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Progress on the transverse SSA in single-inclusive jet production at an EIC at NLO

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University of Tübingen

in collaboration with Patrick Tollkühn, Werner Vogelsang

What is the final goal?

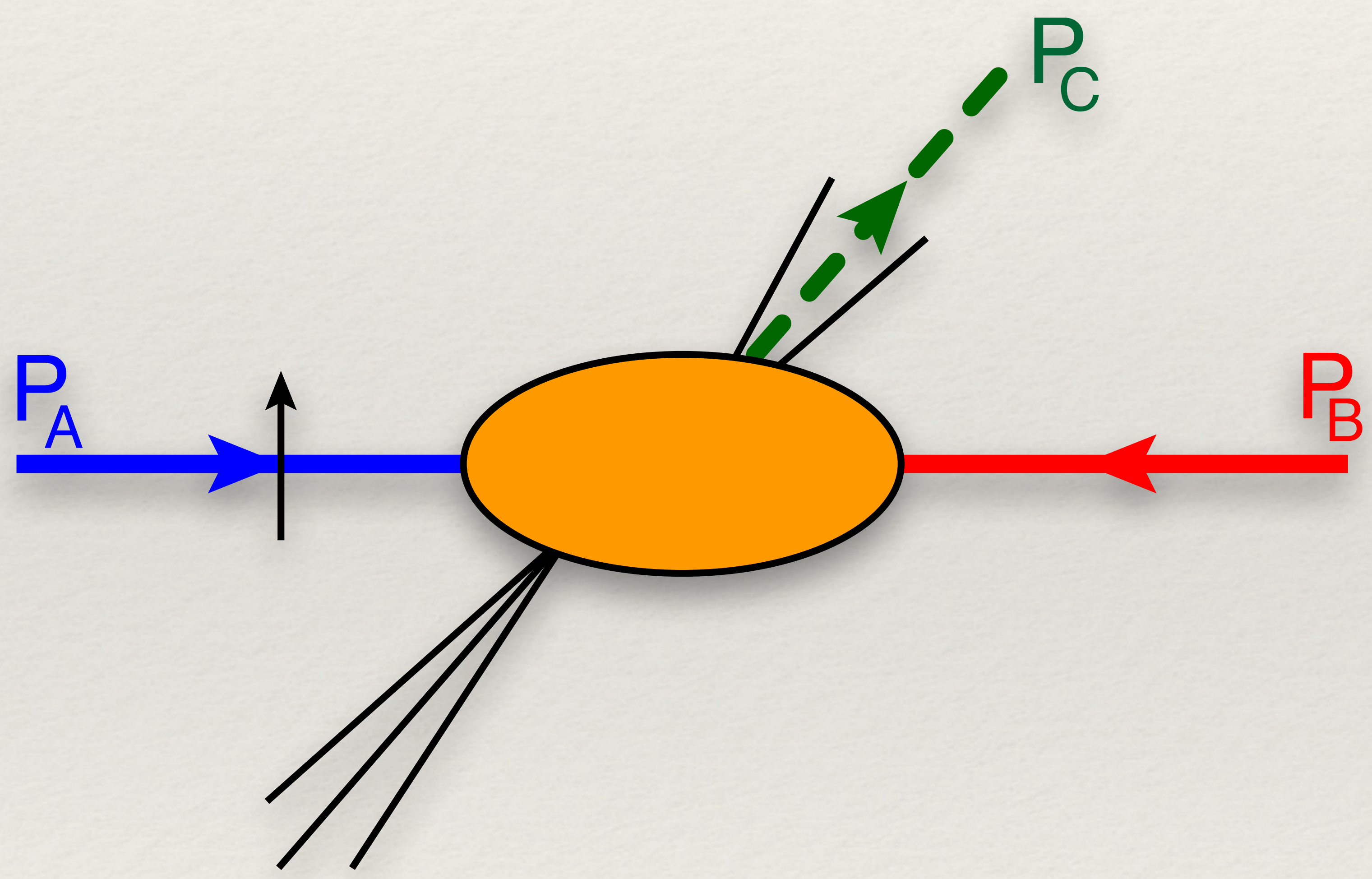
Transverse Single (Double) Spin Asymmetries in single-inclusive processes in collinear pQCD

$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

or

$$\Delta\sigma_T = \frac{\sigma^{\rightarrow\uparrow} - \sigma^{\rightarrow\downarrow}}{\sigma^{\rightarrow\uparrow} + \sigma^{\rightarrow\downarrow}}$$

$$P_A^{\uparrow} + P_B \to P_C + X$$



Experimental data

Polarized proton collisions at RHIC (STAR, PHENIX, BRAHMS)

$$p^{\uparrow}p \to (\pi, \text{jet}\,, \gamma, l, \Lambda, J/\psi, \ldots)X$$

Theory: LO in pQCD

[Koike, Yoshida, Qiu, Metz, Pitonyak, Kang, …]

Polarized eN collisions (HERMES, JLab, COMPASS, EIC)

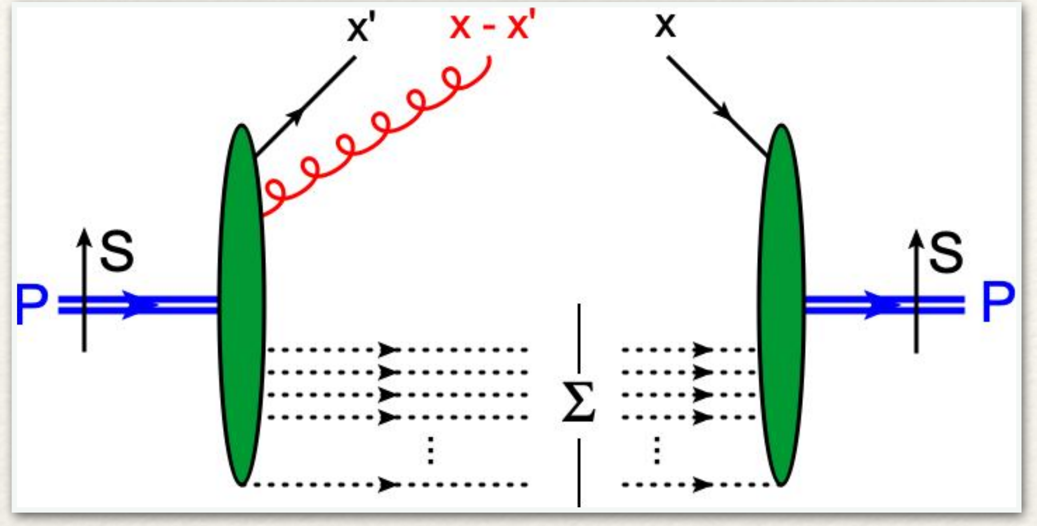
$$l^{(\rightarrow)}N^{\uparrow} \to (l, \pi, \text{jet}\,, \gamma, \Lambda, J/\psi, \ldots)X$$

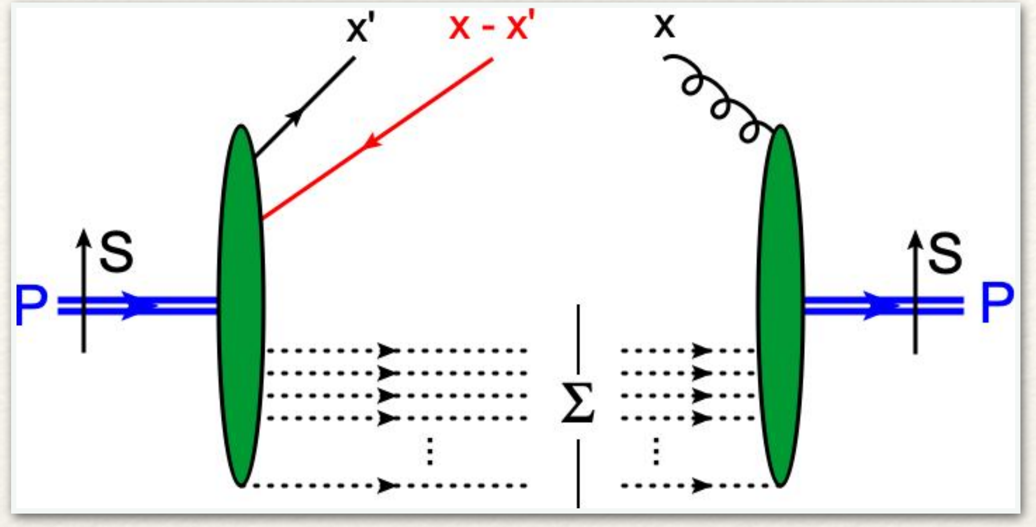
Theory: LO , some NLO

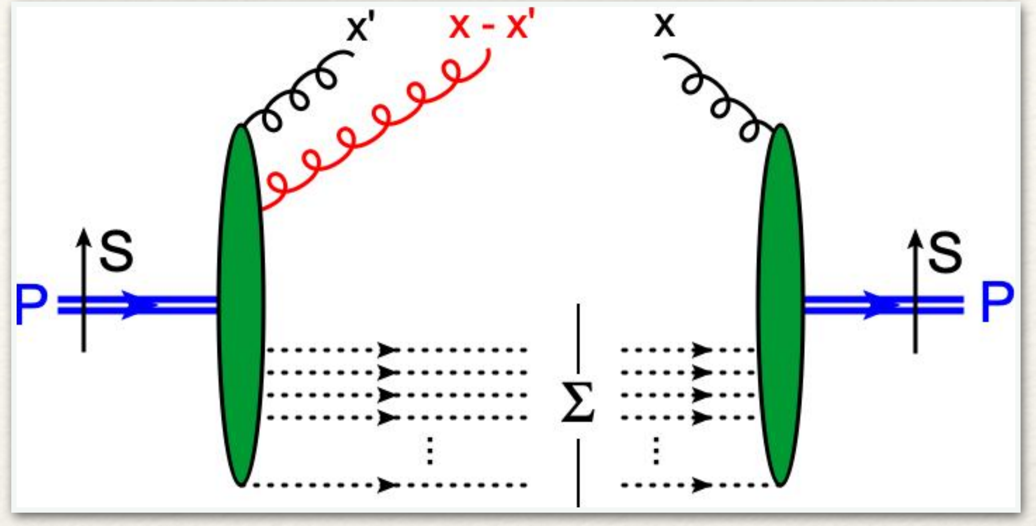
Final goal: global analysis (at NLO) (one day…)

What do we expect to learn (about the nucleon)?

Interference effect of non-valence nucleon LF wave functions







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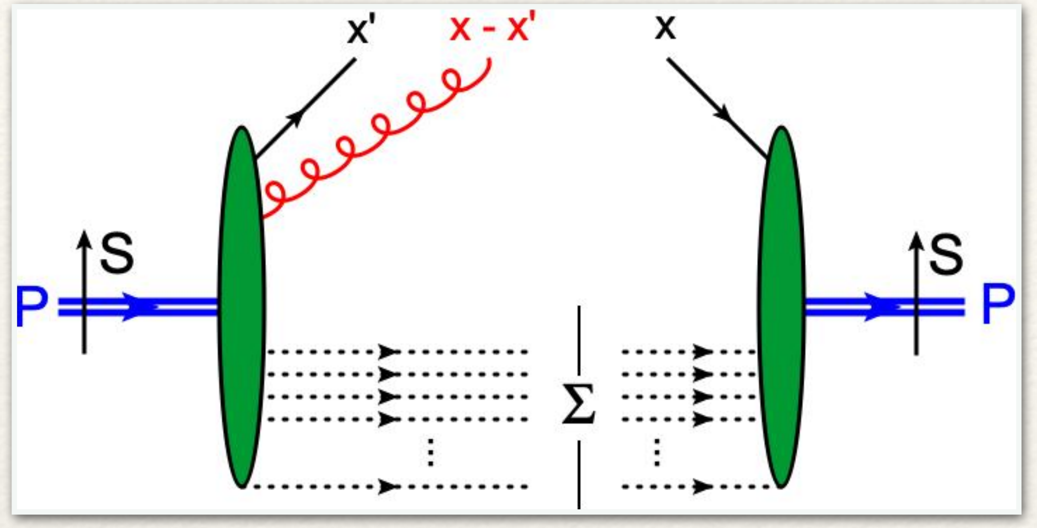

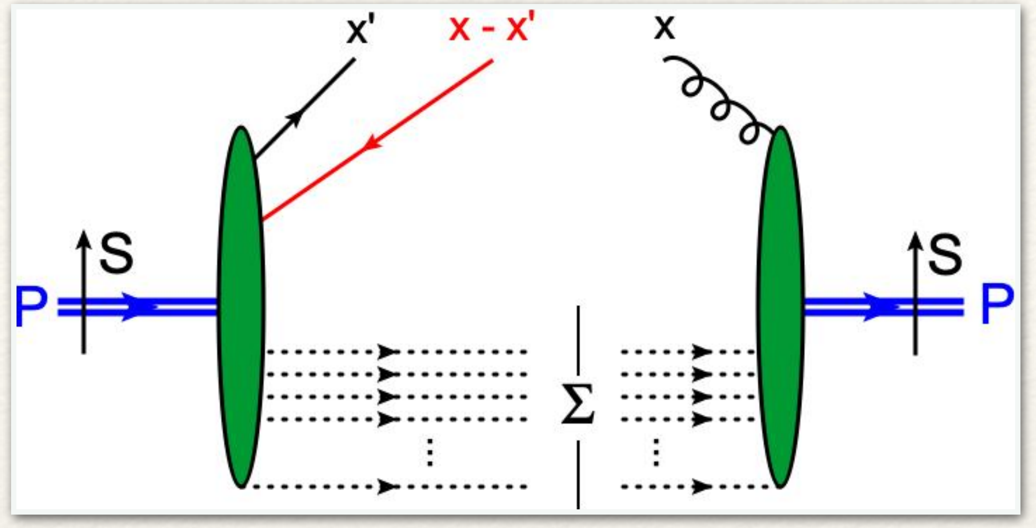

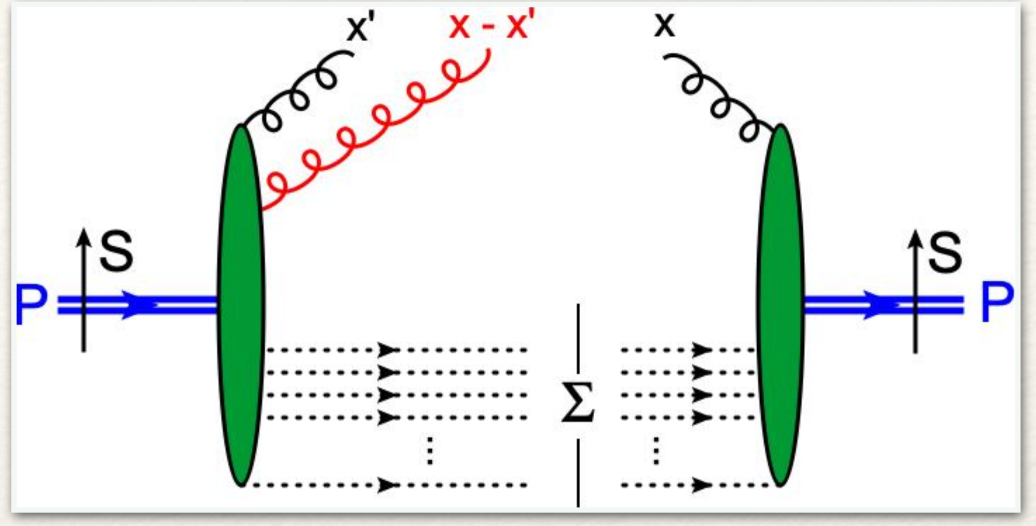

(Chiral-even) Quark - Gluon - Quark correlation functions

$$2M\, i\epsilon^{Pn\rho S} F_{FT}^{q}(x,x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x'} e^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$

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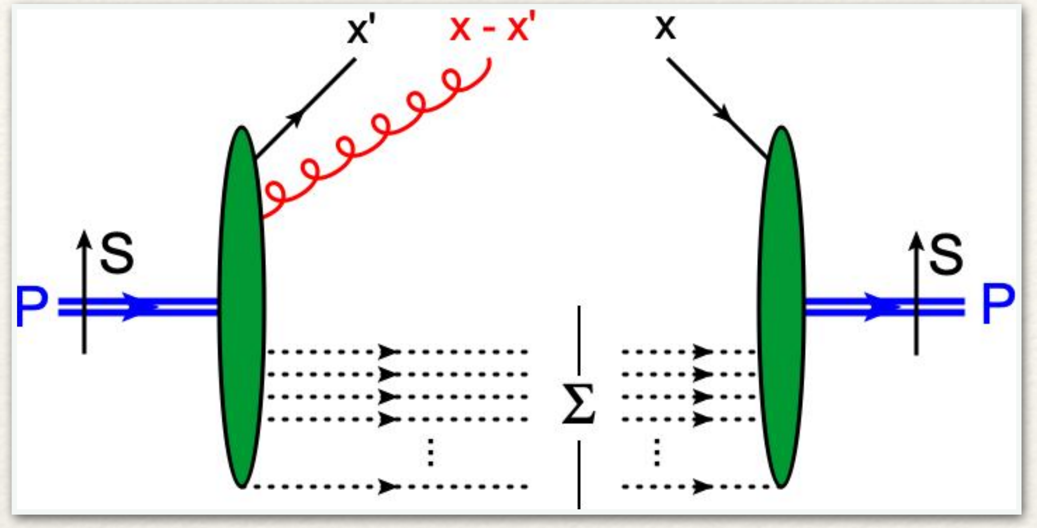

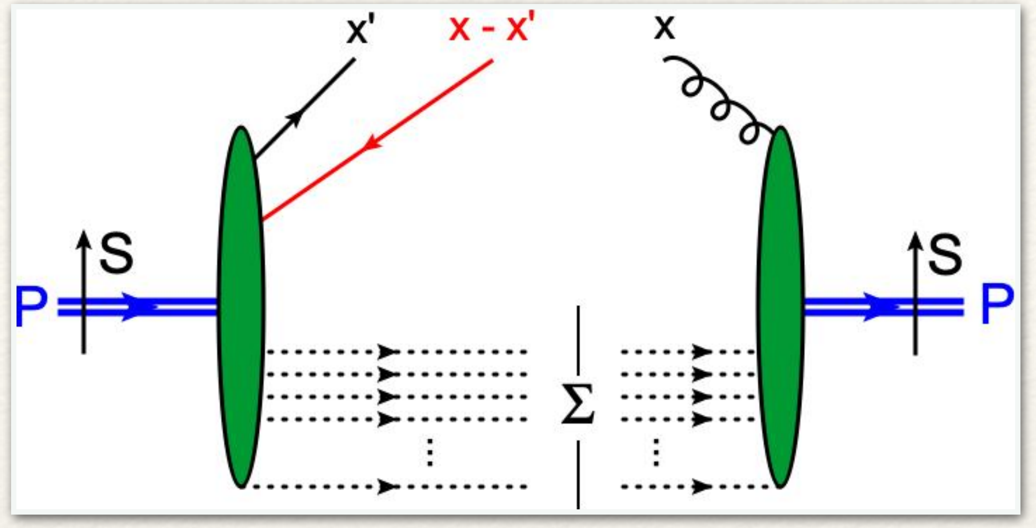

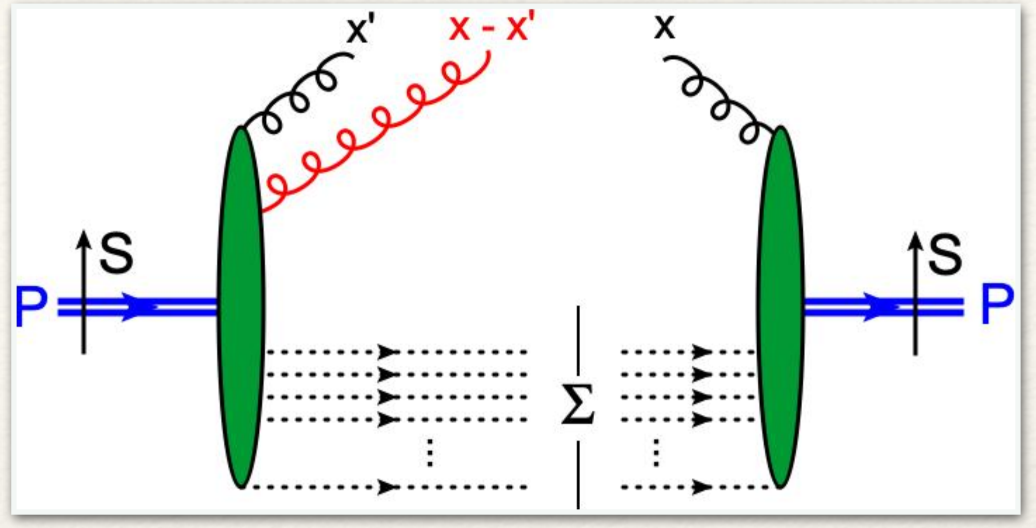

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Why relevant?

→ collinear pQCD: they generate transverse SSA
→ dynamical information: color Lorentz force [M. Burkardt]
→ TMD physics: large transverse momentum behavior of Sivers, Boer-Mulder, worm gear function & their evolution

What do we know about QGQ correlation functions?

Support properties $-1 \leq x, x' \leq 1 \quad |x - x'| \leq 1$ and possibly continuous

Symmetry
$$F^q_{FT}(x,x') = +F^q_{FT}(x',x)$$
$$G^q_{FT}(x,x') = -G^q_{FT}(x',x)$$

charge conjugation
$$F^{\bar{q}}_{FT}(x,x') = +F^q_{FT}(-x,-x')$$
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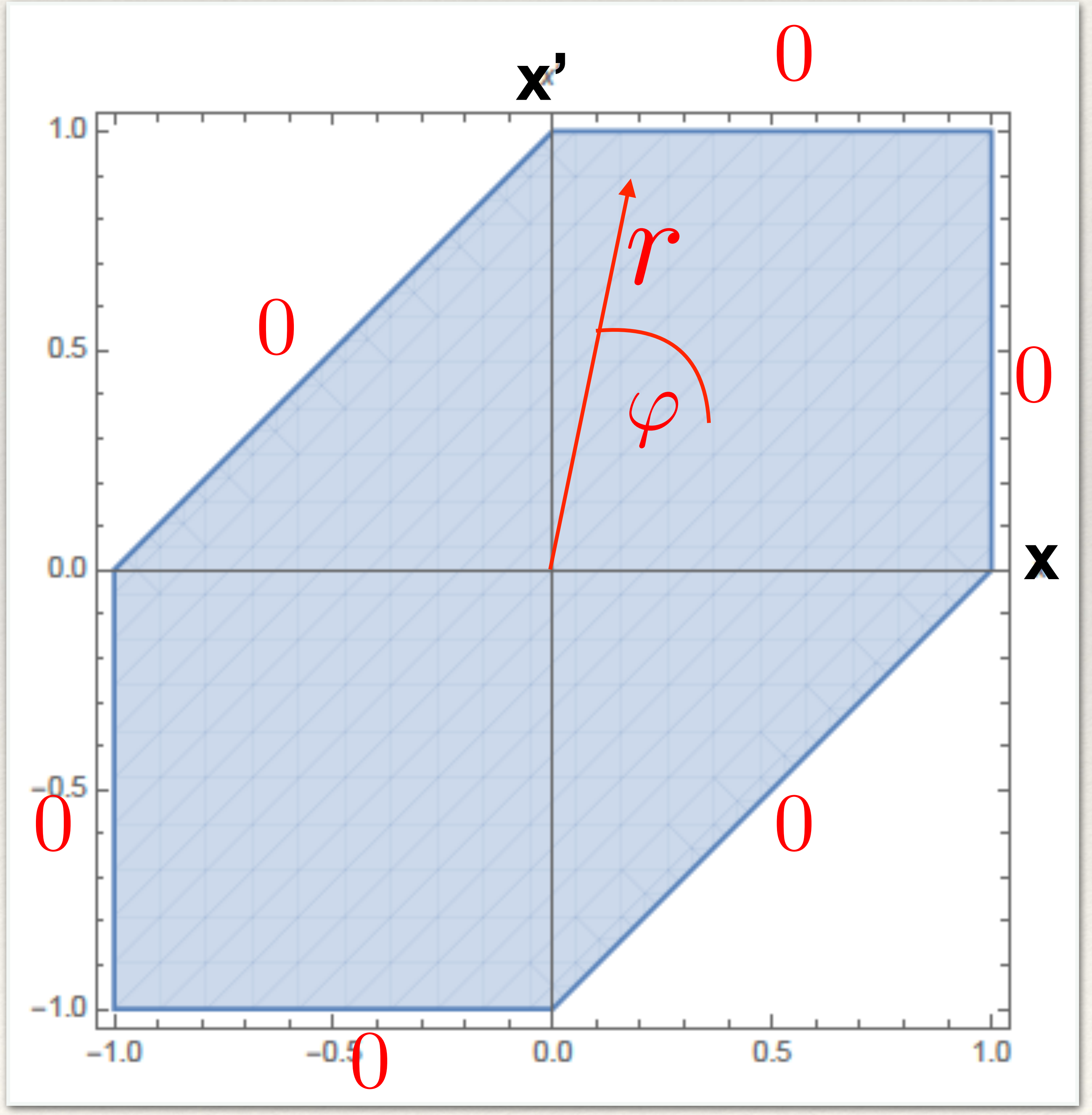

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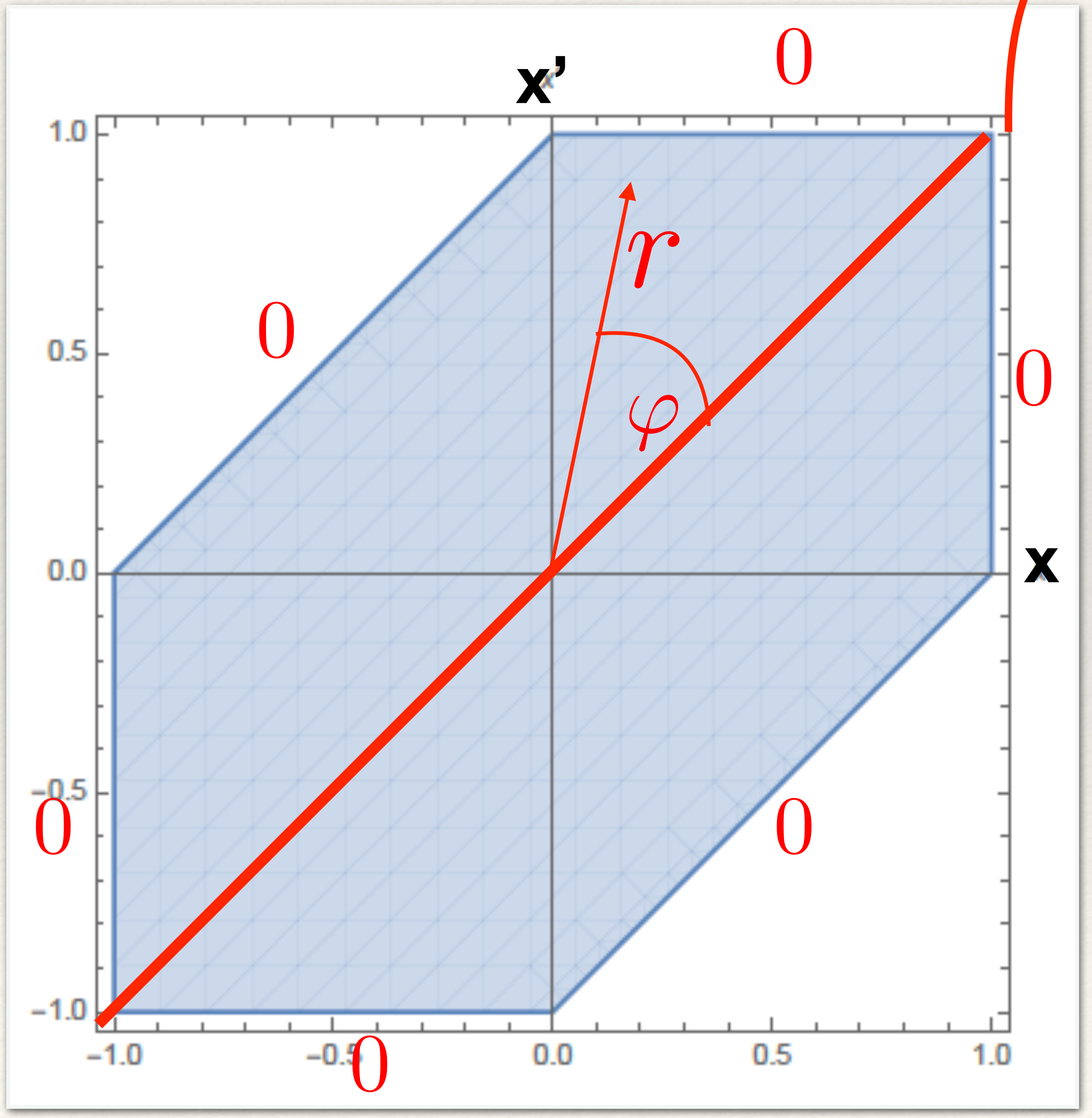
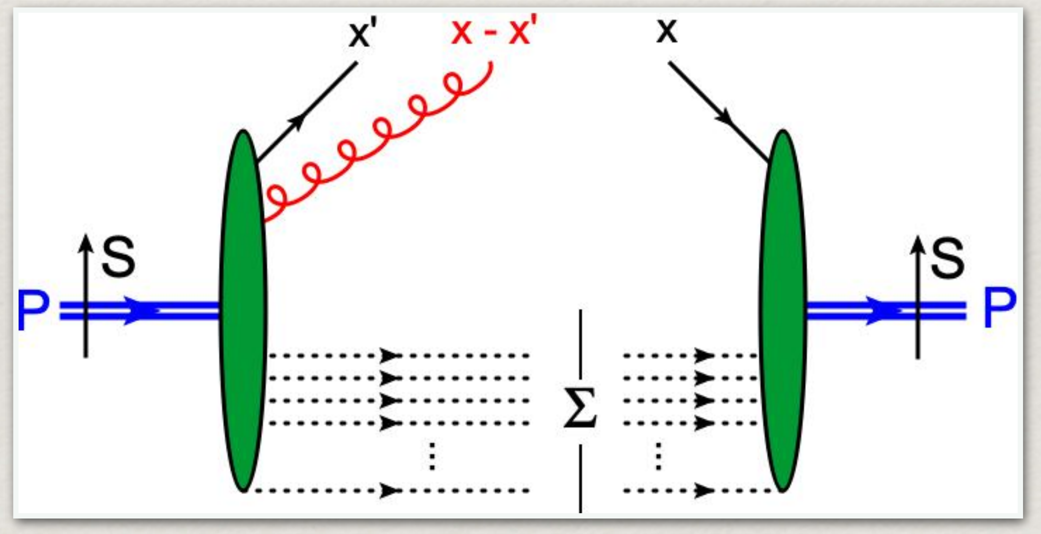


"Soft gluon pole" (SGP)

$$\pi F_{FT}^q(x,x) = f_{1T}^{\perp(1),q}(x)$$
$$G_{FT}^q(x,x) = 0$$



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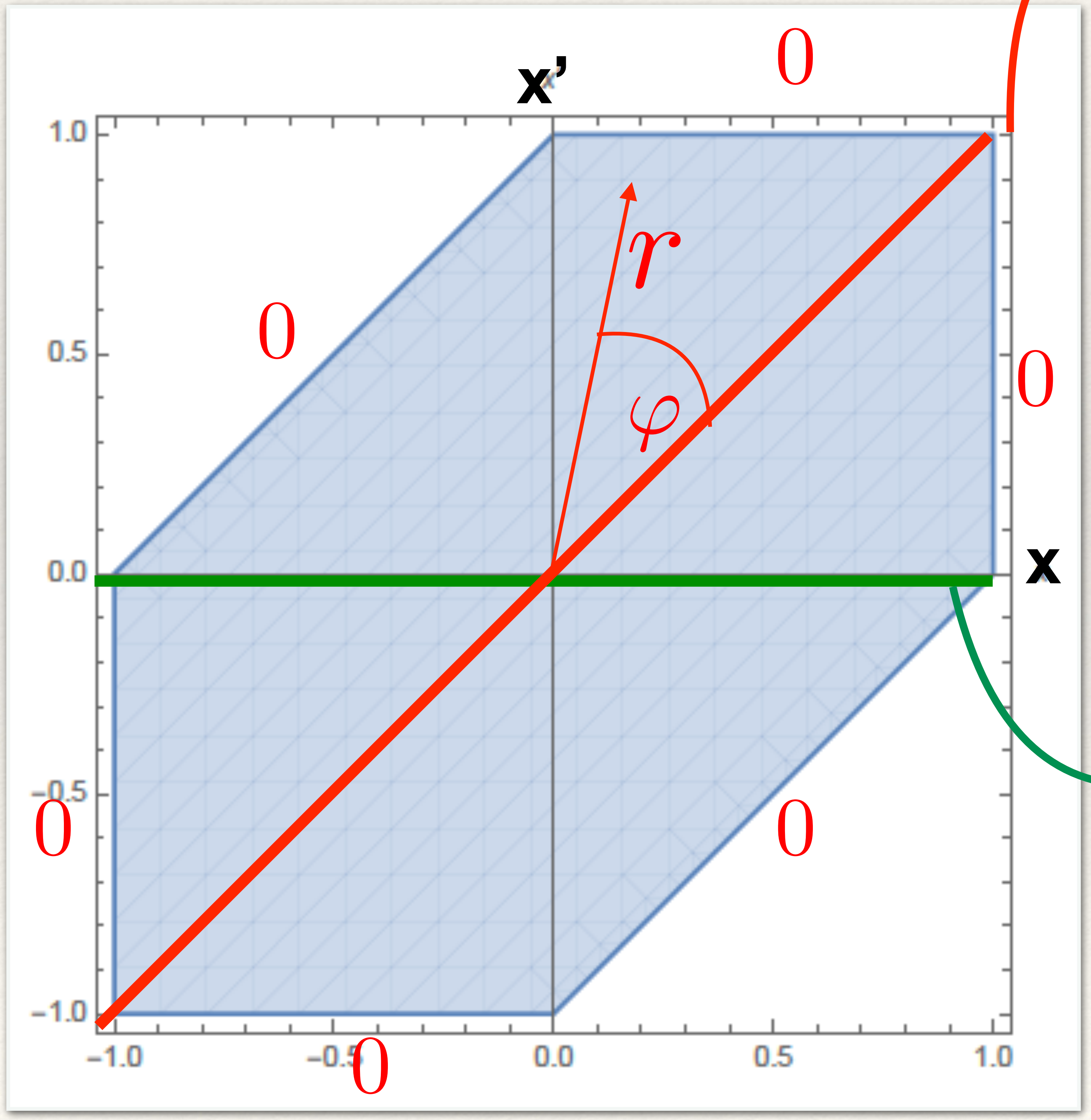

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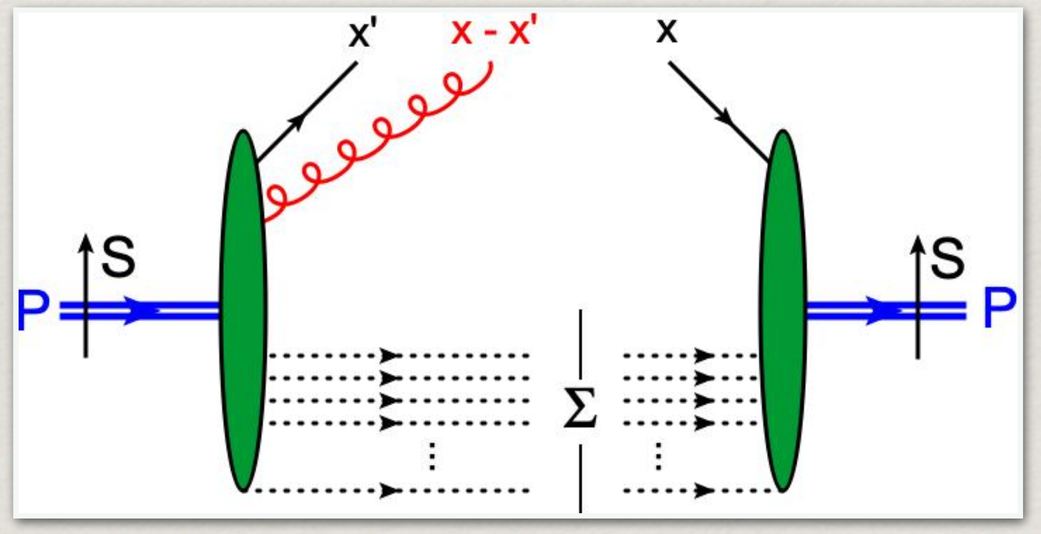

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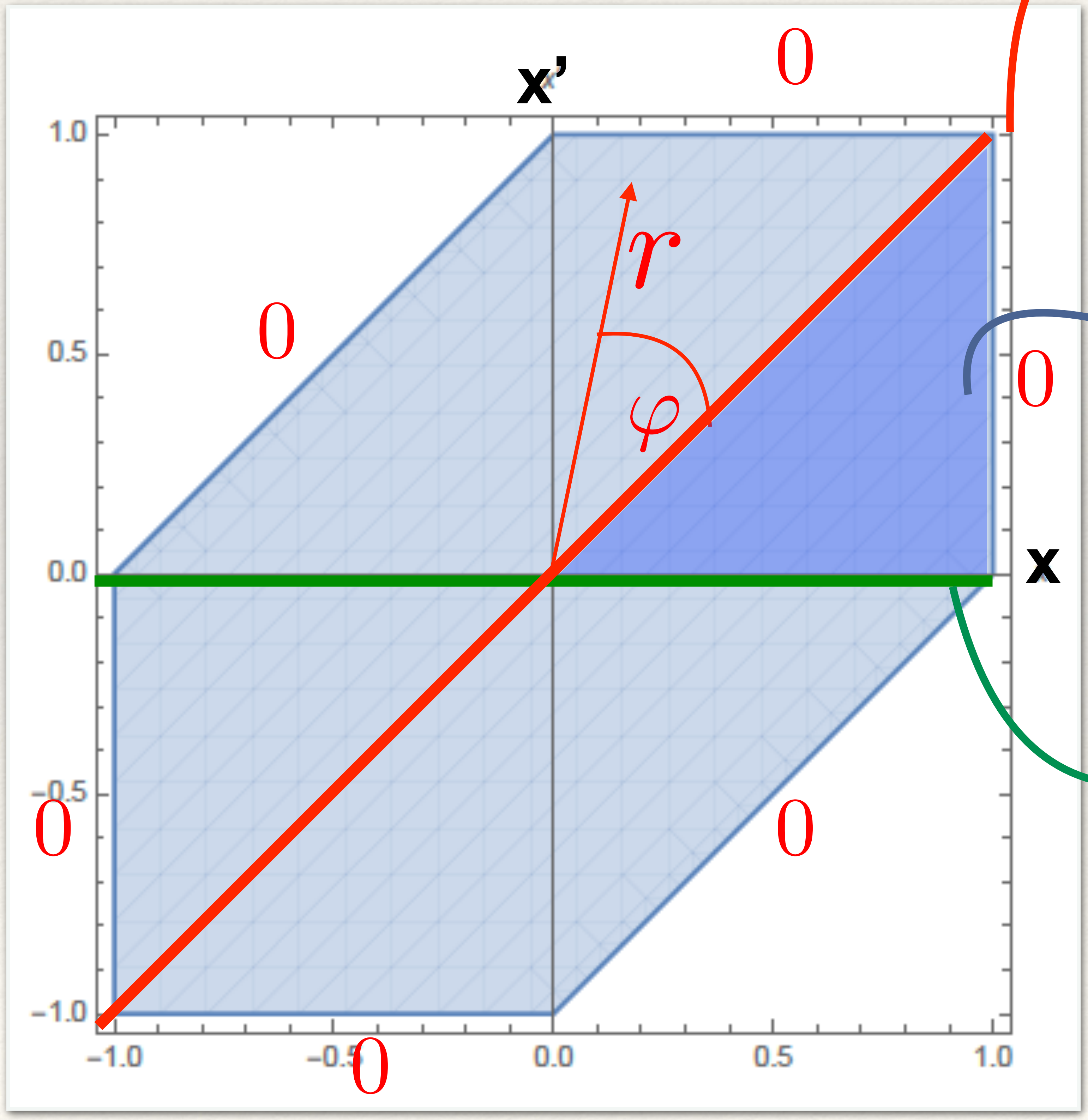

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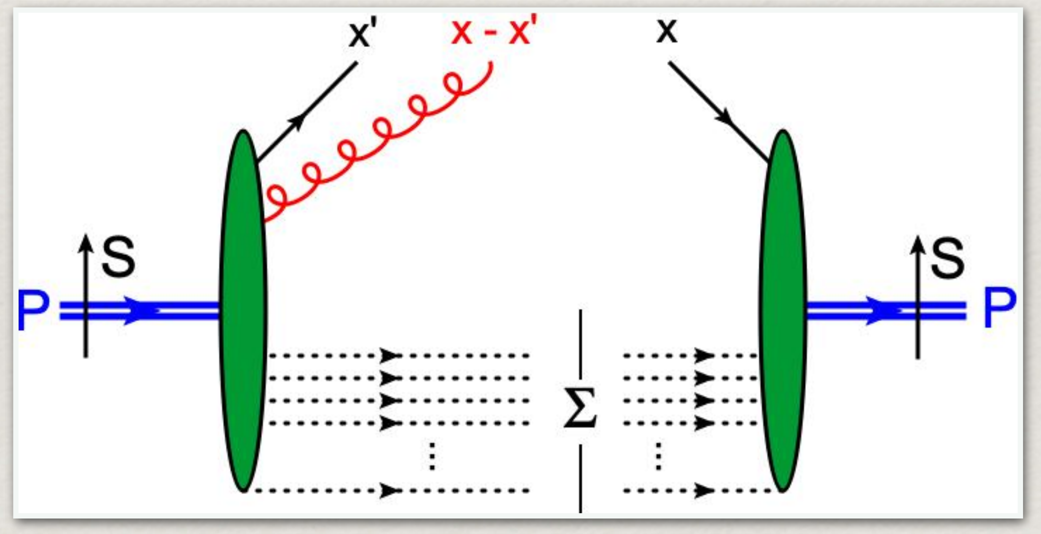

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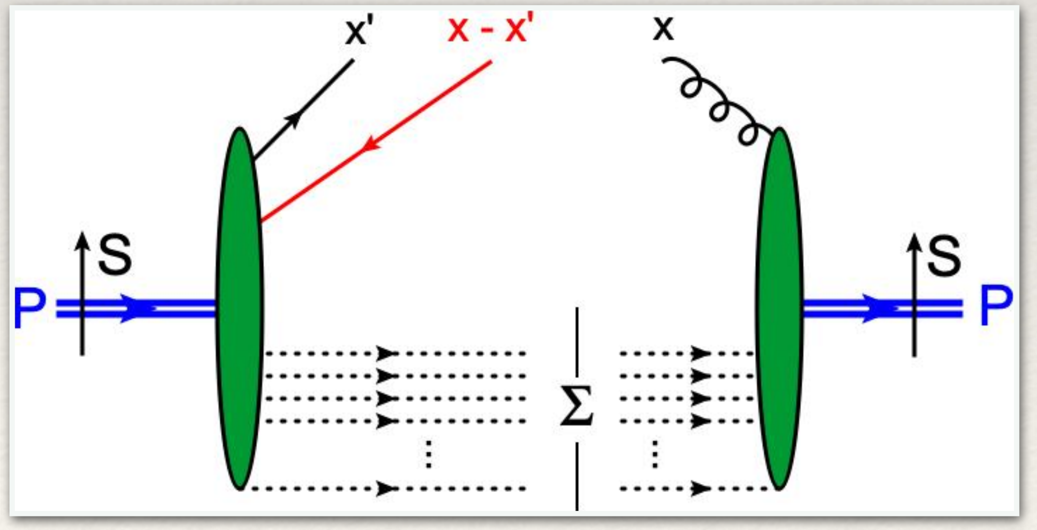
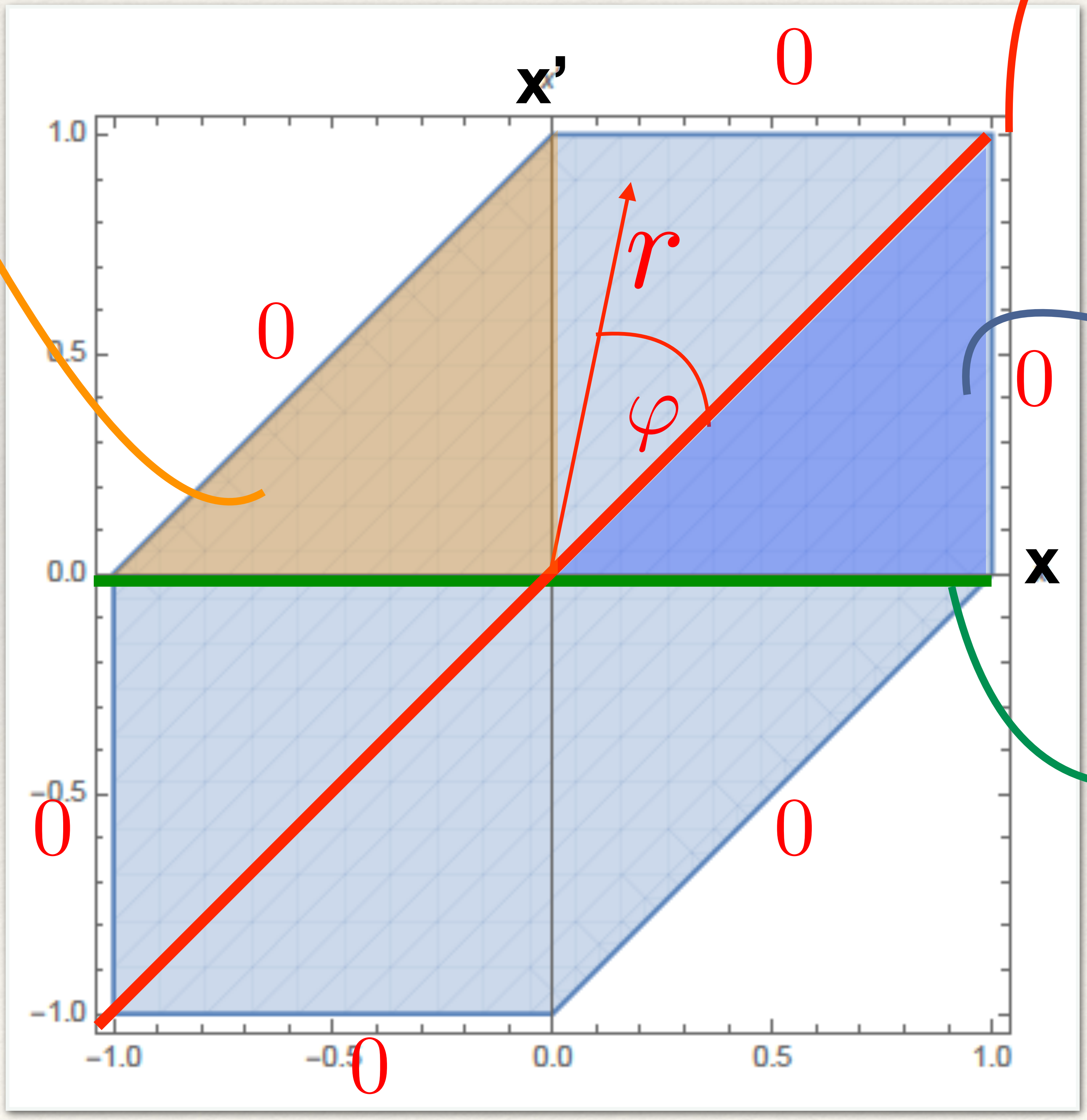
Antiquark-Quark -Gluon

$$F_{FT}(-x', x - x') = ?$$
$$G_{FT}(-x', x - x') = ?$$





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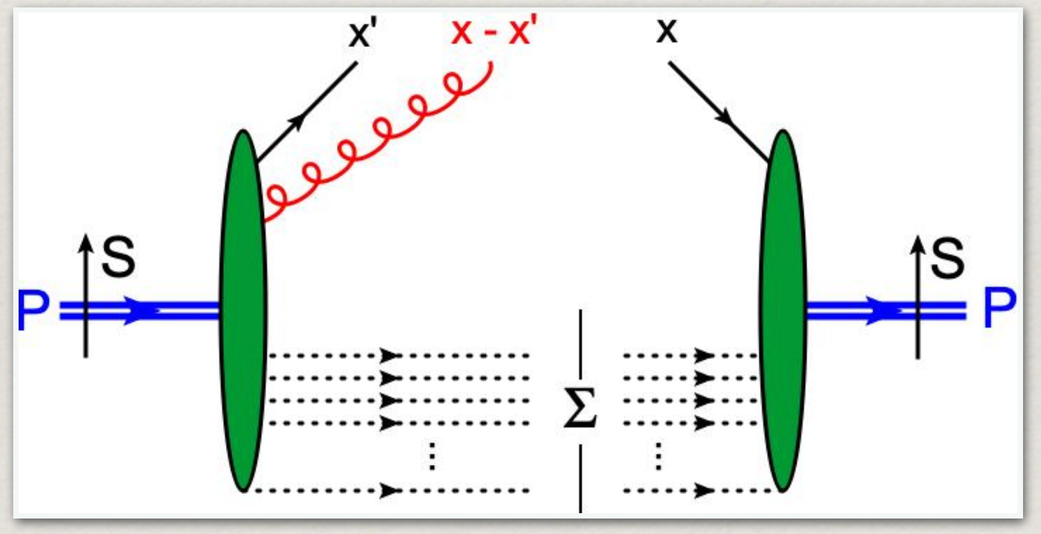

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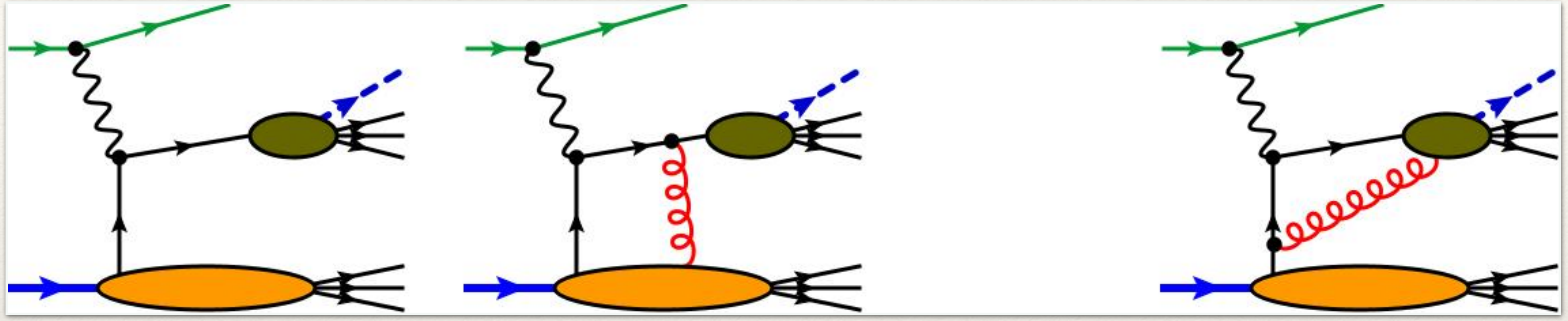
Inclusive π (or jet) production in l+N collisions at LO

Example: Single-inclusive hadron production e $N^{\uparrow} \to \pi X$

[Gamberg, Kang, Metz, Pitonyak, Prokudin; Kanazawa, Koike, Metz, Pitonyak, MS]

final state lepton *not* detected!



$$A_N \propto \sum_q e_q^2 \int dz \left(1 - x\frac{d}{dx}\right) F_{FT}^q(x,x)\, D_1^{\pi/q}(z)$$

$$+ \sum_q e_q^2 \int dz\, h_1^q(x) \int dz_1 \int dz_2\, \Im[\hat{H}_{FU}^{\pi/q}](z_1,z_2)\, f(z_1,z_2)$$

transversity pdf

chiral-odd qgq ff

reduction to jets:

LO: $D_1(z) \to \delta(1-z)$

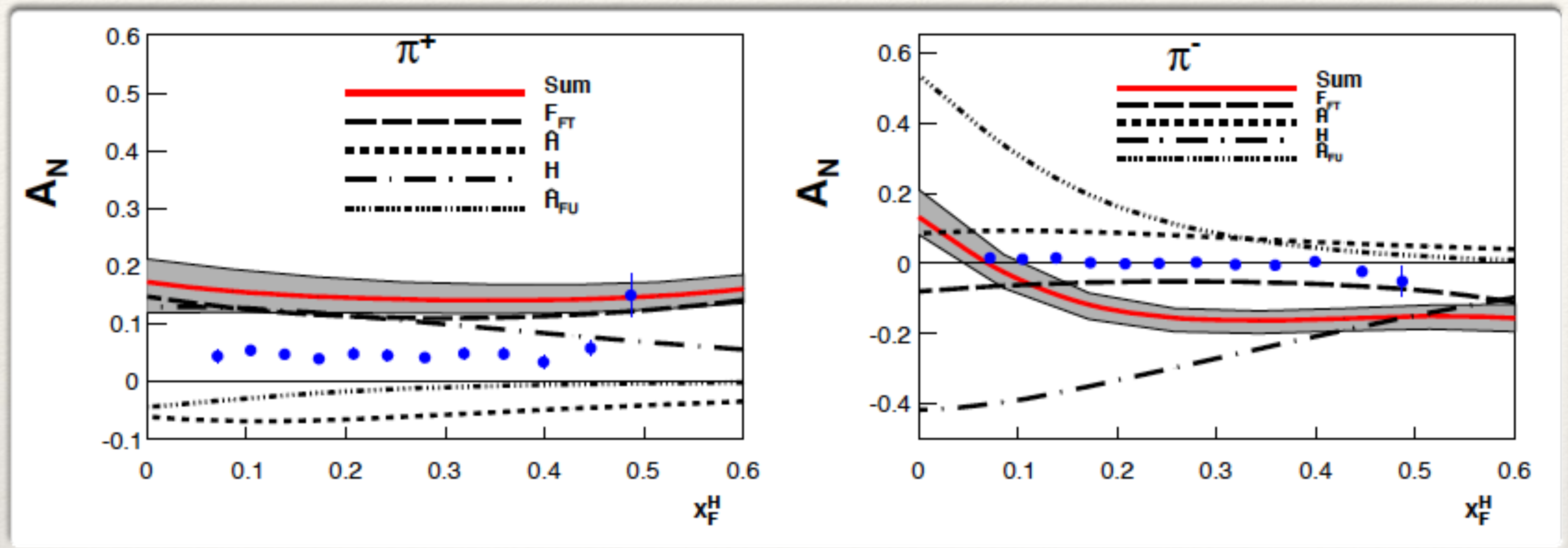
NLO: “small-cone approximation”

Feasible at a future EIC, NLO corrections might be large

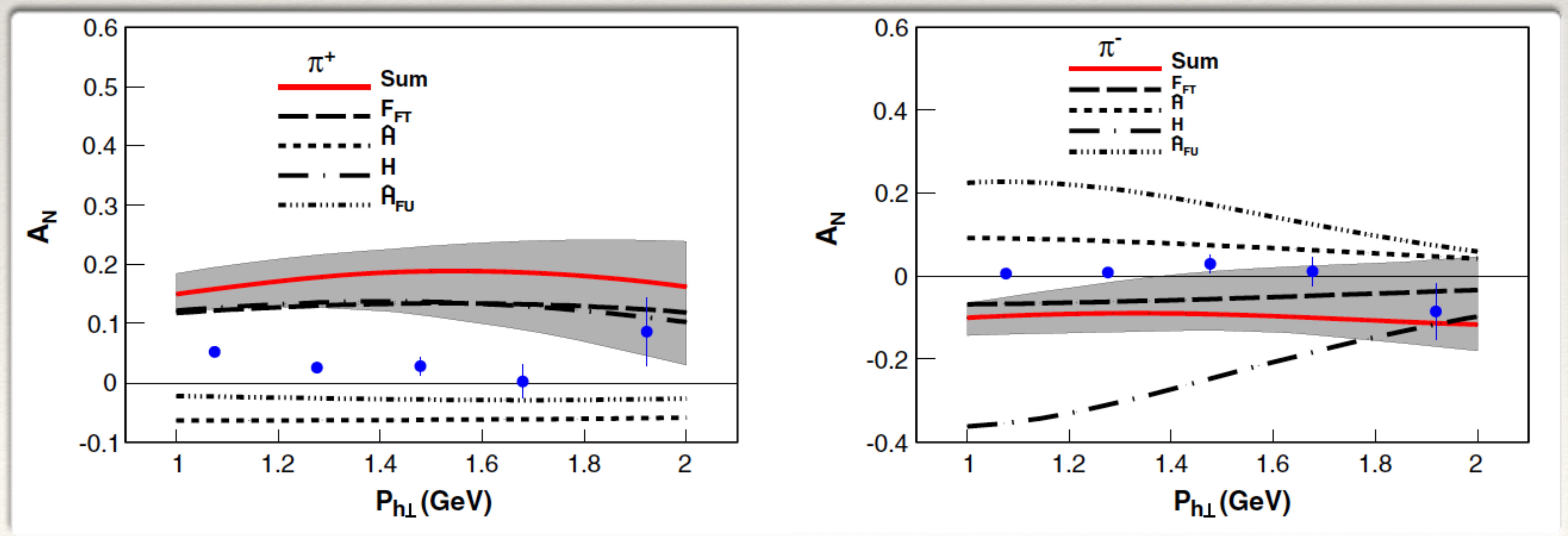
Numerical estimate of the transverse SSA at LO:

[Gamberg, Kang, Metz, Pitonyak, Prokudin; 2014]

x_F - dependence:



P_{hT}- dependence:



Experimental Input:

- Precise data from HERMES & JLab12 for π - production

Theoretical Input:

- Sivers funct. from SIDIS
- Transversity from SIDIS
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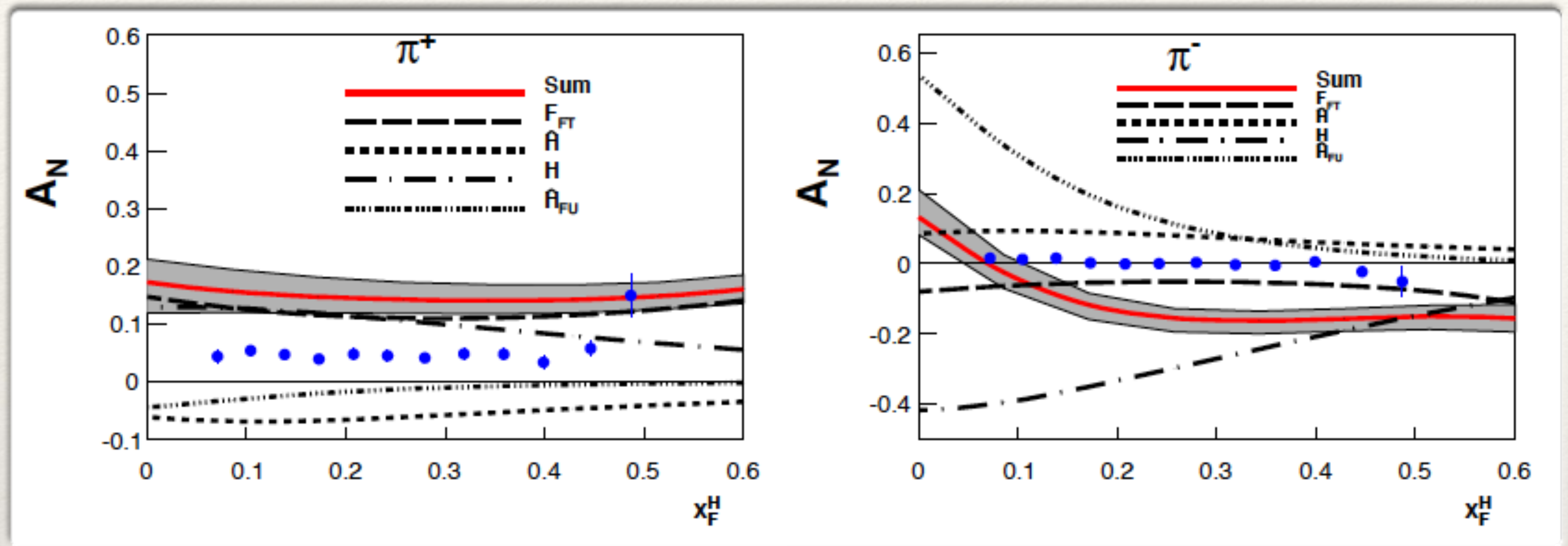

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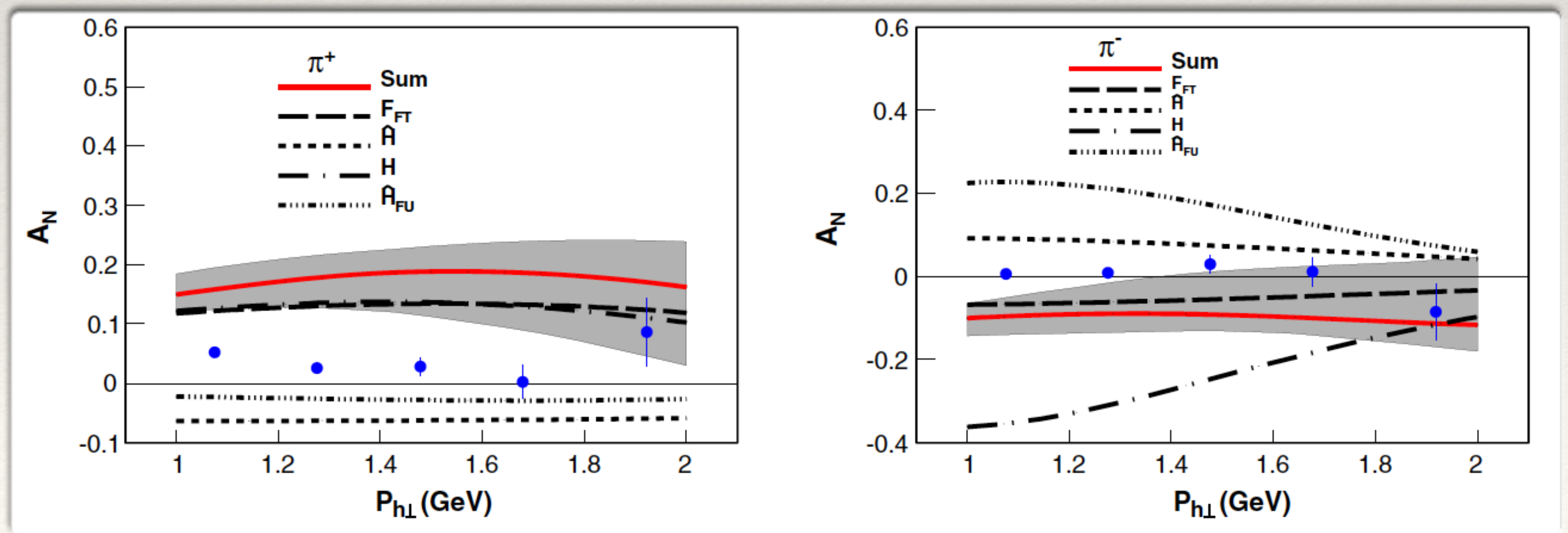

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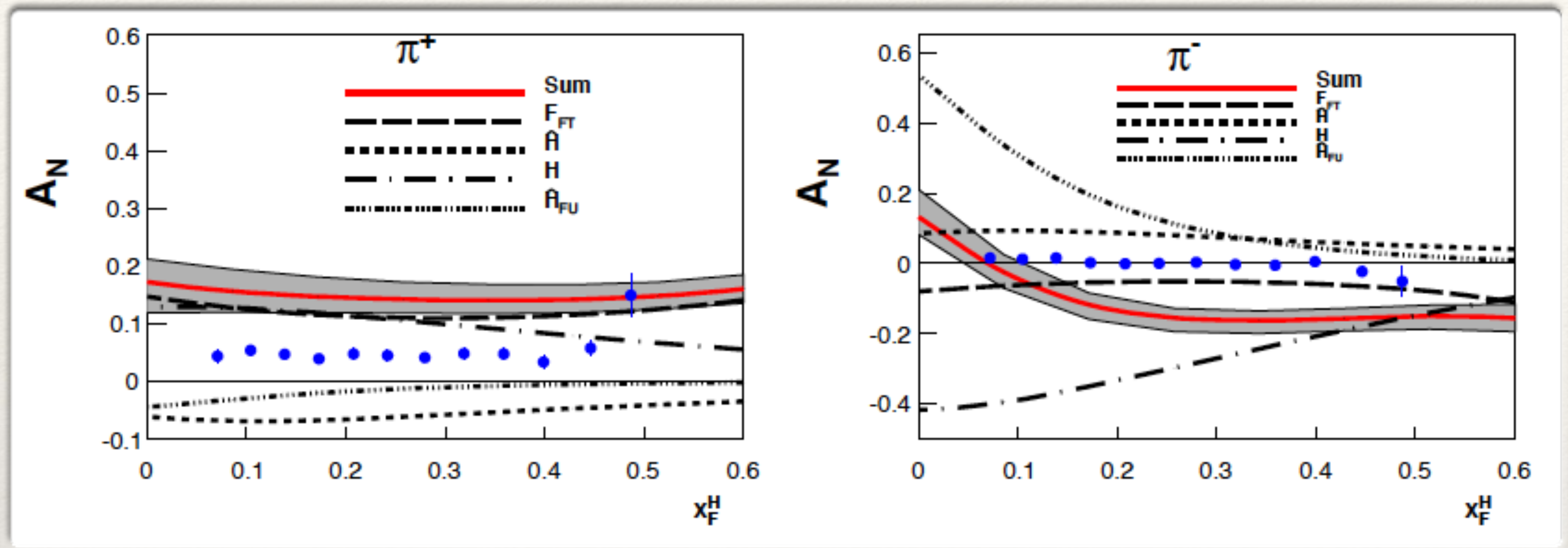

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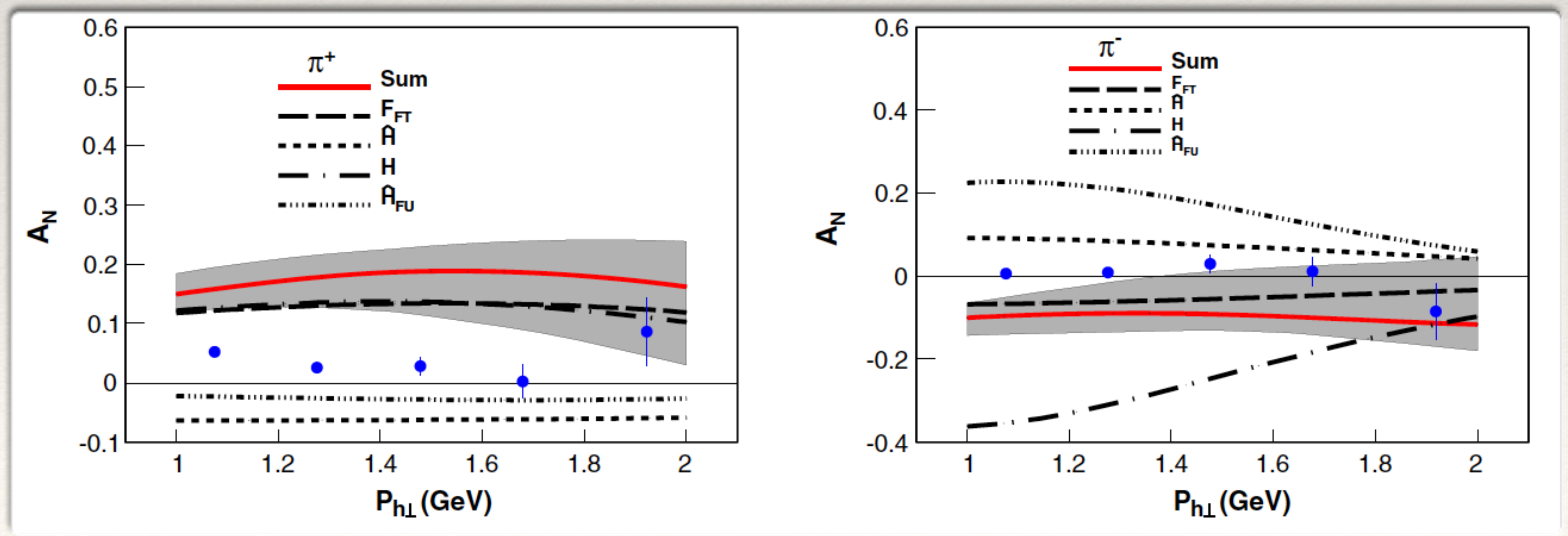

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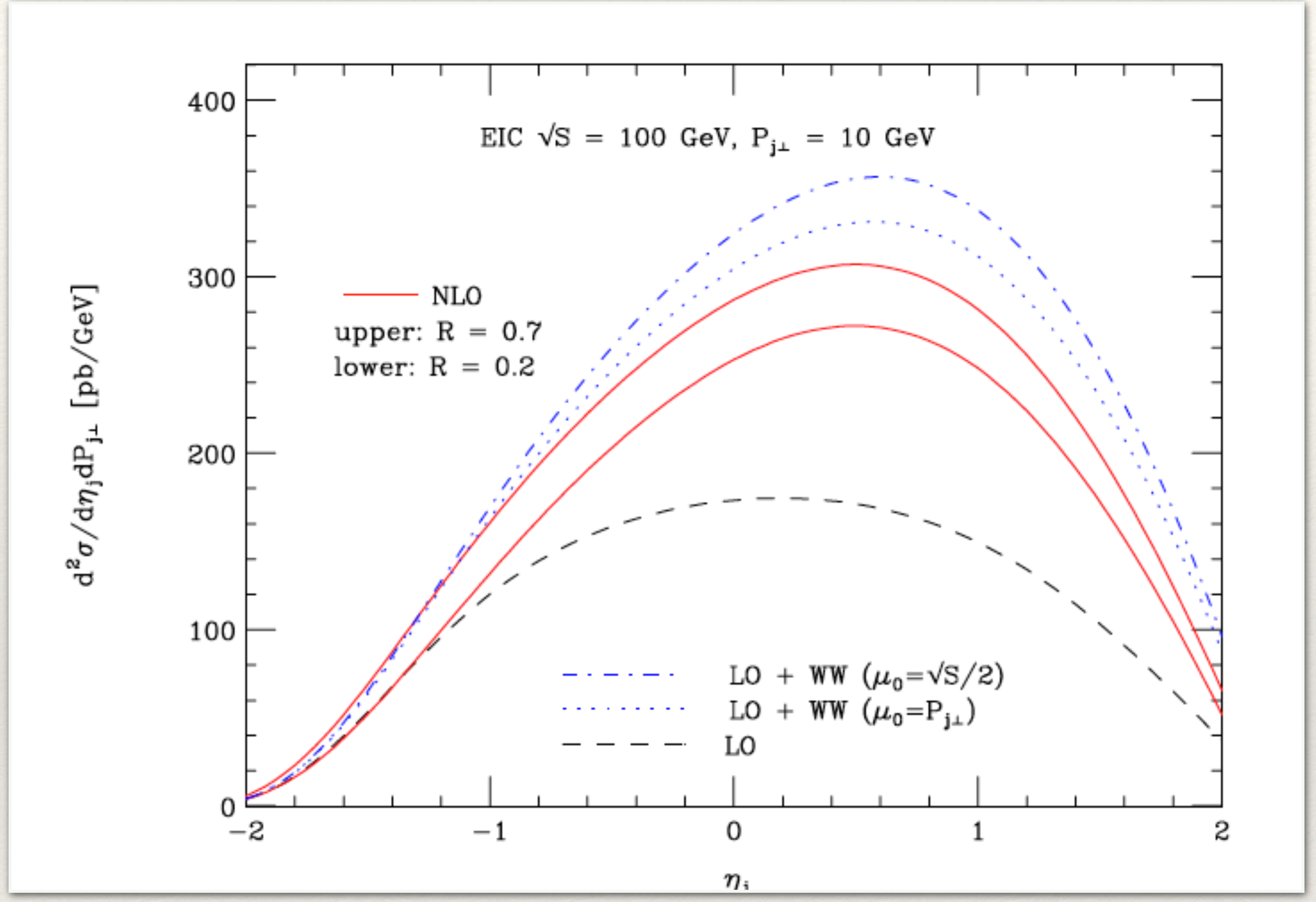
$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

unpolarized cross section

Jet Production at EIC (no fragmentation)

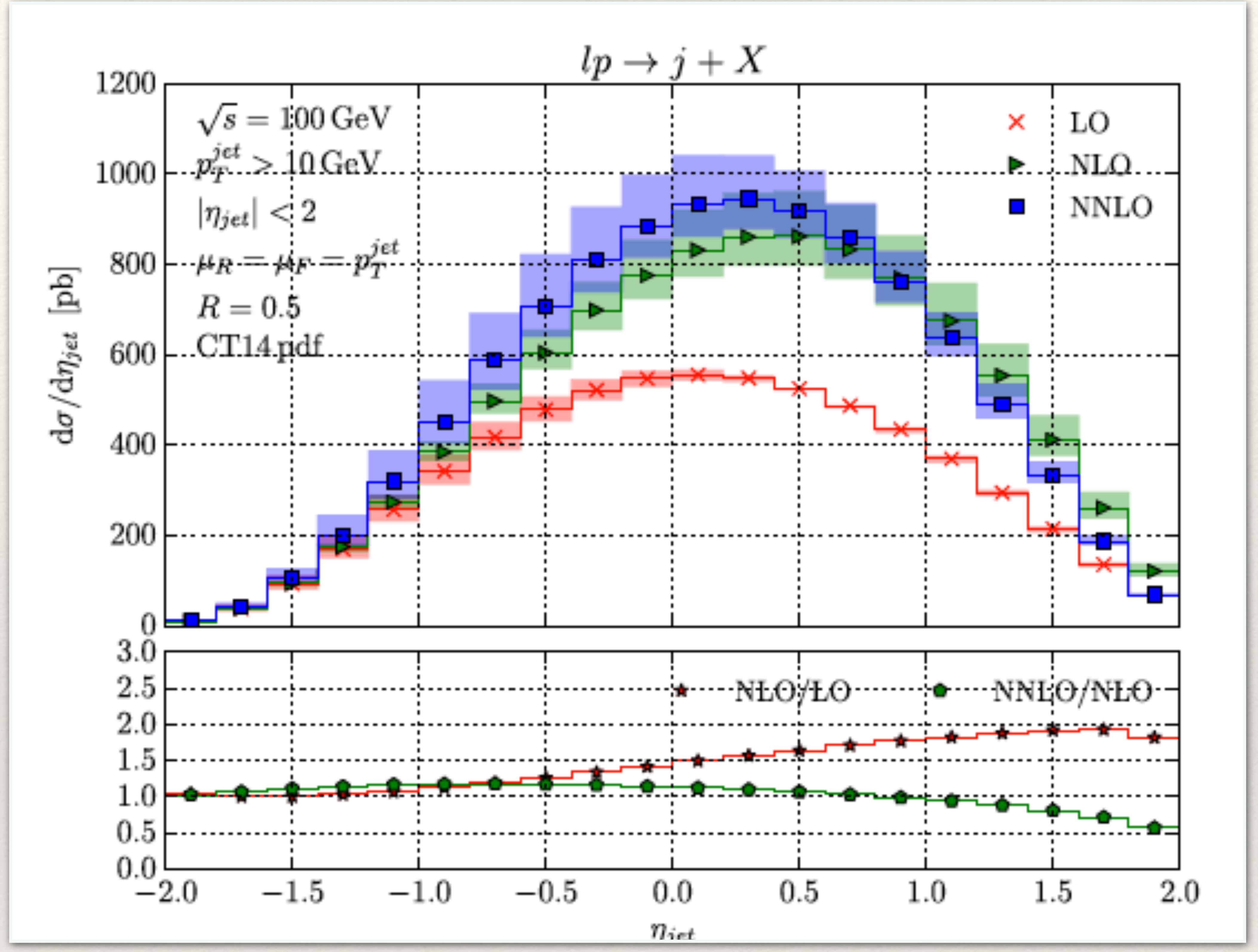
NLO: K ~ 1 - 2

[Hinderer, MS, Vogelsang, PRD, 2015, 2017]



NNLO

[Abelof et al., PLB 763, 52 (2016)]



→ perturbative series converges at NNLO

→ K - factor of *denominator* of SSA may be enough to reduce factor 2 discrepancy to data, NLO corrections of *numerator* of SSA small? Cancellations?

Inclusive π / jet production in transversely polarized $l+N^{\uparrow}$ collisions at NLO

[Tollkühn, MS, Vogelsang, ongoing work]

What do we know so far?

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- "Splitting functions": needed to subtract collinear divergences through factorization

Fragmentation function: DGLAP-type

$$D^{h/q}_{\text{bare}}(z,\mu) = D^{h/q}_{\text{ren}}(z,\mu) + \frac{\alpha_s(\mu)}{2\pi}\frac{S_\varepsilon}{\varepsilon}\left(P_{qq}\otimes D^{h/q}_{\text{ren}}\right)(z,\mu) + \frac{\alpha_s(\mu)}{2\pi}\frac{S_\varepsilon}{\varepsilon}\left(P_{gq}\otimes D^{g/N}_{\text{ren}}\right)(z,\mu) + O(\alpha_s^2)\,,$$

Soft-Gluonic Pole:
derived by
Braun, Manashov, Pirnay,
PRD 80, 114002 (2009)

$$\mu\frac{d}{d\mu}\mathcal{T}_{q,F}(x,x) = \frac{\alpha_s}{\pi}\left\{\int_x^1\frac{d\xi}{\xi}\left[P_{qq}(z)\mathcal{T}_{q,F}(\xi,\xi) + \frac{N_c}{2}\left(\frac{(1+z)\mathcal{T}_{q,F}(x,\xi)-(1+z^2)\mathcal{T}_{q,F}(\xi,\xi)}{1-z} - \mathcal{T}_{\Delta q,F}(x,\xi)\right)\right] - N_c\mathcal{T}_{q,F}(x,x) + \frac{1}{2N_c}\int_x^1\frac{d\xi}{\xi}\left[(1-2z)\mathcal{T}_{q,F}(x,x-\xi) - \mathcal{T}_{\Delta q,F}(x,x-\xi)\right]\right\},$$

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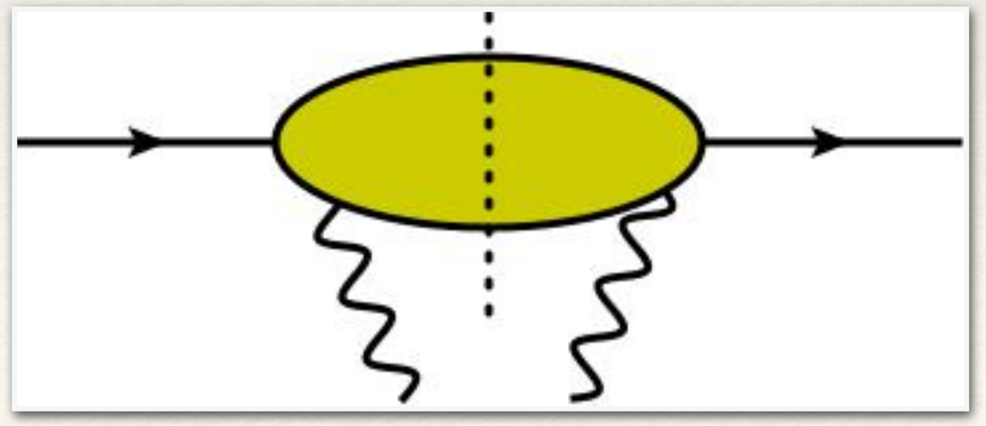
- Collinear divergencies due to *massless* leptons:

"photon-in-lepton" pdf



$$f^{\gamma/\ell}_{\text{ren}}(y,\mu) = f^{\gamma/\ell}_{\text{bare}}(y,\mu) - \frac{\alpha_{\text{em}}}{2\pi}P_{\gamma\ell}(y)\frac{S_\varepsilon}{\varepsilon} + O(\alpha_{\text{em}}^2) = \frac{\alpha_{\text{em}}}{2\pi}P_{\gamma\ell}(y)\left[\ln\left(\frac{\mu^2}{y^2 m_\ell^2}\right)-1\right] + O(\alpha_{\text{em}}^2)$$

combine with hard part

$$\ln\left(\frac{s\,u}{t\,m_e^2}\right)$$

Four partonic channels at NLO:

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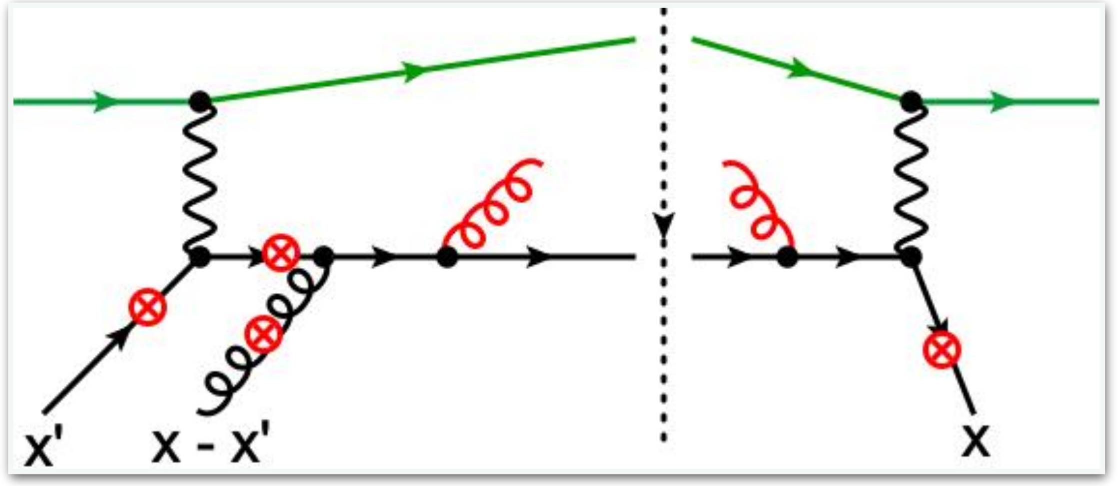
- qg → g: gluon fragmentation ✅

The easiest channel:

-12 real diagrams, no virtual contributions

- no MS - subtraction of collinear divergences from SGP, but from FF



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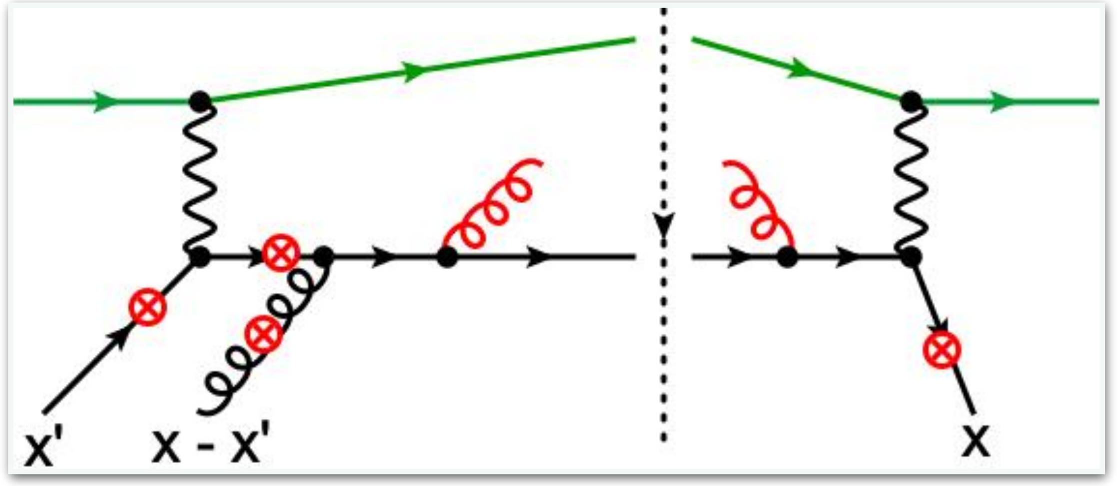

- $gg \to q$: triple gluon correlations 🚫

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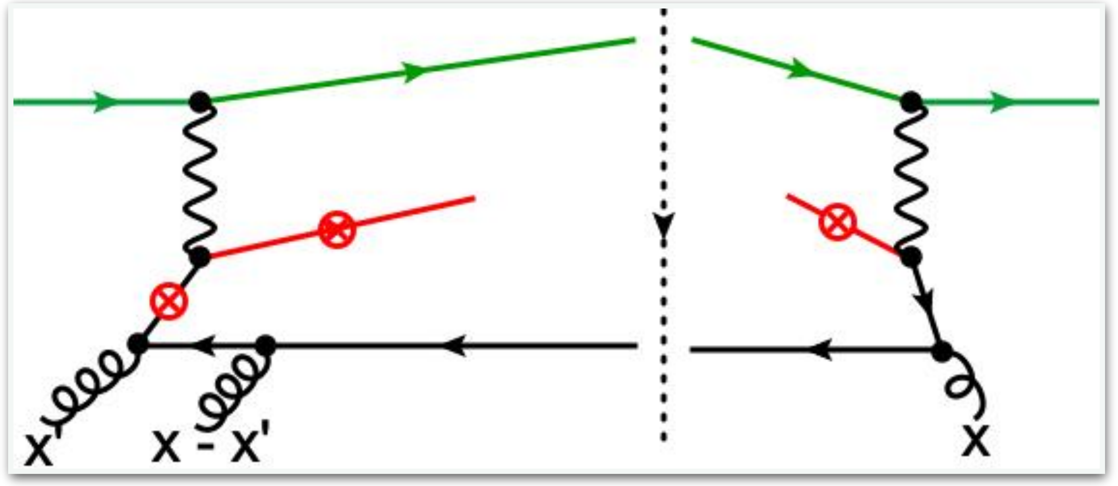

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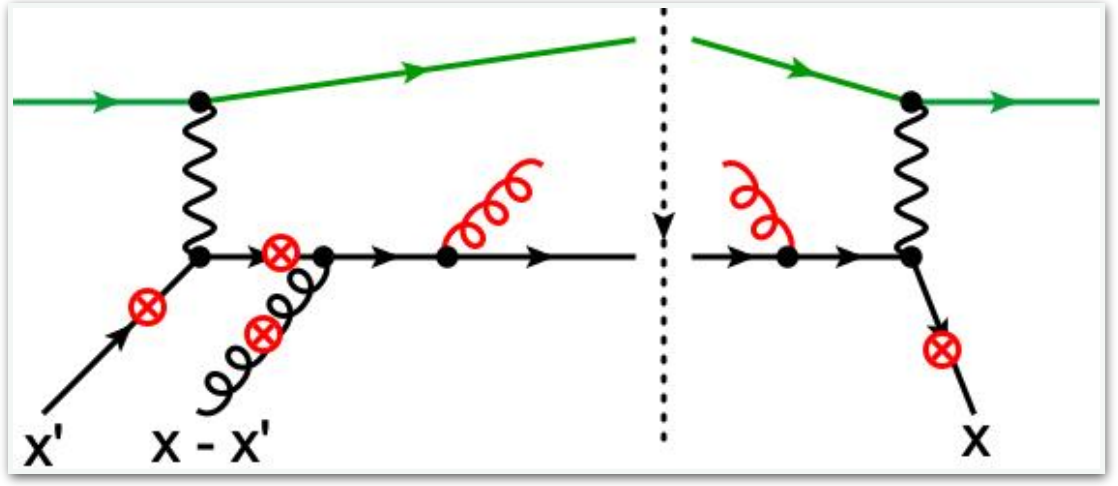

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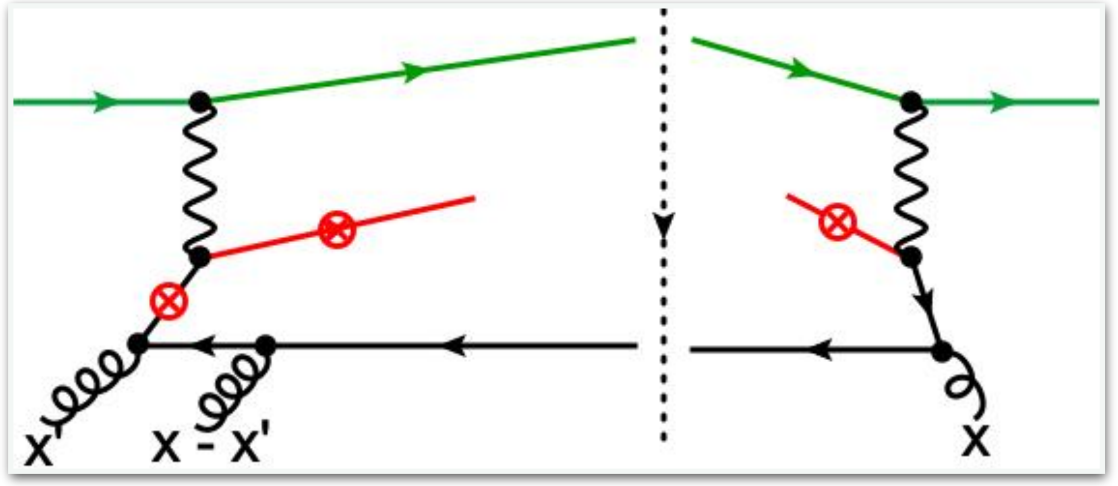

- qq → q: quark-antiquark correlations ✅

  The second most subtle channel: (suppressed in $1/N_c$)

  -12 real diagrams & virtual contributions (x' < 0)

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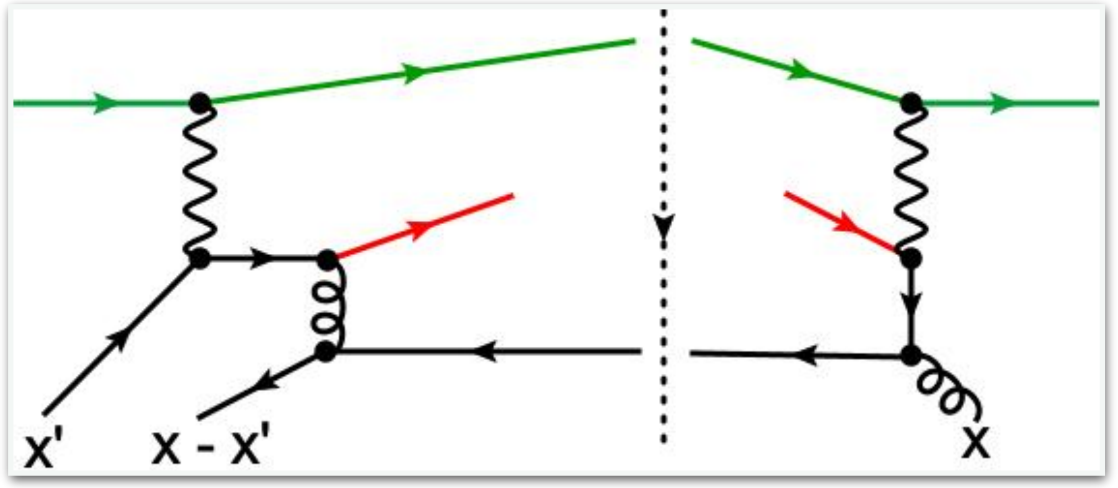


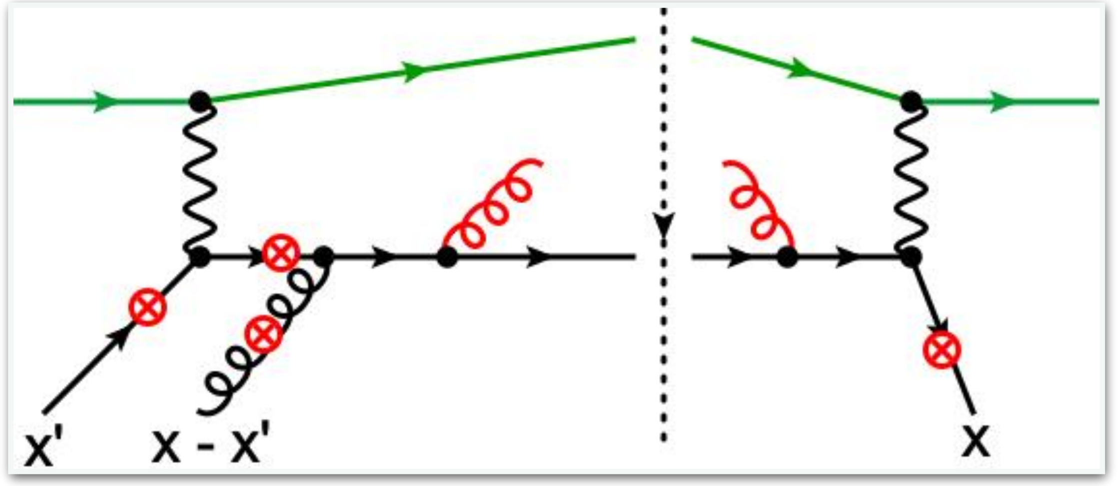
Four partonic channels at NLO:

- qg → g: gluon fragmentation ✅

  The easiest channel:

  -12 real diagrams, no virtual contributions

  - no MS - subtraction of collinear divergences from SGP, but from FF

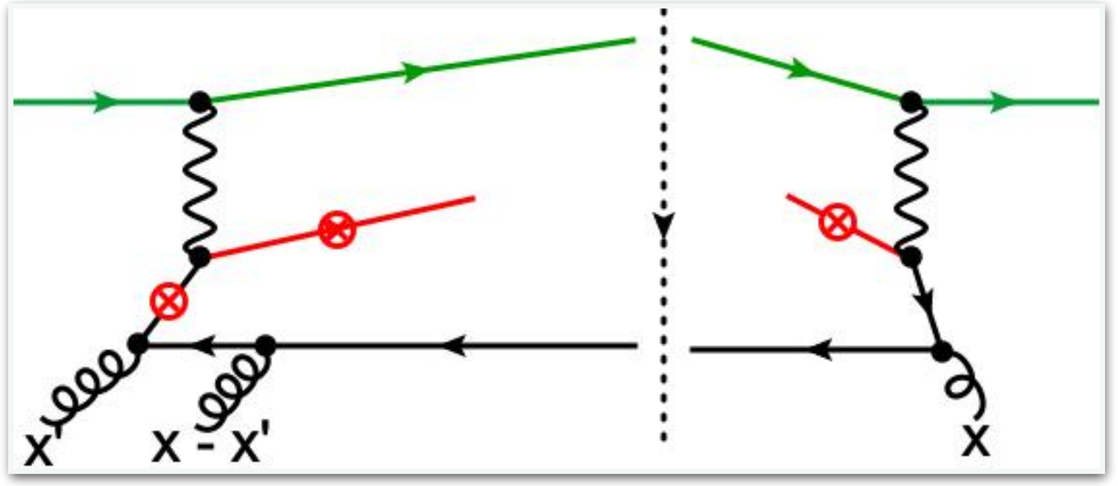


- gg → q: triple gluon correlations 🚫

  The next-to-easiest (?) channel:

  -12 real diagrams, no virtual contributions

  - MS - subtraction of collinear divergences from SGP (?)



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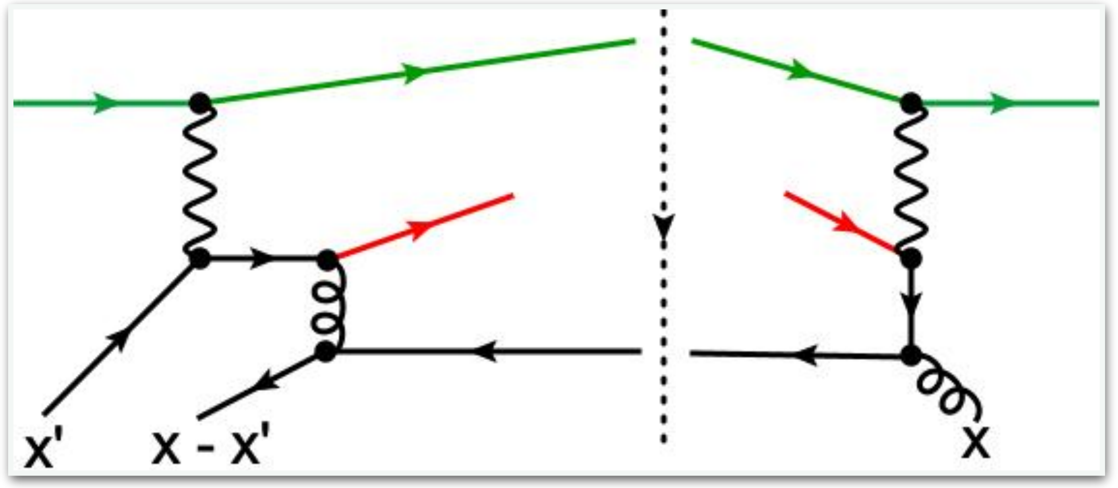

- qg → q: quark-antiquark correlations (partially ✅)

  The most difficult channel:

  -12 real diagrams & virtual contributions (x' > 0)

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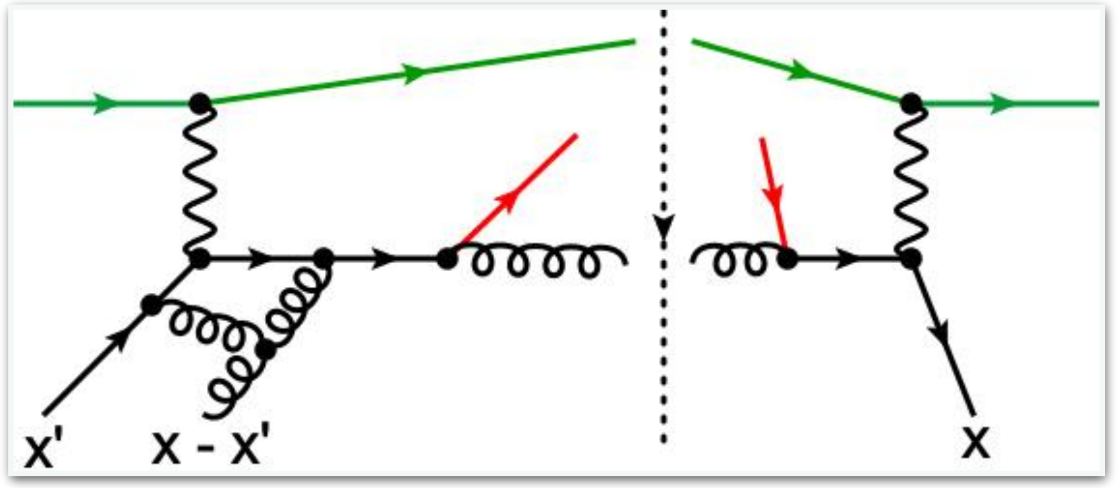

Result for the gluon fragmentation channel / gluonic jet production

$e(l) + N^{\uparrow}(P,S) \to \pi(P_\pi) + X$

Mandelstam variables

$$s = (l+P)^2\,,\; t = (P-P_\pi)^2\,,\; u = (l-P_\pi)^2$$

new integration variables

$$z = \frac{-t}{(1-v)\,s}\,,\; x = \frac{1-v}{v\,w}\frac{u}{t}\,,\; x' = \zeta x\,,\; x_0 = \frac{1-v}{v}\frac{u}{t}$$

tw3 SSA

$$\sigma_0(S) = \frac{8\pi\alpha_{\rm em^2}}{s^2}\frac{M\,\epsilon^{lPP_\pi S}}{u^2}$$

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Soft-Gluon Pole

$$\ln\left(\frac{su}{t\,\mu^2}\right) \quad \ln\left(\frac{su}{t\,m_e^2}\right) \quad \frac{1}{(1-w)_+}$$

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Soft-Fermion Pole

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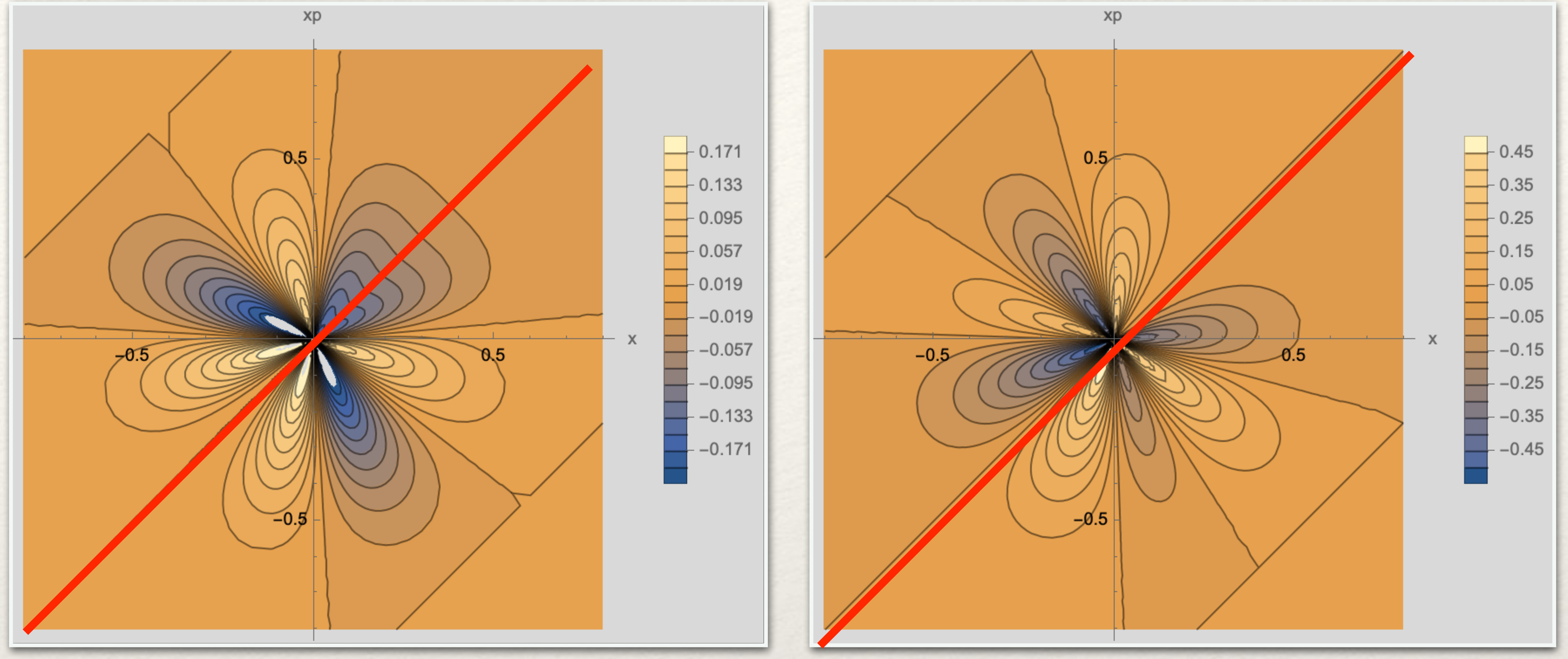
How much does NLO affect the SSA?

- suppose only gluons fragment at NLO
- quark → gluon not very prominent channel for unpol. cross section

- NLO effect on SSA depends on input for Quark-Gluon-Quark correlation functions:

- Model:
  SGP from Sievers function [Anselmino et al., (2008)]
  + extension to full support

$F^u_{FT}(x,x')$ **Scenario 1:** $G^q_{FT}(x,x')$



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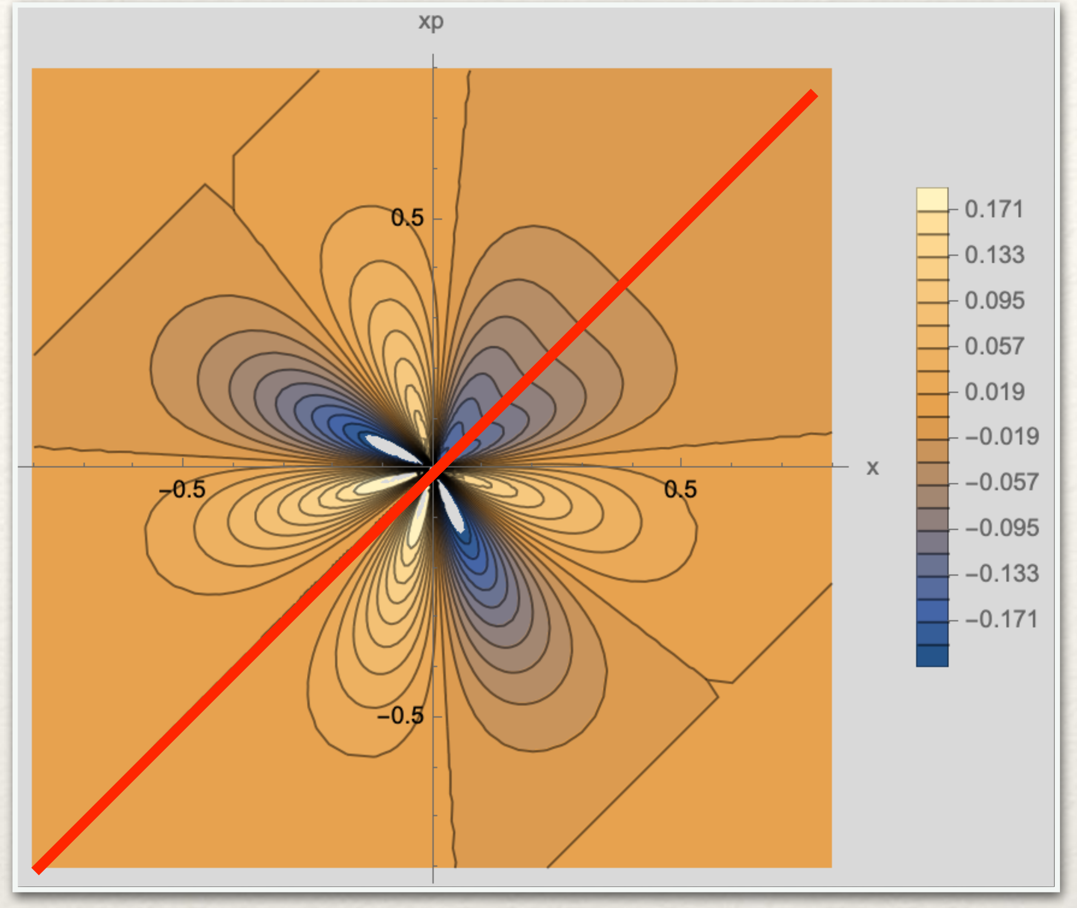
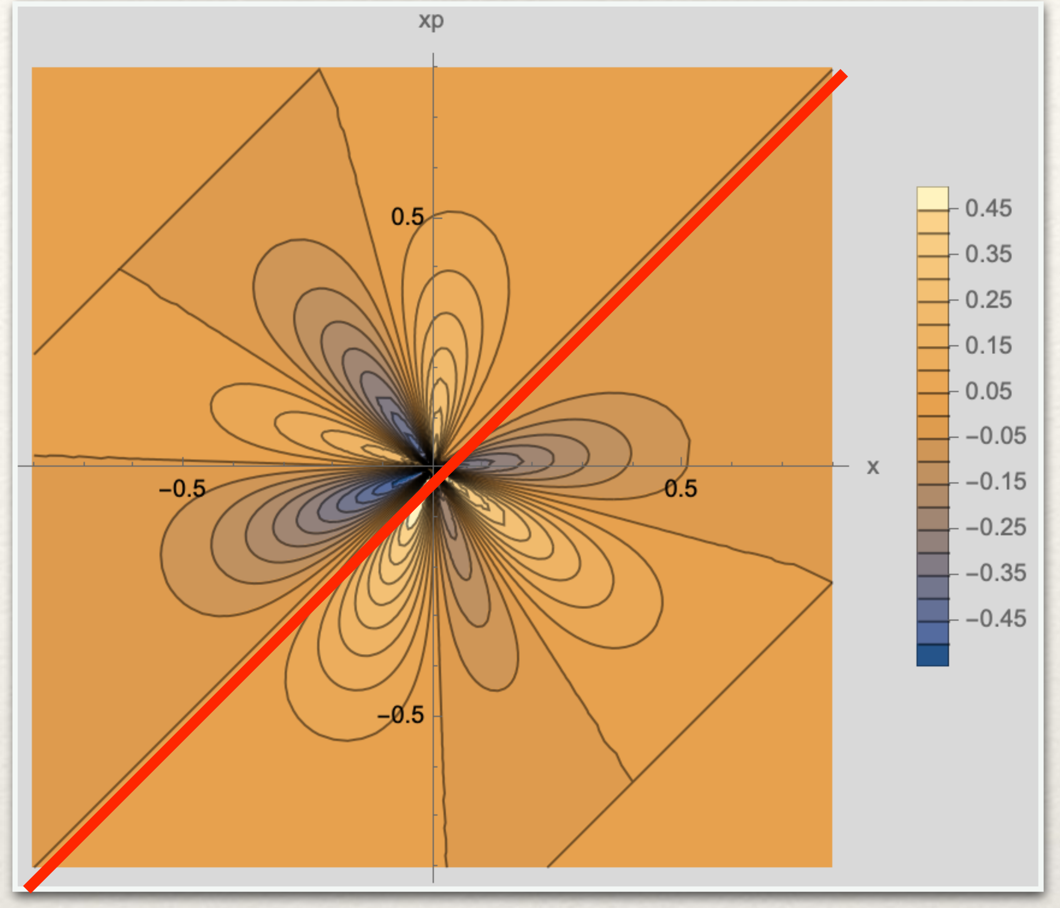
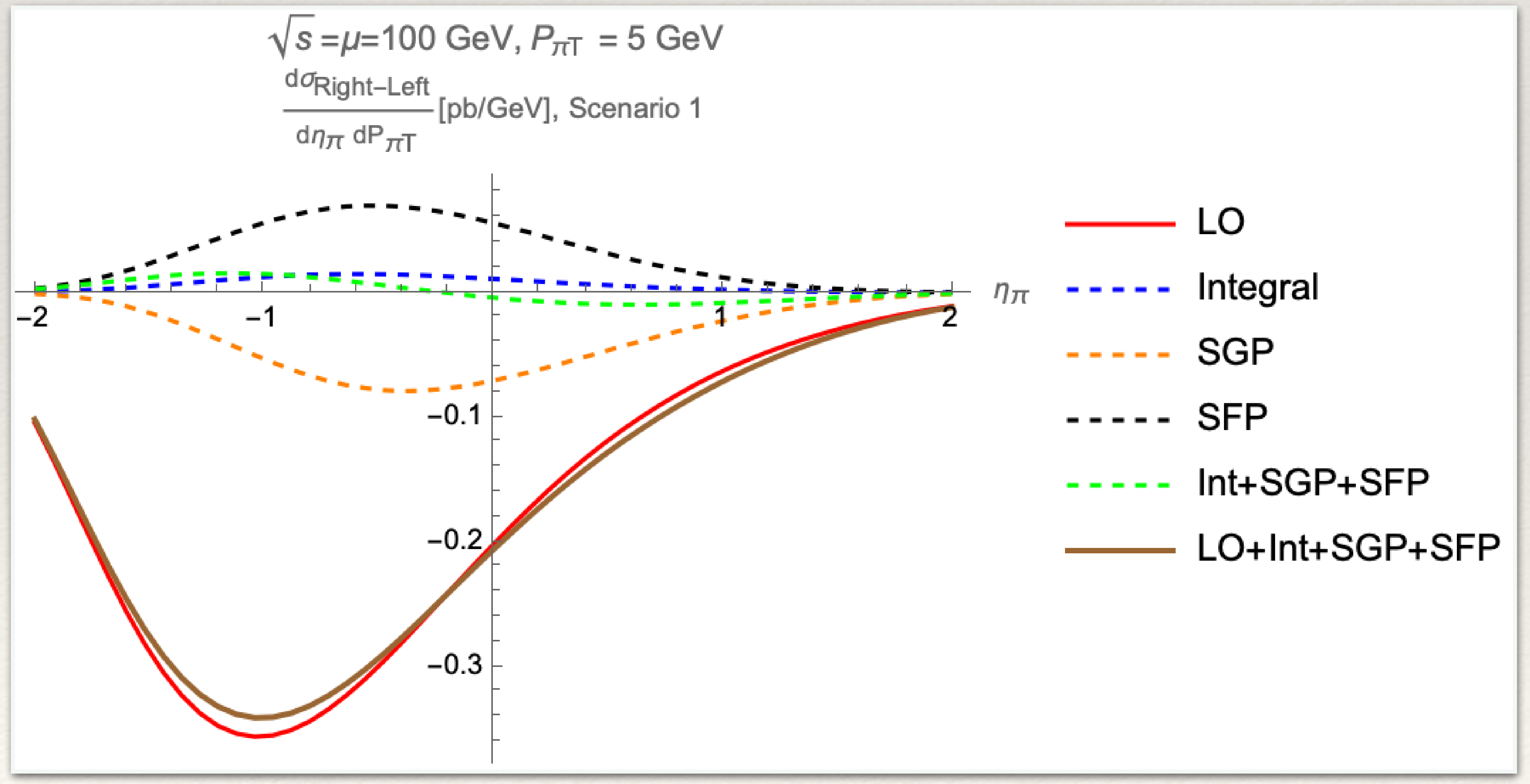
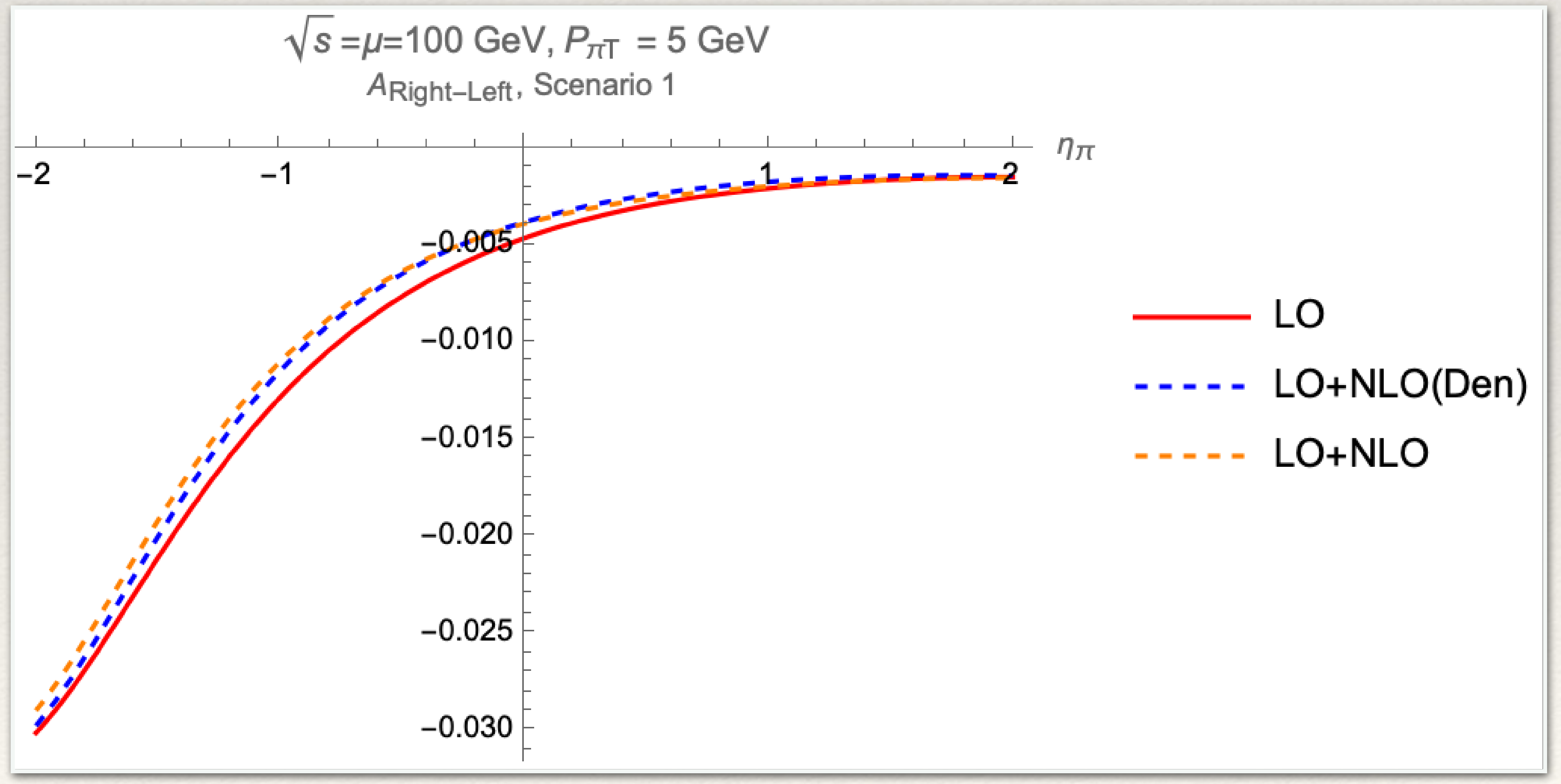




spin-dependent cross section [preliminary]



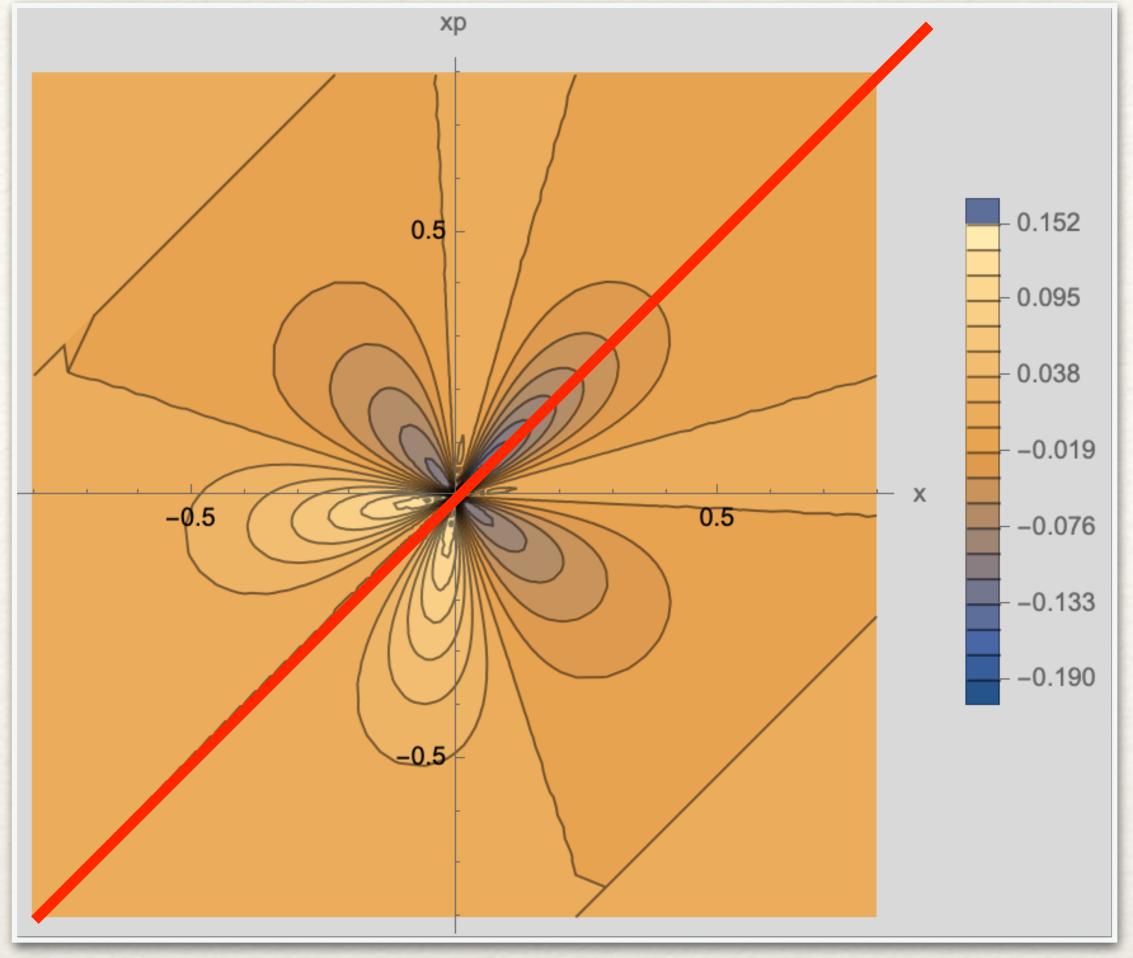
"Right-Left" asymmetry [preliminary]



Scenario 1: NLO corrections of spin-dependent CS cancel!

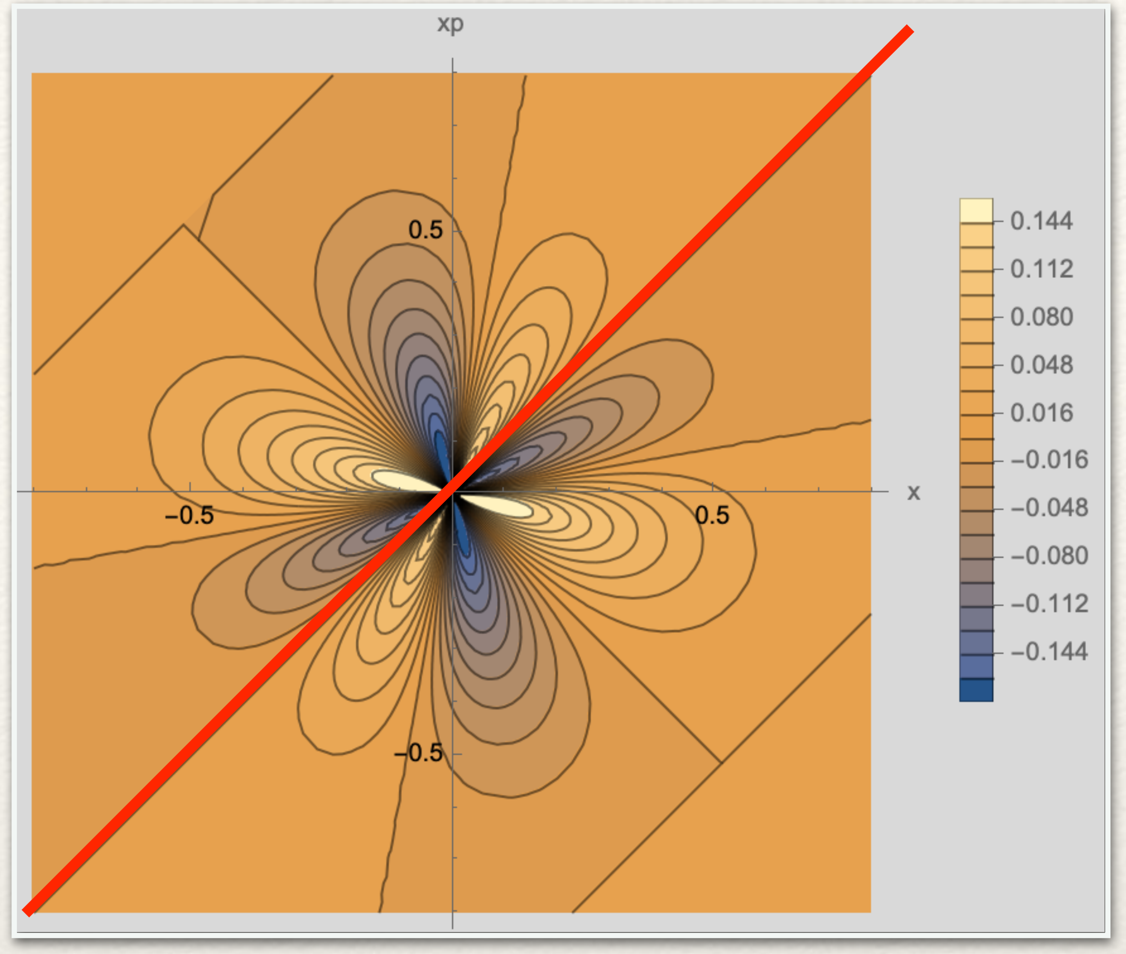
However…

$F^u_{FT}(x,x')$



Scenario 2:

$G^u_{FT}(x,x')$



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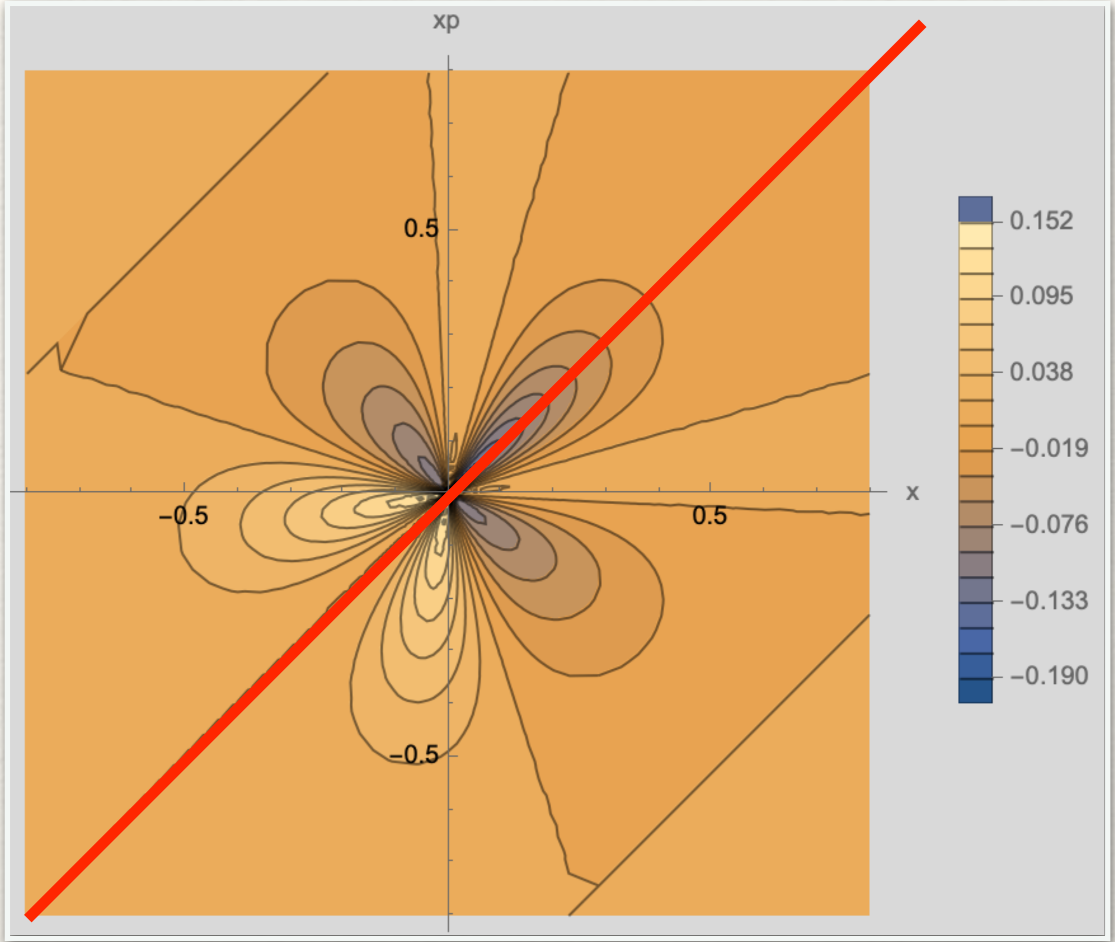

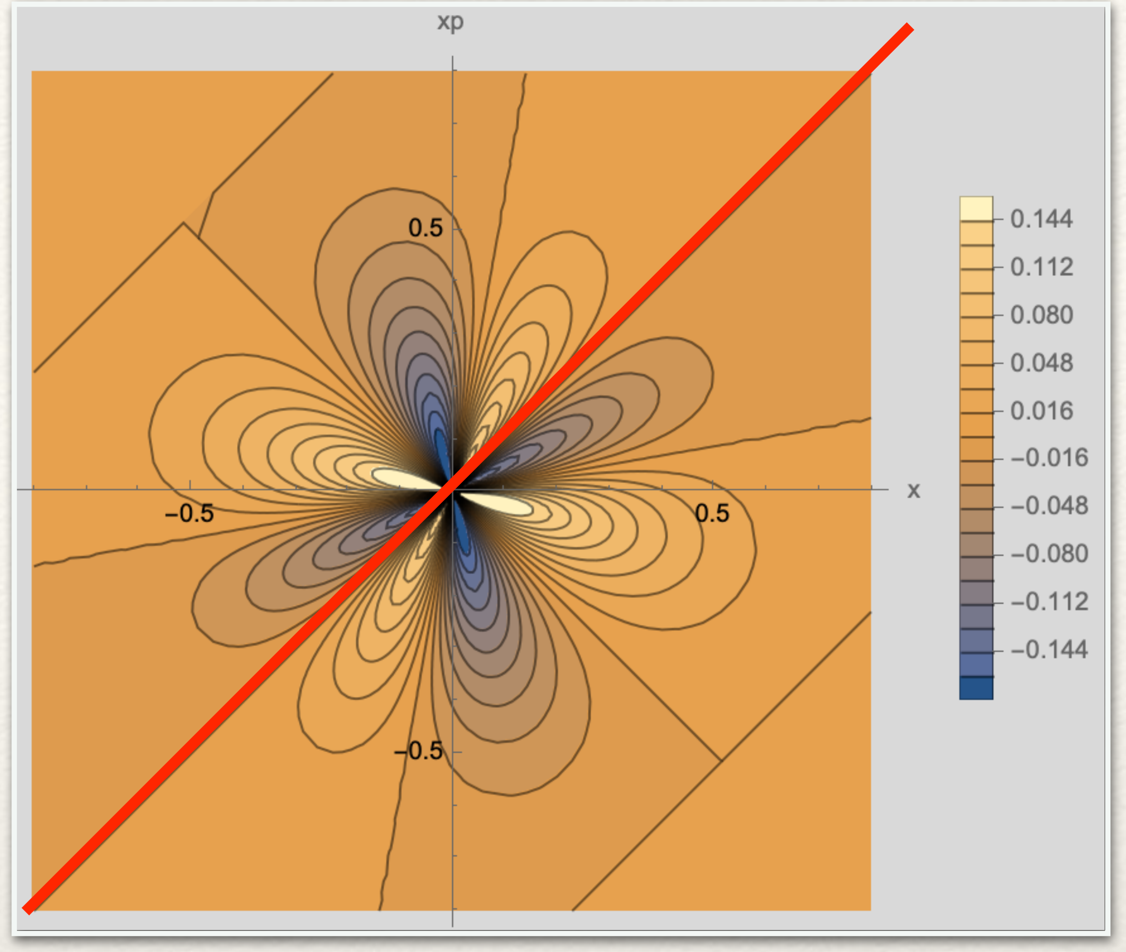

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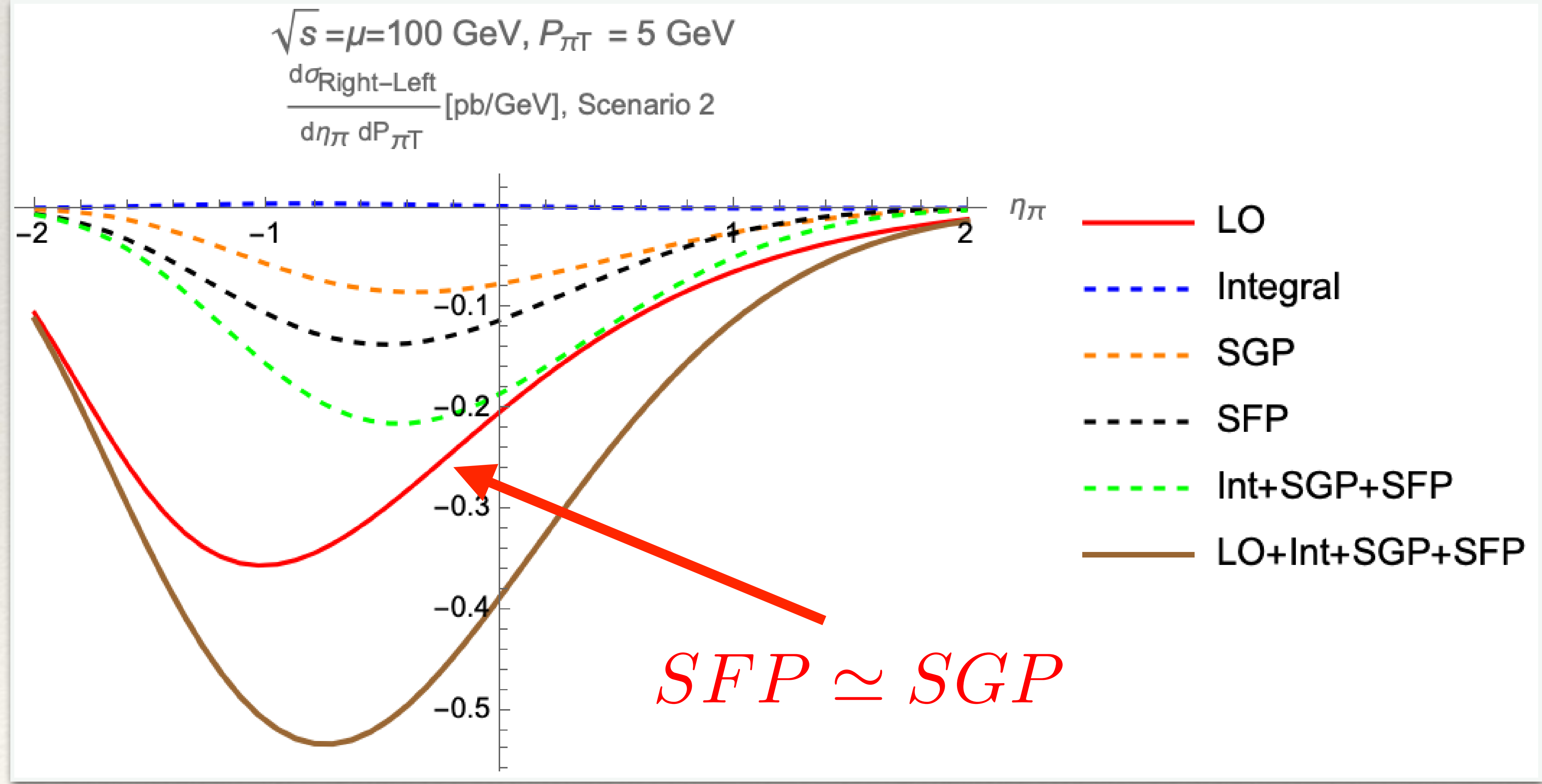

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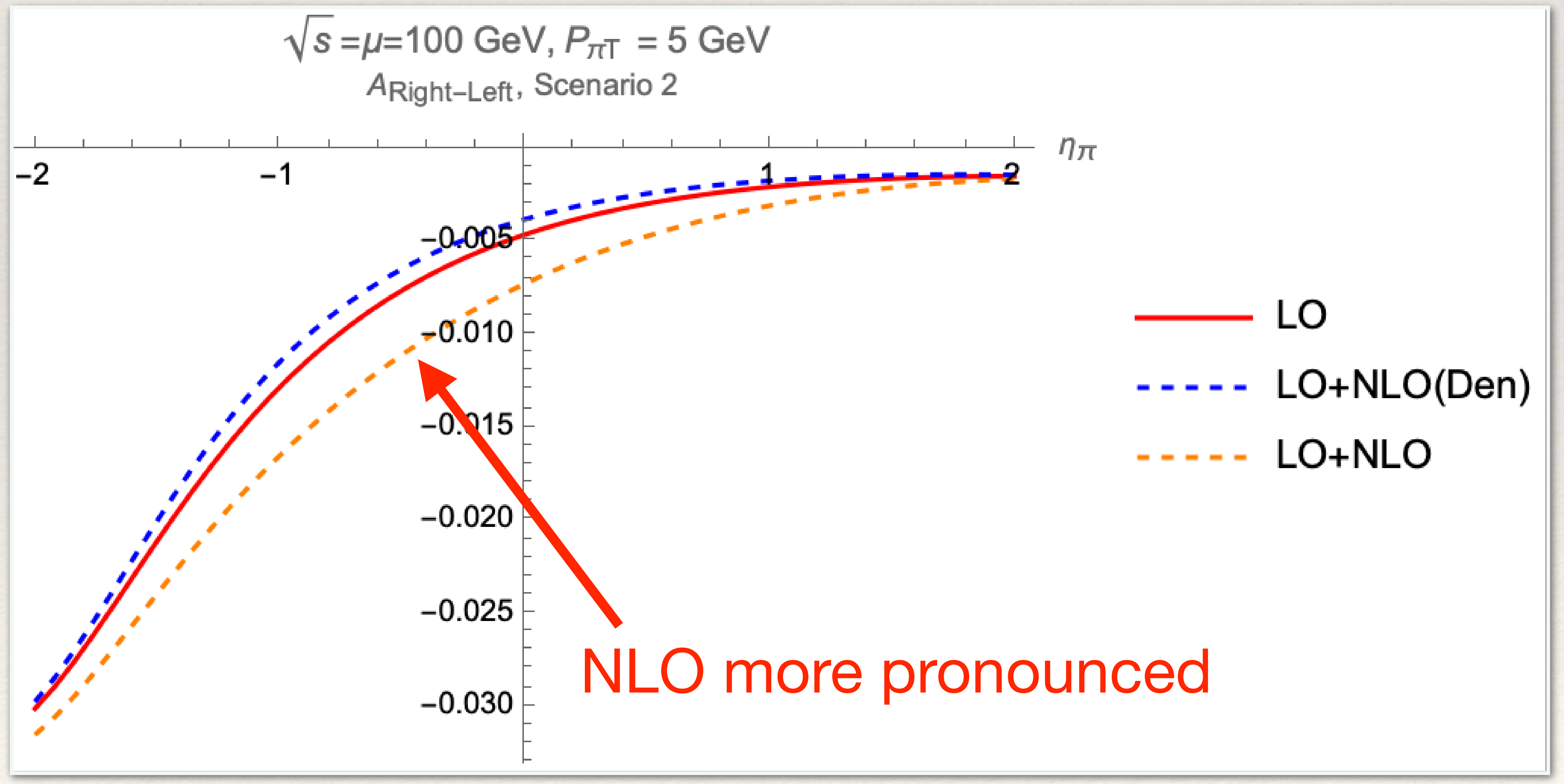


Scenario 2: NLO corrections of spin-dependent CS add up!

Outlook & Summary

- SSA in inclusive π production in eN: HERMES, JLab data exist, EIC will be helpful!
  NLO calculation is needed, 2 out of 4 channels at NLO completed.
  Once finished → Jet production on the way
- Indication from gluon fragmentation channel: NLO fit to data may very well be able to further constrain quark-gluon-quark correlation functions
- Identification of "gluonic" jets at EIC (?) → isolate "quark → gluon" channel
- Inclusive photon production "Abelian version of q →g channel",
  [ongoing work with D. Rein & W. Vogelsang]
- Next logical step: twist-3 fragmentation contribution for π production (whole different story…)
- Work towards more complicated polarized pp - processes…

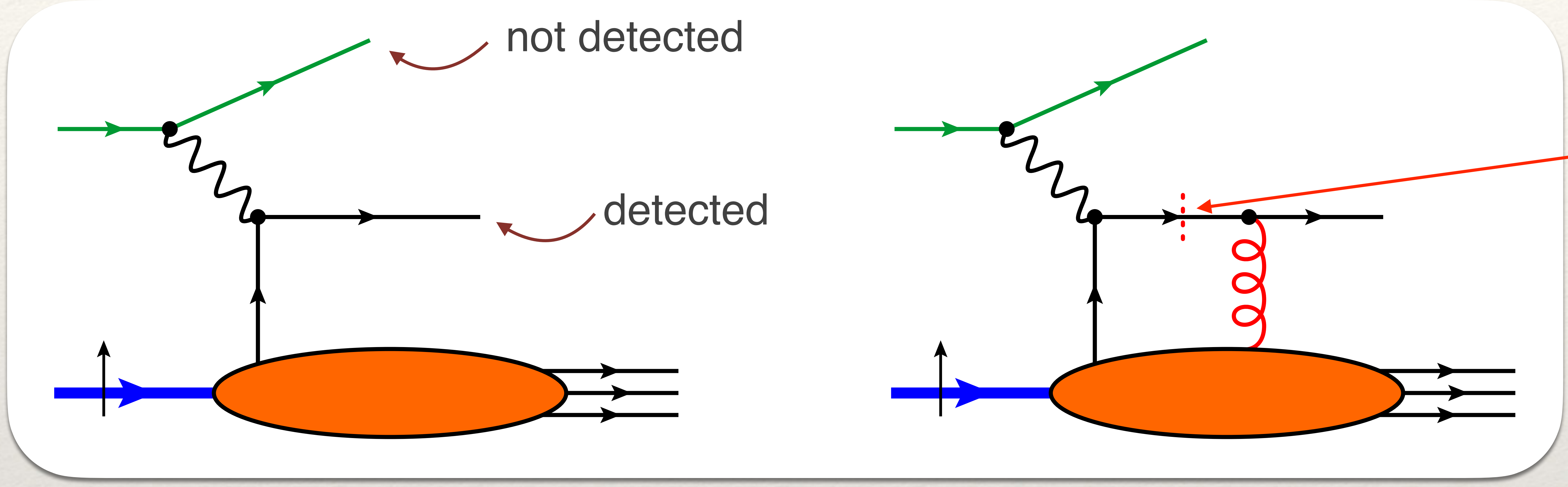
Back up

How do QGQ correlations generate an SSA?

Example: Single-inclusive jet production e $N^{\uparrow} \rightarrow$ jet X

[Gamberg, Kang, Metz, Pitonyak, Prokudin; Kanazawa, Koike, Metz, Pitonyak, MS]

Simple LO diagrams



Soft gluon pole

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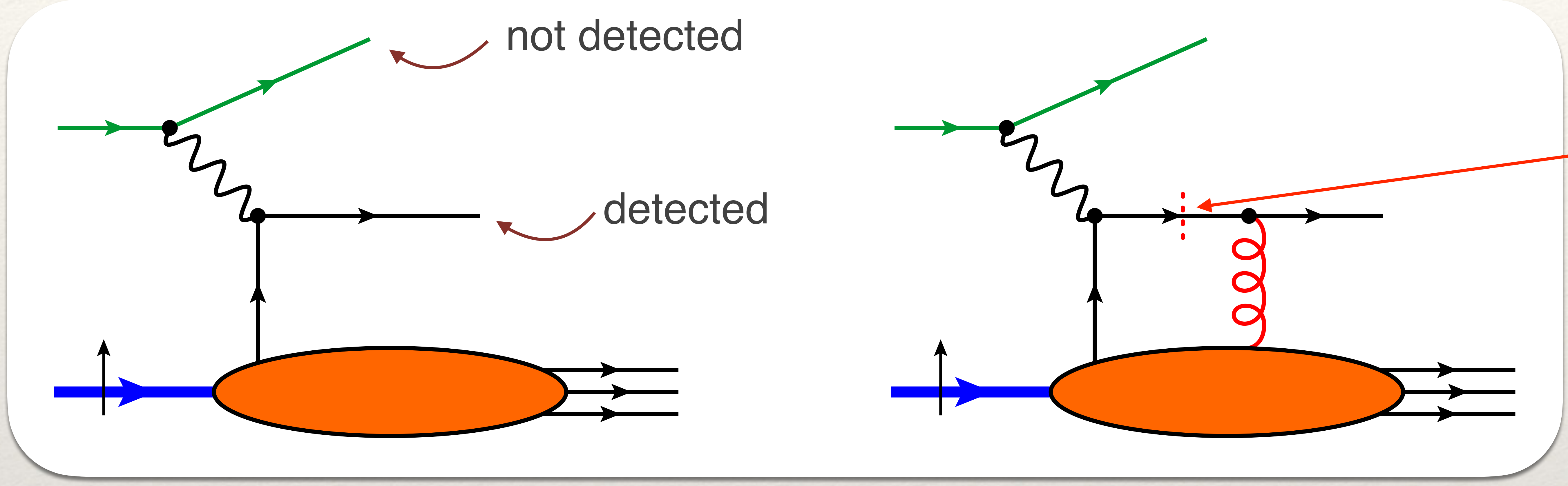

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$$\frac{1}{x - x' + i\epsilon} = \frac{\mathcal{P}}{x - x'} - i\pi\delta(x - x')$$

$$A_N \propto \left(1 - x\frac{d}{dx}\right) F_{FT}^{q}(x, x)$$

SSA generated by soft-gluon pole only

At NLO:

several interdependent sources of imaginary parts

→ Pole terms (Soft-Gluon Poles, Soft-Fermion Poles, Hard Poles) & Integral terms

How to combine these sources & cancel collinear/soft divergences ?

Reexamine integral contributions in gluon fragmentation channel

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reverse order of phase space & x’ integration:

No 1/ε poles,

BUT: endpoint singularities at x’ = x & x’ = 0

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regular partonic factor, but irregular soft function

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regular partonic factor, but irregular soft function

$$\frac{F(x, x')}{x'(x' - x)^2} = \frac{1}{x'} \left[\frac{F(x, x') - F(x, x) - (x' - x)\frac{1}{2}\frac{d}{dx}F(x, x)}{(x' - x)^2} - \frac{F(x, 0) - F(x, x) + x\frac{1}{2}\frac{d}{dx}F(x, x)}{x^2} \right]$$

regular integral contribution, but…

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$$\frac{F(x,x')}{x'(x'-x)^2} = \frac{1}{x'}\left[\frac{F(x,x') - F(x,x) - (x'-x)\frac{1}{2}\frac{d}{dx}F(x,x)}{(x'-x)^2} - \frac{F(x,0) - F(x,x) + x\frac{1}{2}\frac{d}{dx}F(x,x)}{x^2}\right]$$

regular integral contribution, but…

$$+ \int_{x_0}^{1} \frac{dw}{w}\, \hat\sigma^{\text{int}\to SGP}(w)\, F(x,x) \qquad + \int_{x_0}^{1} \frac{dw}{w}\, \hat\sigma^{\text{int}\to dSGP}(w)\, x\frac{d}{dx}F(x,x) \qquad + \int_{x_0}^{1} \frac{dw}{w}\, \hat\sigma^{\text{int}\to SFP}(w)\, F(x,0)$$

… additional contributions to soft gluon poles and soft fermion poles, with 1/ε singularities!

At NLO:

several interdependent sources of imaginary parts

→ Pole terms (Soft-Gluon Poles, Soft-Fermion Poles, Hard Poles) & Integral terms

How to combine these sources & cancel collinear/soft divergences ?

Reexamine integral contributions in gluon fragmentation channel

$$d\sigma^{\mathrm{int}}(S_T) \propto \int_{x_0}^{1} \frac{dw}{w} \int_0^{x} dx'\, \hat{\sigma}^{\mathrm{int}}(x',w)\, F_{FT}(x,x') \Big|_{x=\frac{x_0}{w}}$$

reverse order of phase space & x' integration:
No 1/ε poles,
BUT: endpoint singularities at x' = x & x' = 0

$$d\sigma^{\mathrm{int}}(S_T) \propto \int_{x_0}^{1} \frac{dw}{w} \int_0^{x} dx' \left[x'(x'-x)^2\, \hat{\sigma}^{\mathrm{int}}(x',w)\right] \frac{F_{FT}(x,x')}{x'(x'-x)^2} \Big|_{x=\frac{x_0}{w}}$$

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… additional contributions to soft gluon poles and soft fermion poles, with 1/ε singularities!

Result:

combine all SGP contributions, SFP contributions → eventually all 1/ε singularities cancel!

How could the QGQ correlation functions look like?

Polar coordinates $(x, x') = r(\cos(\varphi + \frac{\pi}{4}), \sin(\varphi + \frac{\pi}{4}))$

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Symmetry

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Fourier series

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charge conjugation

$$F^{\bar{q}}_{FT}(x, x') = +F^q_{FT}(-x, -x')$$
$$G^{\bar{q}}_{FT}(x, x') = +G^q_{FT}(-x, -x')$$

→

$$\sum_{q=u,d,s} e_q^2 \left(F^q_{FT} + F^{\bar{q}}_{FT}\right)(r, \varphi)$$
$$= \sum_q e_q^2 \frac{1}{\pi} \left(f_{1T}^{\perp(1),q} + f_{1T}^{\perp(1),\bar{q}}\right) \left(\frac{r}{\sqrt{2}}\right) \left[1 + \sum_{n=1}^{\infty} \tilde{a}^q_{2n}(r)\ (\cos(2n\varphi) - 1)\right]$$

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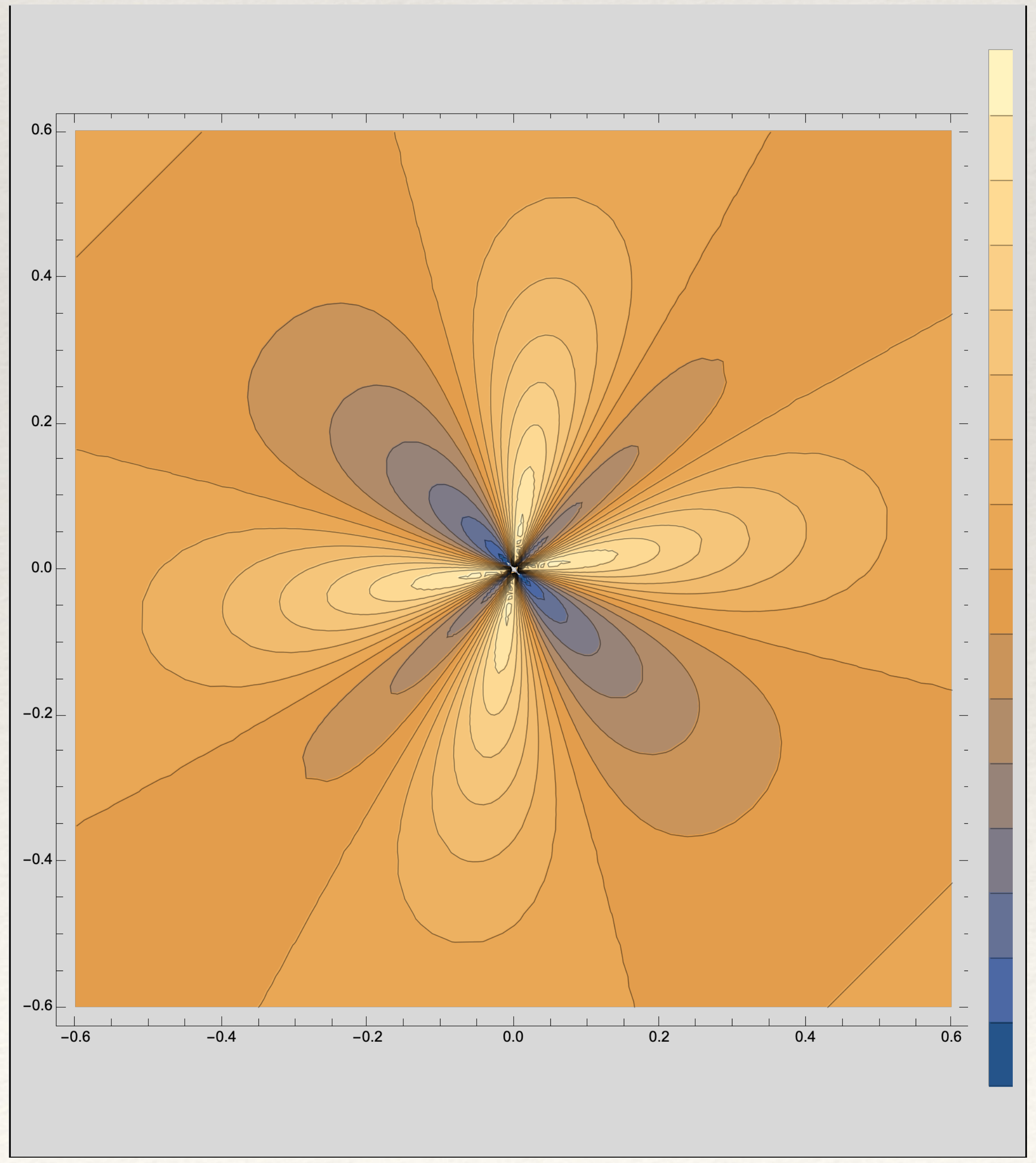

Example for F_{FT} (random choice)

$$\tilde{a}^u_2 = -\frac{1}{2},\ \tilde{a}^u_4 = 1,\ \tilde{a}^u_6 = \frac{1}{3}$$

Ideally: Fit values from experimental data

→ EIC!