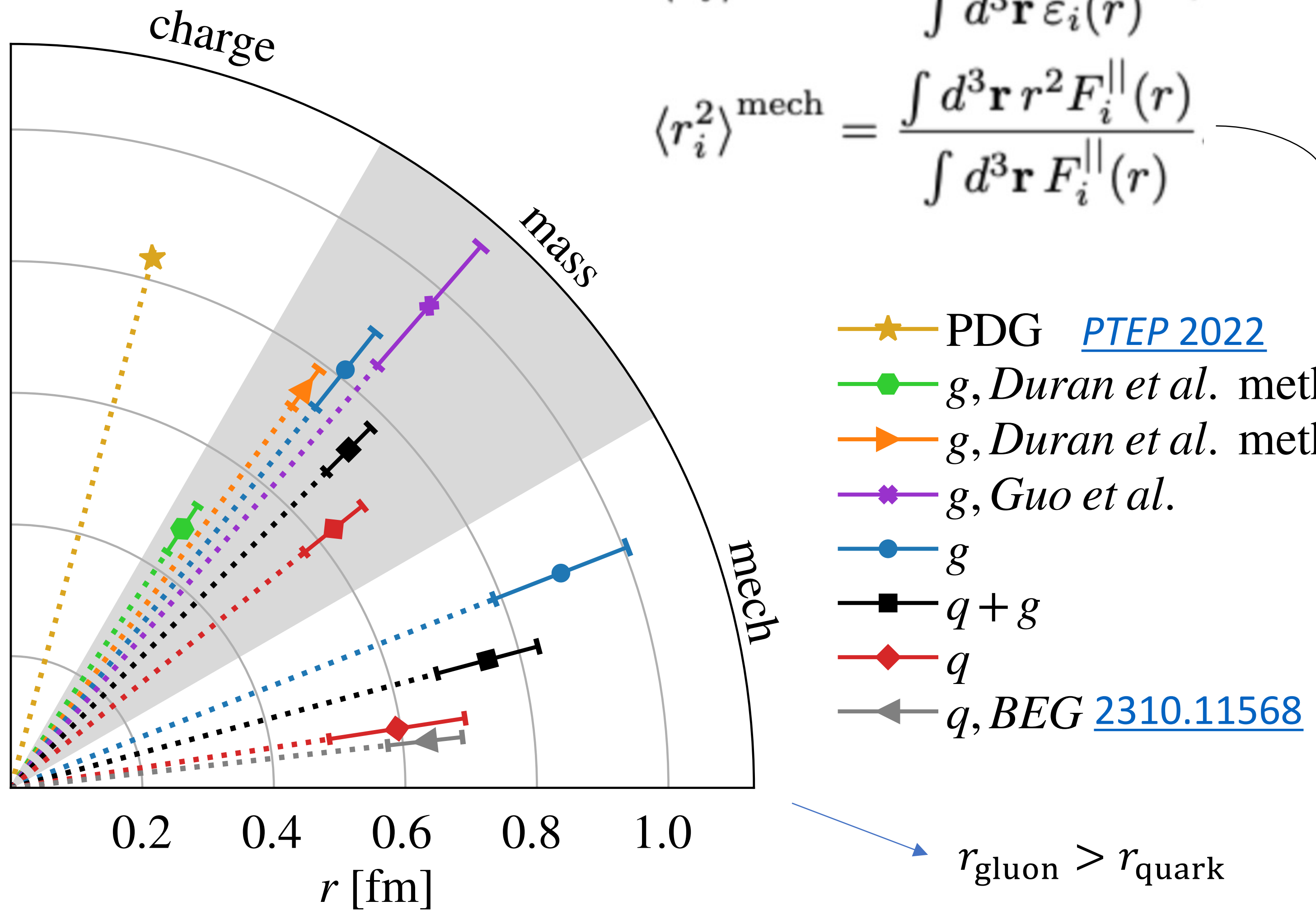
$$p_i(r) = \frac{1}{6m}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}[D_i(t)]_{\text{FT}}$$

$$s_i(r) = -\frac{1}{4m}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}[D_i(t)]_{\text{FT}}$$

$$F_i^{||}(r) = p_i(r) + 2s_i(r)/3$$

FT = Fourier transform
3D Breit frame

Nucleon size



$$\langle r_i^2\rangle^{\text{mass}} = \frac{\int d^3\mathbf{r}\, r^2 \varepsilon_i(r)}{\int d^3\mathbf{r}\, \varepsilon_i(r)}, \longrightarrow \varepsilon_i(r) = m\left[A_i(t) - \frac{t(D_i(t)+A_i(t)-2J_i(t))}{4m^2}\right]_{\text{FT}}$$

$$\langle r_i^2\rangle^{\text{mech}} = \frac{\int d^3\mathbf{r}\, r^2 F_i^{||}(r)}{\int d^3\mathbf{r}\, F_i^{||}(r)}$$

$$\begin{cases} p_i(r) = \frac{1}{6m}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}[D_i(t)]_{\text{FT}} \\ s_i(r) = -\frac{1}{4m}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}[D_i(t)]_{\text{FT}} \\ F_i^{||}(r) = p_i(r) + 2s_i(r)/3 \end{cases}$$

FT = Fourier transform
3D Breit frame

$r_{\text{gluon}} > r_{\text{quark}}$

Summary and outlook

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- First determination of total gravitational form factors of hadrons, including flavor decomposition into gluon, light-quark, and strange-quark contributions.
- Pion GFFs in agreement with chPT predictions
- Nucleon GFFs support certain experimental analyses
- Future work : more ensembles, continuum and physical limit extrapolation, improvements to renormalization

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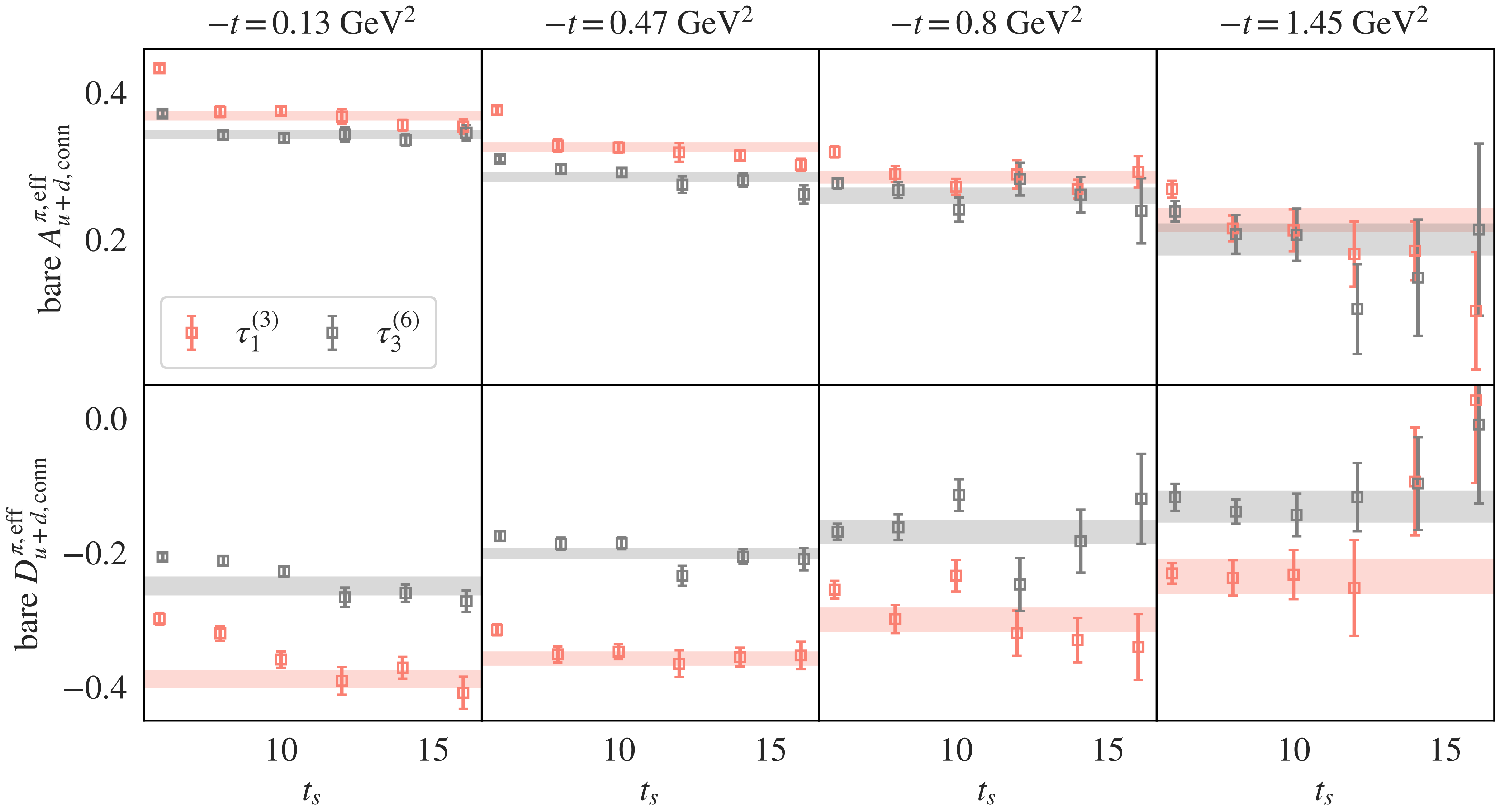
Thank you!

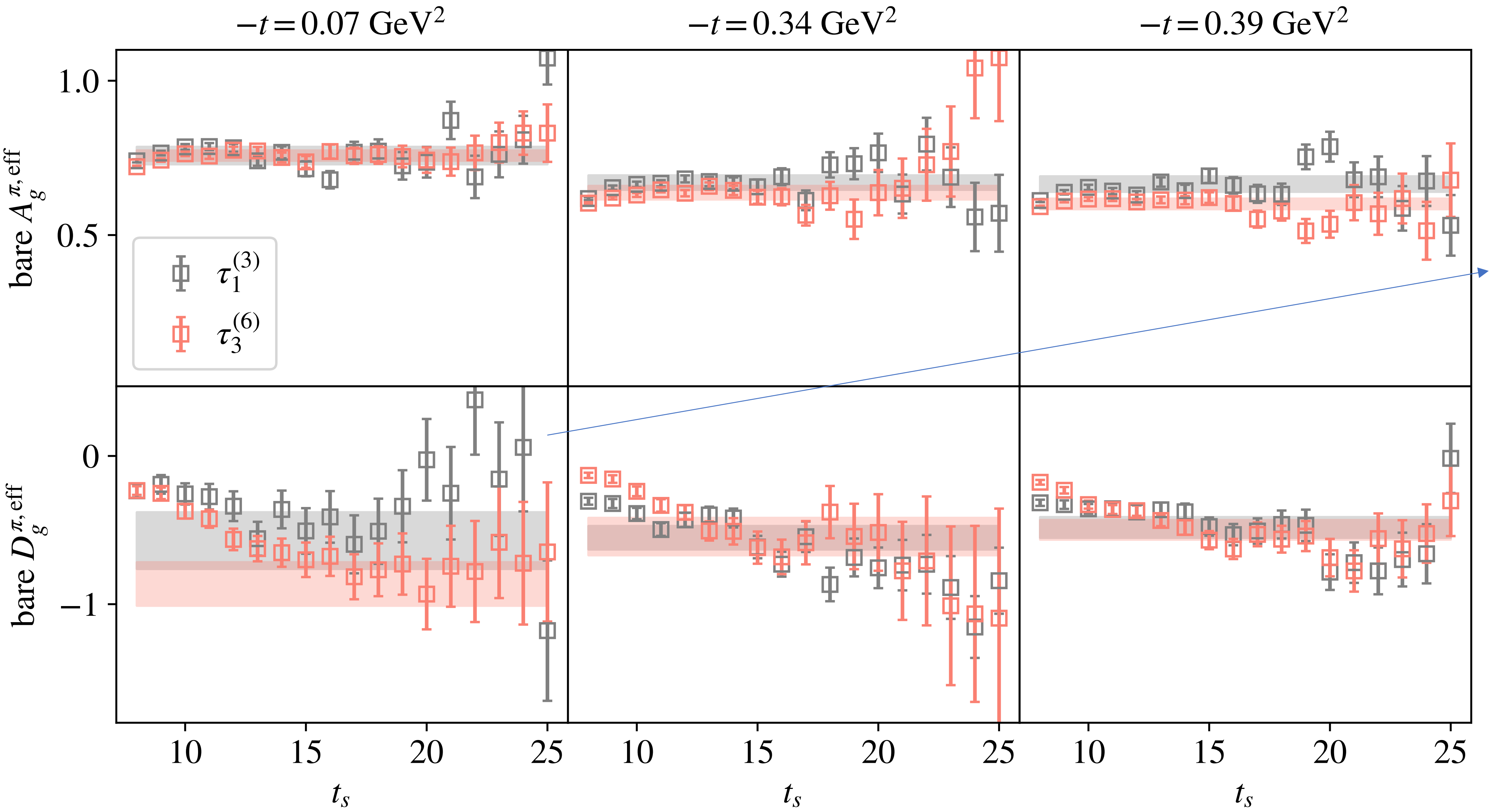
Back-up slides: pion connected quark



bands: bare GFFs from model-averaged summation fits used for the main analysis

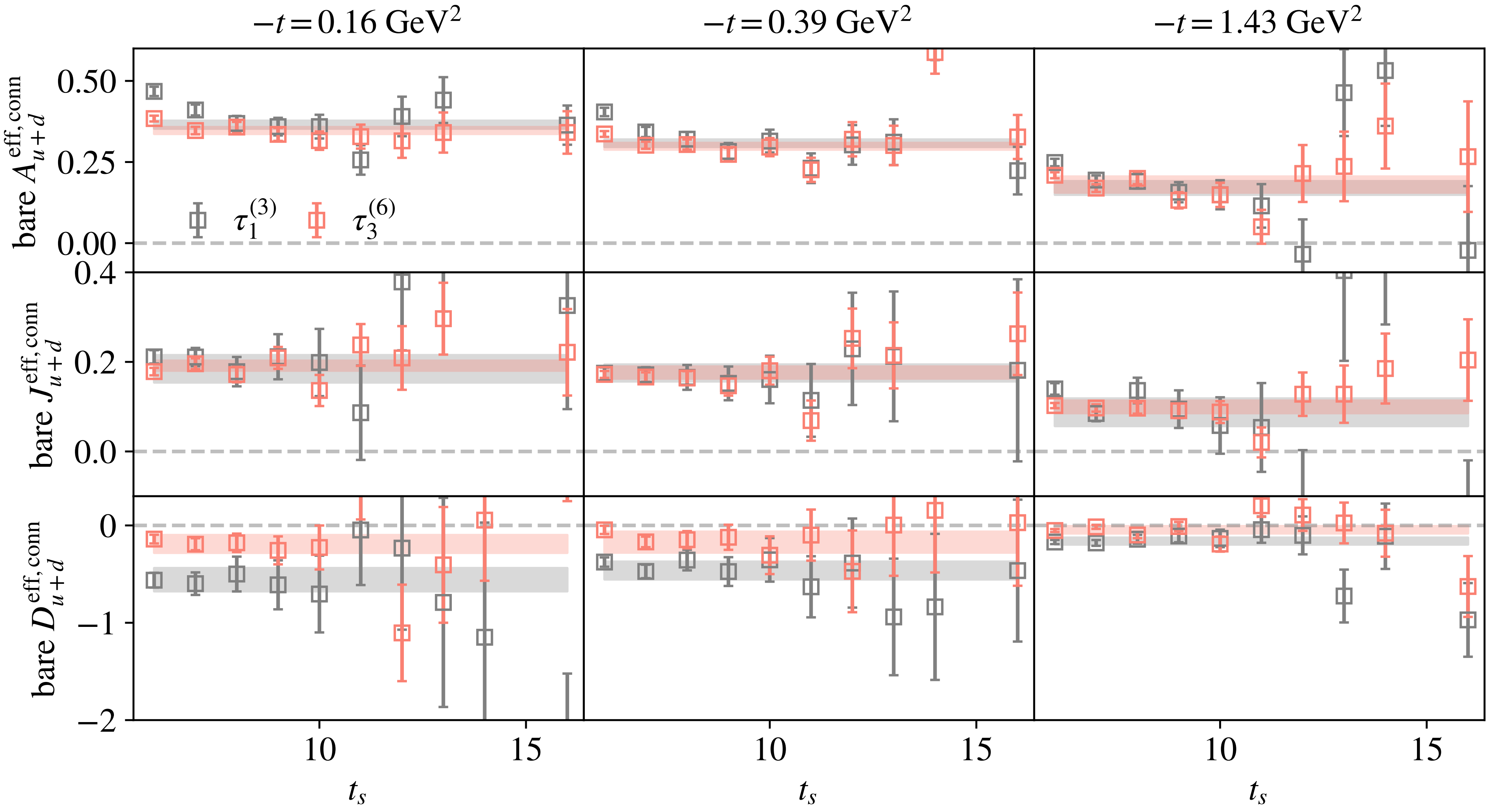
data points: “effective” GFFs used as a cross-check

Back-up slides: pion gluon

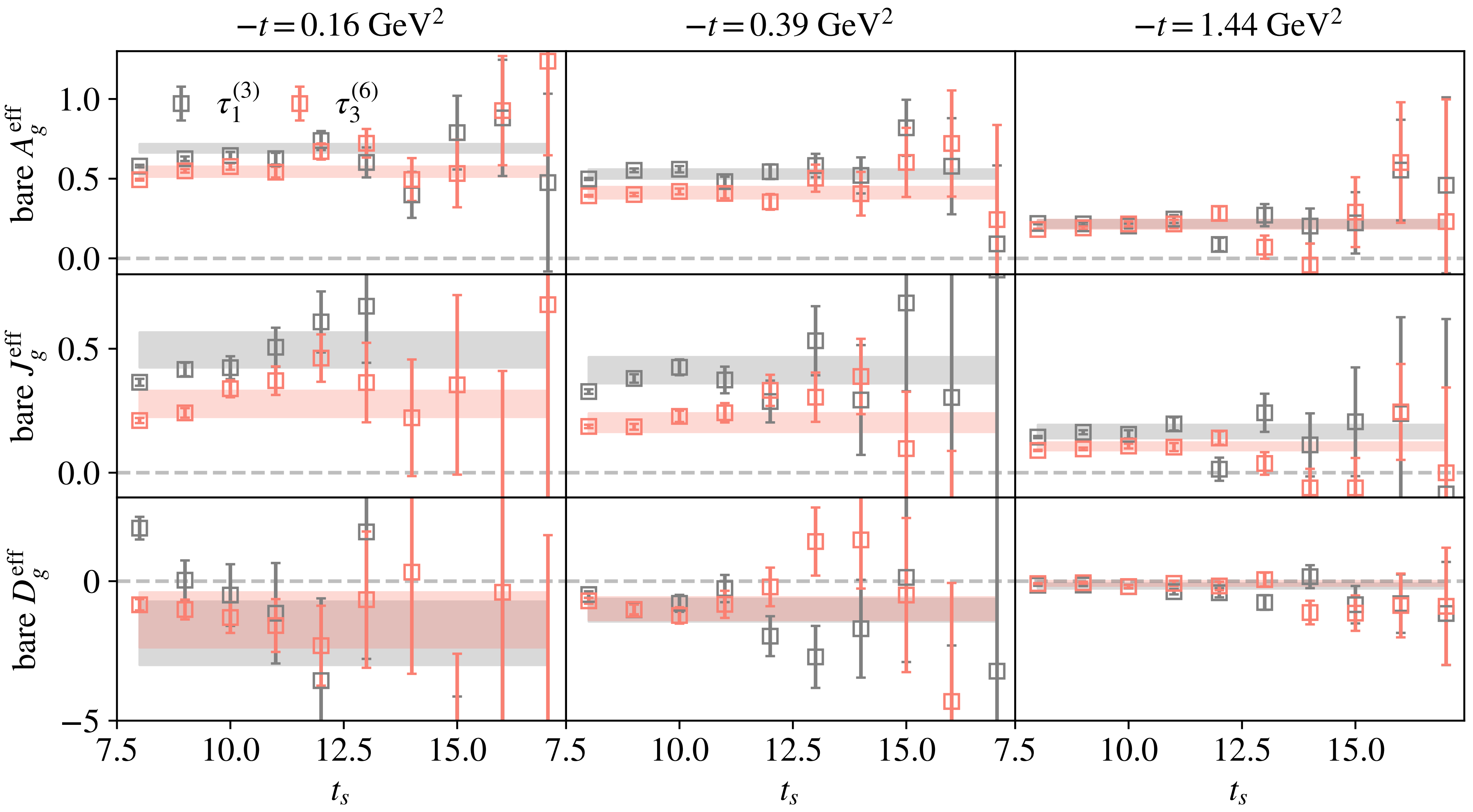


excited states affect extraction of D_g^π more strongly

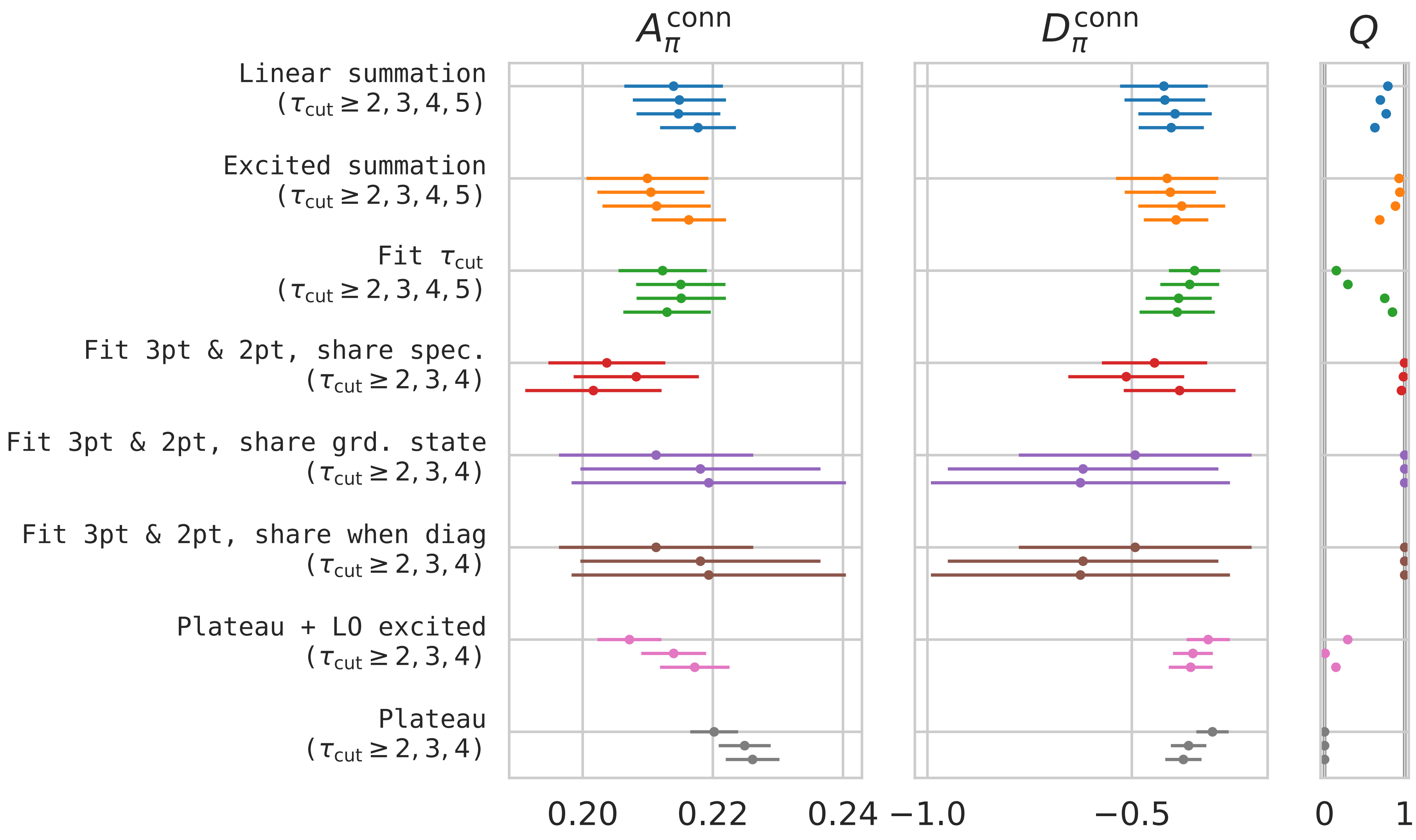
Back-up slides: nucleon u+d connected



Back-up slides: nucleon gluon



Back-up slides



Back-up slides