

Electromagnetic Structure of the Proton from Generalized Polarizabilities

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15th European Research Conference on
Electromagnetic Interactions with Nucleons and Nuclei

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Outline

Introduction to the GPs

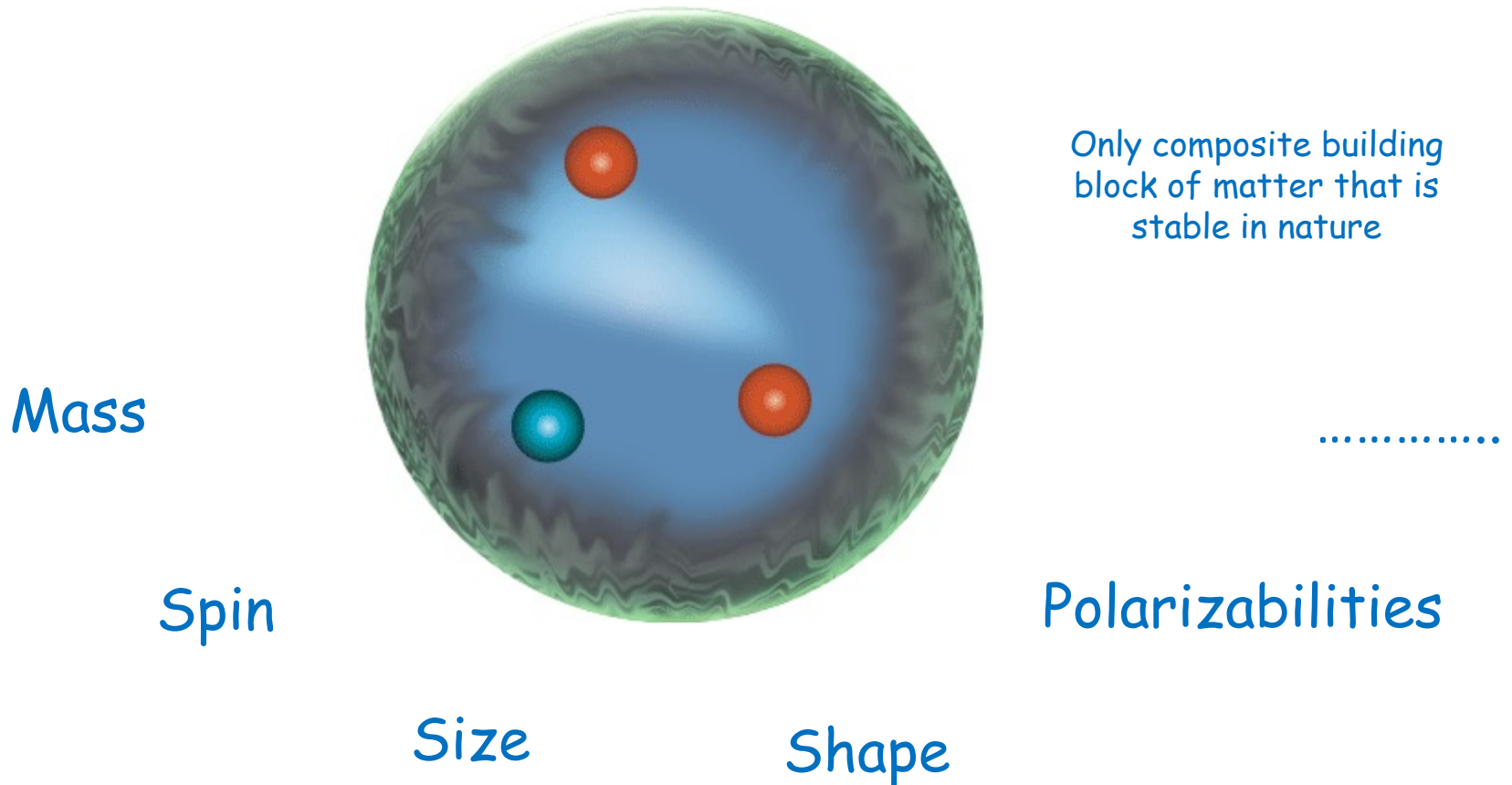
Status & challenges

Recent results / Jlab & MAMI

Prospects / VCS-II @ Jlab, measuring w positrons, ...

Our mission: Explain how the proton emerges from the dynamics of the quark & gluon constituents

How to accomplish: Measure and understand the emergence of the fundamental properties of the proton's bound state



Proton Polarizabilities

Fundamental structure constants
(such as mass, size, shape, ...)

Response of the nucleon to external EM field

Sensitive to the full excitation spectrum

Accessed experimentally through Compton Scattering

RCS: static polarizabilities \rightarrow net effect on the nucleon

Virtual Compton Scattering:

Virtuality of photon gives access to the GPs : $\alpha_E(Q^2)$ & $\beta_M(Q^2)$ + spin GPs

\rightarrow mapping out the spatial distribution of the polarization densities

Fourier transform of densities of electric charges and magnetization of a nucleon deformed by an applied EM field

PDG

150 Baryon Summary Table

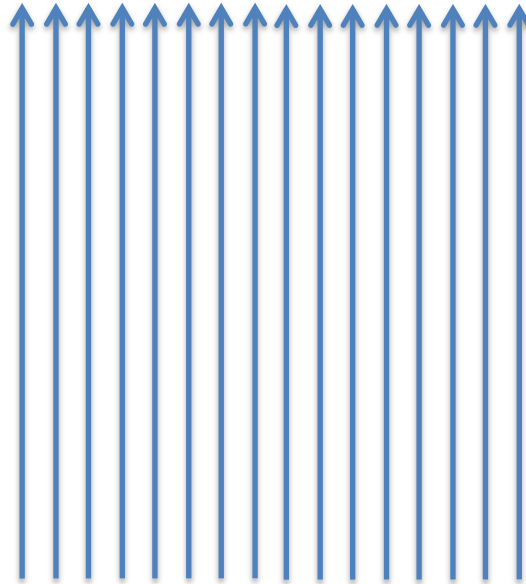
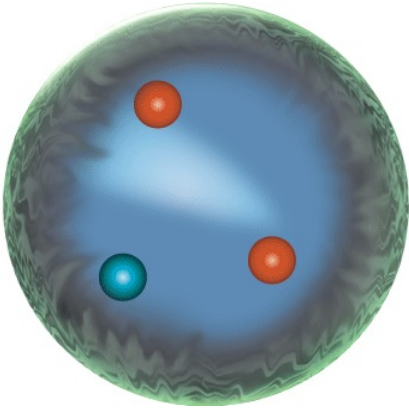
N BARYONS ($S = 0, I = 1/2$) $p, N^+ = uud; n, N^0 = udd$
--

p	$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
Mass $m = 1.00727646681 \pm 0.00000000009$ u	
Mass $m = 938.272046 \pm 0.000021$ MeV [a]	
$ m_p - m_{\bar{p}} /m_p < 7 \times 10^{-10}$, CL = 90% [b]	
$ \frac{q_p}{m_p} /(\frac{q_e}{m_e}) = 0.99999999991 \pm 0.00000000009$	
$ q_p + q_{\bar{p}} /e < 7 \times 10^{-10}$, CL = 90% [b]	
$ q_p + q_e /e < 1 \times 10^{-21}$ [c]	
Magnetic moment $\mu = 2.792847356 \pm 0.000000023 \mu_N$	
$(\mu_p + \mu_{\bar{p}}) / \mu_p = (0 \pm 5) \times 10^{-6}$	
Electric dipole moment $d < 0.54 \times 10^{-23}$ e cm	
Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4} \text{ fm}^3$	
Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4} \text{ fm}^3$ (S = 1.2)	
Charge radius, μp Lamb shift = 0.84087 ± 0.00039 fm [d]	
Charge radius, $e p$ CODATA value = 0.8775 ± 0.0051 fm [d]	
Magnetic radius = 0.777 ± 0.016 fm	
Mean life $\tau > 2.1 \times 10^{29}$ years, CL = 90% [e] ($p \rightarrow$ invisible mode)	
Mean life $\tau > 10^{31}$ to 10^{33} years [e] (mode dependent)	

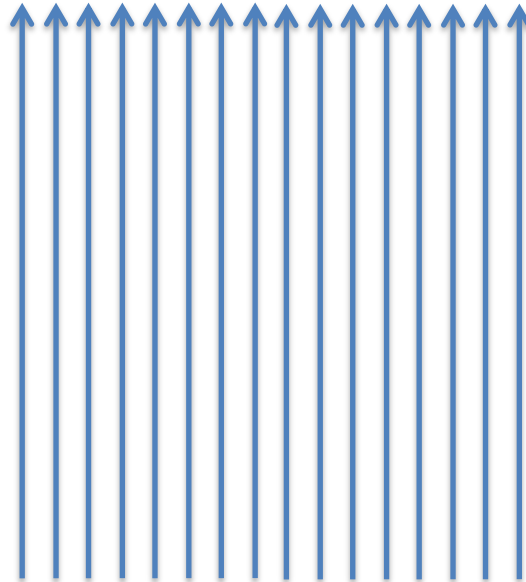
Scalar Polarizabilities

Response of internal structure to an applied EM field

Interaction of the EM field with the internal structure of the nucleon



\vec{E}

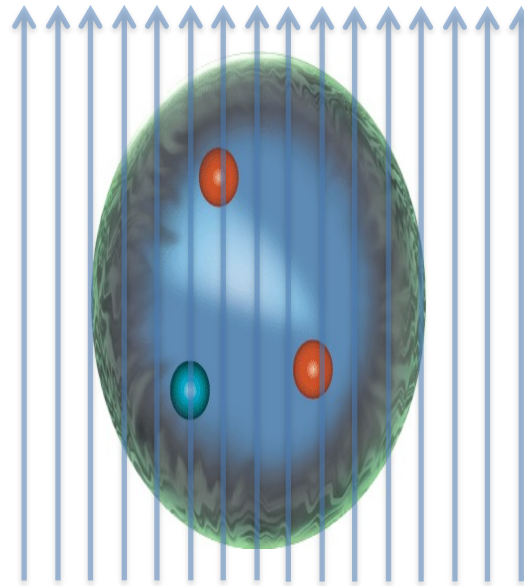
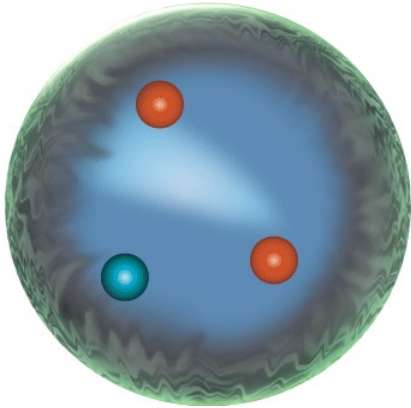


\vec{B}

Scalar Polarizabilities

Response of internal structure to an applied EM field

Interaction of the EM field with the internal structure of the nucleon

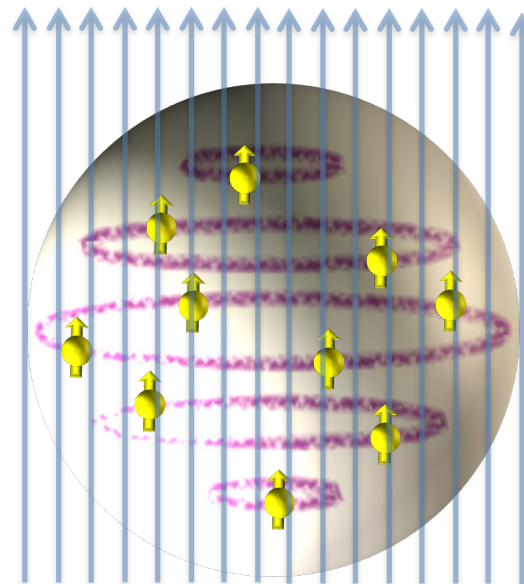


\vec{E}

“stretchability”

$$\vec{d}_{E \text{ induced}} \sim \alpha \vec{E}$$

External field deforms the charge distribution



\vec{B}

“alignability”

$$\vec{d}_{M \text{ induced}} \sim \beta \vec{B}$$

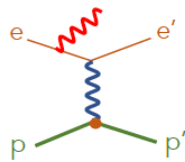
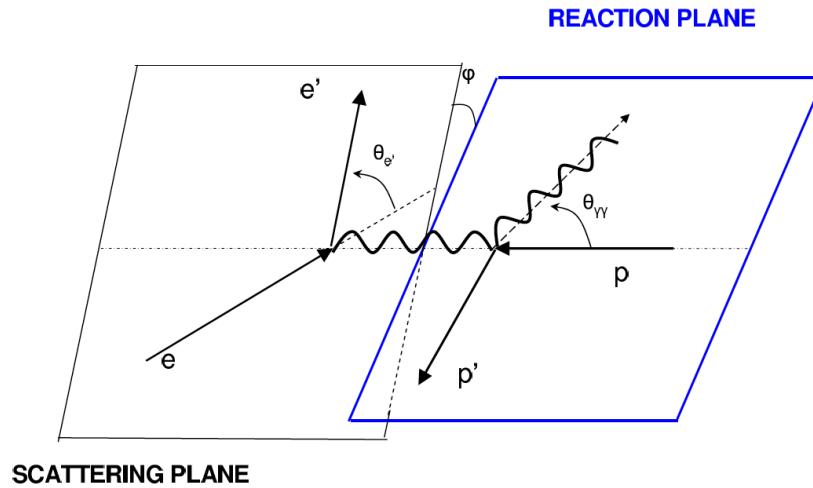
$$\beta_{\text{para}} > 0$$

$$\beta_{\text{diam}} < 0$$

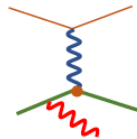
Paramagnetic: proton spin aligns with the external magnetic field

Diamagnetic: π -cloud induction produces field counter to the external perturbation

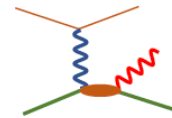
Virtual Compton Scattering



Bethe-Heitler



Born VCS



non-Born VCS

Elastic FFs

GPs

Virtual Compton Scattering

DR

valid below & above
Pion threshold

Dispersive integrals
for Non Born amplitudes

Spin GPs are fixed

Scalar GPs have
an unconstrained part

Fit to the experimental
cross sections at each Q^2

LEX

valid only below
Pion threshold

Response functions

$$d^5\sigma = d^5\sigma^{BH+Born} + q'_{cm} \cdot \phi \cdot \Psi_0 + \mathcal{O}(q'^2_{cm})$$

$$\Psi_0 = v_1 \cdot \left(P_{LL} - \frac{1}{\epsilon} P_{TT} \right) + v_2 \cdot P_{LT}$$

Subtract the spin part

$$P_{TT} = [P_{TT} \text{ spin}]$$

$$P_{LT} = -\frac{2M}{\alpha_{em}} \sqrt{\frac{q'^2_{cm}}{Q^2}} \cdot G_E^p(Q^2) \cdot \beta_M(Q^2) + [P_{LT} \text{ spin}]$$

utilize DR

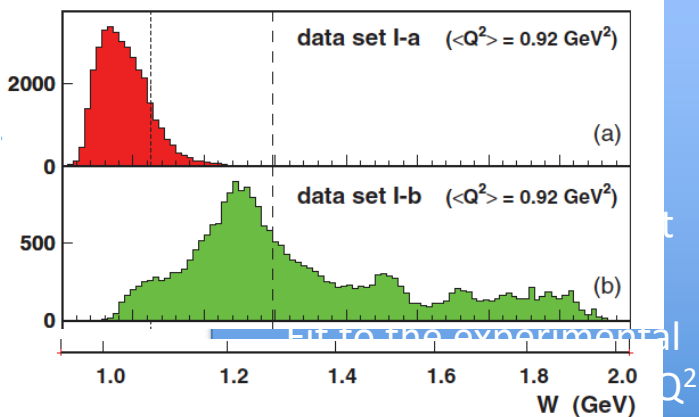
scalar GPs α_E and β_M

Virtual Compton Scattering

DR

valid below & above
Pion threshold

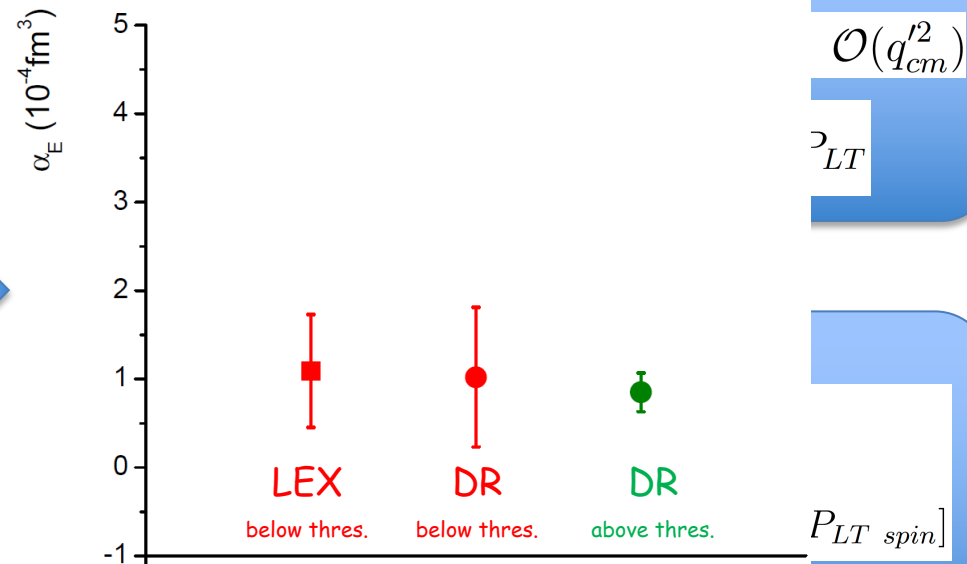
Dispersive integrals
for Non Born amplitudes



LEX

valid only below
Pion threshold

Response functions

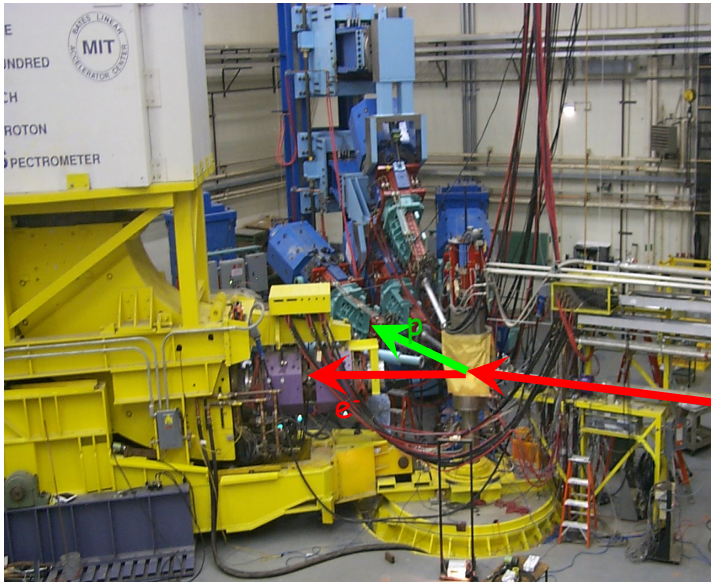


utilize DR

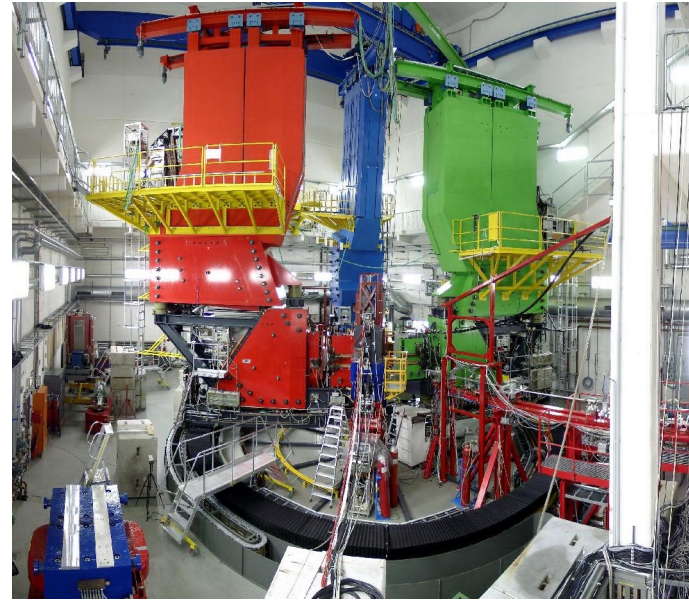
scalar GPs α_E and β_M

Early Experiments

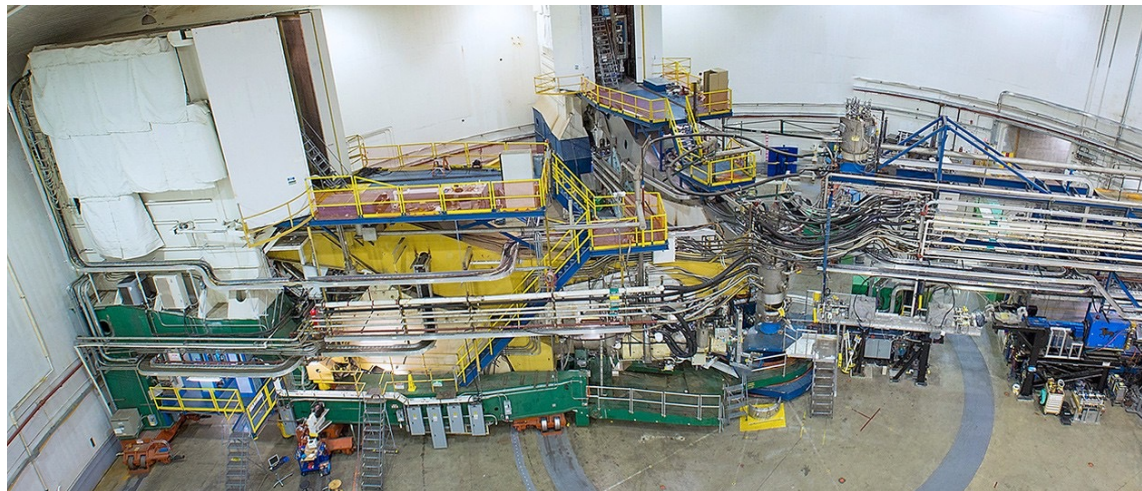
MIT-Bates @ $Q^2=0.06 \text{ GeV}^2$



MAMI-A1 @ $Q^2=0.33 \text{ GeV}^2$



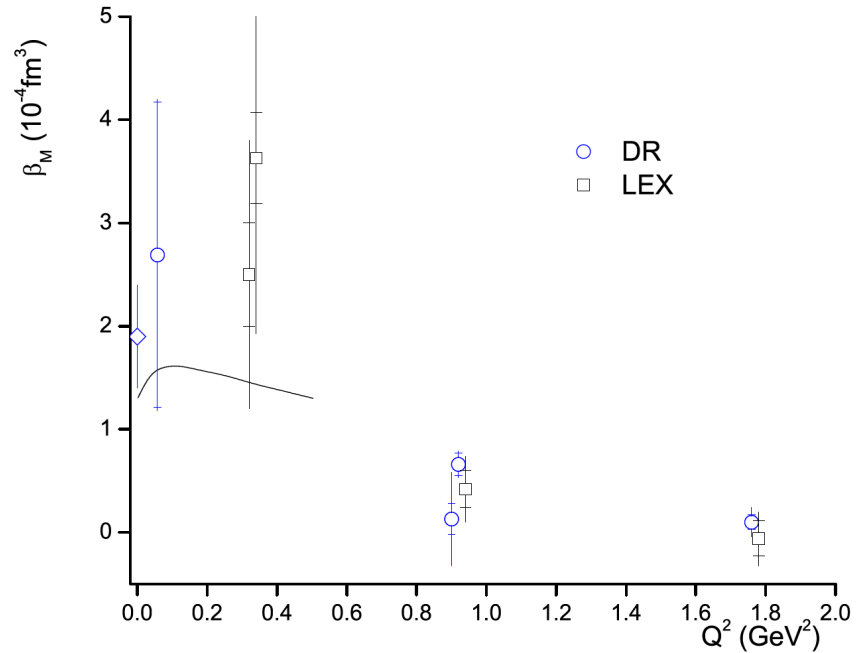
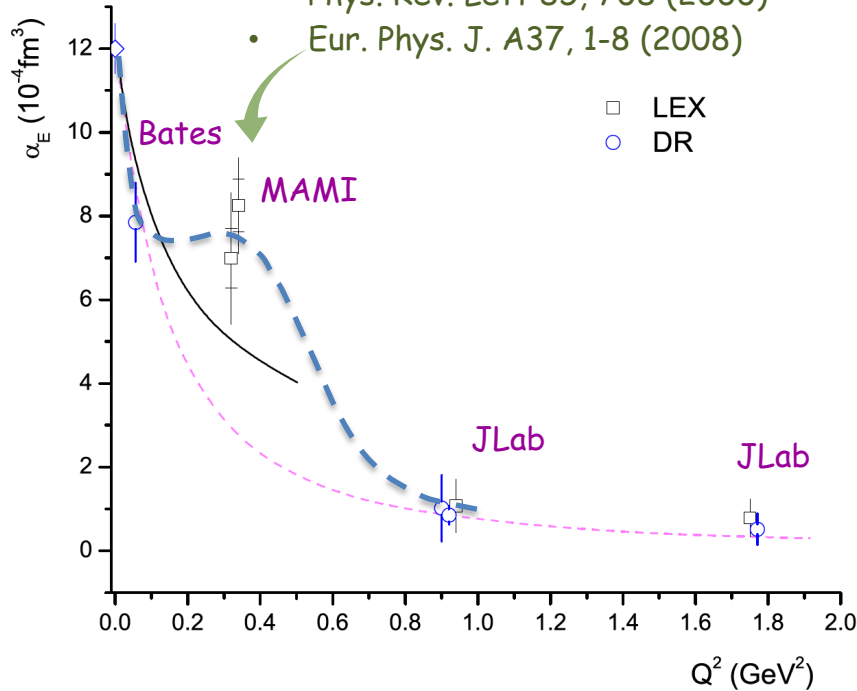
Jlab-Hall A @ $Q^2=0.9 \text{ \& } 1.8 \text{ GeV}^2$



Early Experiments

$Q^2 = 0.33 \text{ (GeV/c)}^2$ measured twice at MAMI:

- Phys. Rev. Lett 85, 708 (2000)
- Eur. Phys. J. A37, 1-8 (2008)



$a_E \approx 10^{-3} V_N$ (stiffness / relativistic character)

Data: non-trivial Q^2 dependence of a_E (?)

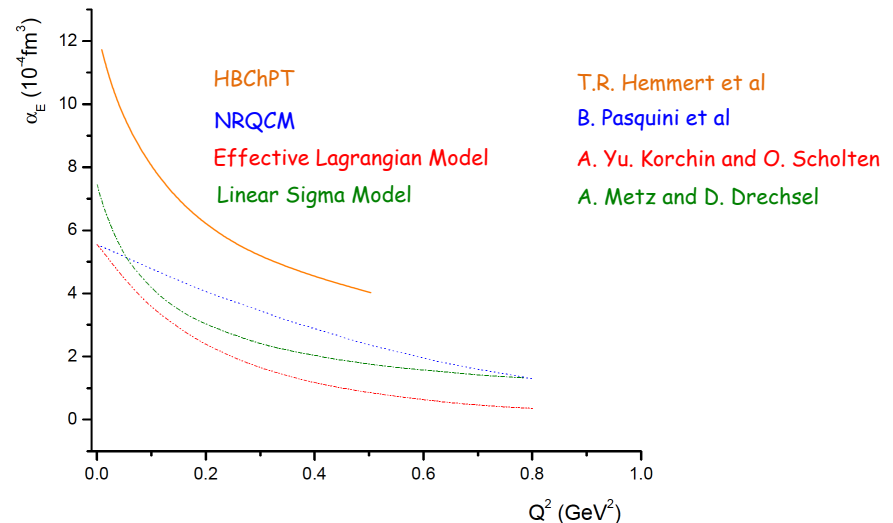
Theory: monotonic fall-off

β_M small \leftrightarrow cancellation of competing mechanisms

Large uncertainties

Higher precision measurements needed

\rightarrow Quantify balance between dia/para-magnetism



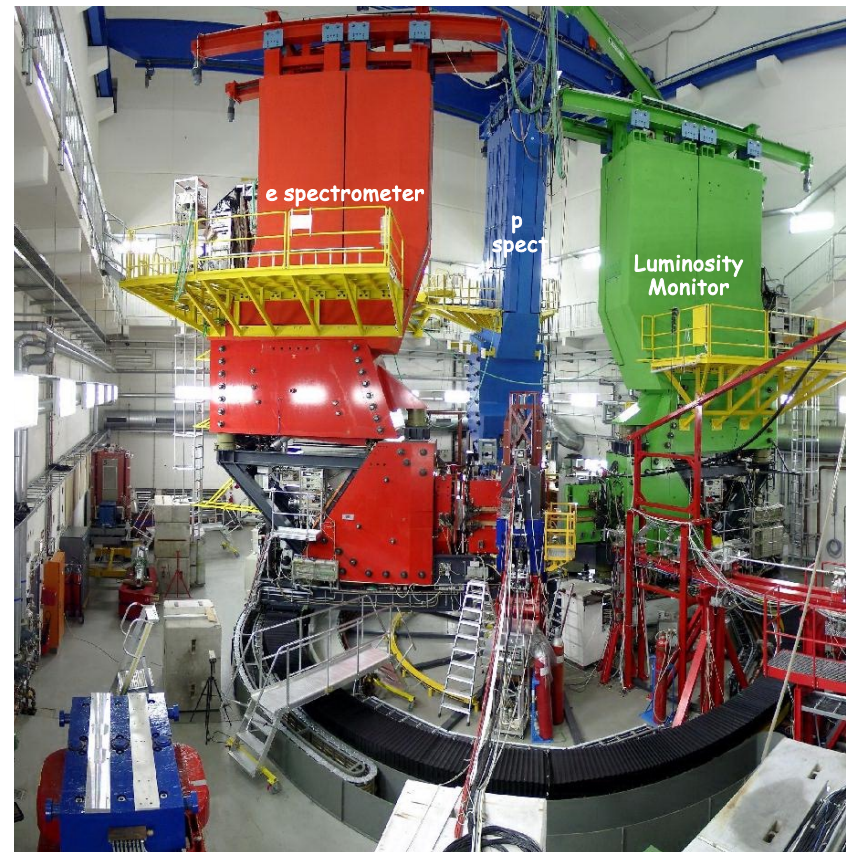
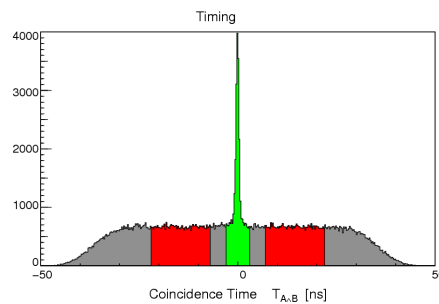
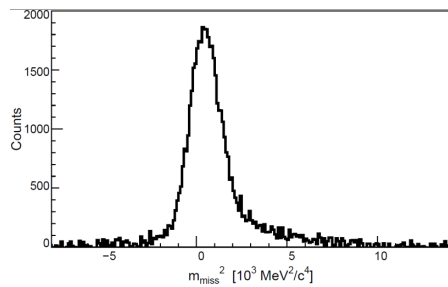
Recent Experiments

Recent Measurements: MAMI

MAMI A1/1-09 (vcsq2) below threshold

MAMI A1/3-12 (vcsdelta) above threshold

Both experiments utilized the A1 setup at MAMI



A1/1-09 @ MAMI

For LEX the higher order terms have to be kept small / under control

$$d^5\sigma = d^5\sigma^{BH+Born} + q'_{cm} \cdot \phi \cdot \Psi_0 + \mathcal{O}(q'^2_{cm})$$

Refined analysis procedure / phase space masking to keep these terms smaller than $\sim 2\%$ - 3% level

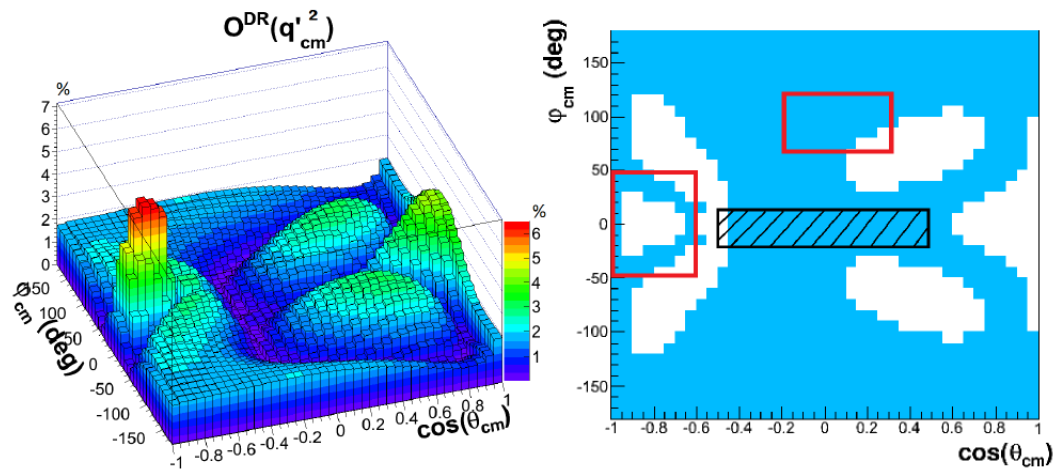
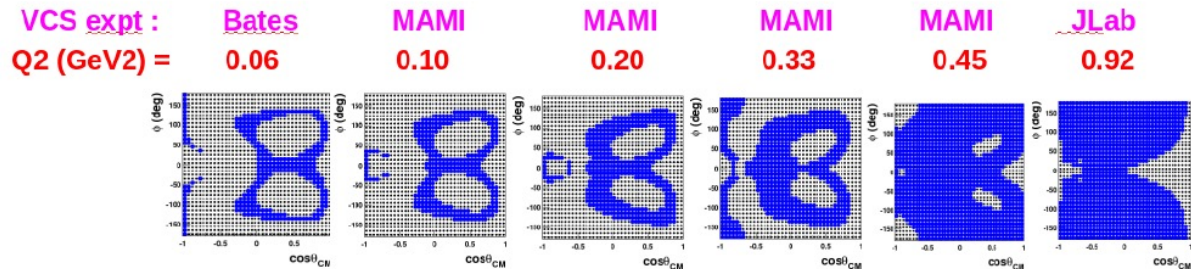
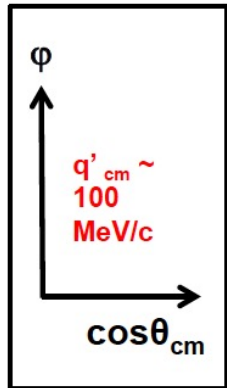


Figure 3.13: (Left) behavior of $\mathcal{O}^{DR}(q'^2_{cm})$ in the $(\cos(\theta_{cm}), \varphi_{cm})$ -plane at $q'_{cm} = 87.5 \text{ MeV}/c$ and (right) two-dimensional representation of the angular region where $\mathcal{O}^{DR}(q'^2_{cm}) < 2\%$ (blue), the red squares correspond to the two areas of interest to perform the GP extraction.

Figure from PhD thesis of L. Correa, Mainz / Cl. Ferrand

Blue bins = where the higher-order estimator is $< 3\%$
(LEX truncation « valid »)



New « vcsq2 » data:

- OOP kinematics (to access the blue region)
- LEX Fit done with bin selection at $Q^2 = 0.1$ and 0.2 GeV^2 .
- was found not necessary at $Q^2 = 0.45 \text{ GeV}^2$.



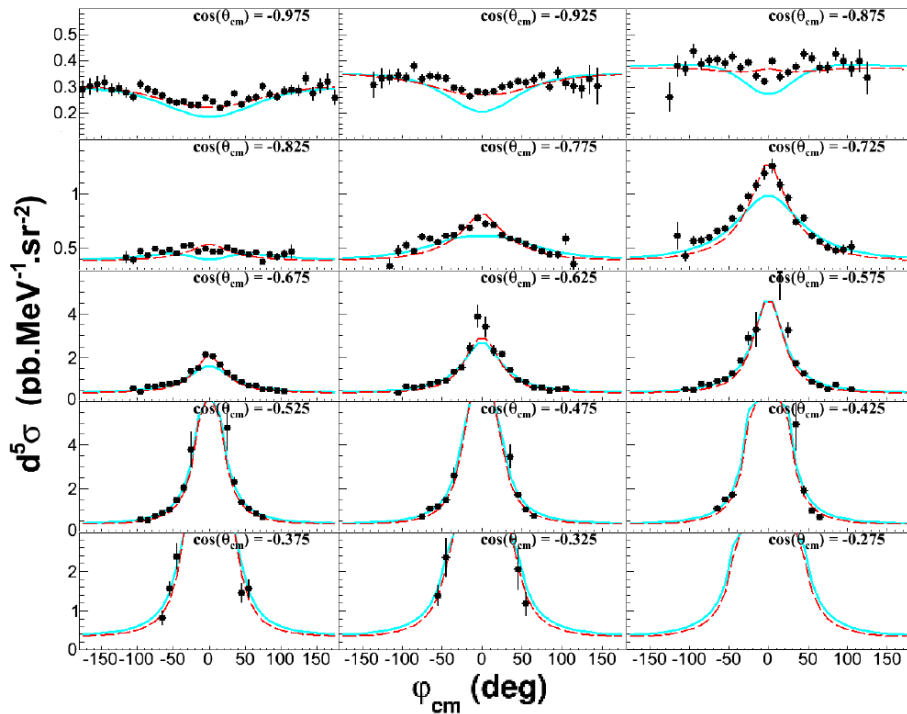
In-plane



8.5 deg OOP

~ 1.0 GeV beam

$Q^2 = 0.1 (GeV/c)^2, 0.2 (GeV/c)^2, \text{ and } 0.45 (GeV/c)^2$



BH+B ---
Polarizability effect ---

GP effect typically 5% - 15% of the cross section

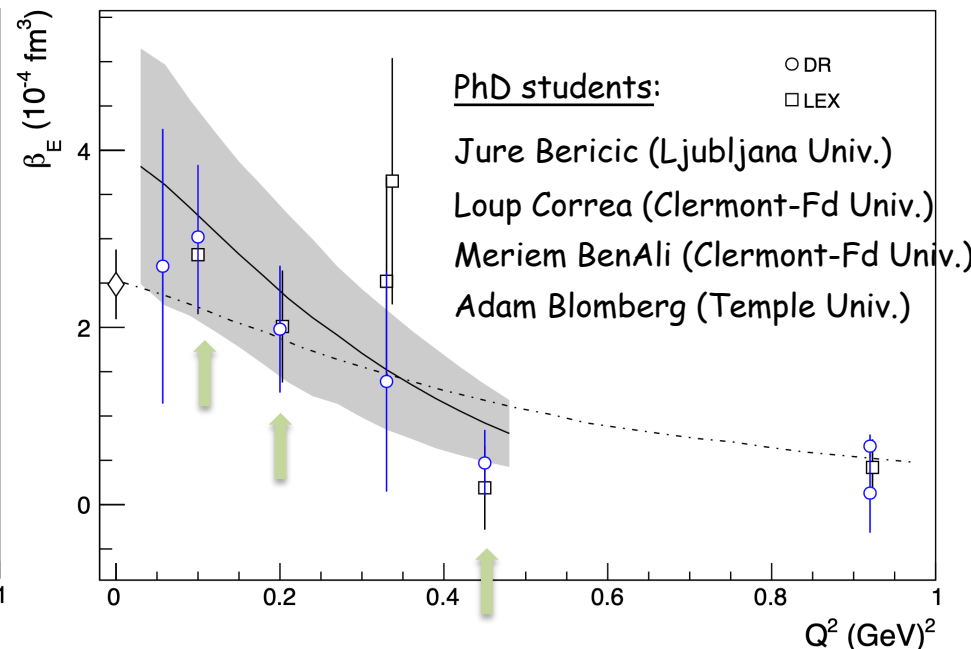
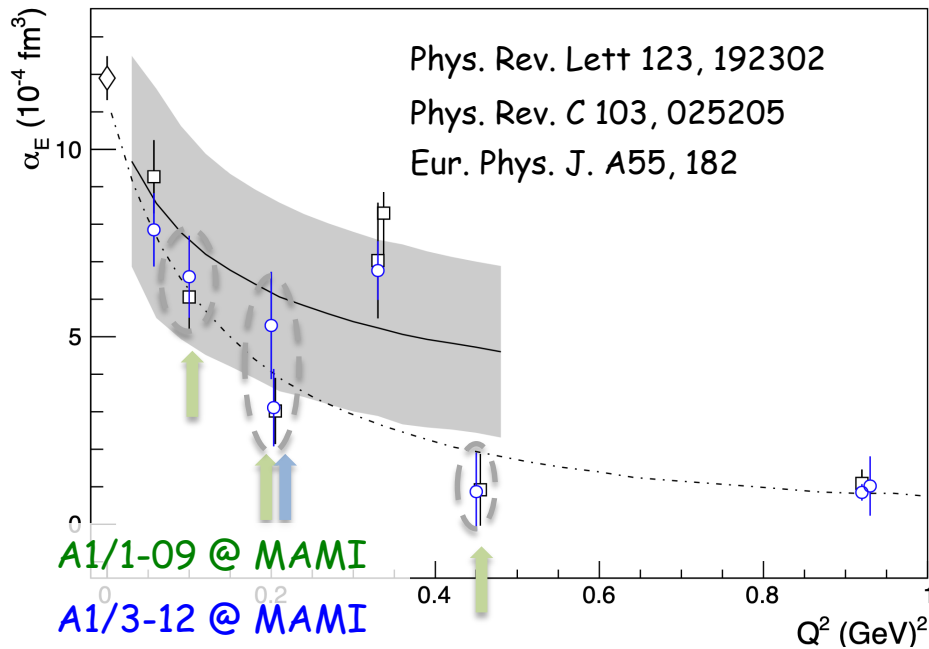
Polarizability fits:

DR fit:
DR calculation includes full dependency in q'_{cm}

LEX fit:
truncated in q'_{cm} . Suppress contribution from higher order terms

Figure 5.8: Setting INP: measured $ep \rightarrow ep\gamma$ cross section at fixed $q'_{cm} = 112.5 MeV/c$ with respect to φ_{cm} for all the $\cos(\theta_{cm})$ -bins. The curves follow the convention of figure 5.6.

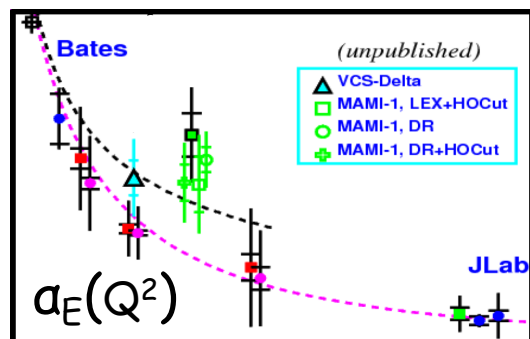
MAMI Results



Revisiting the $Q^2=0.33 \text{ GeV}^2$ data

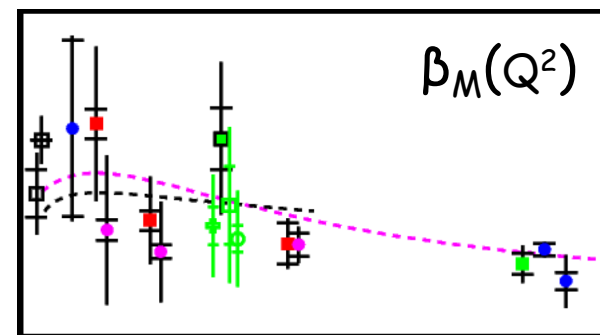
Analysis revisited (unpublished):

The α_E puzzle still holds



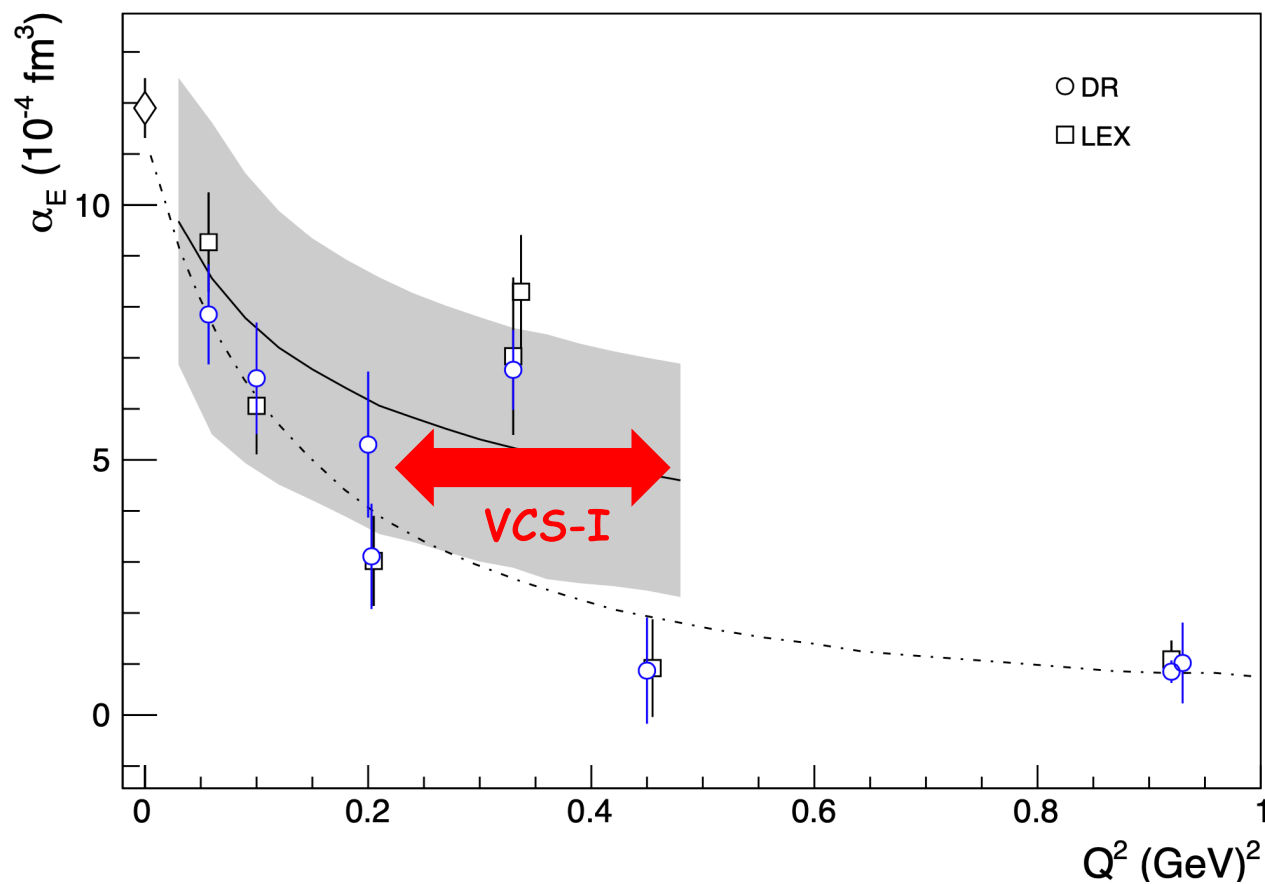
Re-fits at
 $Q^2=0.33$
 GeV^2
 (H.F.)

LEX and DR
 Updated HO-cut

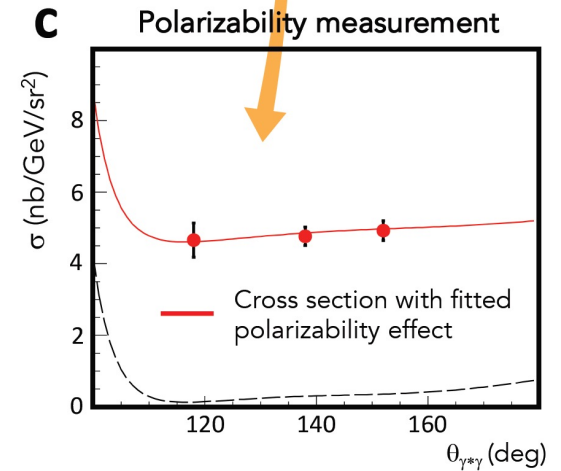
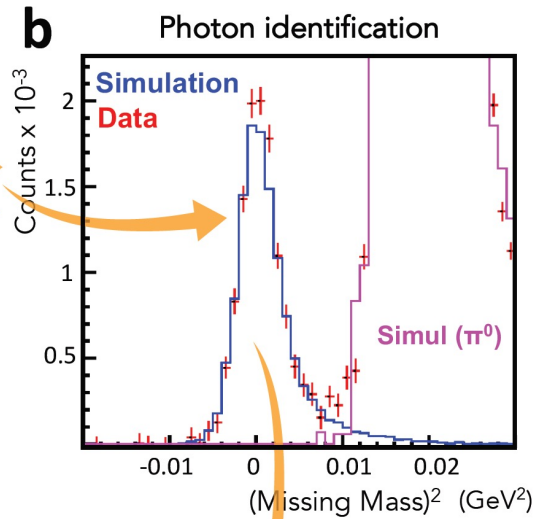
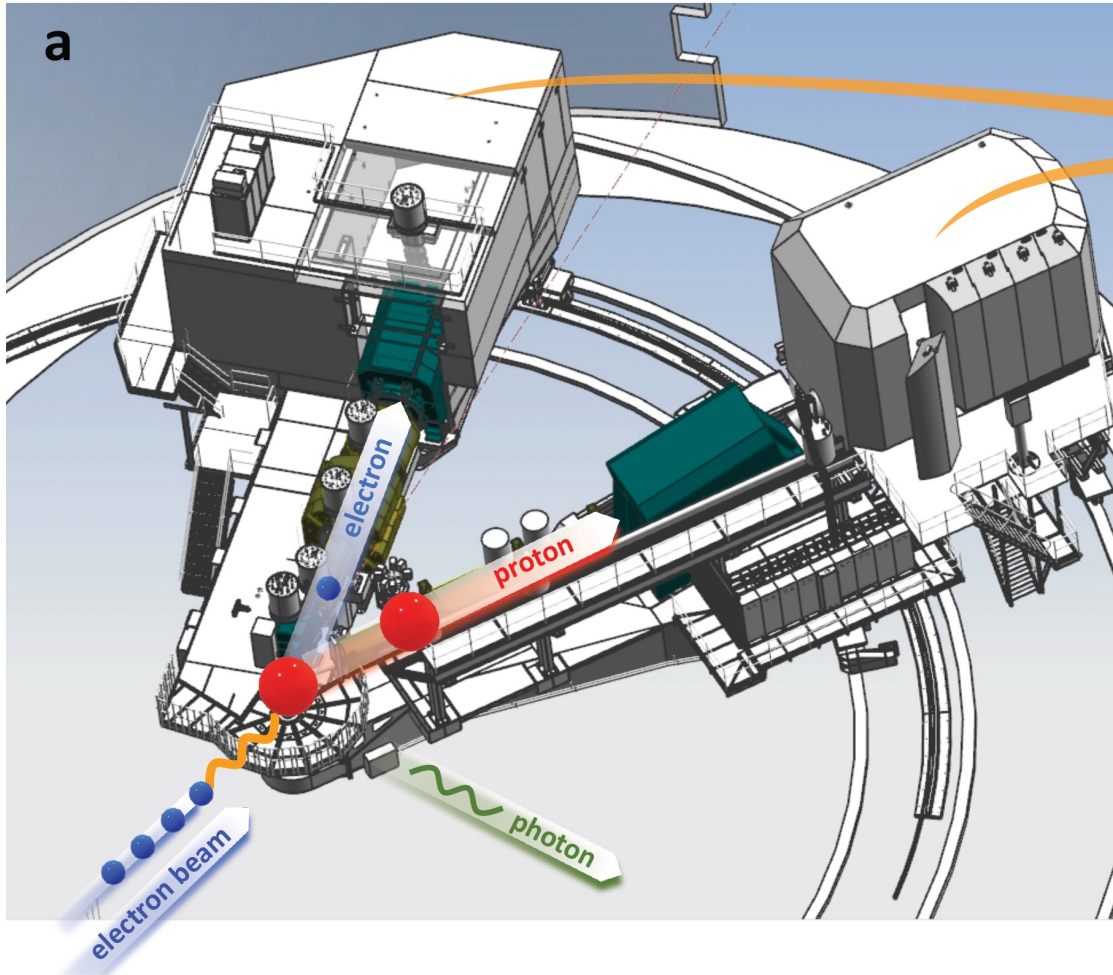


Jlab : VCS-I Experiment (E12-15-001) in Hall C

High precision measurements targeting explicitly the kinematics of interest for a_E



The experiment



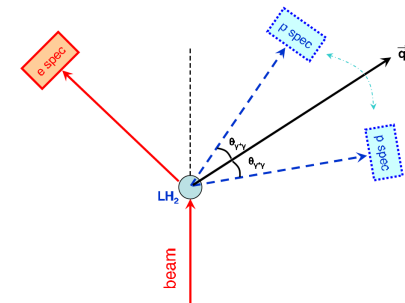
Hall C: SHMS, HMS
 4.56 GeV
 20 μ A
 Liquid hydrogen 10 cm

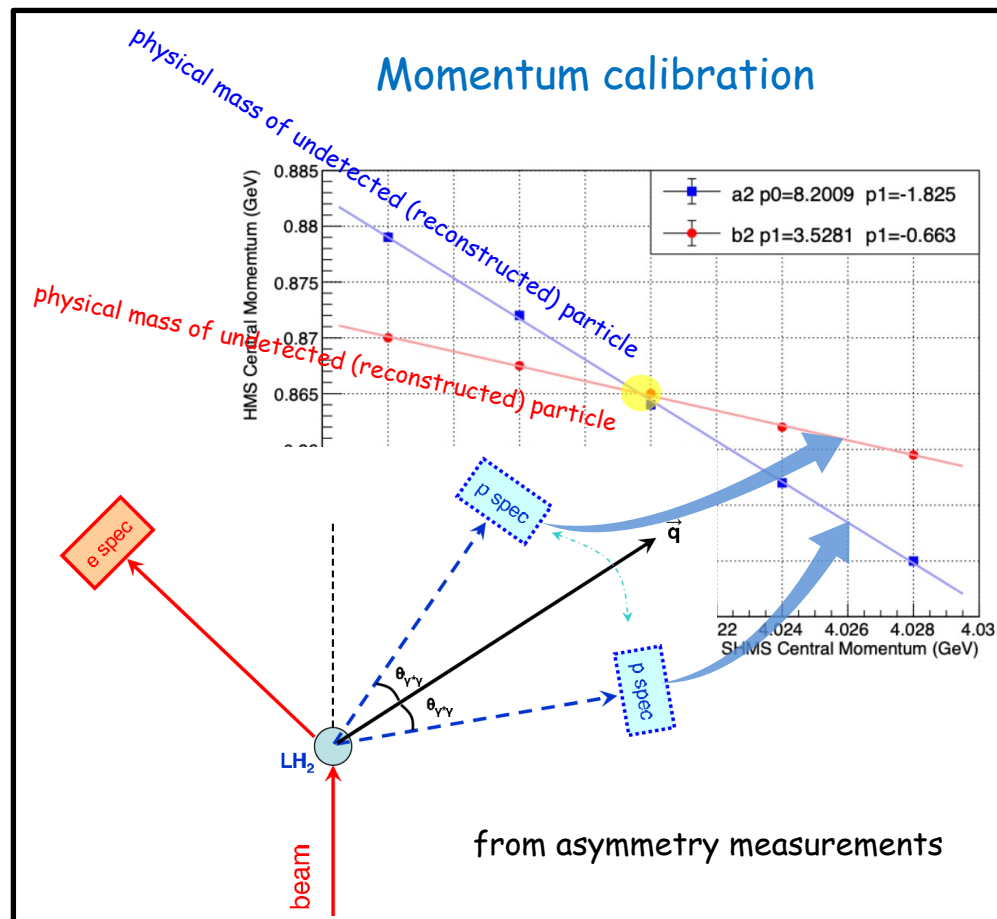
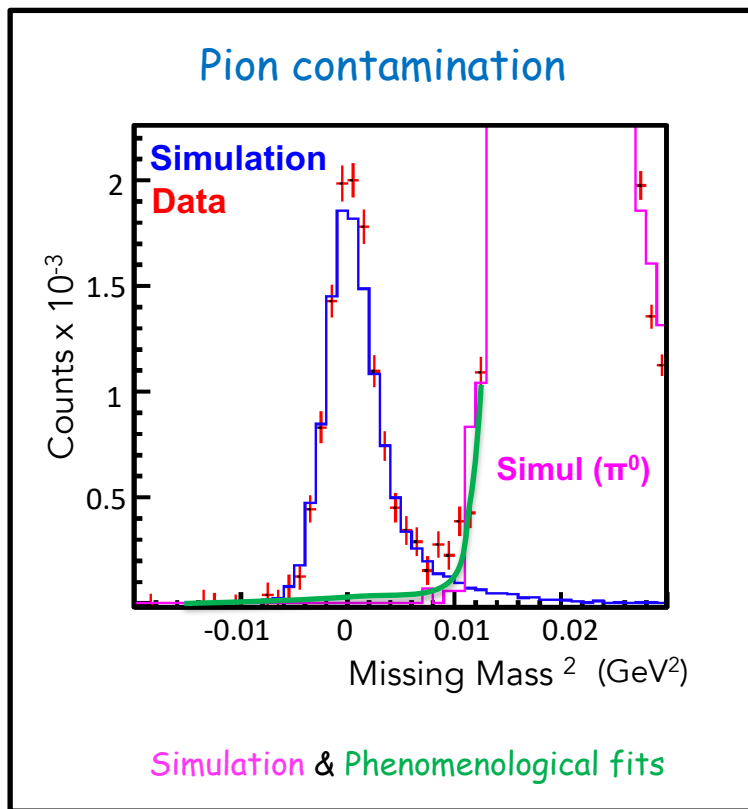
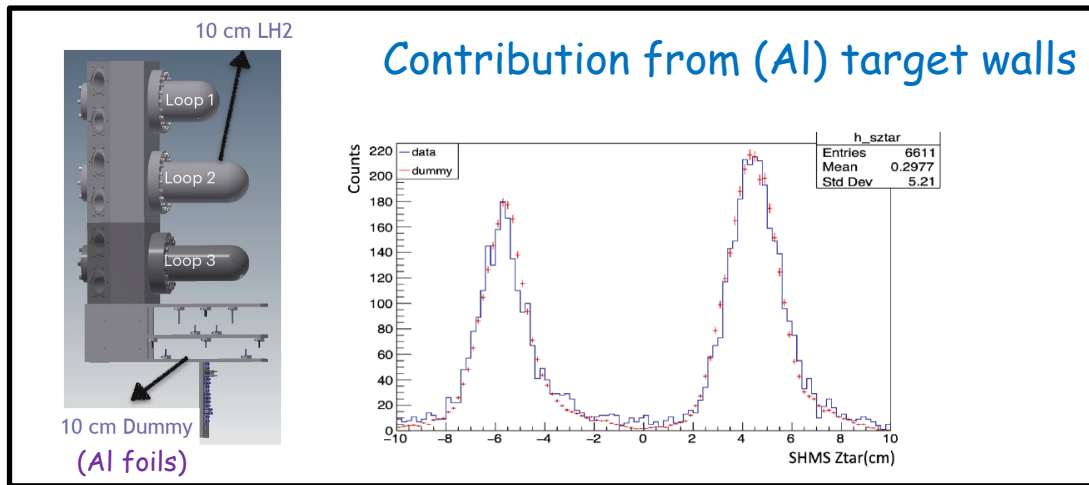
cross sections & azimuthal asymmetries

$$A_{(\phi_{\gamma^*\gamma}=0,\pi)} = \frac{\sigma_{\phi_{\gamma^*\gamma}=0} - \sigma_{\phi_{\gamma^*\gamma}=180}}{\sigma_{\phi_{\gamma^*\gamma}=0} + \sigma_{\phi_{\gamma^*\gamma}=180}}$$

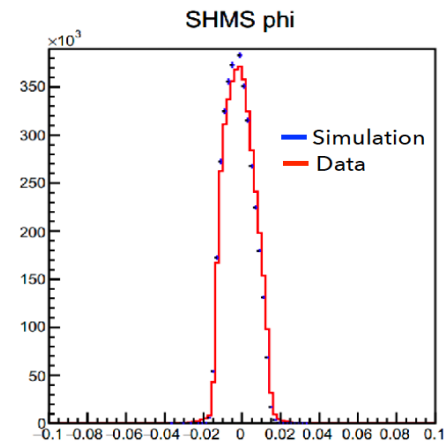
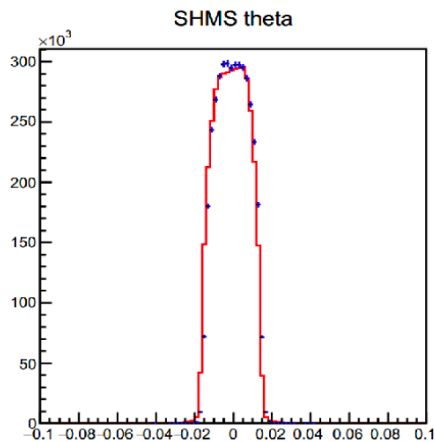
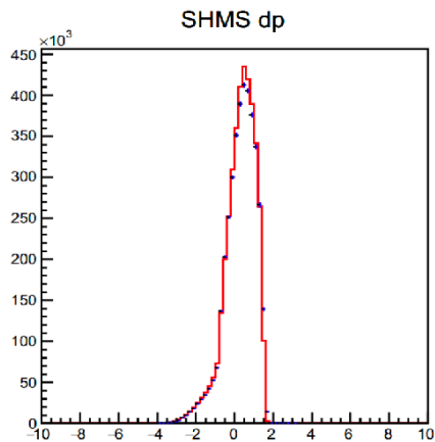
sensitivity to GPs

suppression of systematic asymmetries

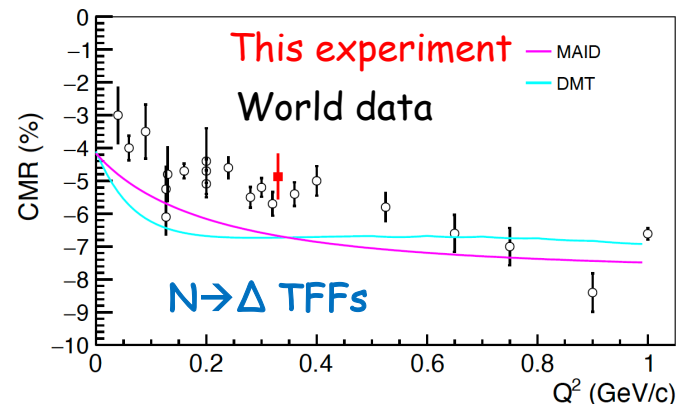
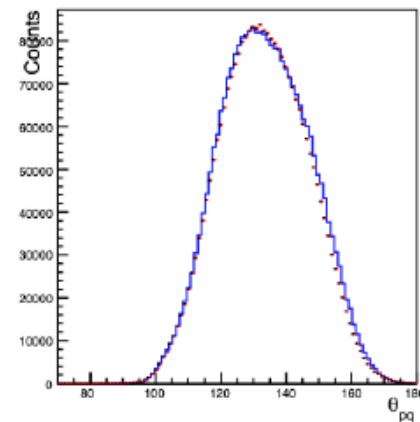
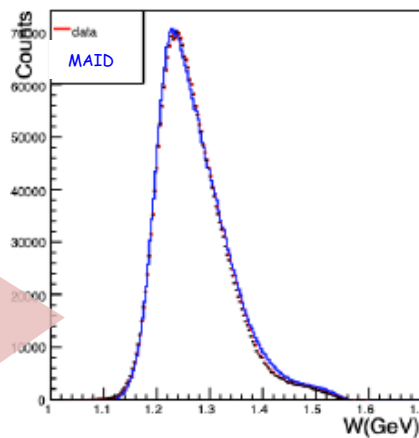
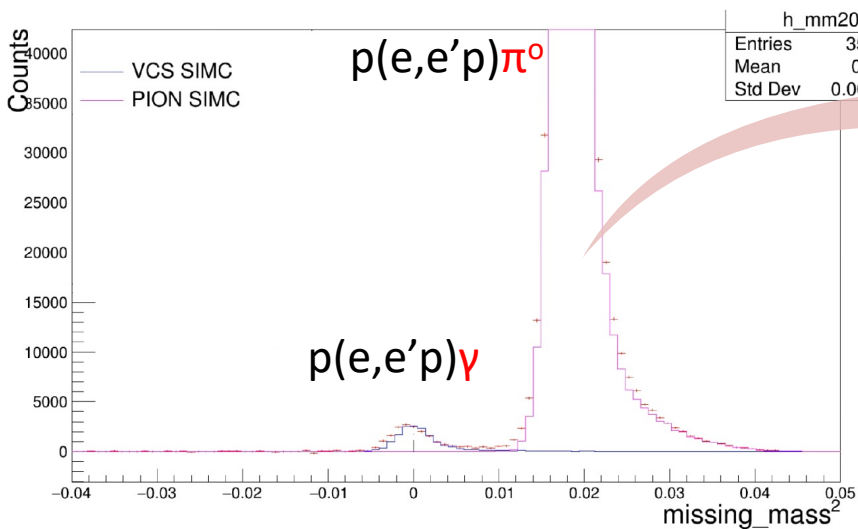




Elastic data

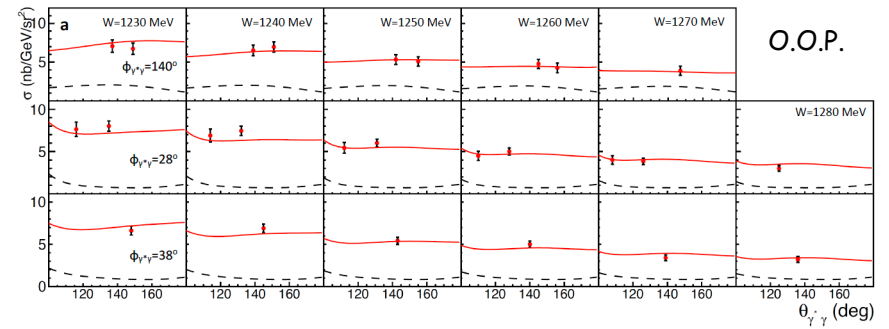
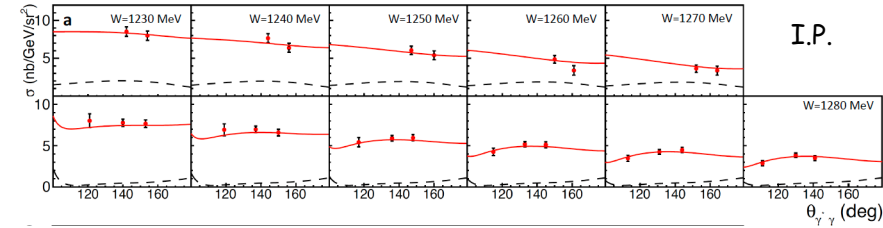


$p(e,e'p)\pi^0$

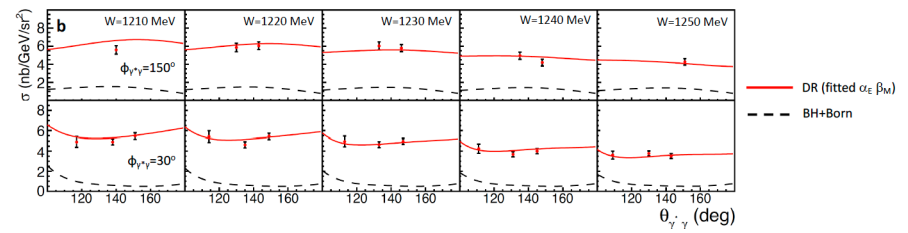
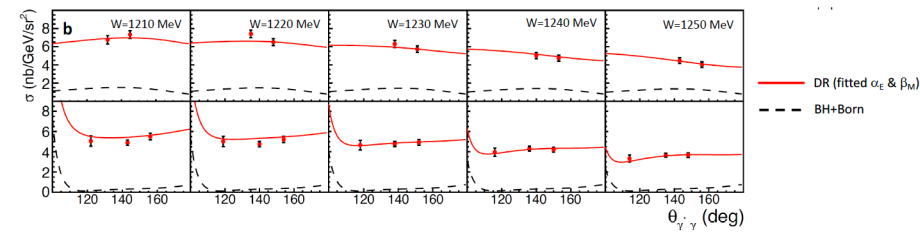


VCS-I results: cross sections

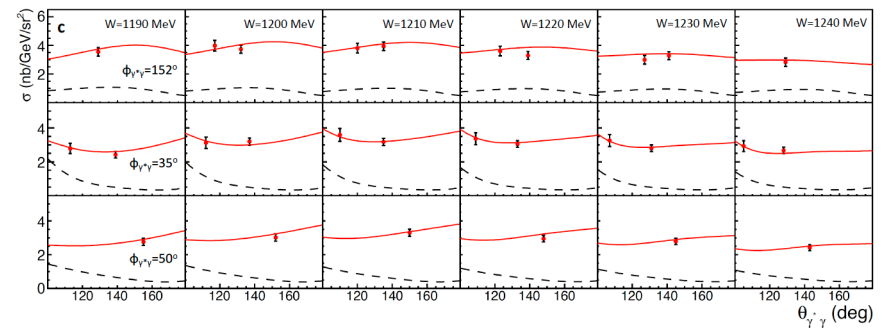
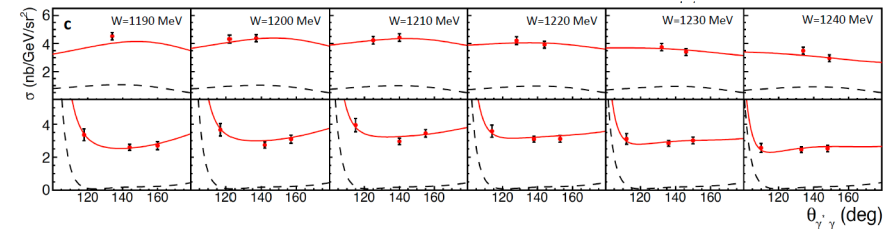
$Q^2=0.27 \text{ GeV}^2$



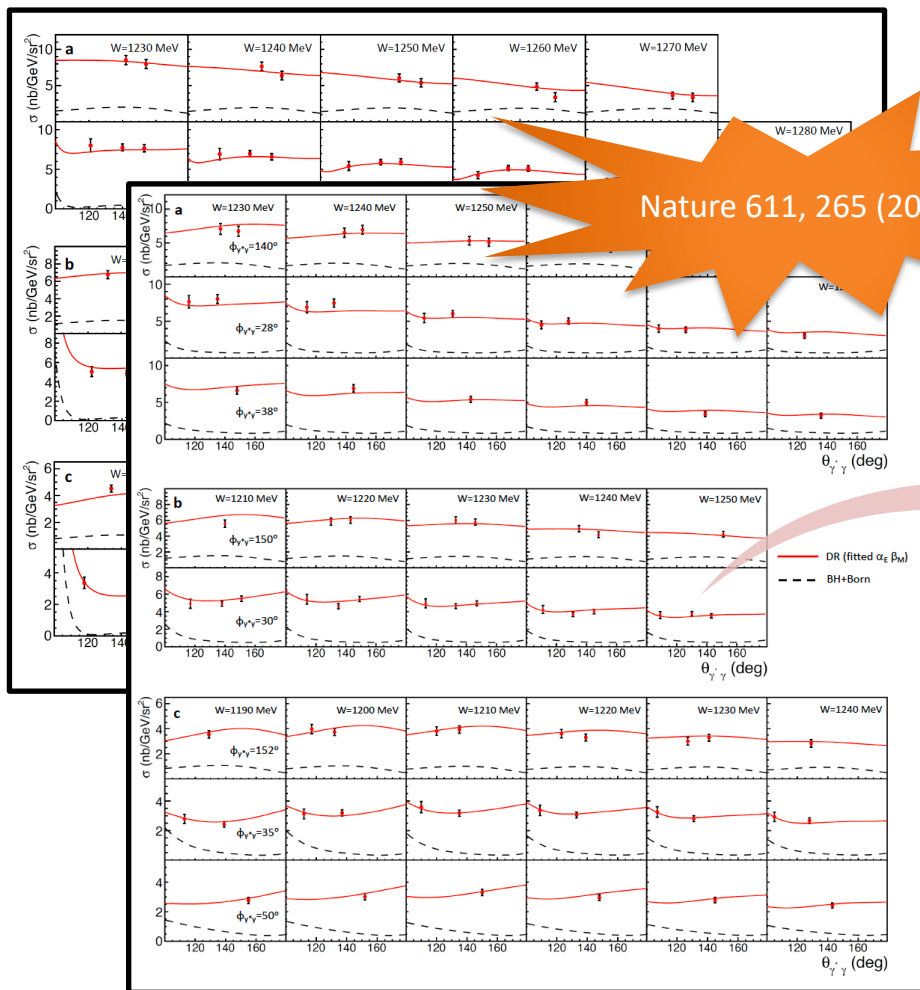
$Q^2=0.33 \text{ GeV}^2$



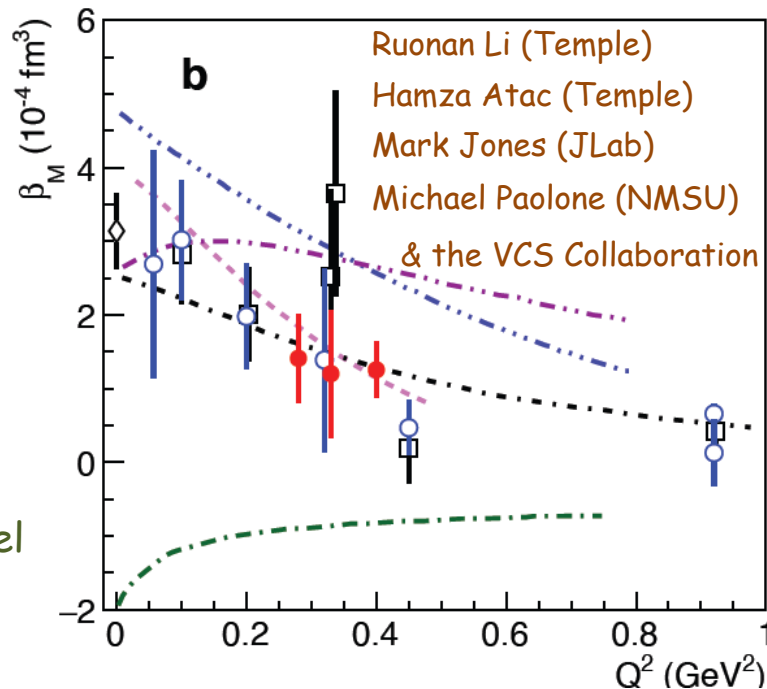
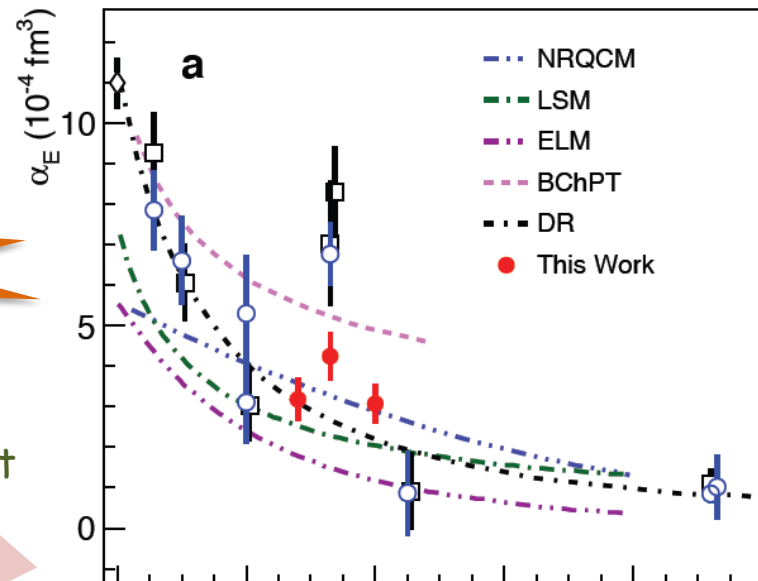
$Q^2=0.40 \text{ GeV}^2$



VCS-I results: GPs



DR fit



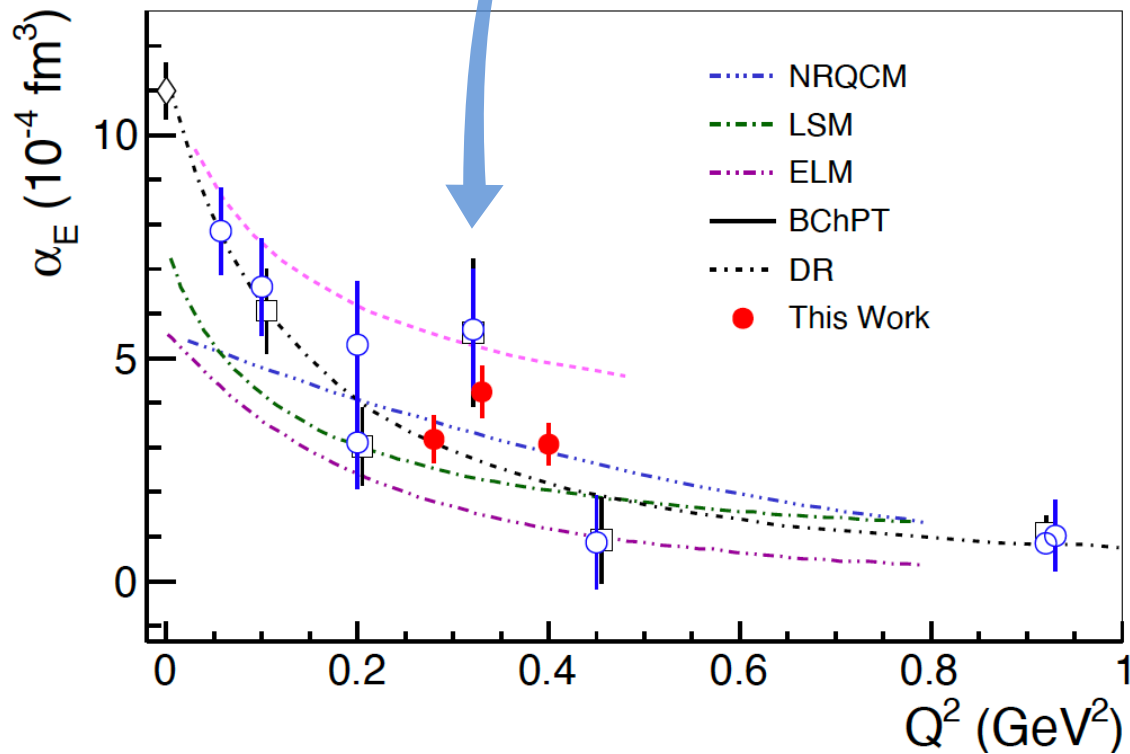
Experimental cross sections are compared to the DR model predictions for all possible values for the GPs

→ $\alpha_E(Q^2)$ and $\beta_M(Q^2)$ are fitted by a χ^2 minimization

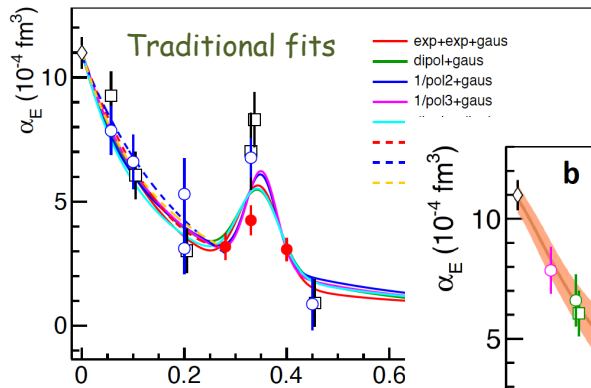
Electric GP (Q^2)

Is there a non-trivial structure?

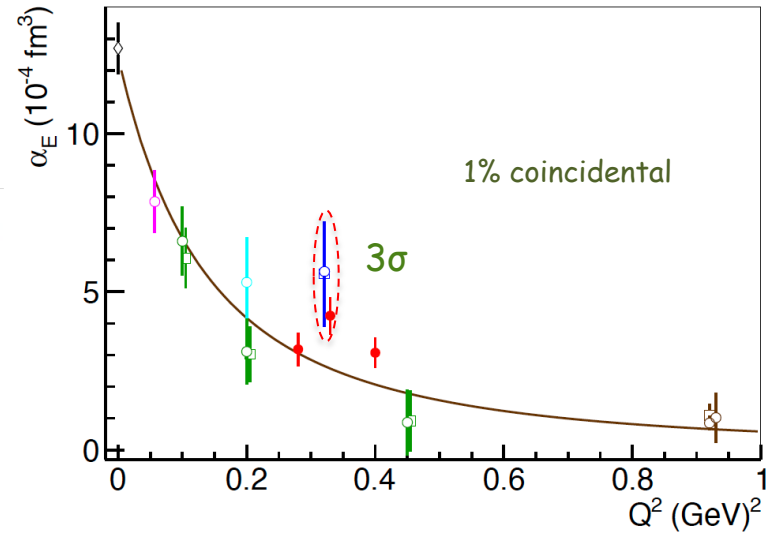
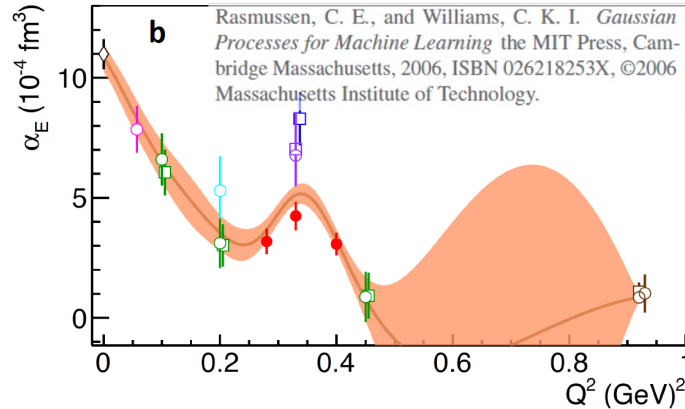
MAMI-I re-analysis (unpublished)



Electric GP



Data-driven techniques:
no underlying functional
form is assumed



Is the observed α_E structure coincidental or not?

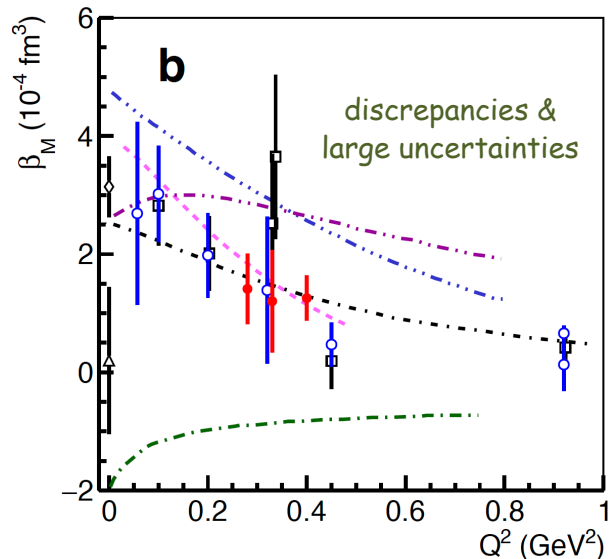
If true: Measure the shape precisely \rightarrow input to theory

If not: We are able to show it with more measurements

Strong tension between world data (?)

Things we do not yet understand well?
Underestimated uncertainties? ...

Magnetic GP



Magnetic GP: Large uncertainties & discrepancies
Needed to disentangle diamagnetism vs
paramagnetism in the proton

Ability to measure α_E and β_M with superb precision
and with consistent systematics across Q^2

Theory: B χ PT

Generalized polarizabilities of the nucleon in baryon chiral perturbation theory

Vadim Lensky^{1,2,3,a}, Vladimir Pascalutsa¹, Marc Vanderhaeghen¹

¹ Institut für Kernphysik, Cluster of Excellence PRISMA, Johannes Gutenberg Universität Mainz, 55128 Mainz, Germany

² Institute for Theoretical and Experimental Physics, Moscow 117218, Russia

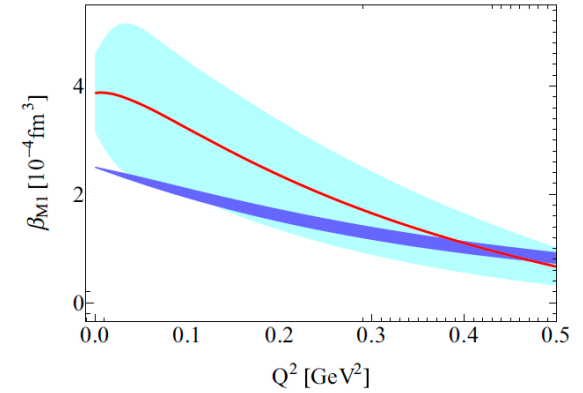
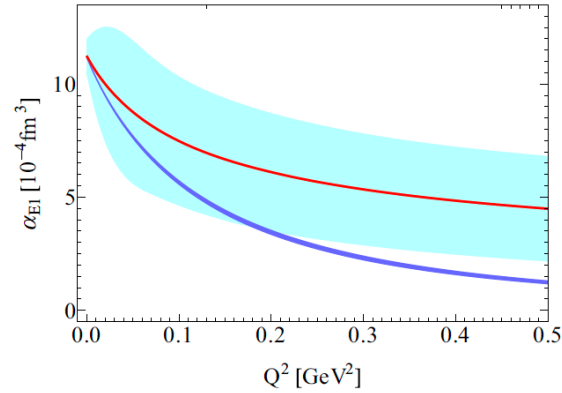
³ National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Moscow 115409, Russia



B χ PT calculation to NLO
in the δ -counting scheme



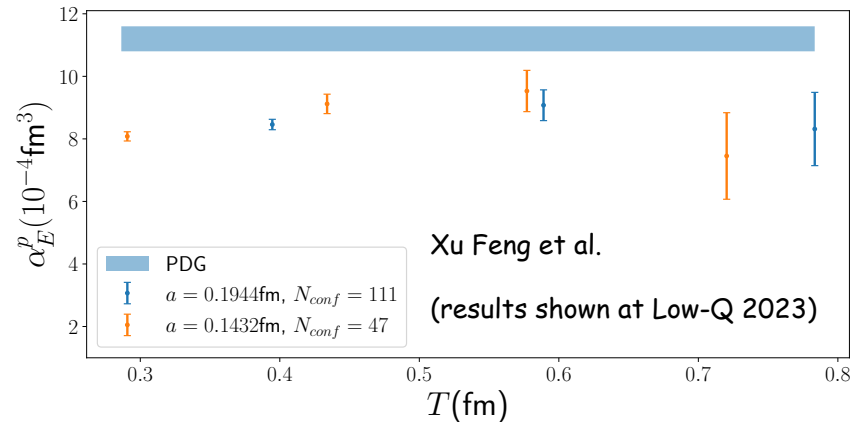
DR calculation
D. Drechsel, B. Pasquini, M. Vanderhaeghen,
Phys. Rep. 378,99 (2003)



Theory: Lattice QCD

Lattice QCD results for
the static polarizabilities

Next step: Lattice QCD
calculations for the GPs



Spatial dependence of induced polarizations

Nucleon form factor data \rightarrow light-front quark charge densities

Formalism extended to the deformation of these quark densities when applying an external e.m. field:

GPs \rightarrow spatial deformation of charge & magnetization densities under an applied e.m. field

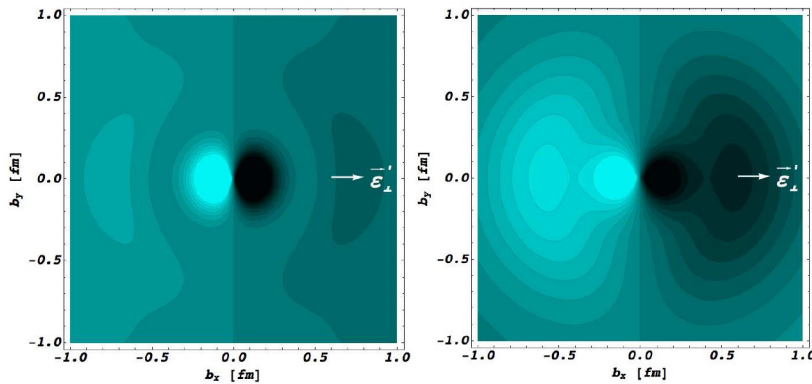
Induced polarization in a proton when submitted to an e.m. field

Phys. Rev. Lett. 104, 112001 (2010)

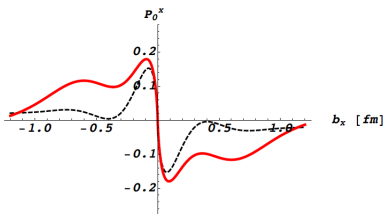
M. Gorchtein, C. Lorce, B. Pasquini, M. Vanderhaeghen

GP I

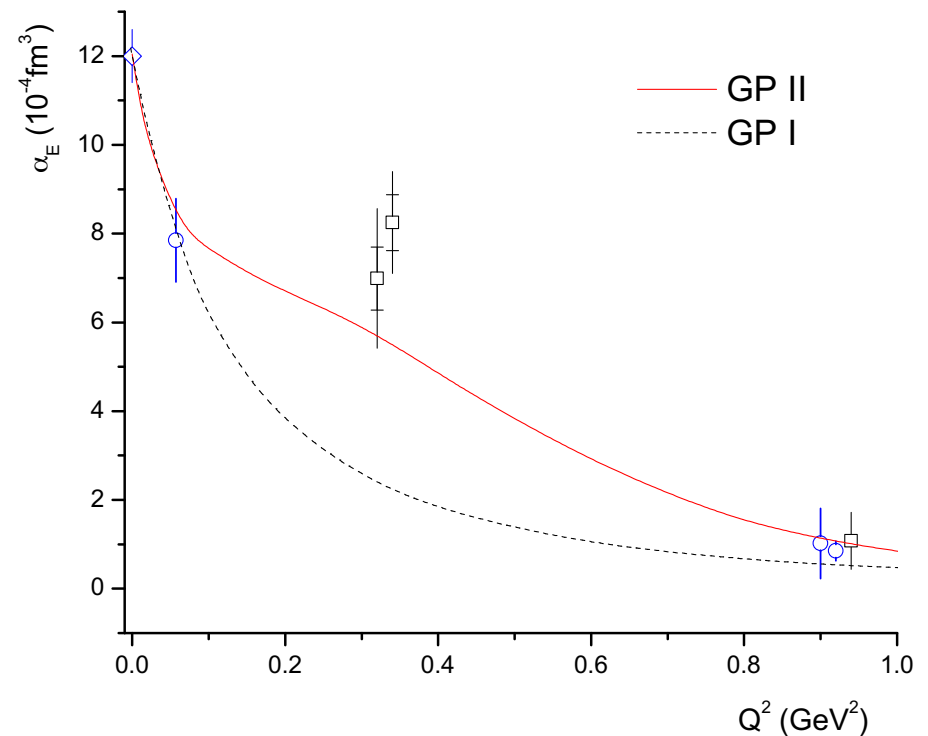
GP II



Light (dark) regions \rightarrow largest (smaller) values
(photon polarization along x-axis, as indicated)



Induced polarization along $b_y=0$

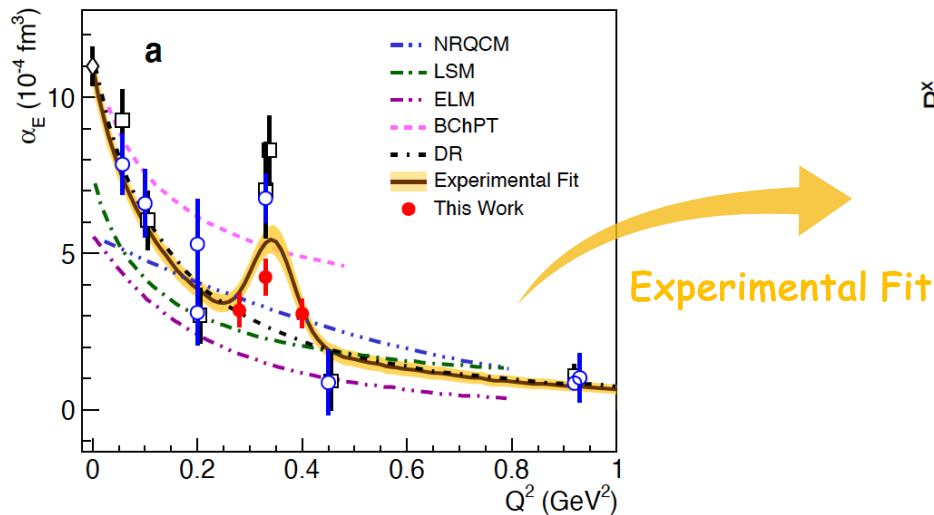


Spatial dependence of induced polarizations

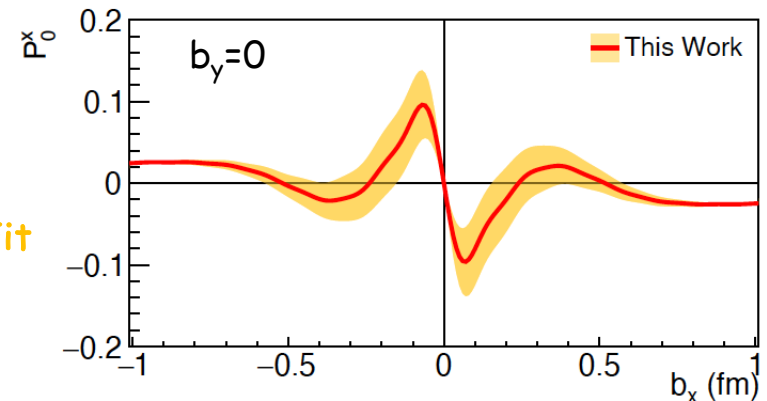
Nucleon form factor data \rightarrow light-front quark charge densities

Formalism extended to the deformation of these quark densities when applying an external e.m. field:

GPs \rightarrow spatial deformation of charge & magnetization densities under an applied e.m. field



Induced polarization in a proton when submitted to an e.m. field

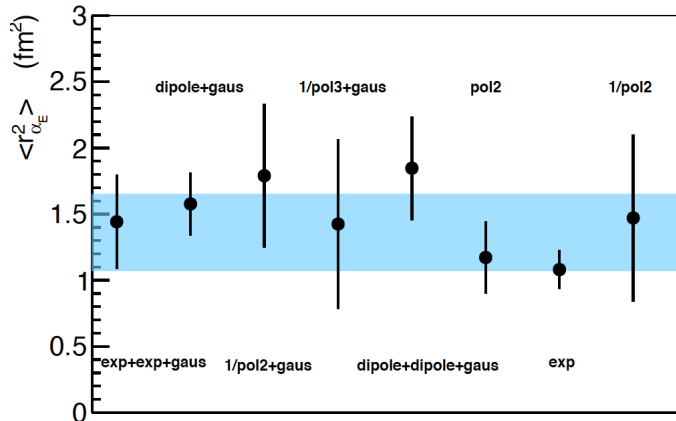
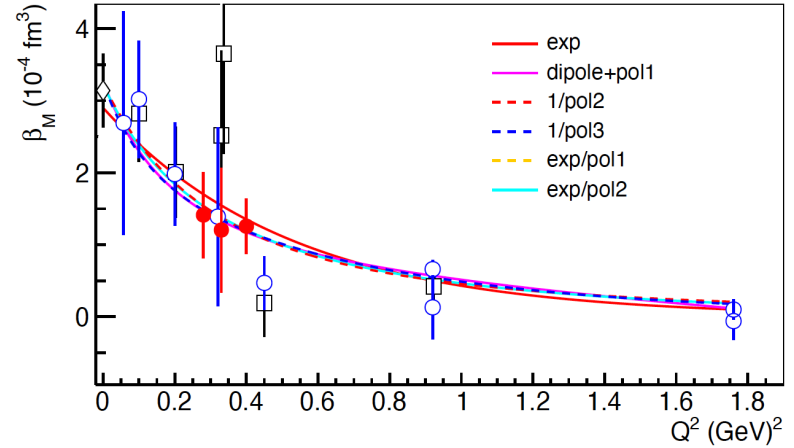
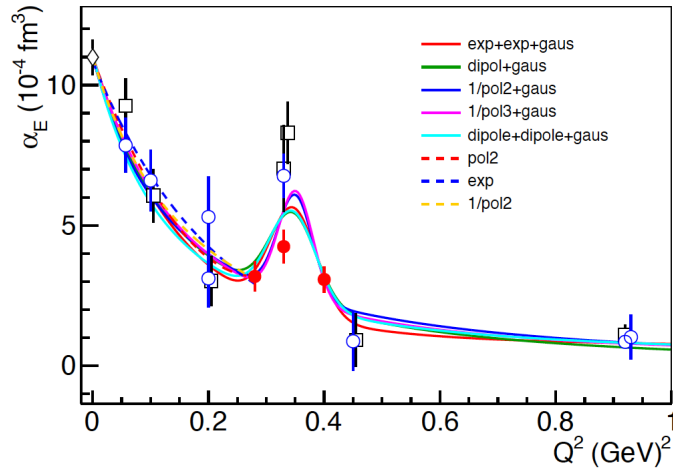


x-y defines the transverse plane with the z-axis being the direction of the fast-moving proton

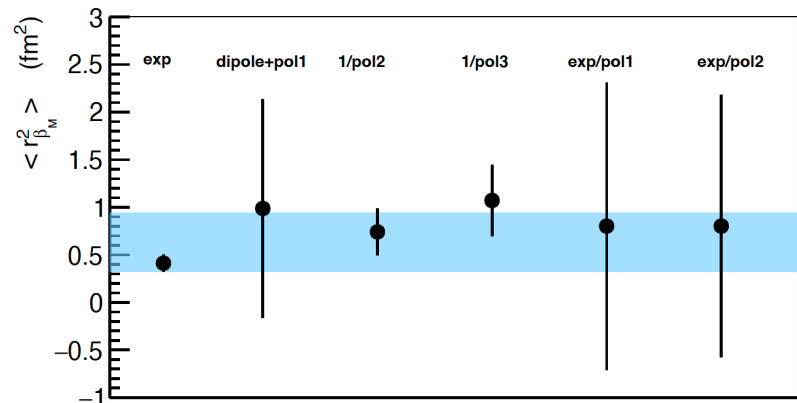
Polarizability radii

$$\langle r_{\alpha_E}^2 \rangle = \frac{-6}{\alpha_E(0)} \cdot \frac{d}{dQ^2} \alpha_E(Q^2) \Big|_{Q^2=0}$$

$$\langle r_{\beta_M}^2 \rangle = \frac{-6}{\beta_M(0)} \cdot \frac{d}{dQ^2} \beta_M(Q^2) \Big|_{Q^2=0}$$



$$\langle r_{\alpha_E}^2 \rangle = 1.36 \pm 0.29 \text{ fm}^2$$

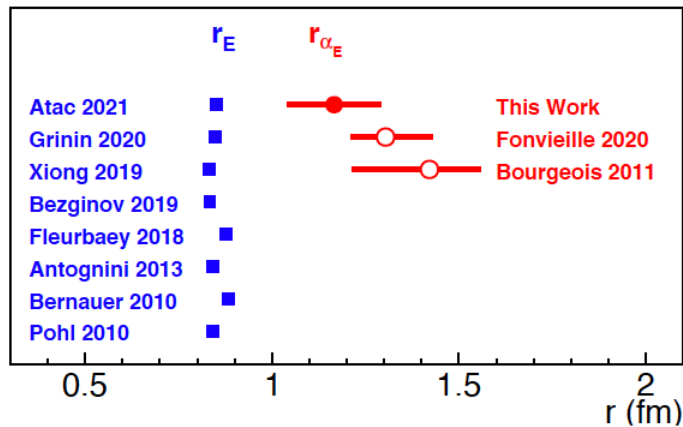
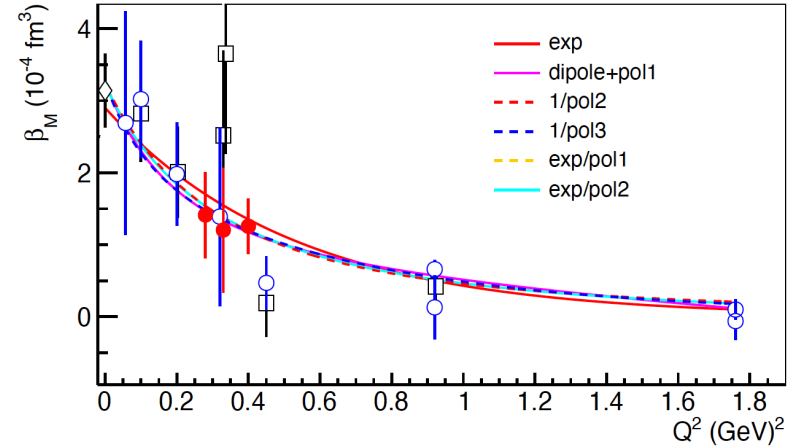
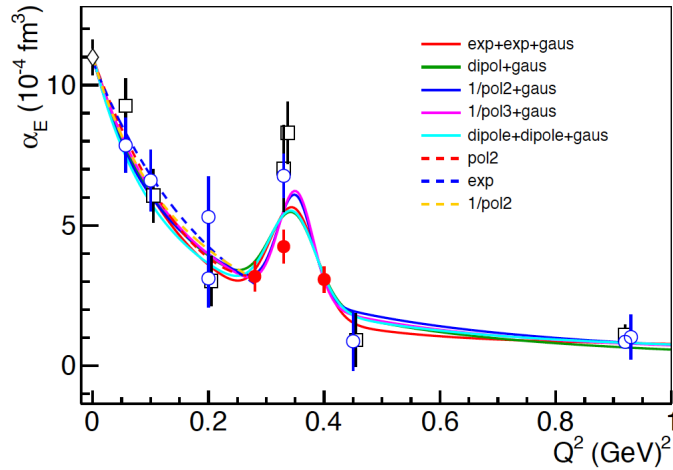


$$\langle r_{\beta_M}^2 \rangle = 0.63 \pm 0.31 \text{ fm}^2$$

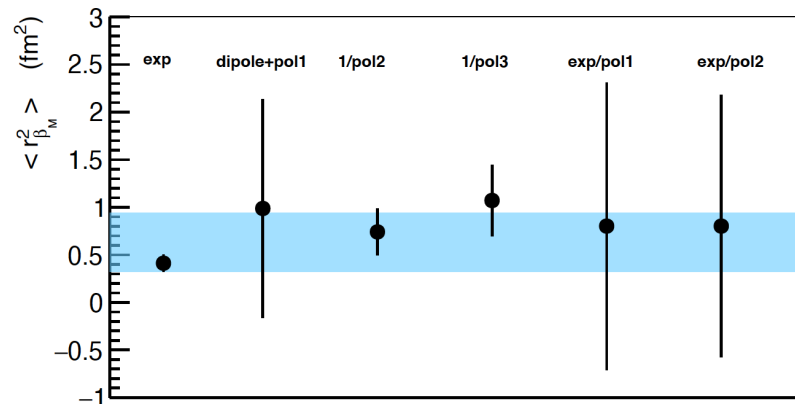
Polarizability radii

$$\langle r_{\alpha_E}^2 \rangle = \frac{-6}{\alpha_E(0)} \cdot \frac{d}{dQ^2} \alpha_E(Q^2) \Big|_{Q^2=0}$$

$$\langle r_{\beta_M}^2 \rangle = \frac{-6}{\beta_M(0)} \cdot \frac{d}{dQ^2} \beta_M(Q^2) \Big|_{Q^2=0}$$

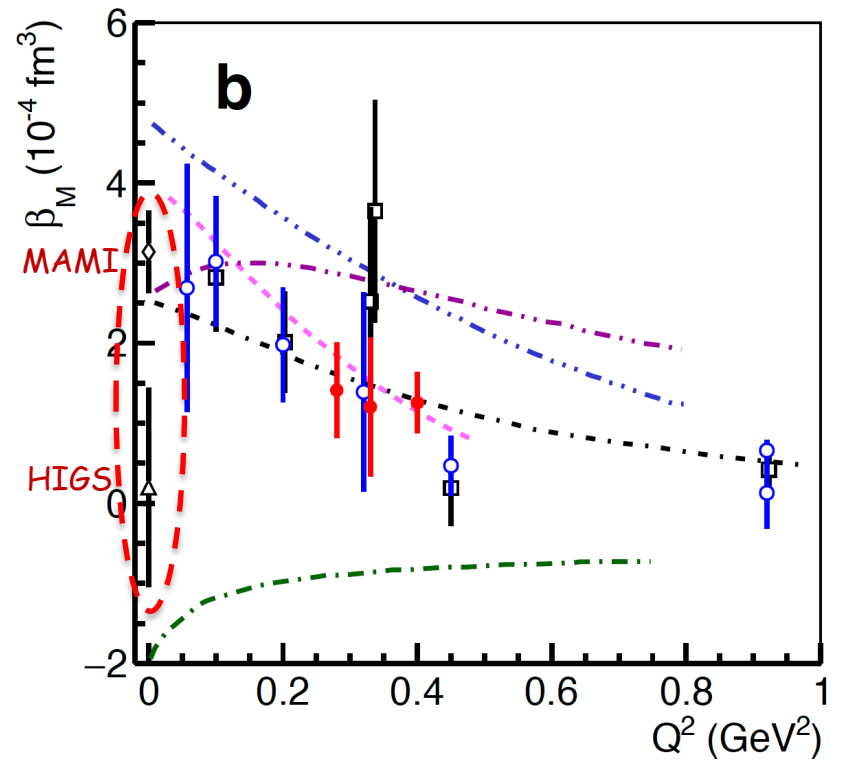
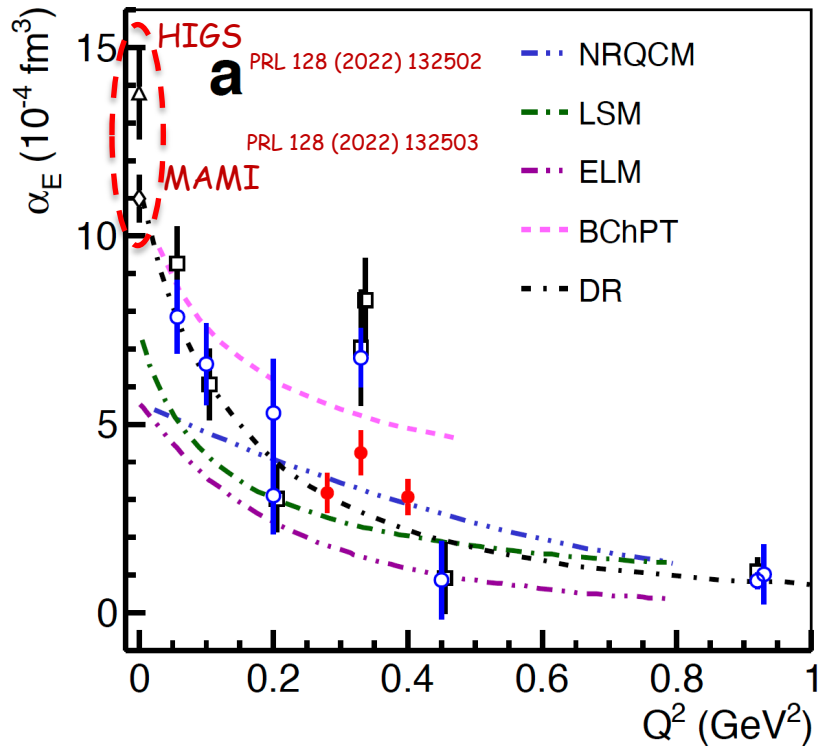


$$\langle r_{\alpha_E}^2 \rangle = 1.36 \pm 0.29 \text{ fm}^2$$

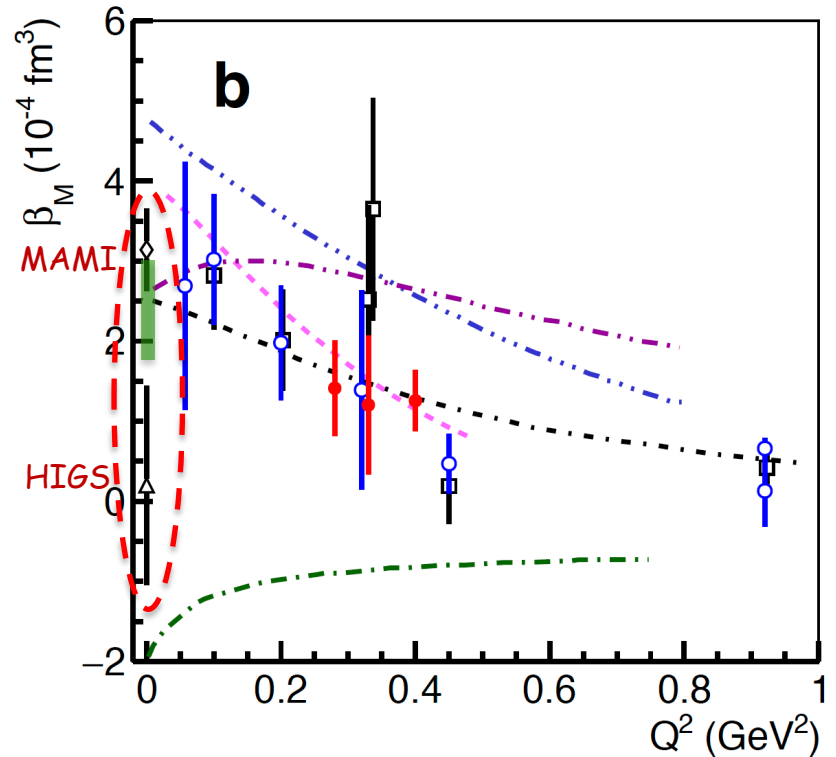
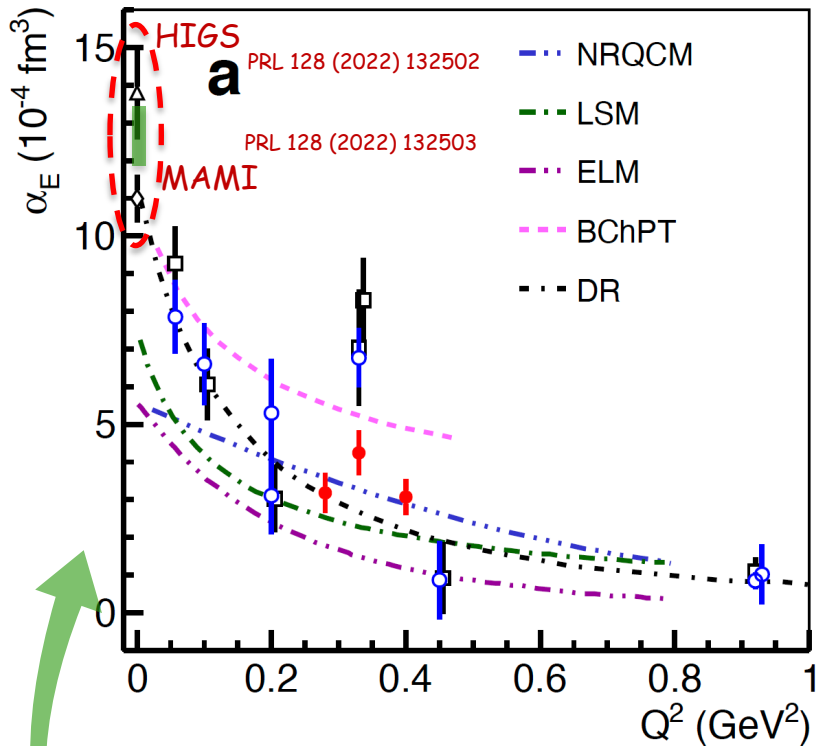


$$\langle r_{\beta_M}^2 \rangle = 0.63 \pm 0.31 \text{ fm}^2$$

Static Polarizabilities



Static Polarizabilities



PHYSICAL REVIEW LETTERS 129, 102501 (2022)

First Concurrent Extraction of the Leading-Order Scalar and Spin Proton Polarizabilities

E. Mornacchi^{1,*}, S. Rodini², B. Pasquini^{3,4} and P. Pedroni⁴

¹Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany

²Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

³Dipartimento di Fisica, Università degli Studi di Pavia, I-27100 Pavia, Italy

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Ⓜ (Received 3 May 2022; revised 11 July 2022; accepted 2 August 2022; published 31 August 2022)

We performed the first simultaneous extraction of the six leading-order proton polarizabilities. We reached this milestone thanks to both new high-quality experimental data and an innovative bootstrap-based fitting method. These new results provide a self-consistent and fundamental benchmark for all future theoretical and experimental polarizability estimates.

$$\alpha_{E1} = [12.7 \pm 0.8(\text{fit}) \pm 0.1(\text{model})] \times 10^{-4} \text{ fm}^3,$$

$$\beta_{M1} = [2.4 \pm 0.6(\text{fit}) \pm 0.1(\text{model})] \times 10^{-4} \text{ fm}^3,$$

$$\gamma_{E1E1} = [-3.0 \pm 0.6(\text{fit}) \pm 0.4(\text{model})] \times 10^{-4} \text{ fm}^4,$$

$$\gamma_{M1M1} = [3.7 \pm 0.5(\text{fit}) \pm 0.1(\text{model})] \times 10^{-4} \text{ fm}^4,$$

$$\gamma_{E1M2} = [-1.2 \pm 1.0(\text{fit}) \pm 0.3(\text{model})] \times 10^{-4} \text{ fm}^4,$$

$$\gamma_{M1E2} = [2.0 \pm 0.7(\text{fit}) \pm 0.4(\text{model})] \times 10^{-4} \text{ fm}^4,$$

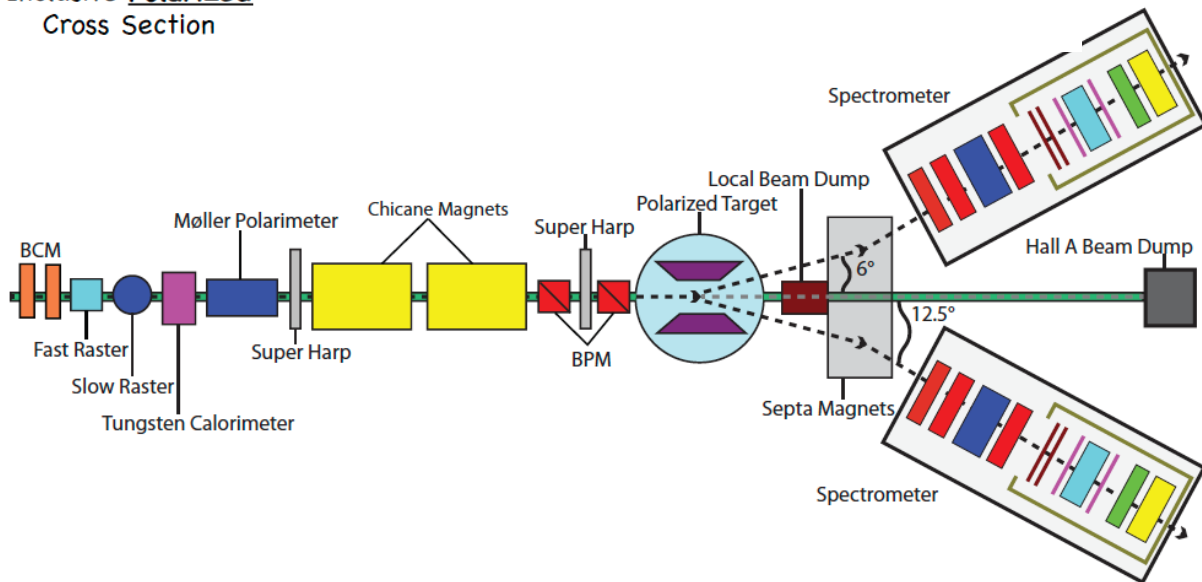
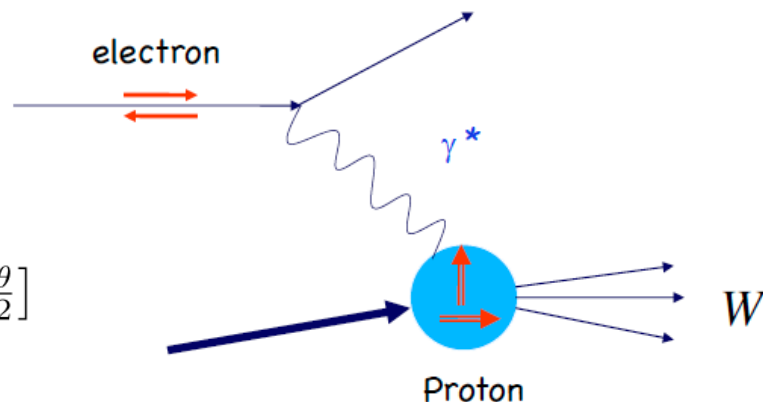
Proton spin structure and generalized polarizabilities

E08-027 (g2p) experiment at JLab

Inclusive measurement at the forward angles of the proton spin-dependent cross sections

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott} \left[\frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right] + \gamma g_1(x, Q^2) + \delta g_2(x, Q^2)$$

Inclusive **Polarized** Cross Section



Hall A

Long. polarized electron beam

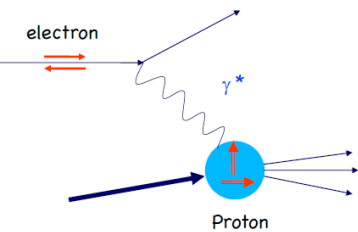
Long. / trans. polarized solid ammonia target

Three beam energies (1.71, 2.25, 3.35 GeV)

Proton spin structure and generalized polarizabilities

g2p results: Structure Functions

SFFs extracted from the polarized cross section differences, i.e. target proton spin parallel and perpendicular to the incoming electron spin



$$\Delta\sigma_{\parallel} = \frac{d^2\sigma}{d\Omega dE'} (\downarrow\uparrow - \uparrow\uparrow)$$

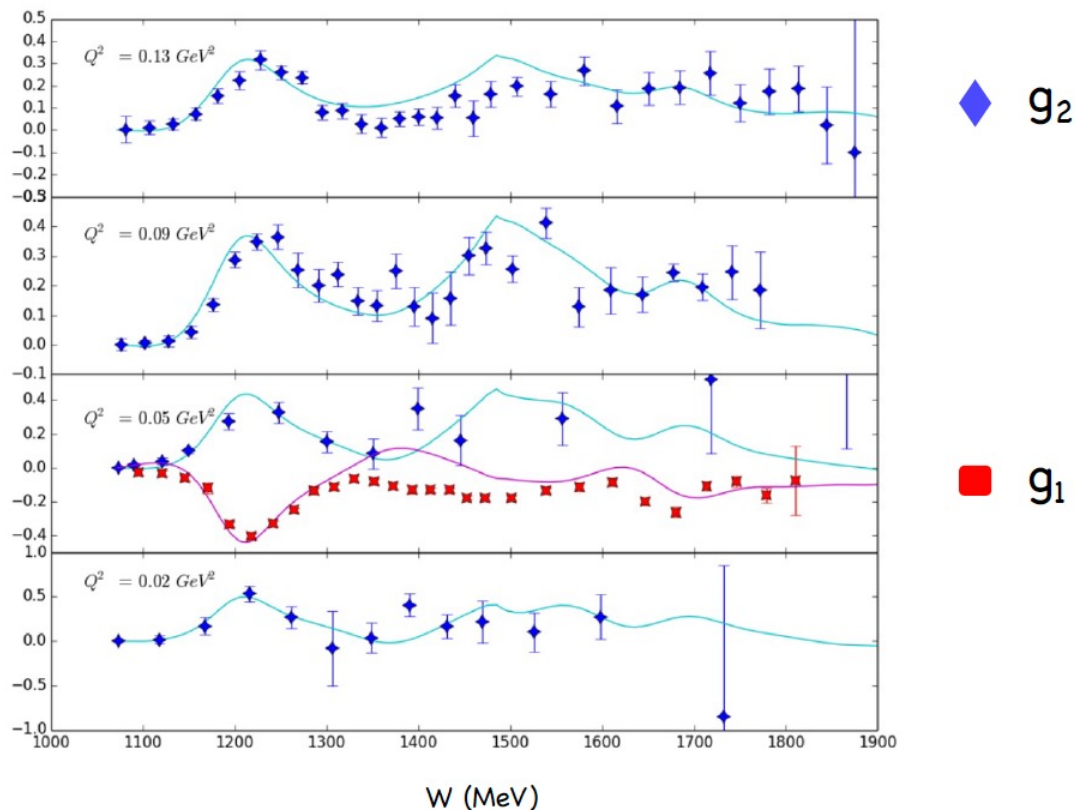
$$\Delta\sigma_{\perp} = \frac{d^2\sigma}{d\Omega dE'} (\downarrow\Rightarrow - \uparrow\Rightarrow)$$



$$g_1(x, Q^2) = K_1 \left[\Delta\sigma_{\parallel} \left(1 + \frac{1}{K_2} \tan \frac{\theta}{2} \right) \right] + \frac{2g_2 \tan \frac{\theta}{2}}{K_2 y}$$

$$g_2(x, Q^2) = \frac{K_1 y}{2} \left[\Delta\sigma_{\perp} \left(K_2 + \tan \frac{\theta}{2} \right) \right] - \frac{g_1 y}{2}$$

Nature Phys. 18, 1441 (2022)



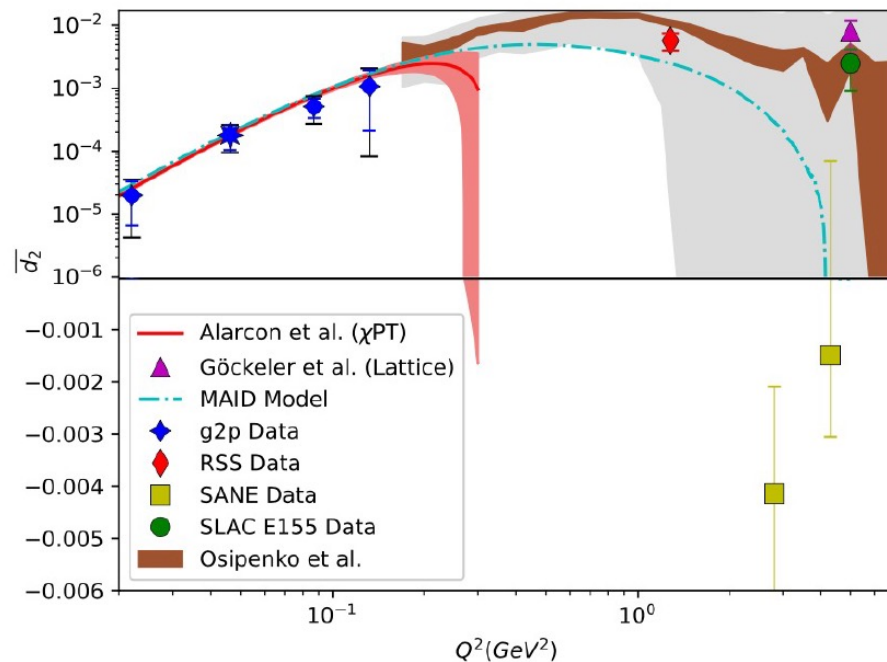
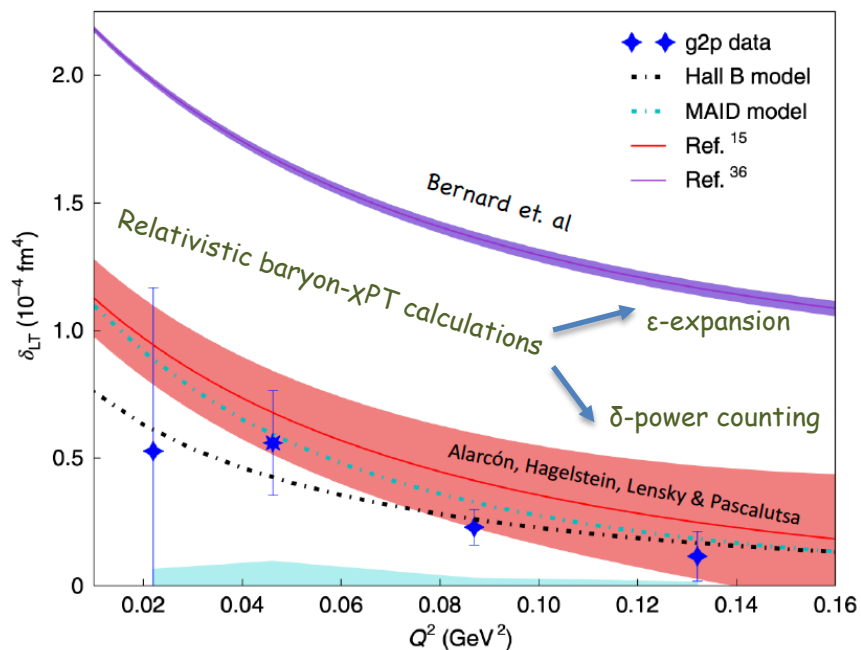
Proton spin structure and generalized polarizabilities

g2p results: spin polarizabilities: higher moments of spin SFs

Nature Phys. 18, 1441 (2022)

$$\delta_{LT}(Q^2) = \frac{4e^2 M^2}{\pi Q^6} \int_0^{x_0} dx x^2 \{g_1(x, Q^2) + g_2(x, Q^2)\}$$

$$\overline{d}_2(Q^2) = \int_0^{x_0} x^2 (2g_1(x, Q^2) + 3g_2(x, Q^2)) dx$$

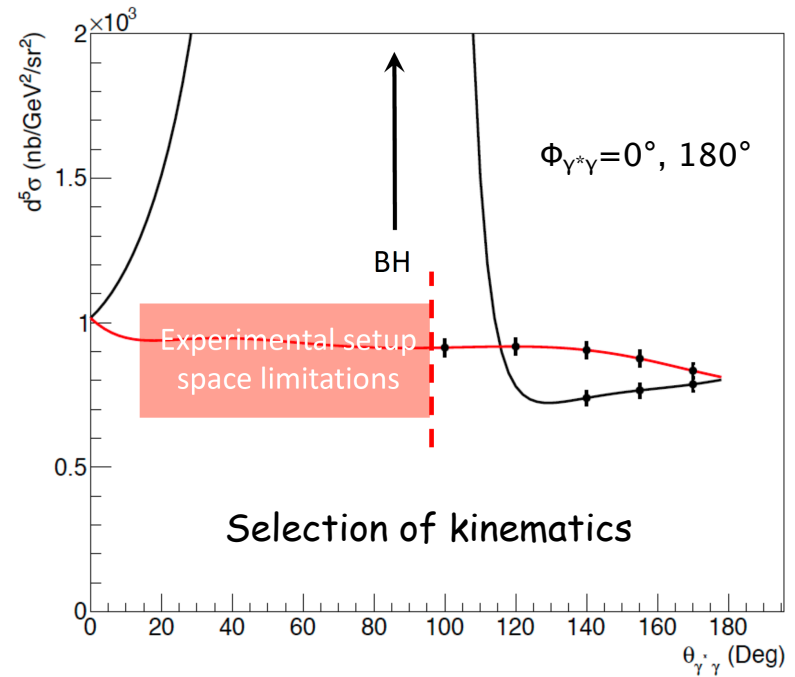
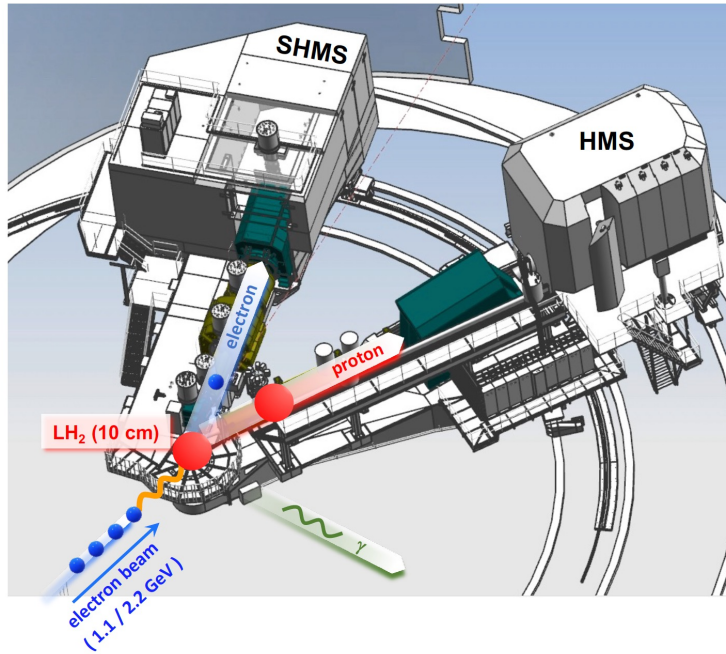


First & high precision measurements

Benchmark for a precise discrimination
of theoretical calculations

Moving Forward

VCS-II (E12-23-001) @ JLab

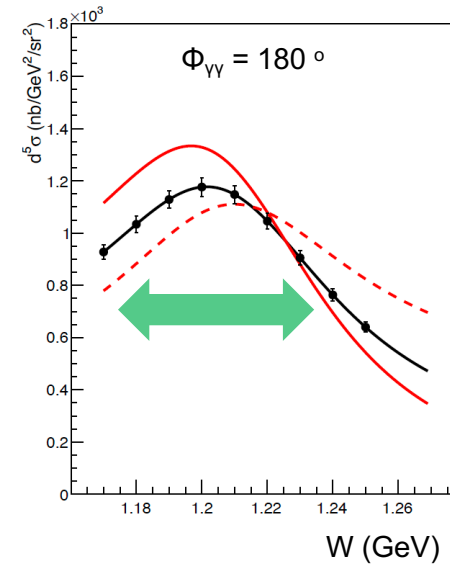
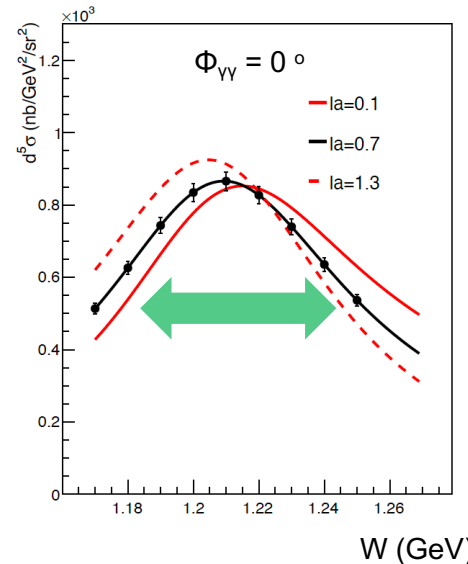


Extend Q^2 range & targeted measurements to fully exploit the sensitivity to the EM GPs

**APPROVED
PAC 51**

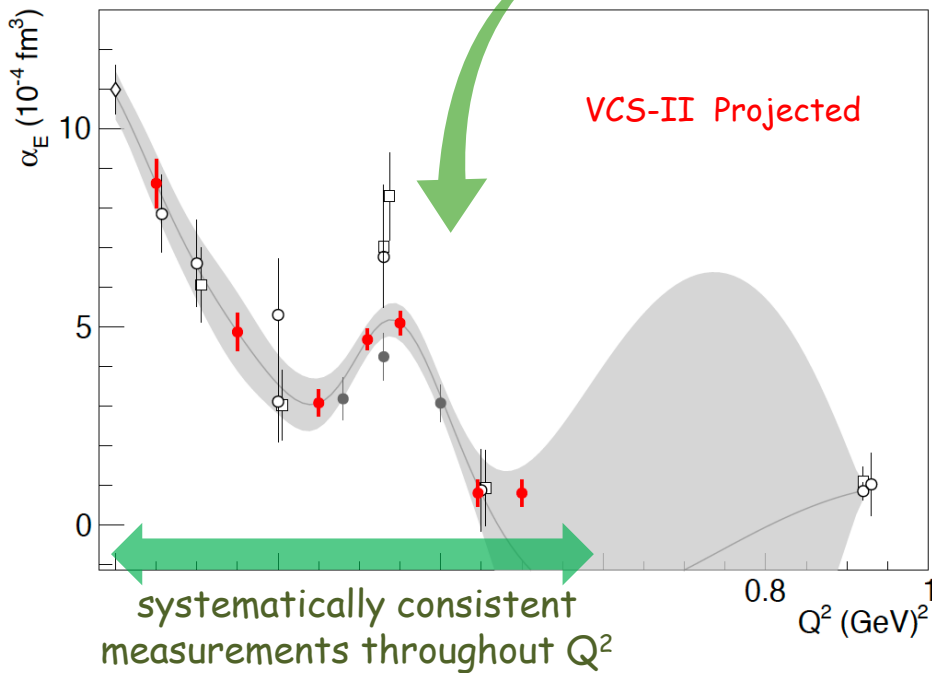
Production ($E_0 = 1.1 \text{ GeV}$): 6 days
 Production ($E_0 = 2.2 \text{ GeV}$): 53 days
 Studies (optics/dummy/calibrations): 3 days

Total: 62 days

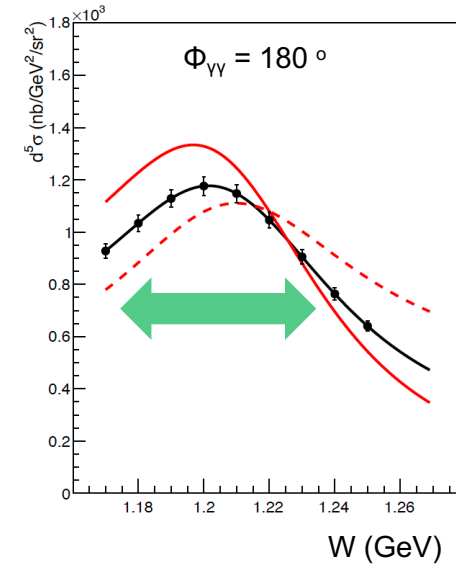
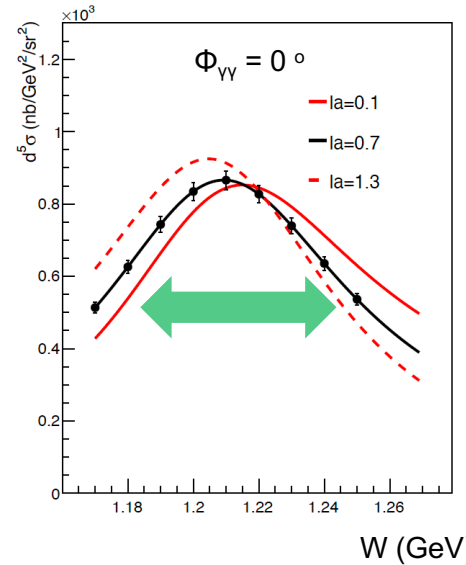


VCS-II Projected Measurements

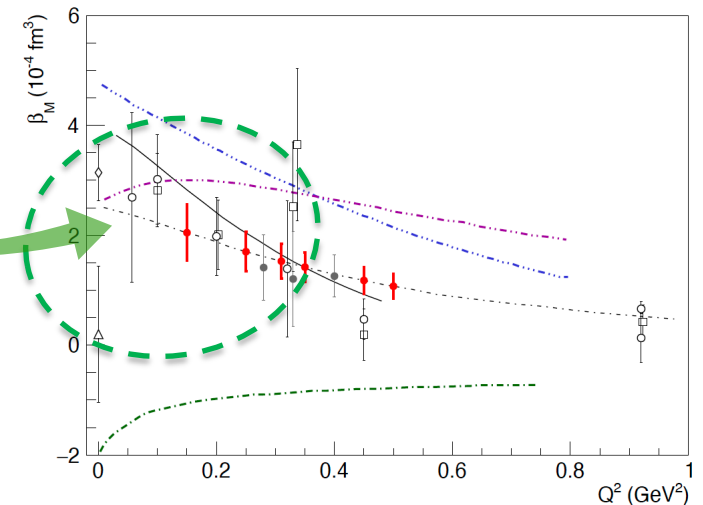
High precision measurements combined with a fine mapping in Q^2



Targeted measurements to fully exploit the sensitivity to the GPs



Improve upon β_M :
Pin down the competing para/dia-magnetic contributions in the nucleon



Can we measure with a different method ?

Yes: positrons and/or beam spin asymmetries

Positrons allow for an independent path to access experimentally the GPs

Eur. Phys. J. A 57 (2021) 11, 316

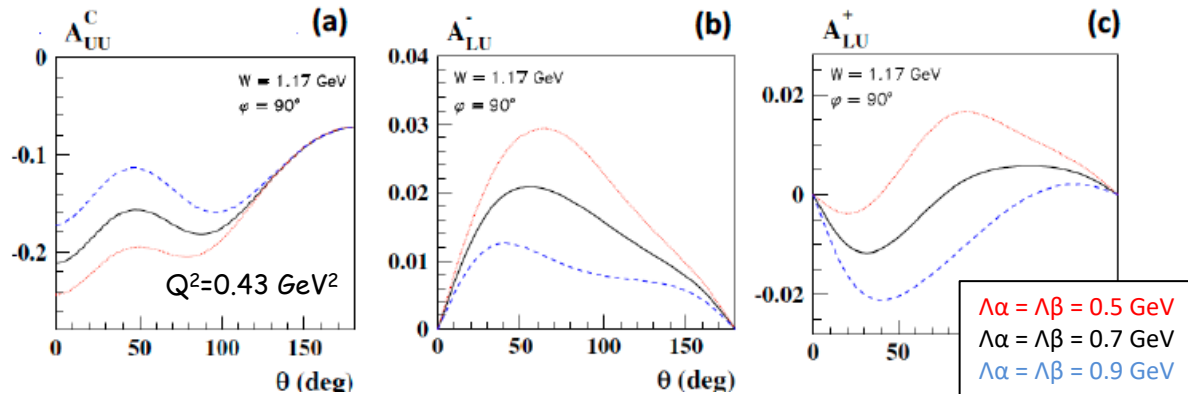
Virtual Compton scattering at low energies with a positron beam

Barbara Pasquini^{a,1,2}, Marc Vanderhaeghen^{b,3}

¹Dipartimento di Fisica, Università degli Studi di Pavia, 27100 Pavia, Italy

²Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, 27100 Pavia, Italy

³Institut für Kernphysik and PRISMA⁺ Cluster of Excellence, Johannes Gutenberg Universität, D-55099 Mainz, Germany



(a): The beam-charge asymmetry as a function of the photon scattering angle at $Q^2 = 0.43 \text{ GeV}^2$.

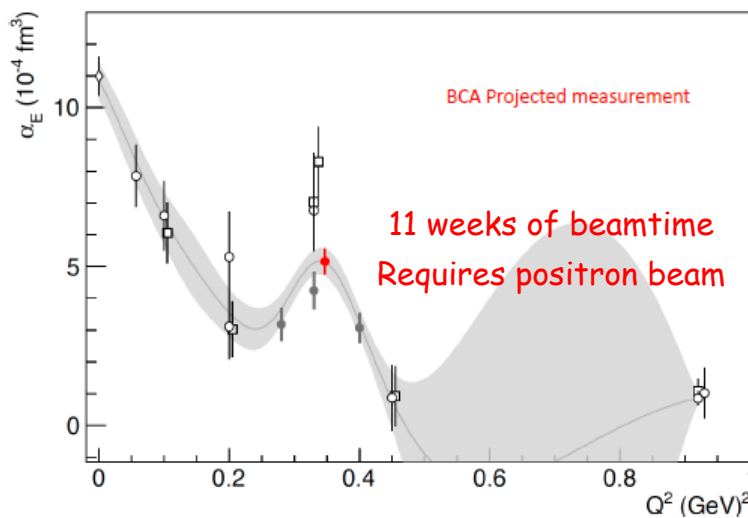
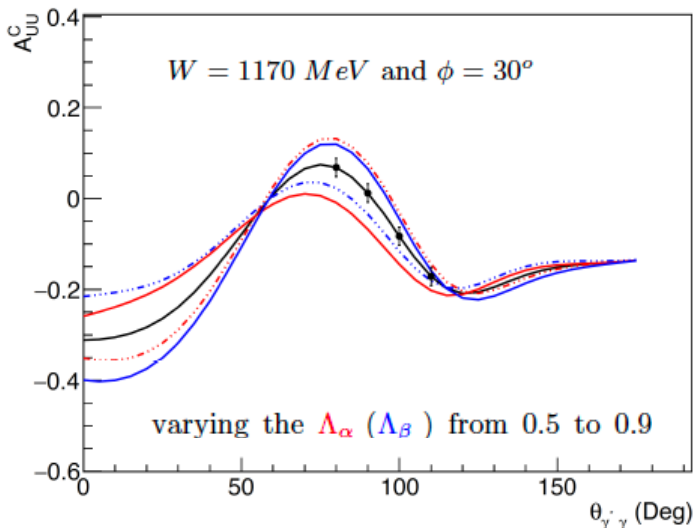
(b) & (c): The electron and positron beam-spin asymmetry as a function of the photon scattering angle for out-of-plane kinematics.

$\Lambda\alpha = \Lambda\beta = 0.5 \text{ GeV}$
 $\Lambda\alpha = \Lambda\beta = 0.7 \text{ GeV}$
 $\Lambda\alpha = \Lambda\beta = 0.9 \text{ GeV}$

Unpolarized beam charge asymmetry (BCA):
$$A_{UU}^C = \frac{(d\sigma_+^+ + d\sigma_-^+) - (d\sigma_+^- + d\sigma_-^-)}{d\sigma_+^+ + d\sigma_-^+ + d\sigma_+^- + d\sigma_-^-}$$

Lepton beam spin asymmetry (BSA):
$$A_{LU}^e = \frac{d\sigma_+^e - d\sigma_-^e}{d\sigma_+^e + d\sigma_-^e}$$

BCA (electrons & positrons)



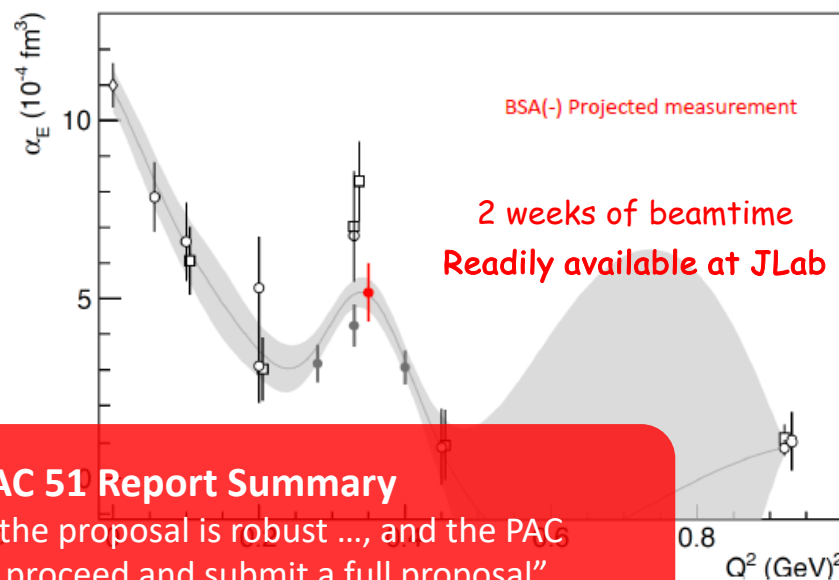
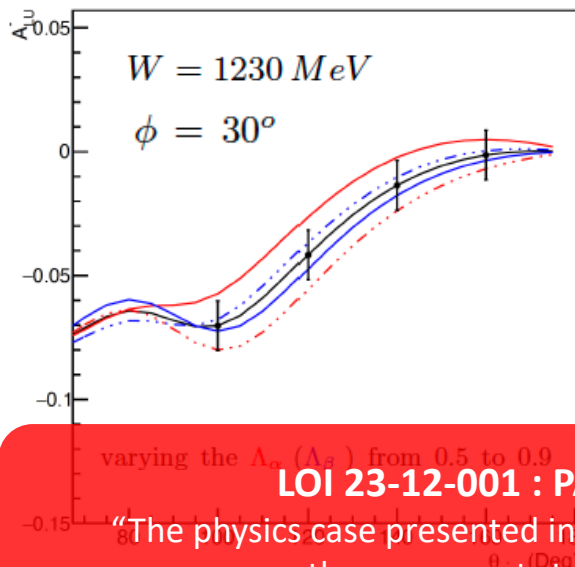
Hall C (SHMS / HMS)

e^- : ~ 1 week @ 50 μA

and

e^+ : ~ 10 weeks @ 5 μA

BSA (electrons or positrons)



e^- (pol. 85% @ 70 μA)

~ 2 weeks of beamtime

or

e^+ (pol. 60% @ 50 nA)

~ 3 orders of magnitude
more beamtime

LOI 23-12-001 : PAC 51 Report Summary

"The physics case presented in the proposal is robust ..., and the PAC encourages the proponents to proceed and submit a full proposal"

Summary

Progress measuring proton's fundamental properties

Insight to spatial deformation of the nucleon densities under an applied EM field, interplay of para/dia-magnetic mechanisms in the proton, spin-dependent response to an EM field, polarizability radii, ...

Electric GP: { possibility for a non-trivial (non-monotonic) behavior in $a_E(Q^2)$
(albeit with a smaller magnitude than originally suggested)
or
at minimum: strong tension between world data

Experiment ahead of theory:

Stringent constraints to theoretical predictions

High precision benchmark data for upcoming LQCD calculations

Future measurements:

Pin down precisely the shape of the a_E structure
(if it exists) - important input for the theory

Independent cross-check

Measure via a different channel (BS asymmetries & positrons)

Thank you!