Pion distribution amplitude from lattice QCD calculation with domain wall and HISQ fermions at physical pion mass

Rui Zhang

Argonne National Laboratory

15th European Research Conference
on Electromagnetic Interactions with Nucleons and Nuclei

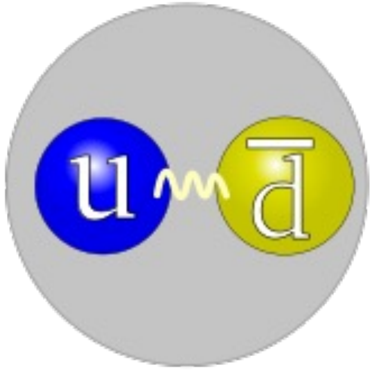
Paphos, Cyprus

In collaboration with E. Baker, D. Bollweg, X. Gao, A. Hanlon,
S. Mukherjee, P. Petreczky, Q. Shi, S.Syritsyn, Y. Zhao

Outline

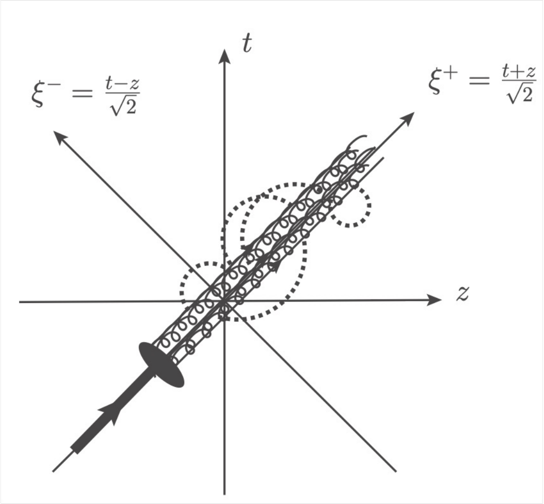
- Introduction of pion distribution amplitude
- Lattice measurements
- Extracting x-dependence from lattice data
- Conclusion and Outlook

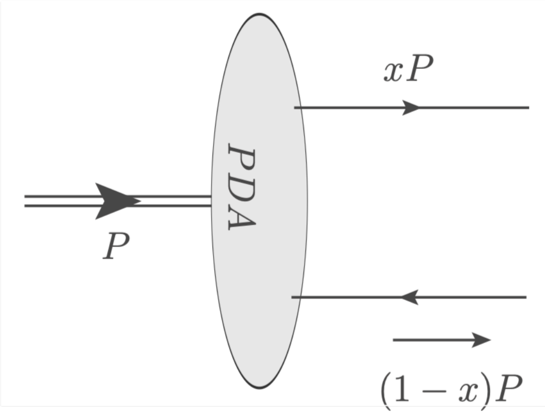
Pion Distribution Amplitude (DA)



Pion lightfront DA $\phi(x)$: probability amplitude of pion in the bound state's minimal Fock component $|q\bar{q}\rangle$

$$\phi(x,\mu) = \frac{1}{if_\pi}\int \frac{d\xi^-}{2\pi} e^{i\left(\frac{1}{2}-x\right)\xi^- p^+} \langle 0|\bar{q}\left(\frac{\xi^-}{2}\right)\gamma^-\gamma_5 U\left(\frac{\xi^-}{2},-\frac{\xi^-}{2}\right) q\left(-\frac{\xi^-}{2}\right)|\pi(p)\rangle$$



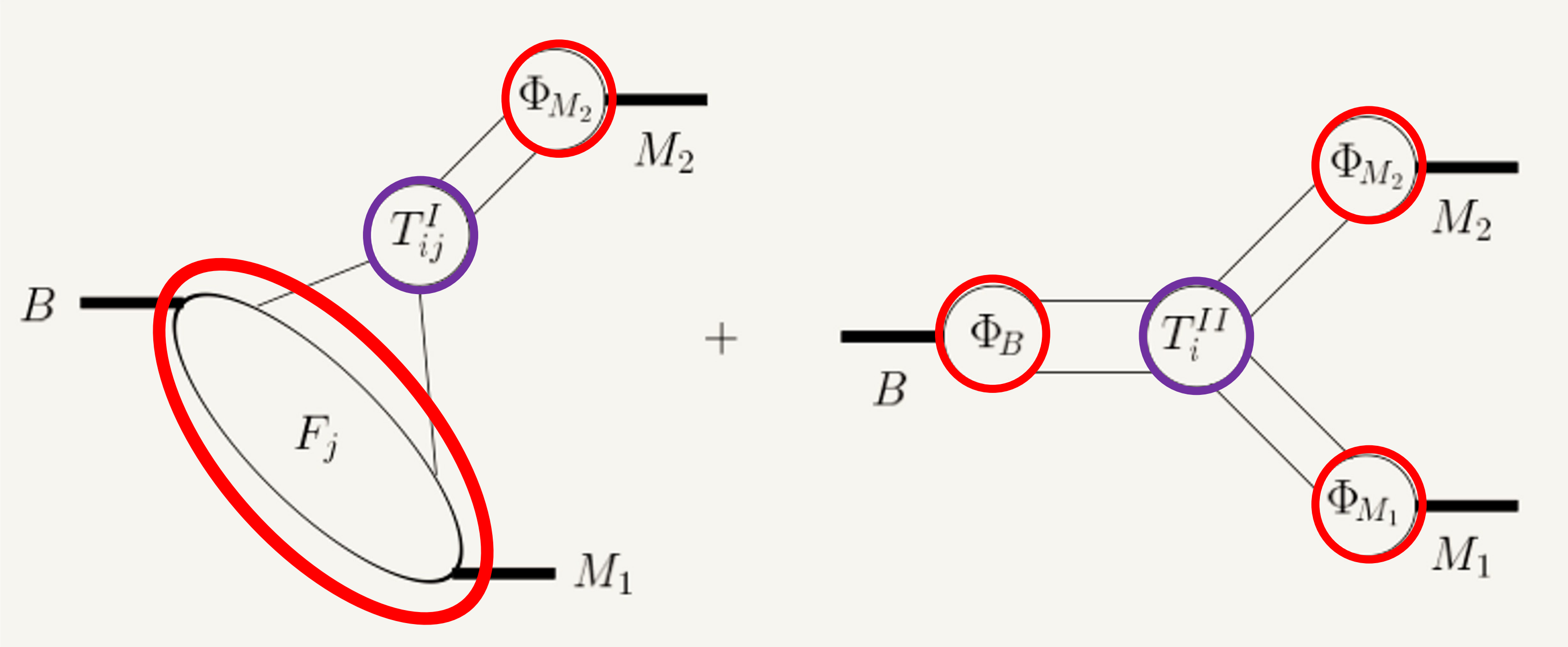


QUARK-GLUON TOMOGRAPHY COLLABORATION

Factorization of hard exclusive process

DA as important input to hard exclusive process at $Q^2 \gg \Lambda_{\mathrm{QCD}}^2$:

Beneke, et al. NPB(2001)



: Nonperturbative, IR

: Perturbative, UV

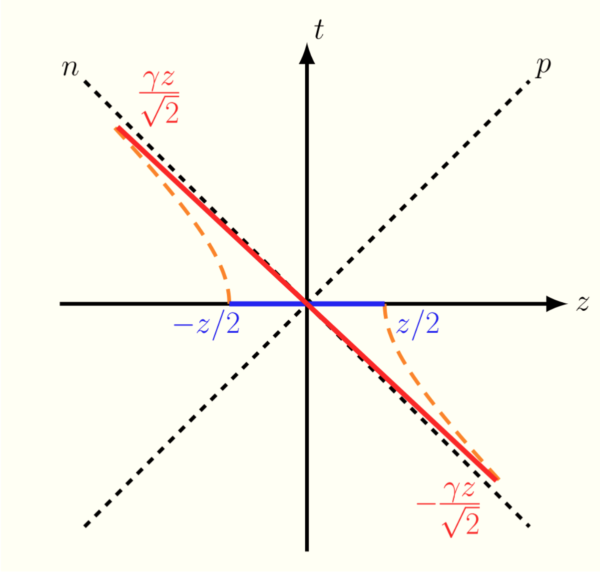
$$\langle \pi K | Q_i | B \rangle = F_0^{B\to\pi}\, T^{\mathrm{I}}_{K,i} * f_K \Phi_K + F_0^{B\to K}\, T^{\mathrm{I}}_{\pi,i} * f_\pi \Phi_\pi + T^{\mathrm{II}}_i * f_B \Phi_B * f_K \Phi_K * f_\pi \Phi_\pi\,,$$

Argonne NATIONAL LABORATORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

X-dependence calculation

Ji, et al., RMP (2021)



- Large Momentum Effective Theory
- quasi-DA: Same IR behavior/ different UV behavior

$$\tilde{\phi}(x, P_z) = \frac{1}{i f_\pi}\int \frac{dz}{2\pi} e^{i\left(\frac{1}{2}-x\right)zP_z}\langle 0|\bar{q}(z)\gamma_z\gamma_5 U(z,-z)q(-z)|\pi(P_z)\rangle$$

- Approach $P \rightarrow \infty$ limit through large P_z expansion
- Matching to lightcone distribution

$$\tilde{\phi}(x, P_z) = \int_0^1 dy\, C(x, y, \mu, P_z)\phi(y, \mu) + O\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

Xiong, et al., PRD (2014)
Ma, et al., PRD (2018)
Izubuchi, et al., PRD (2018)
Liu, et al., PRD, (2019)
Ji, et al., RMP (2021)

Pros: Direct x-dependence calculation, well-controlled systematics in mid-x region

Cons: Large P_z expansion not working near endpoints
More complicated renormalization

Argonne NATIONAL LABORATORY

Outline

Lattice measurements

QUARK-GLUON TOMOGRAPHY COLLABORATION

Lattice Setup

- Physical pion mass
- Chiral symmetric Fermion action – domain wall fermions
- Momentum smeared quark source

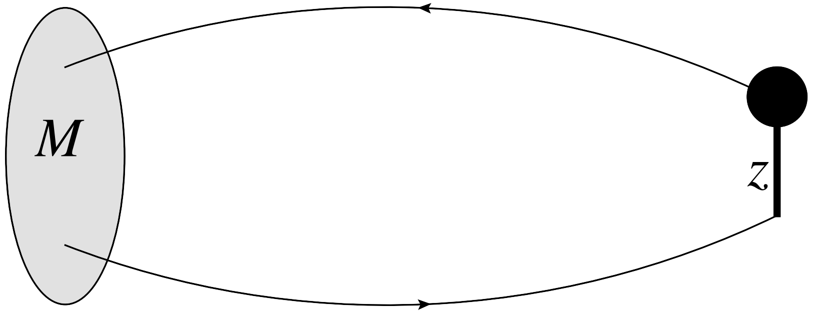
Lattice Spacing-a	Pion Mass	Lattice Volume	$m_\pi L$	Fermion Action
0.0836 fm	137 MeV	$64^3 \times 128$	3.73	2+1f DW

Momentum Smearing	Pion Momentum	Samples	Sources	Effective Statistics
$k = \{0, 1.4\}$ GeV	$P_z = [0, 1.85]$ GeV	55	{32, 128}	Up to 130,000

Argonne NATIONAL LABORATORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

Lattice measurements



- $C_{\pi O_\mu}(t,z,P_z) = e^{-i\frac{zP_z}{2}} \langle u\gamma_5\bar{d}\ (0,P)|\bar{u}(\tau,0)\gamma_\mu\gamma_5 U(0,z)d(z)|\Omega\rangle, \quad \mu = 0,3$
- Spectrum expansion:

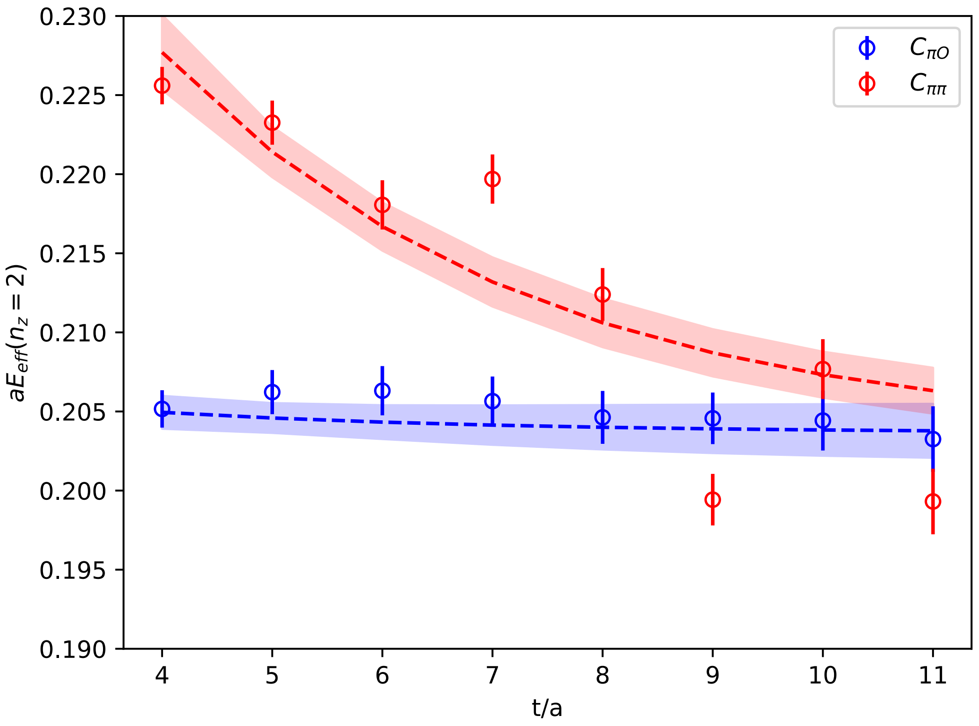
$$C_{\pi O_\mu}(t,z,P_z) = \sum_n \frac{c_n e^{-E_n t}}{2E_n} \langle n|O_{\mu 5}|\Omega\rangle = A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + \cdots$$

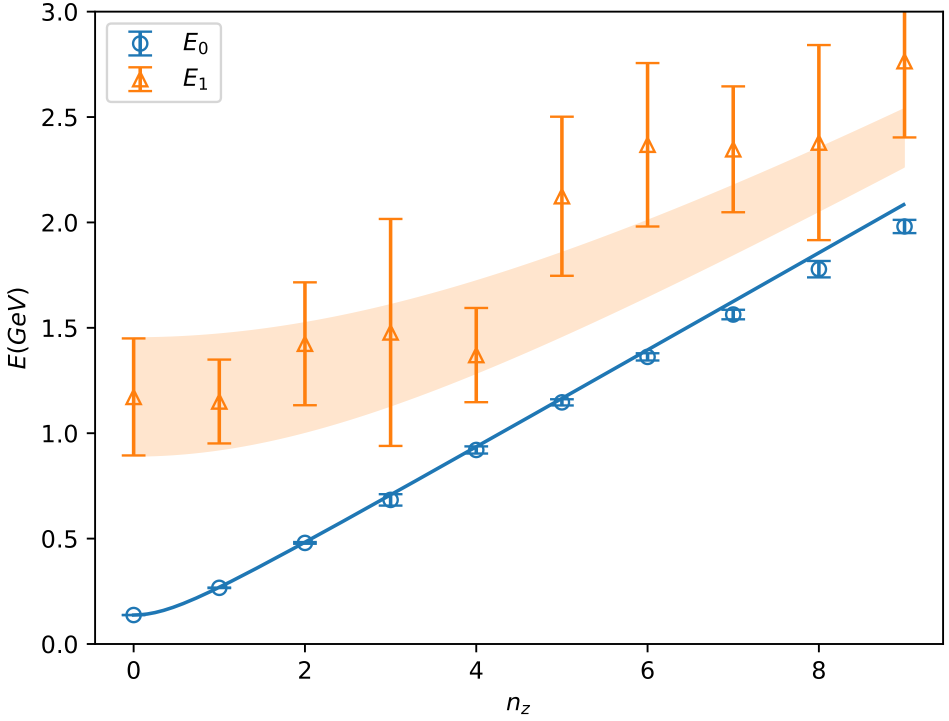
- $C_{\pi\pi}(t,P_z) = \langle u\gamma_5\bar{d}\ (0,P)|\bar{u}\gamma_5 d(t,P)\rangle = \sum_n \frac{|c_n|^2}{2E_n} = B_0 e^{-E_0 t} + B_1 e^{-E_1 t} + \cdots$
- $\langle n|O_\mu|\Omega\rangle = \frac{\sqrt{2E_0}A_0}{B_0} = (i) f_\pi P^\mu h^B(z,P_z)$

Argonne NATIONAL LABORATORY

Two-state fits

We combine fit $C_{\pi O_\mu}(z=0)$ and $C_{\pi\pi}$ to determine the spectrum

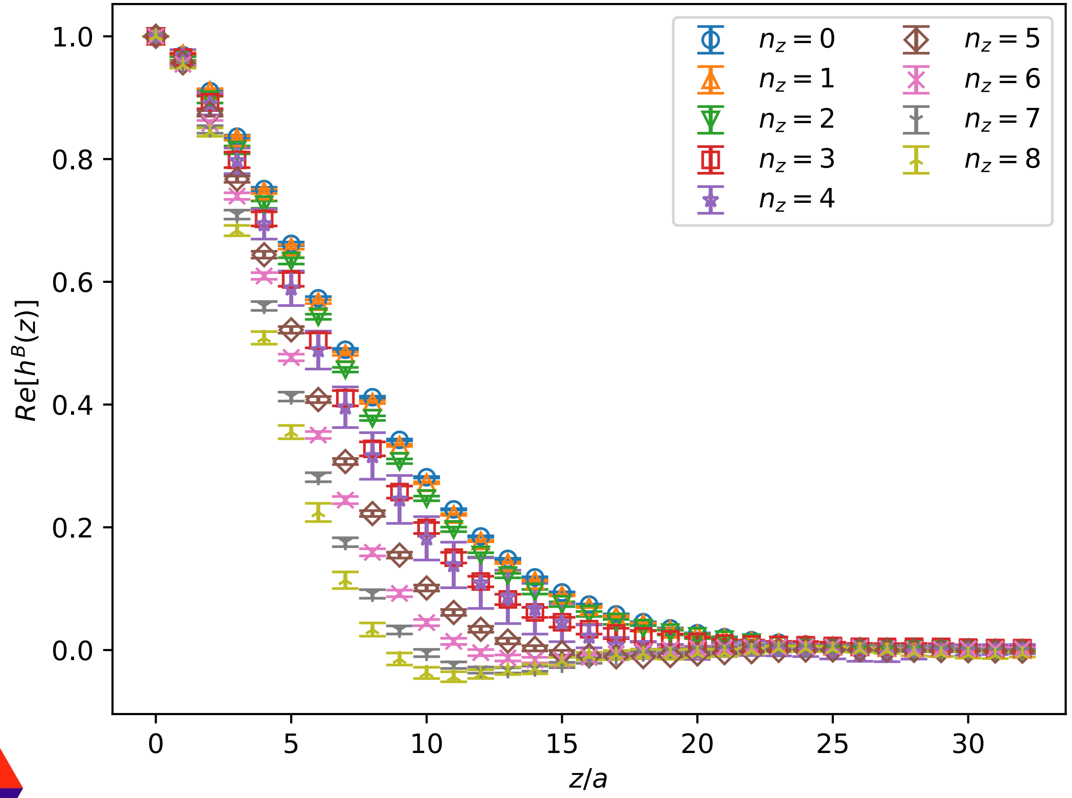


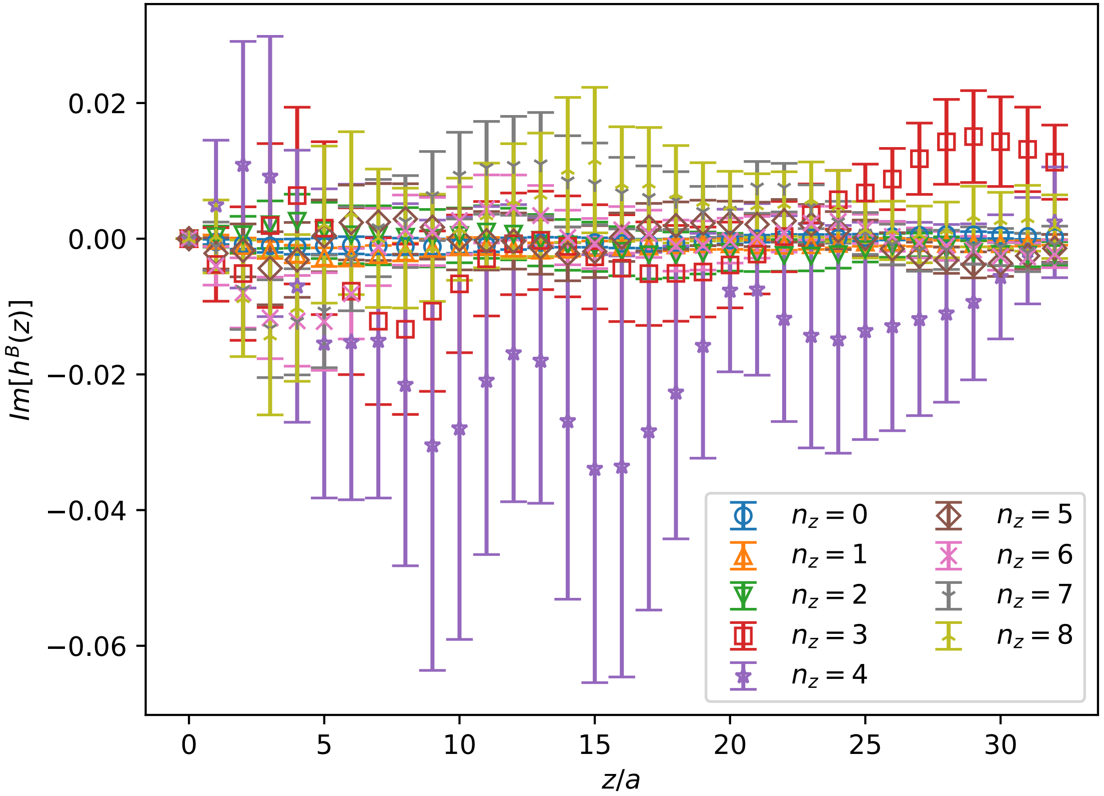


QUARK-GLUON TOMOGRAPHY COLLABORATION

Bare Matrix Elements

Pion DA is symmetric (vanishing imaginary part)



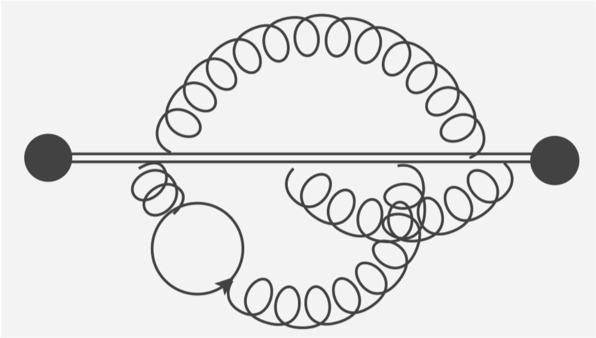


Argonne NATIONAL LABORATORY

Outline

Extracting x-dependence from lattice data

Renormalizing linear divergence



- Non-local operator: $\bar{q}(0)\Gamma U(0,z)q(z)$
- Linearly divergent self-energy $\delta m(a) \sim \frac{1}{a}$
  - $h^B(\mathrm{z}) \sim e^{-\delta m(a)\cdot z}$ Ji, et.al, PRL (2017)
- Renormalon ambiguity in $\Delta\big(\delta m(a)\big) \sim \Lambda_{QCD}$ Beneke, PLB (1995)
  - Renormalon also in the matching kernel Braun, et al., PRD (2018)
- $h^R(\mathrm{z}) \sim h^B(\mathrm{z})e^{\delta m\cdot z}$ uncertain up to $e^{\mathcal{O}(\mathrm{z}\Lambda_{\mathrm{QCD}})} \to \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\mathrm{x}\, P_z}\right)$ in $\tilde{q}$

How to remove linear ambiguity?

$$\ln\left(\frac{C_0(z,z^{-1})\exp(-I(z))}{h^B(z,0)}\right) = \delta m\,|z| + b$$

- Determine $\delta m(a)$ from matching lattice data to pQCD, with a consistently defined regularization of renormalon as the matching.
  - Leading renormalon resummation

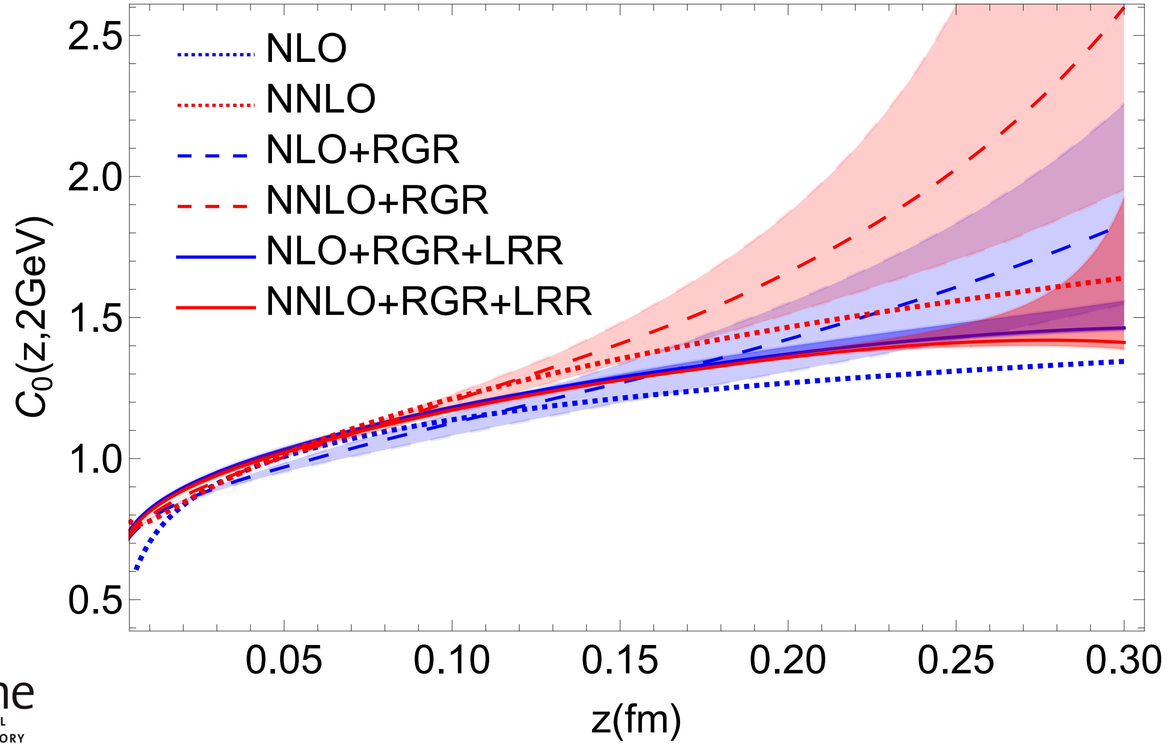
Zhang, et al., PLB (2023)

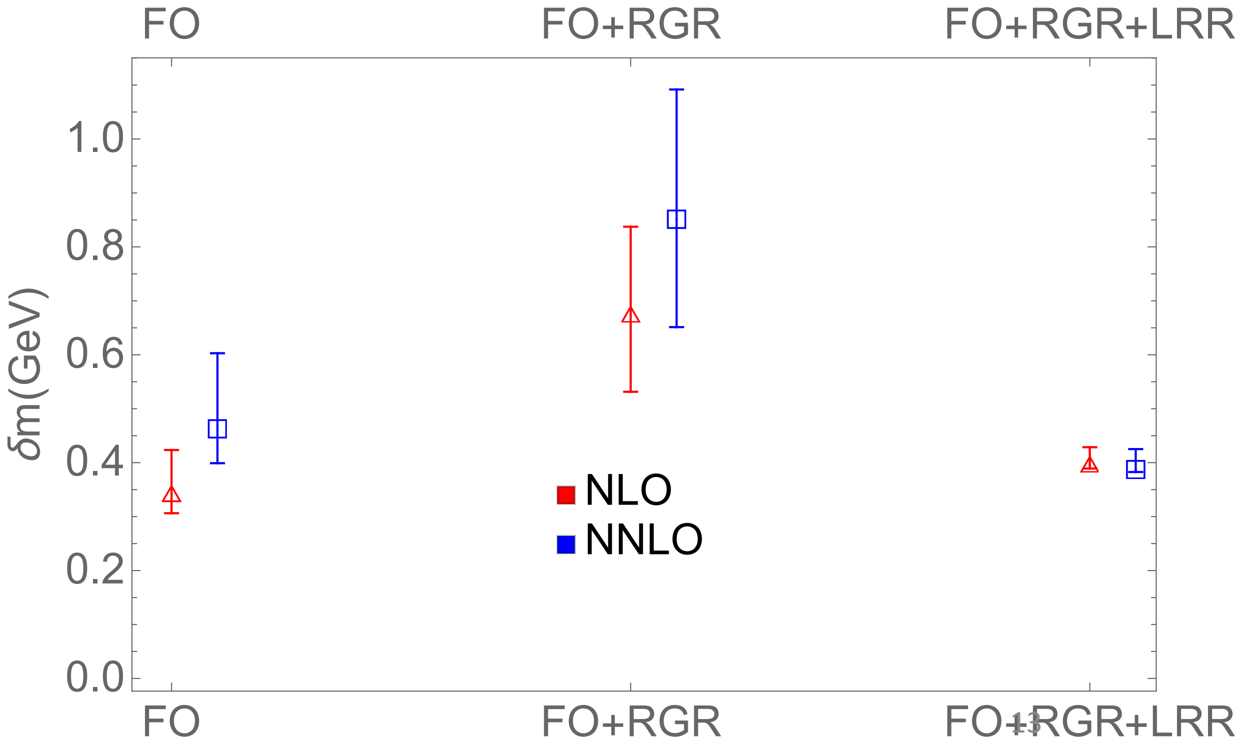
Argonne NATIONAL LABORATORY

δm with leading renormalon resummation

Zhang, et al., PLB (2023)

$$\ln\left(\frac{C_0(z,z^{-1})\exp(-I(z))}{h^B(z,0)}\right)=\delta m\,|z|+b$$



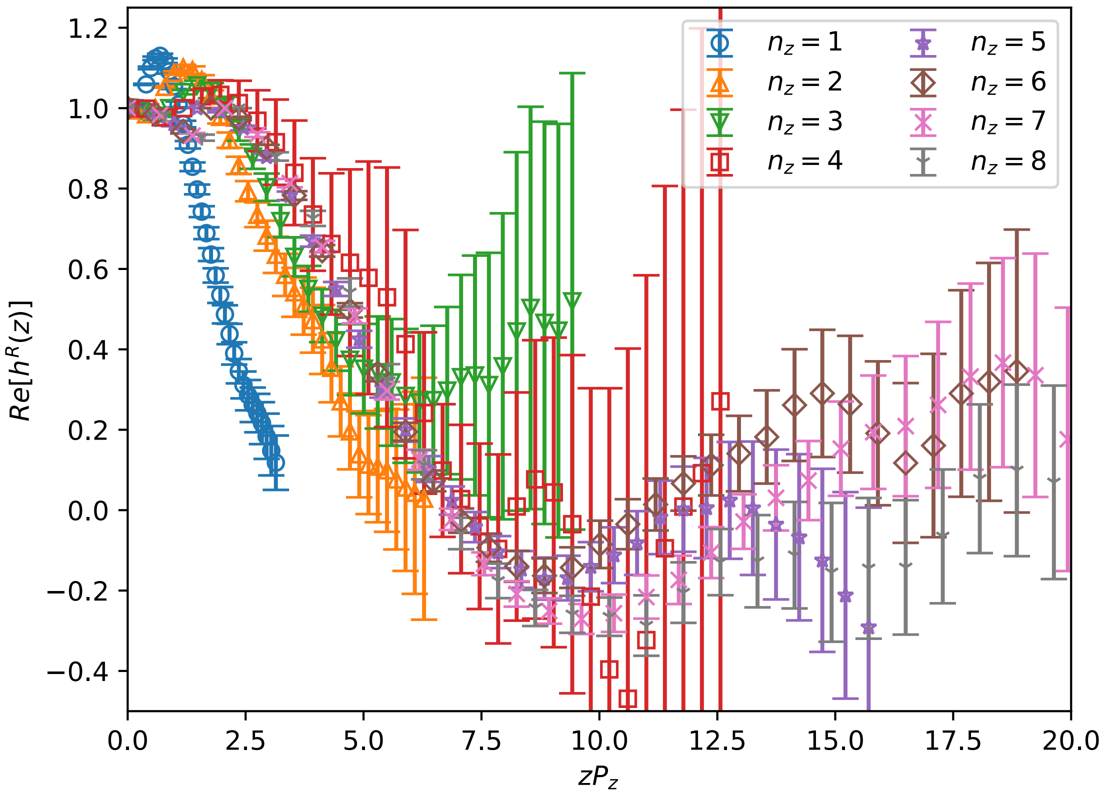


Renormalization in hybrid scheme

Ji, et al., NPB (2020)

$$h^R(z, P_z) = \frac{h^B(z,P_z)}{Z_h(z)}$$

$$Z_h(z) = \begin{cases} h^B(z,0), & |z| < z_s \\ e^{\delta m|z-z_s|} h^B(z_s,0), & |z| > z_s \end{cases}$$



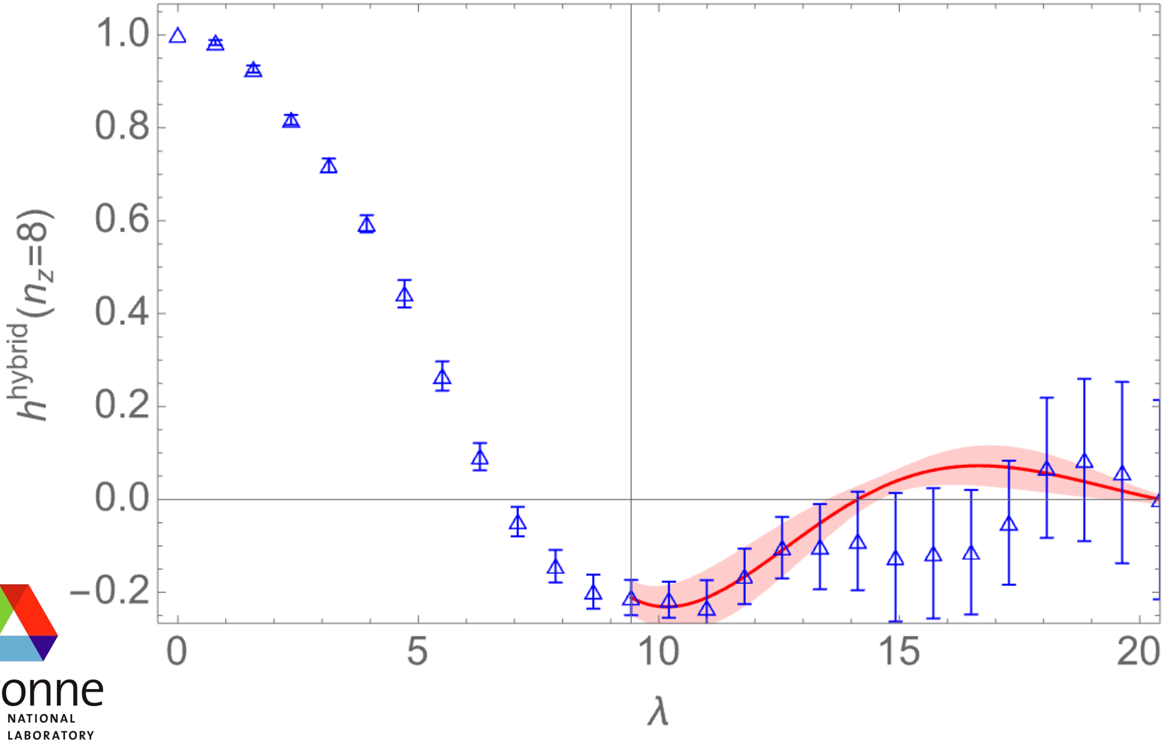
Longtail extrapolation ($\lambda = zP_z \to \infty$)

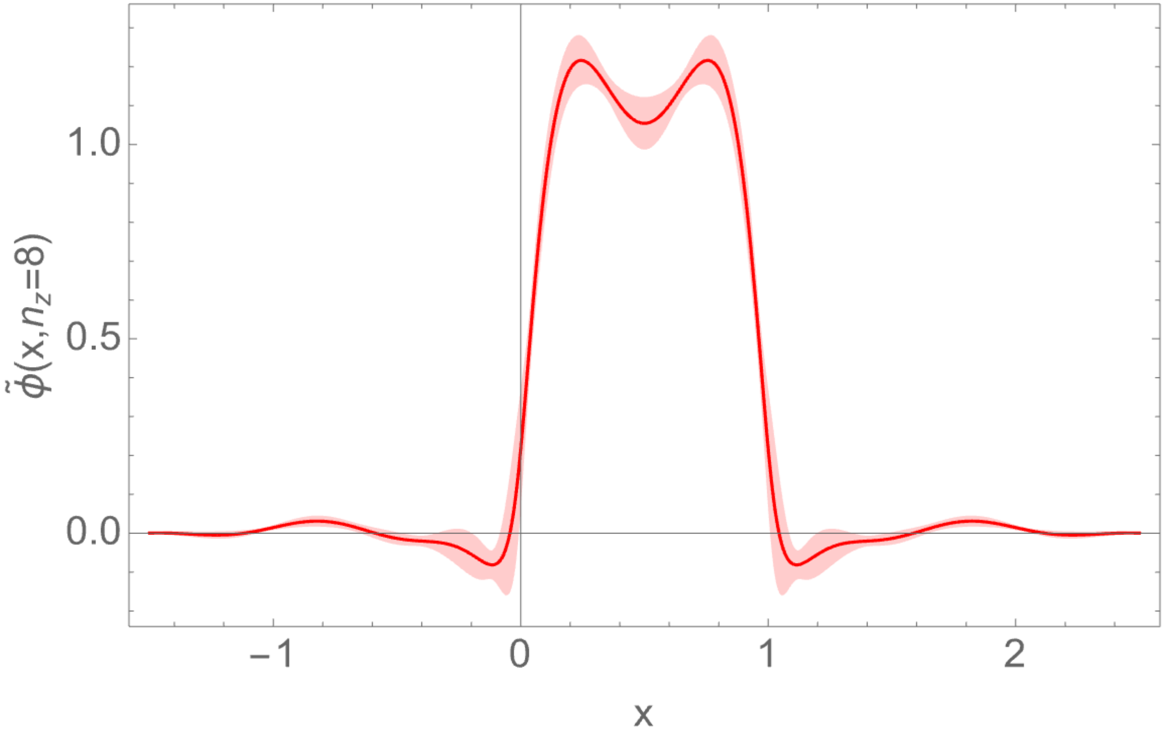
Ji, et al., NPB (2020)

Quasi-DA matrix elements have finite correlation length:

Inferred from Regge behavior

$$h^R(\lambda \to \infty) = e^{-\frac{\lambda}{\lambda_0}}\left(e^{-\frac{i\lambda}{2}}\frac{c_1}{(-i\lambda)^{d_1}} + e^{\frac{i\lambda}{2}}\frac{c_1}{(i\lambda)^{d_1}}\right)$$





QUARK-GLUON TOMOGRAPHY COLLABORATION

Perturbative matching to lightcone DA

- Power correaction

  Braun, et al., PRD (2018)

  - Renormalon resummation

    Zhang, et al., PLB (2023)

$$\tilde{\phi}(x,P_z) = \int_0^1 dy\, C(x,y,\mu,P_z)\phi(y,\mu) + O\left(\frac{\Lambda_{QCD}}{xP_z}\right) + O\left(\frac{\Lambda_{QCD}^2}{x^2P_z^2}\right)$$

$$C_B^{(1)}\left(\Gamma,x,y,\frac{P_z}{\mu}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} [H_1(\Gamma,x,y)]_{+(y)} & x<0<y \\ [H_2(\Gamma,x,y,P^z/\mu)]_{+(y)} & 0<x<y \\ [H_2(\Gamma,1-x,1-y,P^z/\mu)]_{+(y)} & y<x<1 \\ [H_1(\Gamma,1-x,1-y)]_{+(y)} & y<1<x \end{cases}$$

- Higher α_s correaction $(\alpha_s L)^n$
  - Higher order large logs at small $2xP_z$ or $2\bar{x}P_z$

    Holligan, et al., NPB (2023)

$$H_1(\Gamma,x,y) = \begin{cases} \frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} + \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{y-x}{-x} & \Gamma=\gamma^z\gamma_5 \text{ and } \gamma^t \\ \frac{1}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} + \frac{1}{y-x}\frac{x}{y}\ln\frac{y-x}{-x} & \Gamma=\gamma^z\gamma_\perp \end{cases},$$

$$H_2\left(\Gamma,x,y,\frac{P_z}{\mu}\right) = \begin{cases} \frac{1+y-x}{y-x}\frac{x}{y}\boxed{\ln\frac{4x(y-x)(P^z)^2}{\mu^2}} + \frac{1+x-y}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x} - \frac{x}{y}\right) & \Gamma=\gamma^z\gamma_5 \\ \frac{1+y-x}{y-x}\frac{x}{y}\left(\ln\frac{4x(y-x)(P^z)^2}{\mu^2} - 1\right) + \frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} & \Gamma=\gamma^t \\ \frac{1}{y-x}\frac{x}{y}\ln\frac{4x(y-x)(P^z)^2}{\mu^2} + \frac{1}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x} - \frac{x}{y}\right) & \Gamma=\gamma^z\gamma_\perp \end{cases}$$

Liu, et al., PRD (2019)

Argonne NATIONAL LABORATORY

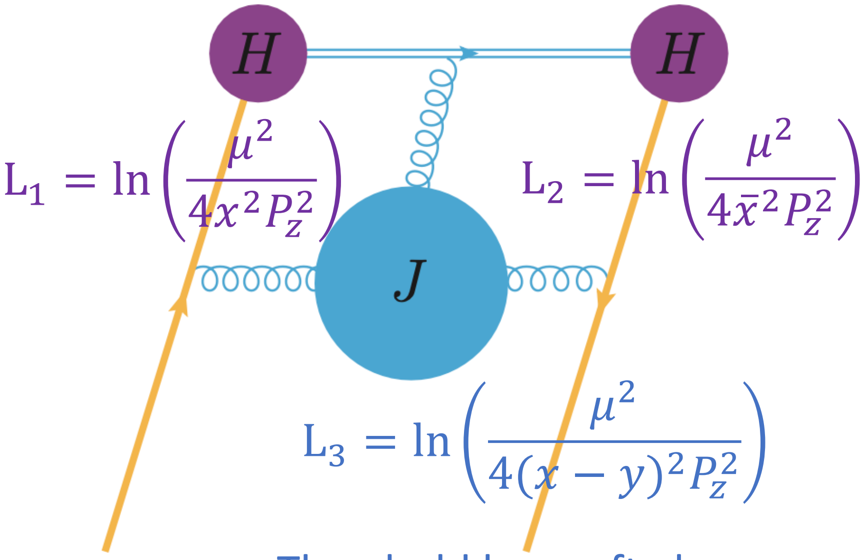
Large logarithm resummation

- The coefficient of log is x/y or $\bar{x}/\bar{y}$
  - Suppressed when far from threshold
- Factorization in threshold limit
- Resumming Sudakov factor
  - $H(x,\mu) = C^{+}(2xP_z,\mu)C^{-}(2\bar{x}P_z,\mu)$
  - $\frac{\partial \ln C^{\pm}(p,\mu)}{\partial \ln \mu^2} = \Gamma_{cusp}\left(\ln\frac{\mu^2}{p^2} \pm i\pi\right) + \gamma_c$
  - Evolve from $\mu_0 = p$ to μ

Avkhadiev, et al., 2307.12359

- Threshold resummation of Jet function is complicated (in progress)

Ji, et al., JHEP (2023)



Threshold log: soft gluon

$$C(x \to y,\mu) \approx H(x,P_z,\mu) \otimes J(x-y,P_z,\mu)$$

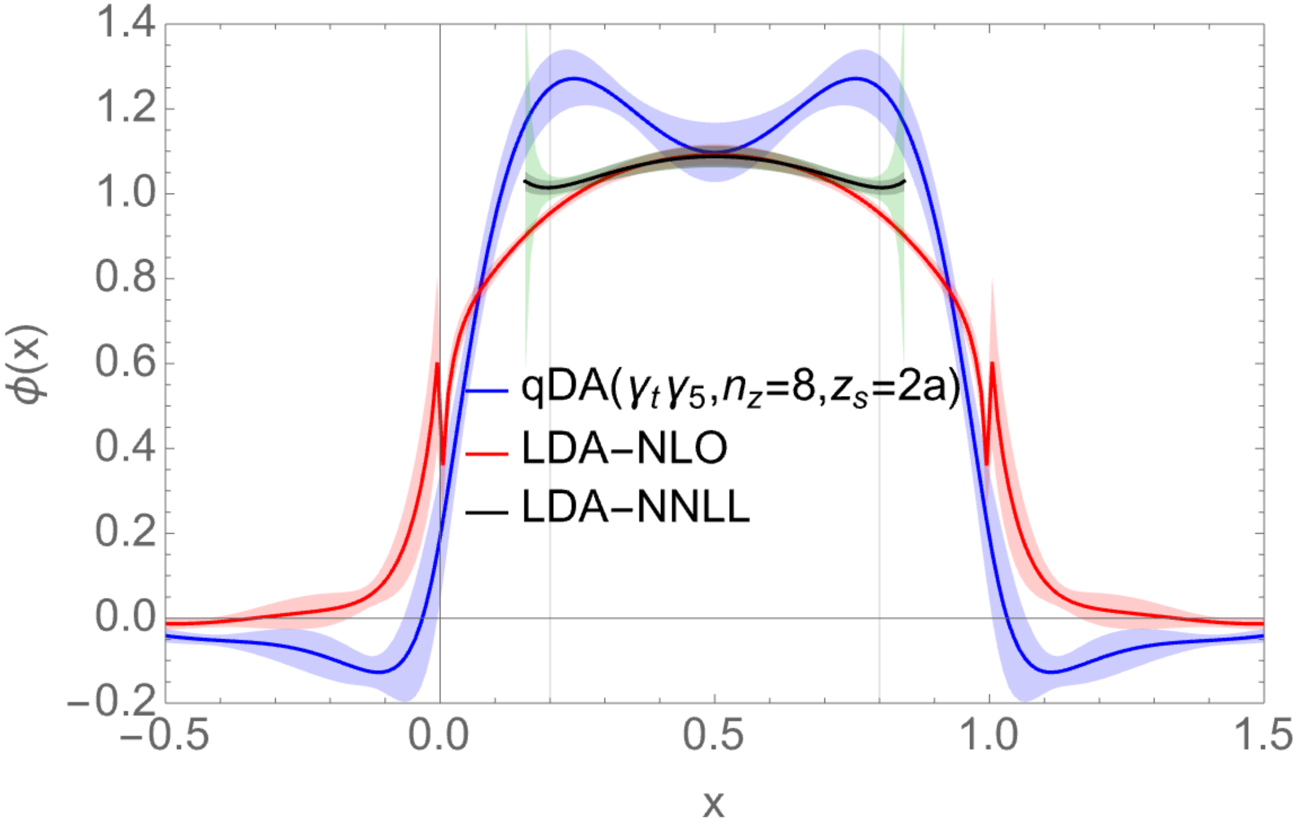
QUARK-GLUON TOMOGRAPHY COLLABORATION

Matching with resummed Sudakov factors

- Scale variation (green)
  - $\mu_0 = c * p$
  - $c = [\frac{1}{\sqrt{2}}, \sqrt{2}]$

- Scale variation is small when $0.2 < x < 0.8$

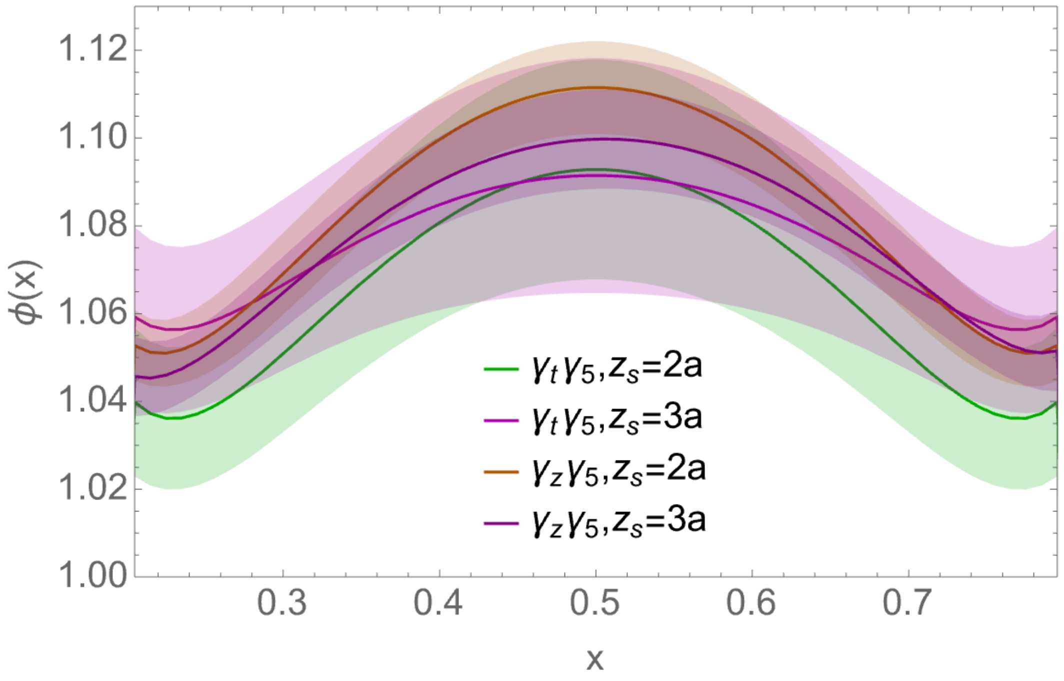
- Region with large scale variations are not reliable



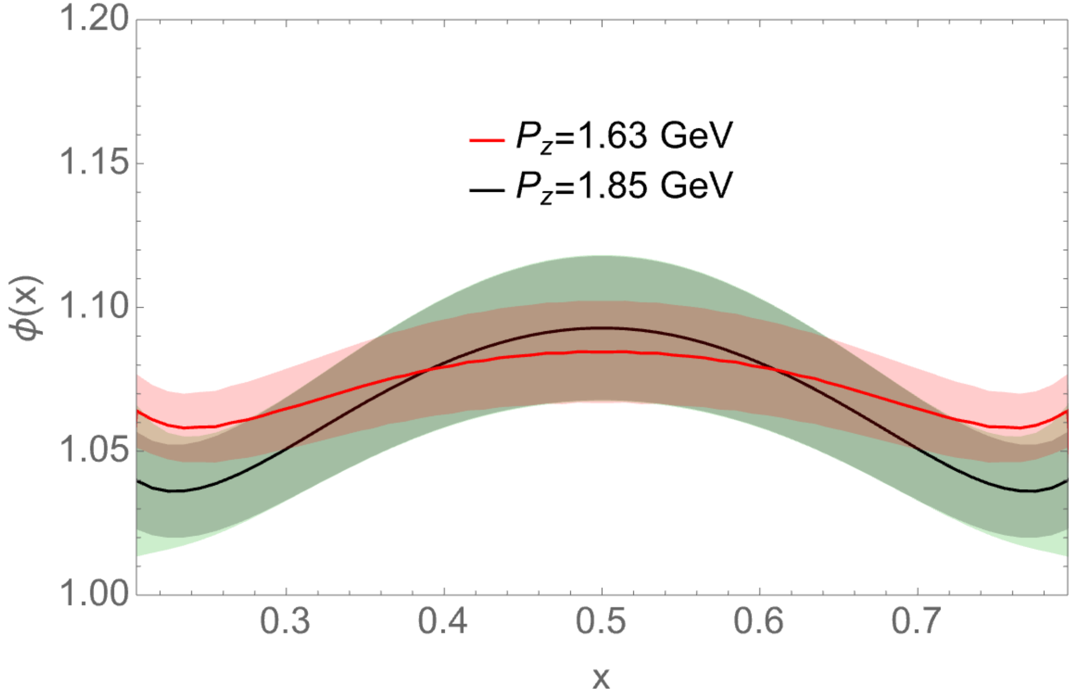
Argonne NATIONAL LABORATORY

Other systematics

- Different operators and z_s

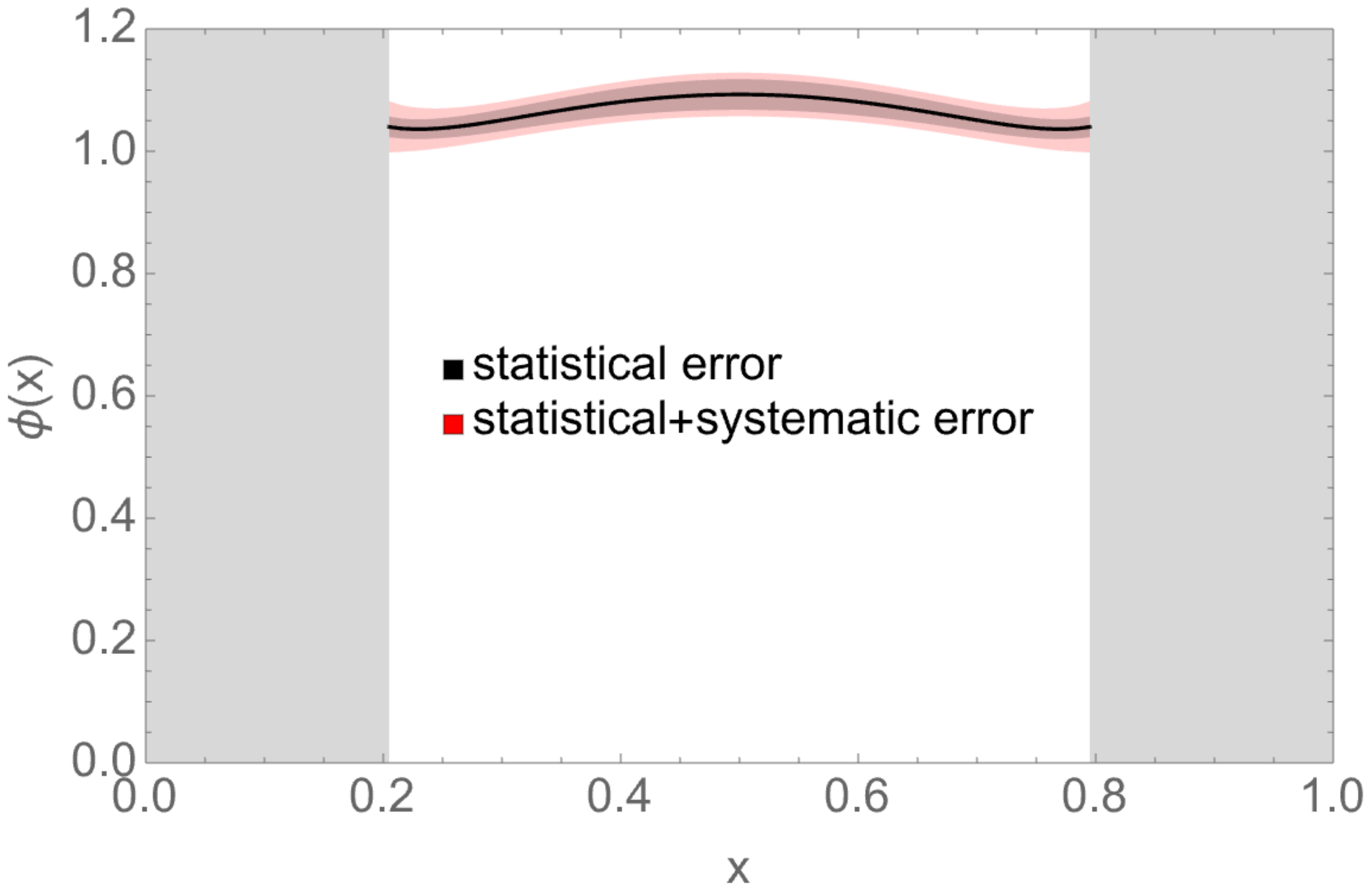


- Momentum dependence



Adding up systematic errors

- We have good control of error in $0.2 < x < 0.8$ for $P_z = 1.85$ GeV
- Statistical error about 3%
- Systematic error about 2%



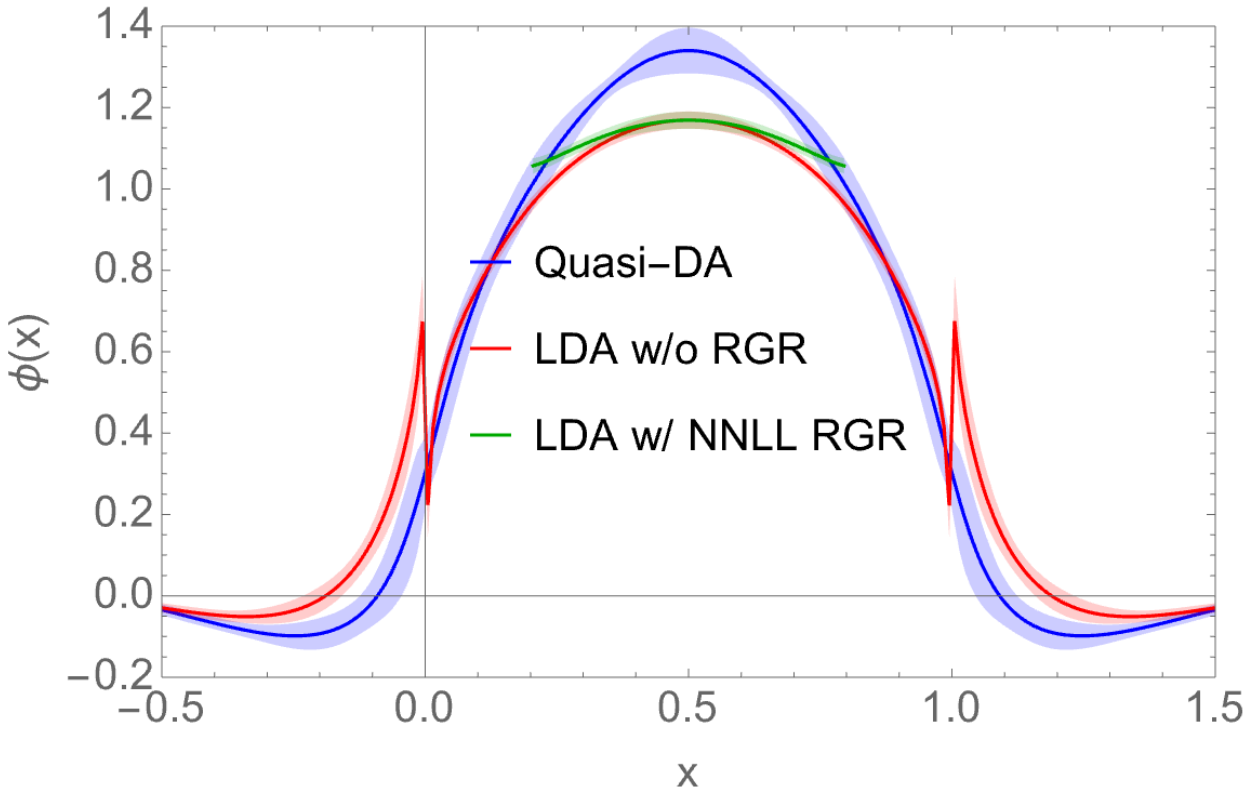
QUARK-GLUON TOMOGRAPHY COLLABORATION

Same analysis with HISQ fermion action

Gao, et al., PRD (2022)

Lattice Spacing-a	Pion Mass	Lattice Volume	$m_\pi L$	Fermion Action	Momemtum
0.076 fm	140 MeV	$64^3 \times 64$	3.39	2+1f Clover	1.78 GeV

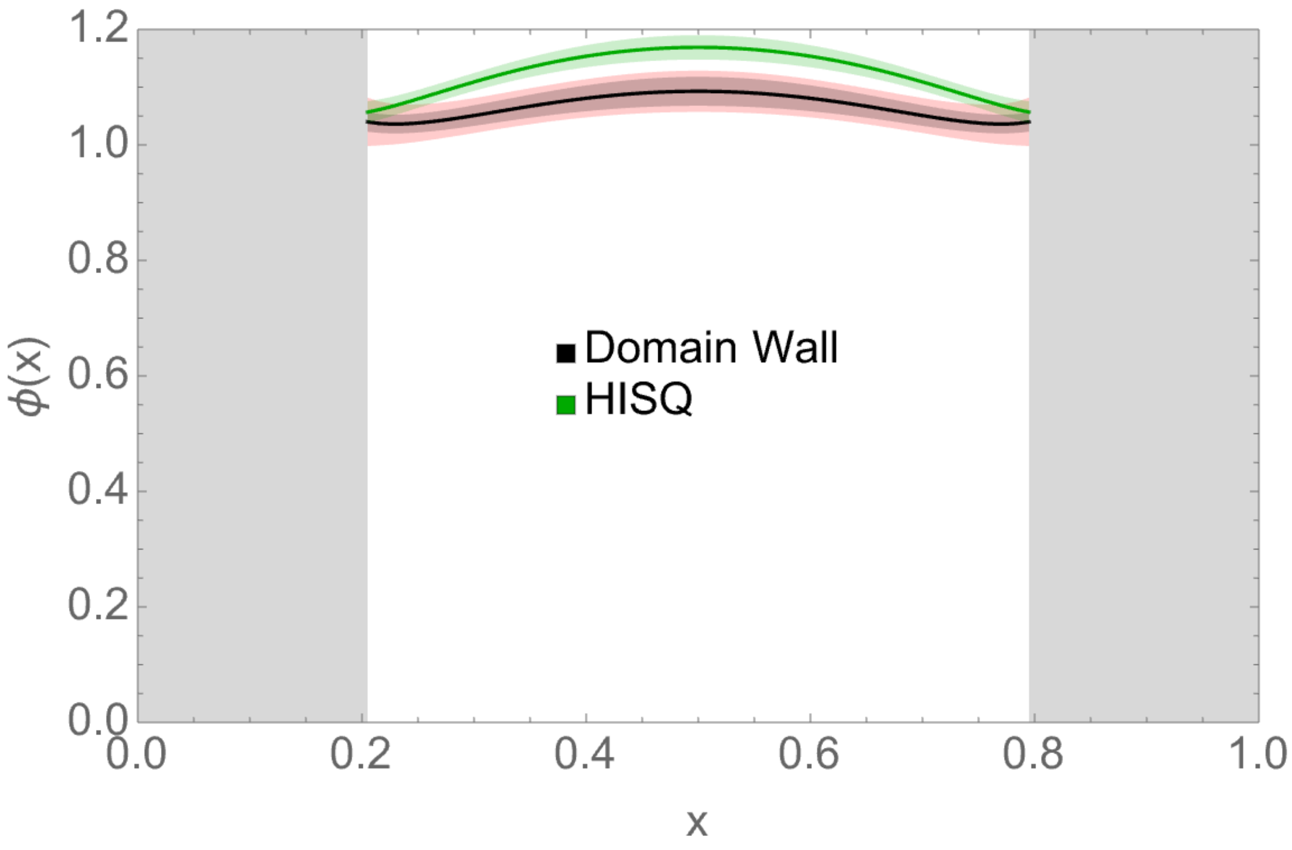
We add the x-dependence analysis to an existing data set with HISQ fermions at physical point, similar lattice spacing and close pion momentum.



Argonne NATIONAL LABORATORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

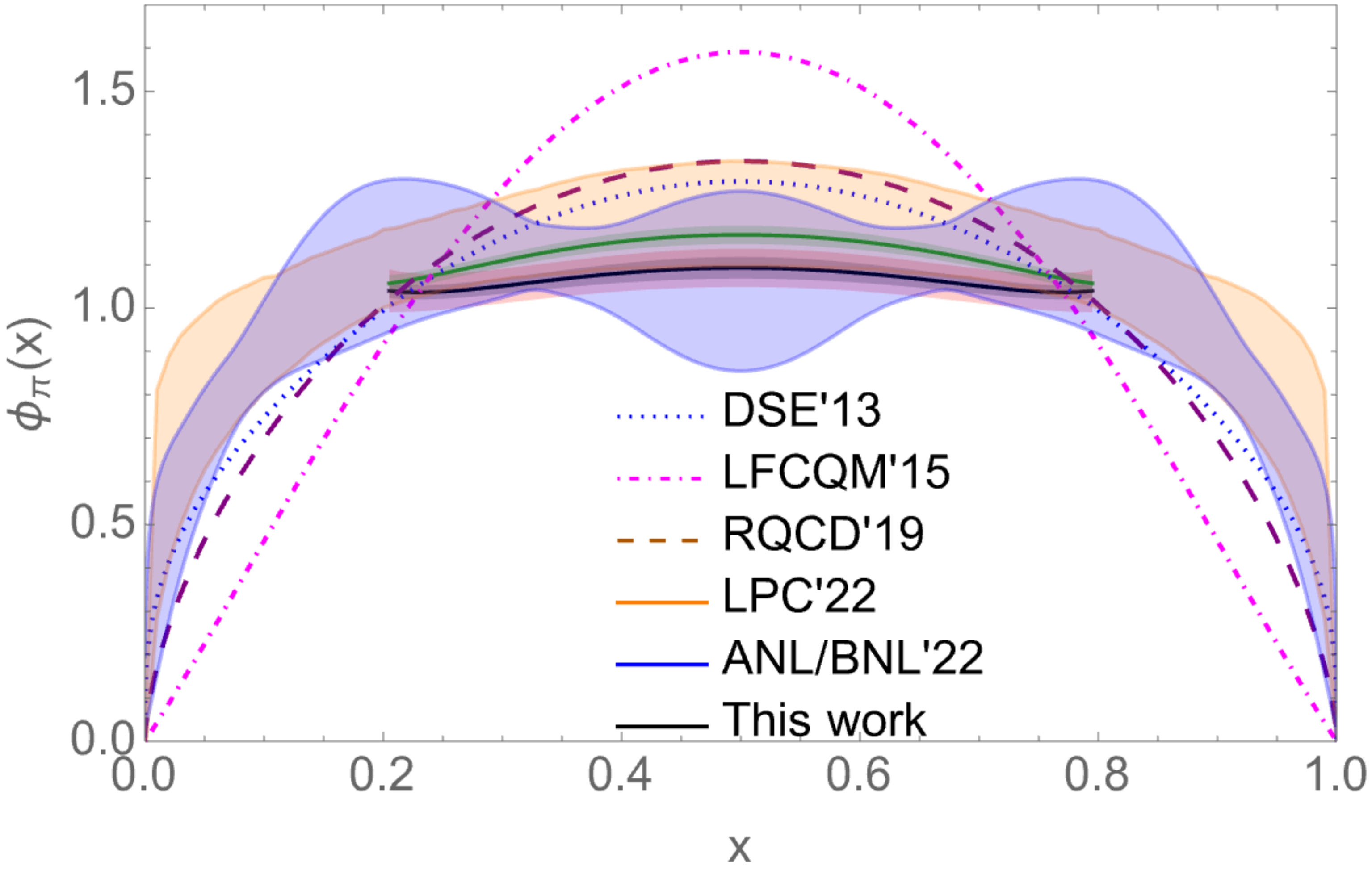
Comparing HISQ and DW results



- The DA obtained from DW results are more flat than HISQ ensembles
- One possible explanation is that the explicit chiral-symmetry breaking term in HISQ action has a similar effect as making the meson heavier (thus has a more narrow distribution)

Argonne NATIONAL LABORATORY

Comparison with previous results



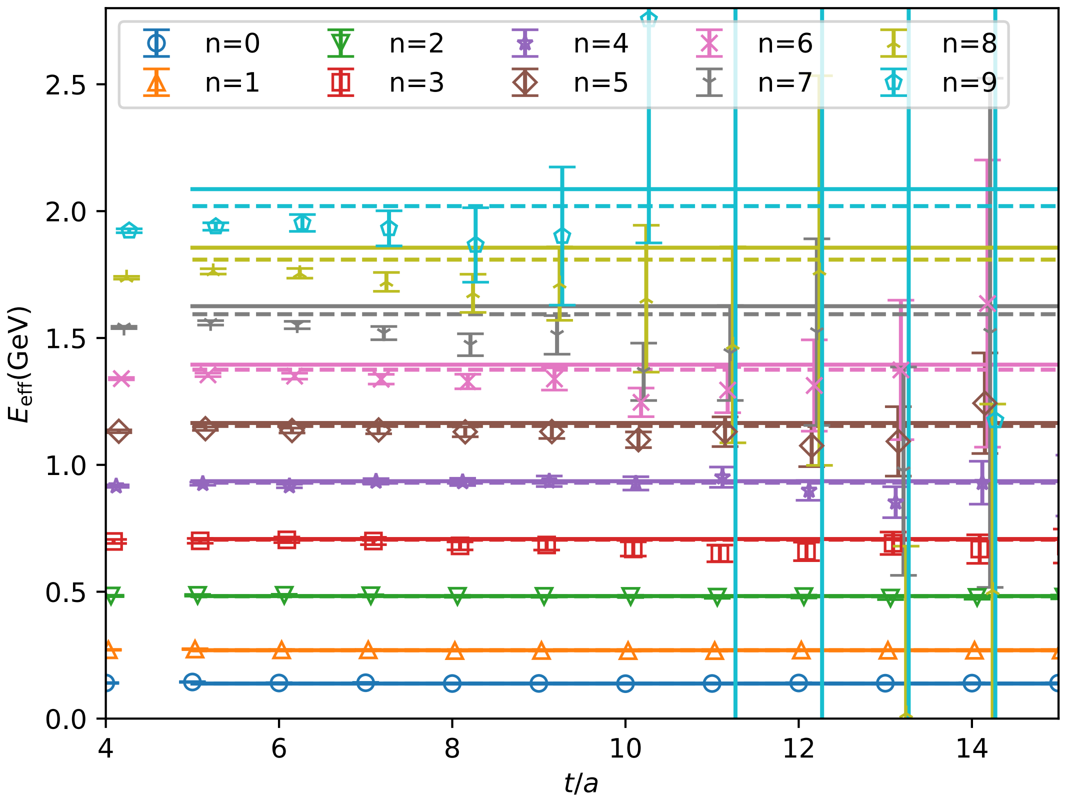
Conclusion and Outlook

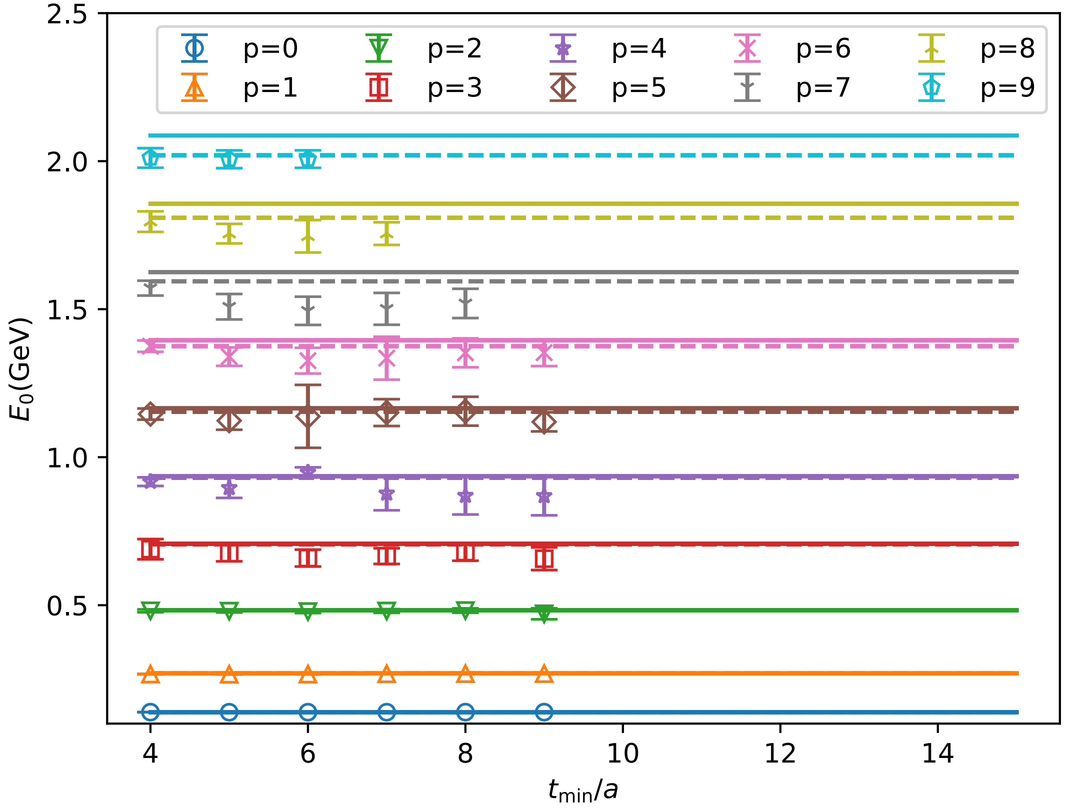
- We present a pion DA calculation using gauge ensembles with domain wall fermions;
- We propose and develop a more robust method to resum the soft-quark-momentum logarithms in the perturbative matching kernel of DA, utilizing the factorization in threshold limit;
- We observe a more flat distribution for domain wall fermions.

- A continuum limit study is needed for a more conclusive comparison
- Threshold resummation is needed for more exact matching

Backup Slides

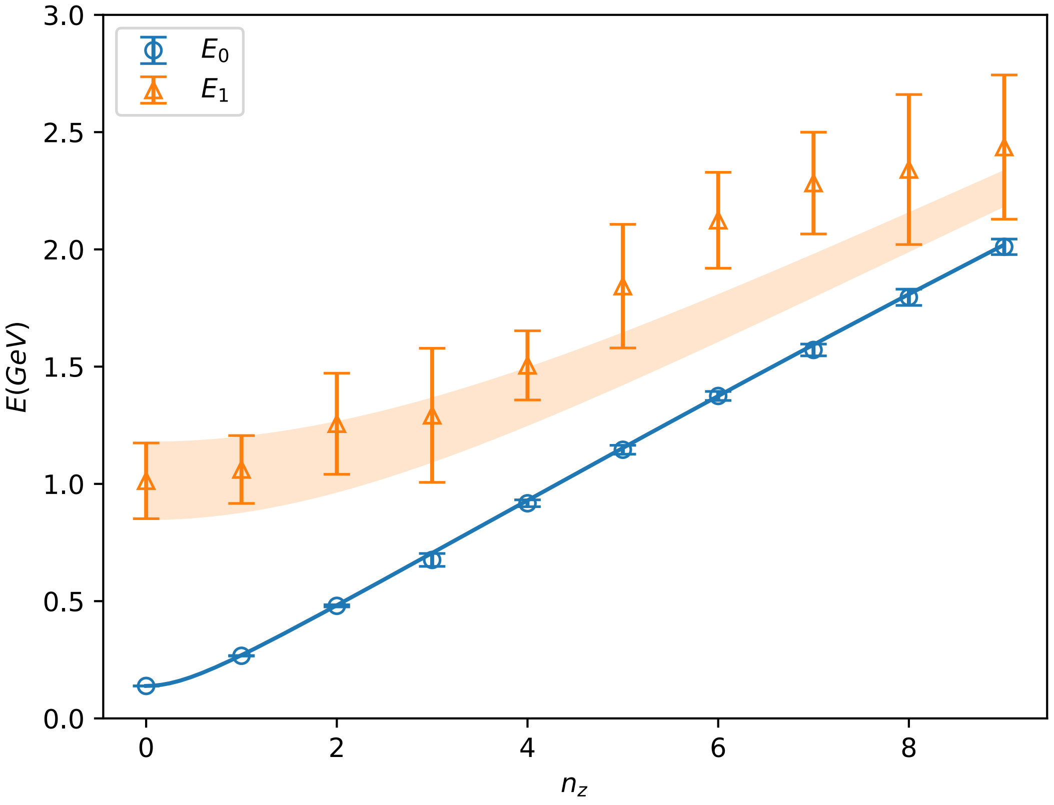
Effective Mass and Energy fits



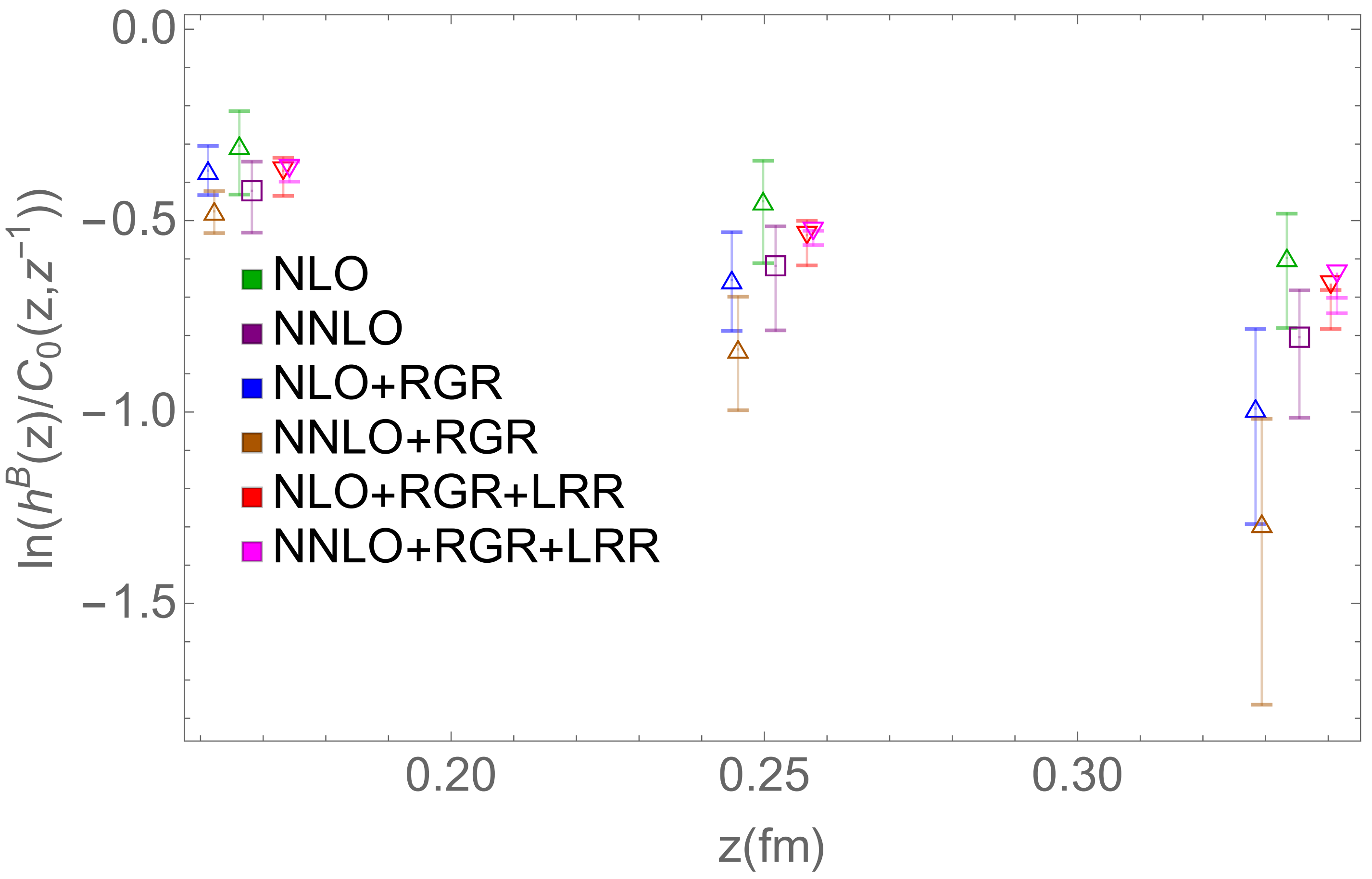


Dispersion relation with discrete momentum

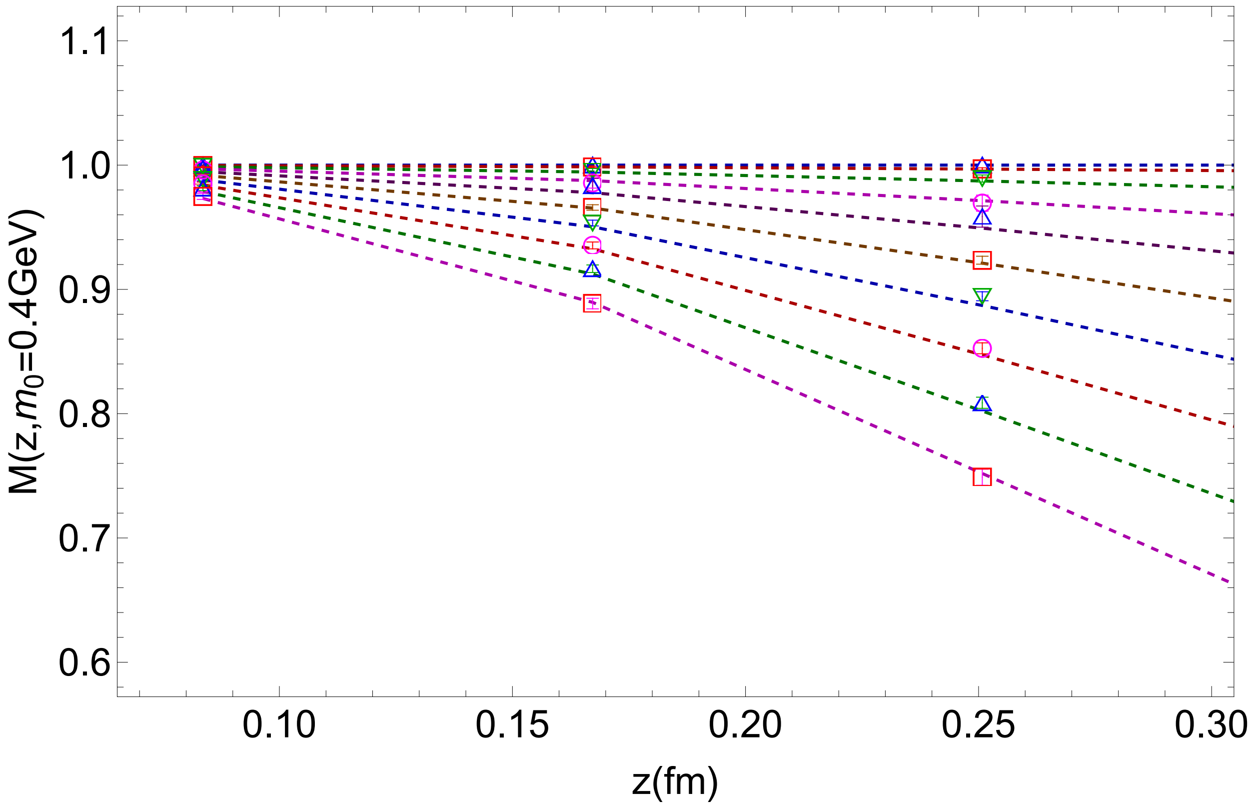
$$E = \sqrt{4\sin^2\frac{p}{2} + m^2}$$



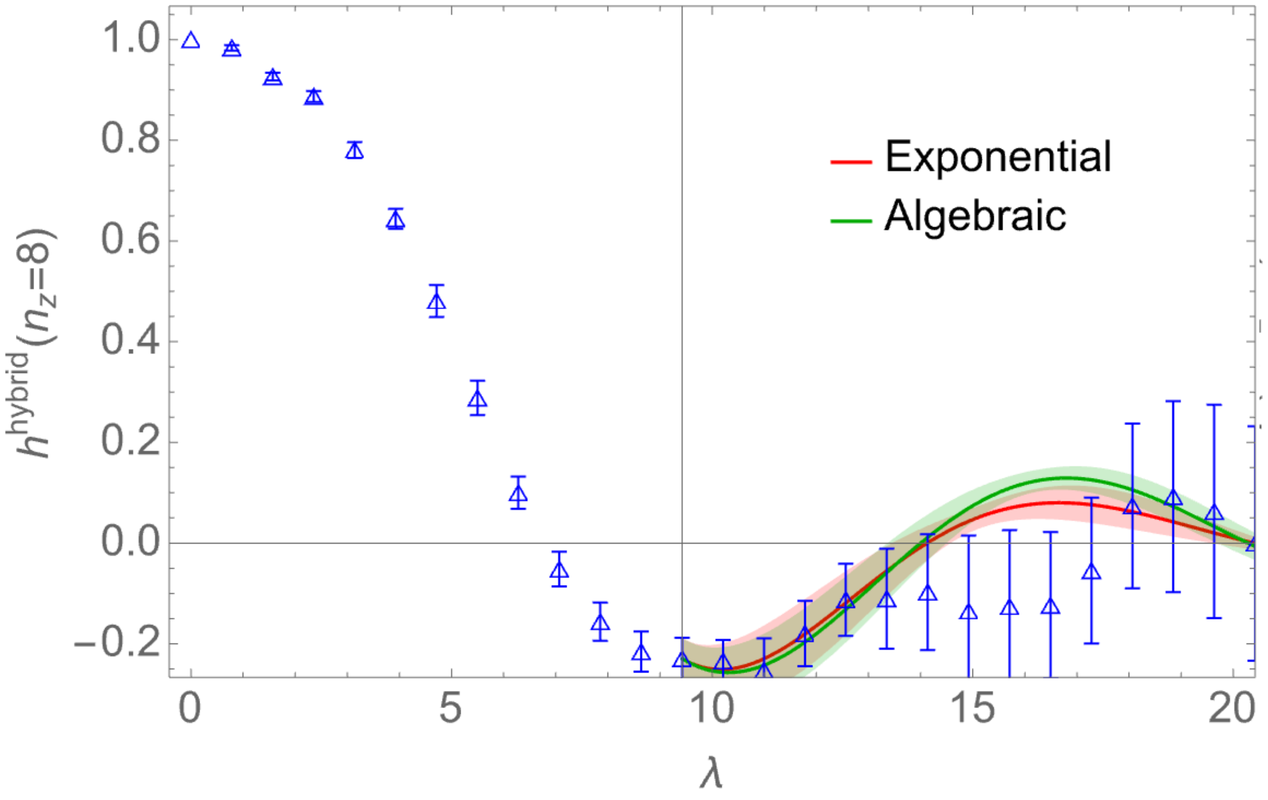
Extracting δm from $P_z = 0$ lattice data

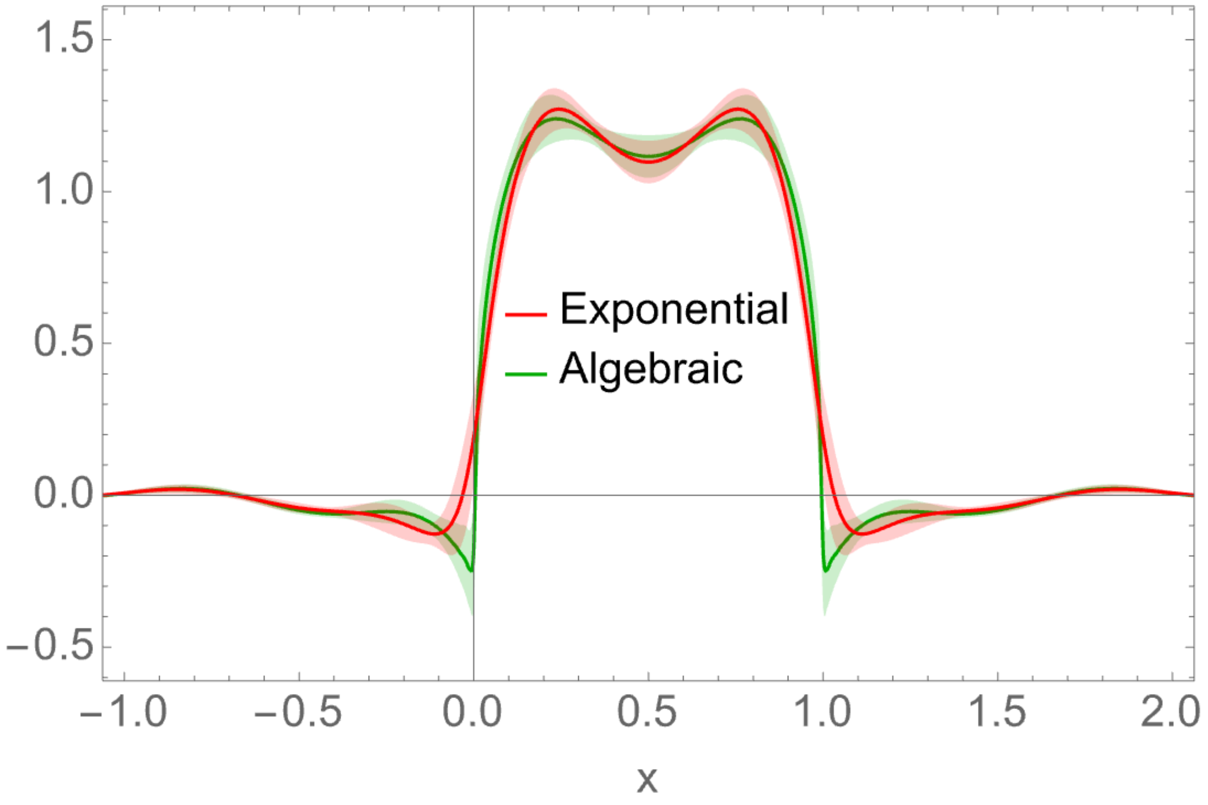


Short-distance correlation vs. OPE



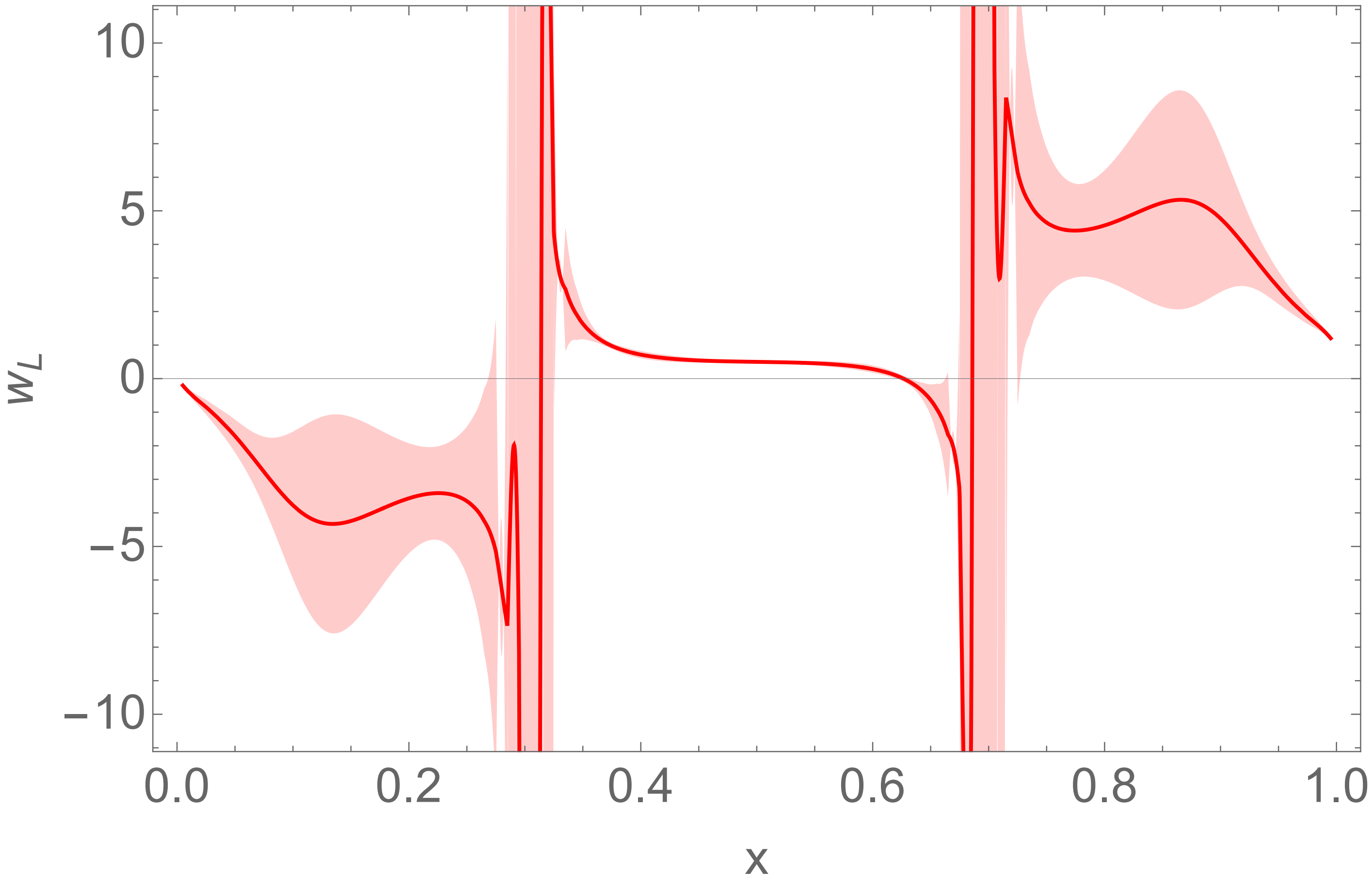
Model dependence in longtail extrapolation



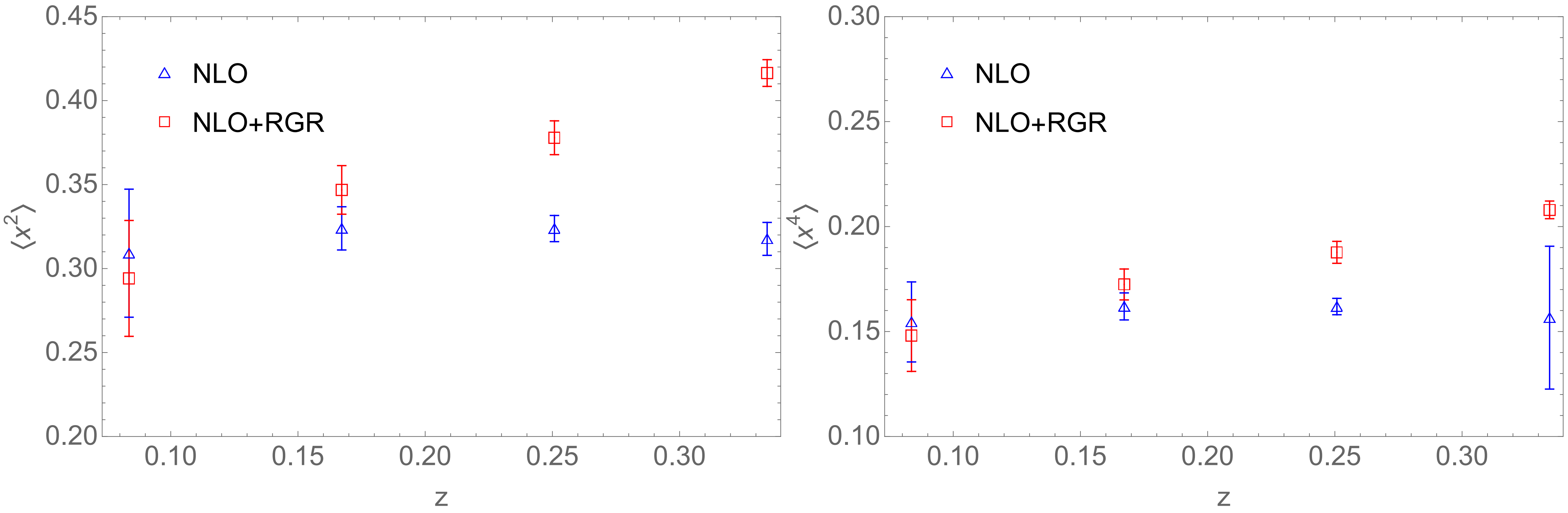


Break down of old resummation method

Weight function already blows up at $x \approx 0.3$



Fit moments with SDF



Preliminary Kaon DA