QCD phase diagram from lattice simulations

A. Yu. Kotov

Forschungszentrum JÜLICH

EINN 2023

QCD at large temperature and density

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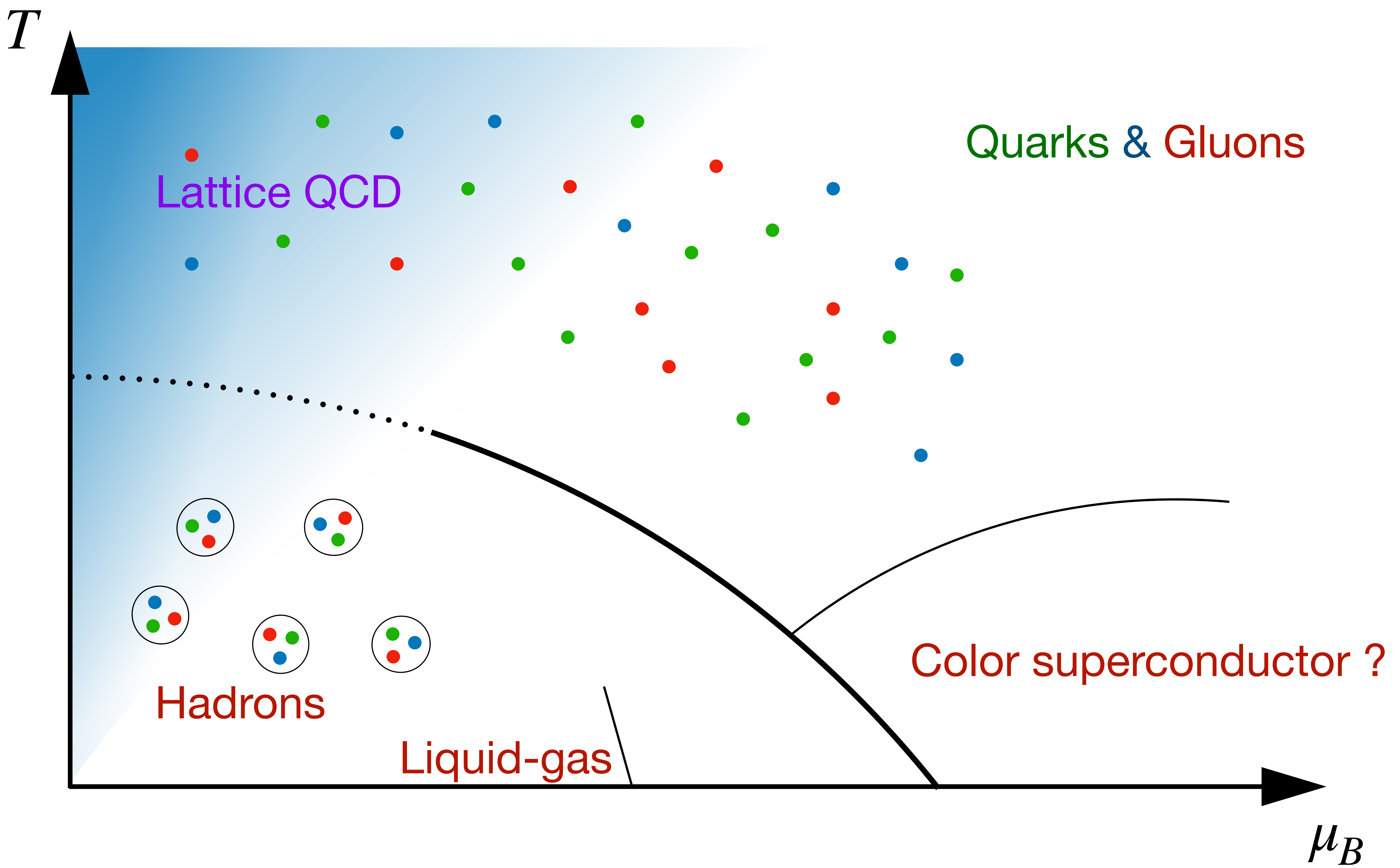
$$T \sim 10^{12-13}\ \mathrm{K} \sim 10^{2-3}\ \mathrm{MeV}$$

$$n \sim n_0 \approx 0.15\ \mathrm{fm}^{-3} \sim 10^6\ \mathrm{MeV}^3$$

Sketch of the phase diagram



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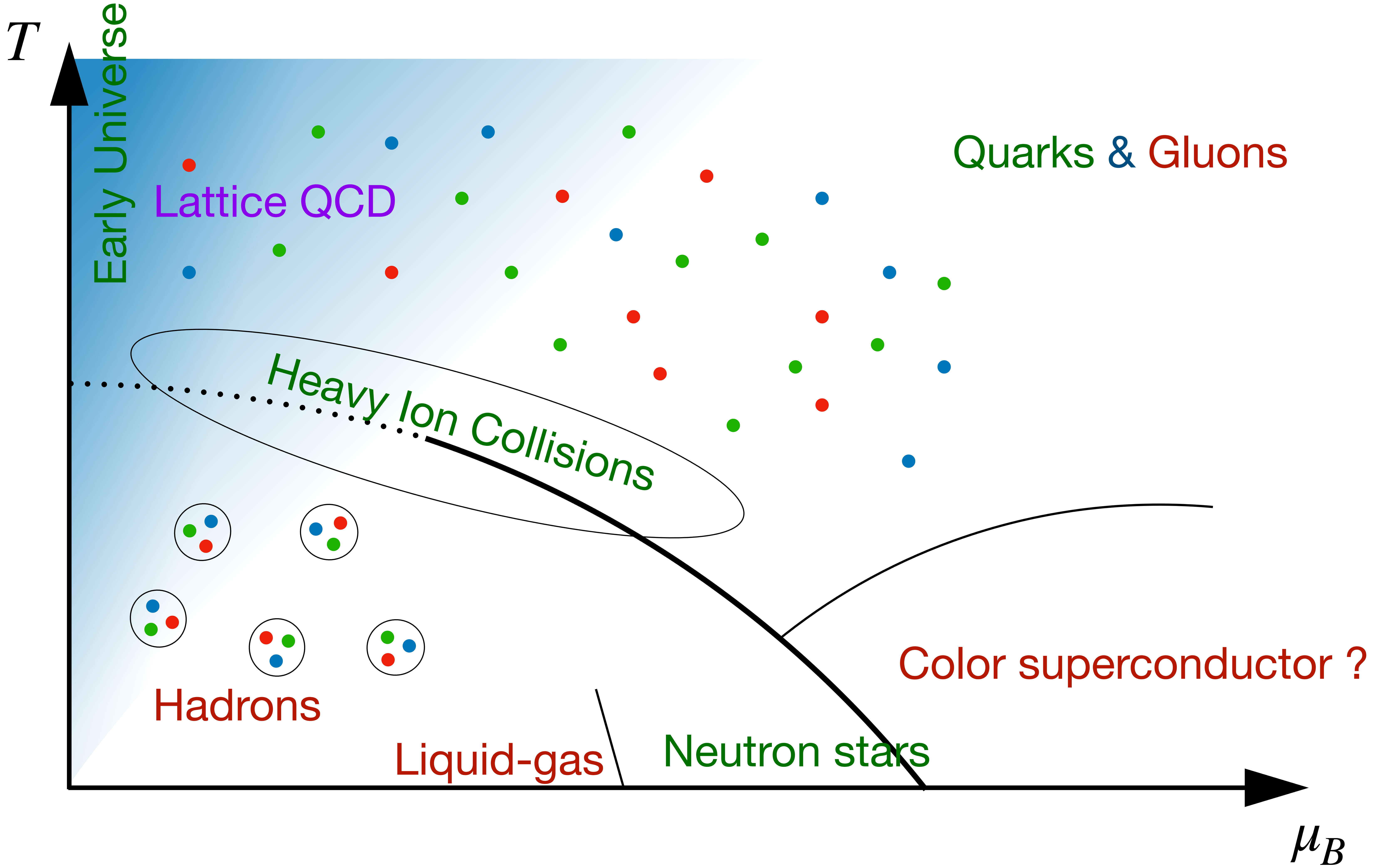
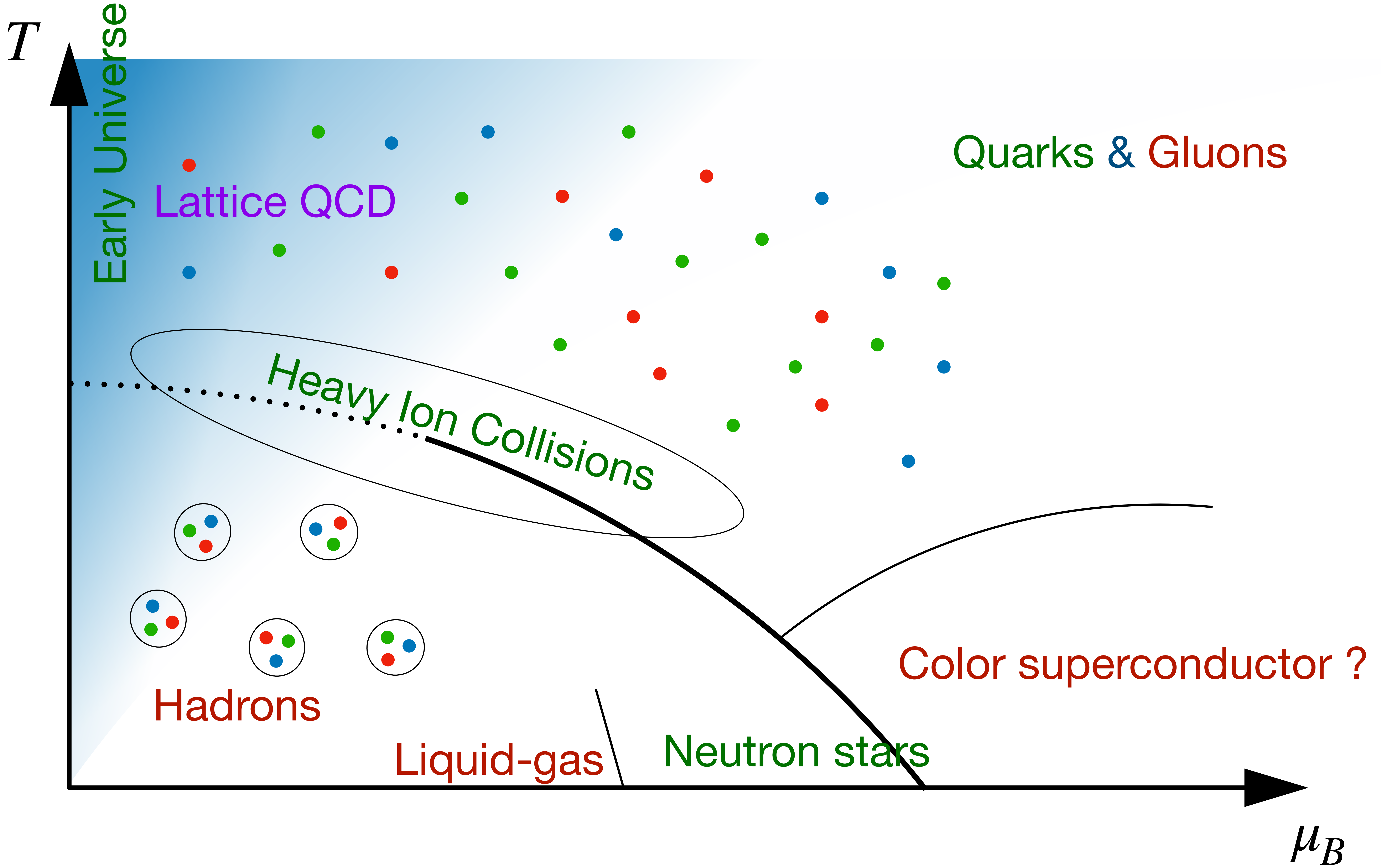


Sketch of the phase diagram

SKETCH!

Precise data: (mainly) lattice QCD in the blue region



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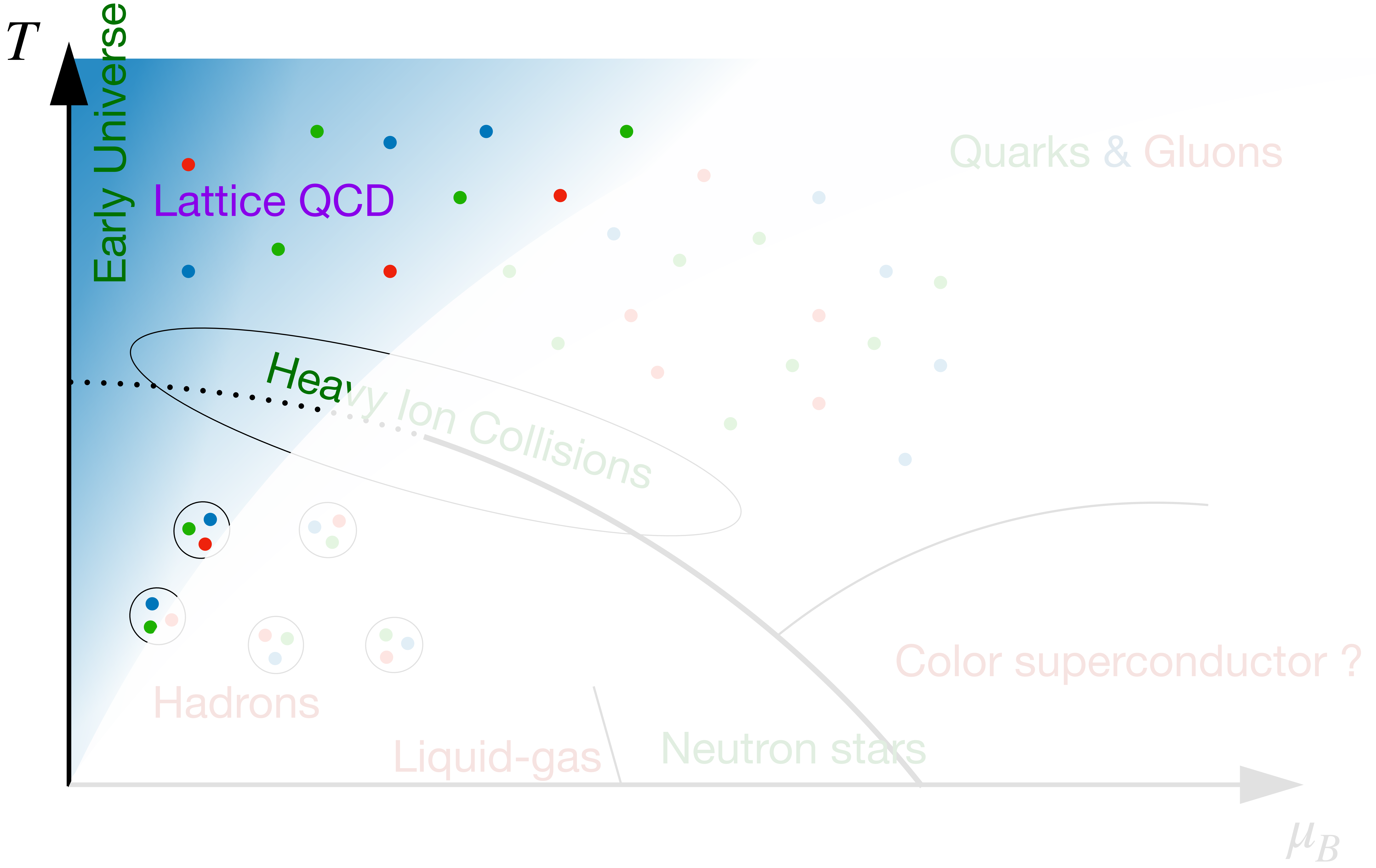
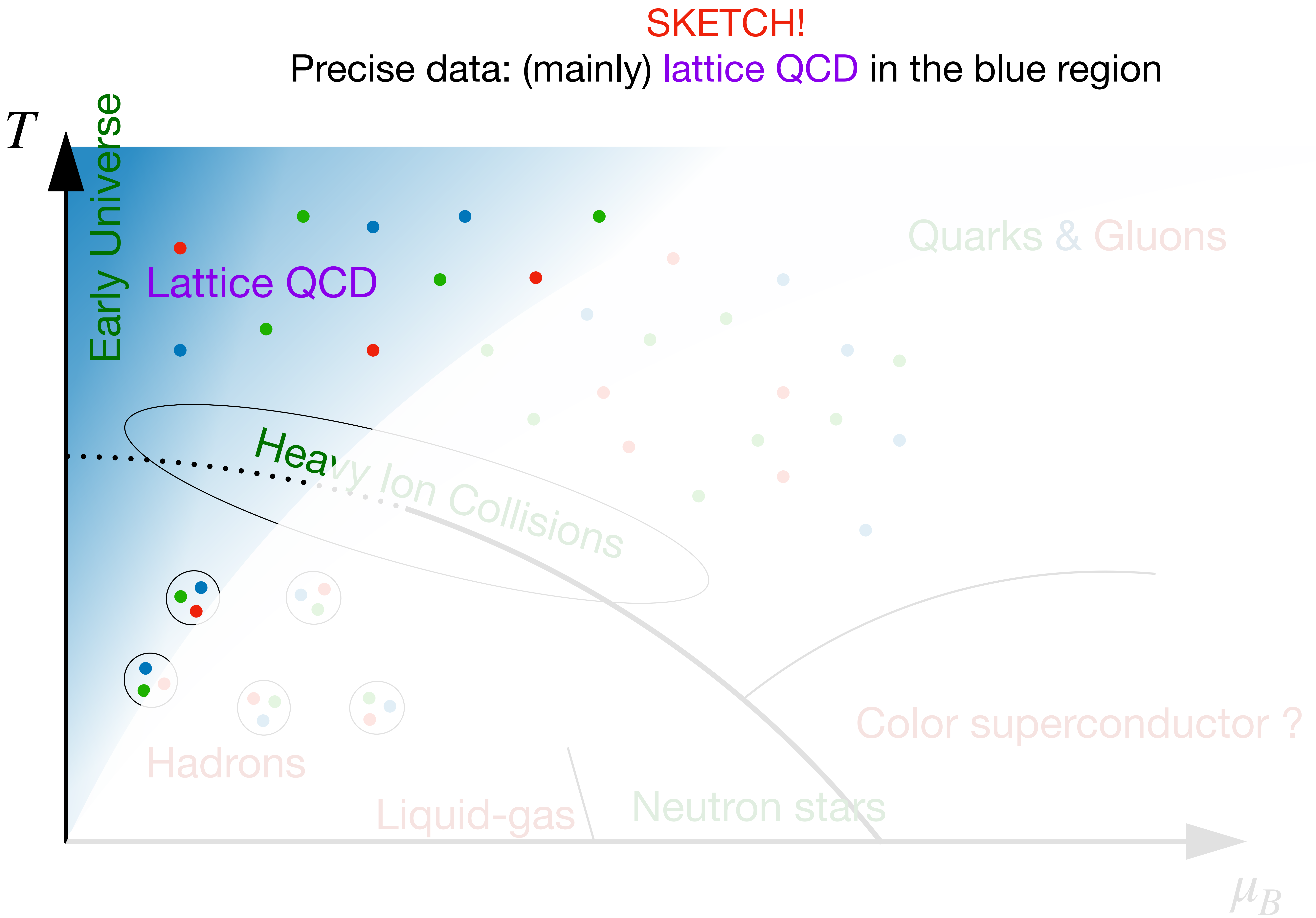


What is covered in this talk?

Overview of the lattice QCD results on:

Sketch of the phase diagram



What is covered in this talk?

Overview of the <u>lattice QCD</u> results on:

- <u>Phase diagram</u> of QCD in (T, μ) plane
- <u>Equation of State</u> (EoS) and <u>conserved charge fluctuations</u>

Thermodynamics and lattice QCD

Grand-canonical partition function

$$Z = \mathrm{tr}\ e^{-(H_{\mathrm{QCD}} - \mu_B B - \mu_Q Q - \mu_S S)/T} \rightarrow \int DA_\mu D\bar{\psi} D\psi e^{-S_{\mathrm{QCD}}} \rightarrow \text{Computer}$$

All field configurations

Thermodynamics and lattice QCD

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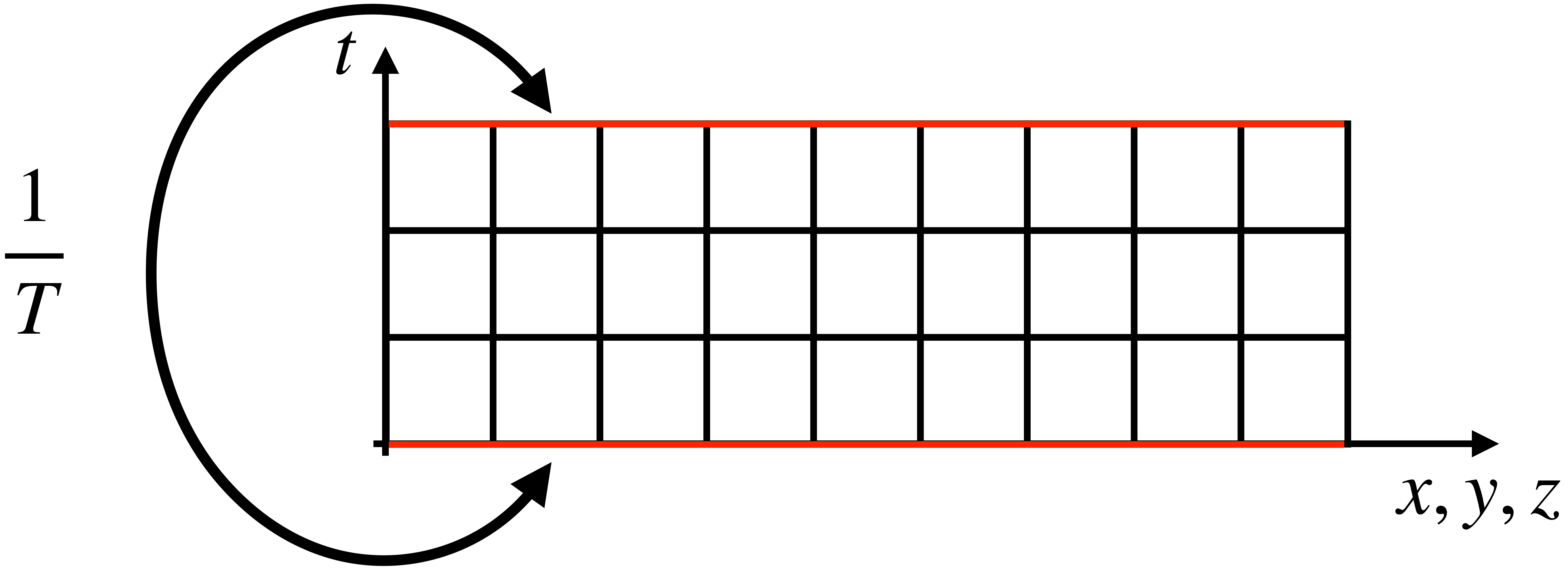
$T \neq 0$: Compact time $A_\mu(t + L_4, x, y, z) = A_\mu(t, x, y, z), \quad L_4 = 1/T$



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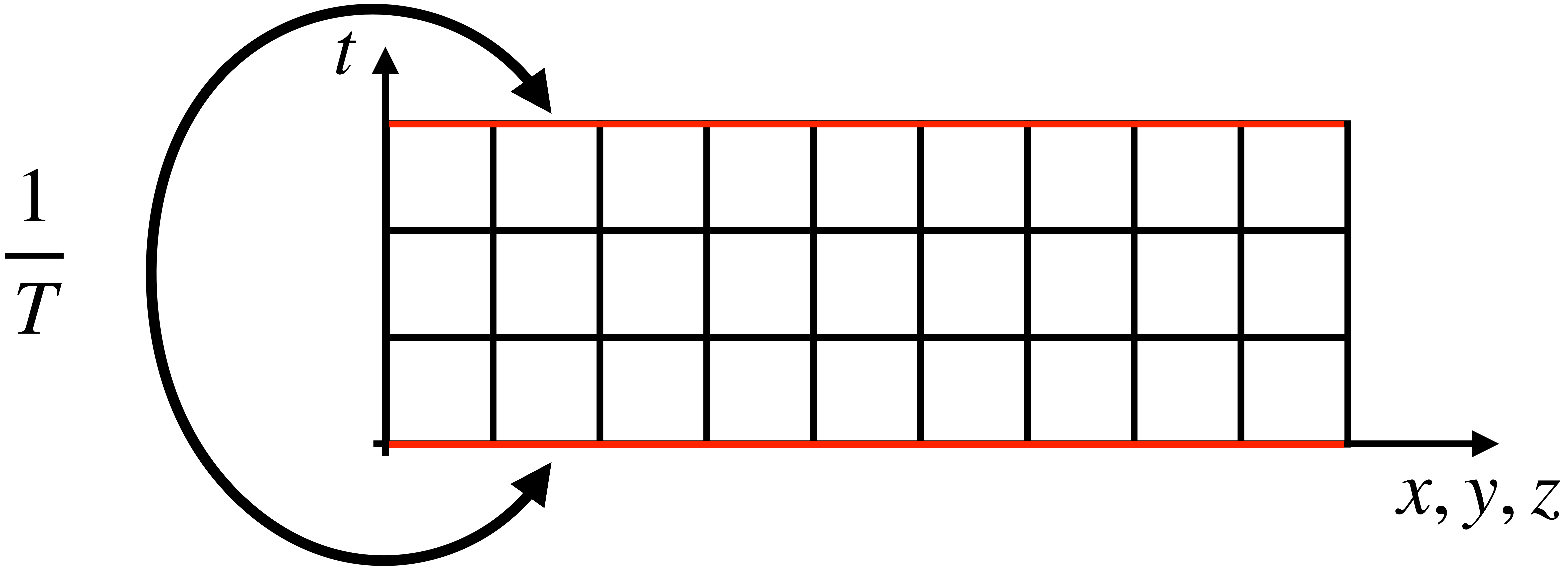

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- Powerful supercomputers (typical cost: 100 years on your gaming computer)
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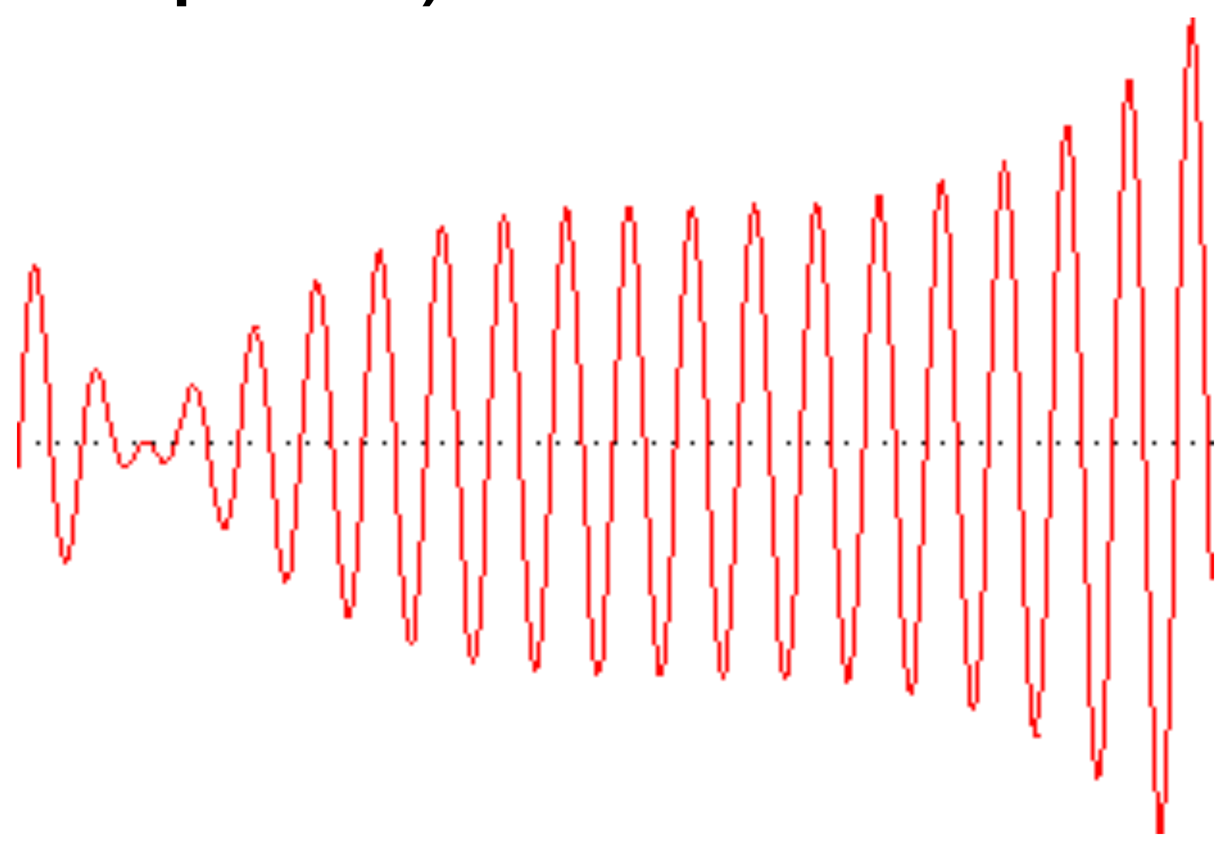

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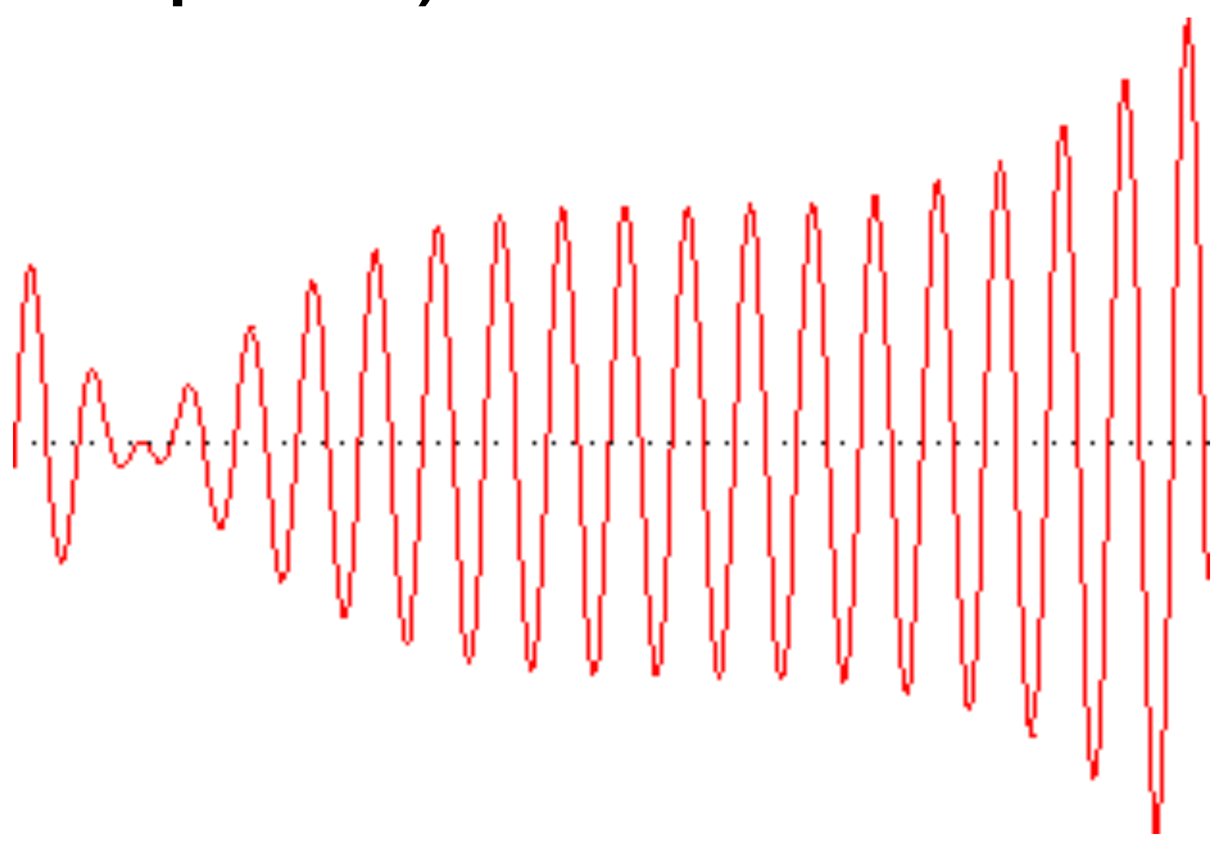

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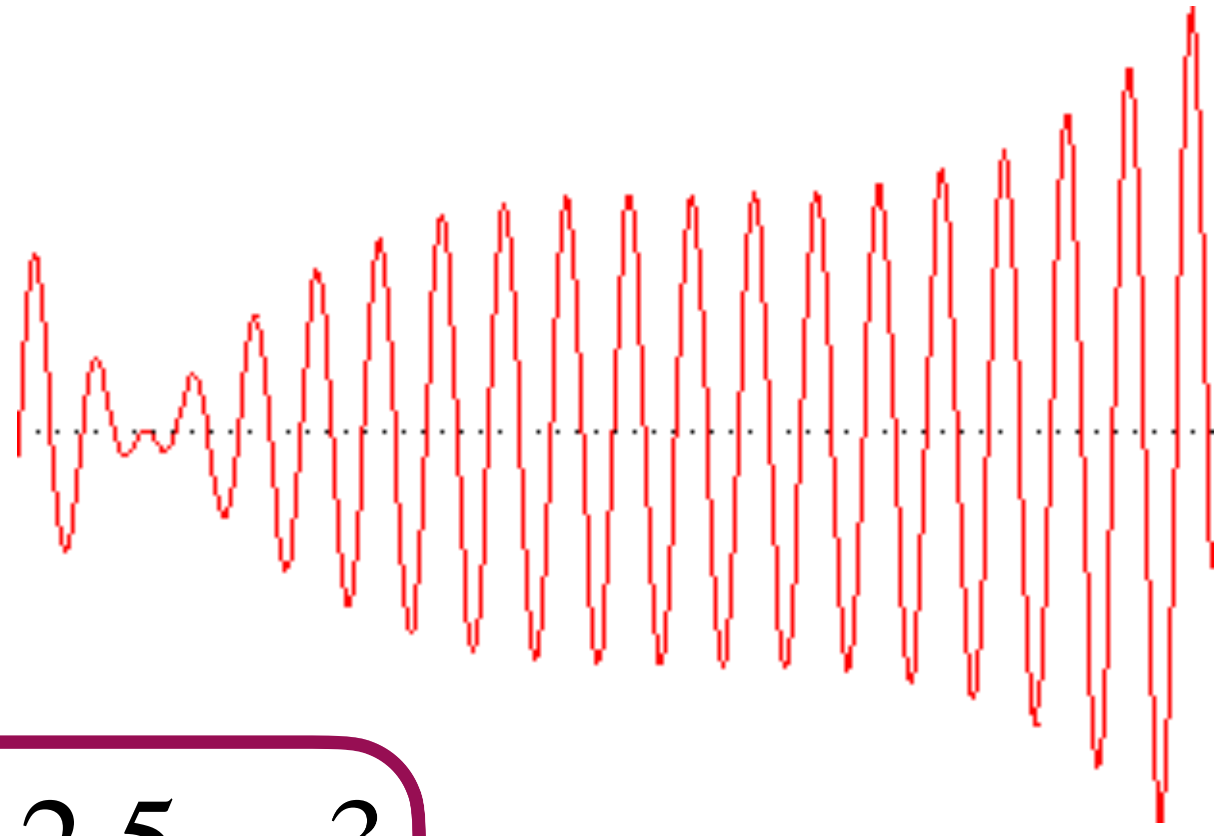
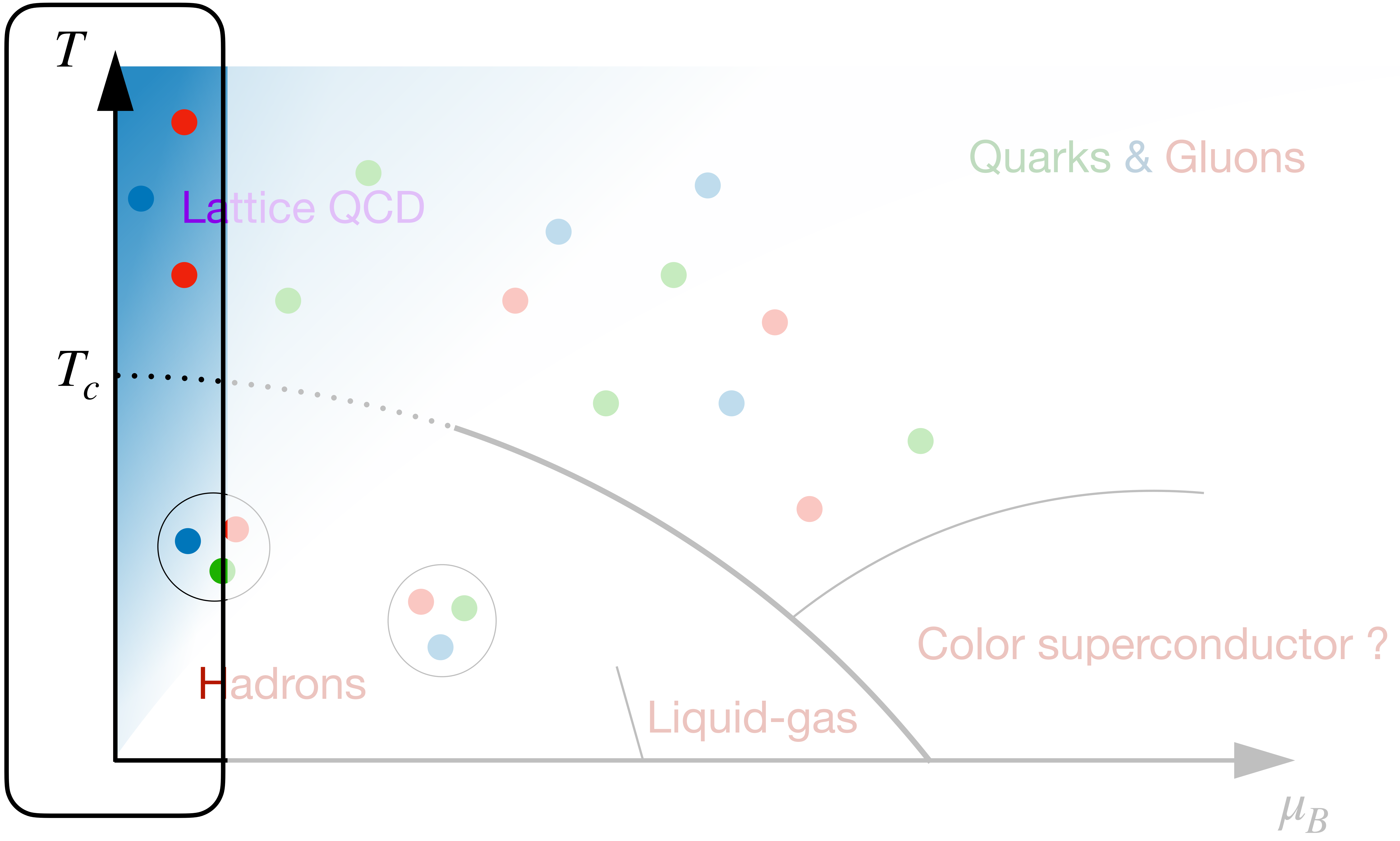


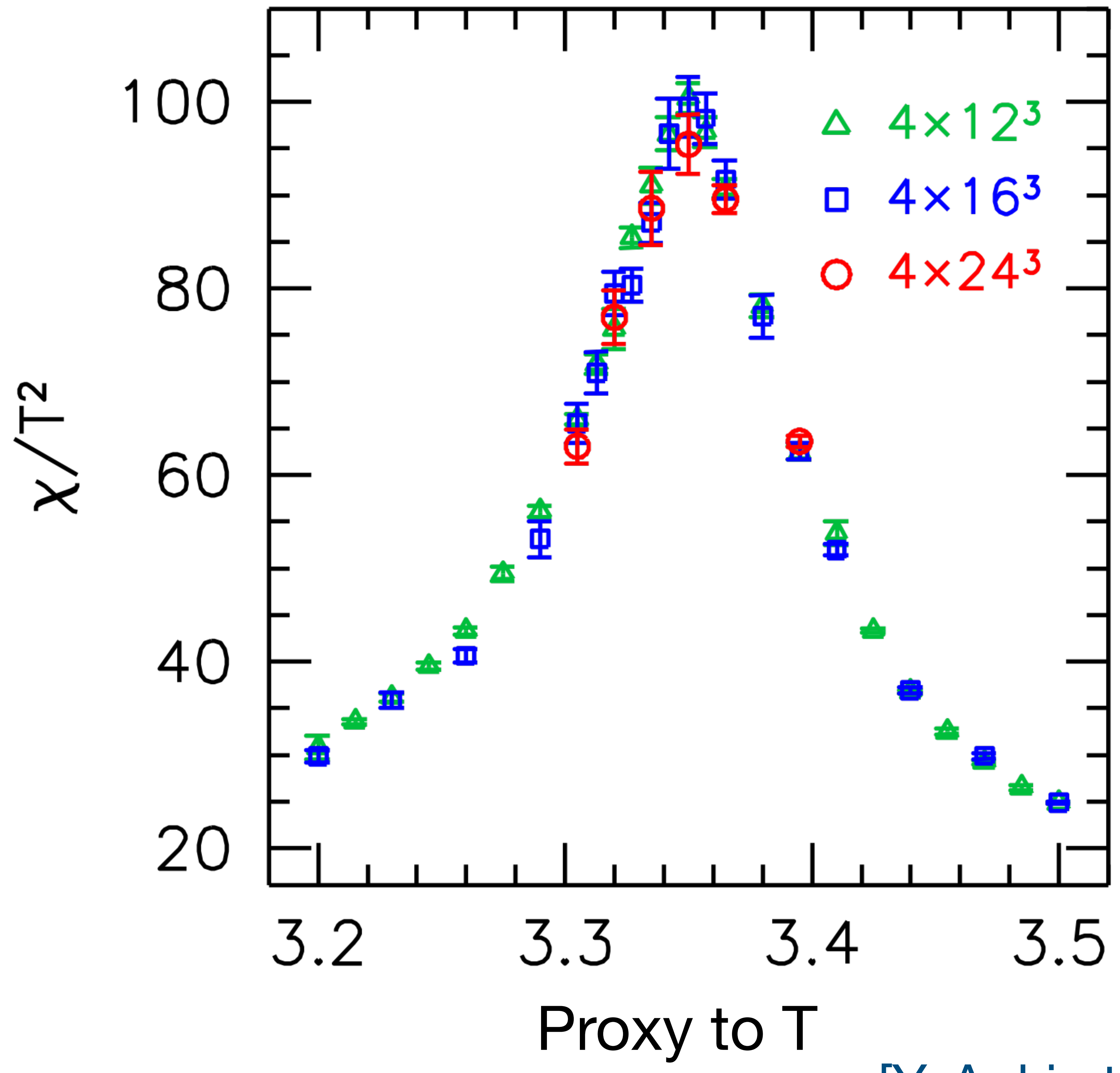
$\boxed{\text{Only work for small } \mu_B/T \lesssim 2.5 - 3}$

Zero μ, nonzero T

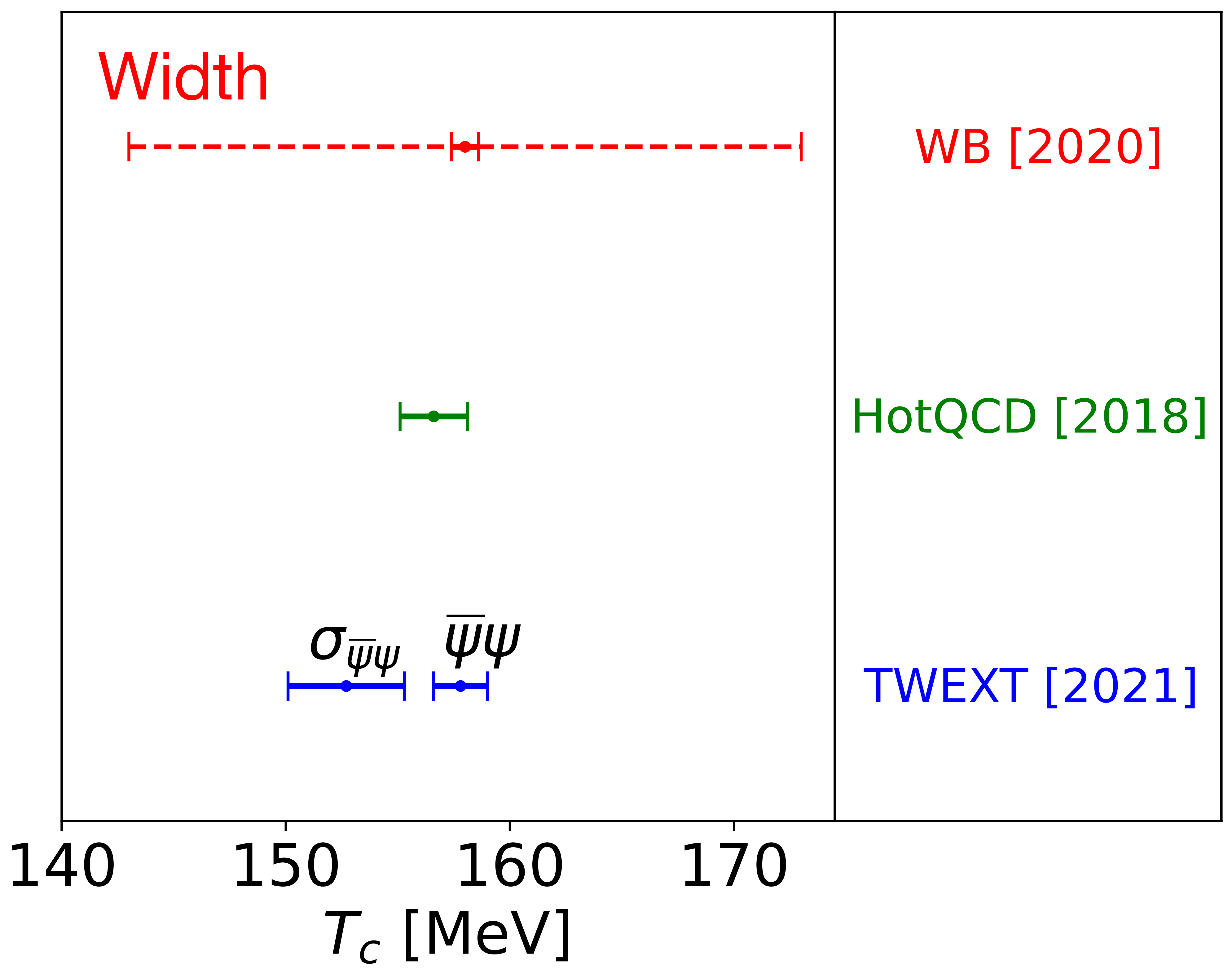


Zero μ, nonzero T

Nature of the chiral phase transition and (pseudo)-critical temperature T_c



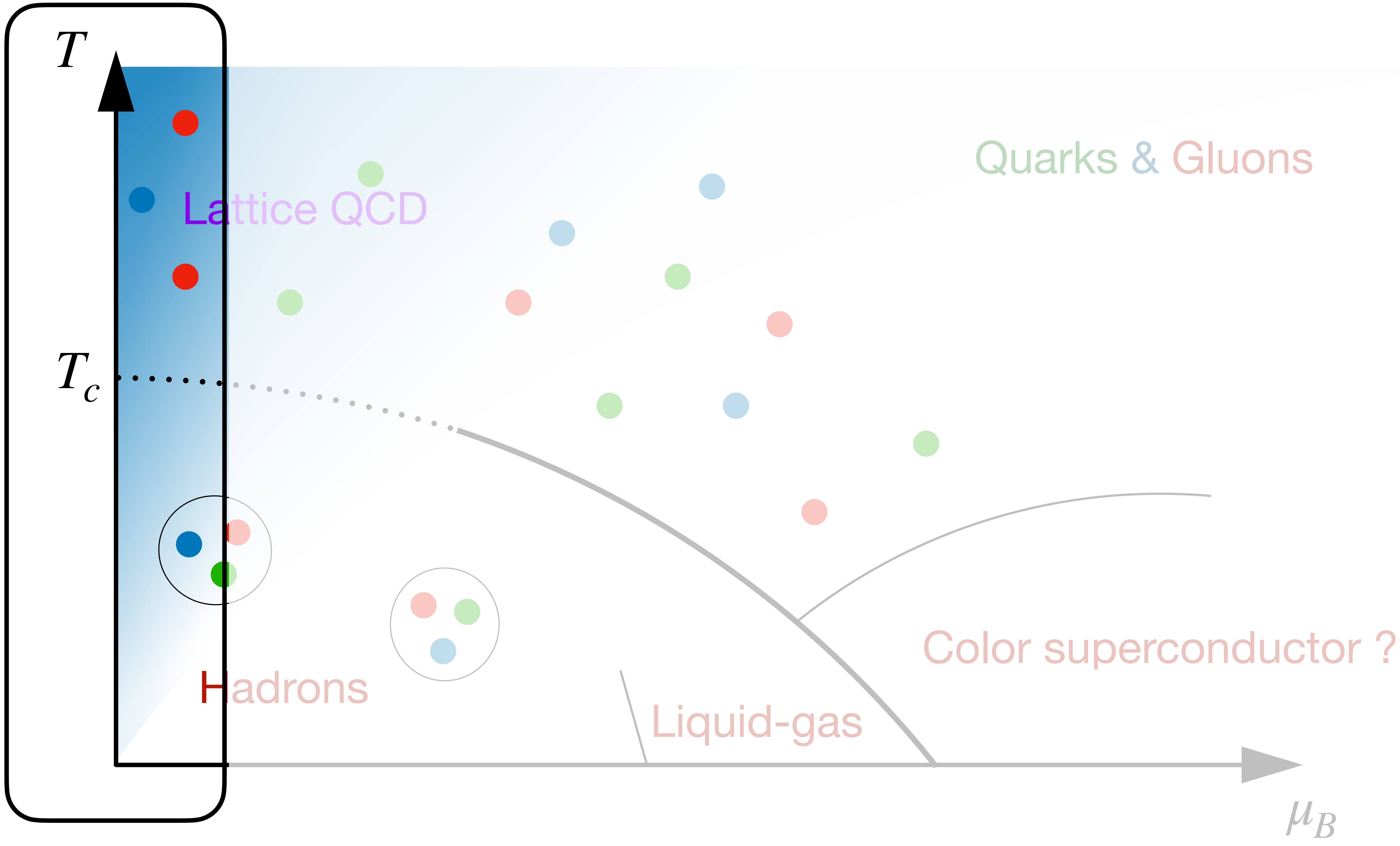
[Y. Aoki et al., 2006]



Crossover: finite width

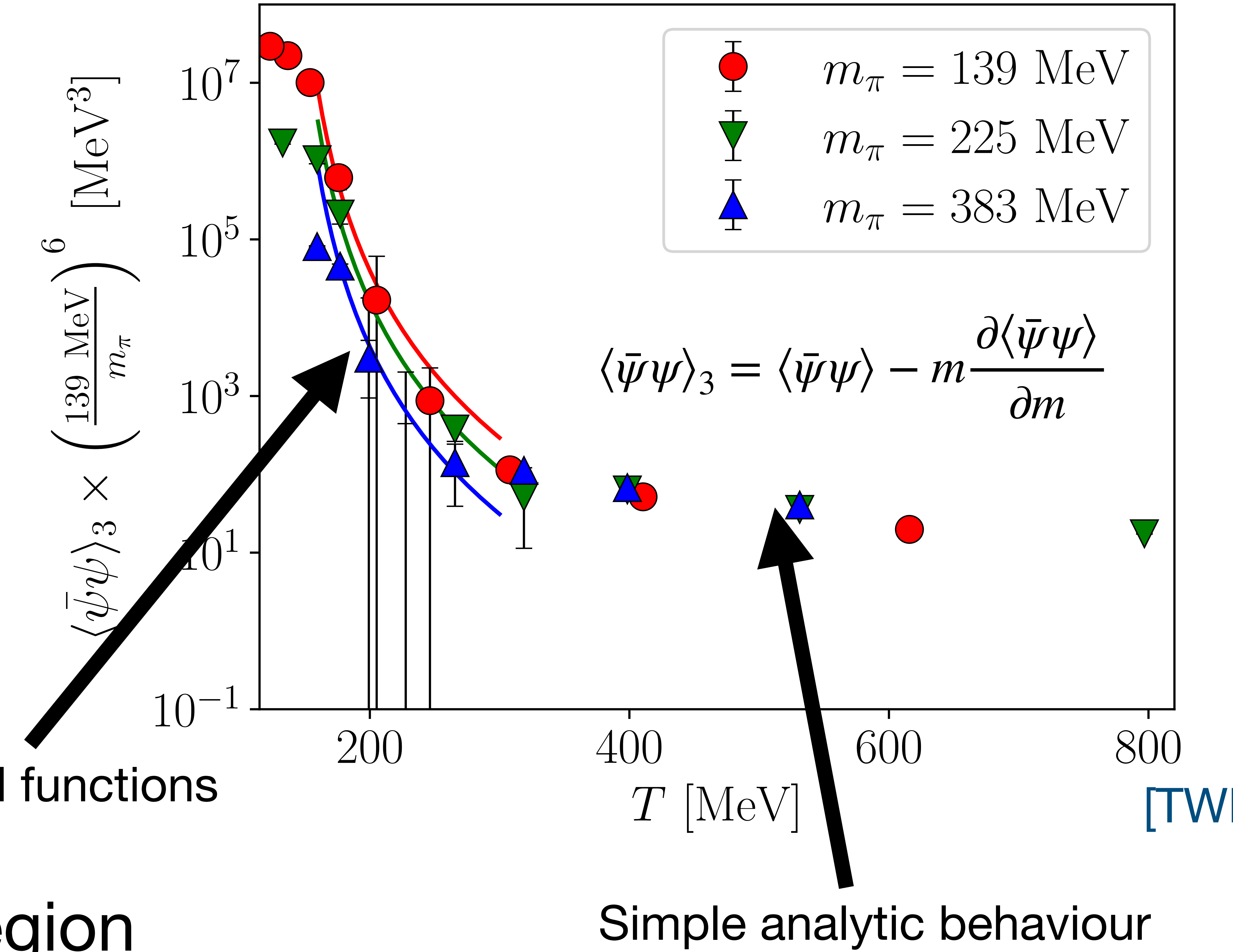
Different observables: different T_c

Zero μ, nonzero T



Zero μ, nonzero T

Possible thresholds/phase transitions and other scenarios



[TWEXT, 2021]

$T \sim 300$ MeV

Scaling window: region near the phase transition with universal behaviour

Same picture in the topological properties: onset of DIGA behaviour

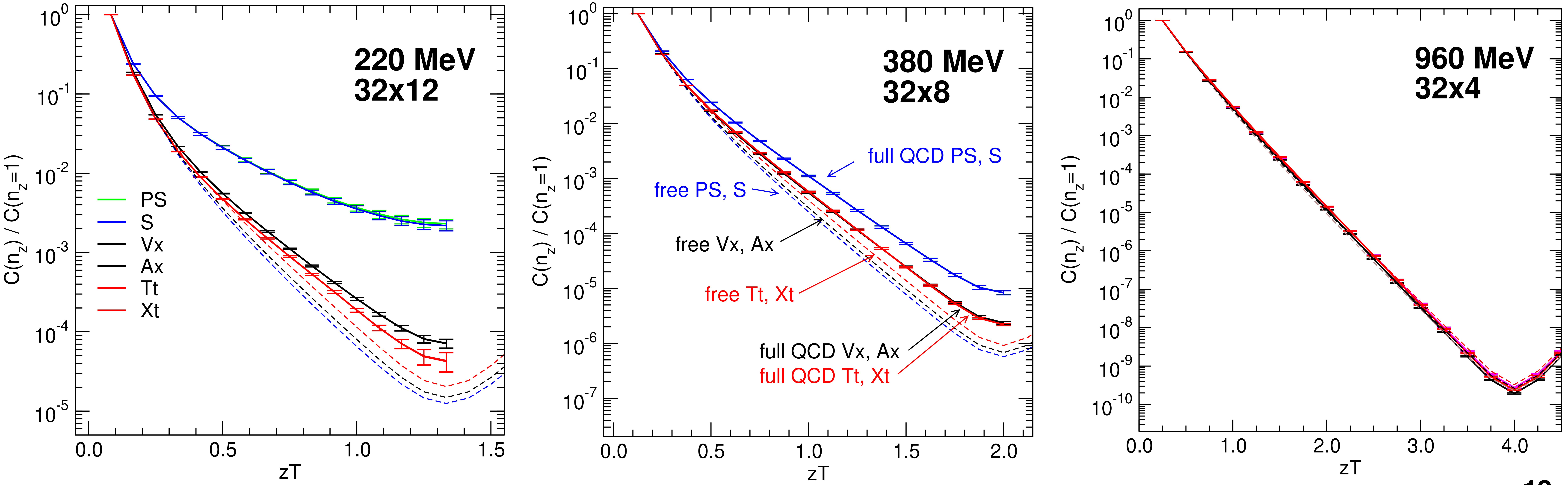
[TWEXT, 2021]

Zero μ, nonzero T

Possible thresholds/phase transitions and other scenarios

New Chiral-Spin $SU(2)_{CS}$ symmetry: $T_{\mathrm{ch}} \lesssim T \lesssim 3T_{\mathrm{ch}}$

[Rohrhofer et al., 2020]



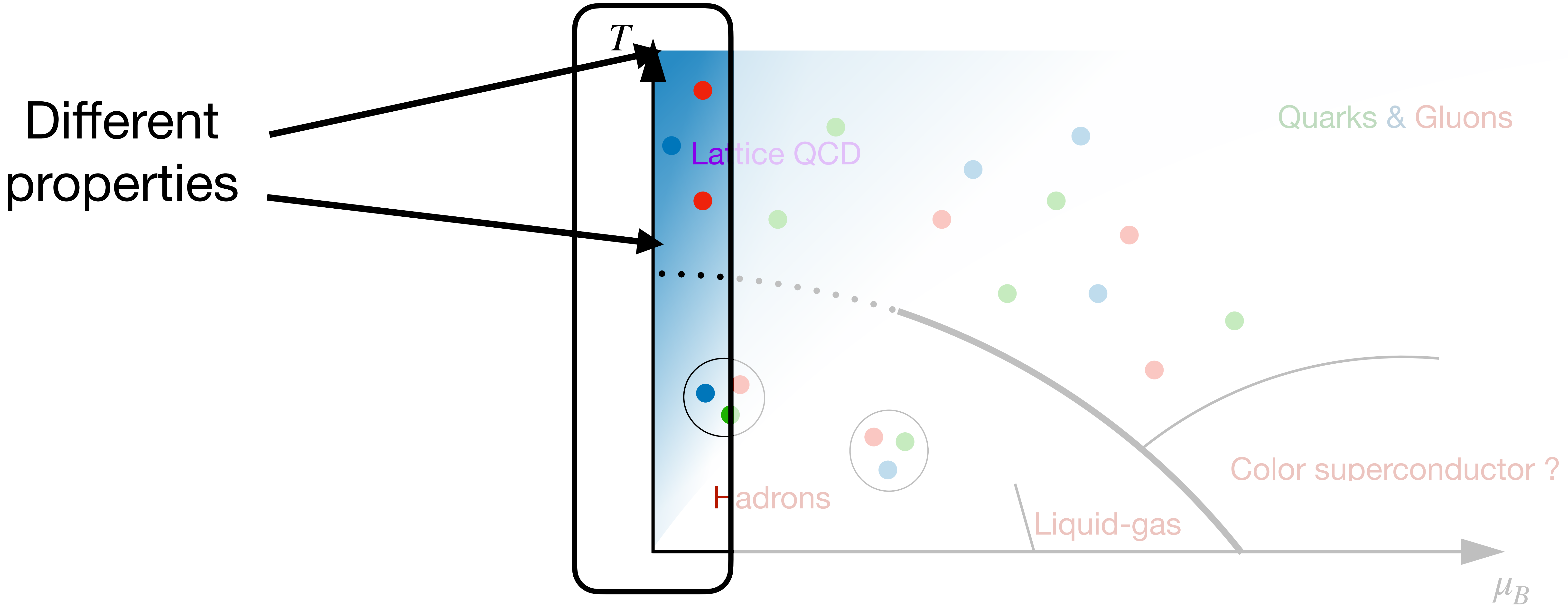
Zero μ, nonzero T

Possible thresholds/phase transitions and other scenarios

- IR phase: $T \lesssim 250\ \mathrm{MeV}$ [A.Alexandru and I. Horvath, 2019]
- Monopole condensation: $T \lesssim 275\ \mathrm{MeV}$ [M.Cardinali, M.D'Elia, A. Pasqui, 2021]

Zero μ, nonzero T

- Several (lattice) indications that regions $T \gtrsim T_c$ and $T \gg T_c$ are different

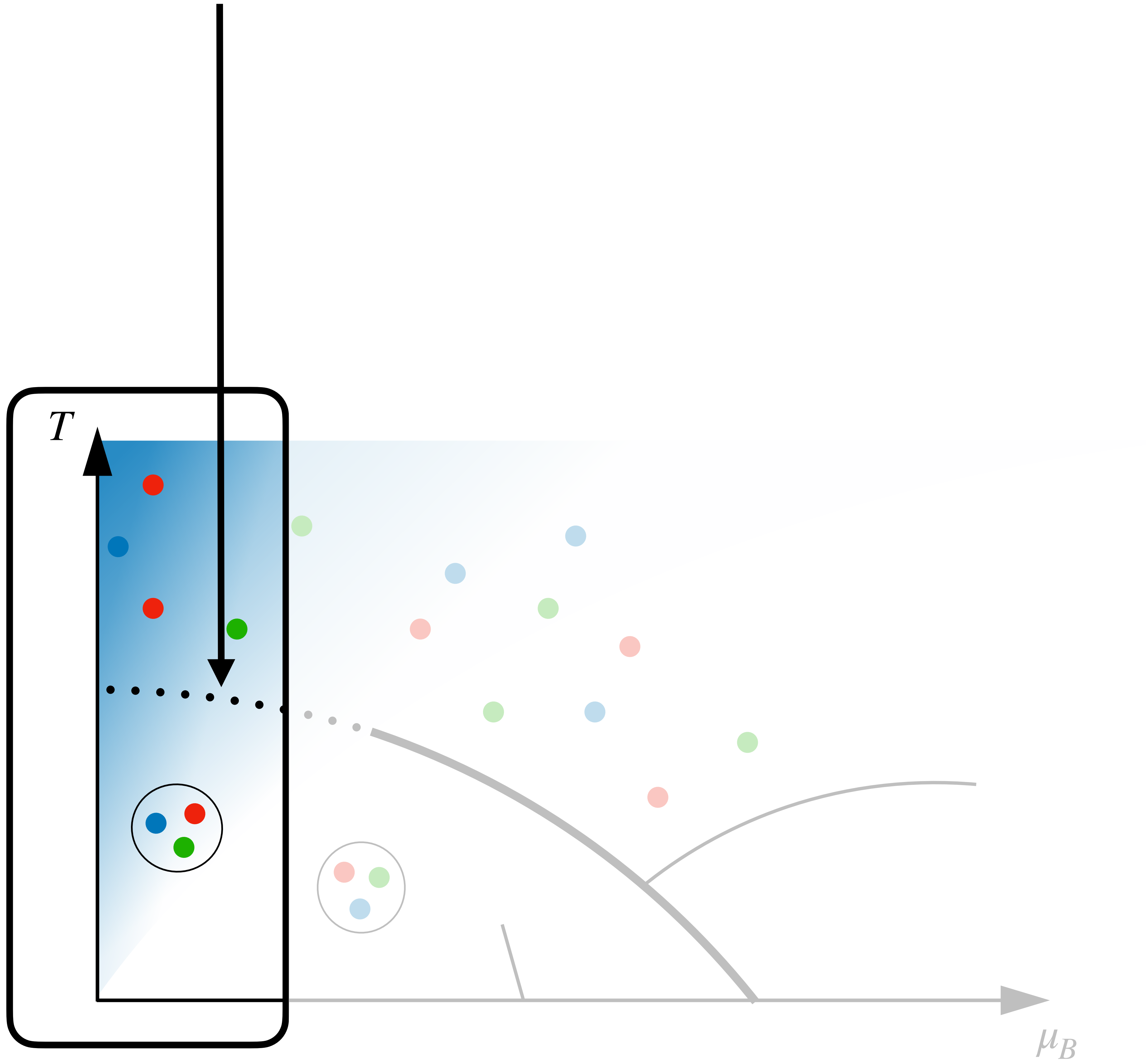


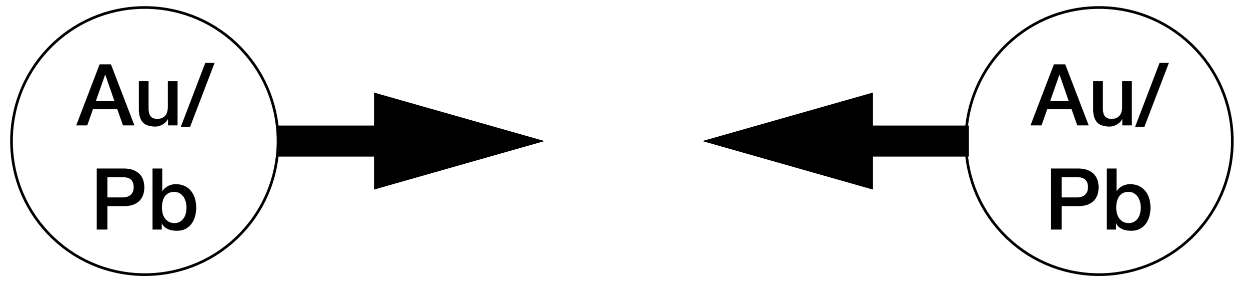
Possible transitions or thresholds: $T \sim 2 - 3\ T_c$
No consensus or full understanding of the physics

Small μ, nonzero T

Taylor expansion

$$T_c(\mu_B) = T_0\left(1 - \kappa_2\left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_0}\right)^4\right), \quad T_0 \equiv T_c(\mu_B = 0)$$





Heavy Ion Collisions:

$$S = 0, \quad Q \approx 0.4B \longrightarrow \mu_{Q,S} \equiv \mu_{Q,S}(\mu_B)$$

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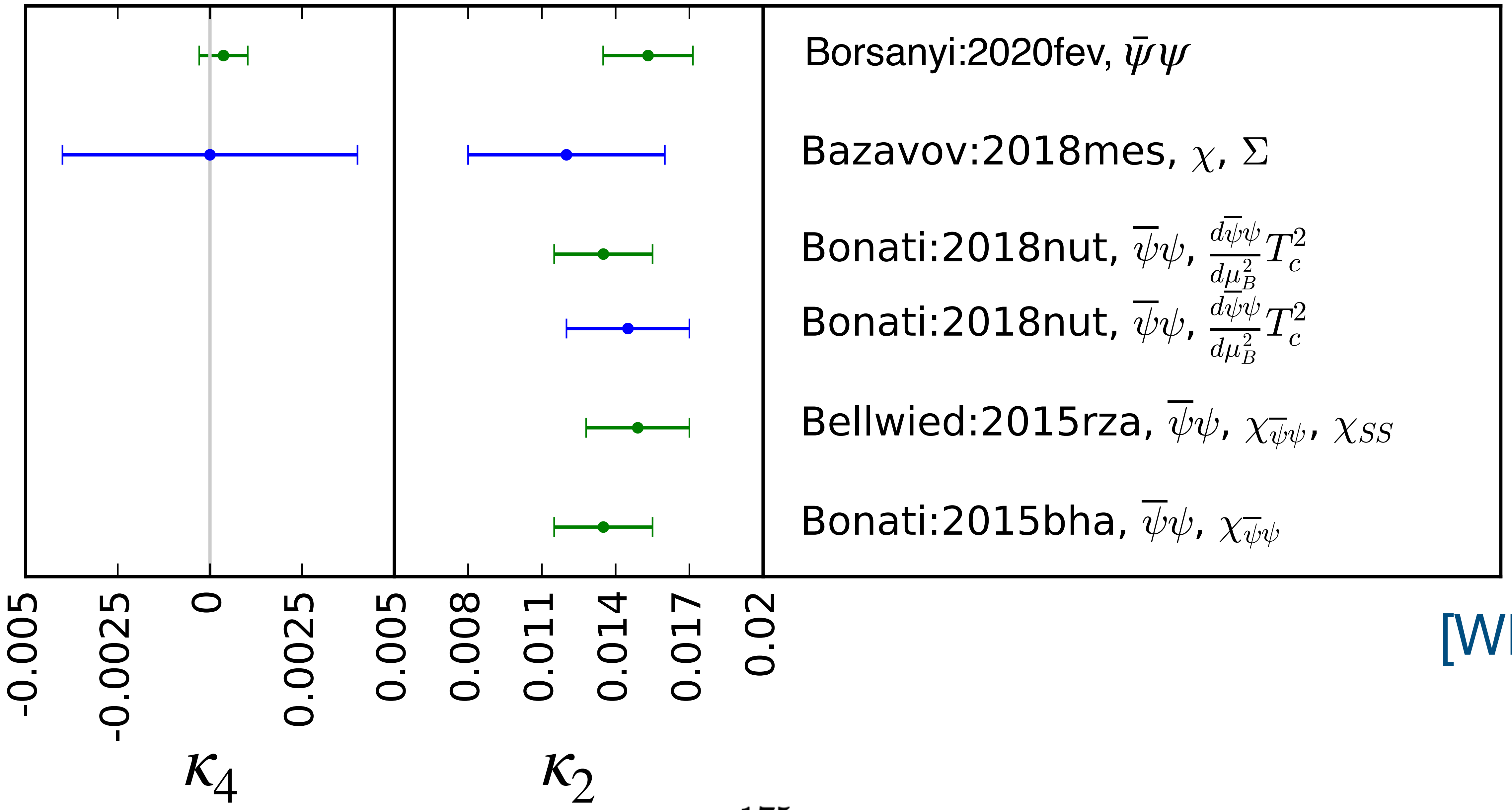
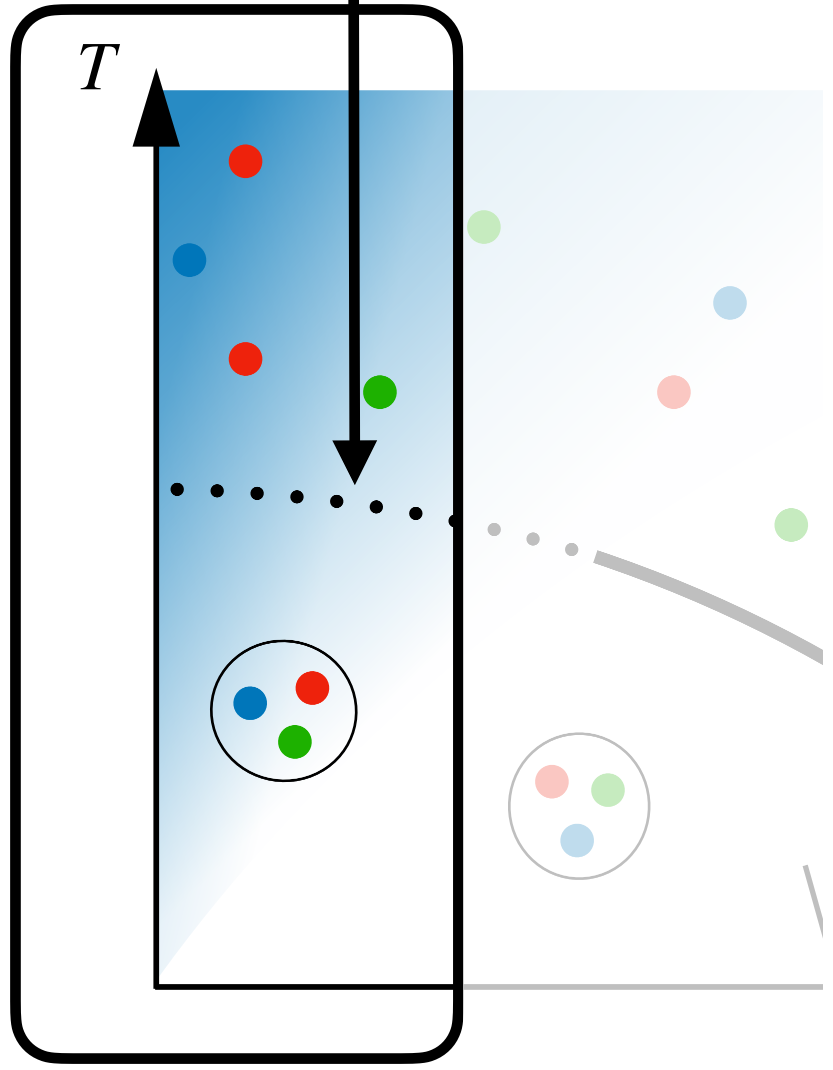
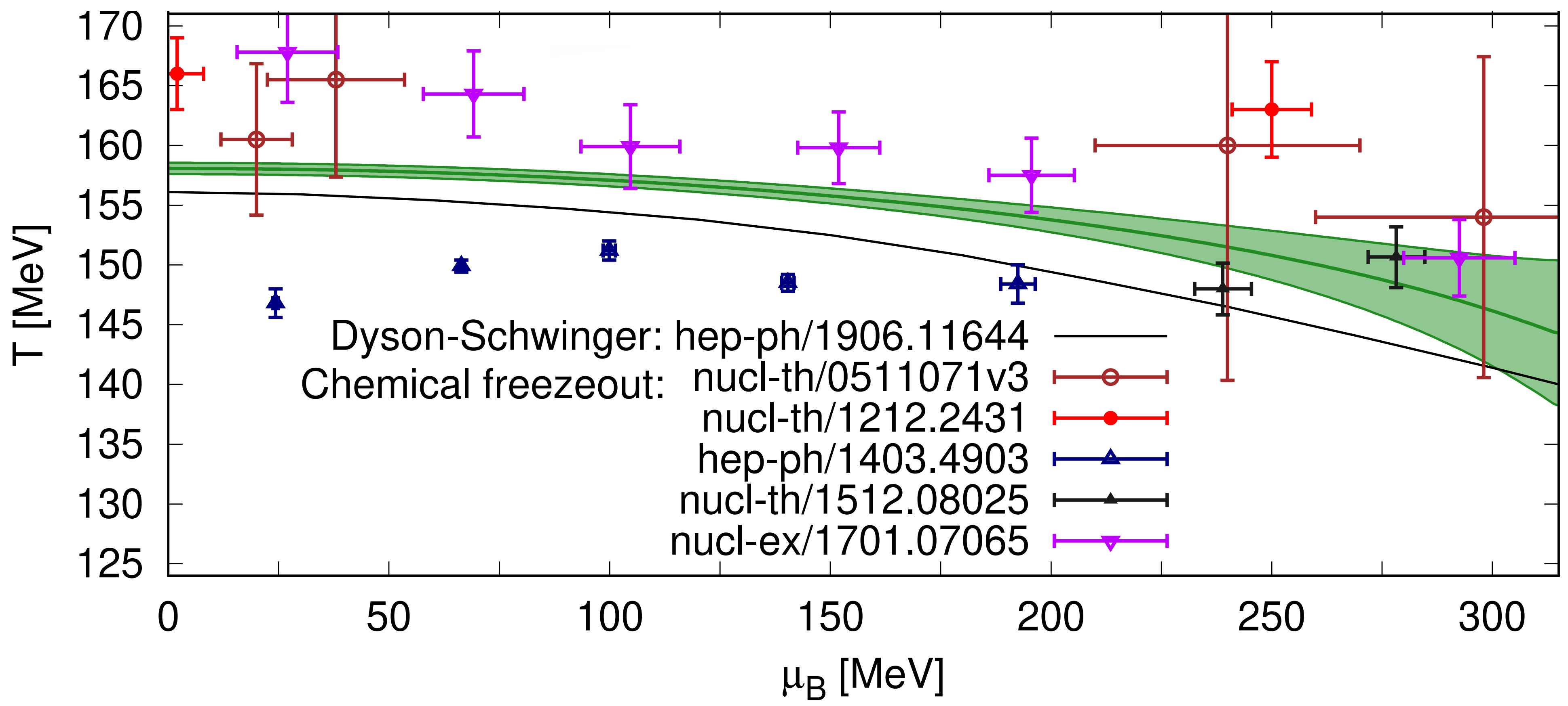


[WB, 2020]





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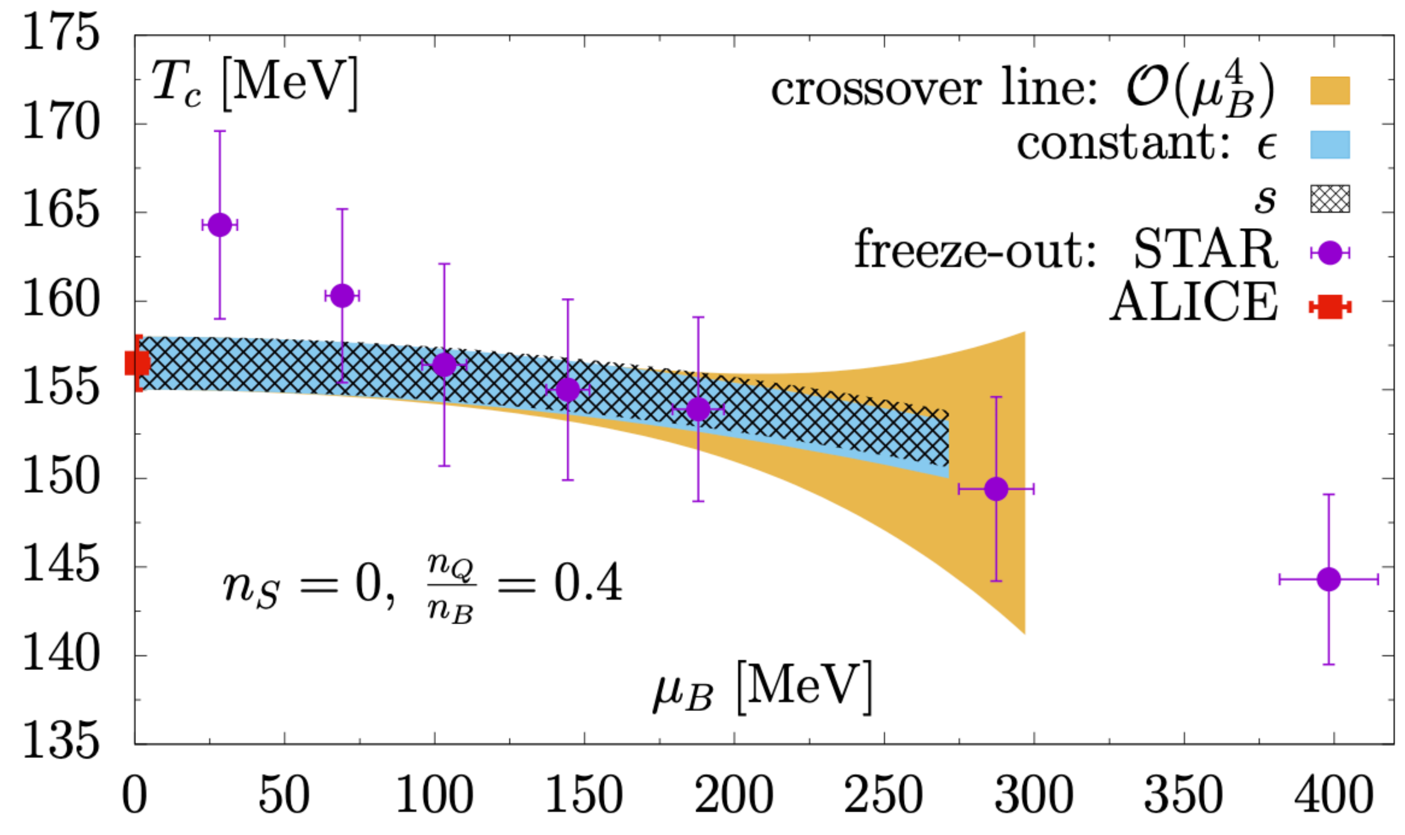

[HotQCD, 2018]

Small μ, nonzero T

Equation of State

- Pressure $p(T, \mu_B)$:
  - Baryon density $n_B = \dfrac{\partial p}{\partial \mu_B}$
  - Entropy density $s = \dfrac{\partial p}{\partial T}$
  - Energy density: $\epsilon = Ts - p + \mu_B B$

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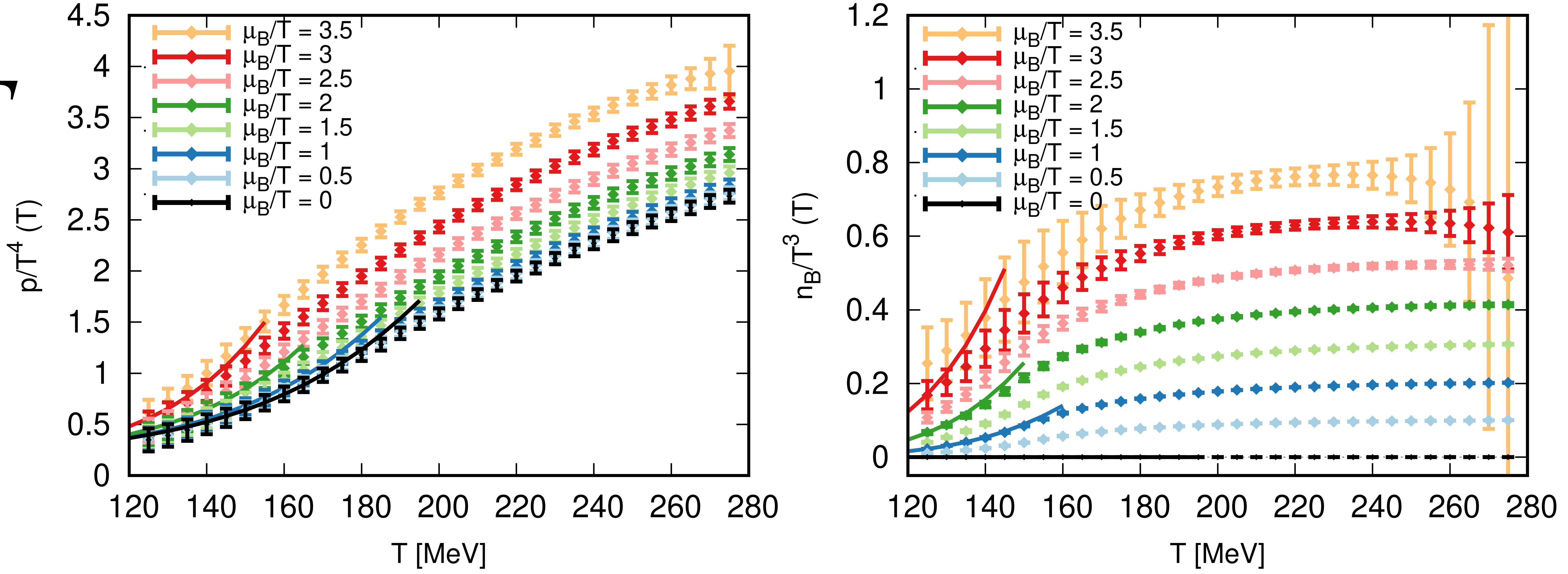
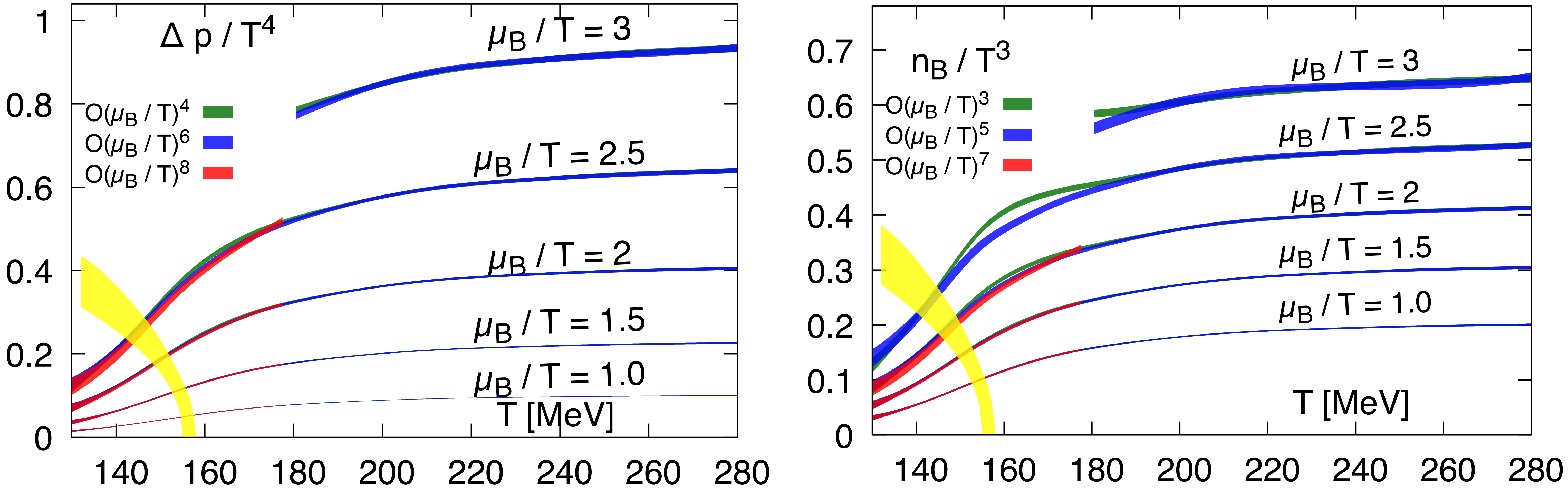
EoS @ $\mu = 0$: [WB, 2014] [HotQCD, 2014]



[WB, 2022]



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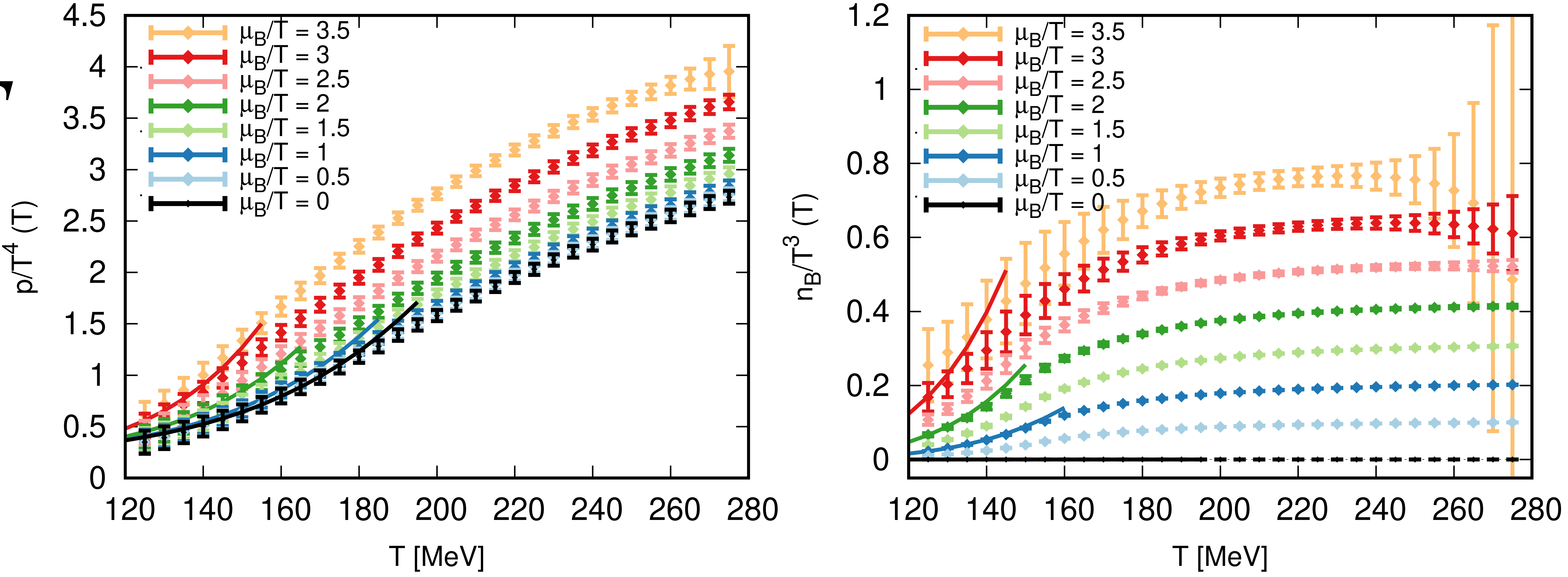
Equation of State:

- $\mu_B/T \lesssim 2.5$ **for** $T \lesssim 200$ MeV
- $\mu_B/T \lesssim 3$ **for** $T \gtrsim 200$ MeV



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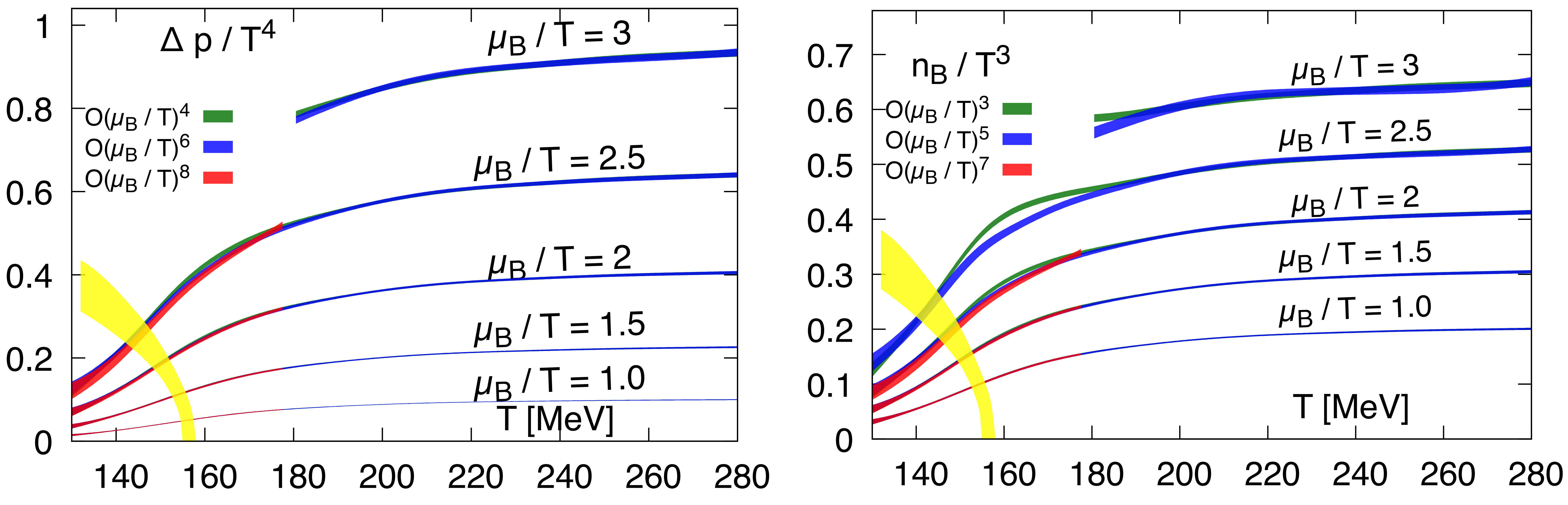
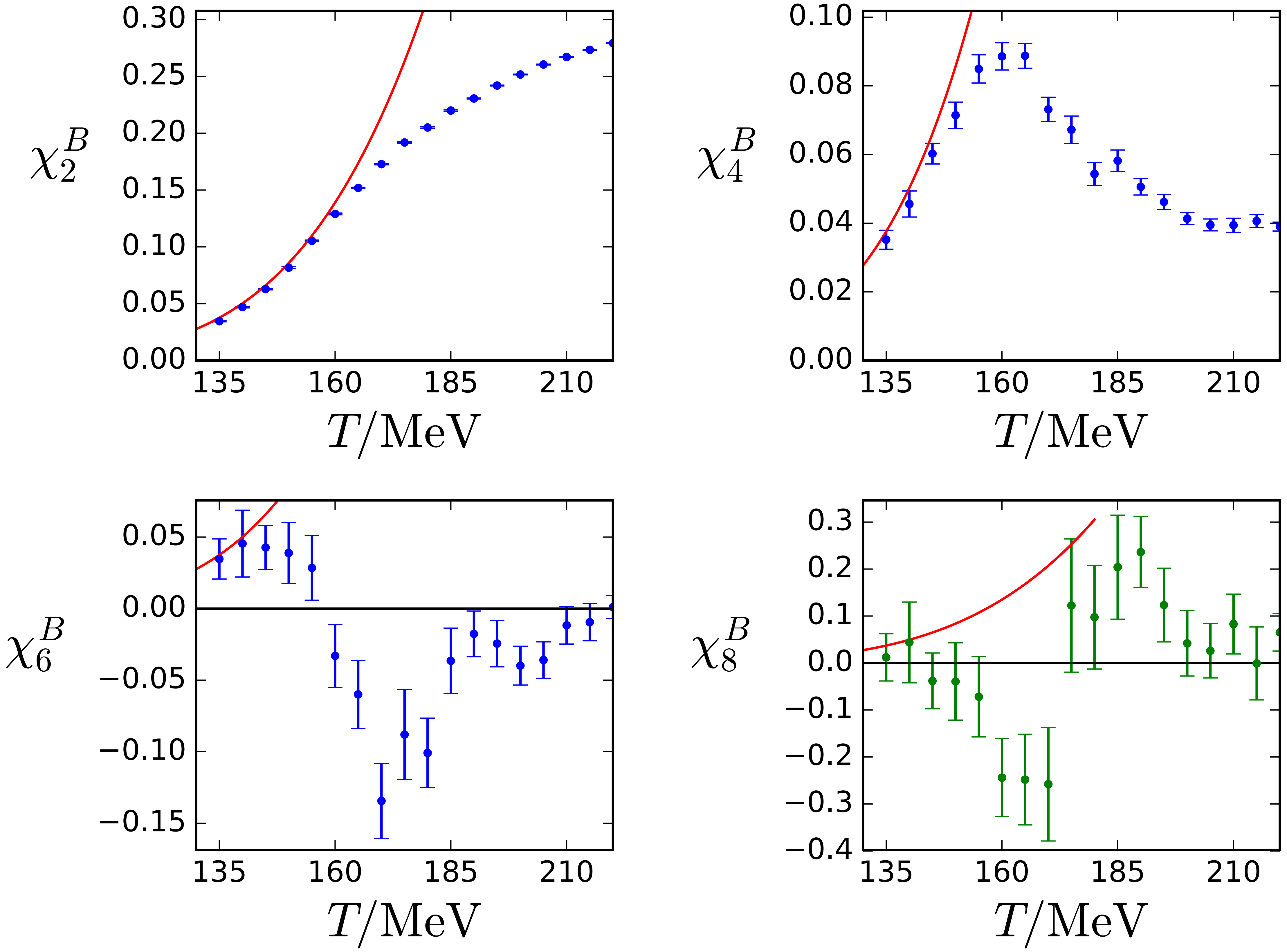


[HotQCD, 2023]

Small $\mu \to 0$, nonzero T

Fluctuations of the conserved charges



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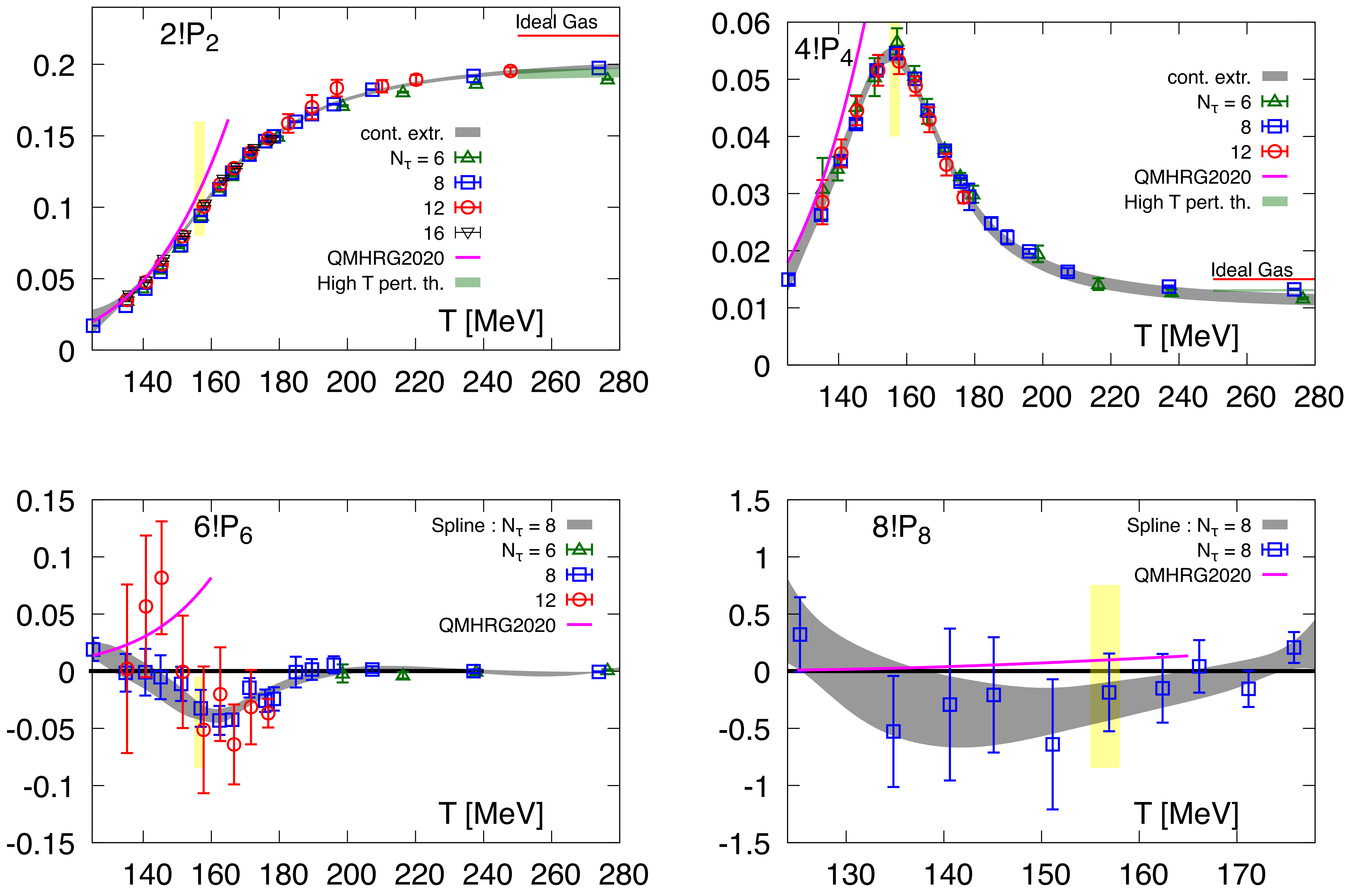


[HotQCD, 2023]

$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{\partial^i(\mu_B/T)\partial^j(\mu_Q/T)\partial^k(\mu_S/T)}$$

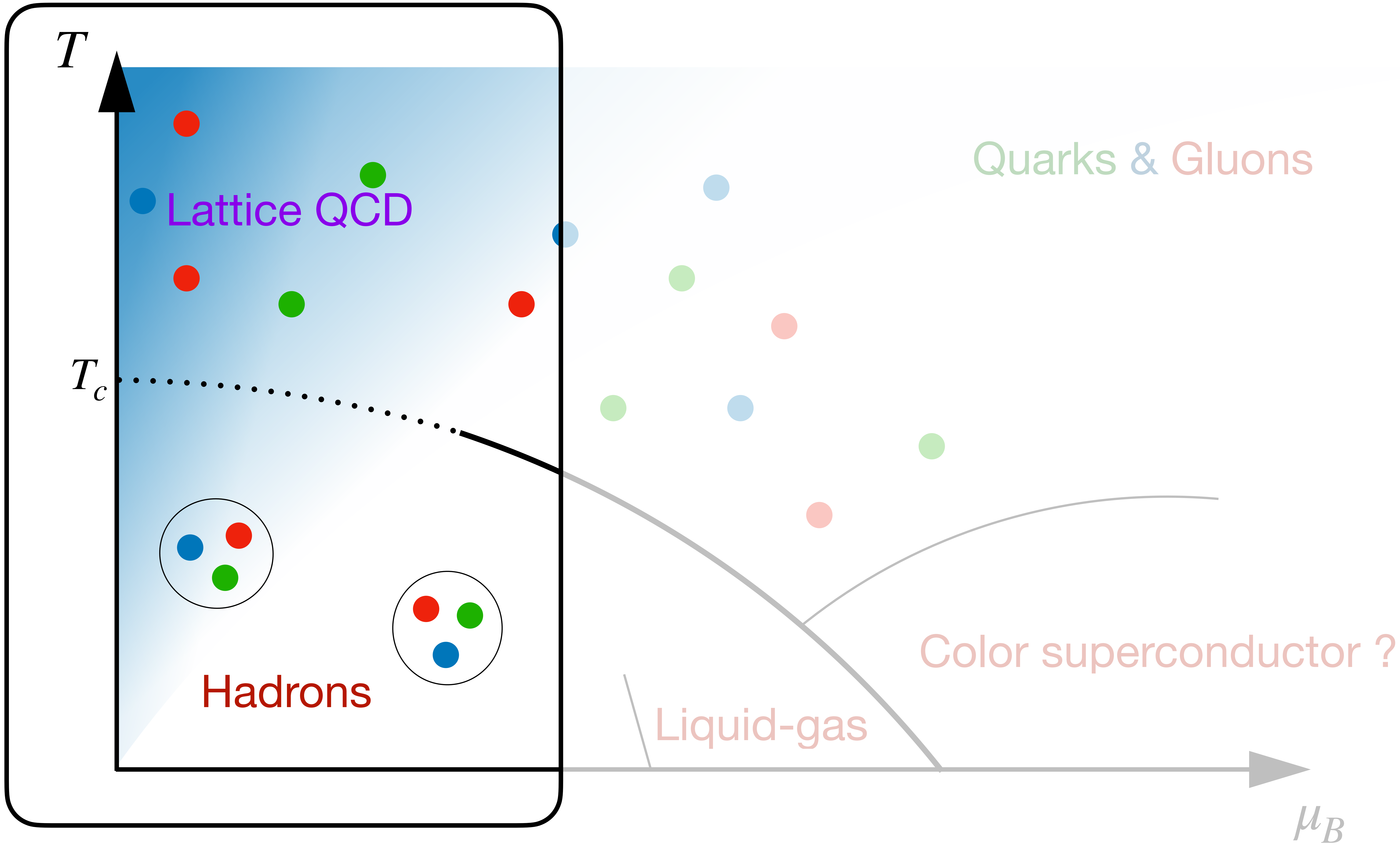
$$\chi_1^B \sim \langle B \rangle \qquad \chi_2^B \sim \langle B^2 \rangle - \langle B \rangle^2$$

B,S: 2,4,6,8 order

Q: large artifacts

Large μ, nonzero T

CEP?



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$$\chi_2^B(T,\mu_B) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} \chi_{2n+2}^B (\mu_B/T)^{2n} \qquad r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2}$$

- Taylor expansion: convergence radius in μ_B

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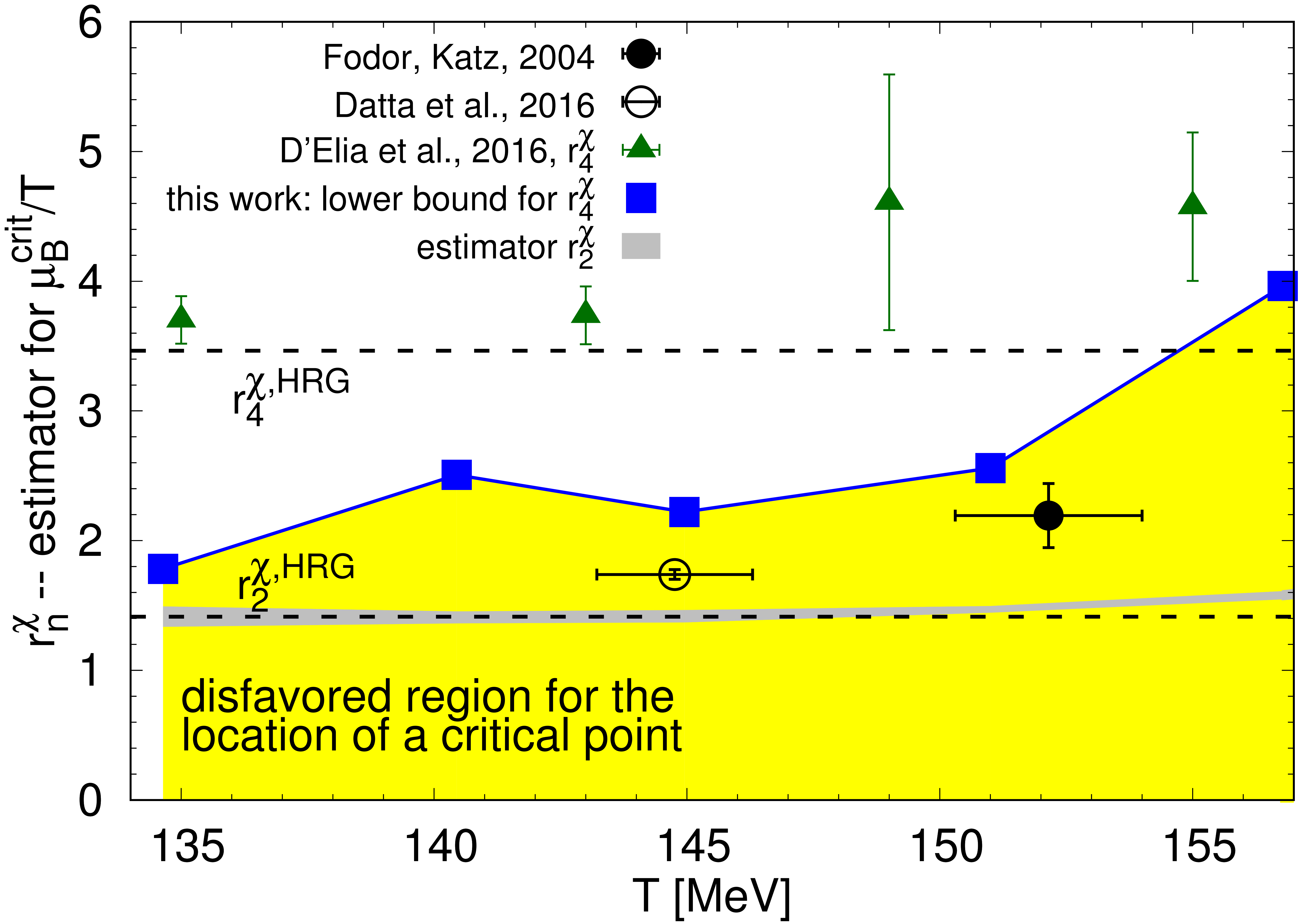

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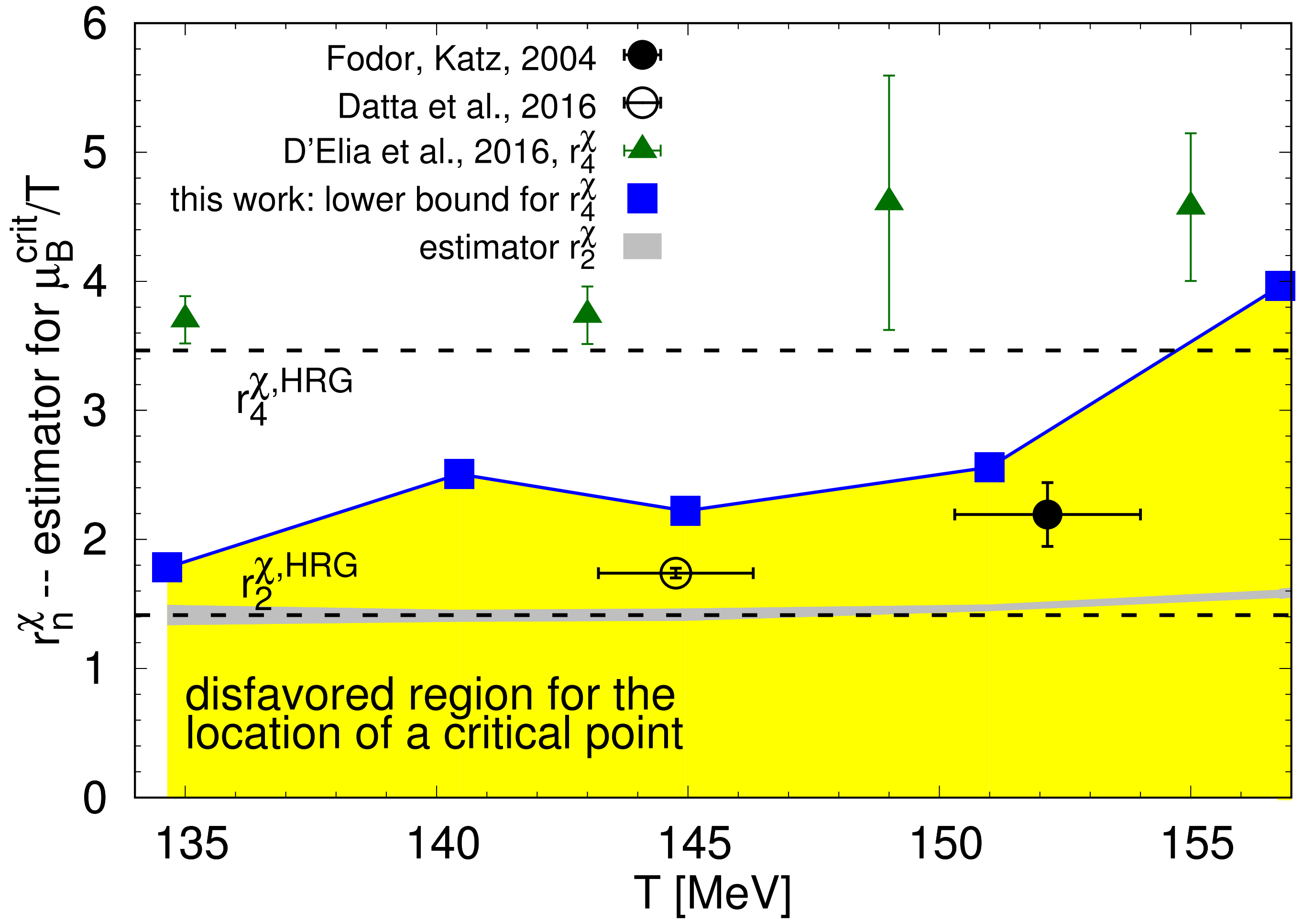

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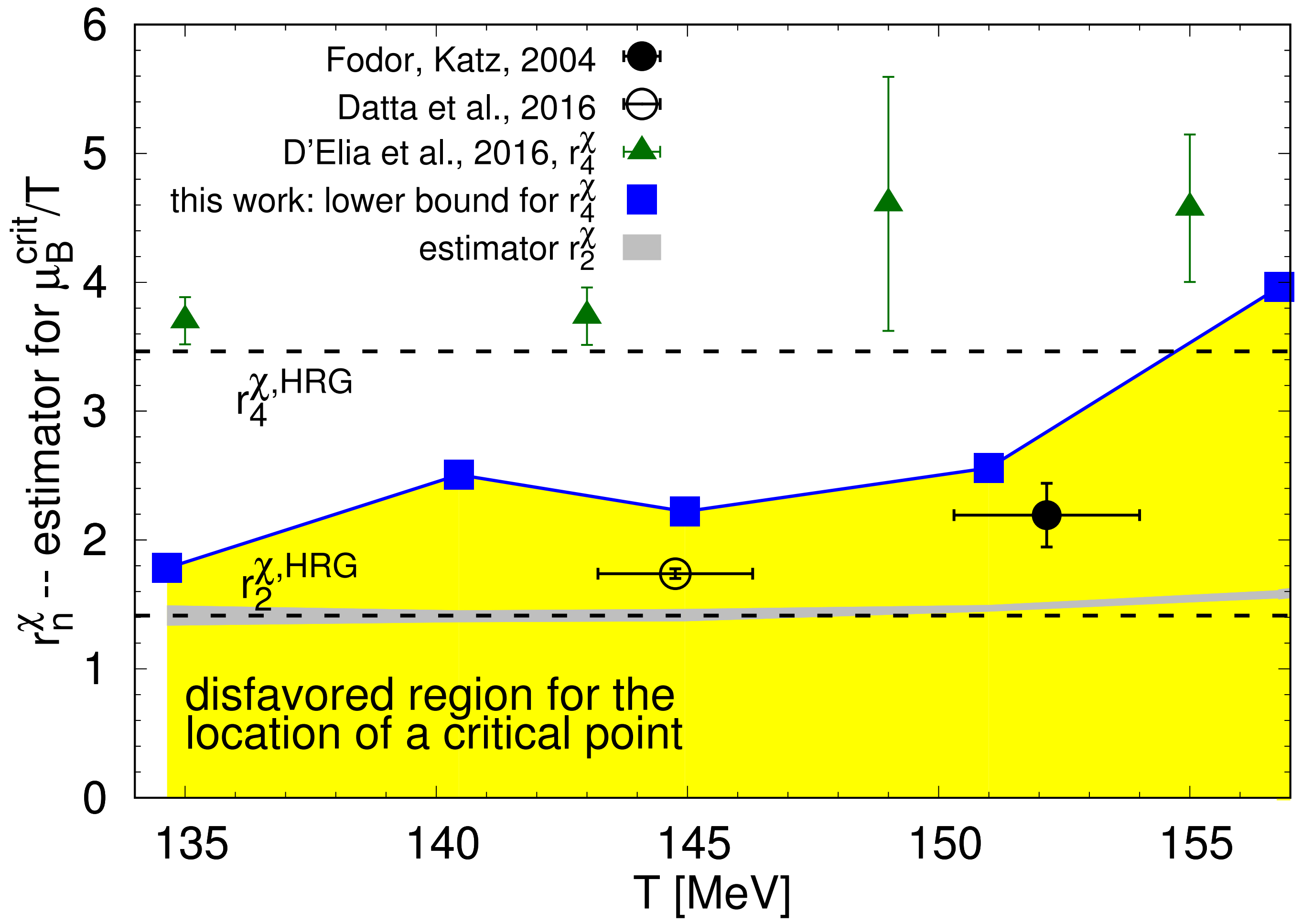

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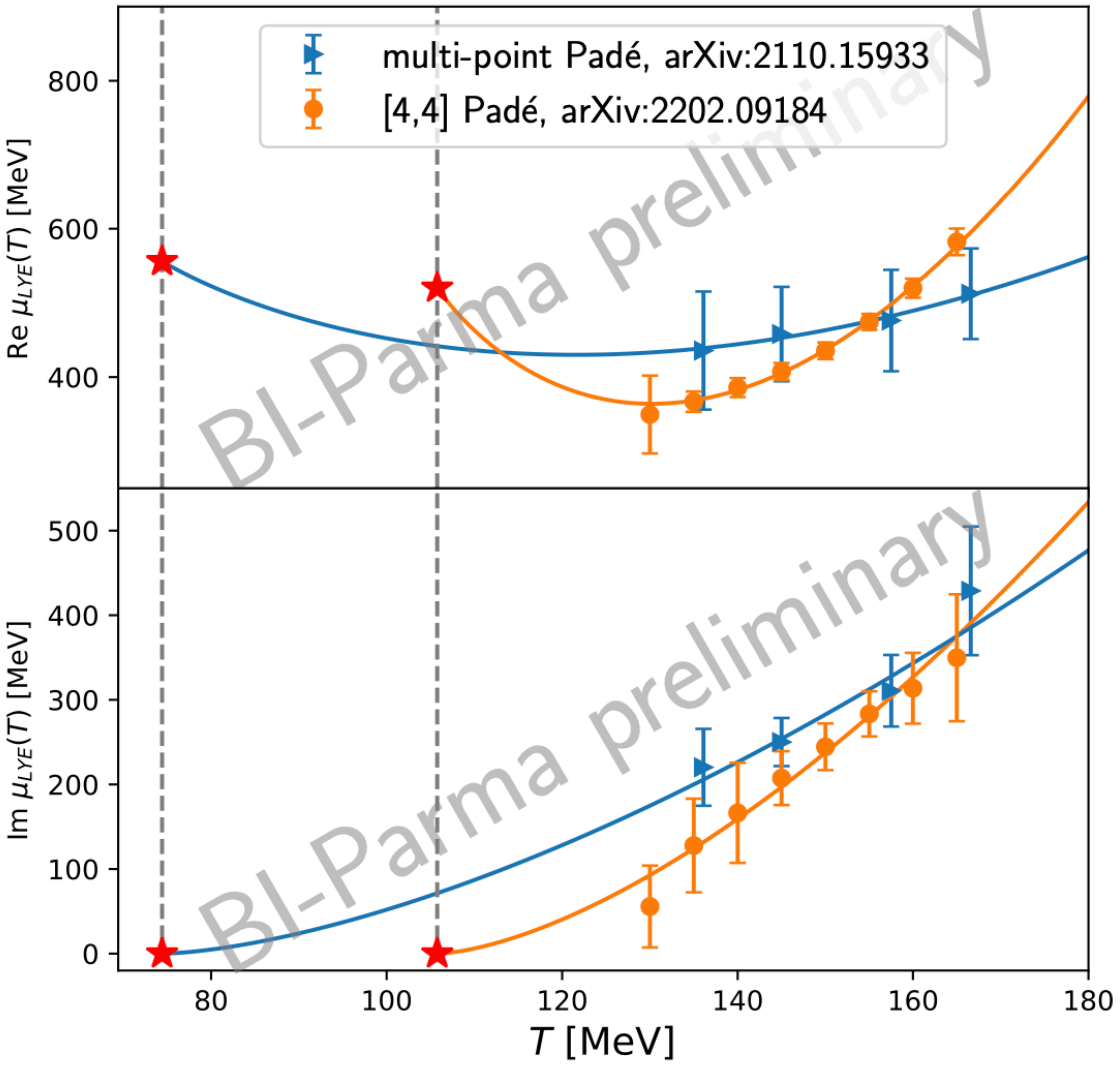
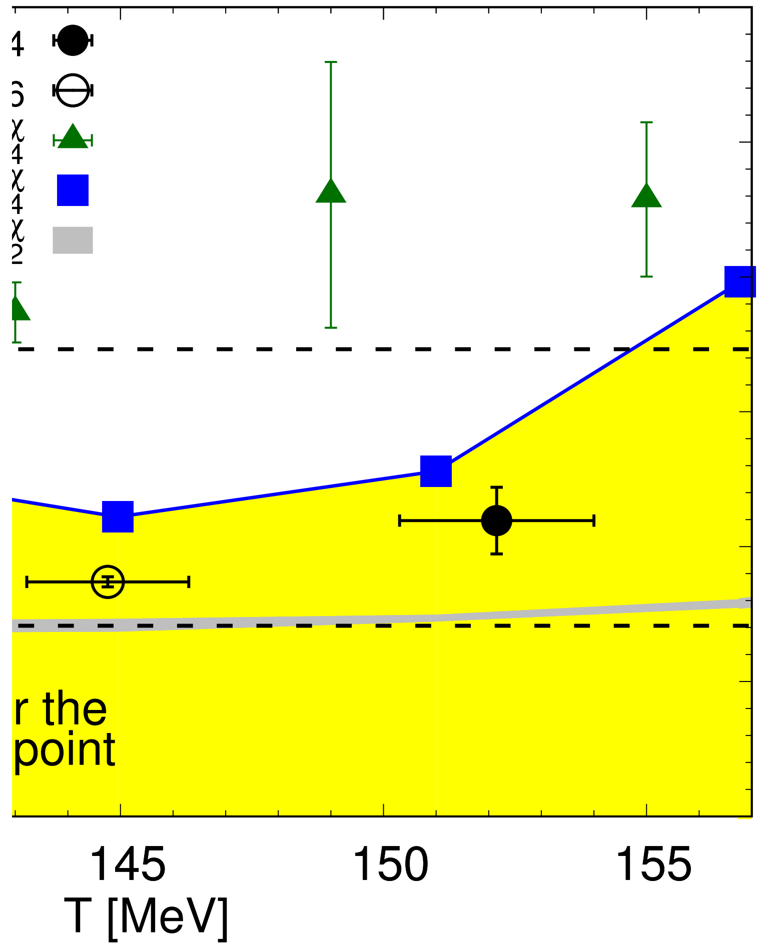
$$= \left| \frac{2n(2n-1)\chi^B_{2n}}{\chi^B_{2n+2}} \right|^{1/2}$$





[Bielefeld-Parma, 2023]

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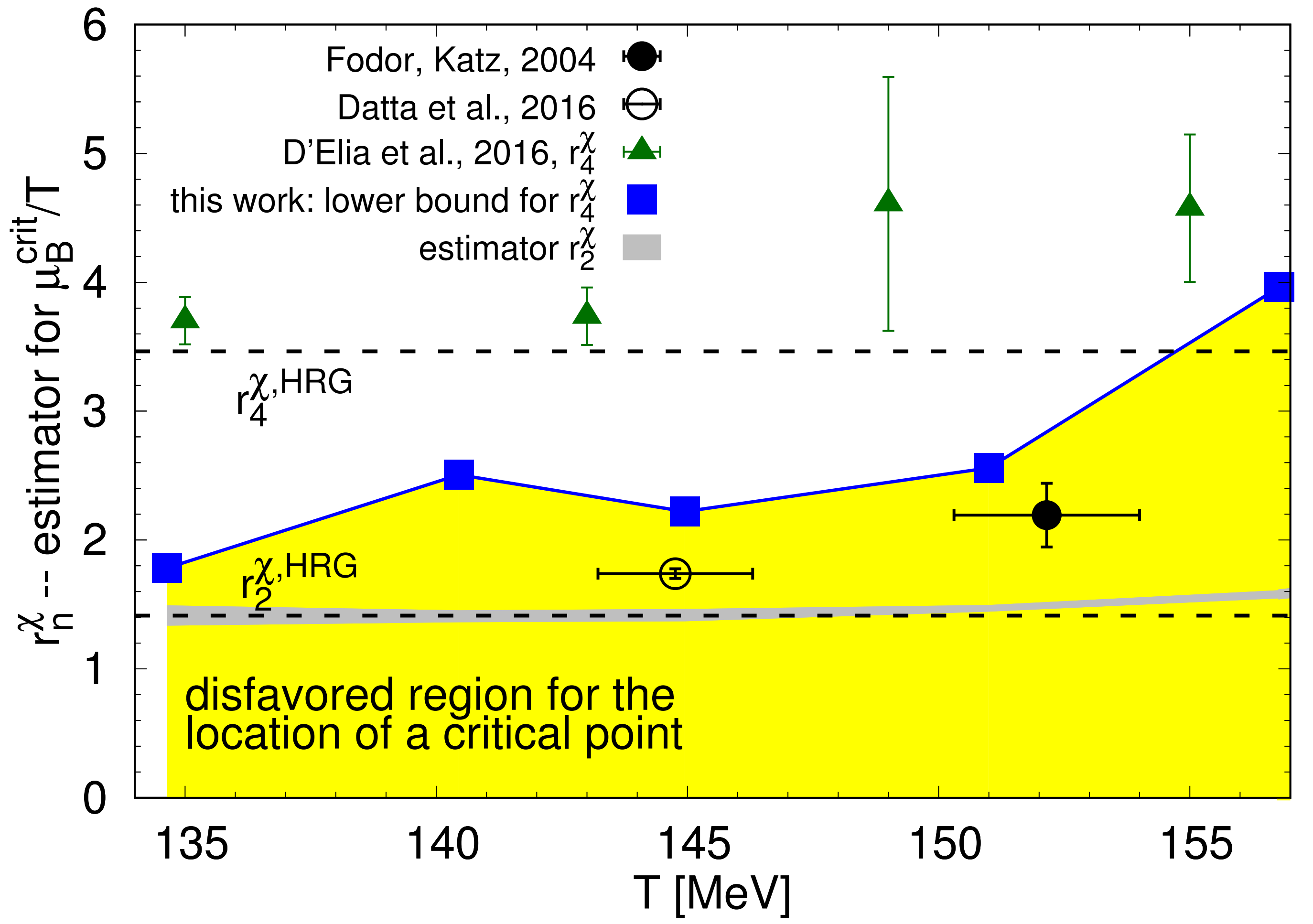

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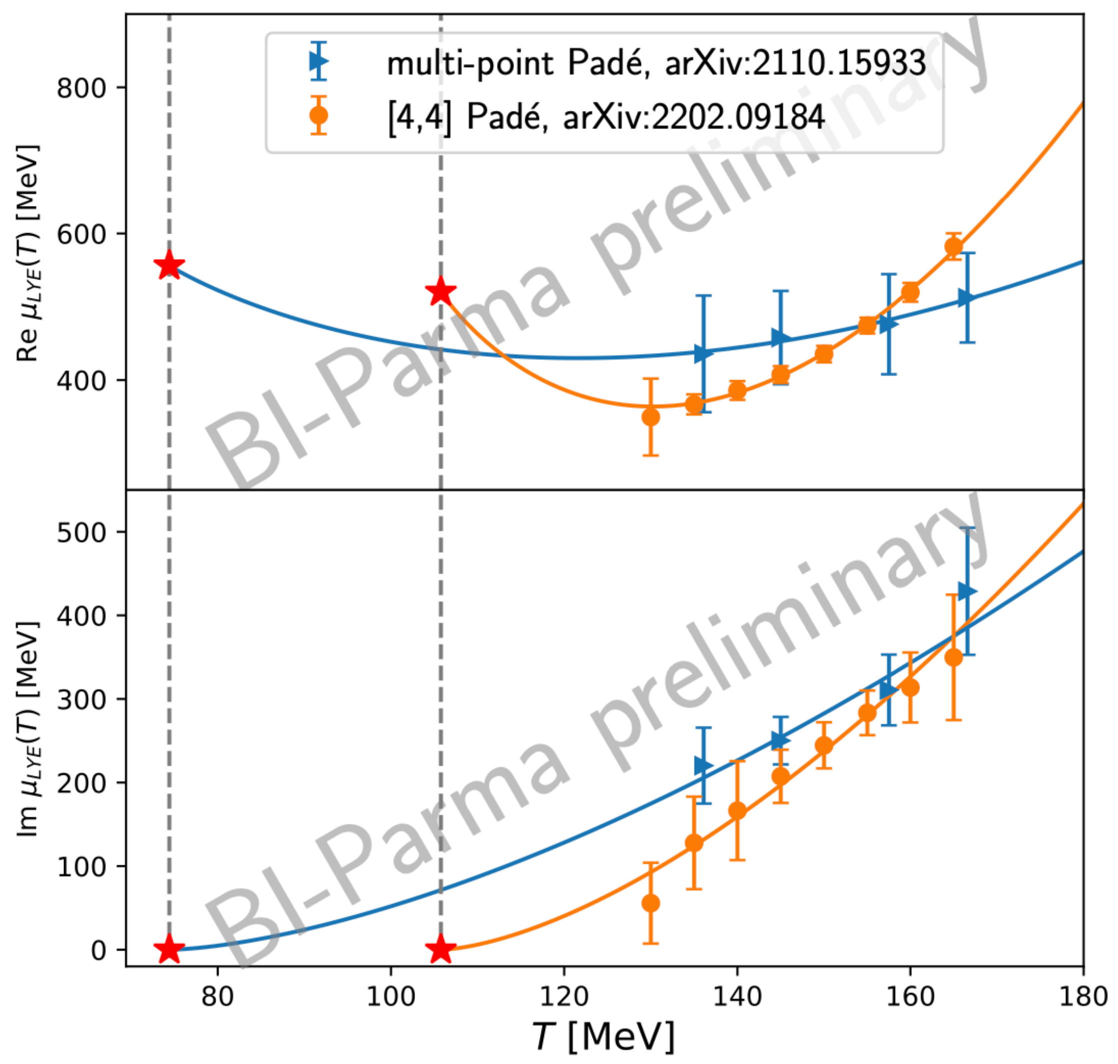

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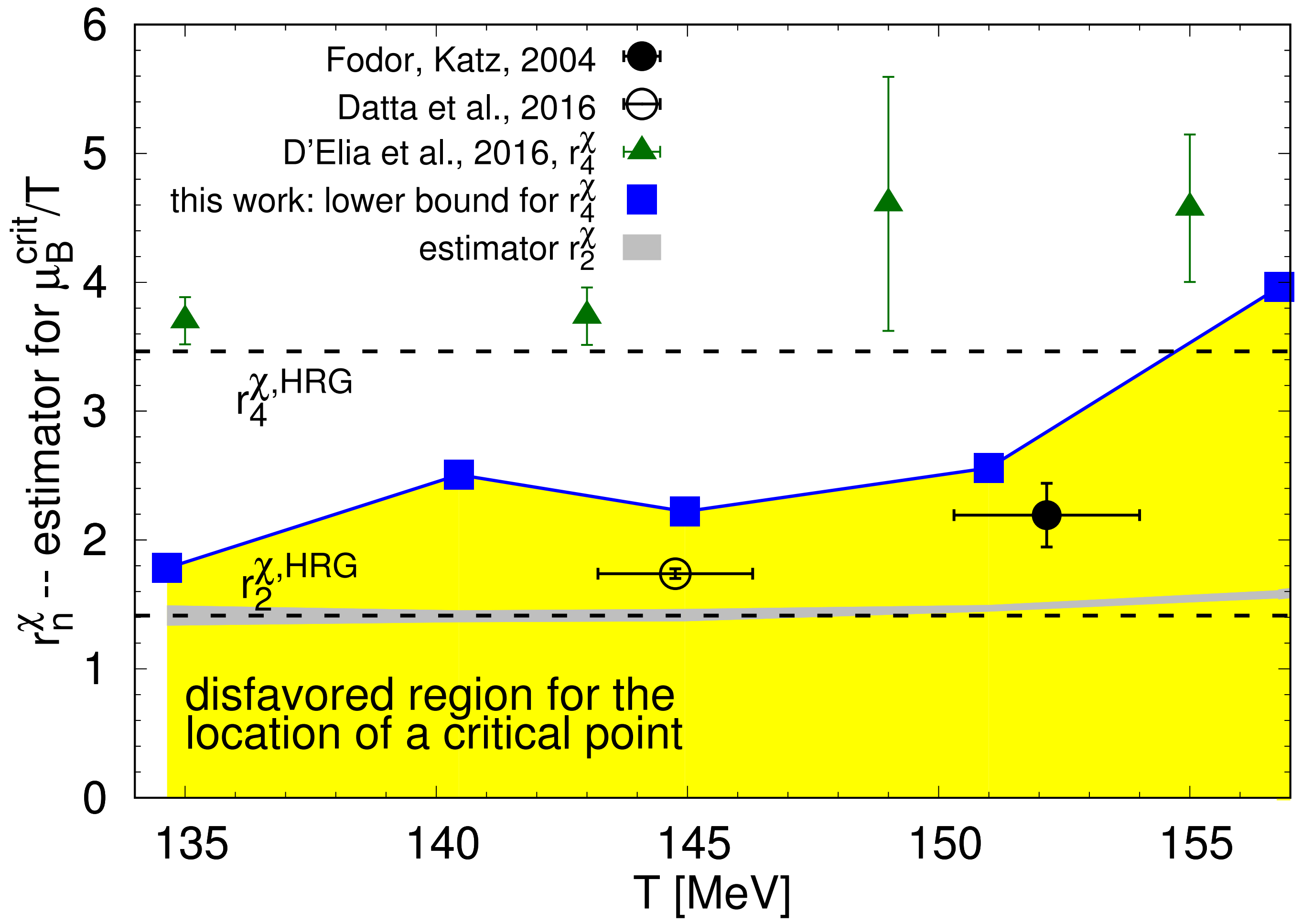
- Taylor expansion: convergence radius in μ_B
- Lee-Yang zeros
- Transition width from $\mu_B^2 < 0$ to $\mu_B^2 > 0$



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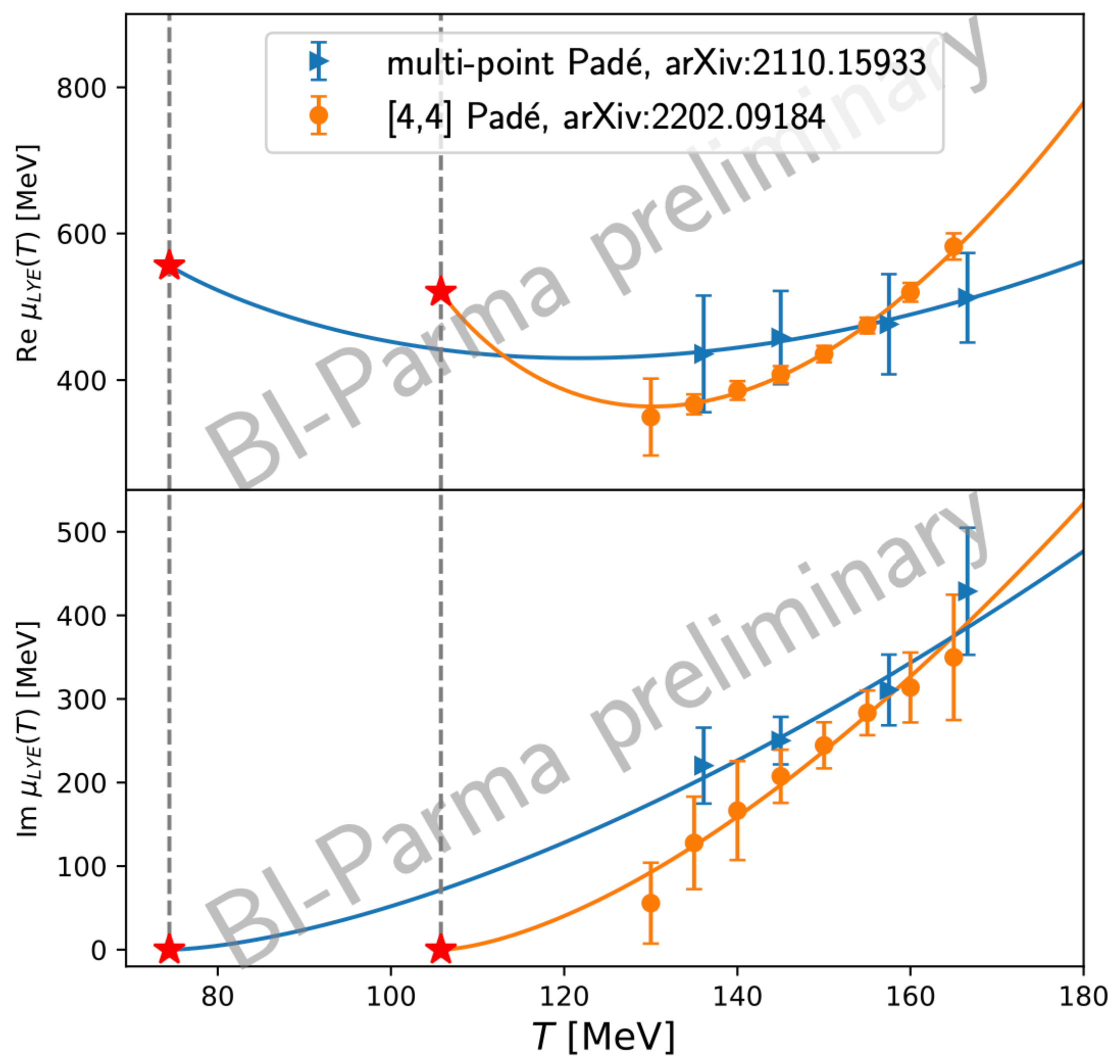
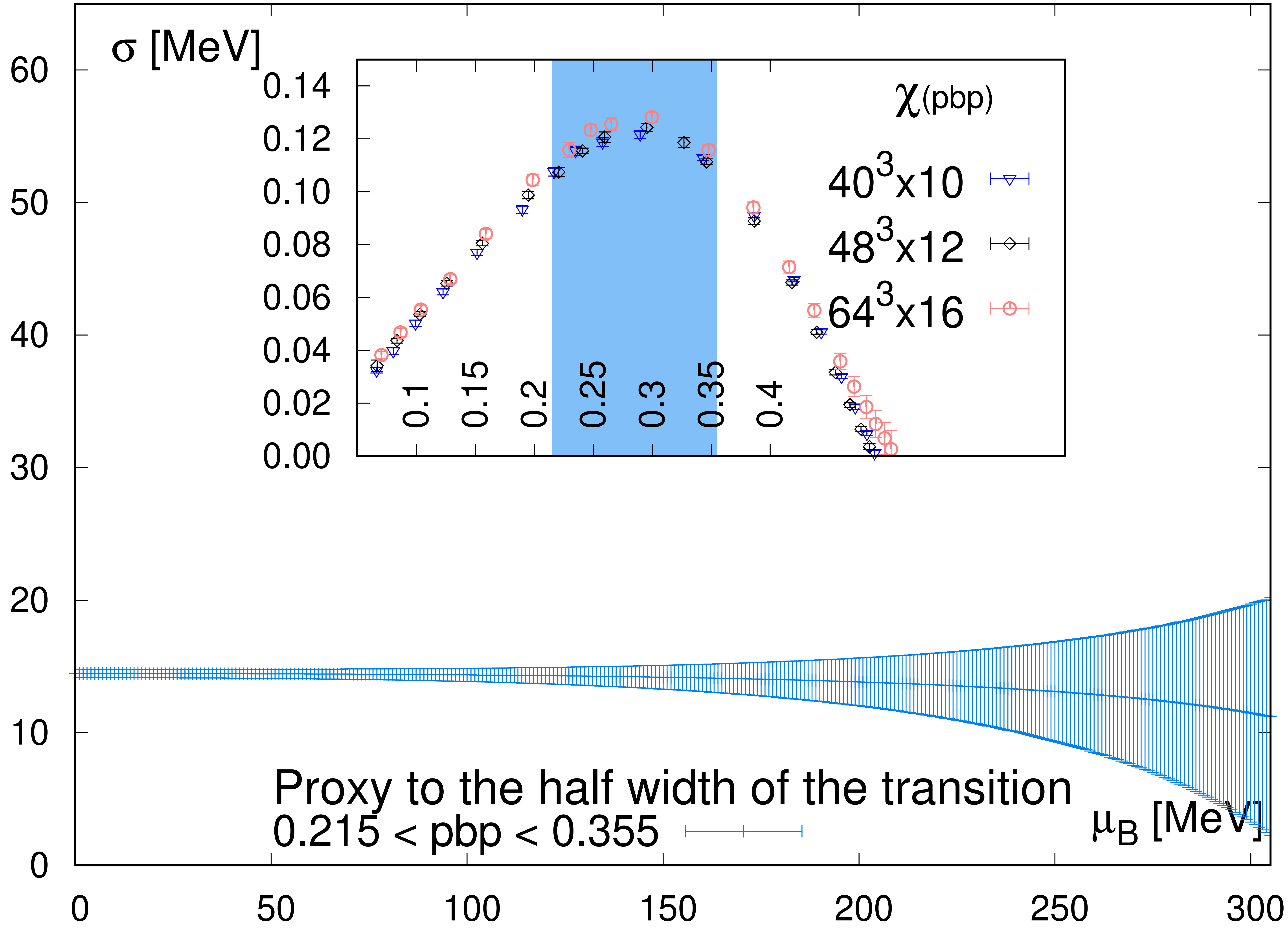


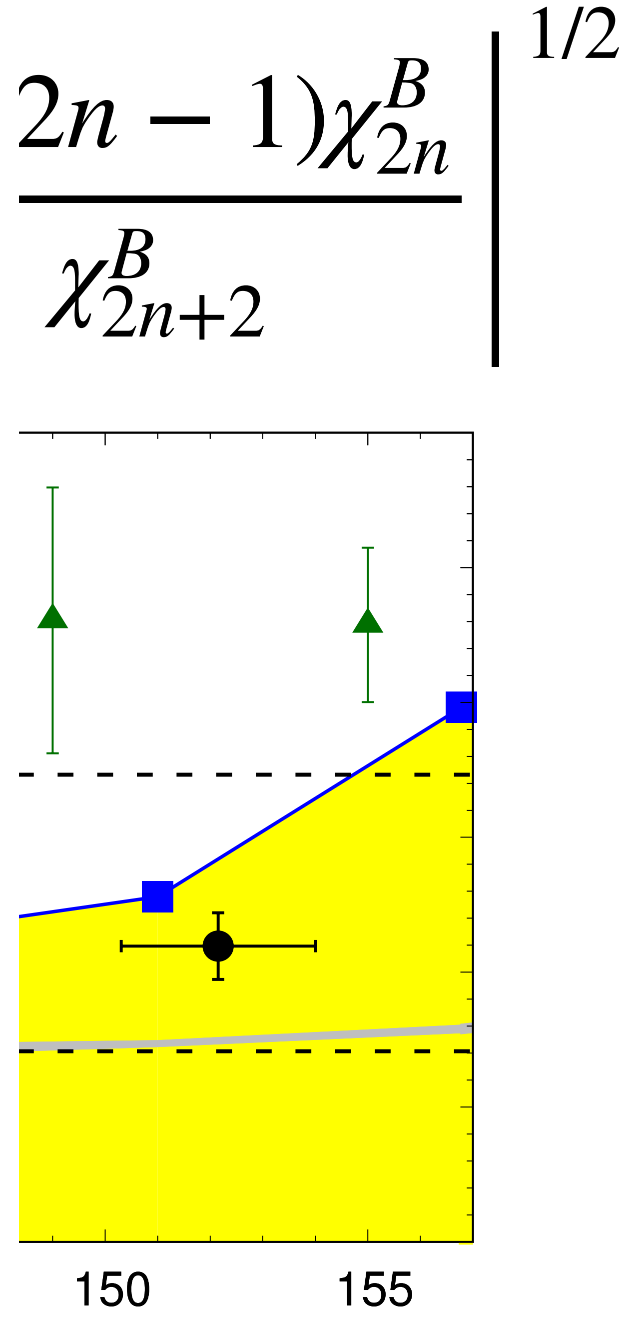
[Bielefeld-Parma, 2023]

Larg

CEP

- Taylo
- Lee-
- Trans





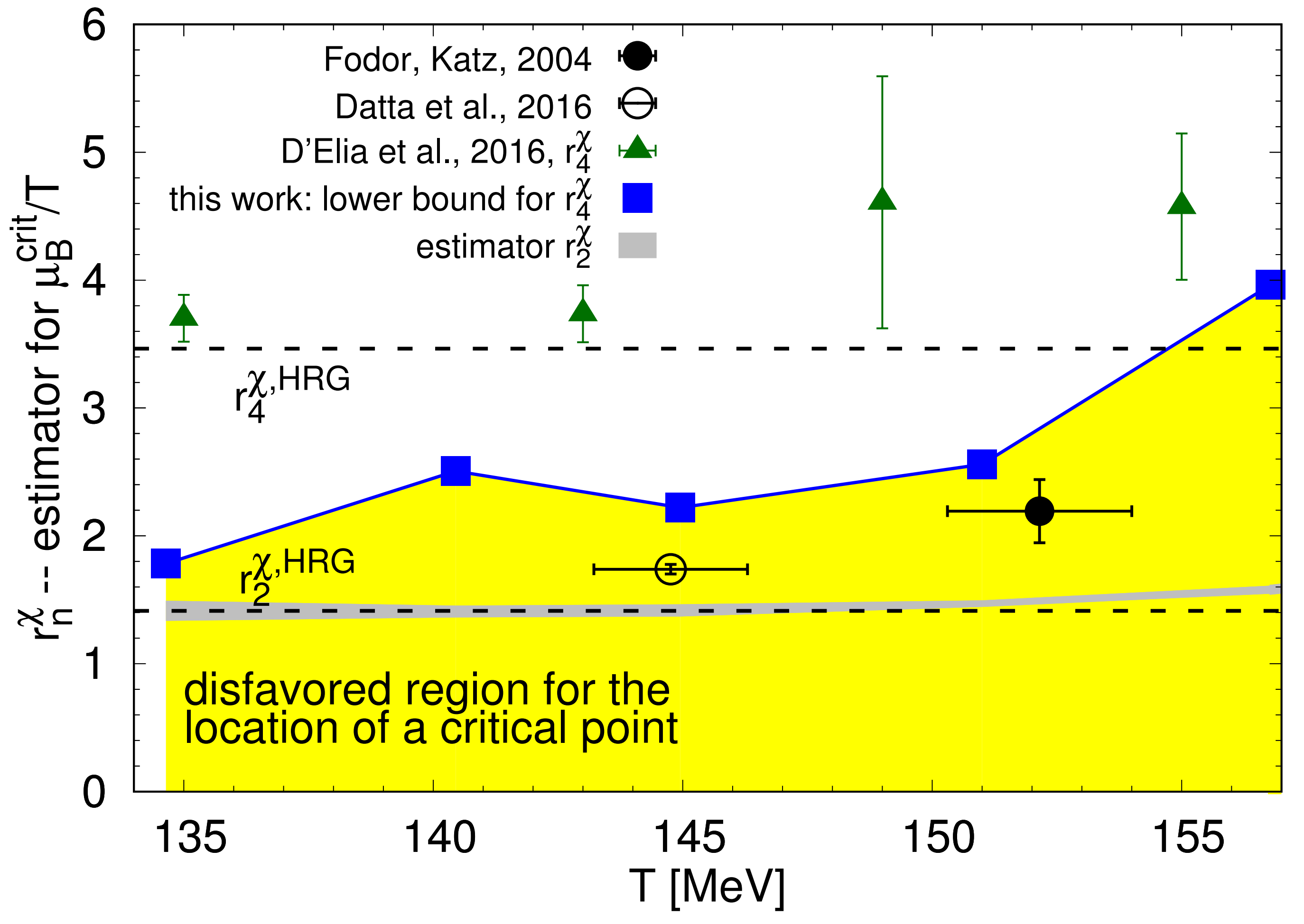
[WB, 2020]

Large μ, nonzero T

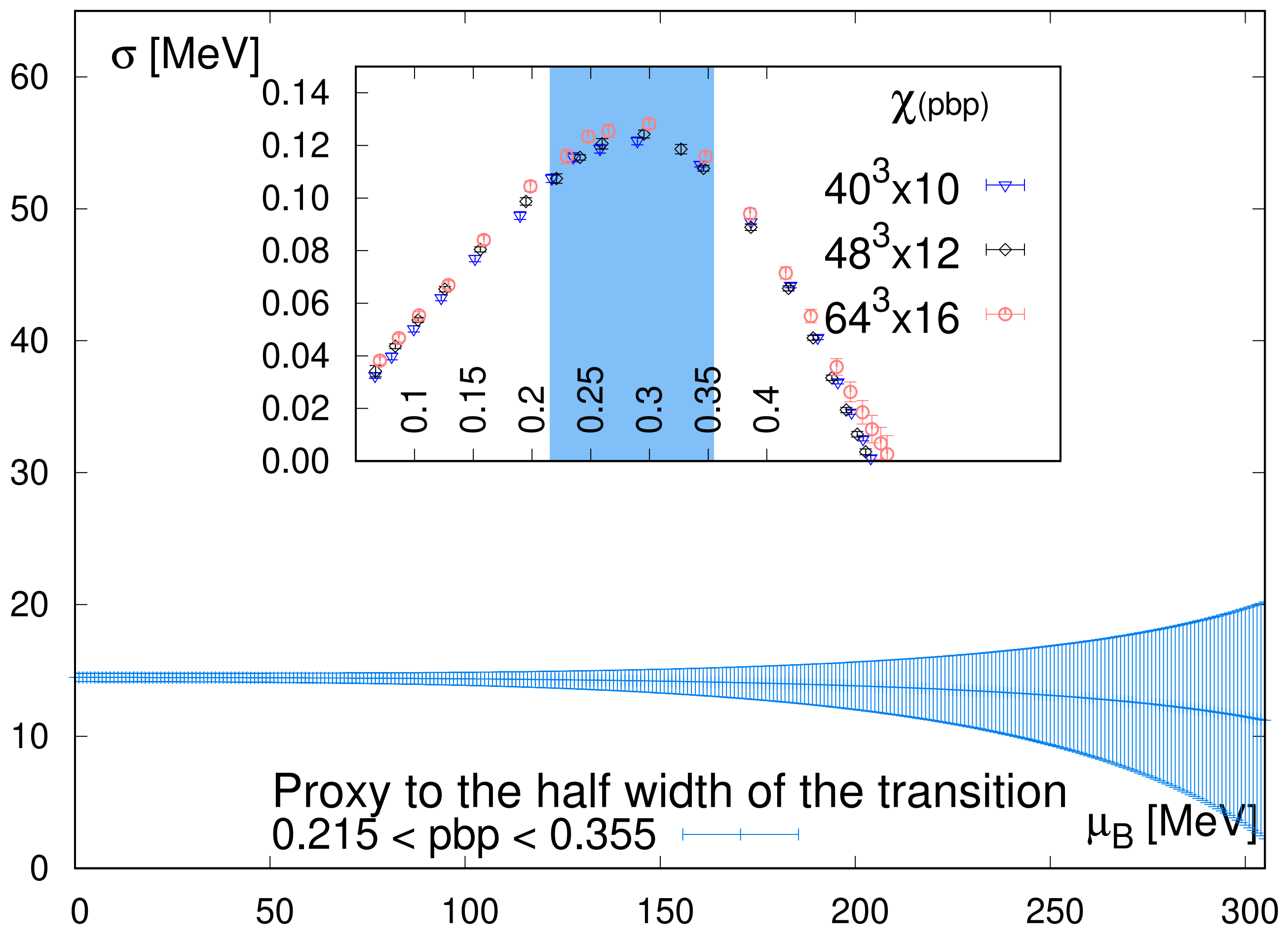
CEP?

$$\chi_2^B(T,\mu_B) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} \chi_{2n+2}^B (\mu_B/T)^{2n} \qquad r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2}$$

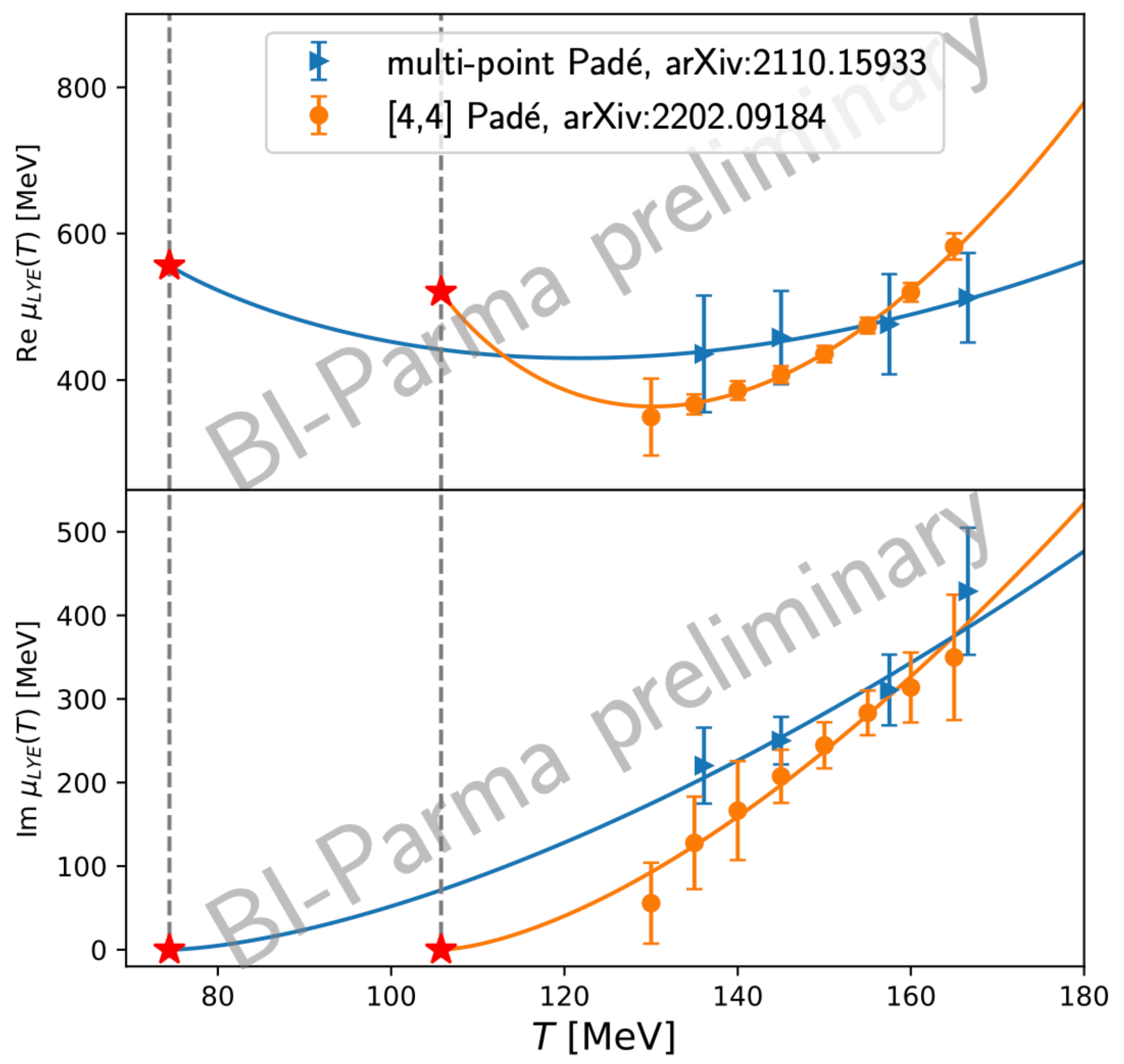
- Taylor expansion: convergence radius in μ_B
- Lee-Yang zeros
- Transition width from $\mu_B^2 < 0$ to $\mu_B^2 > 0$



[HotQCD, 2017]



[WB, 2020]



[Bielefeld-Parma, 2023]

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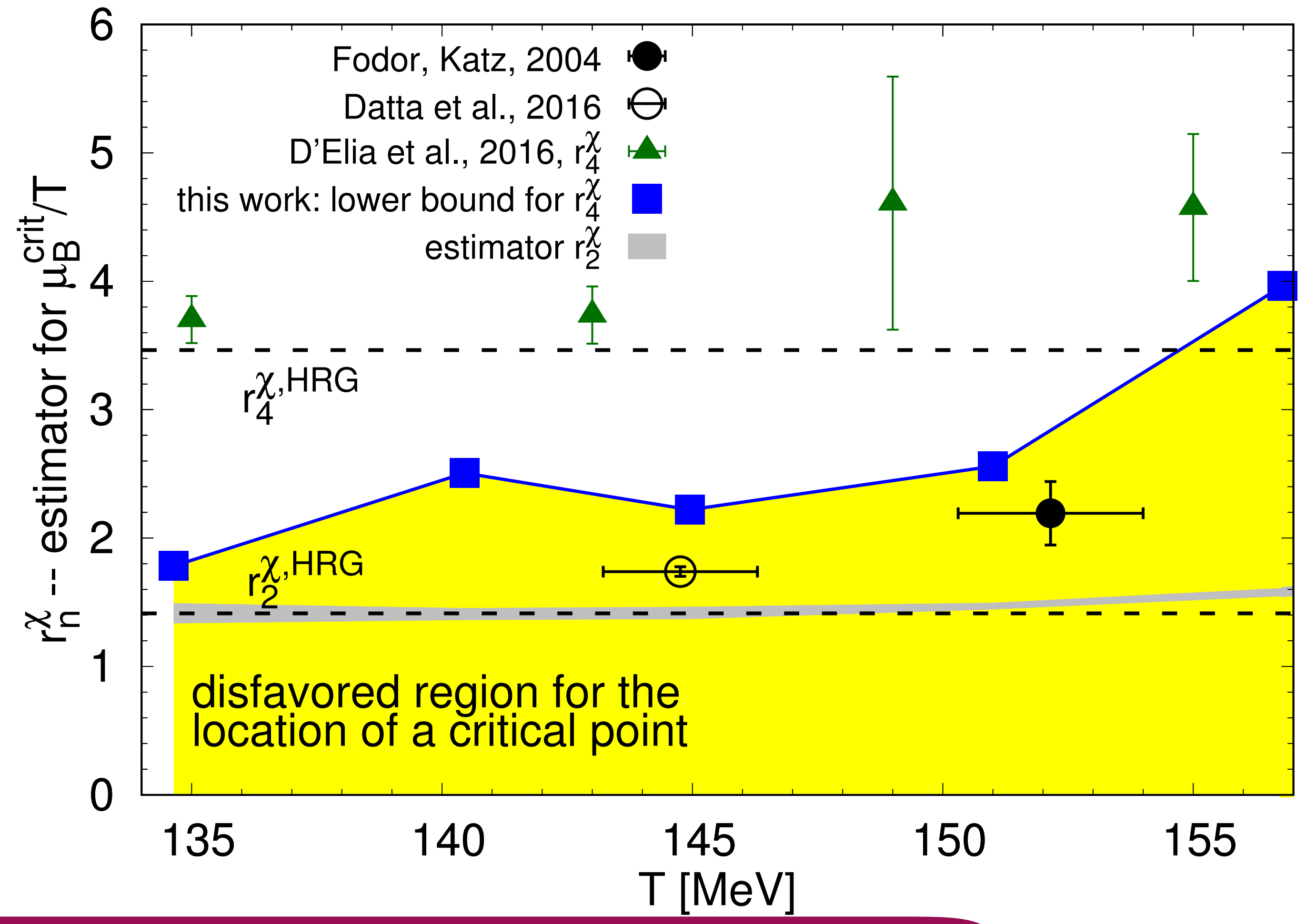

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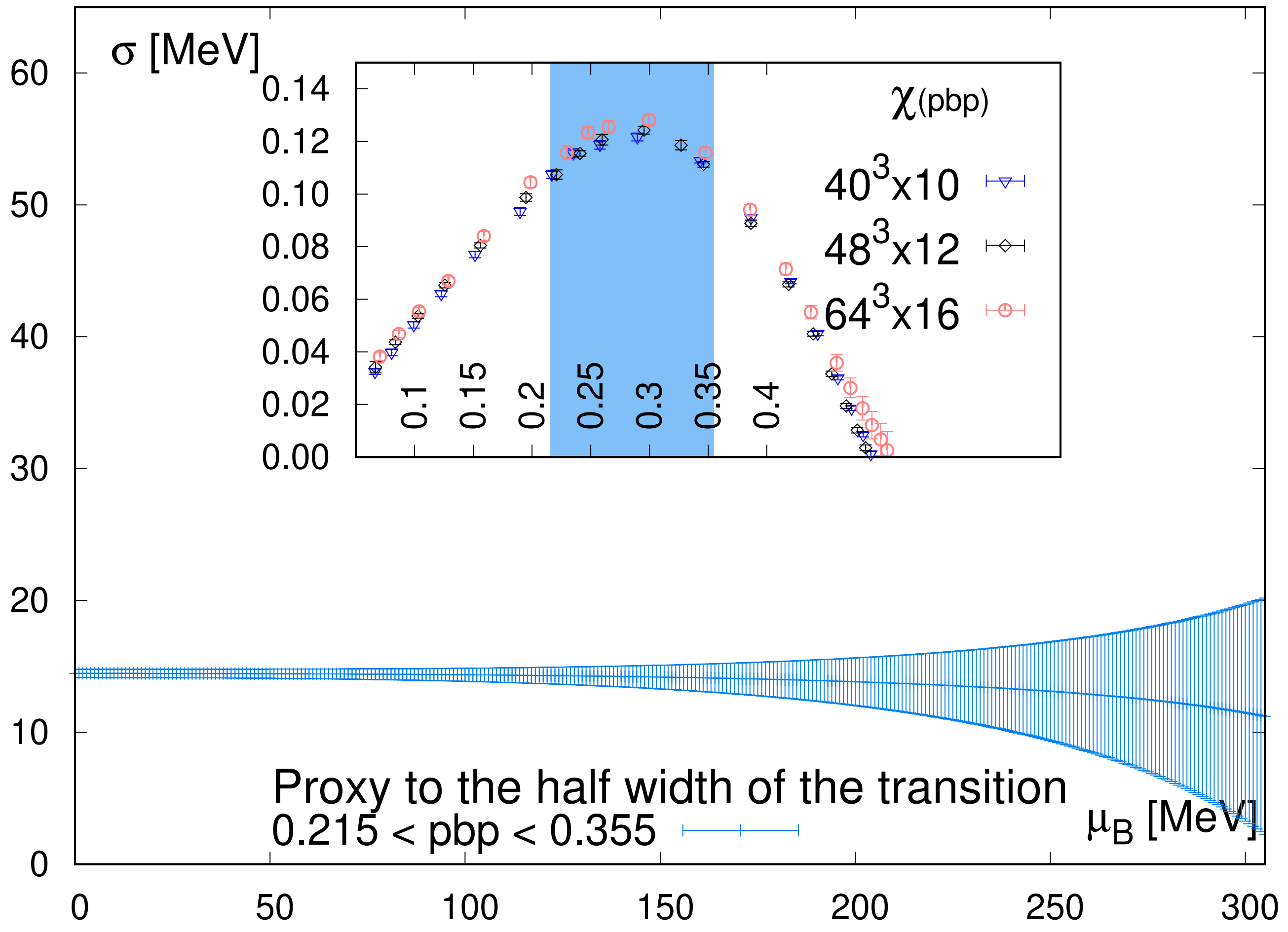

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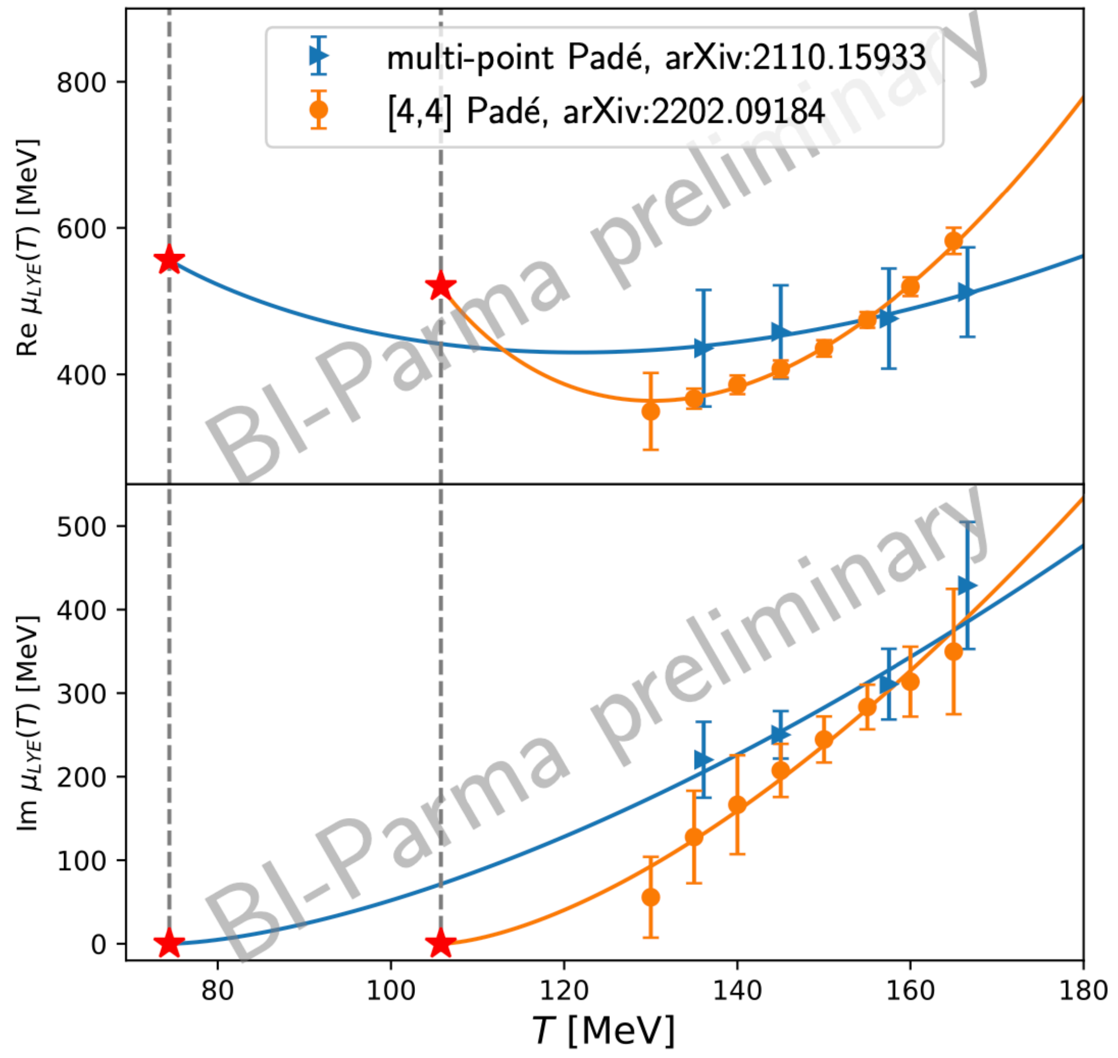


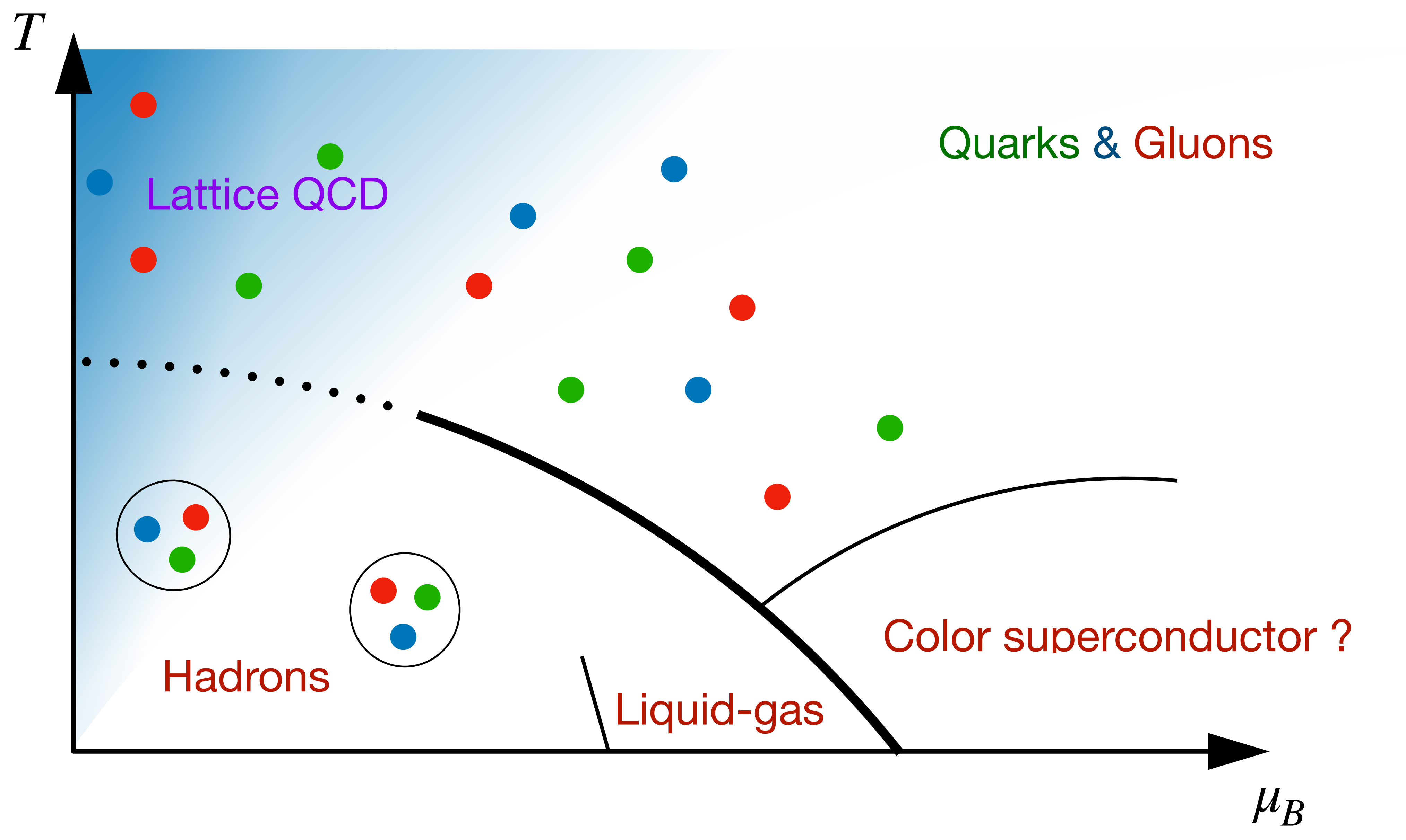
[Bielefeld-Parma, 2023]

Conclusion: lack of controlled estimation, possibly at $\mu_B^c \sim n \times 100$ MeV

FRG: $\mu_B^c \sim 600$ MeV
[Fu et al., 2019]

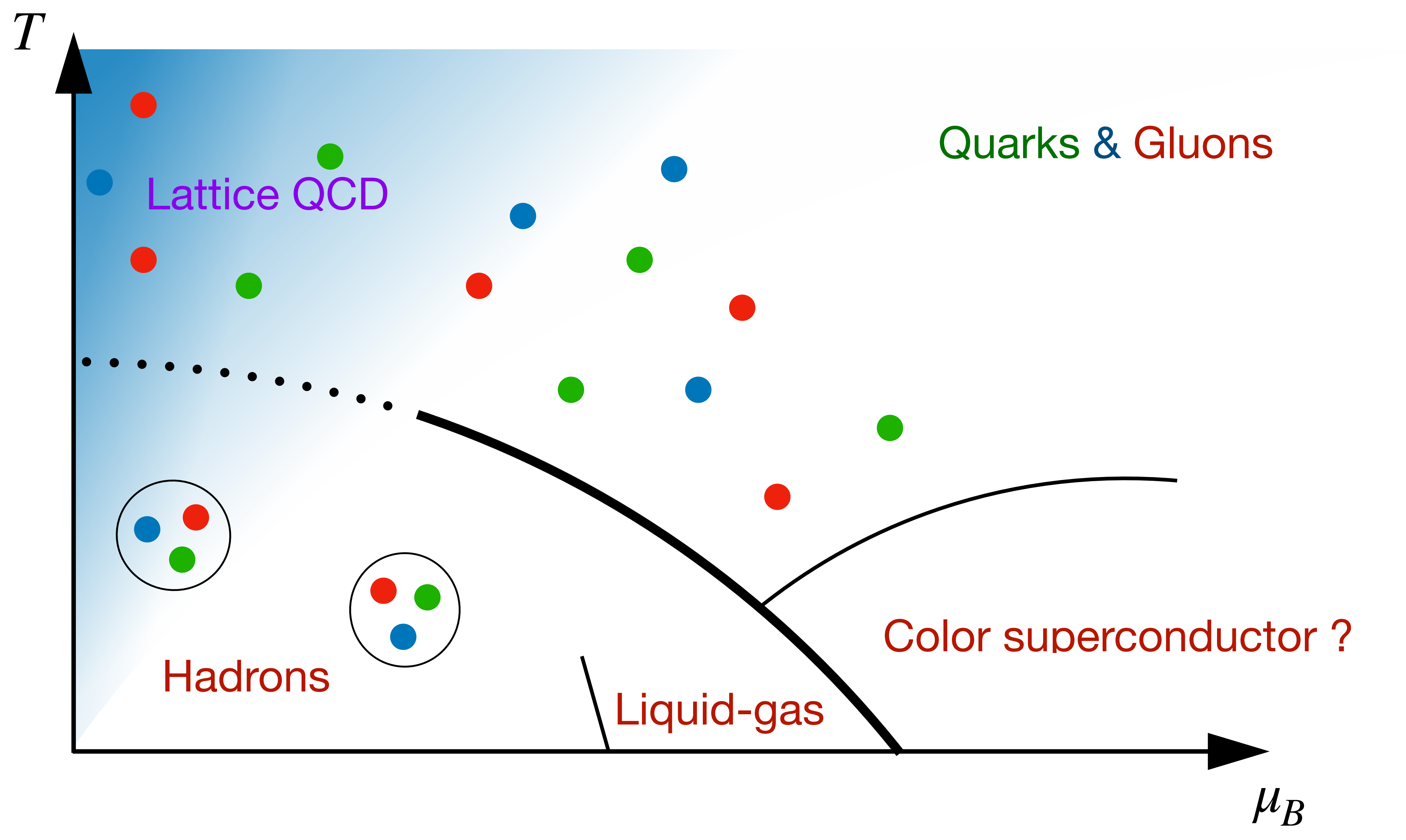
DSE: $\mu_B^c \sim 500$ MeV
[Isserstedt et al., 2019]

Further prospectives



Further prospectives

- Understanding properties (phases, thresholds) in QCD at finite T



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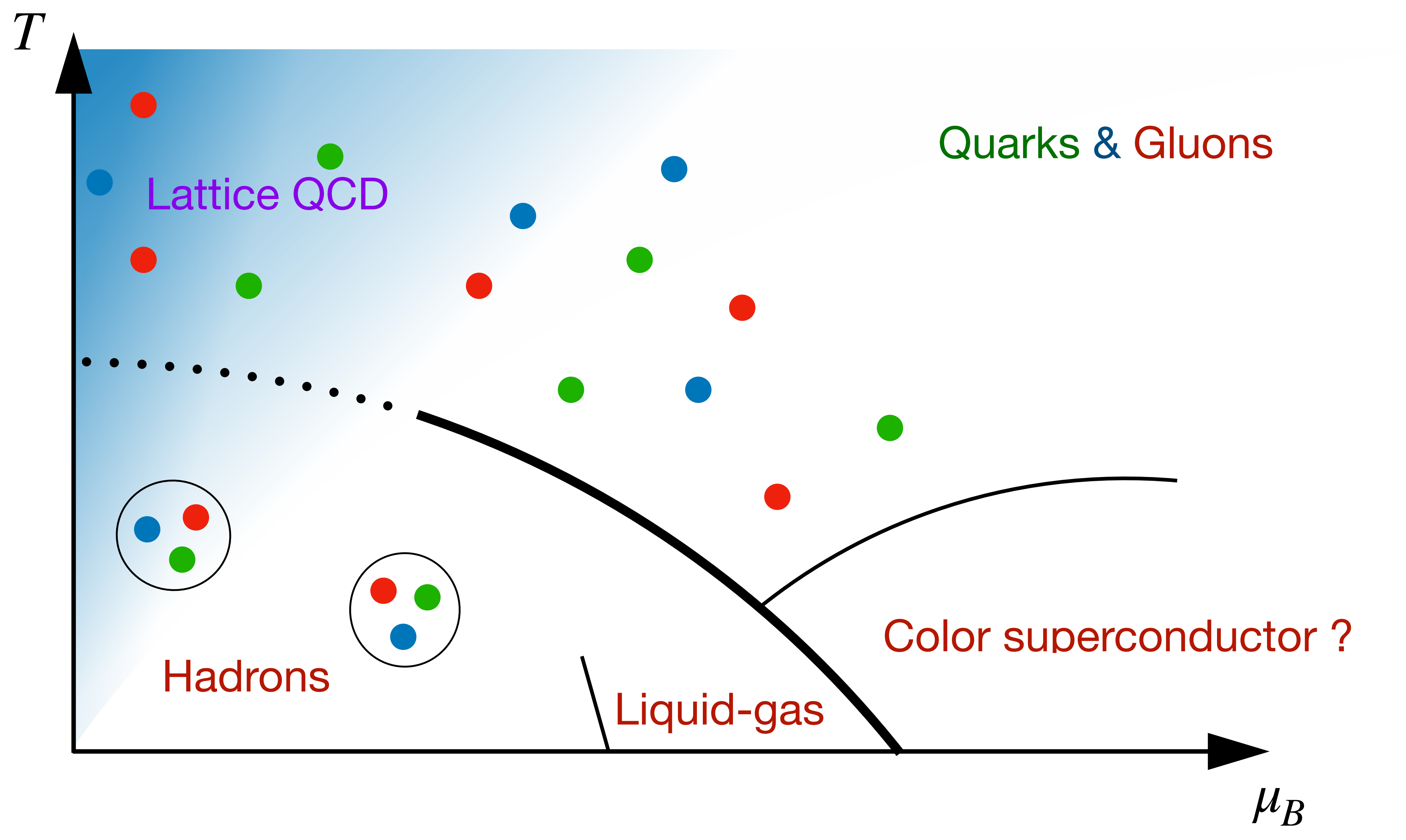

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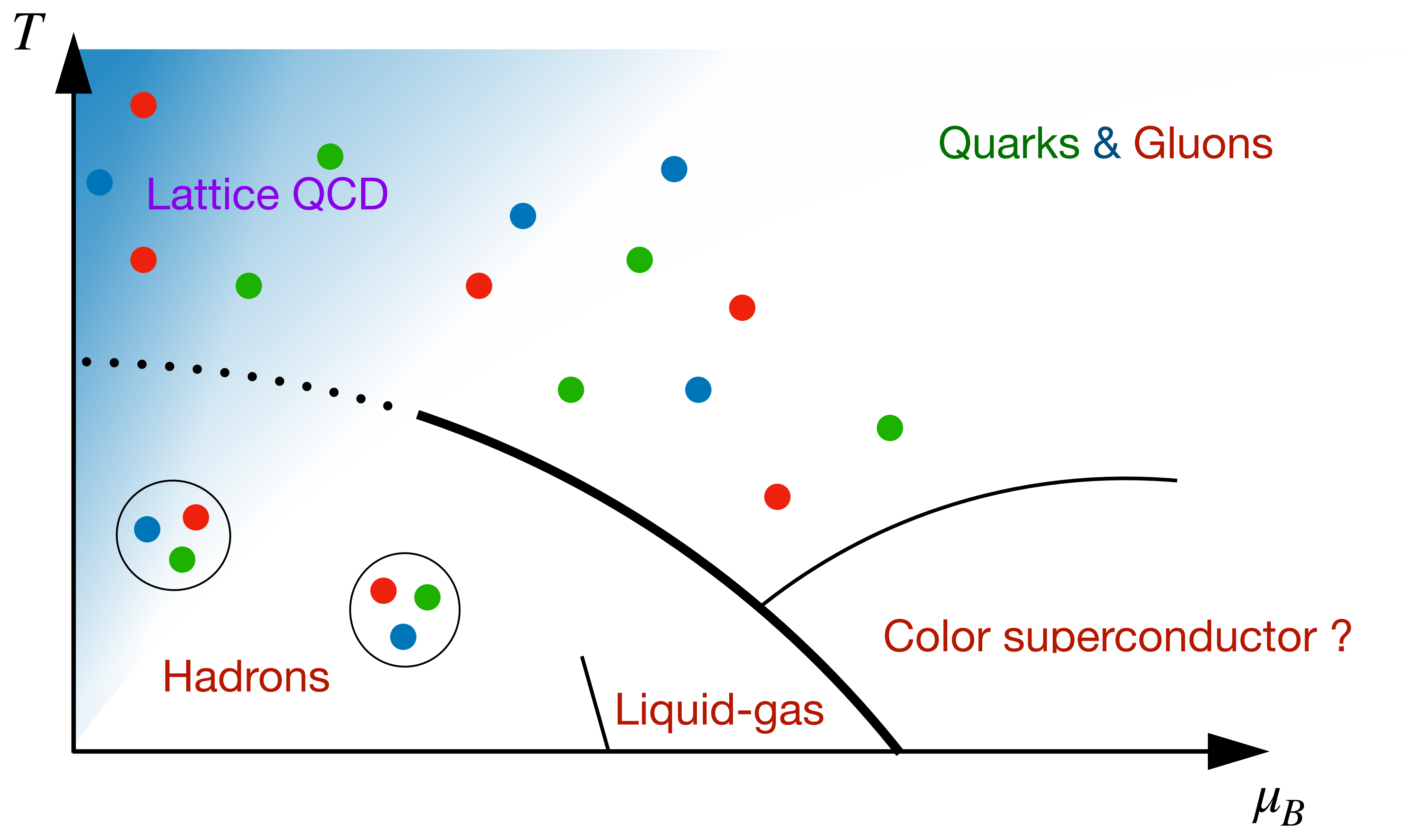

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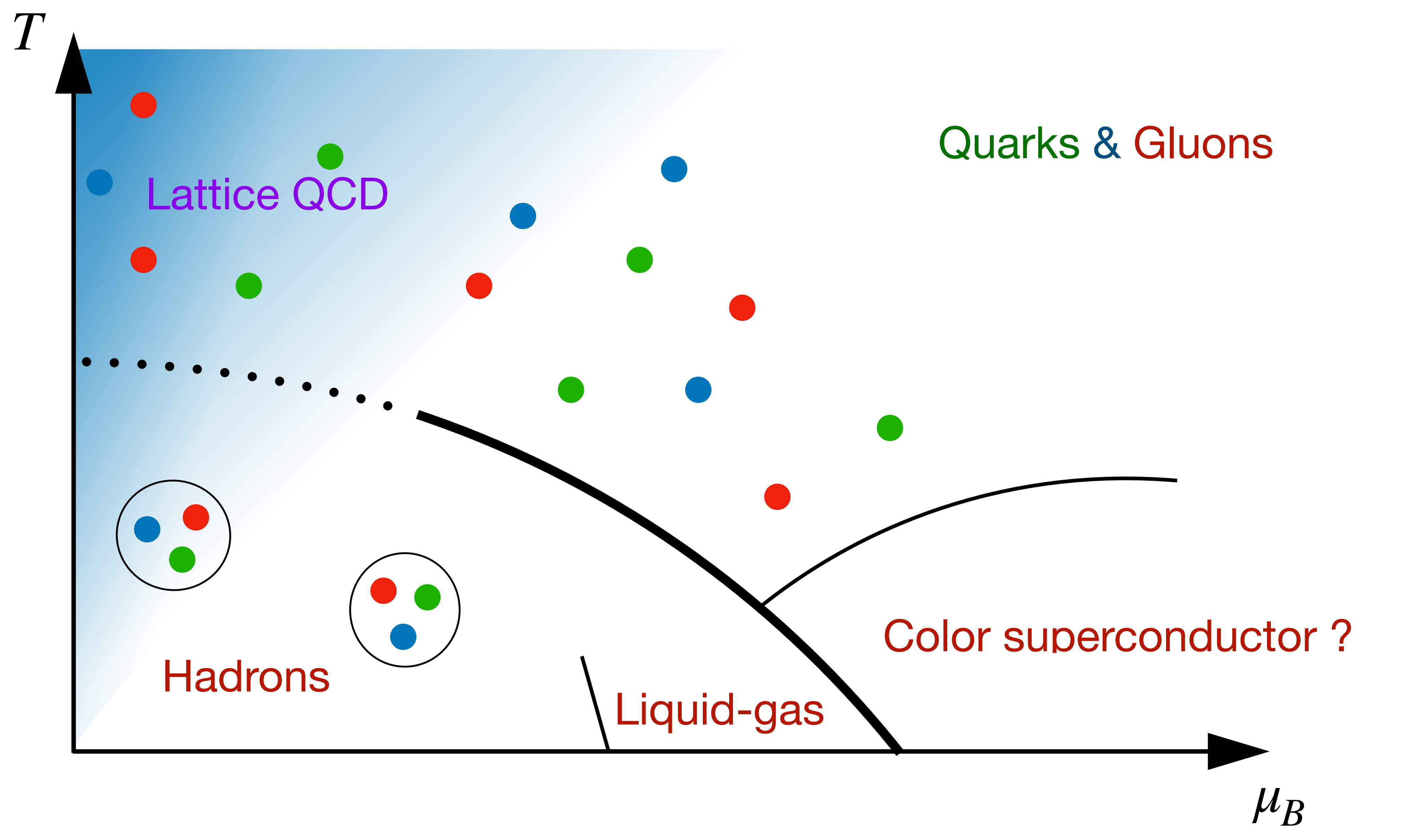
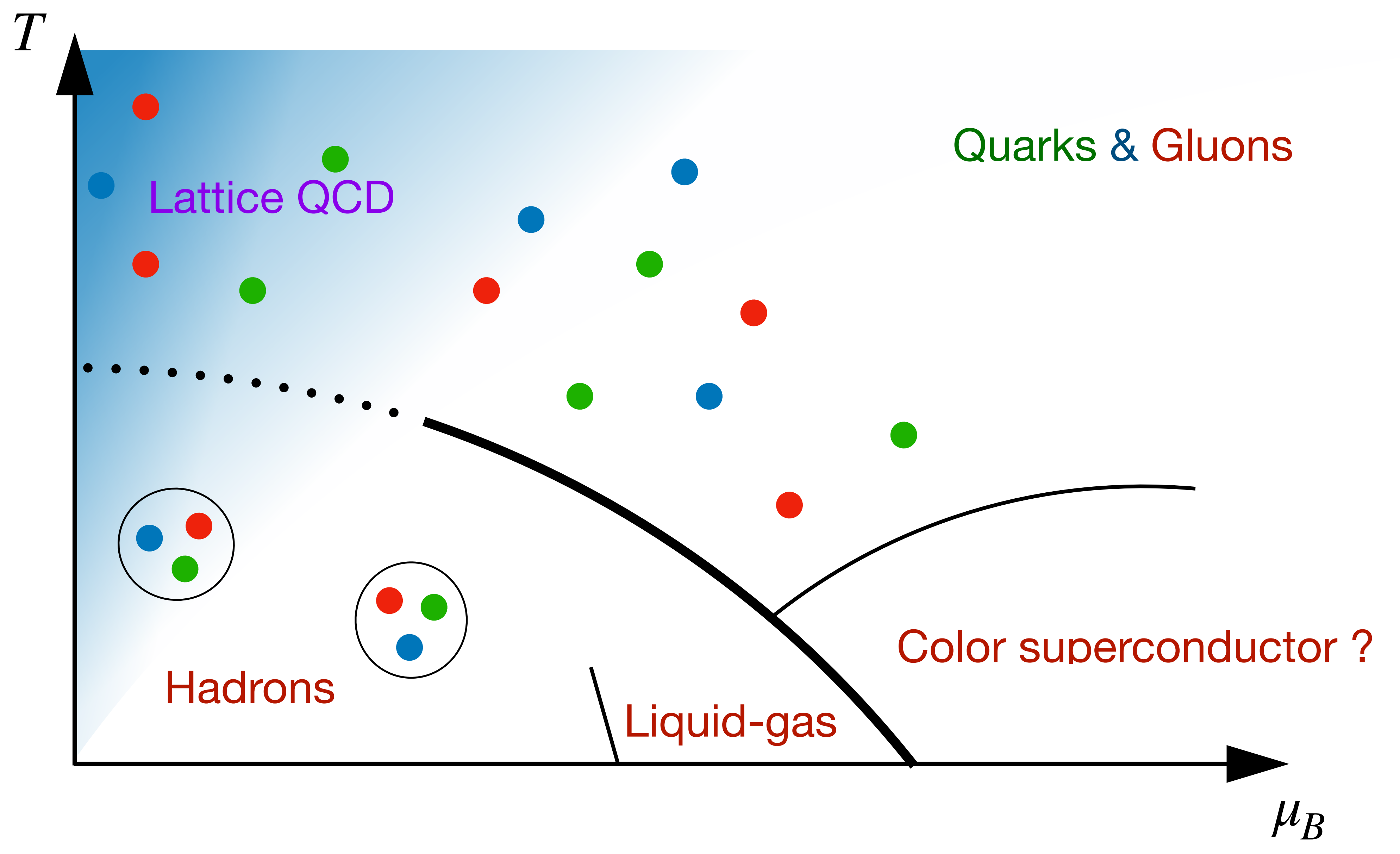


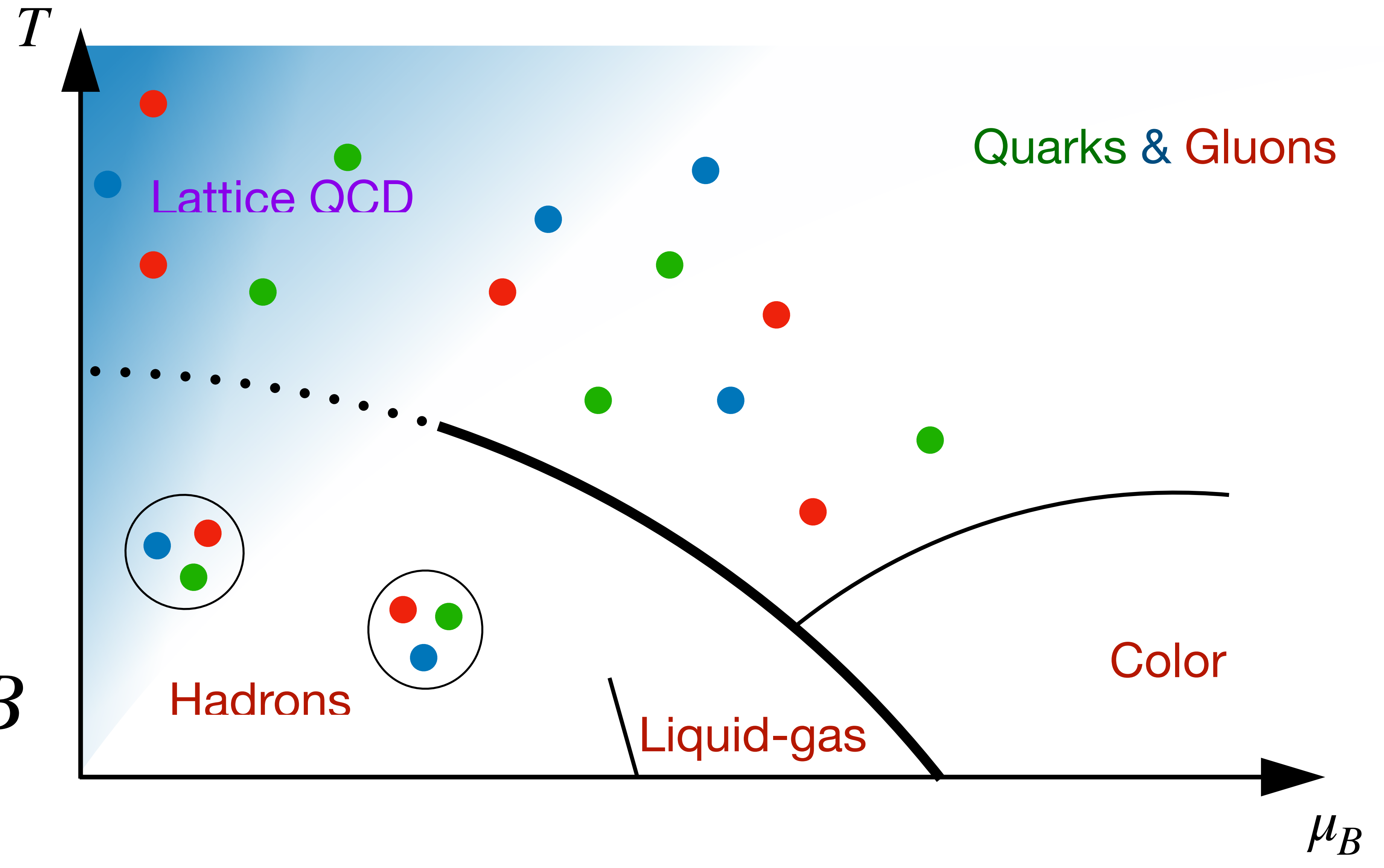
Further prospectives

- Understanding properties (phases, thresholds) in QCD at finite T
- Higher order fluctuations $\chi^n_{B,Q,S}$
- Improving CEP estimations
- Approach to solve or overcome the sign problem
  - Reweighting technique [WB, 2020]
  - Complex Langevin [G. Aarts, 2013] [D.Sexty, 2013]
  - Lefschetz thimbles [M. Cristoforetti et al., 2012]
  - Density of states [K. Langfeld et al., 2012]
  - Machine Learning [S. Lawrence, Y. Yamauchi et al., 2012]

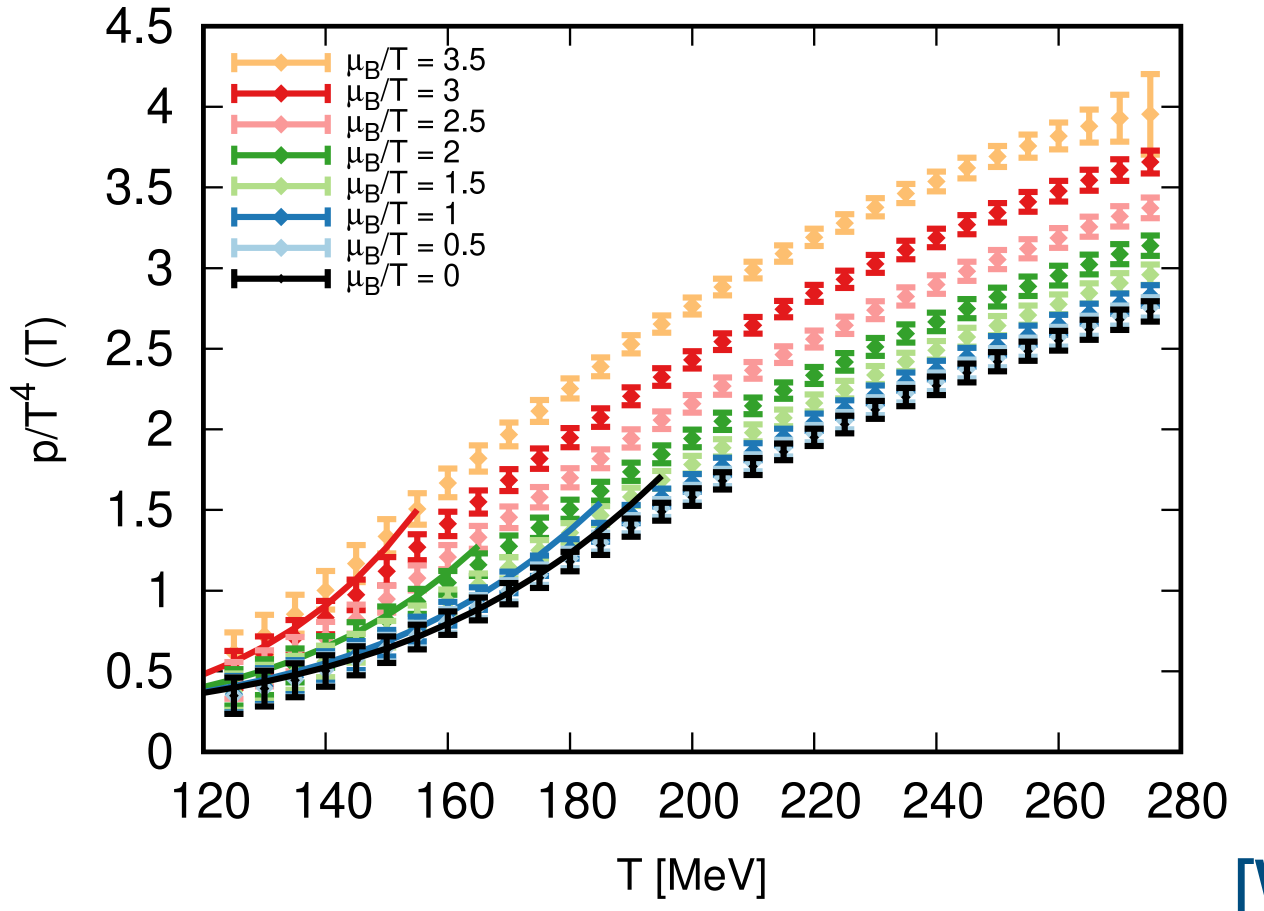


Conclusions

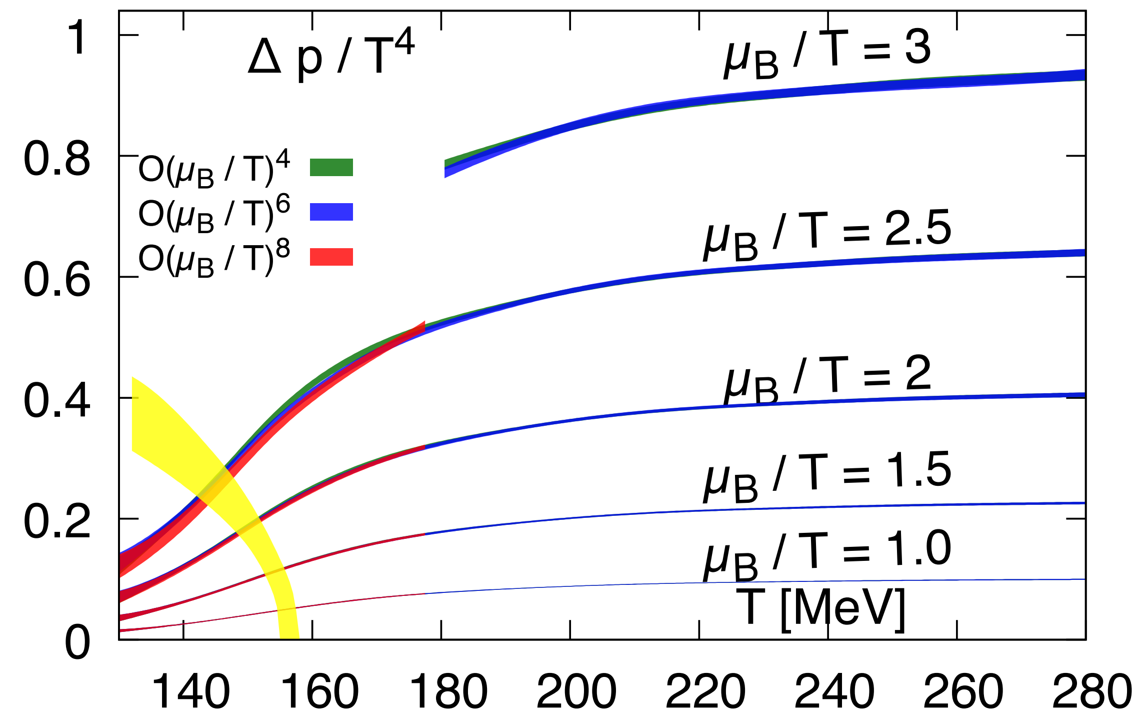
QCD phase diagram on the lattice



- Precise data on the crossover at zero and small μ_B
- Non-trivial properties (phases) of finite-T QCD
- EoS of QCD at $\mu_B/T \lesssim (2.5-3)$, conserved charge fluctuations at $\mu_B = 0$
- CEP, if exists, is most likely at very high μ_B



[WB, 2022]



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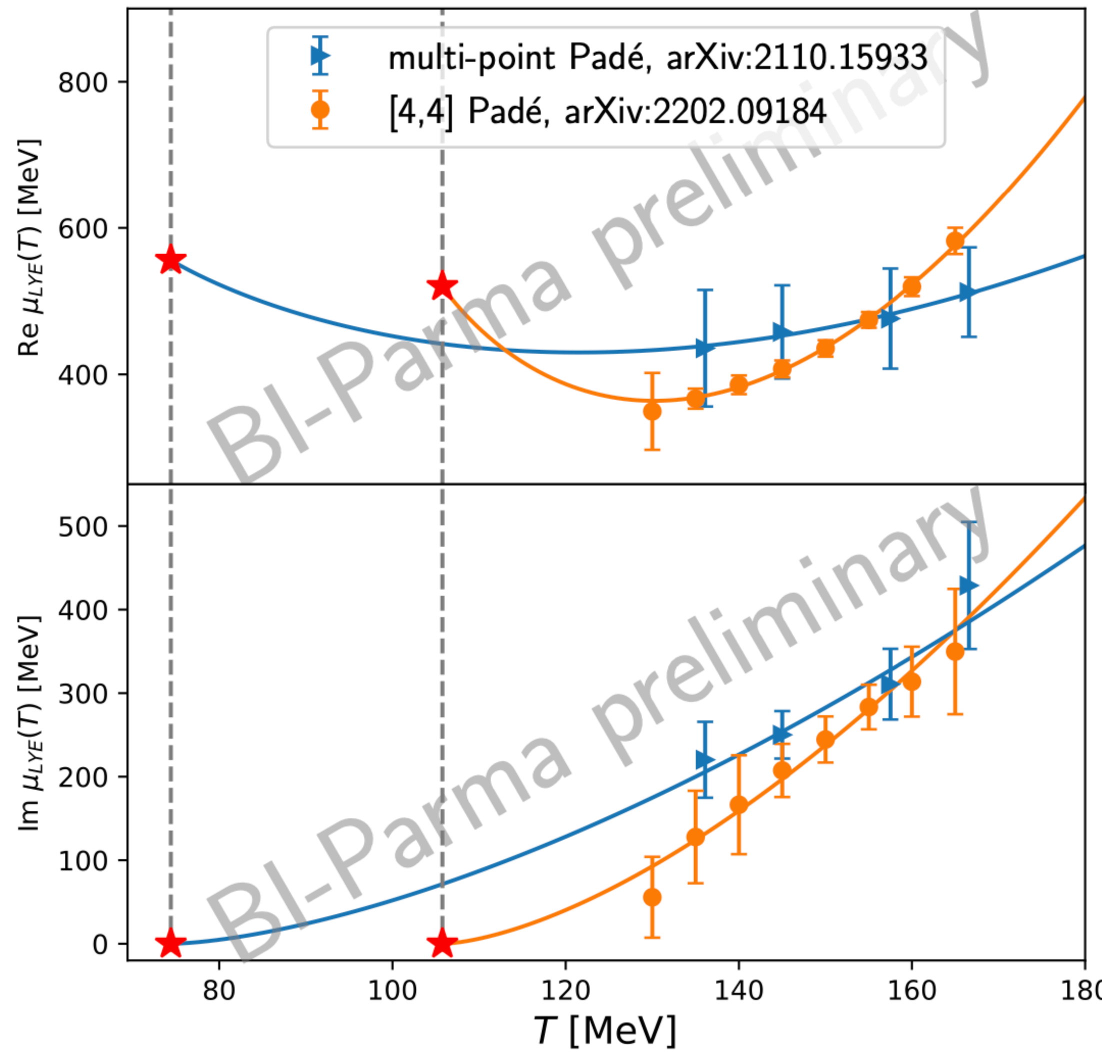


BACKUP

Zero μ, nonzero T

Possible thresholds/phase transitions and other scenarios

Spectrum of the Dirac operator:

$$D|\lambda\rangle = \lambda|\lambda\rangle$$

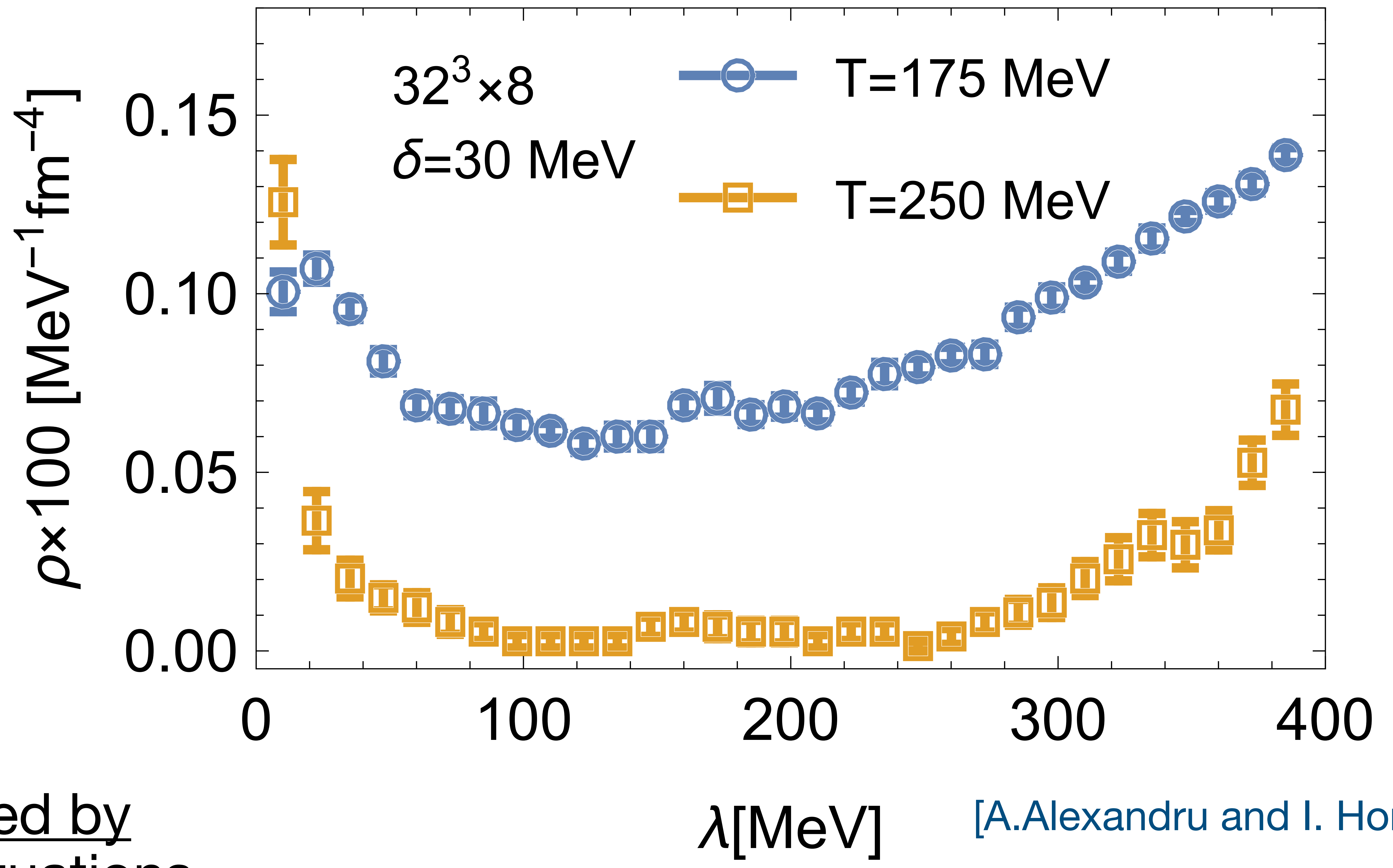
Density $\rho(\lambda)$ contains a lot of information about the system and its symmetries

Banks-Casher relation:

$$\frac{1}{V}\langle\bar{\psi}\psi\rangle \to \int_0^\infty \rho(\lambda)\frac{m}{\lambda^2+m^2} \to \pi\rho(0)$$

IR phase:

$$\rho(\lambda) \sim \lambda^{-1+\delta}$$



[A.Alexandru and I. Horvath, 2019]

Can be described by topological fluctuations

[T. Kovacs, 2023]

$$T \sim 200 - 250 \text{ MeV}$$

Complex Langevin method and Lefschetz thimbles

$$Z = \int_{-\infty}^{\infty} dx e^{-S(x)}$$

- Complexify: $x \to z \in \mathbb{C}$
- Complex Langevin: $\dot{z} = -\partial_z S(z) + \eta$
- Lefschetz thimbles: instead of integral along real axis $z = x \in \mathbb{R}$, deform a contour to get milder sign problem (Machine Learning)

Chiral-spin symmetry

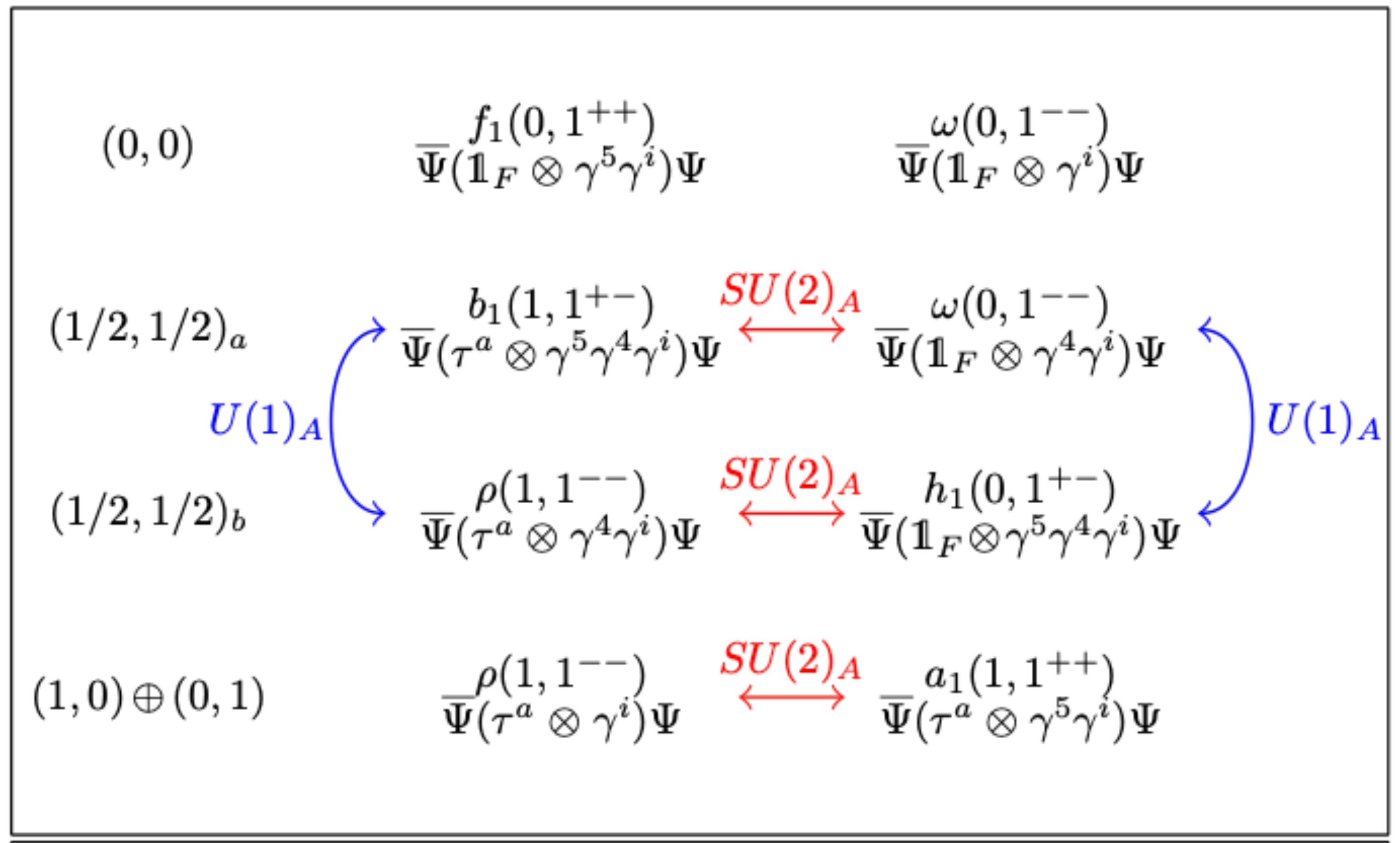
- $\psi \rightarrow \exp\left(i\frac{\epsilon^n \Sigma^n}{2}\right)\psi \qquad \Sigma_n = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$

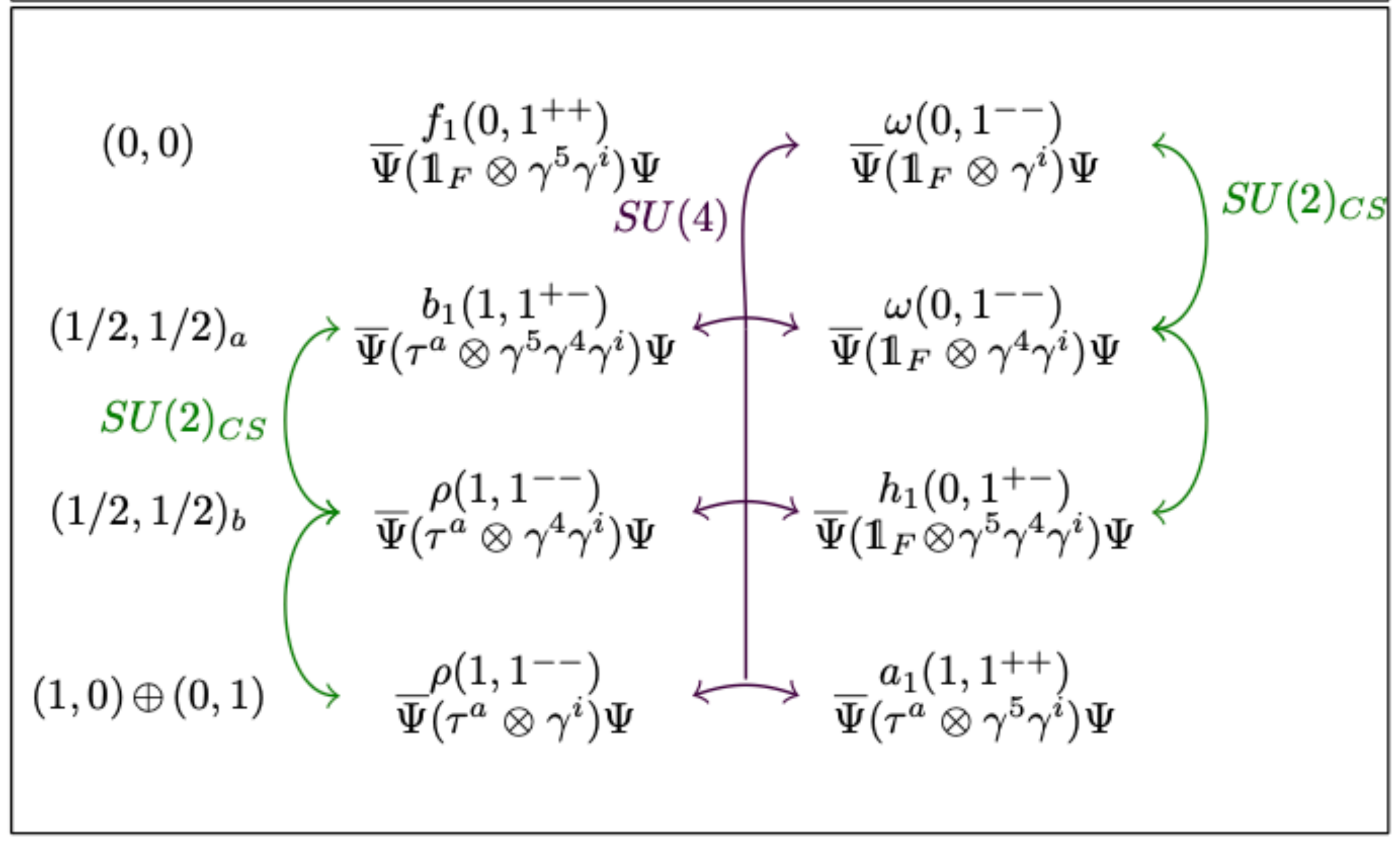
[L. Glozman, 2014]

- Symmetry of the chromo-electric color charge

- $Q^a = \int d^3x q^\dagger(x) T^a q(x)$

- Larger symmetry then in the free theory(!)





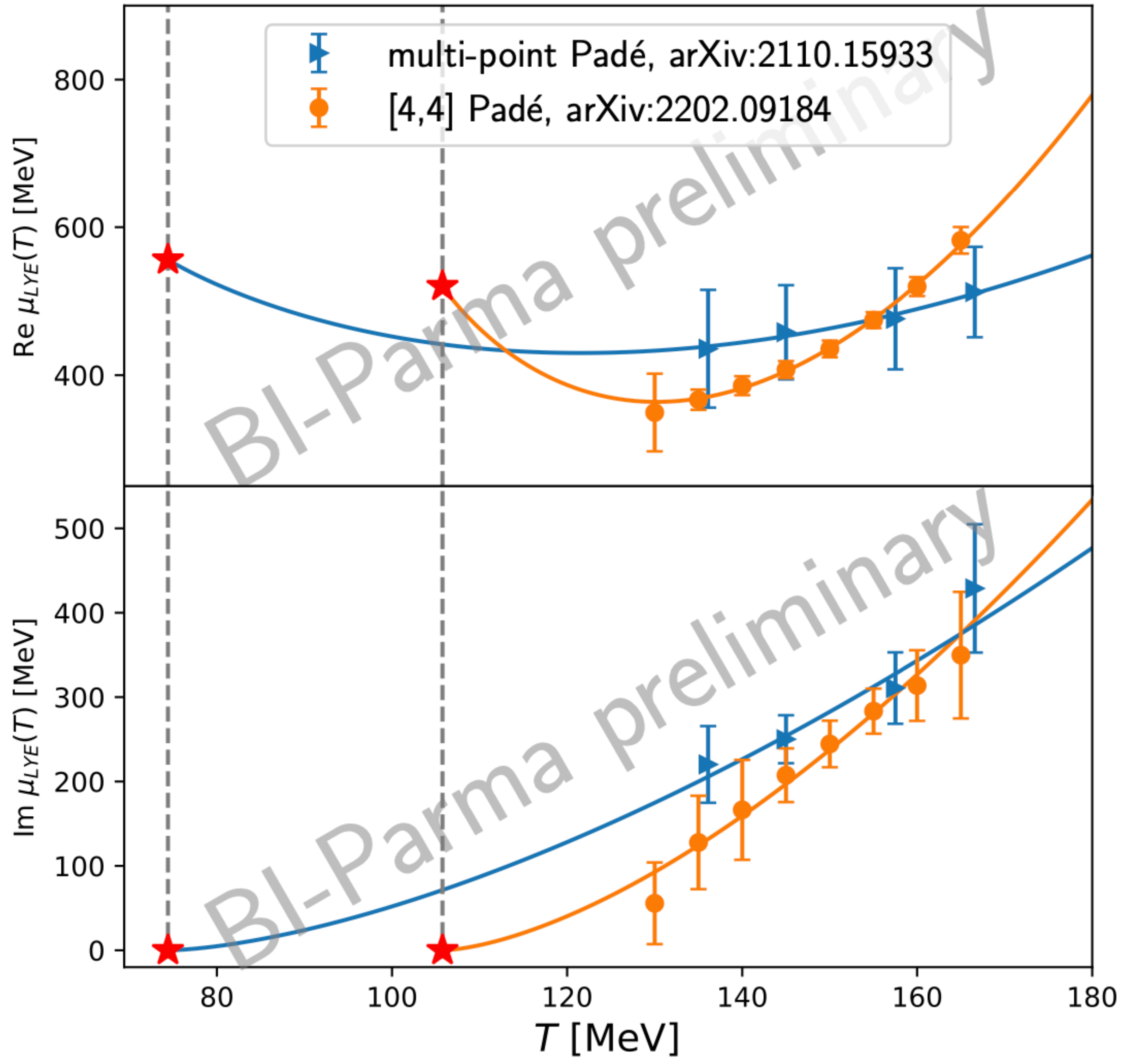
[L. Glozman, 2022]

Lee-Yang zeros

- $Z \equiv Z(V, T, \mu_B)$ is real for $\mu_B \in \mathbb{R}$
- $Z \equiv Z(V, T, \mu_B)$ can have complex μ_B^i roots
- Lee-Yang theorem: phase transition when $\mu_B^i \to \mu_B^c \in \mathbb{R}$

[Lee, Yang, 1952]

- Lattice data: $B(\mu_B) \sim \dfrac{\partial \log Z}{\partial \mu_B}$ at $-i\mu_B^0 \in \mathbb{R}$ or χ_n^B at $\mu_B = 0$
- Reconstruct Pade approximation of $B(\mu_B)$ for all μ_B
- Study its zeros and when they approach real axis



[Bielefeld-Parma, 2023]

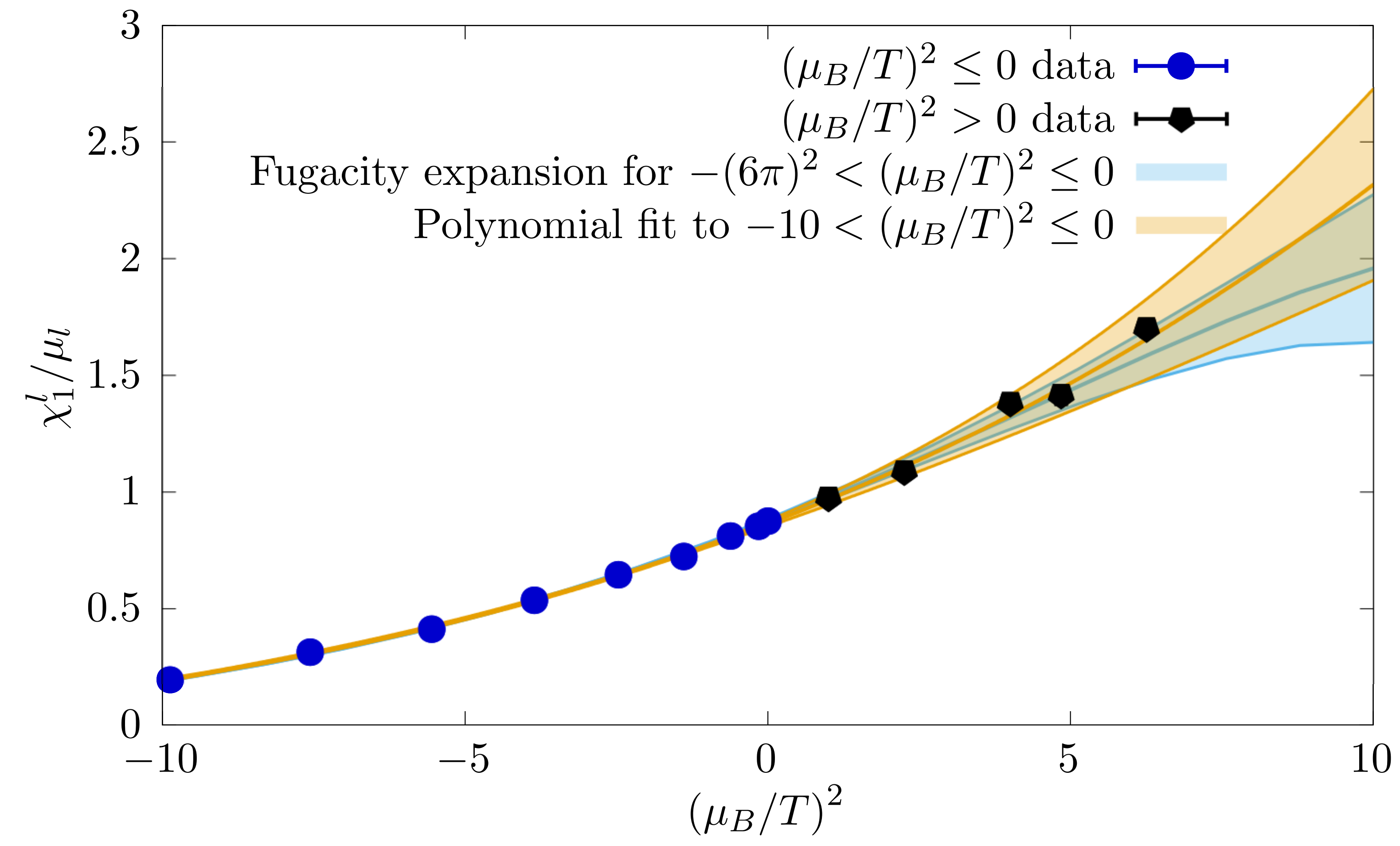
Reweighting for the sign problem

Weight Observable

- $p \sim e^{-S_G} \det D = \boxed{e^{-S_G} |\det D|}\,\boxed{e^{i\theta}}$
- $p \sim e^{-S_G}\,\mathrm{Re}\det D = \boxed{e^{-S_G}\,|\mathrm{Re}\det D|}\,\boxed{\mathrm{sign}(\mathrm{Re}\det D)}$

Weight Observable

- $\int dUp[U]O[U] \to \int dUp_w[U]p_O[U]O[U]$
- Issues with staggered fermions



[WB, 2021]