DVCS at the Precision Frontier

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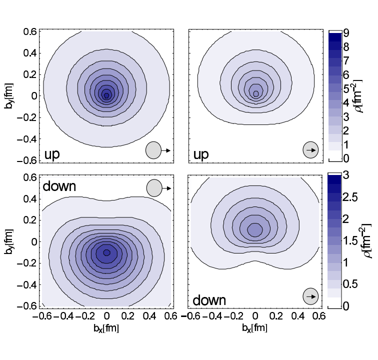
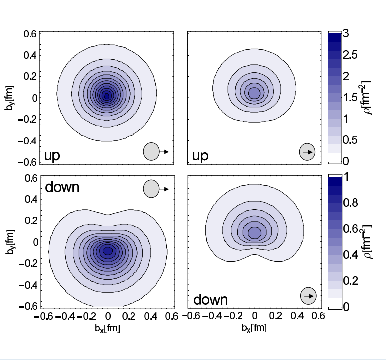
Nucleon Tomography

access to three-dimensional picture of the nucleon (M. Burkardt)



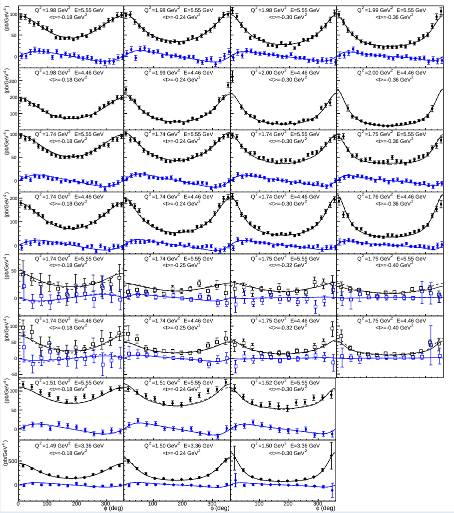


↪ **first two moments of transverse spin parton density**

computer simulations: M. Göckeler *et al.*, Phys.Rev.Lett. 98 (2007) 222001

- **Momentum transfer t defines the resolution of spacial imaging**

Wealth of new data



2010 data of E07-007 and E08-025 [2109.02076]

- **High statistical accuracy**
- **Several beam energies**
- **Neutron/deuteron**
- **Coherent DVCS from ^{4}He**

- **Transverse polarization**

In this talk, a status update:

1. **Towards NNLO accuracy**
   - Two-loop coefficient functions for DVCS ✓
   - Three-loop evolution equations for GPDS (work in progress)

2. **Resummation of threshold logarithms**

3. **Kinematic power corrections** $(\sqrt{-t}/Q)^k$, $(m/Q)^k$
   - Twist-four corrections, $(\sqrt{-t}/Q)^2$, $(m/Q)^2$ ✓
   - Twist-six corrections (work in progress)

NNLO coefficient functions

To the leading-twist accuracy

$$\mathcal{A}_{\mu\nu}^{\rm DVCS} = -g_{\mu\nu}^{\perp} V + \epsilon_{\mu\nu}^{\perp} A + \dots$$
$$V(\xi, Q^2) = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{\xi} C_V(x/\xi, Q^2/\mu^2)\, F_q(x,\xi,t,\mu)\,.$$

$$F_q(x,\xi) = \frac{1}{2P_+}\Big[H_q(x,\xi,t)\bar{u}(p')\gamma_+ u(p) + E_q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2m_N}u(p)\Big]\,.$$

— $C_{V(A)}$ are functions of one variable x/ξ
— real functions for $|x| < \xi$
— can be continued analytically to $|x/\xi| \geq 1$ using $\xi \to \xi - i\epsilon$ prescription
— $C_V(-x/\xi) = -C_V(x/\xi), \quad C_A(-x/\xi) = +C_A(x/\xi)$

In perturbation theory

$$C(x/\xi, Q^2/\mu^2) = C^{(0)}(x/\xi) + a_s C^{(1)}(x/\xi, Q^2/\mu^2) + a_s^2 C^{(2)}(x/\xi, Q^2/\mu^2) + \ldots \qquad a_s = \frac{\alpha_s(\mu)}{4\pi}$$

with, e.g., flavor-nonsinglet — X. D. Ji and J. Osborne, PRD 57, 1337 (1998)

$$C_V^{(0)}(x/\xi) = \frac{\xi}{\xi - x} - \frac{\xi}{\xi + x},$$

$$C_V^{(1)}(x/\xi, 1) = \frac{2C_F\xi}{\xi - x}\left[-\frac{9}{2} - \frac{1}{2}\ln^2 2 + \left[\frac{1}{2}\ln\left(1 - \frac{x}{\xi}\right) - \frac{3}{2}\frac{\xi - x}{\xi + x}\right]\ln\left(1 - \frac{x}{\xi}\right)\right] - (x \leftrightarrow -x).$$

$C_A^{(1)}$ known from — E. Braaten, PRD28, 524 (1983)

New: two-loop CFs

- Flavor-nonsinglet calculated using two different techniques

  $C_V^{(2)}$: V.Braun, A.Manashov, S.Moch, J.Schönleber, JHEP **09**, 117 (2020)
  J. Schönleber, unpublished

  $C_A^{(2)}$: V.Braun, Manashov, Moch, Schönleber, 2106.01437
  J.Gao, T.Huber, Y.Ji and Y.M.Wang, 2106.01390

- Flavor-singlet CFs:

  $C_V^{(2)}$: V.Braun, Y. Ji, J. Schönleber, PRL **129** 172001 (2022)

  $C_A^{(2)}$: Y. Ji, J. Schönleber, e-Print: 2310.05724

- Heavy-quark contrubutions only known to one loop accuracy

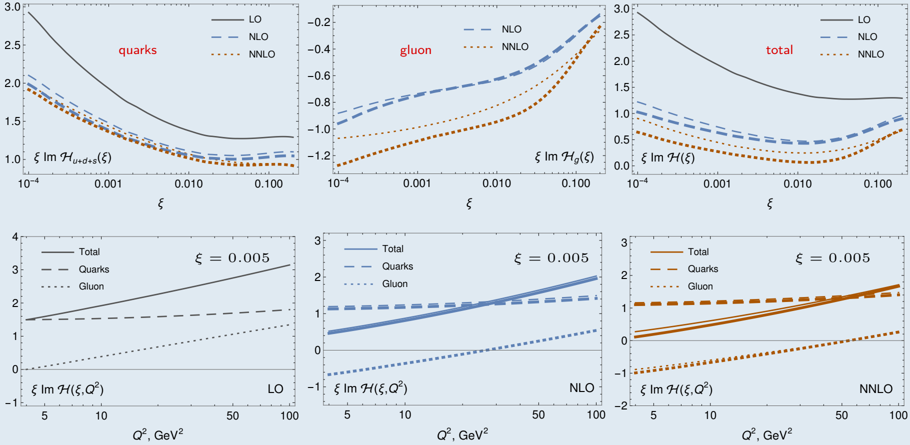
Example (flavor-nonsinglet)

$$C_V^{(2)}(x) = C_F^2 C_P^{(2)}(x) + \frac{C_F}{N_c} C_{NP}^{(2)}(x) + \beta_0 C_F C_\beta^{(2)}(x)$$

$$\begin{aligned}
C_{NP}^{(2)} = {}& 6(1-2\omega)\Big\{ \mathrm{H}_{20} - \mathrm{H}_3 + \mathrm{H}_{110} - \mathrm{H}_{12} + \zeta_2\Big(\mathrm{H}_0 + \mathrm{H}_1\Big) - 3\zeta_3 \Big\} \\
& + 12\Big(\mathrm{H}_{10} - \mathrm{H}_2 - \mathrm{H}_0 - \mathrm{H}_1 + \zeta_2\Big) + \frac{3}{\bar\omega}\mathrm{H}_0 + \frac{3}{\omega}\mathrm{H}_1 \\
& + \Big\{ \frac{1}{\omega}\Big(12\zeta_3 - \frac{3}{2}\zeta_2^2 - \frac{5}{2}\zeta_2 - \frac{73}{24}\Big) - \frac{3}{\omega}\mathrm{H}_{200} - \Big(\frac{2}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_{30} + \Big(\frac{4}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_4 \\
& - \Big(\frac{2}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_{210} + \Big(\frac{3}{\omega} - \frac{2}{\bar\omega}\Big)\mathrm{H}_{22} - \Big(\frac{2}{\omega} - \frac{1}{\bar\omega}\Big)\mathrm{H}_{31} - \frac{5}{\bar\omega}\mathrm{H}_3 + \frac{5}{\bar\omega}\mathrm{H}_{20} \\
& + \Big(\frac{1}{\bar\omega}\Big(\zeta_2 - \frac{9}{2}\Big) + \frac{1}{\omega}\Big(\frac{4}{3} - 2\zeta_2\Big)\Big)\mathrm{H}_{00} - \Big(\frac{2}{\omega}\Big(\zeta_2 - 1\Big) - \frac{1}{\bar\omega}\Big(\zeta_2 + \frac{7}{6}\Big)\Big)\mathrm{H}_2 \\
& + \Big(\frac{1}{\bar\omega}\Big(\frac{19}{6} + 5\zeta_2 - 3\zeta_3\Big) + \frac{1}{\omega}\Big(7\zeta_3 - \frac{16}{9}\Big)\Big)\mathrm{H}_0 - (\omega \leftrightarrow \bar\omega)\Big\}
\end{aligned}$$

where $\omega = (1-x)/2$, $\bar\omega = (1+x)/2$, and $\mathrm{H}_{\vec m} \equiv \mathrm{H}_{\vec m}(\omega)$ are harmonic polylogarithms

Numerical estimates: Imaginary part of the Compton form factor $\mathcal{H}$, $t = -0.1$ GeV^2



GK-model, normalized at input scale $\mu^2 = 4$ GeV^2 to HERAPDF20 (thin lines) and ABMP16 (thick)

— the gluon contribution is large and negative, enhanced at NNLO

Resummation of threshold logarithms

Sudakov-type double logarithms in the CFs:

$$C_V(x/\xi, a_s) \sim \frac{1}{1-x/\xi}\left[1 + a_s C_F \ln^2\left(1-\frac{x}{\xi}\right) + \frac{1}{2}(a_s C_F)^2 \ln^4\left(1-\frac{x}{\xi}\right) + \ldots\right]$$

Resummation to the NNLL accuracy — J. Schoenleber, JHEP **02** (2023), 207

$$C_V(x/\xi, a_s) \sim \frac{1}{1-\frac{x}{\xi}} \exp\left\{\frac{1}{2}\int_{Q^2(1-\frac{x}{\xi})}^{Q^2} \frac{d\mu^2}{\mu^2}\left[-\Gamma_{\rm cusp}(\alpha_s(\mu)) \ln\frac{Q^2(1-\frac{x}{\xi})}{\mu^2} + \gamma_f(\alpha_s(\mu))\right]\right\}$$
$$\times\, H(\alpha_s(Q))\, F(\alpha_s(\sqrt{1-\tfrac{x}{\xi}}\,Q))$$

$\leftarrow \gamma_f$, H and F are known to $\mathcal{O}(\alpha_s^2)$

Evolution equations for GPDs

- Two loops (NLO): singlet + nonsiglet

A. Belitsky, A. Freund, D. Müller, NPB 574, 347 (2000)

— checked by an independent calculation

— evolution code available but can/should be improved

- Three loops much more difficult:
  - Evolution kernels depend on two variables
  - Can be written in terms of HPLs? — likely

Conformal symmetry:

- Make use of the NNLO results for anomalous dimensions
- One loop less compared to direct calculation

Evolution equations for GPDs

Methods:

- Two-loop conformal anomaly — V.B., A.Manashov, S. Moch, M. Strohmaier, JHEP **03** (2016), 142

  $\Rightarrow$ Three-loop evolution equations for flavor-nonsinglet light-ray operators

  V.B., A.Manashov, S. Moch, M. Strohmaier, JHEP **06** (2017), 037
  Y. Ji, A. Manashov, S. Moch, PRD **108** (2023) 054009

- Orthogonality of conformal operators

  $\Rightarrow$ Three-loop mixing matrices for flavor-singlet operators with $N \le 8$

  vector: V.B., K. Chetyrkin, A. Manashov, PLB **834** (2022) 137409

  axial-vector: V.B., K. Chetyrkin, A. Manashov, in preparation

- Numerical impact expected to be moderate because of limited Q^2 range

Kinematic power corrections $(\sqrt{-t}/Q)^k$ and $(m/Q)^k$

- Ambiguity in the choice of collinear directions makes "leading-twist" calculations ambiguous. In addition, electromagnetic Ward identities are violated.
  - Repaired by power-suppressed corrections, $(\sqrt{-t}/Q)^k$ and $(m/Q)^k$
  - "Kinematic" — do not involve new nonperturbative input apart from usual GPDs
  - Factorizable

- Twist-four completed

  V.B., A. Manashov, JHEP **01** (2012), 085 ← method
  V.B., A. Manashov, D. Müller, B. Pirnay, PRD **89** (2014) 074022

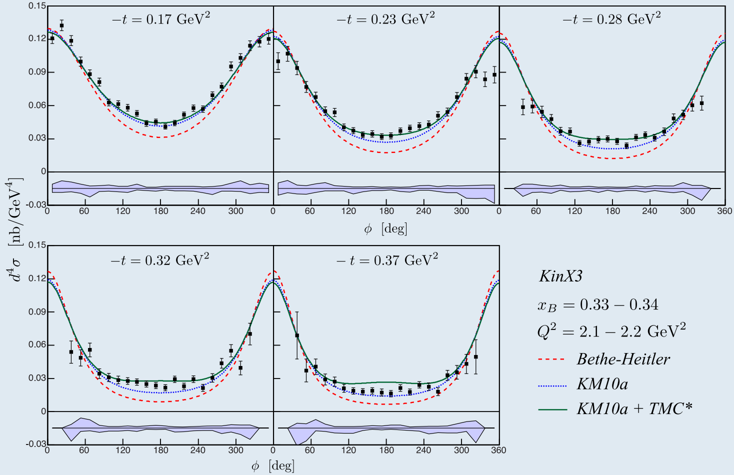
  - Large effects in certain regions of phase space

- Twist-six in progress

  V.B., Y. Ji, A. Manashov, JHEP **03** (2021), 051 ← method
  V.B., Y. Ji, A. Manashov, JHEP **01** (2023), 078 ← scalar target

Large kinematic corrections for the total cross section

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Example: Helicity-flip amplitude to twist-5 accuracy (spin-zero target)

$$\mathcal{A}^{0\pm} = \frac{|P_\perp|}{\sqrt{2}}\frac{4Q}{(qq')}\Big\{D_\xi(T_1\otimes H) \qquad \leftarrow \text{twist 3}$$

$$- \frac{2}{(qq')}\left(\frac{t}{\xi}+|P_\perp|^2 D_\xi\right)D_\xi^2(T_2\otimes H) + \frac{t}{(qq')}D_\xi(T_3\otimes H)\Big\} \qquad \leftarrow \text{twist 5}$$

where

$$D_\xi = \xi^2\partial_\xi$$

$$|\xi P_\perp|^2 = \frac{1-\xi^2}{4}(t_{min}-t) = -\xi^2 m^2 - \frac{1-\xi^2}{4}t\,.$$

and

$$T_1(u) = -\frac{1}{u}\ln(1-u)\,,$$

$$T_2(u) = \frac{\mathrm{Li}_2(u)-\mathrm{Li}_2(1)}{1-u} - \ln(1-u)\,,$$

$$T_3(u) = \frac{\mathrm{Li}_2(u)-\mathrm{Li}_2(1)}{1-u} - \frac{\ln(1-u)}{2u}$$

- Twist-5 corrections are small if expansion parameter changed to $\frac{2}{Q^2}\mapsto\frac{1}{(qq')}=\frac{1}{Q^2+t}$

Summary

1. **Towards NNLO accuracy**
   - Two-loop coefficient functions for DVCS

     — sizeable corrections, completed for light quarks
   - Three-loop evolution equations for GPDS

     — flavor nonsiglet in position space, singlet for the first few moments
     — pressing issue: numerical implementation, also in NLO

2. **Kinematic power corrections** $(\sqrt{-t}/Q)^k$, $(m/Q)^k$
   - Twist-four accuracy, $(\sqrt{-t}/Q)^2$, $(m/Q)^2$

     — complete results available, numerical code (B.Pirnay)
     — large effects for parts of phase space and in collider kinematics
     — Coherent DVCS from nuclei: Target mass corrections do not spoil factorization
   - Higher powers (work in progress)

     — all-order results on OPE level
     — cancellation of IR divergences checked up to twist-6
     — scalar target completed, nucleon in progress

3. **Further issues** — many